SEIR modelling of COVID-19 in the danish population with age mixing

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[1]

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Contents

1 SEIR modelling with mixing matrices

Starting from the 1-community SEIR model, the following diff equations can be set up:

$$\begin{aligned}\frac{d}{dt}S(t) =& -1/N\beta I(t)S(t)\\ \frac{d}{dt}E(t) =& 1/N\beta I(t)S(t) - \eta aE(t)\\ \frac{d}{dt}I(t) =& \eta aE(t) - \gamma I(t)\\ \frac{d}{dt}R(t) =& \gamma I(t)\end{aligned}$$

where a is reciprocal incubation time, β is reciprocal time between contact, γ is reciprocal recovery time, and η is efficiancy of infection (the proportion of exposed population becoming vectors of the diease). Note that we here assume a consant population:

$$S(t) + E(t) + I(t) + R(t) = N \quad , \quad \forall t \in [0, \infty[$$

This model is for one homogenous population, and so to refine it, we now consider a modified model in which wh have several different communities[1], all with their separate evolution system, such that the population in each community, is constant:

$$\begin{aligned}\frac{d}{dt}S_i(t) =& -1/N_i\beta I_i^M(t)S_i(t)\\ \frac{d}{dt}E_i(t) =& 1/N_i\beta I_i^M(t)S_i(t) - \eta aE_i(t)\\ \frac{d}{dt}I_i(t) =& \eta aE_i(t) - \gamma I_i(t)\\ \frac{d}{dt}R_i(t) =& \gamma I_i(t)\end{aligned}$$

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[1] the communities we consider here are different age groups.

where in this model, the rate of exposiure is determined by the new quantity $I_T(t)$ that constitute the wheigted proprtion of the infectious population, given as:

$$I_j^M(t) = \sum_f \sum_i w_j^f (c_{ij}^f)^T I_i(t)$$

where the indices i, j indicates the different age groups, and the index f, indicates the *form of transmition*[2], which can be picked as combinations from the folowing table:

Infection wheights	school (s)	work (w)	home (h)	other (o)
Physical (p) ($\times 1$)	1	0.5	1	0.2
Casual (c) ($\times 0.2$)	0.2	0.1	0.2	0.04

This can now be vectorized as follows:

$$w_i = (w_{ps}, w_{pw}, w_{ph}, w_{po}, w_{cs}, w_{cw}, w_{ch}, w_{co})^T$$

where the subscript indicates the product between the corresponding partial whaights, e.g. $w_{ph} = w_p w_h$. The index indicating the different age groups is necessary if different stategies are emplored by different age groups e.g. the age group 0-9 yrs are allowed in school, while age group 10-19 yrs are not, which corresponds to setting the coorsponding school-wheights to zero for the latter group, but not the first.

With this in place, a script is made and the age-differentiated SEIR-model is is integrated numerically, using scipy-odeint package, with the following results:

[2] or rather the context/sitution/type/incident. pick the one you like best.

The weights described above, used in the scenario is:

Weigts	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-
w_{ps}	1	0	1	1	1	1	1	1
w_{cs}	0.2	0.0	0.2	0.2	0.2	0.2	0.2	0.2
w_{po}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
w_{ph}	1	1	1	1	1	1	1	1
w_{co}	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
w_{ch}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
w_{cw}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_{pw}	0	0	0	0	0	0	0	0

