$$x_k = [b, x_{offset}, \beta, c_0, c_1]^T$$

$$P_k$$

$$U^T U = (m+k)P_k, 其中(m+k) = 4, m为x_k的维度$$

Step1:产生 sigma points。

$$\chi_1 = x_k,\ \ w_1 = \frac{k}{m+k}$$

$$\chi_{i+1} = x_k + u_i, (i = 1,2,3,4,m), w_{i+1} = \frac{1}{2(m+k)}$$

$$\chi_{i+m+1} = x_k - u_i, (i = 1,2,3,4,5), w_{i+m+1} = \frac{1}{2(m+k)}$$

其中，$\mathrm{i} = 1,2, \dots, \mathrm{m}$

令

$$\mathrm{X} = [\chi_1 \quad \cdots \quad \chi_{i+1} \quad \cdots \quad \chi_{i+m+1}]$$

$$\mathrm{W} = \begin{bmatrix} w_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & w_{i+1} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & w_{i+m+1} \end{bmatrix}$$

Step2:UnscentTransform_f

Unscented transform:

$$\mathrm{F} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & v\Delta t & \frac{1}{2}(v\Delta t)^2 & \frac{1}{6}(v\Delta t)^3 \\ 0 & 0 & 1 & v\Delta t & \frac{1}{2}(v\Delta t)^2 \\ 0 & 0 & 0 & 1 & v\Delta t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

对 sigma points 做 unscented transform：

$$X' = \mathrm{FX}$$

$$\hat{x}_k^- = \sum_{i=1}^{2m+1} w_i X_i', \text{其中} X_i' \text{表示} X' \text{的第 i 列}$$

$$P_k^- = \sum_{i=1}^{2m+1} w_i (X_i' - \hat{x}_k^-)(X_i' - \hat{x}_k^-)^T + Q$$

Step3: UnscentTransform_h

Unscented transform:

$$\hat{z}_k = \mathrm{h}\left(X_i^{'}; y\right) = {}^{960}/_2 - {}^{b}/_2 + x_{offset} + \beta y + \frac{1}{2}c_0 y^2 + \frac{1}{6}c_1 y^3, y \in left\_pts$$

$$\hat{z}_k = \mathrm{h}\left(X_i^{'}; y\right) = {}^{960}/_2 + {}^{b}/_2 + x_{offset} + \beta y + \frac{1}{2}c_0 y^2 + \frac{1}{6}c_1 y^3, y \in right\_pts$$

假定当前时刻观测，n_l个车道线左边界点(z_i, y_i)，n_r个右边界点(z_i, y_i)

令$\hat{z}_i = \begin{bmatrix} \hat{z}_i^l \\ \hat{z}_i^r \end{bmatrix}$，表示第 i 个 sigma point($X_i'$)所表示的车道模型，$(n_l + n_r)$

个边界点 y 条件下的观测量的预测值。

$$\mathrm{Z}' = [\hat{z}_1 \quad \dots \quad \hat{z}_{i+1} \quad \dots \quad \hat{z}_{i+m+1}]$$

$$\hat{Z}_k^- = \sum_{i=1}^{2m+1} w_i Z_i', \text{其中} Z_i' \text{表示} Z' \text{的第 i 列}$$

$$Z_i' = Z_i' - \hat{Z}_k^-$$

$$P_z^- = \sum_{i=1}^{2m+1} w_i \left(Z_i' - \hat{Z}_k^-\right)\left(Z_i' - \hat{Z}_k^-\right)^T + R$$

Step4: Compute Kalman gain

$$P_{xz} = \sum_{i=1}^{2m+1} w_i (X_i' - \hat{x}_k^-)\left(Z_i' - \hat{Z}_k^-\right)^T$$

$$K_k = P_{xz} P_z^{-1}$$

Step5:Correct & update covariance matrix

$$x_{k+1} = \hat{x}_k^- + K_k\left(z - \hat{Z}_k^-\right)$$

$$P_{k+1} = P_k^- - K_k P_z {K_k}^T$$