Queueing

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0 Queueing of exponential distributions

Suppose that two exponentially distributed processes are queued. Then the distribution of the total waiting time distribution can be calculated as:

$$
\begin{aligned}
E_{\lambda_1\lambda_2}(t) = &\int_0^t d\tau \lambda_1 \exp\{-\lambda_1\tau\}\lambda_2 \exp\{-\lambda_2(t-\tau)\} \\
=&\lambda_1\lambda_2 \exp\{-\lambda_2 t\} \int_0^t d\tau \exp\{-(\lambda_1-\lambda_2)\tau\} \\
=&\frac{-\lambda_1\lambda_2}{\lambda_1-\lambda_2} \exp\{-\lambda_2 t\}[\exp\{-(\lambda_1-\lambda_2)t\}-1] \\
=&\frac{-\lambda_1\lambda_2}{\lambda_1-\lambda_2}[\exp\{-\lambda_1 t\}-\exp\{-\lambda_2 t\}]
\end{aligned}
$$

Note that in the special case of $\lambda_1 = \lambda_2$, the appearent singularity is removable. This is actually a generalization of the erlang distribution, with gives the distribution of the waiting times of k (identically) queued exponential distributions.