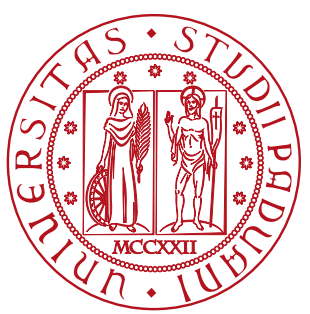
Efficient Low Diameter Clustering

with strong diameter in the CONGEST model

Christian Micheletti



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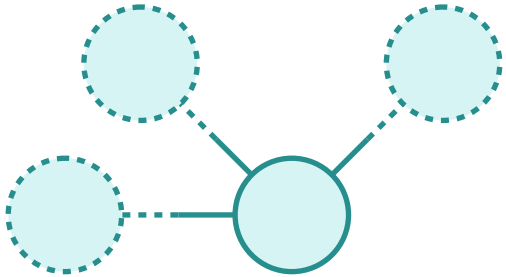
Some graph problems are interesting for **networks of computers**

Distribution ⇒ Parallelism

We'd like to leverage parallelism to relieve computation costs

A First Simple Model

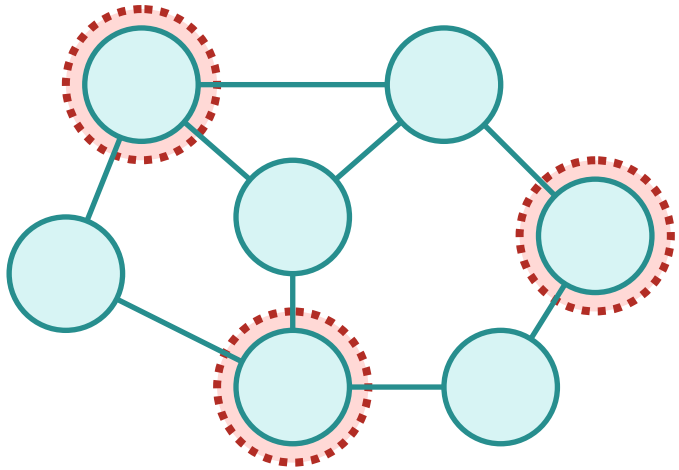


- In the **PN-Network** a node only knows some *Numbered **Ports***
  - Each connected with a **different** node
  - ***def:*** Those are called ***neighbours***
  - There are no **self loops**

A First Simple Model

From the perspective of a single node,
we don't see the whole topology

Example: **Maximal Independent Set** (MIS)



Solving it centralized is an easy greedy algorithm

Each node appears identical to any other

- The only difference *could be* in the number of ports
  - Not enough!
- **We must break this symmetry**

We add **unique identifiers** to the nodes

$$id : V \to \mathbb{N}$$

where $\forall v \in V : id(v) \leq n^c$ for some $c \geq 1$

We choose n^c so that we need $O(\log n)$ bits to represent an identifier, i.e. identifiers are reasonably ***small***

- This seems enough for the model
- We must now define:
  - What **"distributed"** algorithms consist of
  - And the criteria for **complexity** analysis

Distribution ⇒ Collaboration

- Collaborating requires **exchanging messages**
  - …on a medium that is **slow** and **unreliable**

⇒ Communication has the most impact on complexity

⇒ We are intersted in **quantifying** the number of messages that travel across the network

W.l.o.g.[1] we adopt a model of **synchronous communication**

Each round, a node $v \in V$ performs this actions:

1. v **sends** a message $msg \in \mathbb{N}$ to its neighbours;
2. v **receives** messages from its neighbours;
3. ...

[1] Without loss of generality.

3. ***v* executes locally** some algorithm (same for each node).

def: Any message exchange establishes a communication ***round***

🎶 Point (3.) doesn't affect the algorithm's complexity

In Wave, the node with **id(v) = 1** *"waves hello"*

When a node receives the message, forwards it to its neighbours

The running time of this algorithm on a graph G is $O(diam(G))$

Let's leverage **id** to select the first MIS node

- At round #*i*, node v : $\mathbf{id}(v) = i$ executes
  - If no neighbour is in the MIS, add the node
    - And inform the neighbours
  - Otherwise, the node is outside the MIS

A Second Example (Naive MIS)

1: **if** `round` = $id(v)$ **then**
2: $m \leftarrow$ `'want-to-select'`
3: SEND m
4: RECEIVE *messages*
5: **if** `'want-to-select'` $\in$ *messages* **then**
6: **stop** (`result: 'not-in-MIS'`)
7: **if** `round` = $id(v)$ **then**
8: **stop** (`result: 'in-MIS'`)

