in3050_in4050_2023_assignment_2

March 26, 2023

0.1 IN3050/IN4050 Mandatory Assignment 2, 2023: Supervised Learning

USERNAME: chrisrle ————- NAME: Chris René Lerner

0.1.1 Rules

Before you begin the exercise, review the rules at this website:

- https://www.uio.no/english/studies/examinations/compulsory-activities/mn-ifi-mandatory.html

in particular the paragraph on cooperation. This is an individual assignment. You are not allowed to deliver together or copy/share source-code/answers with others. Read also the "Routines for handling suspicion of cheating and attempted cheating at the University of Oslo": - https://www.uio.no/english/studies/examinations/cheating/index.html

By submitting this assignment, you confirm that you are familiar with the rules and the consequences of breaking them.

0.1.2 Delivery

Deadline: Friday, March 24, 2023, 23:59

Your submission should be delivered in Devilry. You may redeliver in Devilry before the deadline, but include all files in the last delivery, as only the last delivery will be read. You are recommended to upload preliminary versions hours (or days) before the final deadline.

0.1.3 What to deliver?

You are recommended to solve the exercise in a Jupyter notebook, but you might solve it in a Python program if you prefer.

Alternative 1 If you choose Jupyter, you should deliver the notebook. You should answer all questions and explain what you are doing in Markdown. Still, the code should be properly commented. The notebook should contain results of your runs. In addition, you should make a pdf of your solution which shows the results of the runs. (If you can't export: notebook -> latex -> pdf on your own machine, you may do this on the IFI linux machines.)

Alternative 2 If you prefer not to use notebooks, you should deliver the code, your run results, and a pdf-report where you answer all the questions and explain your work.

Here is a list of *absolutely necessary* (but not sufficient) conditions to get the assignment marked as passed:

- You must deliver your code (python file or notebook) you used to solve the assignment.
- The code used for making the output and plots must be included in the assignment.
- You must include example runs that clearly shows how to run all implemented functions and methods.
- All the code (in notebook cells or python main-blocks) must run. If you have unfinished code that crashes, please comment it out and document what you think causes it to crash.
- You must also deliver a pdf of the code, outputs, comments and plots as explained above.

Your report/notebook should contain your name and username.

Deliver one single zipped folder (.zip, .tgz or .tar.gz) which contains your complete solution.

Important: if you weren't able to finish the assignment, use the PDF report/Markdown to elaborate on what you've tried and what problems you encountered. Students who have made an effort and attempted all parts of the assignment will get a second chance even if they fail initially. This exercise will be graded PASS/FAIL.

0.1.4 Goals of the assignment

The goal of this assignment is to get a better understanding of supervised learning with gradient descent. It will, in particular, consider the similarities and differences between linear classifiers and multi-layer feed forward networks (multi-layer perceptron, MLP) and the differences and similarities between binary and multi-class classification. A main part will be dedicated to implementing and understanding the backpropagation algorithm.

0.1.5 Tools

The aim of the exercises is to give you a look inside the learning algorithms. You may freely use code from the weekly exercises and the published solutions. You should not use ML libraries like scikit-learn or tensorflow.

You may use tools like NumPy and Pandas, which are not specific ML-tools.

The given precode uses NumPy. You are recommended to use NumPy since it results in more compact code, but feel free to use pure python if you prefer.

0.1.6 Beware

There might occur typos or ambiguities. This is a revised assignment compared to earlier years, and there might be new typos. If anything is unclear, do not hesitate to ask. Also, if you think some assumptions are missing, make your own and explain them!

0.1.7 Initialization

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
import sklearn #for datasets
```

0.2 Datasets

We start by making a synthetic dataset of 2000 datapoints and five classes, with 400 individuals in each class. (See https://scikit-learn.org/stable/modules/generated/sklearn.datasets.make_blobs.html regarding how the data are generated.) We choose to use a synthetic dataset—and not a set of natural occuring data—because we are mostly interested in properties of the various learning algorithms, in particular the differences between linear classifiers and multi-layer neural networks together with the difference between binary and multi-class data.

When we are doing experiments in supervised learning, and the data are not already split into training and test sets, we should start by splitting the data. Sometimes there are natural ways to split the data, say training on data from one year and testing on data from a later year, but if that is not the case, we should shuffle the data randomly before splitting. (OK, that is not necessary with this particular synthetic data set, since it is already shuffled by default by scikit, but that will not be the case with real-world data.) We should split the data so that we keep the alignment between X and t, which may be achieved by shuffling the indices. We split into 50% for training, 25% for validation, and 25% for final testing. The set for final testing *must not be used* till the end of the assignment in part 3.

We fix the seed both for data set generation and for shuffling, so that we work on the same datasets when we rerun the experiments. This is done by the `random_state` argument and the `rng = np.random.RandomState(2022)`.

[2]:
```
from sklearn.datasets import make_blobs
X, t_multi = make_blobs(n_samples=[400,400,400, 400, 400],
                        centers=[[0,1],[4,2],[8,1],[2,0],[6,0]],
                        cluster_std=[1.0, 2.0, 1.0, 0.5, 0.5],
                        n_features=2, random_state=2022)
```

[3]:
```
indices = np.arange(X.shape[0])
rng = np.random.RandomState(2022)
rng.shuffle(indices)
indices[:10]
```

[3]:
```
array([1018, 1295,  643, 1842, 1669,   86,  164, 1653, 1174,  747])
```

[4]:
```
X_train = X[indices[:1000],:]
X_val = X[indices[1000:1500],:]
X_test = X[indices[1500:],:]
t_multi_train = t_multi[indices[:1000]]
t_multi_val = t_multi[indices[1000:1500]]
t_multi_test = t_multi[indices[1500:]]
```

Next, we will make a second dataset by merging classes in (X,t) into two classes and call the new set (X, t2). This will be a binary set. We now have two datasets:

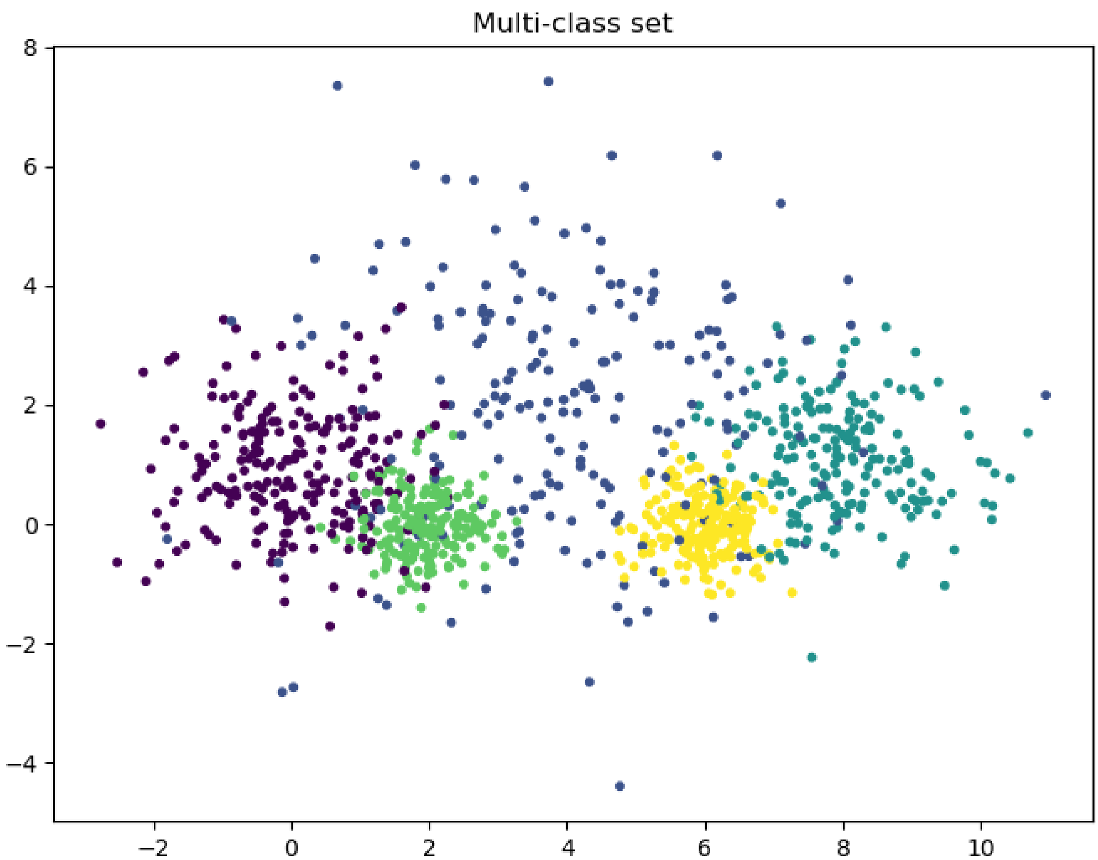
- Binary set: `(X, t2)`
- Multi-class set: `(X, t_multi)`

[5]:
```
t2_train = t_multi_train >= 3
t2_train = t2_train.astype('int')
t2_val = (t_multi_val >= 3).astype('int')
t2_test = (t_multi_test >= 3).astype('int')
```

We can plot the two traning sets.

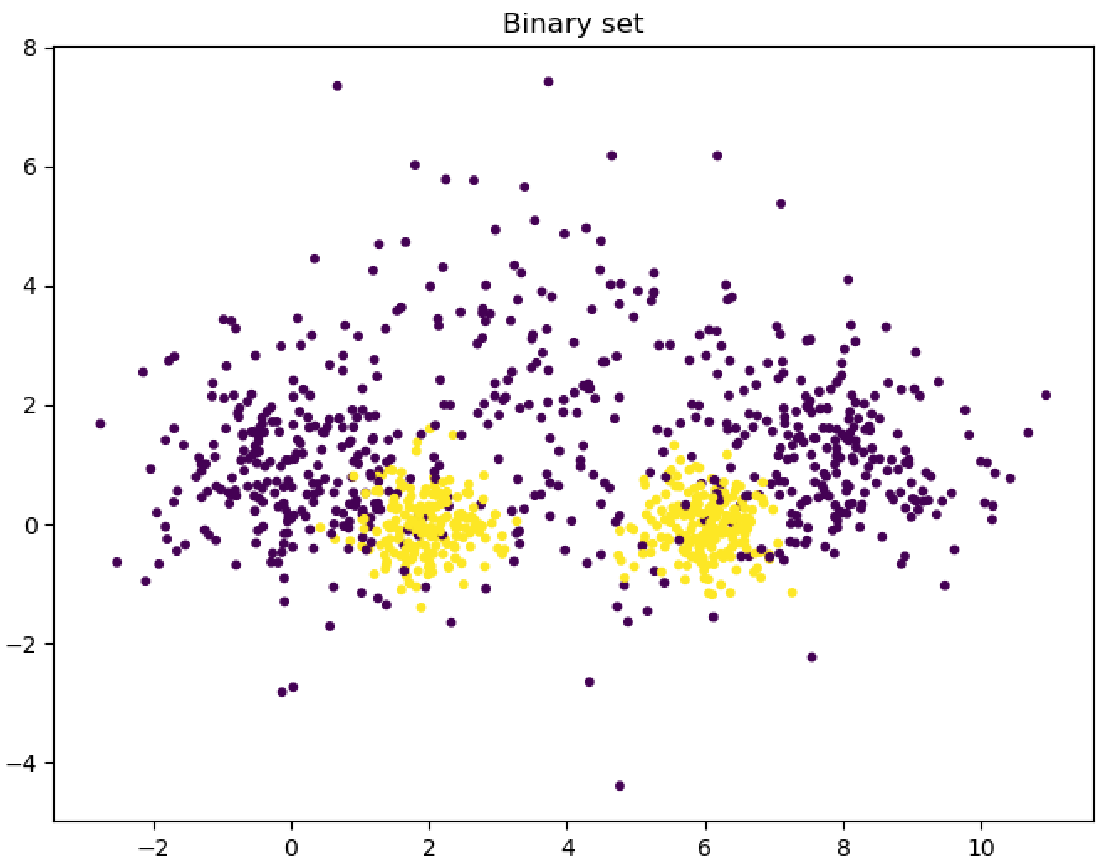
[6]:
```
plt.figure(figsize=(8,6)) # You may adjust the size
plt.scatter(X_train[:, 0], X_train[:, 1], c=t_multi_train, s=10.0)
plt.title('Multi-class set')
```

[6]:
```
Text(0.5, 1.0, 'Multi-class set')
```



[7]:
```
plt.figure(figsize=(8,6))
plt.scatter(X_train[:, 0], X_train[:, 1], c=t2_train, s=10.0)
plt.title('Binary set')
```

[7]:
```
Text(0.5, 1.0, 'Binary set')
```



1 Part I: Linear classifiers

1.1 Linear regression

We see that that set (X, t2) is far from linearly separable, and we will explore how various classifiers are able to handle this. We start with linear regression. You may make your own implementation from scratch or start with the solution to the weekly exercise set 7. We include it here with a little added flexibility.

[8]:
```
def add_bias(X, bias):
    """X is a Nxm matrix: N datapoints, m features
    bias is a bias term, -1 or 1. Use 0 for no bias
    Return a Nx(m+1) matrix with added bias in position zero
    """
    N = X.shape[0]
    biases = np.ones((N, 1))*bias # Make a N*1 matrix of bias-s
    # Concatenate the column of biases in front of the columns of X.
    return np.concatenate((biases, X), axis  = 1)
```

[9]:
```python
class NumpyClassifier():
    """Common methods to all numpy classifiers --- if any"""

    def MSE(self, X, t, weights):
        N = X.shape[0]
        error = X @weights - t
        MSE = (1/N) * error.T @ error
        return MSE
```

[10]:
```python
class NumpyLinRegClass(NumpyClassifier):

    def __init__(self, bias=-1):
        self.bias=bias

    def fit(self, X_train, t_train, eta = 0.1, epochs=10):
        """X_train is a Nxm matrix, N data points, m features
        t_train is avector of length N,
        the targets values for the training data"""
        X_train_old = X_train

        if self.bias:
            X_train = add_bias(X_train, self.bias)

        (N, m) = X_train.shape

        self.weights = weights = np.zeros(m)

        # Used for Task: loss
        self.loss = []
        self.accuracies = []

        for e in range(epochs):
            weights -= eta / N *  X_train.T @ (X_train @ weights - t_train)

            # Calculatin loss with MSE
            error =  t_train - X_train @ weights
            MSE = (1 / N) * (error.T @ error)
            self.loss.append(MSE)