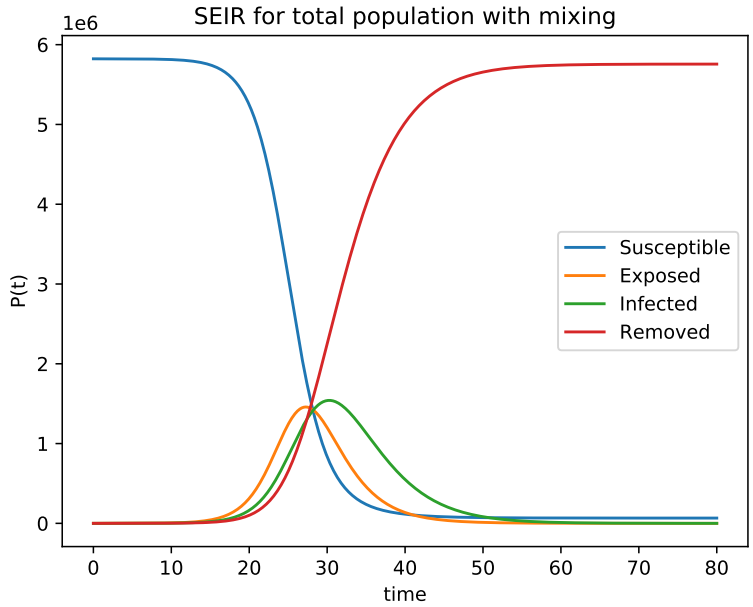
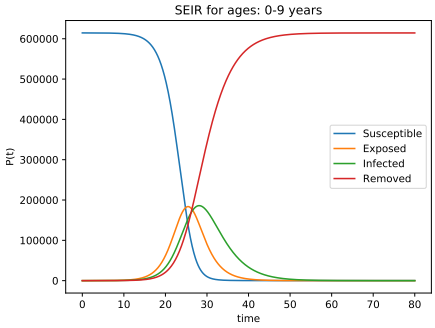
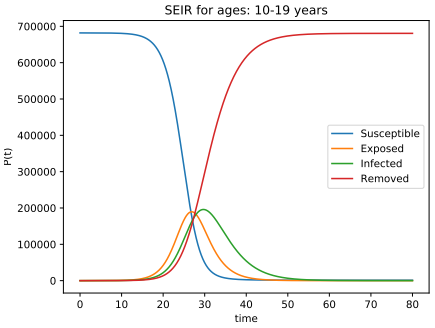


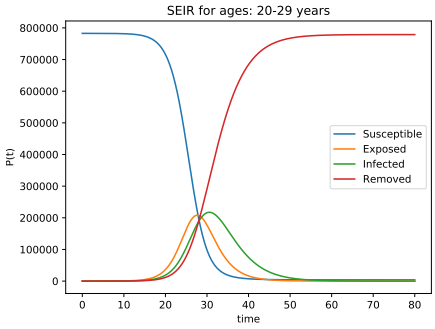
Figure 1: SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 26.0%, at time 30.0 days.



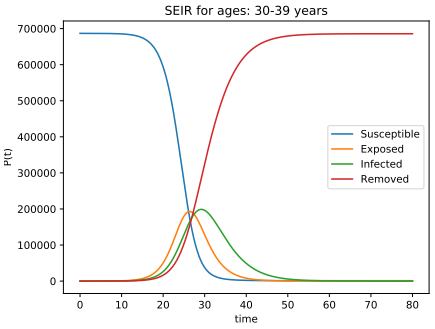
(a) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 30.0%, at time 28.0 days.



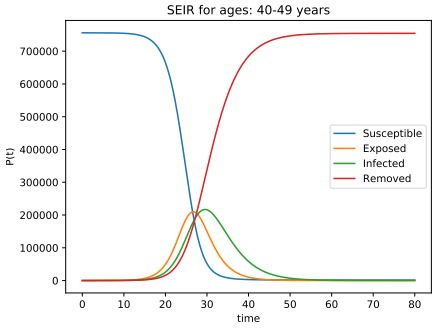
(b) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 29.0%, at time 30.0 days.



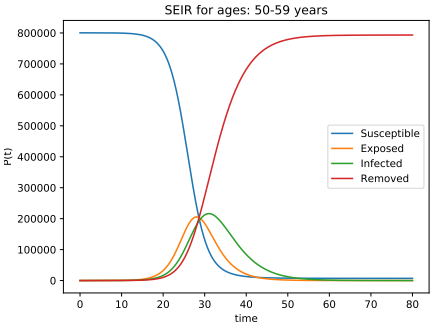
(c) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 28.0%, at time 31.0 days.