- It is correct since no node has the same **id**
- This algorithm runs in $O(n^c)$ (the maximum **id**)
  - **Very bad**

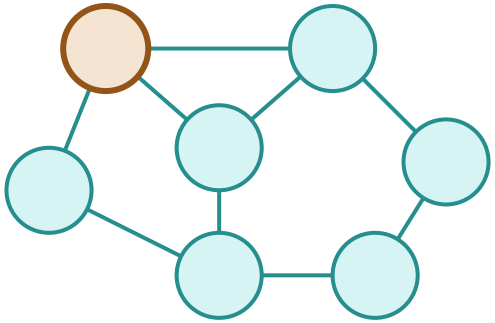
We can be way smarter than that

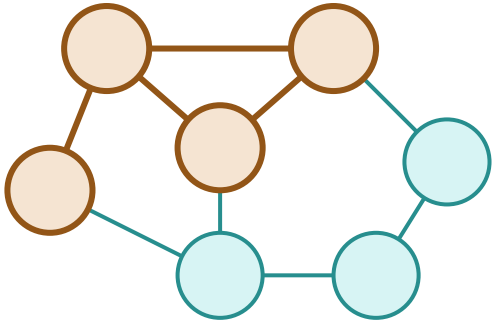
Running a centralized algorithm on a single node would take O(1) rounds

- We'd like to run a MIS algorithm on each node
  - **Centralized** ⇒ each node must have a **local copy** of the **entire** graph

- The algorithm Gather-All makes all nodes build a local copy of the whole graph
  - At round i, each node v knows $ball(i, v)$

$ball(0, v)$ $ball(1, v)$





- All nodes will know the whole graph after $O(diam(G))$ rounds

Graph can be really large

- In a real world setting, it is not always possible to send arbitrary large messages
- We'd like to lift this assumption
  - Messages need to be reasonably ***small***

- We provide an upper bound on messages size
  - Messages larger than will require more rounds to be fully sent

In the CONGEST model, messages can only be in the size of $O(\log n)$

Examples:

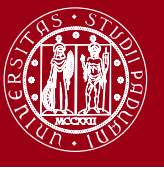
- Sending a single (or a constant amount of) identifier takes $O(1)$ rounds
- Sending a *set* of identifiers can take up to $O(n)$
- Sending the whole graph requires $O(n^2)$ rounds:
  - The adjacency matrix alone reaches that

⇒ We can't use Gather-All in the CONGEST model

- Censor-Hillel et al. provided an algorithm that solves MIS in $O(diam(G)\log^2 n)$ in CONGEST [1]

The diameter can be very large

Worst case: $diam(G) = n$



- A ***Network Decomposition*** groups nodes in **colored clusters**
  - Clusters with the same color are not adjacent
  - We say it to ***"have diameter"*** ***d*** if all of its clusters have diameter at most ***d***
  - It has ***c*** colors

Solving MIS in a color will give a correct partial solution

- We can iterate this action with [1] for all colors
  - (dropping neighbours of different colors)
- This algorithm has complexity $O(c \cdot d \log^2 n)$
  - If $c = O(\log n) = d$ then we would have a MIS algorithm in polylogarithmic time

- Each color induces a **low diameter clustering**

def: A ***low diameter clustering*** $\mathscr{C} \subseteq 2^V$ for a graph G with diameter d is such:

1. $\forall C_1 \neq C_2 \in \mathscr{C} : dist_G(C_1, C_2) \geq 2$
   - **"There are no adjacent clusters"**
2. $\forall C \in \mathscr{C} : diam(G[C]) \leq d$
   - **"Any cluster has diameter at most"** d

A clustering **can not be a partitioning**:
some nodes have to be left out

Main iteration:

1. Find a low diameter clustering
2. Assign a free color to its nodes
3. Repeat to discarded nodes until there are no more left

🎶 To get a $O(\log n)$-colors decomposition, each color has to cluster **at least half** of the uncolored nodes

- Our previous definition of diamter is also called ***strong*** diameter

def: We say a clustering has ***weak*** diameter when:

1. (unchanged) **"There are no adjacent clusters"**
2. "Any cluster has diameter **in** *G* at most" ***d***

- The main accomplishment of [2] is to provide a straightforward algorithm that:
  - Terminates in $O(\log^6 n)$ rounds in the CONGEST model
  - Outputs a clustering with $O(\log^3 n)$ colors
  - The clustering has strong diameter
- Previously [3] provided an algorithm for low diameter clustering with **weak** diameter

