

## What Material Conditionals Mean

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The material conditional (often symbolized as “ $\rightarrow$ ”) expresses the basic if-then relationship in logic. However, its useful meaning beyond “if” and “then” involves a little complexity and is worth spending time to understand. This handout presents that, especially including its relationship to the useful concept of necessary and sufficient conditions.

Logicians typically symbolize the basic if-then relationship “If  $P$ , then  $Q$ ” as  $P \rightarrow Q$ . Sometimes they use the symbol  $\supset$  instead of  $\rightarrow$ . In this formulation,  $P$  and  $Q$  are variables that stand in for sentences.

Here is an example. Let’s associate the variable  $B$  with “You’re in Brooklyn,” and  $N$  with “You’re in New York.” We get:

$B \rightarrow N$ : If you’re in Brooklyn, you’re in New York.

We’ll call  $P \rightarrow Q$  and  $B \rightarrow N$  “material conditionals.” They’re “conditionals” because they express that the left part (like  $P$ ) is a “sufficient condition” for the right part (like  $Q$ ). (More on this below!)

The material conditional is a minimal conditional. It does not capture all the meanings “if-then” can have in English. Accordingly, it is not the only kind of logical conditional. Other kinds of conditionals express *causation* (“ $P$  causes  $Q$ ”) or *temporal sequences* (“If  $P$  happens,  $Q$  will follow”). The material conditional does not include such connotations, though it doesn’t usually conflict with them.

The material conditional simply asserts a *truth-functional* relationship. It being “truth-functional” means that whether the conditional sentence is true is a *function* of just the truth or falsity of its component sentences. It is not committed to them being connected in any further way. In one sense, its meaning is completely captured by its truth table (Table 1).

To think about the meaning of this truth-table all at once:  $P \rightarrow Q$  means that<sup>1</sup> if  $P$  is true,<sup>2</sup>  $Q$  must be also,<sup>3</sup> while if  $P$  is false,<sup>4</sup>  $Q$  can go either way. The conditional is ‘noncommittal’ about  $Q$  in cases where  $P$  is false.  $P \rightarrow Q$  also *does not* assert that  $P$  is true (or that  $Q$  is). It asserts that *if*  $P$  is true,  $Q$  is also true.

Looking at the cases where it is true in Table 1 from top to bottom,  $B \rightarrow N$  tells you that if you’re in Brooklyn then you’re in New York, and if you’re not in Brooklyn, you may or may not be in New York. The one scenario this rules out is that you are in Brooklyn but not in New York. If the conditional is true, that scenario is not possible.<sup>5</sup>

Here’s an example of a false conditional. Let’s now associate  $M$  with “You’re in Minnesota”:

$B \rightarrow M$ : If you’re in Brooklyn, you’re in Minnesota.

If we know that the conditional is false, we know that it is possible to be in Brooklyn and not be in Minnesota (row 2 on Table 1). The conditional is false just when that case can occur.

We will call what’s to the left of the  $\rightarrow$  the “antecedent,” literally meaning “what comes before.” We will call what’s to the right of the  $\rightarrow$  the “consequent,” or literally “what follows.” In this handout, I will keep referring to them as  $P$  and  $Q$ , respectively.

$P$	$Q$	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 1: The truth table that defines the material conditional. For example, on the first row: where  $P$  is true and  $Q$  is true,  $P \rightarrow Q$  is true.

<sup>1</sup> Here I mean that the whole compound sentence “ $P \rightarrow Q$ ” means this. I won’t be careful about putting sentences in quotation marks, but I mean the whole thing.

<sup>2</sup> truth table rows 1 and 2

<sup>3</sup> row 1

<sup>4</sup> rows 3 and 4

<sup>5</sup> An occasional source of confusion is thinking of these sentence-definitions as having the kind of ambiguity English sentences have. What if you’re in Brooklyn, *Iowa*, you might ask? Then it would be possible to be in Brooklyn but not in New York. The answer is that when we translate sentences into symbolized form, we treat them as unambiguous. It’s crucial that we agree on exactly what they mean.