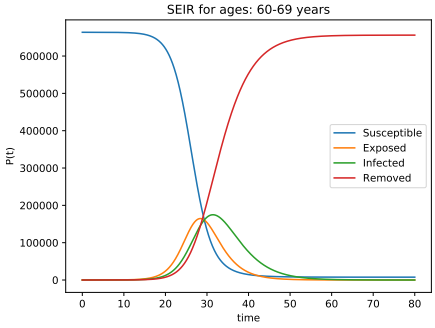
(d) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 29.0%, at time 29.0 days.



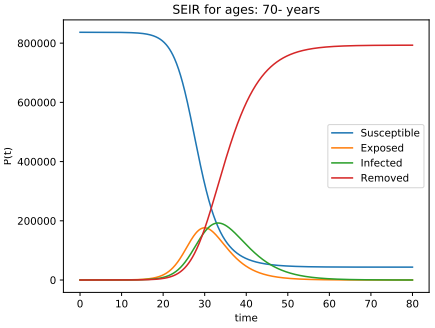
(e) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 29.0%, at time 30.0 days.



(f) SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 27.0%, at time 31.0 days.



(g) SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 26.0%, at time 31.0 days.



(h) SEIR model of COVID-19 with contact mixing. Note that 95.0% becomes infected, with a maximum infectionratio at 23.0%, at time 33.0 days.

The weights described above, used in the scenario is:

Weigts	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-
w_{ps}	0	0	0	0	0	0	0	0
w_{cs}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_{po}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
w_{ph}	1	1	1	1	1	1	1	1
w_{co}	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
w_{ch}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
w_{cw}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_{pw}	0	0	0	0	0	0	0	0

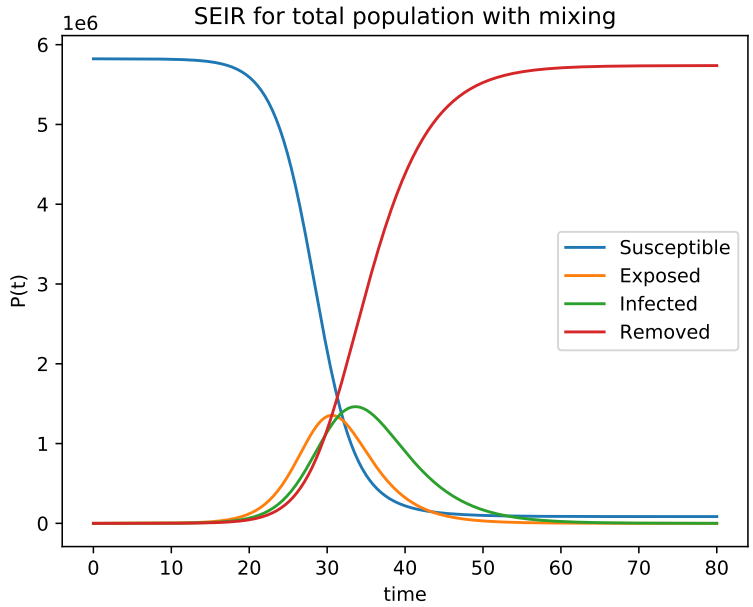
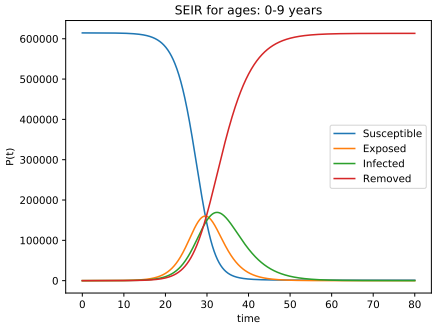
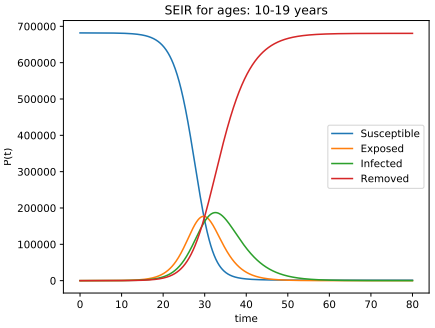