- $O(\log^7 n)$ rounds with $O(\log^3 n)$ colors
- It's possible to turn it into strong diameter

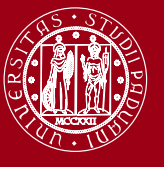
- [4] provided strong diameter in $O(\log^4 n)$ rounds with $O(\log^3 n)$ colors
  - Still has to pass by a weak diameter intermediate solution

Phases

- There are $b = \log(\max i) = O(\log n)$ **phases**
- "One phase for each bit in index"
  - Phase $i \in [0, b-1]$ computes "***terminals*** *set*" Q_i

Notation

- Q_i is the terminals set built *before* phase i
- Q_b is the terminals set built *after* phase $b-1$



Objective 1: Creating **connected components** with low diameters

- Eventually, each connected component will contain exactly one terminal
  - Keep terminals ***close*** to active nodes
    - This ensures polylogarithmic diameter
    - Removing nodes is allowed

Algorithm Outline (informal)

- Q_{i+1} is "*closer*" than Q_i to any node
  - Q_b is ***close*** to any node

Objective 2:

- The algorithm must cluster at least $\frac{n}{2}$ nodes
  - Each phase **removes** at most $\frac{n}{2b}$ nodes

Further notation:

- V_i is the set of **living nodes** at the beginning of phase i ($V_0 = V$)
- V' is the set of **living nodes** after the last phase;
  - $V' = V_b$

1. Q_i is R_i-ruling, i.e. $dist_G(Q_i, v) \leq R_i$ for all $v \in V$
   - We set $R_i = i * O(\log^2 n)$
   - Q_0 is 0-ruling, trivially true with $Q_0 = V$
   - Q_b is $O(\log^3 n)$-ruling

Each node has polylogarithmic distance from Q_b ⇒ each connected component has **at least one** terminal

2. let $q_1, q_2 \in Q_i$ s.t. they are in the same connected component in $G[V_i]$. Then $\mathrm{id}(q_1)[0..i] = \mathrm{id}(q_2)[0..i]$
   - for $i = 0$ it's trivially true
   - for $i = b$ there is ≤ 1 terminal in each c.c.

Along with invariant (1.), it means that each c.c. has polylogarithmic diameter!

3. $|V_i| \geq \left(1 - \frac{i}{2b}\right) |V|$
   - $V_0 \geq V$
   - $V' \geq \frac{1}{2} |V|$

"The algorithm doesn't discard too much vertexes from the graph"

Objective: Keeping only terminals from which is possible to build a **forest** whose **trees** have polylogarithmic diameter

- Leaves of the trees may be connected in G

Outline:

- $2b^2$ **steps**, each computing a forest
- resulting into a sequence of forests $F_0..F_{2b^2}$

Inductive definition:

- F_0 is a BFS forest with roots in Q_i
- let T be any tree in F_j and r its root
  - if $\mathsf{id}(r)[i] = 0$ the whole tree is `red`, if not `blue`
    - `red` vertexes stay `red`
    - some `blue` nodes stay `blue`
    - some others **propose** to join `red` trees

Proposal:

$v \in V_j^{propose} \Leftrightarrow v$ is `blue`

$\wedge\, v$ is the only one in $path(v, root(v))$

that neighbours a `red` node

Define T_v the (`blue`) subtree rooted at v

🎶 v is the only node in T_v that is also in $V_j^{propose}$

Proposal:

- Each node in $V_j^{propose}$ proposes to an arbitrary `red` neighbour
- Each `red` tree decides to grow or not
  - If it grows, it accepts all proposing trees
  - If not, all proposing subtrees are frozen
- **Criteria:** it decides to grow if it would gain at least $\frac{|V(T)|}{2b}$ nodes

🎶🎶 If the `red` tree doesn't decide to grow, it will neighbour `red` nodes only

- This means it will be able to delete nodes only once in the whole phase
  - ⇒ At most $\frac{|V|}{2b}$ nodes are lost in each phase
  - ⇒ After the b phases at most $\frac{|V|}{2}$ nodes are removed

High Level Pseudocode

1: $V_0 \leftarrow V$
2: $Q_0 \leftarrow V$
3: **for** $i \in 0..b-1$ **do**
4: INIT F_0
5: **for** $j \in 0..2b^2-1$ **do**
6: BUILD $V_j^{propose}$ } $O(diam(T_v))$
7: $F_{j+1} \leftarrow$ STEP } $2b^2 = O(\log^2 n)$ } $b = O(\log n)$
8: $V_{i+1} \leftarrow V(F_{2b^2})$
9: $Q_{i+1} \leftarrow roots(F_{2b^2})$

