SEIR modelling of COVID-19 in the danish population with age mixing

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[1]

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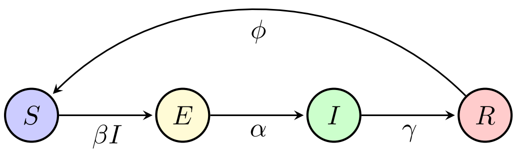
Contents

1 SEIRS modelling with mixing matrices

Starting from the 1-community SEIRS model,



the following diff equations can be set up:

$$
\begin{aligned}
\frac{d}{dt}S(t) =& -\frac{1}{N}\beta I(t)S(t) + \phi R \\
\frac{d}{dt}E(t) =& \frac{1}{N}\beta I(t)S(t) - \alpha E(t) \\
\frac{d}{dt}I(t) =& \alpha E(t) - \gamma I(t) \\
\frac{d}{dt}R(t) =& \gamma I(t) - \phi R
\end{aligned}
$$

where α is reciprocal incubation time, β is reciprocal time between contact, γ is reciprocal recovery time. Note that we here assume a consant population:

$$S(t) + E(t) + I(t) + R(t) = N \quad , \quad \forall t \in [0, \infty[$$

this can now be normalized wrt. the population size N, to get dimentionless populations, and to eliminate the factor N from the couple diff equations. This model is for one homogenous population,

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and so to refine it, we now consider a modified model in which wh have several different communities[1], all with their separate evolution system, such that the population in each community, is constant:

$$
\begin{aligned}
\frac{d}{dt}S_i(t) =& -\frac{1}{N_i}\beta I_i^M(t)S_i(t) + \phi R \\
\frac{d}{dt}E_i(t) =& \frac{1}{N_i}\beta I_i^M(t)S_i(t) - \alpha E_i(t) \\
\frac{d}{dt}I_i(t) =& \alpha E_i(t) - \gamma I_i(t) \\
\frac{d}{dt}R_i(t) =& \gamma I_i(t) - \phi R
\end{aligned}
$$

where in this model, the rate of exposiure is determined by the new quantity $I_T(t)$ that constitute the wheigted proprtion of the infectious population, given as:

$$
I_j^M(t) = \sum_f \sum_i w_j^f (c_{ij}^f)^T I_i(t)
$$

where the indices i, j indicates the different age groups, and the index f, indicates the *form of transmition*[2], which can be picked as combinations from the folowing table:

Infection wheights	school (s)	work (w)	home (h)	other (o)
Physical (p) (×1)	1	0.5	1	0.2
Casual (c) (×0.2)	0.2	0.1	0.2	0.04

This can now be vectorized as follows:

$$
w_i = (w_{ps}, w_{pw}, w_{ph}, w_{po}, w_{cs}, w_{cw}, w_{ch}, w_{co})^T
$$

where the subscript indicates the product between the corresponding partial whaights, e.g. $w_{ph} = w_p w_h$. Again, the populations agegroups copnpartments can be normalized wrt. the size of the agegroup, to eliminate the factor N_i in the couple diff. eqn.s and get dimensionless sub-populations.

$$
S_i(t) + E_i(t) + I_i(t) + R_i(t) = N_i = Np_i \quad , \quad \forall t \in [0, \infty[\qquad \text{and} \qquad \sum_i p_i = 1
$$

The index indicating the different age groups is necessary if different stategies are emplored by different age groups e.g. the age group 0-9 yrs are allowed in school, while age group 10-19 yrs are not, which corresponds to setting the coorsponding school-wheights to zero for the latter group, but not the first.

With this in place, a script is made and the age-differentiated SEIR-model is is integrated numerically, using scipy-odeint package, with the following results:

[1]the communities we consider here are different age groups.

[2]or rather the context/sitution/type/incident. pick the one you like best.

The weights described above, used in the scenario is:

Weigts	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-
w_{ps}	1	0	1	1	1	1	1	1
w_{cs}	0.2	0.0	0.2	0.2	0.2	0.2	0.2	0.2
w_{po}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
w_{ph}	1	1	1	1	1	1	1	1
w_{co}	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
w_{ch}	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
w_{cw}	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
w_{pw}	0	0	0	0	0	0	0	0

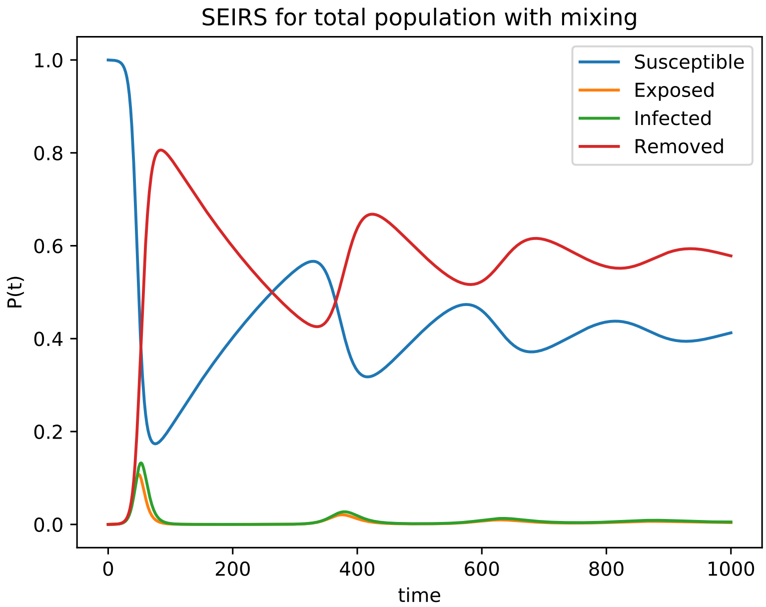
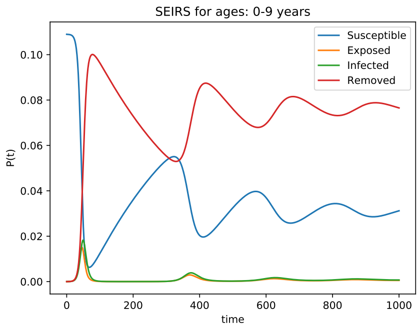
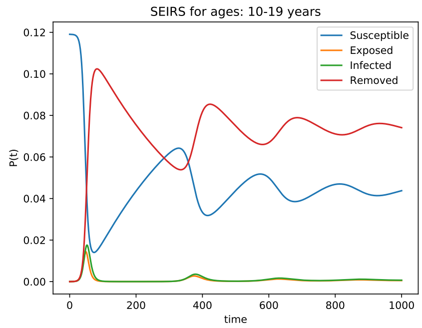


