Extra ND Exercises up through \rightarrow I, \neg I, and \neg E PHI 154 (Eliot) REVISED

Use our natural deduction proof system to prove the validity of each of the following arguments. The first column does not require the negation rules ($\neg I$, and $\neg E$), and then some of the right column does start to require them.

$$\begin{array}{c} 1. \ A \rightarrow (C \vee \neg B) \\ \frac{(R \wedge P) \wedge (R \rightarrow A)}{C \vee \neg B} \end{array}$$

$$\begin{array}{ccc} 2. & N \rightarrow (\neg M \rightarrow \neg O) \\ & \underbrace{(N \leftrightarrow \neg M) \wedge N}_{\neg O} \end{array}$$

3.
$$\neg H \leftrightarrow I$$

$$G \leftrightarrow \neg H$$

$$\underline{I \leftrightarrow \neg E}$$
 $\neg E \rightarrow G$

4.
$$\frac{P \wedge R}{C \to (R \wedge P)}$$

5.
$$\frac{M \leftrightarrow (D \land N)}{(N \land D) \to M}$$

6.
$$(B \to C) \land (U \to D)$$

 $\frac{B \land (U \land F)}{D \land C}$

7.
$$\underline{H \wedge L}$$
 $(B \to L) \wedge (A \to H)$

8.
$$\frac{K}{K \to K}$$

9.
$$\frac{F}{R \to (A \to F)}$$

10.
$$N \lor P$$

$$\frac{P \land (H \leftrightarrow P)}{(C \lor B) \to H}$$

11.
$$\underline{E}$$
 $B \to (E \lor F)$

12.
$$C \lor \neg A$$

 $(T \leftrightarrow \neg E) \land M$
 $\underline{M \to \neg E}$
 $T \lor B$

13.
$$K \wedge J$$

 $M \rightarrow (\neg C \rightarrow E)$
 $\underbrace{(J \vee A) \rightarrow M}_{\neg C \rightarrow E}$

14.
$$\frac{\neg E \wedge E}{A}$$

15.
$$\frac{\neg F \wedge F}{B \wedge \neg B}$$

16.
$$\frac{F \wedge (\neg J \wedge K)}{\neg (K \wedge \neg F)}$$

17.
$$R \leftrightarrow \neg H$$

$$\frac{R \land M}{H \rightarrow \neg P}$$

18.
$$\frac{Q \leftrightarrow R}{\neg (R \land \neg Q)}$$

19.
$$S \to \neg D$$

 $\neg (S \to E)$
 $E \leftrightarrow \neg D$
 R

20.
$$R \to M$$

 $P \land (\neg F \lor \neg G)$
 $M \lor Q$
 $\frac{\neg M}{\neg R}$

21.
$$\frac{P \wedge \neg R}{\neg (P \to R)}$$

22.
$$\neg (L \lor M) \to O$$

$$\frac{\neg E \land \neg O}{L \lor M}$$

23.
$$(B \lor A) \to D$$

 $D \to E$
 $B \to \neg \neg E$

24.
$$\underline{B \wedge C}$$
 $\neg (B \rightarrow \neg C)$

25. $\neg (P \land \neg P)$ (prove without premises)