- Recall invariant (1.)
  - $\forall v \in V : dist_G(Q_i, v) = O(\log^3 n)$, for all $i \in 0..b$
  - Hence, $diam(T_v) = O(\log^3 n)$, for all $v \in V$
- Complexity is $\#steps \times \#phases \times O(diam(T_v))$
  - $= O(\log n) \times O(\log^2 n) \times O(\log^3 n)$

The algorithm runs in $O(\log^6 n)$ communication steps

Bibliography

[1] K. Censor-Hillel, M. Parter, and G. Schwartzman, "Derandomizing Local Distributed Algorithms under Bandwidth Restrictions." [Online]. Available: https://arxiv.org/abs/1608.01689

[2] V. Rozhoň, B. Haeupler, and C. Grunau, "A Simple Deterministic Distributed Low-

Diameter Clustering." [Online]. Available: https://arxiv.org/abs/2210.11784

[3] V. Rozhoň and M. Ghaffari, "Polylogarithmic-Time Deterministic Network Decomposition and Distributed Derandomization." [Online]. Available: https://arxiv.org/abs/1907.10937

[4] V. Rozhoň, M. Elkin, C. Grunau, and B. Haeupler, "Deterministic Low-Diameter Decompositions for Weighted Graphs and Distributed and Parallel Applications." [Online]. Available: https://arxiv.org/abs/2204.08254

This presentation is supposed to briefly showcase what you can do with this package.

For a full documentation, read the online book.

Let's explore what we have here.

On the top of this slide, you can see the slide title.

We used the `title` argument of the `#slide` function for that:

```
#slide(title: "First slide")[
    ...
]
```

(This works because we utilise the `clean` theme; more on that later.)

Titles are not mandatory, this slide doesn't have one.

But did you notice that the current section name is displayed above that top line?

We defined it using `#new-section-slide("Introduction")`.

This helps our audience with not getting lost after a microsleep.

You can also spot a short title above that.

Now, look down!

There we have some general info for the audience about what talk they are actually attending right now.

You can also see the slide number there.

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim aeque doleamus animo, cum corpore dolemus, fieri tamen permagna accessio potest, si aliquod aeternum et infinitum impendere malum nobis opinemur. Quod idem licet transferre in voluptatem, ut postea variari

voluptas distinguique possit, augeri amplificarique non possit. At etiam Athenis, ut e.

Sometimes we don’t want to display everything at once.

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That's what the `#pause` function is there for!

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That's what the `#pause` function is there for!

It makes everything after it appear at the next subslide.

(Also note that the slide number does not change while we are here.)

When `#pause` does not suffice, you can use more advanced commands to show or hide content.

These are some of your options: - `#uncover`

- `#only`
- `#alternatives`
- `#one-by-one`
- `#line-by-line`

Let’s explore them in more detail!

With `#uncover`, content still occupies space, even when it is not displayed.

For example, are only visible on the second"subslide".

In `()` behind `#uncover`, you specify *when* to show the content, and in `[]` you then say *what* to show:

`#uncover`: Reserving space

```
#uncover(3)[Only visible on the third "subslide"]
```

With `#uncover`, content still occupies space, even when it is not displayed.

For example, these words are only visible on the second"subslide".

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```

Only visible on the third“subslide”

So far, we only used single subslide indices to define when to show something.

We can also use arrays of numbers …

```
#uncover((1, 3, 4))[Visible on subslides 1, 3, and 4]
```

Visible on subslides 1, 3, and 4

…or a dictionary with `beginning` and/or `until` keys:


```
#uncover((beginning: 2, until: 4))[Visible on
subslides 2, 3, and 4]
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```

Visible on subslides 2, 3, and 4

As as short hand option, you can also specify rules as strings in a special syntax.

Comma separated, you can use rules of the form

1-3	from subslide 1 to 3 (inclusive)
-4	all the time until subslide 4 (inclusive)
2-	from subslide 2 onwards
3	only on subslide 3

Everything that works with `#uncover` also works with `#only`.

However, content is completely gone when it is not displayed.

For example, the rest of this sentence moves.

Again, you can use complex string rules, if you want.

`#only`: Reserving no space

```
#only("2-4, 6")[Visible on subslides 2, 3, 4, and 6]
```

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However, content is completely gone when it is not displayed.

For example, see how the rest of this sentence moves.

Again, you can use complex string rules, if you want.

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Visible on subslides 2, 3, 4, and 6

You might be tempted to try

```
#only(1)[Ann] #only(2)[Bob] #only(3)[Christopher]
likes #only(1)[chocolate] #only(2)[strawberry]
#only(3)[vanilla] ice cream.
```

Ann

likes chocolate

ice cream.

But it is hard to see what piece of text actually changes because everything moves around.
Better:

```
#alternatives[Ann][Bob][Christopher] likes
#alternatives[chocolate][strawberry][vanilla] ice
cream.
```

Ann likes chocolate ice cream.

You might be tempted to try

```
#only(1)[Ann] #only(2)[Bob] #only(3)[Christopher]
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Bob

likes strawberry

ice cream.