### Sufficient conditions

Another way of expressing the if-then inference the material conditional licenses is to say that the truth of its antecedent ( $P$ ) is *sufficient* for the truth of its consequent ( $Q$ ). Or, the truth of  $P$  offers sufficient grounds for inferring the truth of  $Q$ . Or, in epistemic<sup>6</sup> terms, the truth of  $P$  is sufficient evidence for concluding  $Q$ . If you know  $P$ , you can conclude  $Q$ . We say  $P$  is a “sufficient” condition for  $Q$  because in these ways  $P$  is *enough* for  $Q$ . You don’t need anything more. If you find that you’re in Brooklyn, that’s sufficient grounds for inferring that you’re in New York.

Besides “If  $P$  then  $Q$ ” some of the ways sufficient conditions are expressed in English are:  $P$  is grounds for  $Q$ ;  $P$  entails  $Q$ ; given  $P$ ,  $Q$ ; provided that  $P$ ,  $Q$ ;  $Q$  if  $P$ ;  $Q$ , given  $P$ ;  $Q$  follows from  $P$ ;  $Q$  is entailed by  $P$ .<sup>7</sup>

Note that  $P \rightarrow Q$  does *not* say that  $Q$  is sufficient for  $P$ . In  $B \rightarrow N$ , being in New York is not sufficient for being in Brooklyn. You might be in Poughkeepsie or Schenectady or Montauk. Knowing that  $Q$  is true leaves open whether  $P$  is true (row 1) or false (row 3). If you’re in New York, you might be in Brooklyn, but you also might not be. So again,  $P \rightarrow Q$  does not say that  $Q$  is sufficient for  $P$ .

### Necessary conditions

A “necessary condition for  $X$ ” is a condition that needs to be true for  $X$  to be true. Some examples: being a dog is a necessary condition for being a good dog; being human is a necessary condition for being a woman; being over 16 is a necessary condition for having a driver’s license in New York; understanding what a line is is a necessary condition for understanding what a triangle is.

Being a necessary condition is *not* the same as being a sufficient condition, and the two are often not true at the same time. Being a dog is a necessary condition for being a good dog, but it is not sufficient. Being a good dog is sufficient for being a dog, but it is not necessary.

Here is the point I have been leading up to: the material conditional expresses *both* sufficient and necessary conditions, but in *opposite directions*. As above,  $P \rightarrow Q$  says that  $P$  is sufficient for  $Q$ , and does not say that  $Q$  is sufficient for  $P$ . Equivalently, the antecedent is sufficient for the consequent, but the consequent is not sufficient for the antecedent. However, it also says that  $Q$  is necessary for  $P$ . That is, in addition to saying “If  $P$  then  $Q$ ” it says “ $P$  only if  $Q$ .” The consequent is necessary for the antecedent, though the antecedent is not sufficient for the consequent.

Why? Look again at the truth table. If  $P \rightarrow Q$  is true (rows 1, 3, and 4),  $P$  is true on those rows only when  $Q$  is (row 1). So  $Q$  needs to be true for  $P$  to be true, and  $Q$  is a necessary condition for  $P$ . That is,  $P$  only if  $Q$ .

Some of the ways we express that  $Q$  is a necessary condition for  $P$  in English are:  $P$  only if  $Q$ ;  $Q$  is necessary for  $P$ ;  $Q$  is essential for  $P$ ; you can’t

<sup>6</sup> i.e., having to do with knowledge and its justification

<sup>7</sup> Two related sentences *not* on this list are “Because  $P$ ,  $Q$ ” and “Since  $P$ ,  $Q$ .” I think those sentences assert, in addition to  $P \rightarrow Q$ , that  $P$  is true. Moreover, they say that  $P$  being true is the reason  $Q$  is true. The material conditional does not say that. It does not say there is any relationship between  $P$  and  $Q$  that goes beyond how their truth values are related. By the same token, it also doesn’t say that they have no such relationship!