            # Calculating accuracy
            accuracy = np.mean(self.predict(X_train_old) == t_train)
            self.accuracies.append(accuracy)

    def predict(self, X, threshold=0.5):
        """X is a Kxm matrix for some K>=1
        predict the value for each point in X"""
```

```
        if self.bias:
            X = add_bias(X, self.bias)
        ys = X @ self.weights
        return ys > threshold
```

We can train and test a first classifier.

[11]:
```
def accuracy(predicted, gold):
    return np.mean(predicted == gold)
```

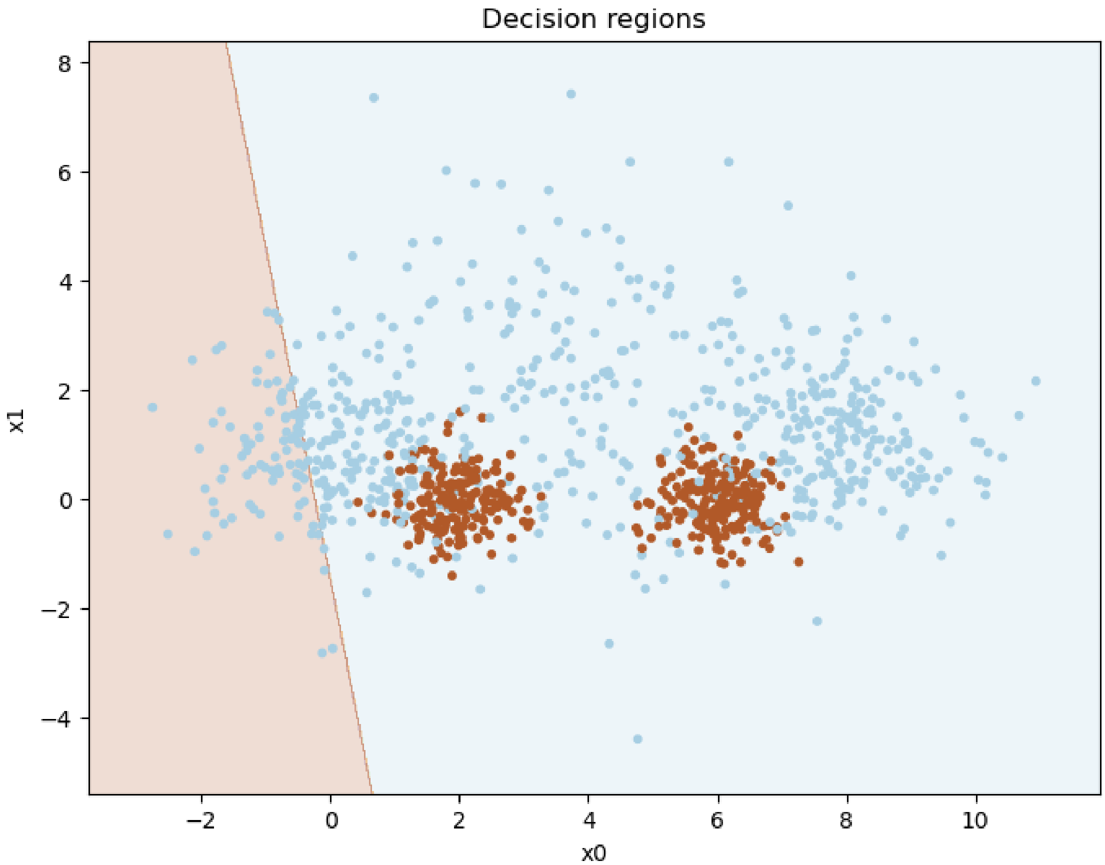
[12]:
```
cl = NumpyLinRegClass()
cl.fit(X_train, t2_train, eta=0.1, epochs=10)
accuracy(cl.predict(X_val), t2_val)
```

[12]:
```
0.522
```

The following is a small procedure which plots the data set together with the decision boundaries. You may modify the colors and the rest of the graphics as you like. The procedure will also work for multi-class classifiers

[13]:
```
def plot_decision_regions(X, t, clf=[], size=(8,6)):
    """Plot the data set (X,t) together with the decision boundary of the␣
 ↪classifier clf"""
    # The region of the plane to consider determined by X
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1

    # Make a make of the whole region
    h = 0.02  # step size in the mesh
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    # Classify each meshpoint.
    Z = Z.reshape(xx.shape)

    plt.figure(figsize=size) # You may adjust this

    # Put the result into a color plot
    plt.contourf(xx, yy, Z, alpha=0.2, cmap = 'Paired')

    plt.scatter(X[:,0], X[:,1], c=t, s=10.0, cmap='Paired')

    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.title("Decision regions")
    plt.xlabel("x0")
    plt.ylabel("x1")

#    plt.show()
```

[14]:
```
plot_decision_regions(X_train, t2_train, cl)
```



1.1.1 Task: Tuning

The result is far from impressive. Remember that a classifier which always chooses the majority class will have an accuracy of 0.6 on this data set.

Your task is to try various settings for the two training hyper-parameters, *eta* and *epochs*, to get the best accuracy on the validation set.

Report how the accuracy vary with the hyper-parameter settings. It it not sufficient to give the final hyperparemters. You must also show how you found them and results for alternative values you tried aout.

When you are satisfied with the result, you may plot the decision boundaries, as above.

1.1.2 Answer: Tuning manually

Initial hyper-parameter values: eta = 0.1, epochs = 10, with an accuracy of 0.522.

With keeping epochs = 10, but changing eta: The accuracy falls immediately upon increasing eta, but never goes below 0.516. Putting eta = 0.01 yields an accuracy of 0.502, which is the worst so far. Putting eta = 0.09 yields an accuracy of 0.55. Putting eta = 0.08 yields an accuracy of 0.576. Putting eta = 0.07 yields an accuracy of 0.532. Every eta below 0.07 down to 0.01 is worste than 0.532.

With keeping eta = 0.08, but changing epochs: It seems that the accuracy stays the same if the number of epochs is an even number, with an accuracy for 0.576. The accuracy of epochs = 1 is 0.496. The accuracy of any odd number under 15 is worse than for the even numbers. Epochs = 15 yields an accuracy of 0.574. Epochs = 17 yields an accuracy of 0.596. It is now better than the initial value. Epochs = 19 yields an accuracy of 0.606. Epochs = 21 and epochs = 23 yields an accuracy of 0.624, after this it only gets worse.

With epochs = 21, try changing eta again: Any other value than eta = 0.08 goes below 0.6.

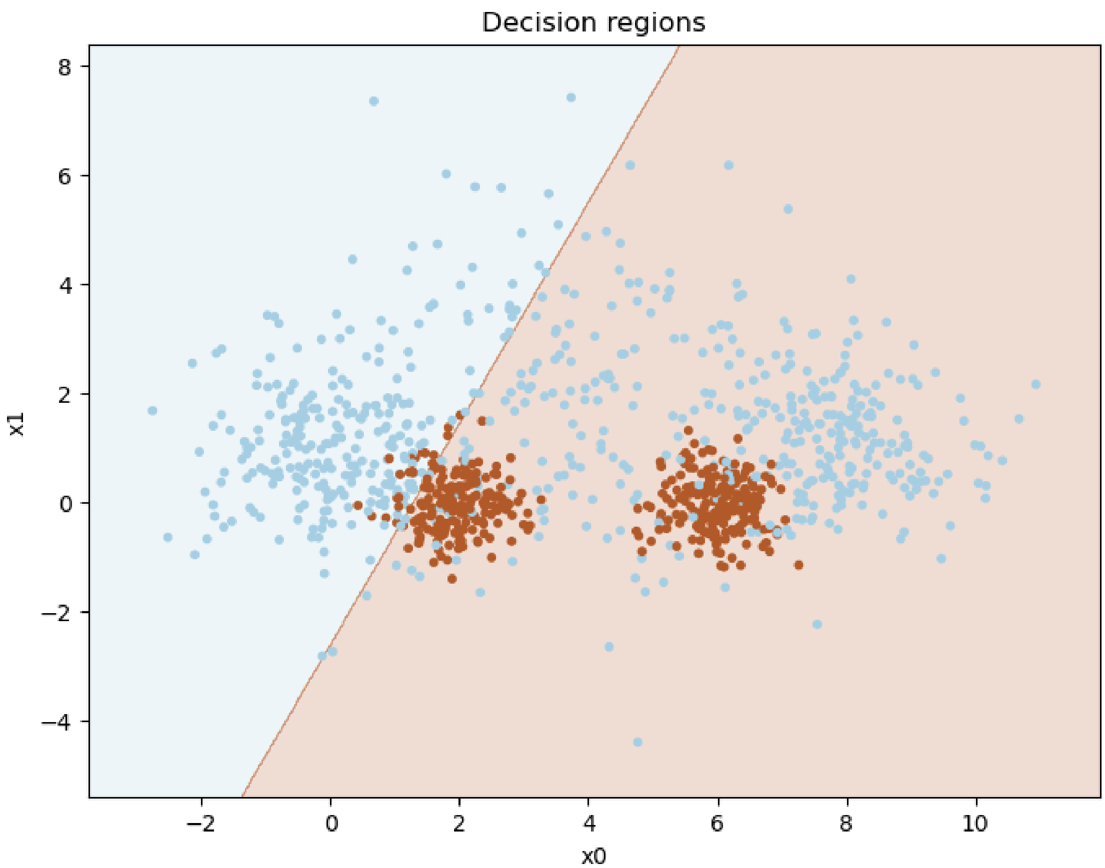
Final values: eta = 0.08, epochs = 21, with an accuracy of 0.624

1.1.3 The decision boundaries for the manually tuned classifier

[15]:
```
cl = NumpyLinRegClass()
cl.fit(X_train, t2_train, eta=0.08, epochs=21)
accuracy(cl.predict(X_val), t2_val)
```

[15]: 0.624

[16]:
```
plot_decision_regions(X_train, t2_train, cl)
```



These decision regions are a lot better than the original one, but it is still higly incorrect. It would be best to tune the hyperparameters automatically.

1.1.4 Answer: Tuning automatically with code inspired from solution to weekly exercises

[17]:
```python
## Tuning automatically
eta_start = 0.01
for epoch in [1, 2, 5, 10, 50, 100, 1000, 10000, 100000]:
    cl_tune = NumpyLinRegClass()
    cl_tune.fit(X_train, t2_train, eta=eta_start, epochs=epoch)
    print("Learning rate: {}  Epochs: {:7}  Accuracy: {}".format(
        eta_start, epoch, accuracy(cl_tune.predict(X_val), t2_val)))
```

```
Learning rate: 0.01  Epochs:       1  Accuracy: 0.576
Learning rate: 0.01  Epochs:       2  Accuracy: 0.576
Learning rate: 0.01  Epochs:       5  Accuracy: 0.574
Learning rate: 0.01  Epochs:      10  Accuracy: 0.502
Learning rate: 0.01  Epochs:      50  Accuracy: 0.504
Learning rate: 0.01  Epochs:     100  Accuracy: 0.56
```

```
Learning rate: 0.01  Epochs:    1000  Accuracy: 0.686
Learning rate: 0.01  Epochs:   10000  Accuracy: 0.704
Learning rate: 0.01  Epochs:  100000  Accuracy: 0.704
```

[18]:
```
best_epochs = 10000
for eta in [0.01, 0.001, 0.0001, 0.00001]:
    cl_tune = NumpyLinRegClass()
    cl_tune.fit(X_train, t2_train, eta=eta, epochs=best_epochs)
    print("Learning rate: {:7}  Epochs: {:7}  Accuracy: {}".format(
        eta, best_epochs, accuracy(cl_tune.predict(X_val), t2_val)))
```

