TFL Natural Deduction with Biconditionals Exercise
PHI 154 (Eliot) Fall 2020

For each argument, construct a proof using the natural deduction system described in Chapter 16. The premises are separated by commas, and the conclusion comes after the "therefore" symbol, which is "$\therefore$" (as introduced on page 2). The inference rules we have learned at this point are R, $\wedge$I, $\wedge$E, $\rightarrow$I, $\rightarrow$E, $\leftrightarrow$E, $\leftrightarrow$E.

1. $P \leftrightarrow N, (P \leftrightarrow U) \wedge N \therefore N \wedge P$
2. $V \rightarrow X, V \leftrightarrow E \therefore E \rightarrow X$
3. $L \wedge O, F \leftrightarrow (O \wedge L), F \leftrightarrow (T \wedge Z) \therefore T \wedge Z$
4. $C \leftrightarrow R, P \leftrightarrow T \therefore (C \wedge T) \rightarrow (R \wedge P)$
5. $\neg R \leftrightarrow H, \neg R \wedge \neg D, \neg D \leftrightarrow L \therefore L \wedge H$
6. $B \rightarrow P, (\neg M \wedge P) \rightarrow C, (B \rightarrow P) \leftrightarrow P, \neg M \therefore C$
7. $S, Y \therefore Y \leftrightarrow S$
8. $F \rightarrow Q, Q \rightarrow (G \wedge F) \therefore Q \leftrightarrow F$

Some additional, completely optional, relatively easy proofs

9. $J \leftrightarrow K, K \leftrightarrow L, L \leftrightarrow M, J \therefore M \wedge J$
10. $Y \vee Z, \neg E \leftrightarrow (Y \vee Z) \therefore B \rightarrow \neg E$
11. $C \leftrightarrow F, M \therefore F \leftrightarrow (M \wedge C)$
12. $\therefore P \leftrightarrow P$*

Some additional, completely optional, slightly more challenging proofs

13. $U \leftrightarrow (G \wedge \neg M), C \rightarrow \neg M, (C \wedge Y) \wedge G \therefore U$
14. $M \leftrightarrow (O \leftrightarrow I), Z \wedge I \therefore O \rightarrow M$
15. $\therefore [(P \rightarrow Q) \wedge (Q \rightarrow P)] \leftrightarrow (P \leftrightarrow Q)$*

* prove from no premises