

Natural Deduction Exercises with Derived Rules

PHI 154 (Eliot) Fall 2022, version 2022-11-14

On average (but not necessarily), the second half of these is more difficult than the second half. I have tried to include problems that encourage you to practice the derived rules. It will benefit you to do as many as you can until you feel that proving them feels fairly quick and natural.

The following are valid arguments in TFL or are theorems that can be proved without premises. Prove them using the natural deduction system for TFL.

1. $\neg N$

$$\frac{(\neg N \rightarrow L) \wedge [D \leftrightarrow (\neg N \vee A)]}{L \wedge D}$$
2. Derive $(A \wedge A) \leftrightarrow A$ without premises.
3. $M \rightarrow (A \rightarrow R)$
 $\neg A \rightarrow \neg M$

$$\frac{L \wedge M}{R}$$
4. $\neg(\neg J \vee K) \rightarrow \neg(L \wedge M)$
 $\neg(\neg J \vee K) \wedge M$

$$\neg(L \leftrightarrow M)$$
5. $R \rightarrow (\neg C \rightarrow D)$
 $\neg C \wedge R$

$$D \wedge R$$
6. $J \wedge K$

$$K \wedge (J \vee P)$$
7. $F \wedge \neg I$
 $H \vee I$

$$H \wedge F$$
8. $[M \vee (C \rightarrow T)] \wedge P$
 $(P \rightarrow \neg M) \wedge C$

$$T$$
9. $\neg(\neg M \vee \neg B) \wedge P$

$$\neg(\neg M \vee \neg B) \vee \neg P$$
10. E

$$(R \vee E) \wedge (E \vee \neg(N \vee M))$$
11. $(O \vee Q) \rightarrow R$
 $(F \wedge Q) \wedge C$

$$F \wedge R$$
12. $M \wedge (O \vee N)$

$$(O \vee N) \wedge (M \vee O)$$
13. $A \vee D$
 $\neg S \rightarrow C$
 $A \rightarrow \neg S$

$$\frac{\neg D}{C}$$
14. $\neg\neg O \wedge \neg\neg S$
 $\neg O \vee \neg T$

$$\neg T$$
15. $\neg R \rightarrow P$
 $(F \vee R) \wedge (O \wedge M)$
 $O \rightarrow \neg R$

$$M \wedge P$$
16. $(M \wedge L) \rightarrow (P \vee R)$
 $(L \wedge M) \wedge \neg R$

$$P$$
17. $L \rightarrow [M \rightarrow (\neg N \vee I)]$
 $(M \wedge \neg I) \wedge L$

$$\neg N$$
18. $[(B \vee M) \wedge R] \rightarrow T$
 $M \wedge R$

$$T$$
19. $K \wedge \neg L$
 $[\neg(\neg G \wedge \neg H) \vee R] \wedge F$
 $F \rightarrow \neg R$

$$Q \vee \neg(\neg G \wedge \neg H)$$
20. $(A \wedge \neg C) \wedge (E \wedge F)$

$$(A \wedge F) \wedge E$$
21. $C \wedge (\neg R \wedge S)$
 $M \wedge (\neg R \rightarrow P)$

$$P \wedge C$$
22. $C \rightarrow [(C \wedge D) \rightarrow (A \rightarrow B)]$
 $D \rightarrow C$
 $D \wedge B$

$$A \rightarrow B$$
23. $E \rightarrow (F \wedge \neg G)$
 $J \vee \neg H$
 $E \wedge F$

$$\neg G$$
24. $(L \wedge B) \wedge (Q \vee F)$
 $\neg Q \wedge [(F \wedge L) \rightarrow R]$

$$R$$
25. $\neg L \vee M$
 $M \rightarrow N$

$$L \rightarrow N$$

$$\begin{array}{l}
26. \ A \rightarrow B \\
\quad B \rightarrow \neg C \\
\quad \neg D \rightarrow E \\
\quad \frac{\neg C \rightarrow \neg D}{\neg E \rightarrow \neg A}
\end{array}$$

$$\begin{array}{l}
27. \ \neg(H \vee U) \\
\quad \neg(H \vee \neg C) \\
\quad \frac{\neg(C \wedge E)}{\neg(H \vee E)}
\end{array}$$

