Extra ND Exercises up through $\to$I, $\neg$I, and $\neg$E

PHI 154 (Eliot)

Use our natural deduction proof system to prove the validity of each of the following arguments. The first column does not require the negation rules ($\neg$I, and $\neg$E), and then some of the right column does start to require them.

1. $A \to (C \lor \neg B)$
   $\underline{(R \land P) \land (R \to A)}$
   $C \lor \neg B$

2. $N \to (\neg M \to \neg O)$
   $\underline{(N \leftrightarrow \neg M) \land N}$
   $\neg O$

3. $\neg H \leftrightarrow I$
   $G \leftrightarrow \neg H$
   $\underline{I \leftrightarrow \neg E}$
   $\neg E \to G$

4. $\underline{P \land R}$
   $C \to (R \land P)$

5. $\underline{M \leftrightarrow (D \land N)}$
   $(N \land D) \to M$

6. $\underline{E}$
   $B \to (E \lor F)$

7. $\underline{H \land L}$
   $(B \to L) \land (A \to H)$

8. $\underline{K}$
   $K \to K$

9. $\underline{F}$
   $R \to (A \to F)$

10. $N \lor P$
    $\underline{P \land (H \leftrightarrow P)}$
    $(C \lor B) \to H$

11. $\underline{E}$
    $B \to (E \lor F)$

12. $C \lor \neg A$
    $(T \leftrightarrow \neg E) \land M$
    $\underline{M \to \neg E}$
    $T \lor B$

13. $K \land J$
    $M \to (\neg C \to E)$
    $\underline{(J \lor A) \to M}$
    $\neg C \to E$

14. $\underline{\neg E \land E}$
    A

15. $\underline{\neg F \land F}$
    $B \land \neg B$

16. $\underline{F \land (\neg J \land K)}$
    $\neg(K \land \neg F)$

17. $R \leftrightarrow \neg H$
    $\underline{R \land M}$
    $H \to \neg P$

18. $\underline{Q \leftrightarrow R}$
    $\neg(R \land \neg Q)$

19. $S \to \neg D$
    $\neg(S \to E)$
    $\underline{E \leftrightarrow \neg D}$
    R

20. $R \to M$
    $P \land (\neg F \lor \neg G)$
    $M \lor Q$
    $\underline{\neg M}$
    $\neg R$

21. $\underline{P \land \neg R}$
    $\neg(P \to R)$

22. $\neg(L \lor M) \to O$
    $\underline{\neg E \land \neg O}$
    $L \lor M$

23. $(B \lor A) \to D$
    $\underline{D \to E}$
    $B \to \neg\neg E$

24. $\underline{B \land C}$
    $\neg(B \to \neg C)$

25. $\neg(P \land \neg P)$ (prove without premises)