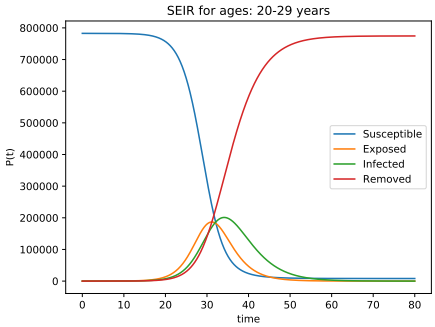
Figure 3: SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 25.0%, at time 34.0 days.



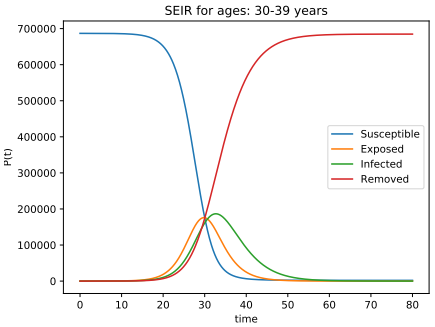
(a) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 28.0%, at time 32.0 days.



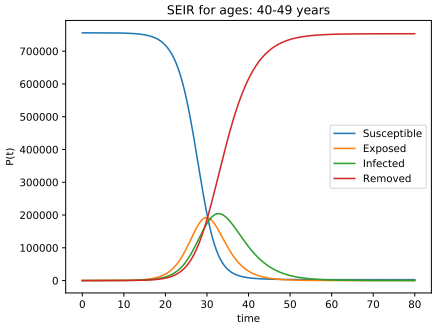
(b) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 27.0%, at time 33.0 days.



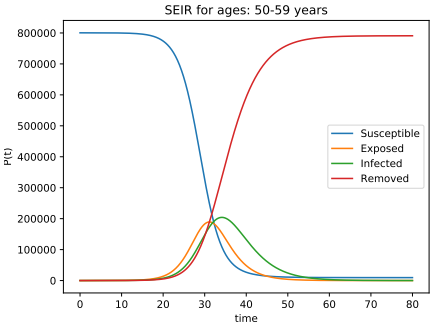
(c) SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 26.0%, at time 34.0 days.



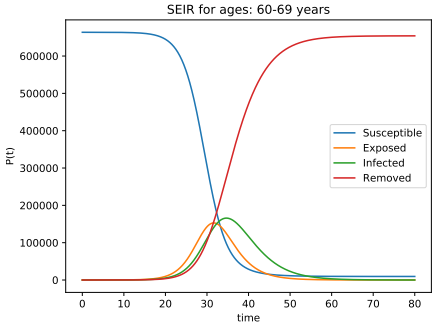
(d) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 27.0%, at time 33.0 days.



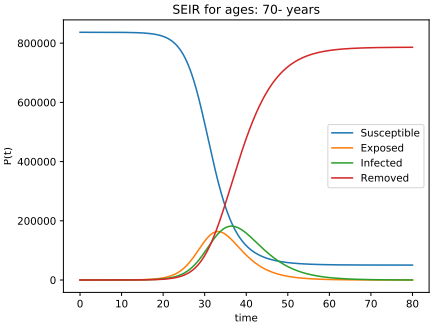
(e) SEIR model of COVID-19 with contact mixing. Note that 100.0% becomes infected, with a maximum infectionratio at 27.0%, at time 33.0 days.



(f) SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 26.0%, at time 34.0 days.



(g) SEIR model of COVID-19 with contact mixing. Note that 99.0% becomes infected, with a maximum infectionratio at 25.0%, at time 35.0 days.



(h) SEIR model of COVID-19 with contact mixing. Note that 94.0% becomes infected, with a maximum infectionratio at 22.0%, at time 36.0 days.

A Contact Matrices