Normalizing videos with different illuminations.

Here, we quickly discuss how to normalize behavior videos with different background illuminations, such that subject-tracking is illumination independent.

Let's assume that video frames are denoted by F_j. The mean frame F_b (video background) is calculated by

$$F_b = \frac{1}{N} \sum_j F_j \qquad , \tag{1}$$

where N denotes the number of frames in the video.

The difference frame $F_j - F_b$ is optimally zeros everywhere except for the pixels that show a moving object (the subject, whose behavior is tracked)[1].

We define the intensity of a difference frame as

$$I_j = \frac{1}{n} \sum_{x,y} \left(F_j - F_b\right) \qquad , \tag{2}$$

where n denotes the number of pixels in a frame.

We define the mean difference intensity $\bar{I}$ of a video as

$$\bar{I} = \frac{1}{N} \sum_j I_j \qquad . \tag{3}$$

The goal of the normalization should be, that $\bar{I}$ is constant across videos, because the subjects should look the same in all videos, even though the background might differ. Note, depending on the camera view and the posture of the subject in each frame (and thus different size), the single frame difference intensity I_j is not a good measure to look at, which is why we consider the mean I_j.

Assume we want to normalize a given video, such that $\bar{I}$ matches a given reference $\bar{I}_{ref}$ (e.g., the mean difference intensity $\bar{I}$ of the first video). We introduce the following scaling factor α, and then show that we can scale each frame individually by α to achieve the desired $\bar{I} \overset{!}{=} \bar{I}_{ref}$.

$$\alpha = \frac{\bar{I}_{ref}}{\bar{I}} \tag{4}$$

Now we consider the video F', in which each individual frame is scaled by α.

[1]Note, that the subject pixels in this difference formulation are negative if the subject is darker than the background.

$$\begin{aligned}
\bar{I}' &= \frac{1}{N}\sum_j I'_j && (5)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y}\left(F'_j - F'_b\right) && (6)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y}\left(F'_j - \frac{1}{N}\sum_i F'_i\right) && (7)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y}\left(\alpha\cdot F_j - \frac{1}{N}\sum_i \alpha\cdot F_i\right) && (8)\\
&= \alpha\cdot\frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y}\left(F_j - \frac{1}{N}\sum_i F_i\right) && (9)\\
&= \alpha\cdot\bar{I} = \frac{\bar{I}_{ref}}{\bar{I}}\cdot\bar{I} = \bar{I}_{ref} && (10)\\
& && (11)
\end{aligned}$$

Hence, by rescaling individual frames with α, we can normalize all videos such that their mean difference intensity matches the desired reference value.

1 Problems because of limited value range and the solution

The above scheme is not that easily transferable to positive valued video frames because $F_j - F_b$ may incorporate positive and negative pixel values. I.e., pixel values are typically in $[0, 255]$, meaning that the difference frame $F_j - F_b$ pixel values lay within an interval $[-255, 255]$ and thus have to be rescaled.

$$f(x) = \frac{x + 255}{2} \tag{12}$$

Hence, instead of working with the difference frame $F_j - F_b$ directly, we should work with $f\big(F_j - F_b\big)$.

$$
\begin{aligned}
\bar{I}' &= \frac{1}{N}\sum_j I'_j && (13)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} f\big(F'_j - F'_b\big) && (14)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} f\big(F'_j - \frac{1}{N}\sum_i F'_i\big) && (15)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} f\big(\alpha \cdot F_j - \frac{1}{N}\sum_i \alpha \cdot F_i\big) && (16)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} \frac{1}{2}\big(\alpha \cdot F_j - \frac{1}{N}\sum_i \alpha \cdot F_i + 255\big) && (17)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} \alpha\frac{1}{2}\big(F_j - \frac{1}{N}\sum_i F_i + \frac{255}{\alpha}\big) && (18)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} \alpha\frac{1}{2}\big(F_j - \frac{1}{N}\sum_i F_i + (255-255) + \frac{255}{\alpha}\big) && (19)\\
&= \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} \alpha\frac{1}{2}\big(F_j - \frac{1}{N}\sum_i F_i + 255\big) + \frac{1}{N}\sum_j \frac{1}{n}\sum_{x,y} \alpha\frac{1}{2}\big(\frac{255}{\alpha} - 255\big) && (20)\\
&= \alpha \cdot \bar{I} + \frac{1}{2}\frac{1}{N}\frac{1}{n}\sum_j\sum_{x,y}\big(255 - \alpha \cdot 255\big) && (21)\\
&= \alpha \cdot \bar{I} + C - \alpha \cdot C && (22)
\end{aligned}
$$

where C in above equation corresponds to

$$C = \frac{1}{2}\frac{1}{N}\frac{1}{n}\sum_j\sum_{x,y} 255 \tag{23}$$

The goal is then to find an α value that satisfies the following equation

$$
\begin{aligned}
\bar{I}_{ref} &= \alpha \cdot \bar{I} - \alpha \cdot C + C && (24)\\
\Leftrightarrow \alpha &= \frac{\bar{I}_{ref} - C}{\bar{I} - C} && (25)
\end{aligned}
$$