*Here we arrive at the main common point of confusion. Helping some students get clear on this idea was my motivation for writing this.*

$P$	$Q$	$P \rightarrow Q$
T	<span style="border: 1px solid black; padding: 2px;">T</span>	T
T	F	F
F	T	T
F	F	T

Table 2: Where the conditional and the antecedent are true, the consequent necessarily is, too. It is a necessary condition for the antecedent.

have/have evidence for/be  $P$  without having/having evidence for/being  $Q$ .<sup>8</sup>

### *Necessary and sufficient conditions*

We're now in a position to put necessary and sufficient conditions together. Determining necessary and sufficient conditions for things is a very common task in philosophy, as well as law, science, and other disciplines. When we have both necessary and sufficient conditions for  $X$ , we have a definition for  $X$ . We know what  $X$  is equivalent to.

For example, the necessary and sufficient conditions for being beer are: being a malted beverage made from water, malt, hops, and yeast. Being a lager<sup>9</sup> is a sufficient but not necessary condition for being a beer; there are other kinds of beer like ale, stout, and porter, so a beer doesn't need to be a lager. Being a malted beverage is a necessary but not sufficient condition for being a beer. Wine coolers are also malted beverages, so while beer needs to be a malted beverage, being a malted beverage isn't enough to make something a beer.<sup>10</sup> When we know necessary and sufficient conditions for something, we know exactly what it is.

A philosophical example: Plato proposes that we have knowledge exactly whenever we have justified true beliefs.<sup>11</sup> Another way of saying this is that we have knowledge if and only if we have justified true beliefs. Having a true belief is necessary but not on its own sufficient for having knowledge. The belief must also be justified. Knowing that Earth is round is sufficient for having knowledge, but not necessary (and accordingly no specific facts appear in the definition of knowledge). Plato would say that having a belief, the belief being true, and the belief being justified are jointly sufficient conditions for knowledge. They are also all required for knowledge, and so are each necessary conditions.<sup>12</sup>

To express necessary *and* sufficient conditions for some one thing in terms of material conditionals, we need two conditionals.  $P \rightarrow Q$  says that  $P$  is a sufficient condition for  $Q$ .  $Q \rightarrow P$  says that  $P$  is a necessary condition for  $Q$ . So  $P$  is necessary *and* sufficient for  $Q$  when  $(P \rightarrow Q)$  and  $(Q \rightarrow P)$ . Since it is so often useful to express necessary and sufficient conditions, symbolic logic adopts a new symbol to express them. It's the "biconditional" ( $\leftrightarrow$ ). The "bi-" prefix means "two," so "biconditional" effectively means "two conditionals."  $P \leftrightarrow Q$  means " $P$  if *and* only if  $Q$ ."

Phrases that express biconditionals in English include:  $P$  if and only if  $Q$ ;  $P$  just in case  $Q$ ;  $P$  exactly when  $Q$ ;  $P$  if but only if  $Q$ ;  $P$  is the definition of  $Q$ ;  $P$  is equivalent to  $Q$ .

So: In "If you're in Brooklyn, you're in New York," what's sufficient for what? What's necessary for what?

<sup>8</sup> This is the basis for the argument form called *modus tollens* or denying the consequent, which we will encounter shortly.

<sup>9</sup> a lighter-colored style of beer including Budweiser, Coors Light, and Miller Genuine Draft

<sup>10</sup> Other features of some beers, like added fruit or chocolate, are neither necessary nor sufficient for being beer.

<sup>11</sup> In Plato's *Theaetetus*, a dialogue which is all about knowledge, Socrates says at 210b: "So, it seems, the answer to the question 'What is knowledge?' will be 'Correct judgement accompanied by ... an account.'" The common interpretation of 'account' is 'justification.' The Hackett version of *Theaetetus* (1990) translated by M.J. Levett contains an intellectually exciting commentary by Myles Burnyeat.

<sup>12</sup> It's common for definitions to involve several jointly sufficient but individually necessary conditions, like this one does.

Other ways to write the biconditional are 'IFF' (short for 'if and only if') and the symbol " $\equiv$ " which is often used in texts which use " $\supset$ " instead of " $\rightarrow$ " for the material conditional.

$P$	$Q$	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Table 3: The characteristic truth table for the material *biconditional*.