

**Natural Deduction Exercises with Derived Rules**  
PHI 154 (Eliot) Fall 2021, version 2020-11-28

On average (but not necessarily), the first half of these is more difficult than the second half. I have tried to include problems that encourage you to practice the derived rules. It will benefit you to do as many as you can until you feel that proving them feels fairly quick and natural.

The following are valid arguments in TFL or are theorems that can be proved without premises. Prove them using the natural deduction system for TFL.

1.  $\frac{\neg L \vee M}{\frac{M \rightarrow N}{L \rightarrow N}}$
2.  $\frac{A \rightarrow B}{\frac{B \rightarrow \neg C}{\frac{\neg D \rightarrow E}{\frac{\neg C \rightarrow \neg D}{\neg E \rightarrow \neg A}}}}$
3.  $\frac{\neg(H \vee U)}{\frac{\neg(H \vee \neg C)}{\frac{\neg(C \wedge E)}{\neg(H \vee E)}}}$
4.  $\frac{F \vee F}{F \wedge F}$
5.  $\frac{O}{M \rightarrow (Q \rightarrow (L \rightarrow O))}$
6.  $\frac{(P \rightarrow Q) \rightarrow (R \rightarrow S)}{\frac{R \wedge \neg S}{\neg Q}}$
7.  $\frac{R \rightarrow (U \wedge P)}{\frac{E \vee R}{\frac{E \rightarrow F}{\frac{F \leftrightarrow U}{U}}}}$
8.  $\frac{C \wedge \neg F}{\frac{(P \rightarrow H) \leftrightarrow I}{\frac{H \leftrightarrow \neg F}{I}}}$
9.  $\frac{A \wedge (B \vee C)}{(A \wedge B) \vee (A \wedge C)}$
10.  $\frac{(L \rightarrow \neg M) \rightarrow P}{\frac{O \wedge \neg P}{M \wedge O}}$
11.  $\frac{\neg(Q \wedge C)}{\frac{D \rightarrow C}{\frac{\neg Q \rightarrow \neg D}{\neg D}}}$
12.  $\frac{E \rightarrow (F \rightarrow G)}{\frac{E}{\frac{\neg F \rightarrow \neg E}{G}}}$
13.  $\frac{\neg(M \vee N)}{\neg(M \leftrightarrow \neg N)}$
14.  $\frac{(D \rightarrow E) \rightarrow D}{D}$
15.  $\frac{B \rightarrow C}{\neg C \rightarrow \neg(B \wedge G)}$
16.  $\frac{R \rightarrow S}{\frac{\neg R \rightarrow \neg S}{R \leftrightarrow S}}$
17.  $\frac{(Q \rightarrow S) \wedge \neg N}{\frac{(\neg Q \rightarrow O) \vee N}{\frac{(\neg S \rightarrow O) \rightarrow L}{L}}}$
18.  $\frac{B \vee \neg C}{\frac{\neg B \vee \neg C}{\neg C}}$
19.  $\frac{H \vee (F \vee D)}{D \vee (F \vee H)}$
20.  $\frac{(C \vee E) \vee F}{\neg C \rightarrow (\neg E \rightarrow F)}$
21.  $\frac{J \wedge (\neg K \rightarrow L)}{(J \wedge K) \vee (J \wedge L)}$
22.  $\frac{R \leftrightarrow S}{\frac{(S \vee T) \wedge \neg T}{R}}$
23.  $\frac{G \leftrightarrow \neg H}{\frac{\neg H \leftrightarrow I}{\frac{I \leftrightarrow \neg E}{\neg E \rightarrow G}}}$
24.  $\frac{[M \vee (O \leftrightarrow T)] \wedge N}{\frac{N \leftrightarrow \neg M}{B \rightarrow (O \leftrightarrow T)}}$

25.  $\neg S \leftrightarrow C$   
 $(F \vee S) \wedge (G \wedge M)$   
 $\frac{G \rightarrow \neg S}{M \wedge C}$
26.  $(A \wedge B) \leftrightarrow (P \vee D)$   
 $\frac{(B \wedge A) \wedge \neg D}{P}$
27.  $\neg N$   
 $\frac{(\neg N \rightarrow L) \wedge [D \leftrightarrow (\neg N \vee A)]}{L \wedge D}$
28. Derive  $(A \wedge A) \leftrightarrow A$  without premises.
29.  $M \rightarrow (A \rightarrow R)$   
 $\neg A \rightarrow \neg M$   
 $\frac{L \wedge M}{R}$
30.  $\neg(\neg J \vee K) \rightarrow \neg(L \wedge M)$   
 $\frac{\neg(\neg J \vee K) \wedge M}{\neg(L \leftrightarrow M)}$
31.  $R \rightarrow (\neg C \rightarrow D)$   
 $\frac{\neg C \wedge R}{D \wedge R}$
32.  $\frac{J \wedge K}{K \wedge (J \vee P)}$
33.  $F \wedge \neg I$   
 $\frac{H \vee I}{H \wedge F}$
34.  $[M \vee (C \rightarrow T)] \wedge P$   
 $\frac{(P \rightarrow \neg M) \wedge C}{T}$
35.  $\neg(\neg M \vee \neg B) \wedge P$   
 $\frac{\neg(\neg M \vee \neg B) \vee \neg P}{P}$
36.  $\frac{E}{(R \vee E) \wedge (E \vee \neg(N \vee M))}$
37.  $(O \vee Q) \rightarrow R$   
 $\frac{(F \wedge Q) \wedge C}{F \wedge R}$
38.  $M \wedge (O \vee N)$   
 $\frac{M \wedge (O \vee N)}{(O \vee N) \wedge (M \vee O)}$
39.  $A \vee D$   
 $\neg S \rightarrow C$   
 $A \rightarrow \neg S$   
 $\frac{\neg D}{C}$
40.  $\neg\neg O \wedge \neg\neg S$   
 $\frac{\neg O \vee \neg T}{\neg T}$
41.  $\neg R \rightarrow P$   
 $(F \vee R) \wedge (O \wedge M)$   
 $\frac{O \rightarrow \neg R}{M \wedge P}$
42.  $(M \wedge L) \rightarrow (P \vee R)$   
 $\frac{(L \wedge M) \wedge \neg R}{P}$
43.  $L \rightarrow [M \rightarrow (\neg N \vee I)]$   
 $\frac{(M \wedge \neg I) \wedge L}{\neg N}$
44.  $[(B \vee M) \wedge R] \rightarrow T$   
 $\frac{M \wedge R}{T}$
45.  $K \wedge \neg L$   
 $[\neg(\neg G \wedge \neg H) \vee R] \wedge F$   
 $\frac{F \rightarrow \neg R}{Q \vee \neg(\neg G \wedge \neg H)}$
46.  $\frac{(A \wedge \neg C) \wedge (E \wedge F)}{(A \wedge F) \wedge E}$
47.  $C \wedge (\neg R \wedge S)$   
 $\frac{M \wedge (\neg R \rightarrow P)}{P \wedge C}$
48.  $C \rightarrow [(C \wedge D) \rightarrow (A \rightarrow B)]$   
 $D \rightarrow C$   
 $\frac{D \wedge B}{A \rightarrow B}$
49.  $E \rightarrow (F \wedge \neg G)$   
 $J \vee \neg H$   
 $\frac{E \wedge F}{\neg G}$
50.  $(L \wedge B) \wedge (Q \vee F)$   
 $\frac{\neg Q \wedge [(F \wedge L) \rightarrow R]}{R}$