```
Learning rate:    0.01  Epochs:   10000  Accuracy: 0.704
Learning rate:   0.001  Epochs:   10000  Accuracy: 0.686
Learning rate:  0.0001  Epochs:   10000  Accuracy: 0.56
Learning rate:   1e-05  Epochs:   10000  Accuracy: 0.524
```

1.1.5 Answer: tuning automatically

I first tuned with regards to the number of epochs, having the learning rate set as 0.01 and not 0.1 since it would cause overflows. The best accuracy of 0.704 came from a number of 10 000 epochs

Then after finding a good number of epochs, I tuned the learning rate with regards to this number of 10 000 epochs. The best accuracy of 0.704 came from a learning rate of 0.01

The decision regions from these parameters also makes the most sense when you look at the plot.

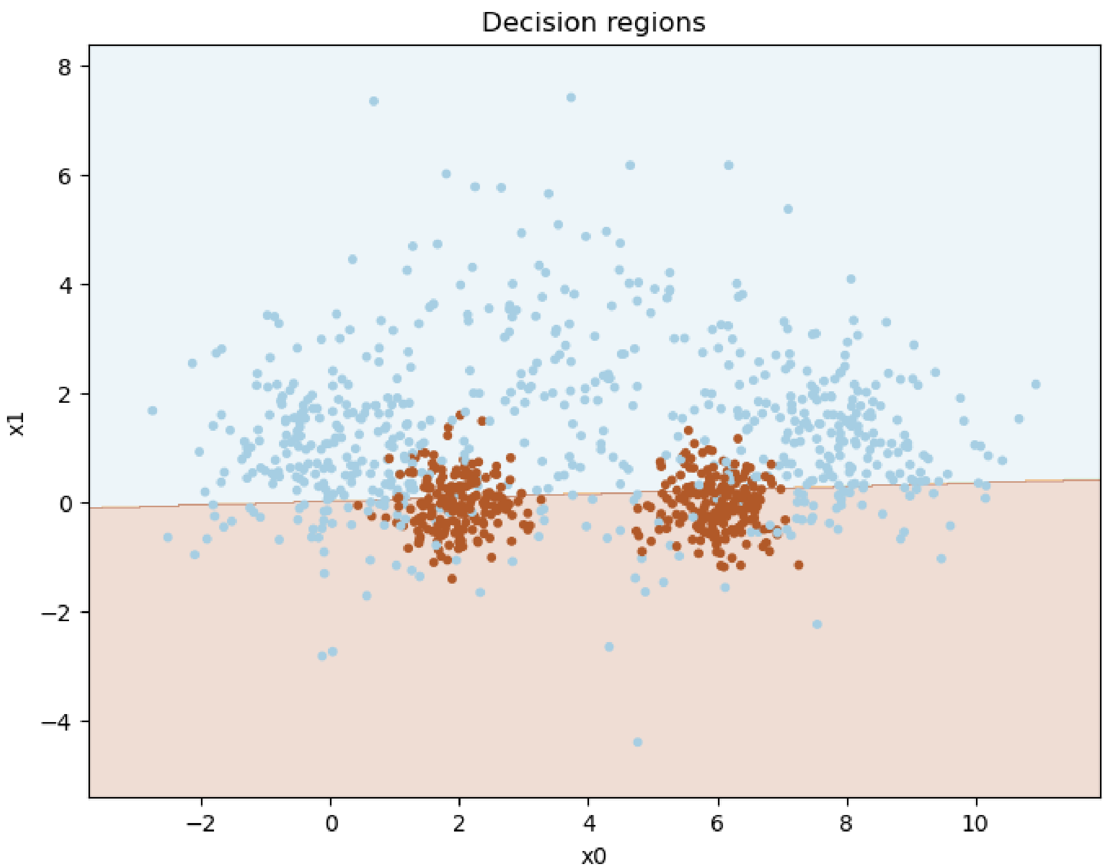
Final values: eta = 0.01, epochs = 10 000, with an accuracy of 0.704.

1.1.6 The decision boundaries for the tuned classifier

[19]:
```
cl_linreg = NumpyLinRegClass()
cl_linreg.fit(X_train, t2_train, eta=0.01, epochs=10000)
accuracy(cl_linreg.predict(X_val), t2_val)
```

[19]: 0.704

[20]:
```
plot_decision_regions(X_train, t2_train, cl_linreg)
```



1.1.7 Task: Loss

The linear regression classifier is trained with mean squared error loss. So far, we have not calculated the loss explicitly in the code. Extend the code to calculate the loss on the training set for each epoch and to store the losses such that the losses can be inspected after training.

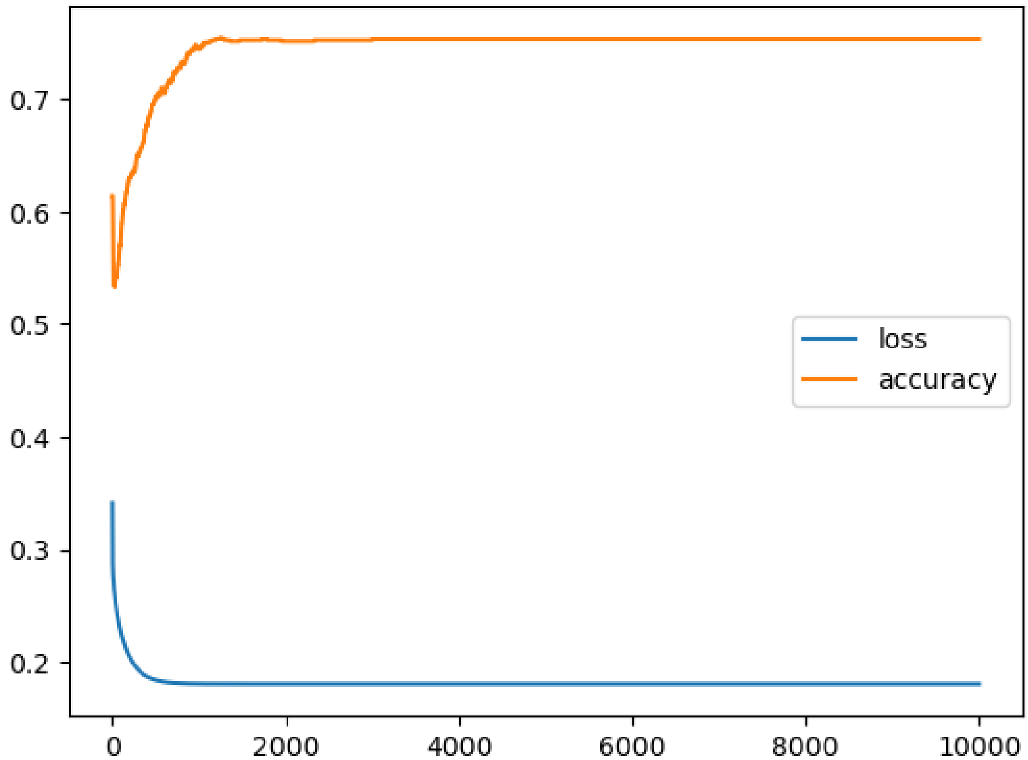
Also extend the classifier to calculate the accuracy on the training data after each epoch.

Train a classifier with your best settings from last point. After training, plot the loss as a function of the number of epochs. Then plot the accuracy as a function of the number of epochs.

Comment on what you see: Are the function monotone? Is this as expected?

[21]:
```
## Trains a classifier wit the best settings from last point.
cl_linreg2 = NumpyLinRegClass()
eta = 0.01
epochs = 10000
cl_linreg2.fit(X_train, t2_train, eta=eta, epochs=epochs)

## PLotting the loss and accuracy as a function of the number of epochs.
loss_list = cl_linreg2.loss
```

```
accuracy_list = cl_linreg2.accuracies
epochs_axis = range(0, epochs)

plt.plot(epochs_axis, loss_list, label="loss")
plt.plot(epochs_axis, accuracy_list, label="accuracy")
plt.legend()
plt.show()
```



1.1.8 Answer:

The accuracy function appears to not be monotone as expected, bouncing in the beginning, and then stabilizing at the top.

The loss function appears to be monotone, and decreasing, which is also expected.

1.1.9 Task: Scaling

we have seen in the lectures that scaling the data may improve training speed.

- Implement a scaler, either standard scaler (normalizer) or max-min scaler
- Scale the data
- Train the model on the scaled data
- Experiment with hyper-parameter settings and see whether you can speed up the training.
- Report final hyper-meter settings and show how you found them.

- Plot the loss curve and the accuracy curve for the classifier trained on scaled data with the best settings you found.

[22]:
```
## Implementation of a max-min scaler.
def max_min_scaler(training_data):

    len_j = training_data.shape[0]
    len_i = training_data.shape[1]
    result = training_data.copy()

    for i in range(len_i):
        # All observations, but only feature i.
        min_i = min(training_data[:, i])
        max_i = max(training_data[:, i])

        # Scaling the data.
        for j in range(len_j):
            xji = training_data[j, i]
            scaled_xji = (xji - min_i) / (max_i - min_i)
            result[j][i] = scaled_xji

    return result

X_train_scaled = max_min_scaler(X_train)
X_val_scaled = max_min_scaler(X_val)
```

[23]:
```
# Experimenting with new hyper-parameters to be used with scaled data.
eta_start = 1
for epoch in [1, 2, 5, 10, 50, 100, 1000, 10000, 100000]:
    cl_tune = NumpyLinRegClass()
    cl_tune.fit(X_train_scaled, t2_train, eta=eta_start, epochs=epoch)
    print("Learning rate: {}  Epochs: {:7}  Accuracy: {}".format(
        eta_start, epoch, accuracy(cl_tune.predict(X_val_scaled), t2_val)))
```

```
Learning rate: 1  Epochs:       1  Accuracy: 0.432
Learning rate: 1  Epochs:       2  Accuracy: 0.576
Learning rate: 1  Epochs:       5  Accuracy: 0.576
Learning rate: 1  Epochs:      10  Accuracy: 0.576
Learning rate: 1  Epochs:      50  Accuracy: 0.77
Learning rate: 1  Epochs:     100  Accuracy: 0.742
Learning rate: 1  Epochs:    1000  Accuracy: 0.69
Learning rate: 1  Epochs:   10000  Accuracy: 0.69
Learning rate: 1  Epochs:  100000  Accuracy: 0.69
```

[24]:
```
best_epochs = 100
for eta in [1, 0.1, 0.01, 0.001, 0.0001, 0.00001]:
    cl_tune = NumpyLinRegClass()
    cl_tune.fit(X_train_scaled, t2_train, eta=eta, epochs=best_epochs)
```

```
    print("Learning rate: {:7}  Epochs: {:7}  Accuracy: {}".format(
        eta, best_epochs, accuracy(cl_tune.predict(X_val_scaled), t2_val)))
```

```
Learning rate:       1  Epochs:     100  Accuracy: 0.742
Learning rate:     0.1  Epochs:     100  Accuracy: 0.576
Learning rate:    0.01  Epochs:     100  Accuracy: 0.576
Learning rate:   0.001  Epochs:     100  Accuracy: 0.576
Learning rate:  0.0001  Epochs:     100  Accuracy: 0.576
Learning rate:   1e-05  Epochs:     100  Accuracy: 0.576
```

1.1.10 Results from tuning hyperparameter with scaled sets

It appears that the learning rate can have a value of 1 without causing an overflow, and that this also speeds up the training hundredfold, achieving an even better accuracy than before aswell. An eta of 10 will cause an overflow, which would not work in this case, just like an eta of 0.1 for the unscaled set.

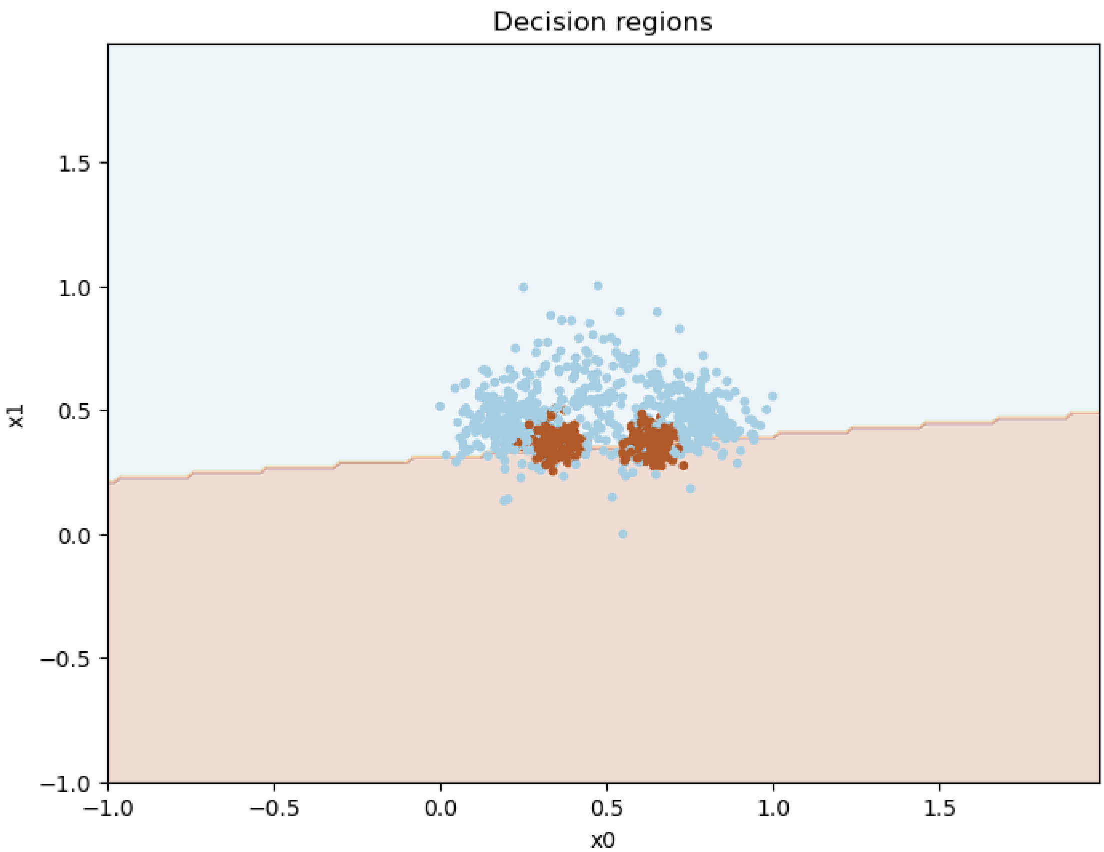
Final hyperparameter values: eta = 1, epochs = 100, with an accuracy of 0.742

1.1.11 Training the model with these parameters and plotting the decision regions

[25]:
```
# Trains the model on the scaled data.
cl_linreg3 = NumpyLinRegClass()
eta = 1
epochs = 100
cl_linreg3.fit(X_train_scaled, t2_train, eta=eta, epochs=epochs)

# Checks the accuracy on the scaled validation set
X_val_scaled = max_min_scaler(X_val)
accuracy(cl_linreg3.predict(X_val_scaled), t2_val)
```

[25]: 0.742

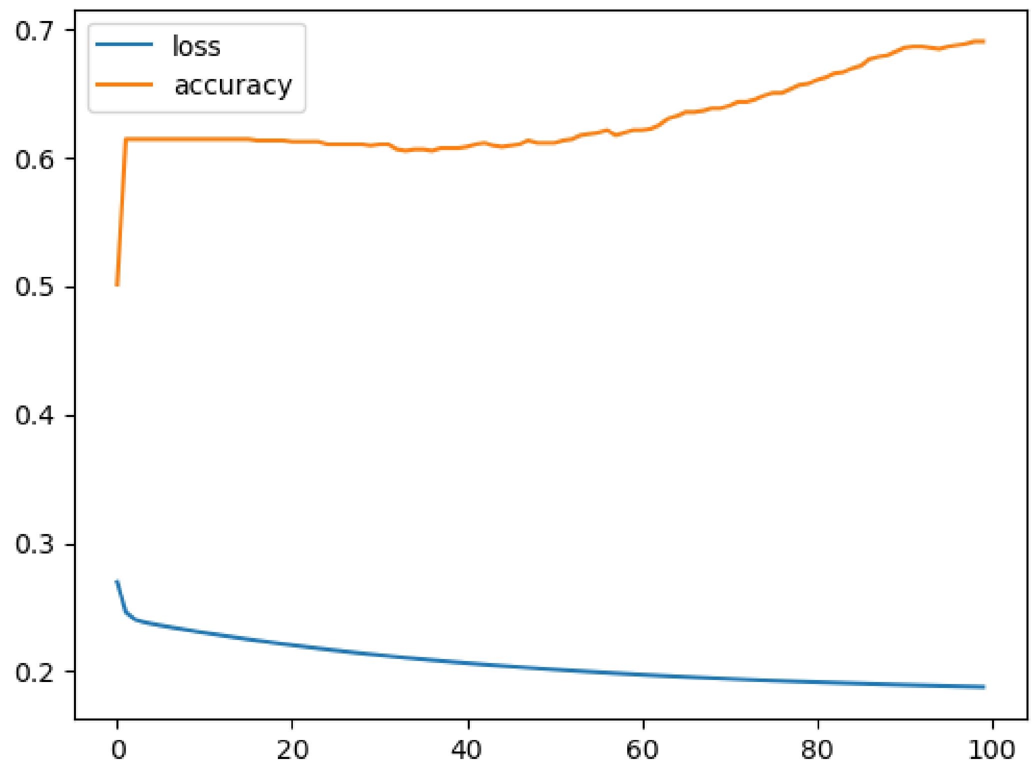
[26]:
```
plot_decision_regions(X_train_scaled, t2_train, cl_linreg3)
```



An experimental value I found outside of these automatically generated parameters is 0.77. This can be reached with eta = 0.001, and 50 000 epochs.

1.1.12 Plotting loss and accuracy as a function of the number of epochs

[27]:
```
## PLotting the loss and accuracy as a function of the number of epochs.
loss_list = cl_linreg3.loss
accuracy_list = cl_linreg3.accuracies
epochs_axis = range(0, epochs)

