Probability 2

PHI 152 Scientific Reasoning (Eliot)

Combining probabilities

To combine probabilities we need to remember two concepts that constrain when we can: **independence** and **mutual exclusivity**:

Independence (again)

independence (events)

Two events are independent when the occurrence of one does not influence the probability of the occurrence of the other. (41)

independence (propositions)

Two propositions are independent when the truth of one does not make the truth of the other any more or less probable.

example?

Independent:

- it will rain today
- I will get a good grade on my test

Not independent:

- I caught my train this morning on my way to work.
- I was late for work.

(assuming a scenario)

Mutual exclusivity

mutual exclusivity (events)

Two events which cannot occur at the same time are mutually exclusive (or “disjoint”).

mutual exclusivity (propositions)

Two propositions are mutually exclusive if they cannot be true at the same time.

example?

Mutually exclusive: getting a good grade on this test, getting a bad grade on this test
Not mutually exclusive: I studied for my test, I got a good grade on the test
Not mutually exclusive: I got a good grade on the first test question, and I got a bad grade on the second test question.

Adding probabilities

Pr(rolling a 1 OR rolling a 5) = 1/6 + 1/6 = 1/3

$Pr(A \vee B) = Pr(A) + Pr(B)$ if A and B are mutually exclusive.

example: What is the probability of rolling a 1 or a 5 on a six-sided die?

This gets us into trouble in cases like this, where events aren't mutually exclusive:

Pr(rolling a 1 OR rolling odd OR rolling even) = 1/6 + 1/2 + 1/2 = 1 1/6 (greater than 1!) X

Adding probabilities enhanced!

So, what if events A and B are not mutually exclusive?

General addition rule for probabilities

$$Pr(A \vee B) = Pr(A) + Pr(B) - Pr(A \& B)$$

Probability of rolling 1 or odd:

Pr(rolling 1) + Pf(rolling odd) – Pr(rolling 1 and odd)

= 1/6 + 1/2 – 1/6
= 1/2

Multiplying probabilities

remember:

OR * ANY * ADD (beware of not mutually exclusive)
AND * ALL * MULTIPLY (beware of their not being independent)

$Pr(A\&B) = Pr(A) \times Pr(B)$ if A and B are independent.

example: What is the probability of rolling a 5 and then another 5 on a six-sided die? = (1/6 x 1/6) = 1/36

(By the way, the general multiplication rule will need to wait.)

Question

When two cards are drawn in succession from a standard pack of cards,[1] what are the probabilities of drawing:

a. two hearts in a row, with replacement, $1/4 \times 1/4 = 1/16$
b. two hearts in a row, without replacement, $1/4 \times 12/51$
c. two cards, neither of which is a heart, with replacement, $3/4 \times 3/4$
d. and two cards, neither of which is a heart, without replacement?

$3/4 \times 38/51$

[1] 52 cards, divided into four equal suits (hearts, diamonds, clubs, and spades), of 13 cards each; hearts and diamonds are red suits, while clubs and spades are black suits; the cards in each suit are numbered 1 to 10 plus a jack, queen, and king.

Two ways of expressing probabilities

categorical probability

A probability is **categorical** when it is expressed as not depending on anything else.

conditional probability

A probability is **conditional** when it is expressed as depending something else (or some other things).

example 1: “There’s a 30% chance of snow on Friday.”

example 2: “There’s a 60% chance of snow if the warm front passes us on Friday.”

Calculating conditional probabilities

What if we want to combine the probability of an event with the probability of an event on which it is conditional?

example: There's a 60% chance of snow if the warm front passes us on Friday, and the front is 50% likely to reach us by then.

We normally express "the probability of A conditional on B (or 'given' B)" as Pr(A|B).[2]

conditional probability

$Pr(A|B) = Pr(A\&B)/Pr(B)$ so long as $Pr(B) > 0$

[2] Hacking writes "Pr(A/B)." That strikes me as unnecessarily confusing, because "/" suggests division. I prefer the more standard "|" (the "vertical bar" or "pipe").

Having defined conditional probability, we can now express the rule for combining probabilities with multiplication whether or not they're independent or correlated:

General multiplication rule for probabilities

$$Pr(A\&B) = Pr(A|B) \times Pr(B)$$

A simple conditional probability example

A fair, six-sided die is labeled on each side with one of the numbers one through six.

What's the probability that the outcome of a roll is 6, given that the outcome is even?

You can figure it out easily, but let's also calculate it with $Pr(A|B) = Pr(A\&B)/Pr(B)$.

That says that the probability of 6 given even is equal to the probability of both six and even divided by the probability of even. So:

$Pr(six \mid even) = \frac{1}{6}/\frac{1}{2} = \frac{1}{3}$

A slightly richer problem

Suppose Dell laptops use hard drives from two different manufacturers, Seagate and Toshiba, with 40% of their drives coming from Seagate and 60% from Toshiba. Now suppose these drives aren't perfect, such that 3 percent of Seagate laptops have bad sectors, while 6 percent of Toshiba laptops have bad sectors.

1. What is the probability that a random Dell laptop has a Seagate drive and bad sectors?
2. What is the probability that a random Dell laptop has bad sectors?
3. What is the probability that a Dell laptop which has bad sectors has a Seagate drive?

Two perspectives on probability

belief-type probability

Probabilities are about the degree to which someone *believes* or *should believe*.

related concepts: confidence, credibility, evidence

frequency-type probability

Probabilities are about how often things happen, sometimes as a function of how they are.

related concepts: tendency, propensity, disposition, symmetry

(Alternatively: subjective/objective; epistemic/aleatory; $probability_1$ and $probability_2$)

Personal probability

This is the first of five theories of probability or interpretations of probability we'll consider.

Consider: "I am very confident that the dinosaurs were extinguished after a giant asteroid hit the Earth."

This expresses *the speaker's own personal confidence or degree of belief*.

Consider: “One can be very confident that the dinosaurs were extinguished after a giant asteroid hit Earth.”

This expresses what is *endorsed as reasonable or rational to believe.*

Consider: "Relative to the available evidence, it is reasonable to have a high degree of belief in the dinosaur/asteroid hypothesis."

This expresses what *hypothesis is logical to accept given a certain body of evidence* and/or a set of possibilities for how things could be.

Relative frequency

Consider: "In the long run, the number of heads in coin flips approaches $\frac{1}{2}$."

In an infinite sequence of trials, the relative frequency of outcome S converges on limit L.

This expresses an *idealization of actual results of trials on a chance setup*, perhaps supported by working back and forth between actual data and an idealized model.

Propensity

Consider: “Because this coin-flipping machine is fair, it tends to produce heads half the time.”

This expresses the *propensity or tendency or disposition of a chance setup based on physical properties* known or conjectured to describe the setup.

Talking in each perspective

How do we talk about the probability of drawing ball #434 from an urn of 1000 balls in terms of:

1. personal probability
2. interpersonal probability
3. logical probability
4. relative frequency, and
5. propensity

Additional question

What might be harder to do while talking in terms of:

1. personal probability
2. interpersonal probability
3. logical probability
4. relative frequency, and
5. propensity?