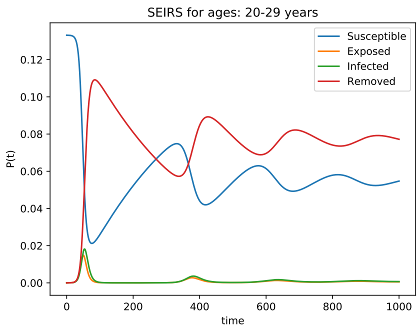
Figure 1: SEIRS model of COVID-19 with contact mixing. Note that 80.61% becomes infected, with a maximum infectionratio at 13.24%, at time 53.0 days.



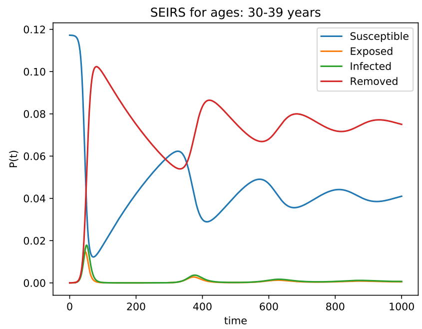
(a) SEIRS model of COVID-19 with contact mixing. Note that 10.01% becomes infected, with a maximum infectionratio at 1.83%, at time 49.0 days.



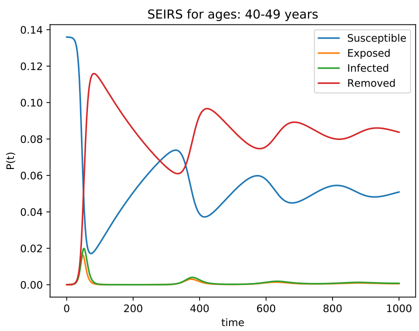
(b) SEIRS model of COVID-19 with contact mixing. Note that 10.24% becomes infected, with a maximum infectionratio at 1.76%, at time 52.0 days.



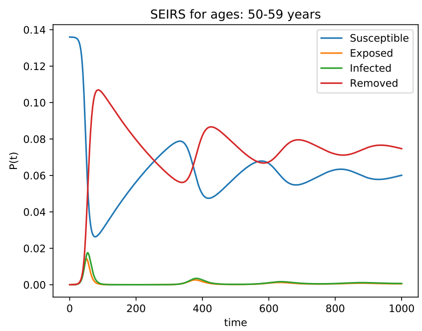
(c) SEIRS model of COVID-19 with contact mixing. Note that 10.92% becomes infected, with a maximum infectionratio at 1.82%, at time 54.0 days.



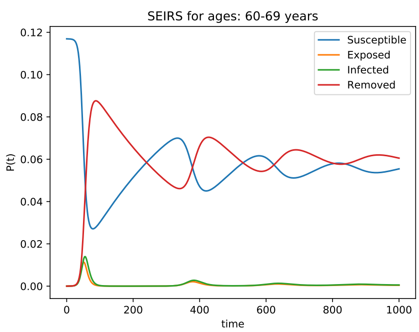
(d) SEIRS model of COVID-19 with contact mixing. Note that 10.23% becomes infected, with a maximum infectionratio at 1.79%, at time 51.0 days.



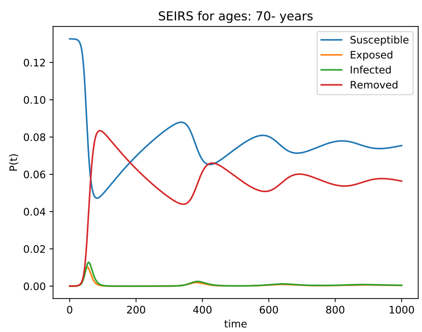
(e) SEIRS model of COVID-19 with contact mixing. Note that 11.59% becomes infected, with a maximum infectionratio at 1.99%, at time 52.0 days.



(f) SEIRS model of COVID-19 with contact mixing. Note that 10.7% becomes infected, with a maximum infectionratio at 1.76%, at time 54.0 days.



(g) SEIRS model of COVID-19 with contact mixing. Note that 8.76% becomes infected, with a maximum infectionratio at 1.4%, at time 55.0 days.



(h) SEIRS model of COVID-19 with contact mixing. Note that 8.34% becomes infected, with a maximum infectionratio at 1.27%, at time 57.0 days.

A Contact Matrices