plt.plot(epochs_axis, loss_list, label="loss")
plt.plot(epochs_axis, accuracy_list, label="accuracy")
plt.legend()
plt.show()
```



1.2 Logistic regression

a) You should now implement a logistic regression classifier similarly to the classifier based on linear regression. You may use code from the solution to weekly exercise set week07.

b) In addition to the method `predict` which predicts a class for the data, include a method `predict_probability` which predicts the probability of the data belonging to the positive class.

c) As with the classifier based on linear regression, we want to calculate loss and accuracy after each epoch. The prefered loss for logistic regression is binary cross-entropy. You could have used mean squared error. The most important is that your implementation of the loss corresponds to your implementation of the gradient descent.

d) In addition, extend the fit-method with optional arguments for a validation set (X_val, t_val). If a validation set is included in the call to fit, calculate the loss and the accuracy for the validation set after each epoch.

e) The training runs for a number of epochs. We cannot know beforehand for how many epochs it is reasonable to run the training. One possibility is to run the training until the learning does not improve much. Extend the fit-method with two keyword arguments, `tol` and `n_epochs_no_update` and stop training when the loss has not improved with more than `tol` after running `n_epochs_no_update` epochs. A possible default value for `n_epochs_no_update` is 5. Also, add an attribute to the classifier which tells us after fitting how many epochs were ran.

f) Train classifiers with various learning rates, and with varying values for `tol` for finding optimal values. Also consider the effect of scaling the data.

g) After a succesful training, plot both training loss and validation loss as functions of the number of epochs in one figure, and both accuracies as functions of the number of epochs in another figure. Comment on what you see.

1.2.1 Implementing the logistic regression classifier using code from solutions to weekly exercises

Implementation using MSE

[28]:
```
## a) Implementing the logistic regression classifier. Code taken from solution␣
↪to weekly exercise set week07

def logistic(x):
    return 1/(1+np.exp(-x))

class NumpyLogReg(NumpyClassifier):

    def __init__(self, bias=-1):
        self.bias=bias

    def fit(self, X_train, t_train, eta = 0.1, epochs=10, X_val=[], t_val=[],␣
↪tol=5, n_epochs_no_update=5):
        """X_train is a Nxm matrix, N data points, m features
        t_train is avector of length N,
        the targets values for the training data"""

        X_train_old = X_train
        X_val_old = X_val

        val_check = 0
        if (len(X_val) != 0 and len(t_val) != 0):
            val_check = 1
            if self.bias:
                X_val = add_bias(X_val, self.bias)

        (N, m) = X_train.shape
        if self.bias:
            X_train = add_bias(X_train, self.bias)

        self.weights = weights = np.zeros(m+1)

        # Stores loss and accuracy data.
        self.loss_train = []
        self.loss_val = []
```

```
        self.accuracies_train = []
        self.accuracies_val = []

        # Used to control the tolerance of the training.
        self.number_of_epochs = 0
        current_n_epochs_no_update = 0
        current_asserted_value = 9999999

        for e in range(epochs):
            weights -= eta / N *  X_train.T @ (self.forward(X_train) - t_train)

            # Loss and accuracy for training sets.
            error = t_train - self.forward(X_train)
            MSE_train = (1 / N) * (error.T @ error)
            self.loss_train.append(MSE_train)
            self.accuracies_train.append(accuracy(self.predict(X_train_old),␣
↪t_train))

            self.number_of_epochs += 1

            # Loss and accuracy for validation sets if given.
            if (val_check):
                error = t_val - self.forward(X_val)
                MSE_val = (1 / N) * (error.T @ error)
                self.loss_val.append(MSE_val)
                self.accuracies_val.append(accuracy(self.predict(X_val_old),␣
↪t_val))

                # Checks the tolerance for the training.
                if ((current_asserted_value - MSE_val) > tol):
                    current_n_epochs_no_update = 0
                    current_asserted_value = MSE_val
                else:
                    current_n_epochs_no_update += 1

                if (current_n_epochs_no_update == n_epochs_no_update):
                    #pass
                    break


    def forward(self, X):
        return logistic(X @ self.weights)

    def predict(self, X, threshold=0.5):
        """X is a Kxm matrix for some K>=1
        predict the value for each point in X"""
        if self.bias:
```

```
            z = add_bias(X, self.bias)
        return (self.forward(z) > threshold).astype('int')

    def predict_probability(self, X):
        if self.bias:
            z = add_bias(X, self.bias)
        return self.forward(z)

    # Does not add bias term. NOT USED
    def cross_entropy_loss(self, X, t):
        sum = 0
        N = t.shape[0]
        probabilities = np.zeros(N)

        for j in range(0, N):
            xj = X[j]
            wj = self.weights[j]
            yj = self.logistic(x @ w)
            tj = t[j]
            probability = (y**tj) * ((1 - yj)**(1 - tj))
            probabilities[j] = probability

        for j in range(0, N):
            Xj = X[j]
            tj = t[j]
            probability = probabilities[j]
            logarithm = np.log(probability)
            sum -= logarithm

        return sum
```

1.2.2 Training various classifiers without scaling

I have decided to use a total number of epochs = 100 000, a starting learning rate of 0.1, and 10 epochs without any updates.

[29]:

```
epochs = 100000
eta_start = 0.1

for tol in [5, 0.5, 0.05, 0.005, 0.0005, 0.00005, 0.000005]:
    cl_tune = NumpyLogReg()
    cl_tune.fit(X_train, t2_train, eta=eta_start, epochs=epochs, X_val=X_val,␣
 ↪t_val = t2_val, tol=tol, n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}  No_epochs: {}".format(
        eta_start, tol, accuracy(cl_tune.predict(X_val), t2_val), cl_tune.
 ↪number_of_epochs))
```

```
Learning rate:     0.1  tol:       5  Accuracy: 0.604  No_epochs: 11
Learning rate:     0.1  tol:     0.5  Accuracy: 0.604  No_epochs: 11
Learning rate:     0.1  tol:    0.05  Accuracy: 0.604  No_epochs: 11
Learning rate:     0.1  tol:   0.005  Accuracy: 0.668  No_epochs: 32
Learning rate:     0.1  tol:  0.0005  Accuracy: 0.69  No_epochs: 80
Learning rate:     0.1  tol:   5e-05  Accuracy: 0.704  No_epochs: 195
Learning rate:     0.1  tol:   5e-06  Accuracy: 0.71  No_epochs: 589
```

[30]:
```
best_tol = 0.000005

for eta in [1, 0.1, 0.01, 0.001, 0.0001, 0.00001]:
    cl_tune = NumpyLogReg()
    cl_tune.fit(X_train, t2_train, eta=eta, epochs=epochs, X_val=X_val, t_val =␣
↪t2_val, tol=best_tol, n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}  No_epochs: {}".format(
        eta, best_tol, accuracy(cl_tune.predict(X_val), t2_val), cl_tune.
↪number_of_epochs))
```

```
Learning rate:       1  tol:   5e-06  Accuracy: 0.684  No_epochs: 14
Learning rate:     0.1  tol:   5e-06  Accuracy: 0.71  No_epochs: 589
Learning rate:    0.01  tol:   5e-06  Accuracy: 0.704  No_epochs: 1829
Learning rate:   0.001  tol:   5e-06  Accuracy: 0.684  No_epochs: 6710
Learning rate:  0.0001  tol:   5e-06  Accuracy: 0.642  No_epochs: 19178
Learning rate:   1e-05  tol:   5e-06  Accuracy: 0.544  No_epochs: 4353
```

1.2.3 Results from training without scaling:

With a tolerance of 5e-06 and learning rate of 0.1, the classifier has achieved an accuracy of 0.71.

1.2.4 Training various classifiers with scaling

[31]:
```
epochs = 100000
eta_start = 1

for tol in [5, 0.5, 0.05, 0.005, 0.0005, 0.00005, 0.000005]:
    cl_tune = NumpyLogReg()
    cl_tune.fit(X_train_scaled, t2_train, eta=eta_start, epochs=epochs,␣
↪X_val=X_val_scaled, t_val = t2_val, tol=tol, n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}  No_epochs: {}".format(
        eta_start, tol, accuracy(cl_tune.predict(X_val_scaled), t2_val),␣
↪cl_tune.number_of_epochs))
```

```
Learning rate:       1  tol:       5  Accuracy: 0.576  No_epochs: 11
Learning rate:       1  tol:     0.5  Accuracy: 0.576  No_epochs: 11
Learning rate:       1  tol:    0.05  Accuracy: 0.576  No_epochs: 11
Learning rate:       1  tol:   0.005  Accuracy: 0.576  No_epochs: 11
Learning rate:       1  tol:  0.0005  Accuracy: 0.786  No_epochs: 178
Learning rate:       1  tol:   5e-05  Accuracy: 0.752  No_epochs: 309
Learning rate:       1  tol:   5e-06  Accuracy: 0.75  No_epochs: 336
```

[32]:
```
best_tol_scaled = 0.0005

for eta in [1, 0.1, 0.01, 0.001, 0.0001, 0.00001]:
    cl_tune = NumpyLogReg()
    cl_tune.fit(X_train_scaled, t2_train, eta=eta, epochs=epochs,␣
 ↪X_val=X_val_scaled, t_val = t2_val, tol=best_tol_scaled, n_epochs_no_update␣
 ↪= 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}  No_epochs: {}".format(
        eta, best_tol_scaled, accuracy(cl_tune.predict(X_val_scaled), t2_val),␣
 ↪cl_tune.number_of_epochs))
```

```
Learning rate:       1  tol:  0.0005  Accuracy: 0.786  No_epochs: 178
Learning rate:     0.1  tol:  0.0005  Accuracy: 0.576  No_epochs: 40
Learning rate:    0.01  tol:  0.0005  Accuracy: 0.576  No_epochs: 11
Learning rate:   0.001  tol:  0.0005  Accuracy: 0.576  No_epochs: 11
Learning rate:  0.0001  tol:  0.0005  Accuracy: 0.576  No_epochs: 11
Learning rate:   1e-05  tol:  0.0005  Accuracy: 0.576  No_epochs: 11
```

1.2.5 Results from training with scaling:

With a tolerance of 5e-04 and learning rate of 1, the classifier has achieved an accuracy of 0.786. This is the all time best.

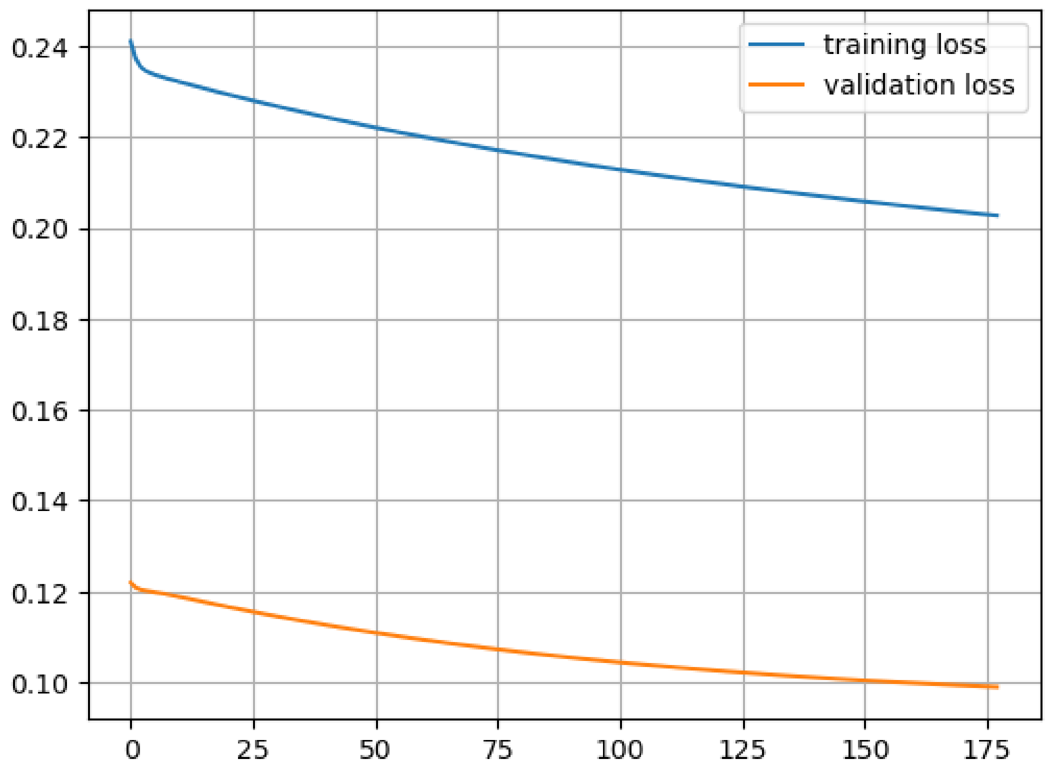
Final values: eta = 1, tolerance = 05e-04, epochs = 100 000, n_epochs_no_update = 10, SCALED SETS.

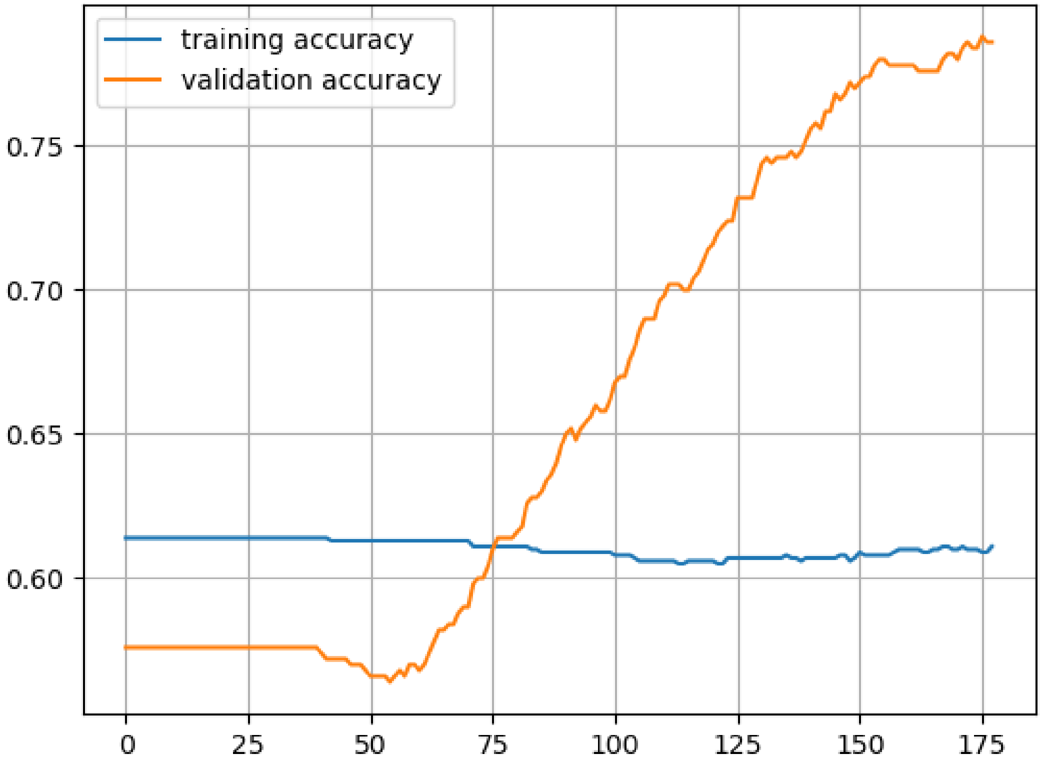
1.2.6 Plotting the results from scaled training with these final values

[33]:
```
#
eta = 1
tol = 0.0005
epochs = 100000

