Queueing

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0 Queueing of exponential distributions

Suppose that two exponentially distributed processes are queued. Then the distribution of the total waiting time distribution can be calculated as:

$$E(t) = \int_0^t d\tau \lambda_1 \exp\{-\lambda_1 \tau\} \lambda_2 \exp\{-\lambda_2 (t - \tau)\}$$

$$= \lambda_1 \lambda_2 \exp\{-\lambda_2 t\} \int_0^t d\tau \exp\{-(\lambda_1 - \lambda_2)\tau\}$$

$$= \frac{-\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \exp\{-\lambda_2 t\}[\exp\{-(\lambda_1 - \lambda_2)t\} - 1]$$

$$= \frac{-\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} [\exp\{-\lambda_1 t\} - \exp\{-\lambda_2 t\}]$$

Note that in the special case of $\lambda_1 = \lambda_2$, the appearent singularity is removable.