$$\begin{array}{l}
28. \ \frac{F \vee F}{F \wedge F}
\end{array}$$

$$\begin{array}{l}
29. \ \frac{O}{M \rightarrow (Q \rightarrow (L \rightarrow O))}
\end{array}$$

$$\begin{array}{l}
30. \ (P \rightarrow Q) \rightarrow (R \rightarrow S) \\
\quad \frac{R \wedge \neg S}{\neg Q}
\end{array}$$

$$\begin{array}{l}
31. \ R \rightarrow (U \wedge P) \\
\quad E \vee R \\
\quad E \rightarrow F \\
\quad \frac{F \leftrightarrow U}{U}
\end{array}$$

$$\begin{array}{l}
32. \ C \wedge \neg F \\
\quad (P \rightarrow H) \leftrightarrow I \\
\quad \frac{H \leftrightarrow \neg F}{I}
\end{array}$$

$$\begin{array}{l}
33. \ \frac{A \wedge (B \vee C)}{(A \wedge B) \vee (A \wedge C)}
\end{array}$$

$$\begin{array}{l}
34. \ (L \rightarrow \neg M) \rightarrow P \\
\quad \frac{Q \wedge \neg P}{M \wedge O}
\end{array}$$

$$\begin{array}{l}
35. \ \neg(Q \wedge C) \\
\quad D \rightarrow C \\
\quad \frac{\neg Q \rightarrow \neg D}{\neg D}
\end{array}$$

$$\begin{array}{l}
36. \ E \rightarrow (F \rightarrow G) \\
\quad E \\
\quad \frac{\neg F \rightarrow \neg E}{G}
\end{array}$$

$$\begin{array}{l}
37. \ \frac{\neg(M \vee N)}{\neg(M \leftrightarrow \neg N)}
\end{array}$$

$$\begin{array}{l}
38. \ \frac{(D \rightarrow E) \rightarrow D}{D}
\end{array}$$

$$\begin{array}{l}
39. \ \frac{B \rightarrow C}{\neg C \rightarrow \neg(B \wedge G)}
\end{array}$$

$$\begin{array}{l}
40. \ R \rightarrow S \\
\quad \frac{\neg R \rightarrow \neg S}{R \leftrightarrow S}
\end{array}$$

$$\begin{array}{l}
41. \ (Q \rightarrow S) \wedge \neg N \\
\quad (\neg Q \rightarrow O) \vee N \\
\quad \frac{(\neg S \rightarrow O) \rightarrow L}{L}
\end{array}$$

$$\begin{array}{l}
42. \ B \vee \neg C \\
\quad \frac{\neg B \vee \neg C}{\neg C}
\end{array}$$

$$\begin{array}{l}
43. \ \frac{H \vee (F \vee D)}{D \vee (F \vee H)}
\end{array}$$

$$\begin{array}{l}
44. \ \frac{(C \vee E) \vee F}{\neg C \rightarrow (\neg E \rightarrow F)}
\end{array}$$

$$\begin{array}{l}
45. \ \frac{J \wedge (\neg K \rightarrow L)}{(J \wedge K) \vee (J \wedge L)}
\end{array}$$

$$\begin{array}{l}
46. \ R \leftrightarrow S \\
\quad \frac{(S \vee T) \wedge \neg T}{R}
\end{array}$$

$$\begin{array}{l}
47. \ G \leftrightarrow \neg H \\
\quad \neg H \leftrightarrow I \\
\quad \frac{I \leftrightarrow \neg E}{\neg E \rightarrow G}
\end{array}$$

$$\begin{array}{l}
48. \ [M \vee (O \leftrightarrow T)] \wedge N \\
\quad \frac{N \leftrightarrow \neg M}{B \rightarrow (O \leftrightarrow T)}
\end{array}$$

$$\begin{array}{l}
49. \ \neg S \leftrightarrow C \\
\quad (F \vee S) \wedge (G \wedge M) \\
\quad \frac{G \rightarrow \neg S}{M \wedge C}
\end{array}$$

$$\begin{array}{l}
50. \ (A \wedge B) \leftrightarrow (P \vee D) \\
\quad \frac{(B \wedge A) \wedge \neg D}{P}
\end{array}$$