

FOL Natural Deduction Exercise 2

PHI 154 (Eliot) Fall 2022

Write natural deduction proofs for the following valid arguments using the rules of FOL. All the basic and derived natural deduction rules of TFL and FOL (except identity, which we didn't cover) are available to you. See *forall x* Appendix C 3.4–3.7, pages 406–411 for a summary of all the natural deduction rules.

A few notes:

- Proofs 1–3 look very similar at first glance, but number 2 and number 3 require you to do more, in different ways. The idea is that you learn something from how they vary.
- Proof 7 is the Aristotelian argument called “Darapti.” (It’s also on *fx* page 306.)

$$\begin{array}{l} 1. \quad \forall y M(y) \rightarrow \forall y O(y) \\ \quad \neg \forall y O(y) \\ \hline \quad \exists (y) \neg M(y) \end{array}$$

$$\begin{array}{l} 2. \quad \forall y M(y) \rightarrow \forall y O(y) \\ \quad \neg \forall y O(y) \\ \hline \quad \exists (z) \neg M(z) \end{array}$$

$$\begin{array}{l} 3. \quad \forall y M(y) \rightarrow \forall z O(z) \\ \quad \neg \forall y O(y) \\ \hline \quad \exists (y) \neg M(y) \end{array}$$

$$\begin{array}{l} 4. \quad \forall x \forall y (T(x, y) \rightarrow \neg T(y, x)) \\ \hline \quad \forall x \neg T(z, z) \end{array}$$

$$\begin{array}{l} 5. \quad \exists x C(x) \\ \quad \forall x (D(x) \leftrightarrow C(x)) \\ \hline \quad \exists x (C(x) \wedge D(x)) \end{array}$$

$$\begin{array}{l} 6. \quad \forall x (M(x) \rightarrow N(x)) \\ \quad \forall x (M(x) \vee \neg F(x)) \\ \quad \exists x [P(x) \wedge \neg N(x)] \\ \hline \quad \exists x [\neg F(x) \wedge Px] \end{array}$$

$$\begin{array}{l} 7. \quad \exists x G(x) \\ \quad \forall x (G(x) \rightarrow F(x)) \\ \quad \forall x (G(x) \rightarrow H(x)) \\ \hline \quad \exists x (H(x) \wedge F(x)) \end{array}$$

$$\begin{array}{l} 8. \quad \exists x \neg C(x) \rightarrow \neg \exists x \neg (Y(x) \wedge L(x)) \\ \quad \forall x (C(x) \rightarrow P(x)) \\ \quad \forall y (\neg P(y) \wedge \neg V(y)) \\ \hline \quad \neg \forall x \neg L(x) \end{array}$$

$$\begin{array}{l} 9. \quad \neg \exists x (H(x) \vee J(x)) \\ \quad \forall y (M(x) \vee H(x)) \\ \hline \quad M(e) \end{array}$$

$$\begin{array}{l} 10. \quad \exists x W(x) \\ \quad \forall y \neg W(y) \vee W(n) \\ \hline \quad W(n) \end{array}$$