cl_logreg = NumpyLogReg()
cl_logreg.fit(X_train_scaled, t2_train, eta=eta, epochs=epochs,
                   X_val=X_val_scaled, t_val = t2_val, tol=tol,␣
 ↪n_epochs_no_update = 10)
```

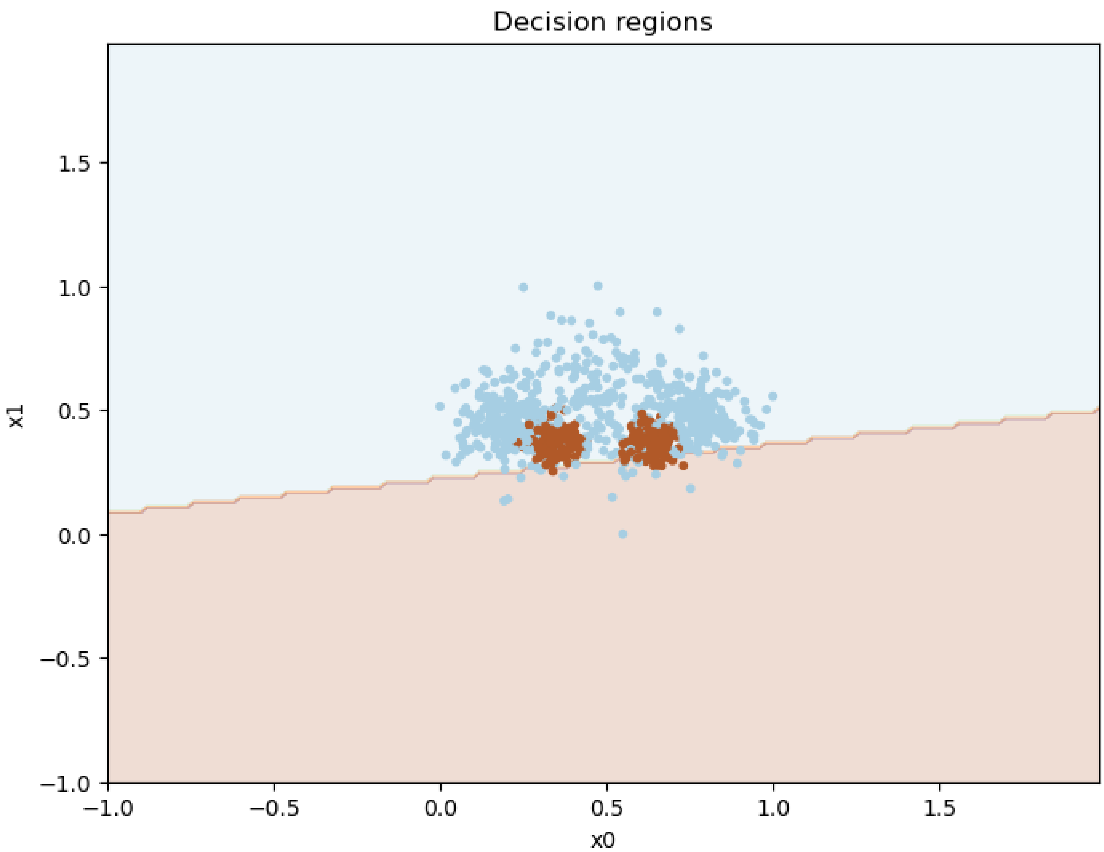
[34]:
```
loss_train_list = cl_logreg.loss_train
loss_val_list = cl_logreg.loss_val

accuracies_train_list = cl_logreg.accuracies_train
accuracies_val_list = cl_logreg.accuracies_val

epochs_axis = range(cl_logreg.number_of_epochs)

plt.plot(epochs_axis, loss_train_list, label="training loss")
plt.plot(epochs_axis, loss_val_list, label="validation loss")
```

```
plt.legend()
plt.grid()
plt.show()

plt.plot(epochs_axis, accuracies_train_list, label="training accuracy")
plt.plot(epochs_axis, accuracies_val_list, label="validation accuracy")
plt.legend()
plt.grid()
plt.show()
```





[35]:
```
plot_decision_regions(X_train_scaled, t2_train, cl_logreg)
```



1.3 Multi-class classifiers

We turn to the task of classifying when there are more than two classes, and the task is to ascribe one class to each input. We will now use the set (X, t_multi).

1.3.1 "One-vs-rest" with logistic regression

We saw in the lecture how a logistic regression classifier can be turned into a multi-class classifier using the one-vs-rest approach. We train one logistic regression classifier for each class. To predict the class of an item, we run all the binary classifiers and collect the probability score from each of them. We assign the class which ascribes the highest probability.

Build such a classifier. Train the resulting classifier on (X_train, t_multi_train), test it on (X_val, t_multi_val), tune the hyper-parameters and report the accuracy.

Also plot the decision boundaries for your best classifier similarly to the plots for the binary case.

[36]:
```
class OneVsRest(NumpyClassifier):

    def fit(self, X_train, t_multi_train, eta = 0.01, epochs=100000, X_val=[],
 ↪t_multi_val=[], tol=0.000005, n_epochs_no_update=10):
```

```
        self.classifiers = []

        for decision in range(0, 5):
            # Makes the training sets binary.
            t_train_inner = self.split_training_set(t_multi_train, decision)
            t_val_inner = self.split_training_set(t_multi_val, decision)

            # Trains a binary logistic regression classifier on the binary sets.
            cl_logi_inner = NumpyLogReg()
            cl_logi_inner.fit(X_train, t_train_inner, eta=eta, epochs=epochs,
                              X_val=X_val, t_val=t_val_inner, tol=tol,␣
↪n_epochs_no_update=n_epochs_no_update)

            # Appends the weights to the total list of weights.
            self.classifiers.append(cl_logi_inner)

    def split_training_set(self, t_multi_train, decision):
        t2_train = t_multi_train == decision
        t2_train = t2_train.astype('int')
        return t2_train

    def predict(self, X):

        list_of_probabilities_list = []

        for i in range(0, 5):
            list_of_probabilities = self.classifiers[i].predict_probability(X)
            list_of_probabilities_list.append(list_of_probabilities)

        len_X = X.shape[0]
        resulting_y = np.zeros(len_X)

        for j in range(0, len_X):
            observation_x = X[j]

            highest_probability = 0
            highest_probability_index = 0  # important

            for i in range(0, 5):
                list_of_probabilities = list_of_probabilities_list[i]
                if list_of_probabilities[j] > highest_probability:
                    highest_probability = list_of_probabilities[j]
                    highest_probability_index = i

            # Use the highest_probability_index to decide X[j]
            resulting_y[j] = highest_probability_index
```

```
        return resulting_y
```

1.3.2 Tuning the hyperparameters for One vs Rest without scaling

[37]:
```
epochs = 100000
eta_start = 0.1

for tol in [5, 0.5, 0.05, 0.005, 0.0005, 0.00005, 0.000005]:
    cl_tune = OneVsRest()
    cl_tune.fit(X_train, t_multi_train, eta=eta_start, epochs=epochs,
                X_val=X_val, t_multi_val=t_multi_val, tol=tol,␣
 ↪n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}".format(
        eta_start, tol, accuracy(cl_tune.predict(X_val), t_multi_val)))
```

```
Learning rate:     0.1  tol:       5  Accuracy: 0.356
Learning rate:     0.1  tol:     0.5  Accuracy: 0.356
Learning rate:     0.1  tol:    0.05  Accuracy: 0.356
Learning rate:     0.1  tol:   0.005  Accuracy: 0.422
Learning rate:     0.1  tol:  0.0005  Accuracy: 0.686
Learning rate:     0.1  tol:   5e-05  Accuracy: 0.788
Learning rate:     0.1  tol:   5e-06  Accuracy: 0.822
```

[38]:
```
best_tol = 0.000005

for eta in [1, 0.1, 0.01, 0.001, 0.0001, 0.00001]:
    cl_tune = OneVsRest()
    cl_tune.fit(X_train, t_multi_train, eta=eta, epochs=epochs,
                X_val=X_val, t_multi_val=t_multi_val, tol=best_tol,␣
 ↪n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}".format(
        eta, best_tol, accuracy(cl_tune.predict(X_val), t_multi_val)))
```

```
Learning rate:       1  tol:   5e-06  Accuracy: 0.426
Learning rate:     0.1  tol:   5e-06  Accuracy: 0.822
Learning rate:    0.01  tol:   5e-06  Accuracy: 0.788
Learning rate:   0.001  tol:   5e-06  Accuracy: 0.682
Learning rate:  0.0001  tol:   5e-06  Accuracy: 0.378
Learning rate:   1e-05  tol:   5e-06  Accuracy: 0.264
```

1.3.3 Results from the training without scaling:

With a tolerance of 5e-06 and learning rate of 0.1, we can achieve the accuracy of 0.822 with multi-class classifying, without scaling.

1.3.4 Tuning the hyperparameters for One vs Rest with scaling

[39]:
```
epochs = 100000
eta_start = 1

for tol in [5, 0.5, 0.05, 0.005, 0.0005, 0.00005, 0.000005, 0.0000005, 0.
 ↪00000005]:
    cl_tune = OneVsRest()
    cl_tune.fit(X_train_scaled, t_multi_train, eta=eta_start, epochs=epochs,
                X_val=X_val_scaled, t_multi_val=t_multi_val, tol=tol,␣
 ↪n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}".format(
        eta_start, tol, accuracy(cl_tune.predict(X_val_scaled), t_multi_val)))
```

```
Learning rate:       1  tol:       5  Accuracy: 0.418
Learning rate:       1  tol:     0.5  Accuracy: 0.418
Learning rate:       1  tol:    0.05  Accuracy: 0.418
Learning rate:       1  tol:   0.005  Accuracy: 0.414
Learning rate:       1  tol:  0.0005  Accuracy: 0.64
Learning rate:       1  tol:   5e-05  Accuracy: 0.818
Learning rate:       1  tol:   5e-06  Accuracy: 0.822
Learning rate:       1  tol:   5e-07  Accuracy: 0.832
Learning rate:       1  tol:   5e-08  Accuracy: 0.832
```

[40]:
```
best_tol_scaled = 0.0000005

for eta in [1, 0.1, 0.01, 0.001, 0.0001, 0.00001]:
    cl_tune = OneVsRest()
    cl_tune.fit(X_train_scaled, t_multi_train, eta=eta, epochs=epochs,
                X_val=X_val_scaled, t_multi_val=t_multi_val,␣
 ↪tol=best_tol_scaled, n_epochs_no_update = 10)
    print("Learning rate: {:7}  tol: {:7}  Accuracy: {}".format(
        eta, best_tol_scaled, accuracy(cl_tune.predict(X_val_scaled),␣
 ↪t_multi_val)))
```

