

Extra ND Exercises up through \rightarrow I, \neg I, and \neg E
PHI 154 (Eliot)

Use our natural deduction proof system to prove the validity of each of the following arguments. Roughly, these progress from easier to slightly more difficult, mostly just in the sense that earlier ones do not require any of the subproof rules (\rightarrow I, \neg I, and \neg E), and then they start to require them.

1.
$$\frac{A \rightarrow (C \vee \neg B) \quad (R \wedge P) \wedge (R \rightarrow A)}{C \vee \neg B}$$
2.
$$\frac{N \rightarrow (\neg M \rightarrow \neg O) \quad (N \leftrightarrow \neg M) \wedge N}{\neg O}$$
3.
$$\frac{C \vee \neg A \quad (T \leftrightarrow \neg E) \wedge M \quad \underline{M \rightarrow \neg E}}{T \vee B}$$
4.
$$\frac{K \wedge J \quad M \rightarrow (\neg C \rightarrow E) \quad (J \vee A) \rightarrow M}{\neg C \rightarrow E}$$
5.
$$\frac{\neg H \leftrightarrow I \quad G \leftrightarrow \neg H \quad \underline{I \leftrightarrow \neg E}}{\neg E \rightarrow G}$$
6.
$$\frac{\underline{P \wedge R}}{C \rightarrow (R \wedge P)}$$
7.
$$\frac{M \leftrightarrow (D \wedge N)}{(N \wedge D) \rightarrow M}$$
8.
$$\frac{\underline{E}}{B \rightarrow (E \vee F)}$$
9.
$$\frac{\underline{H \wedge L}}{(B \rightarrow L) \wedge (A \rightarrow H)}$$
10.
$$\frac{\underline{K}}{K \rightarrow K}$$
11.
$$\frac{\underline{F}}{R \rightarrow (A \rightarrow F)}$$
12.
$$\frac{N \vee P \quad \underline{P \wedge (H \leftrightarrow P)}}{(C \vee B) \rightarrow H}$$
13.
$$\frac{\underline{\neg E \wedge E}}{A}$$
14.
$$\frac{\underline{\neg F \wedge F}}{B \wedge \neg B}$$
15.
$$\frac{F \wedge (\neg J \wedge K)}{\neg(K \wedge \neg F)}$$
16.
$$\frac{R \leftrightarrow \neg H \quad \underline{R \wedge M}}{H \rightarrow \neg P}$$
17.
$$\frac{Q \leftrightarrow R}{\neg(R \wedge \neg Q)}$$
18.
$$\frac{S \rightarrow \neg D \quad \neg(S \rightarrow E) \quad \underline{E \leftrightarrow \neg D}}{R}$$
19.
$$\frac{R \rightarrow M \quad P \wedge (\neg F \vee \neg G) \quad M \vee Q \quad \underline{\neg M}}{\neg R}$$
20.
$$\frac{\underline{P \wedge \neg R}}{\neg(P \rightarrow R)}$$
21.
$$\frac{\neg(L \vee M) \rightarrow O \quad \underline{\neg E \wedge \neg O}}{L \vee M}$$
22.
$$\frac{(B \vee A) \rightarrow D \quad \underline{D \rightarrow E}}{B \rightarrow \neg \neg E}$$
23.
$$\frac{\underline{B \wedge C}}{\neg(B \rightarrow \neg C)}$$
24. $\neg(P \wedge \neg P)$ (prove without premises)