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#alternatives[Ann][Bob][Christopher] likes
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Bob likes strawberry ice cream.

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likes #only(1)[chocolate] #only(2)[strawberry]
#only(3)[vanilla] ice cream.
```

Christopher

likes vanilla

ice cream.

But it is hard to see what piece of text actually changes because everything moves around.
Better:

```
#alternatives[Ann][Bob][Christopher] likes
#alternatives[chocolate][strawberry][vanilla] ice
cream.
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Christopher likes vanilla ice cream.

`#alternatives` is to `#only` what `#one-by-one` is to `#uncover`.

`#one-by-one` behaves similar to using `#pause` but you can additionally state when uncovering should start.

```
#one-by-one(start: 2)[one ][by ][one]
```

`start` can also be omitted, then it starts with the first subside:

```
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one

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```

onebyone

`#line-by-line`: syntactic sugar for `#one-by-one`

Sometimes it is convenient to write the different contents to uncover one at a time in subsequent lines.

This comes in especially handy for bullet lists, enumerations, and term lists.

```
#line-by-line(start: 2)[
    - first
```

```
    - second
    - third
]
```

`start` is again optional and defaults to 1.

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`start` is again optional and defaults to 1.

`#line-by-line`: syntactic sugar for `#one-by-one`

Sometimes it is convenient to write the different contents to uncover one at a time in subsequent lines.

This comes in especially handy for bullet lists, enumerations, and term lists.

```
#line-by-line(start: 2)[
    - first
```

- first
- second

`#line-by-line`: syntactic sugar for `#one-by-one`

```
    - second
    - third
]
```

- third

`start` is again optional and defaults to 1.

`#list-one-by-one` and Co: when `#line-by-line` doesn't suffice

While `#line-by-line` is very convenient syntax-wise, it fails to produce more sophisticated bullet lists, enumerations or term lists. For example, non-tight lists are out of reach.

For that reason, there are `#list-one-by-one`, `#enum-one-by-one` , and `#terms-one-by-one`, respectively.

`#list-one-by-one` and Co: when `#line-by-line` doesn't suffice


```
#enum-one-by-one(start: 2,
tight: false, numbering:
"i)")[first][second]
[third]
```

i)
ii)
iii)

Note that, for technical reasons, the bullet points, numbers, or terms are never covered.

`start` is again optional and defaults to 1.

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```
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tight: false, numbering:
"i)")[first][second]
[third]
```

i) first

ii)

iii)

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"i)")[first][second]
[third]
```

i) first
ii) second
iii) third

Note that, for technical reasons, the bullet points, numbers, or terms are never covered.

`start` is again optional and defaults to 1.

… is defined by the *theme* of the presentation.

This demo uses the `unipd` theme.

Because of it, the title slide and the decoration on each slide (with section name, short title, slide number etc.) look the way they do.

Themes can also provide variants, for example …

... this one!

It's very minimalist and helps the audience focus on an important point.

Your own theme?

If you want to create your own design for slides, you can define custom themes!

The book explains how to do so.

Polylux ships a `utils` module with solutions for common tasks in slide building.

You can scale content such that it has a certain height using `#fit-to-height(height, content)`:

This function also allows you to fill the remaining space by using fractions as heights, i.e. `fit-to-height(1fr)[...]`:

Often you want to put different content next to each other. We have the function `#side-by-side` for that:

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do.

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do

Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do

Side by side content

eiusmod tempor incididunt ut labore.

eiusmod tempor.

Why not include an outline?

1. Overview
2. Models
3. Models
4. LOCAL Algorithms

5. CONGEST Algorithms
6. CONGEST Algorithms
7. Clusterings
8. The Algorithm
9. The Algorithm

Typst gives us so many cool things[2] . Use them!

[2] For example footnotes!

Hopefully you now have some kind of idea what you can do with this template.

Consider giving it a GitHub star or open an issue if you run into bugs or have feature requests.