```
Learning rate:       1  tol:   5e-07  Accuracy: 0.832
Learning rate:     0.1  tol:   5e-07  Accuracy: 0.822
Learning rate:    0.01  tol:   5e-07  Accuracy: 0.818
Learning rate:   0.001  tol:   5e-07  Accuracy: 0.62
Learning rate:  0.0001  tol:   5e-07  Accuracy: 0.592
Learning rate:   1e-05  tol:   5e-07  Accuracy: 0.4
```

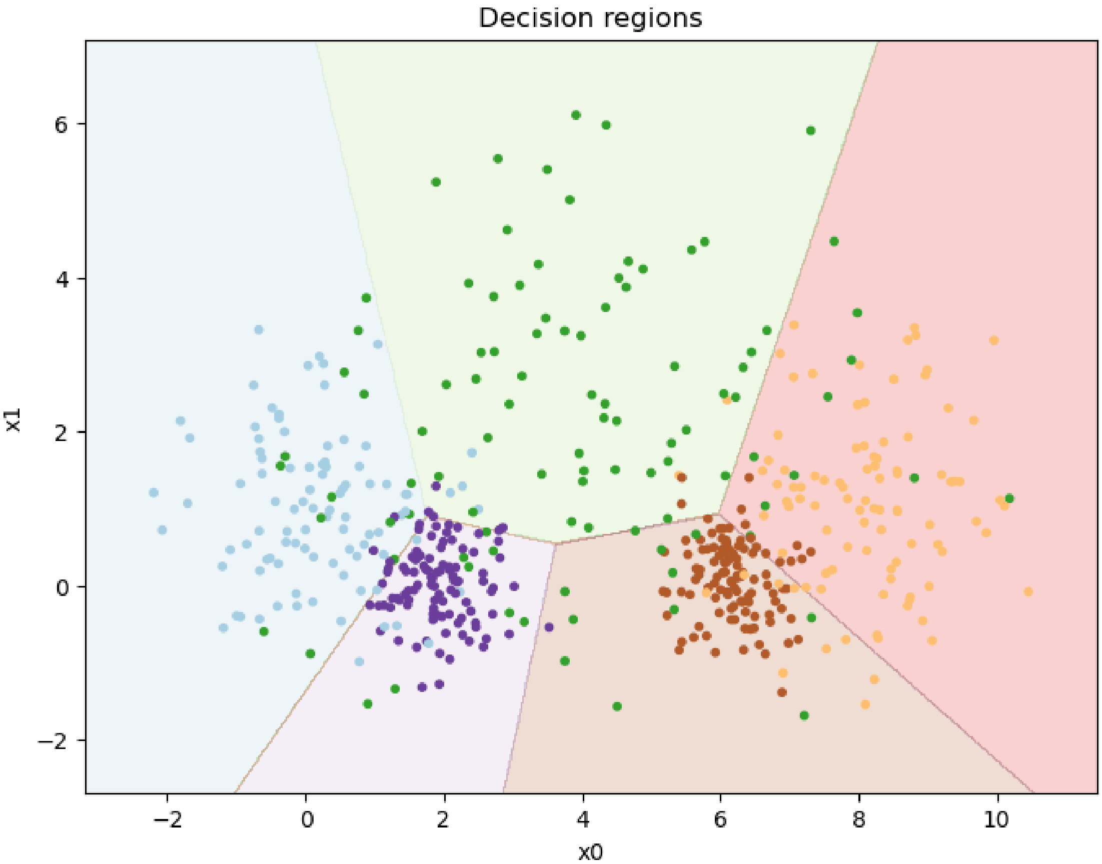
1.3.5 Results from the training with scaling only:

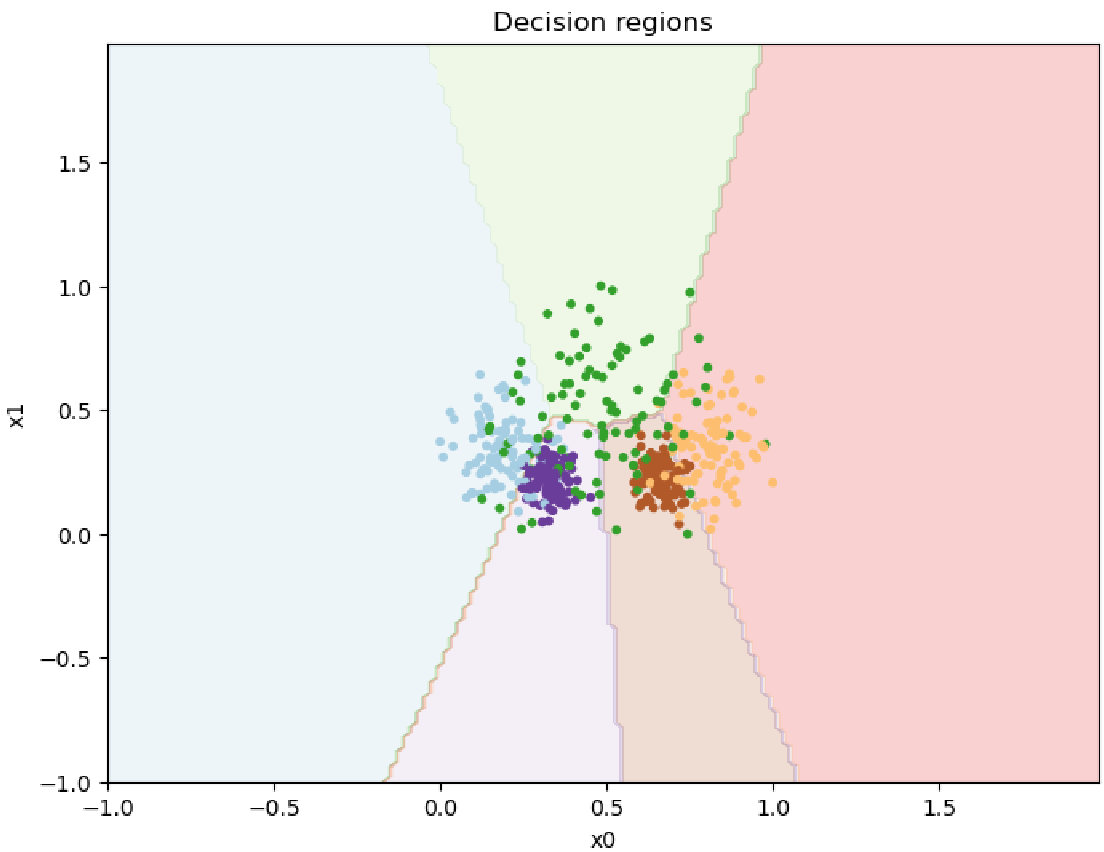
With a tolerance of 5e-07 and learning rate of 1, we can achieve the accuracy of 0.832 with multi-class classifying, with scaling.

1.3.6 Plotting the decision boundaries on both unscaled and scaled

[41]:
```
eta = 0.1
eta_scaled = 1
tol = 0.000005
tol_scaled = 0.0000005
epochs = 100000


cl_multi_unscaled = OneVsRest()
cl_multi_unscaled.fit(X_train, t_multi_train, eta=eta, epochs=epochs,
             X_val=X_val, t_multi_val=t_multi_val, tol=tol, n_epochs_no_update␣
 ↪= 10)


cl_multi_scaled = OneVsRest()
cl_multi_scaled.fit(X_train_scaled, t_multi_train, eta=eta_scaled,␣
 ↪epochs=epochs,
             X_val=X_val_scaled, t_multi_val=t_multi_val, tol=tol_scaled,␣
 ↪n_epochs_no_update = 10)
```

[42]:
```
plot_decision_regions(X_val, t_multi_val, cl_multi_unscaled)
plot_decision_regions(X_val_scaled, t_multi_val, cl_multi_scaled)
```


Decision regions
x1
x0
6
4
2
0
−2
−2
0
2
4
6
8
10



The resulting decision boundaries seem highly accurate!

1.3.7 For in4050-students: Multi-nominal logistic regression

The following part is only mandatory for in4050-students. In3050-students are also welcome to make it a try. Everybody has to return for the part 2 on multi-layer neural networks.

In the lecture, we contrasted the one-vs-rest approach with the multinomial logistic regression, also called softmax classifier. Implement also this classifier, tune the parameters, and compare the results to the one-vs-rest classifier.

Remember that this classifier uses softmax in the forward phase. For loss, it uses categorical cross-entropy loss. The loss has a somewhat simpler form than in the binary case. To calculate the gradient is a little more complicated. The actual gradient and update rule is simple, however, as long as you have calculated the forward values correctly.

2 Part II Multi-layer neural networks

2.1 A first non-linear classifier

The following code it a simple implementation of a multi-layer perceptron. It is quite restricted. There is only one hidden layer. It can only handle binary classification. In addition, it uses a simple final layer similar to the linear regression classifier above. One way to look at it is what happens when we add a hidden layer to the linear regression classifier.

It can be used to make a non-linear classifier for the set (X, t2). Experiment with settings for learning rate and epochs and see how good results you can get. Report results for variouse settings. Be prepared to train for a looooong time. Plot the training set together with the decision regions as in part I.

[43]:
```
class MLPBinaryLinRegClass(NumpyClassifier):
    """A multi-layer neural network with one hidden layer"""

    def __init__(self, bias=-1, dim_hidden = 6):
        """Intialize the hyperparameters"""
        self.bias = bias
        self.dim_hidden = dim_hidden

        def logistic(x):
            return 1/(1+np.exp(-x))
        self.activ = logistic

        def logistic_diff(y):
            return y * (1 - y)
        self.activ_diff = logistic_diff

    def fit(self, X_train, t_train, eta=0.001, epochs = 100, X_val=[],␣
↪t_val=[], tol=0.5, n_epochs_no_update=10):
        """Intialize the weights. Train *epochs* many epochs.

        X_train is a Nxm matrix, N data points, m features
        t_train is a vector of length N of targets values for the training␣
↪data,
        where the values are 0 or 1.
        """
        # This time, no bias will be added. Code written by myself
        val_check = 0
        X_val_bias = 0
        T_val = 0
        if (len(X_val) != 0 and len(t_val) != 0):
            val_check = 1
            X_val_bias = add_bias(X_val, self.bias)
            T_val = t_val.reshape(-1,1)

        self.eta = eta

        T_train = t_train.reshape(-1,1)

        dim_in = X_train.shape[1]
        dim_out = T_train.shape[1]
```

```
        # Stores loss and accuracy data. Code written by myself
        self.loss_train = []
        self.loss_val = []
        self.accuracies_train = []
        self.accuracies_val = []

        # Used to control the tolerance of the training. Code written by myself
        self.number_of_epochs = 0
        current_n_epochs_no_update = 0
        current_asserted_value = 9999999


        # Itilaize the wights
        self.weights1 = (np.random.rand(
            dim_in + 1,
            self.dim_hidden) * 2 - 1)/np.sqrt(dim_in)
        self.weights2 = (np.random.rand(
            self.dim_hidden+1,
            dim_out) * 2 - 1)/np.sqrt(self.dim_hidden)
        X_train_bias = add_bias(X_train, self.bias)


        for e in range(epochs):
            # One epoch
            hidden_outs, outputs = self.forward(X_train_bias)
            # The forward step
            out_deltas = (outputs - T_train)
            # The delta term on the output node
            hiddenout_diffs = out_deltas @ self.weights2.T
            # The delta terms at the output of the jidden layer
            hiddenact_deltas = (hiddenout_diffs[:, 1:] *
                                self.activ_diff(hidden_outs[:, 1:]))
            # The deltas at the input to the hidden layer
            self.weights2 -= self.eta * hidden_outs.T @ out_deltas
            self.weights1 -= self.eta * X_train_bias.T @ hiddenact_deltas
            # Update the weights


            # Calculates loss and accuracy. Code written by myself
            SE_train = out_deltas.T @ out_deltas
            SE_train = SE_train.reshape(-1)
            #print(SE_train.reshape)
            self.loss_train.append(SE_train)
            self.accuracies_train.append(accuracy(self.predict(X_train),␣
↪t_train))
```

```python
            self.number_of_epochs += 1

            # Loss and accuracy for validation sets if given. Code written by␣
↪myself
            if (val_check):
                _, val_outputs = self.forward(X_val_bias)
                val_out_deltas = (val_outputs - T_val)
                SE_val = val_out_deltas.T @ val_out_deltas
                SE_val = SE_val.reshape(-1)
                self.loss_val.append(SE_val)
                self.accuracies_val.append(accuracy(self.predict(X_val), t_val))

                # Checks the tolerance for the training. Code written by myself
                #print(f"{current_asserted_value - MSE_train}")
                if ((current_asserted_value - SE_val) > tol):
                    #print("Tolerance broken!!!")
                    current_n_epochs_no_update = 0
                    current_asserted_value = SE_val
                else:
                    current_n_epochs_no_update += 1

                if (current_n_epochs_no_update == n_epochs_no_update):
                    #pass
                    break



    def forward(self, X):
        """Perform one forward step.
        Return a pair consisting of the outputs of the hidden_layer
        and the outputs on the final layer"""
        hidden_activations = self.activ(X @ self.weights1)
        hidden_outs = add_bias(hidden_activations, self.bias)
        outputs = hidden_outs @ self.weights2
        return hidden_outs, outputs

    def predict(self, X):
        """Predict the class for the mebers of X"""
        Z = add_bias(X, self.bias)
        forw = self.forward(Z)[1]
        score= forw[:, 0]
        return (score > 0.5)

    # Code written by myself
    def predict_probability(self, X):
        Z = add_bias(X, self.bias)
        forw = self.forward(Z)[1]
```

```
        score= forw[:, 0]
        return score
```

2.2 Improving the classifier

You should now make changes to the classifier similarly to what you did with the logistic regression classifier in part 1.

a) In addition to the method predict, which predicts a class for the data, include a method predict_probability which predict the probability of the data belonging to the positive class. The training should be based on this value as with logistic regression.

b) Calculate the loss and the accuracy after each epoch and store them for inspection after training.

c) In addition, extend the fit-method with optional arguments for a validation set (X_val, t_val). If a validation set is included in the call to fit, calculate the loss and the accuracy for the validation set after each epoch.

d) The training runs for a number of epochs. We cannot know beforehand for how many epochs it is reasonable to run the training. One possibility is to run the training until the learning does not improve much. Extend the fit method with two keyword arguments, `tol` and `n_epochs_no_update` and stop training when the loss has not improved with more than tol after `n_epochs_no_update`. A possible default value for `n_epochs_no_update` is 5. Also, add an attribute to the classifier which tells us after fitting how many epochs were ran.

e) Tune the hyper-parameters:`eta`, `tol`and `dim-hidden`. Also consider the effect of scaling the data.

f) After a succesful training with a best setting for the hyper-parameters, plot both training loss and validation loss as functions of the number of epochs in one figure, and both accuracies as functions of the number of epochs in another figure. Comment on what you see.

g) The algorithm contains an element of non-determinism. Hence, train the classifier 10 times with the optimal hyper-parameters and report the mean and standard deviation of the accuracies over the 10 runs.

2.2.1 Tuning the parameters without scaling

For this classifier, I've chosen n_epochs_no_update = 100.

[44]:
```
epochs = 100000
eta_start = 0.001
dim_hidden_start = 6

for tol in [500, 50, 5, 0.5, 0.05, 0.005, 0.0005, 0.00005]:
    cl_tune = MLPBinaryLinRegClass(dim_hidden=dim_hidden_start)
    cl_tune.fit(X_train, t2_train, eta=eta_start, epochs=epochs,
                X_val=X_val, t_val=t2_val, tol=tol, n_epochs_no_update = 100)
    print("Learning rate: {:7}  tol: {:7}  dim_hidden: {:3}  Accuracy: {}␣
↪no_epochs: {}".format(
```

```
        eta_start, tol, dim_hidden_start, accuracy(cl_tune.
↪predict(X_val_scaled), t2_val), cl_tune.number_of_epochs))
```

```
Learning rate:   0.001  tol:     500  dim_hidden:   6  Accuracy: 0.574
no_epochs: 101
Learning rate:   0.001  tol:      50  dim_hidden:   6  Accuracy: 0.576
no_epochs: 144
Learning rate:   0.001  tol:       5  dim_hidden:   6  Accuracy: 0.576
no_epochs: 265
Learning rate:   0.001  tol:     0.5  dim_hidden:   6  Accuracy: 0.548
no_epochs: 243
Learning rate:   0.001  tol:    0.05  dim_hidden:   6  Accuracy: 0.536
no_epochs: 264
Learning rate:   0.001  tol:   0.005  dim_hidden:   6  Accuracy: 0.576
no_epochs: 20847
Learning rate:   0.001  tol:  0.0005  dim_hidden:   6  Accuracy: 0.57 no_epochs:
321
Learning rate:   0.001  tol:   5e-05  dim_hidden:   6  Accuracy: 0.576
no_epochs: 50573
```

[45]:
```
best_tol = 0.5

for eta in [0.0001, 0.00001, 0.000001, 0.0000001]:
    cl_tune = MLPBinaryLinRegClass(dim_hidden=dim_hidden_start)
    cl_tune.fit(X_train, t2_train, eta=eta, epochs=epochs,
                X_val=X_val, t_val=t2_val, tol=best_tol, n_epochs_no_update =␣
↪100)
    print("Learning rate: {:7}  tol: {:4}  dim_hidden: {:3}  Accuracy: {}␣
↪no_epochs: {}".format(
        eta, best_tol, dim_hidden_start, accuracy(cl_tune.predict(X_val),␣
↪t2_val), cl_tune.number_of_epochs))
```

```
Learning rate:  0.0001  tol:  0.5  dim_hidden:   6  Accuracy: 0.814 no_epochs:
1774
Learning rate:   1e-05  tol:  0.5  dim_hidden:   6  Accuracy: 0.714 no_epochs:
2769
Learning rate:   1e-06  tol:  0.5  dim_hidden:   6  Accuracy: 0.57 no_epochs:
1110
Learning rate:   1e-07  tol:  0.5  dim_hidden:   6  Accuracy: 0.576 no_epochs:
4051
```

[46]:
```
best_eta = 0.0001

for dim_hidden in range(6, 100, 6):
    cl_tune = MLPBinaryLinRegClass(dim_hidden=dim_hidden)
    cl_tune.fit(X_train, t2_train, eta=best_eta, epochs=epochs,
                X_val=X_val, t_val=t2_val, tol=best_tol, n_epochs_no_update =␣
↪100)
```

```
    print("Learning rate: {:7}  tol: {:4}  dim_hidden: {:3}  Accuracy: {}␣
↪no_epochs: {}".format(
        eta, best_tol, dim_hidden, accuracy(cl_tune.predict(X_val), t2_val),␣
↪cl_tune.number_of_epochs))
```

```
Learning rate:   1e-07  tol:  0.5  dim_hidden:   6  Accuracy: 0.816 no_epochs:
1518
Learning rate:   1e-07  tol:  0.5  dim_hidden:  12  Accuracy: 0.82 no_epochs:
1673
Learning rate:   1e-07  tol:  0.5  dim_hidden:  18  Accuracy: 0.84 no_epochs:
878
Learning rate:   1e-07  tol:  0.5  dim_hidden:  24  Accuracy: 0.848 no_epochs:
1166
Learning rate:   1e-07  tol:  0.5  dim_hidden:  30  Accuracy: 0.834 no_epochs:
1059
Learning rate:   1e-07  tol:  0.5  dim_hidden:  36  Accuracy: 0.846 no_epochs:
716
Learning rate:   1e-07  tol:  0.5  dim_hidden:  42  Accuracy: 0.844 no_epochs:
921
Learning rate:   1e-07  tol:  0.5  dim_hidden:  48  Accuracy: 0.836 no_epochs:
716
Learning rate:   1e-07  tol:  0.5  dim_hidden:  54  Accuracy: 0.848 no_epochs:
867
Learning rate:   1e-07  tol:  0.5  dim_hidden:  60  Accuracy: 0.85 no_epochs:
1035
Learning rate:   1e-07  tol:  0.5  dim_hidden:  66  Accuracy: 0.85 no_epochs:
775
Learning rate:   1e-07  tol:  0.5  dim_hidden:  72  Accuracy: 0.85 no_epochs:
857
Learning rate:   1e-07  tol:  0.5  dim_hidden:  78  Accuracy: 0.854 no_epochs:
673
Learning rate:   1e-07  tol:  0.5  dim_hidden:  84  Accuracy: 0.844 no_epochs:
992
Learning rate:   1e-07  tol:  0.5  dim_hidden:  90  Accuracy: 0.828 no_epochs:
1268
Learning rate:   1e-07  tol:  0.5  dim_hidden:  96  Accuracy: 0.824 no_epochs:
870
```

2.2.2 Results from tuning without scaling

It seems that with tolerance, it does not matter much, and I therefore chose 0.5 as a good middleground. For the learning rate, it seems that higher values dominate, and i chose eta = 0.0001. Also, eta as little as 0.001 causes overflow. For dim_hidden the accuracy seems to generally max out around dim_hidden = 36.

2.2.3 Tuning the hyperparameters with scaling

[47]:
```python
epochs = 100000
eta_start = 0.001
dim_hidden_start = 6

for tol in [500, 50, 5, 0.5, 0.05, 0.005, 0.0005, 0.00005, 0.000005]:
    cl_tune = MLPBinaryLinRegClass(dim_hidden=dim_hidden_start)
    cl_tune.fit(X_train_scaled, t2_train, eta=eta_start, epochs=epochs,
                X_val=X_val_scaled, t_val=t2_val, tol=tol, n_epochs_no_update =␣
 ↪100)
    print("Learning rate: {:7}  tol: {:7}  dim_hidden: {:3}  Accuracy: {}␣
 ↪no_epochs: {}".format(
        eta_start, tol, dim_hidden_start, accuracy(cl_tune.
 ↪predict(X_val_scaled), t2_val), cl_tune.number_of_epochs))
```

```
Learning rate:   0.001  tol:     500  dim_hidden:   6  Accuracy: 0.576
no_epochs: 101
Learning rate:   0.001  tol:      50  dim_hidden:   6  Accuracy: 0.576
no_epochs: 109
Learning rate:   0.001  tol:       5  dim_hidden:   6  Accuracy: 0.576
no_epochs: 109
Learning rate:   0.001  tol:     0.5  dim_hidden:   6  Accuracy: 0.718
no_epochs: 474
Learning rate:   0.001  tol:    0.05  dim_hidden:   6  Accuracy: 0.722
no_epochs: 496
Learning rate:   0.001  tol:   0.005  dim_hidden:   6  Accuracy: 0.71 no_epochs:
559
Learning rate:   0.001  tol:  0.0005  dim_hidden:   6  Accuracy: 0.724
no_epochs: 593
Learning rate:   0.001  tol:   5e-05  dim_hidden:   6  Accuracy: 0.706
no_epochs: 378
Learning rate:   0.001  tol:   5e-06  dim_hidden:   6  Accuracy: 0.722
no_epochs: 480
```

[48]:
```python
best_tol = 0.05

for eta in [0.001, 0.0001, 0.00001, 0.000001, 0.0000001]:
    cl_tune = MLPBinaryLinRegClass(dim_hidden=dim_hidden_start)
    cl_tune.fit(X_train_scaled, t2_train, eta=eta, epochs=epochs,
                X_val=X_val_scaled, t_val=t2_val, tol=best_tol,␣
 ↪n_epochs_no_update = 100)
    print("Learning rate: {:7}  tol: {:4}  dim_hidden: {:3}  Accuracy: {}␣
 ↪no_epochs: {}".format(
        eta, best_tol, dim_hidden_start, accuracy(cl_tune.
 ↪predict(X_val_scaled), t2_val), cl_tune.number_of_epochs))
```

```
Learning rate:   0.001  tol: 0.05  dim_hidden:   6  Accuracy: 0.732 no_epochs:
```

```
807
Learning rate:  0.0001  tol: 0.05  dim_hidden:   6  Accuracy: 0.75 no_epochs:
2420
Learning rate:   1e-05  tol: 0.05  dim_hidden:   6  Accuracy: 0.764 no_epochs:
23528
Learning rate:   1e-06  tol: 0.05  dim_hidden:   6  Accuracy: 0.576 no_epochs:
1557
Learning rate:   1e-07  tol: 0.05  dim_hidden:   6  Accuracy: 0.576 no_epochs:
735
```

[49]:

```
best_eta = 0.00001

for dim_hidden in range(6, 100, 6):
    cl_tune = MLPBinaryLinRegClass(dim_hidden=dim_hidden)
    cl_tune.fit(X_train_scaled, t2_train, eta=best_eta, epochs=epochs,
                X_val=X_val_scaled, t_val=t2_val, tol=best_tol,␣
 ↪n_epochs_no_update = 100)
    print("Learning rate: {:7}  tol: {:4}  dim_hidden: {:3}  Accuracy: {}␣
 ↪no_epochs: {}".format(
        eta, best_tol, dim_hidden, accuracy(cl_tune.predict(X_val_scaled),␣
 ↪t2_val), cl_tune.number_of_epochs))
```

```
Learning rate:   1e-07  tol: 0.05  dim_hidden:   6  Accuracy: 0.758 no_epochs:
20558
Learning rate:   1e-07  tol: 0.05  dim_hidden:  12  Accuracy: 0.762 no_epochs:
17026
Learning rate:   1e-07  tol: 0.05  dim_hidden:  18  Accuracy: 0.768 no_epochs:
22175
Learning rate:   1e-07  tol: 0.05  dim_hidden:  24  Accuracy: 0.576 no_epochs:
122
Learning rate:   1e-07  tol: 0.05  dim_hidden:  30  Accuracy: 0.778 no_epochs:
14221
Learning rate:   1e-07  tol: 0.05  dim_hidden:  36  Accuracy: 0.764 no_epochs:
10601
Learning rate:   1e-07  tol: 0.05  dim_hidden:  42  Accuracy: 0.76 no_epochs:
9211
Learning rate:   1e-07  tol: 0.05  dim_hidden:  48  Accuracy: 0.76 no_epochs:
10261
Learning rate:   1e-07  tol: 0.05  dim_hidden:  54  Accuracy: 0.774 no_epochs:
9549
Learning rate:   1e-07  tol: 0.05  dim_hidden:  60  Accuracy: 0.762 no_epochs:
8639
Learning rate:   1e-07  tol: 0.05  dim_hidden:  66  Accuracy: 0.76 no_epochs:
8882
Learning rate:   1e-07  tol: 0.05  dim_hidden:  72  Accuracy: 0.758 no_epochs:
8120
Learning rate:   1e-07  tol: 0.05  dim_hidden:  78  Accuracy: 0.768 no_epochs:
6858
```

```
Learning rate:   1e-07  tol: 0.05  dim_hidden:  84  Accuracy: 0.754 no_epochs:
5559
Learning rate:   1e-07  tol: 0.05  dim_hidden:  90  Accuracy: 0.756 no_epochs:
5330
Learning rate:   1e-07  tol: 0.05  dim_hidden:  96  Accuracy: 0.758 no_epochs:
5433
```

2.2.4 Results from tuning with scaling

It seems that with tolerance, it seems to favour values under 0.5, and I therefore chose tol = 0.05. For the learning rate, it seems that eta = 0.00001 is generally the best. For dim_hidden the accuracy seems to again generally max out around dim_hidden = 36.

The scaling ends up being worse with this classifier.

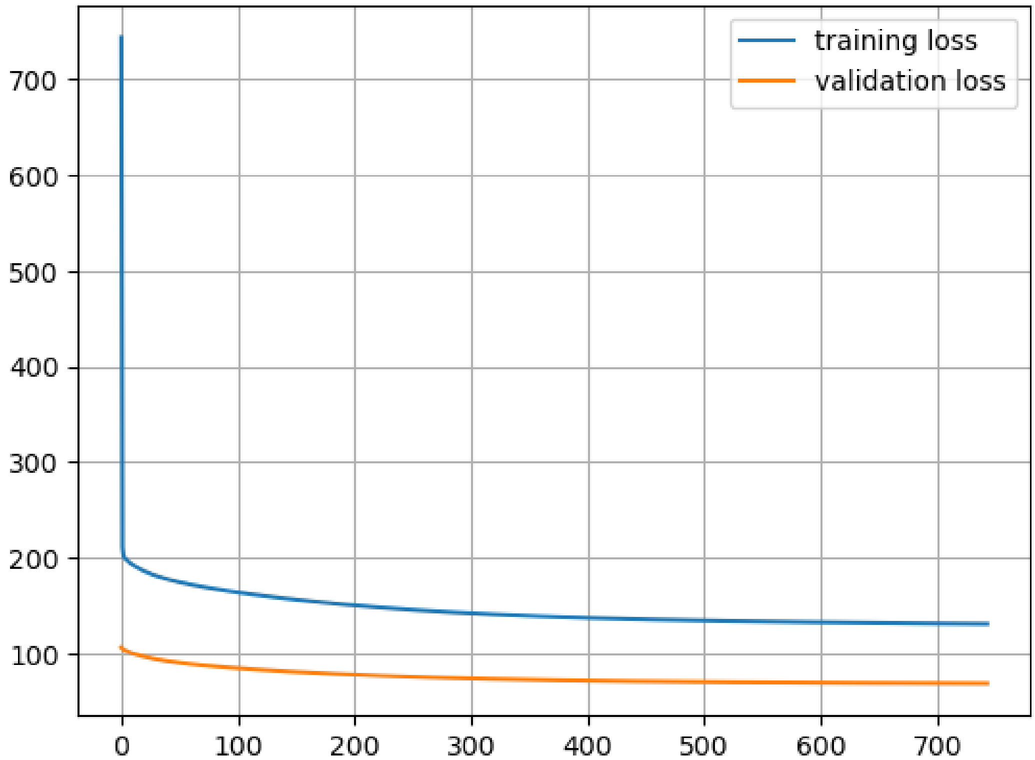
2.2.5 Training with tuned hyperparameters for unscaled sets, and plotting loss and accuracy

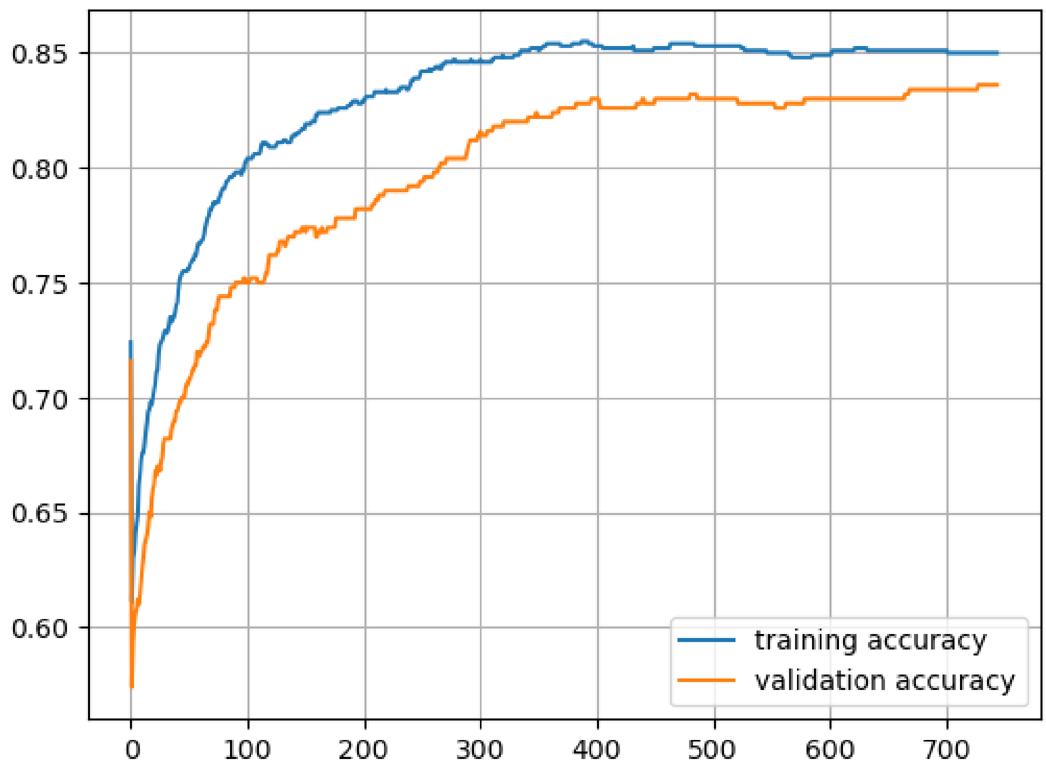
[50]:
```python
epochs = 100000
eta = 0.0001
tol = 0.5
dim_hidden = 36

cl_mlp = MLPBinaryLinRegClass(dim_hidden=dim_hidden)
cl_mlp.fit(X_train, t2_train, eta=eta, epochs=epochs,
                X_val=X_val, t_val=t2_val, tol=tol, n_epochs_no_update = 100)

print(accuracy(cl_mlp.predict(X_train), t2_train))
```

```
0.85
```

[51]:
```python
loss_train_list = cl_mlp.loss_train
loss_val_list = cl_mlp.loss_val

#print(loss_train_list)

accuracies_train_list = cl_mlp.accuracies_train
accuracies_val_list = cl_mlp.accuracies_val

epochs_axis = range(cl_mlp.number_of_epochs)

plt.plot(epochs_axis, loss_train_list, label="training loss")
plt.plot(epochs_axis, loss_val_list, label="validation loss")
plt.legend()
plt.grid()
plt.show()

plt.plot(epochs_axis, accuracies_train_list, label="training accuracy")
plt.plot(epochs_axis, accuracies_val_list, label="validation accuracy")
```

```
plt.legend()
plt.grid()
plt.show()
```

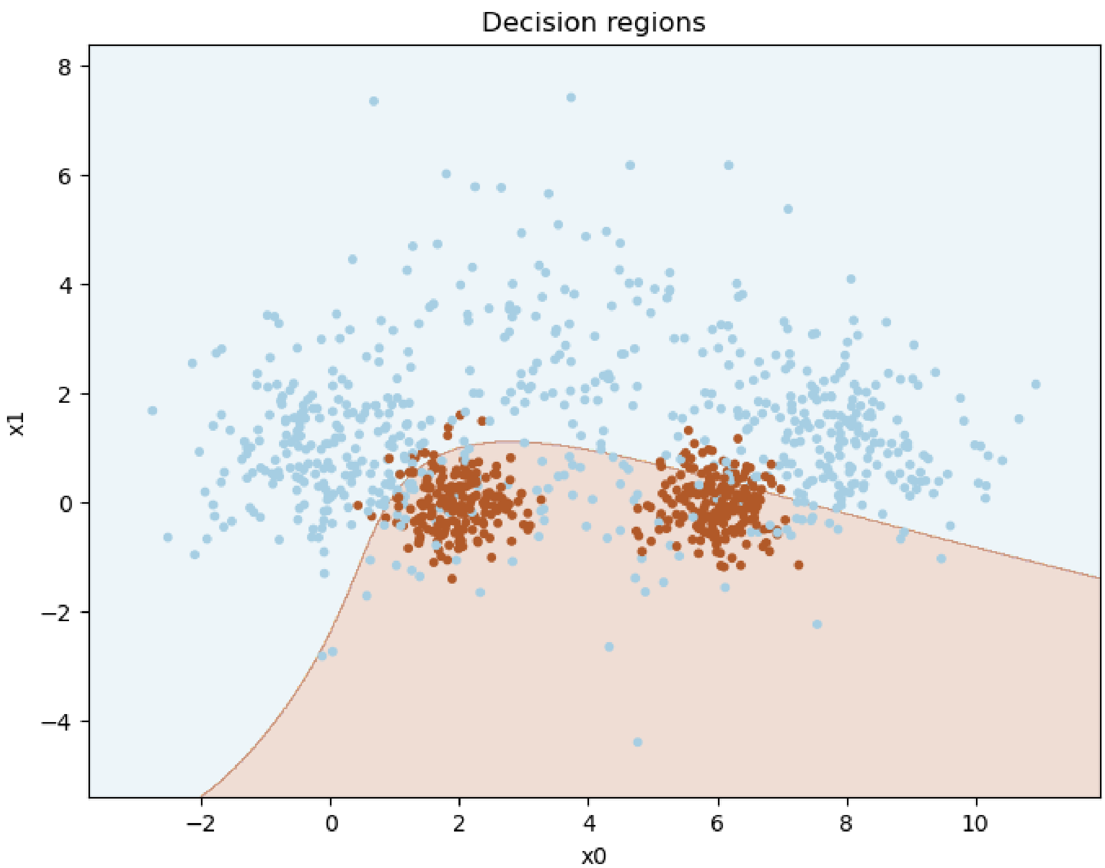




For the loss curves, it seems that the validation loss is always half the amount of the training loss.

For the accuracy curves, it seems that the curves aree quite similar.

[52]:
```
plot_decision_regions(X_train, t2_train, cl_mlp)
```



2.2.6 Training 10 times with optimal hyperparameters and reporting accuracy

[53]:
```
epochs = 100000
eta = 0.0001
tol = 0.5
dim_hidden = 36

list_of_accuracies = []

for i in range(0, 10):
    cl_mlp = MLPBinaryLinRegClass(dim_hidden=dim_hidden)
    cl_mlp.fit(X_train, t2_train, eta=eta, epochs=epochs,
               X_val=X_val, t_val=t2_val, tol=tol, n_epochs_no_update = 100)

    list_of_accuracies.append(accuracy(cl_mlp.predict(X_val), t2_val))

mean_accuracy = sum(list_of_accuracies) / len(list_of_accuracies)
max_accuracy = max(list_of_accuracies)
```

```
print(f"Mean accuracy: {mean_accuracy} --- Max accuracy: {max_accuracy}")
```

```
Mean accuracy: 0.8405999999999999 --- Max accuracy: 0.852
```

2.3 For IN4050-students: Multi-class neural network

The following part is only mandatory for in4050-students. In3050-students are also welcome to make it a try. (This is the most fun part of the set :))

The goal is to use a feed-forward network for non-linear multi-class classfication and apply it to set (X, t_multi).

Modify the netork to become a multi-class classifier. As a check of your implementation, you may apply it to (X, t_2) and see whether you get similar results as above.

Train the resulting classifier on (X_train, t_multi_train), test it on (X_val, t_multi_val), tune the hyper-parameters and report the accuracy.

Plot the decision boundaries for your best classifier..

3 Part III: Final testing

We can now perform a final testing on the held-out test set.

3.1 Binary task (X, t2)

Consider the linear regression classifier, the logistic regression classifier and the multi-layer network with the best settings you found. Train each of them on the training set and calculate accuracy on the held-out test set, but also on the validation set and the training set. Report in a 3 by 3 table.

Comment on what you see. How do the three different algorithms compare? Also, compare the results between the different data sets. In cases like these, one might expect slightly inferior results on the held-out test data compared to the validation data. Is that the case here?

Also report precision and recall for class 1.

3.2 For IN4050-students: Multi-class task (X, t_multi)

The following part is only mandatory for in4050-students. In3050-students are also welcome to do it. It requires the earlier IN4050-only tasks.

Compare the three multi-class classfiers, the one-vs-rest and the multinomial logistic regression from part one, and the multi-class neural network from part two. Evaluate on test, validation and training set as above.

Comment on what you see

3.2.1 Binary task:

For this final testing, I have decided to train and test only with unscaled sets. The optimal hyperparameters for all the binary classifiers given unscaled sets are as following:

Linreg eta = 0.01, epochs = 10 000

Logreg eta = 0.1, tol = 5e-06, n_epochs_no_updates = 10

MLP eta = 0.0001, tol = 0.5, n_epochs_no_update = 100, dim_hidden = 36

[54]:
```
# Training linear regression classifier
eta_linreg = 0.01
epochs_linreg = 10000
cl_linreg_final = NumpyLinRegClass()
cl_linreg_final.fit(X_train, t2_train, eta=eta_linreg, epochs=epochs_linreg)

linreg_train_accuracy = accuracy(cl_linreg_final.predict(X_train), t2_train)
linreg_val_accuracy = accuracy(cl_linreg_final.predict(X_val), t2_val)
linreg_test_accuracy = accuracy(cl_linreg_final.predict(X_test), t2_test)

# Training logistic regression classifier
eta_logreg = 0.1
tol_logreg = 0.000005
cl_logreg_final = NumpyLogReg()
cl_logreg_final.fit(X_train, t2_train, eta=eta_logreg, epochs=100000,
                    X_val=X_val, t_val=t2_val, tol=tol_logreg,␣
 ↪n_epochs_no_update=10)

logreg_train_accuracy = accuracy(cl_logreg_final.predict(X_train), t2_train)
logreg_val_accuracy = accuracy(cl_logreg_final.predict(X_val), t2_val)
logreg_test_accuracy = accuracy(cl_logreg_final.predict(X_test), t2_test)

# Training multi-layer perceptron classifier
dim_hidden = 36
eta_mlp = 0.0001
tol_mlp = 0.5
cl_mlp_final = MLPBinaryLinRegClass(dim_hidden=dim_hidden)
cl_mlp_final.fit(X_train, t2_train, eta=eta_mlp, epochs=100000,
                X_val=X_val, t_val=t2_val, tol=tol_mlp, n_epochs_no_update =␣
 ↪100)

mlp_train_accuracy = accuracy(cl_mlp_final.predict(X_train), t2_train)
mlp_val_accuracy = accuracy(cl_mlp_final.predict(X_val), t2_val)
mlp_test_accuracy = accuracy(cl_mlp_final.predict(X_test), t2_test)

# Reporting in a three by three table
three_by_three = "Classifier        train        val        test\n"
three_by_three += f"Linreg:           {linreg_train_accuracy}       ␣
 ↪{linreg_val_accuracy}        {linreg_test_accuracy}\n"
three_by_three += f"Logreg:           {logreg_train_accuracy}       ␣
 ↪{logreg_val_accuracy}        {logreg_test_accuracy}\n"
three_by_three += f"MLP    :          {mlp_train_accuracy}       ␣
 ↪{mlp_val_accuracy}        {mlp_test_accuracy}\n"
```

```
print(three_by_three)
```

```
Classifier        train       val        test
Linreg:           0.753       0.704        0.724
Logreg:           0.76       0.71        0.732
MLP    :          0.852       0.836        0.844
```

3.2.2 Regarding the three by three table

It seems that for the linear regression classifier and logistic regrestion classifier, they are almost equally as good. With scaling they both would've been better, but then logistic regression would've dominated a little.

MLP is the best in all the three different sets.

Another observation is that for all the three sets, the accuracy is the highest for the training set. This does make sense though, since the classifiers are trained using that set. In contrast to the question about the validation set vs test set, it seems that the test set wins.

3.2.3 Calculating precision and recall for class 1

[55]:
```
def precision(y, t):
    len_t = t.shape[0]
    true_results = []
    false_results = []
    for i in range(len_t):
        if y[i] == t[i]:
            true_results.append(y[i])
        elif y[i] != t[i]:
            false_results.append(y[i])
    tp, fp = 0, 0
    for i in true_results:
        if i == 1:
            tp += 1
    for i in false_results:
        if i == 1:
            fp += 1
    precision = tp / (tp + fp)
    return precision

def recall(y, t):
    len_t = t.shape[0]
    true_results = []
    false_results = []
    for i in range(len_t):
        if y[i] == t[i]:
            true_results.append(y[i])
        elif y[i] != t[i]:
```

```
            false_results.append(y[i])
    tp, fn = 0, 0
    for i in true_results:
        if i == 1:
            tp += 1
    for i in false_results:
        if i == 0:
            fn += 1
    recall = tp / (tp + fn)
    return recall
```

[56]:
```
def print_precisions_and_recalls(classifier, name):
    train_precision = precision(classifier.predict(X_train), t2_train)
    val_precision = precision(classifier.predict(X_val), t2_val)
    test_precision = precision(classifier.predict(X_test), t2_test)

    train_recall = recall(classifier.predict(X_train), t2_train)
    val_recall = recall(classifier.predict(X_val), t2_val)
    test_recall = recall(classifier.predict(X_test), t2_test)

    print(f"\nClassifier: {name}\n")
    print("            train                     val                     ␣
 ↪test")
    print(f"precision:  {train_precision}        {val_precision}       ␣
 ↪{test_precision}")
    print(f"recall   :  {train_recall}        {val_recall}       ␣
 ↪{test_recall}\n")
    print("----------------------------------------------------")

print_precisions_and_recalls(cl_linreg_final, "linreg")
print_precisions_and_recalls(cl_logreg_final, "logreg")
print_precisions_and_recalls(cl_mlp_final, "mlp")
```

```
Classifier: linreg

            train                       val                       test
precision:  0.6893732970027248        0.6818181818181818
0.6839080459770115
recall   :  0.655440414507772        0.5660377358490566
0.5891089108910891

----------------------------------------------------

Classifier: logreg

            train                       val                       test
```

```
precision:  0.6931216931216931        0.6871508379888268
0.6888888888888889
recall   :  0.6787564766839378        0.5801886792452831
0.6138613861386139

----------------------------------------------------

Classifier: mlp

            train                      val                       test
precision:  0.7793427230046949        0.7954545454545454
0.7952380952380952
recall   :  0.8601036269430051        0.8254716981132075
0.8267326732673267

----------------------------------------------------
```