$$\Phi = \frac{Q}{2\pi} \ln r_1 - \frac{Q}{2\pi} \ln r_2 - Q_0 x + C$$

$$r_1 = \sqrt{(x-d)^2 + y^2} \qquad r_2 = \sqrt{(x+d)^2 + y^2}$$

$$r_1 = 2d \qquad r_2 = 4d \qquad x = 3d \qquad h = h_0 \rightarrow$$

$$\Phi = \Phi_0 = T h_0$$

$$\Phi = \frac{Q}{2\pi} \ln \frac{r_1}{2d} - \frac{Q}{2\pi} \ln \frac{r_2}{4d} - Q_0 (x - 3d) + \Phi_0$$

$$Q_x = -\frac{\partial \Phi}{\partial x}$$

$$= -\frac{Q}{2\pi} \frac{x-d}{r_1^2} + \frac{Q}{2\pi} \frac{x+d}{r_2^2} + Q_0$$

$$Q_x(x, y=0) = 0$$

$$-\frac{Q}{2\pi} \frac{\cancel{x-d}}{(x-d)^{\cancel{2}}} + \frac{Q}{2\pi} \frac{\cancel{x+d}}{(x+d)^{\cancel{2}}} + Q_0 = 0$$

$$\frac{Q}{2\pi} \frac{-(\cancel{x} + d) + (\cancel{x} - d)}{(x-d)(x+d)} + Q_0 = 0$$

$$\frac{Q}{2\pi} \frac{-2d}{x^2 - d^2} = -Q_0$$

$$\frac{Q}{\cancel{2}\pi} \frac{+\cancel{2}d}{x^2 - d^2} = +Q_0$$

$$x^2 - d^2 = \frac{Qd}{\pi Q_0}$$

$$x_s = \pm \sqrt{d^2 + \frac{Qd}{\pi Q_0}} = \pm d \sqrt{1 + \frac{Q}{\pi Q_0 d}}$$

$x \geq 0 \qquad h = h_L + A e^{-x/\lambda} + B e^{x/\lambda}$

$x \to \infty \quad h \to h_L \quad \to B = 0$

$x = 0 \quad Q_x = Q_0$

$$Q_x = -T \frac{dh}{dx}$$

$$= +T \left(+\frac{A}{\lambda} e^{-x/\lambda} \right)$$

$$Q_x(x=0) = \frac{TA}{\lambda} = Q_0 \to A = \frac{Q_0 \lambda}{T}$$

$$\boxed{h = h_L + \frac{Q_0 \lambda}{T} e^{-x/\lambda} \qquad x \geq 0}$$

$x \leq 0 \qquad \Phi = -Q_0 x + C$

$$h = -\frac{Q_0 x}{T} + D$$

$$x = 0 \to h_0 = h_L + \frac{Q_0 \lambda}{T} \to D = h_0$$

$$h = -\frac{Q_0 x}{T} + h_L + \frac{Q_0 \lambda}{T}$$

$$\boxed{h = -\frac{Q_0 (x - \lambda)}{T} + h_L \qquad x \leq 0}$$

$$h = -Q_0 \frac{(x-\lambda)}{T} + h_2$$

$$h(x=-L) = h_1 \qquad \lambda = \sqrt{Tc}$$

$$+Q_0 \frac{(+L+\lambda)}{T} + h_2 = h_1$$

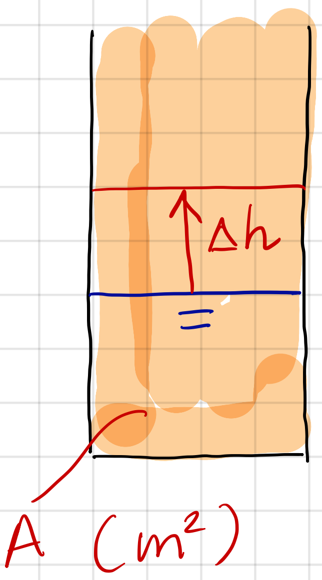
$$L + \lambda = \frac{T(h_1 - h_2)}{Q_0}$$

$$\lambda^2 = \left(\frac{T(h_1-h_2)}{Q_0} - L\right)^2$$

$$\cancel{T}c = \frac{\left(\frac{T(h_1-h_2)}{Q_0} - L\right)^2}{T}$$

TRANSIENT FLOW

Unconfined



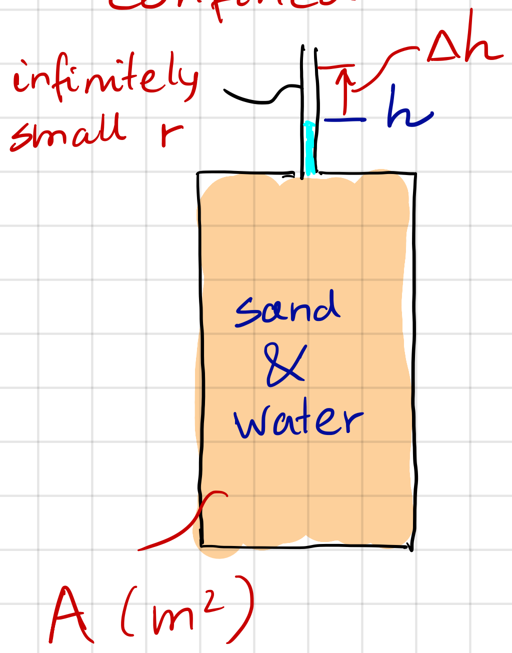
$$\Delta V = S_y \Delta h A$$

S_y : specific yield

storage coef. of unconfined (phreatic) aquifer

S_y : 0.1 – 0.2

Confined



1. compression of water
2. expansion of soil skeleton

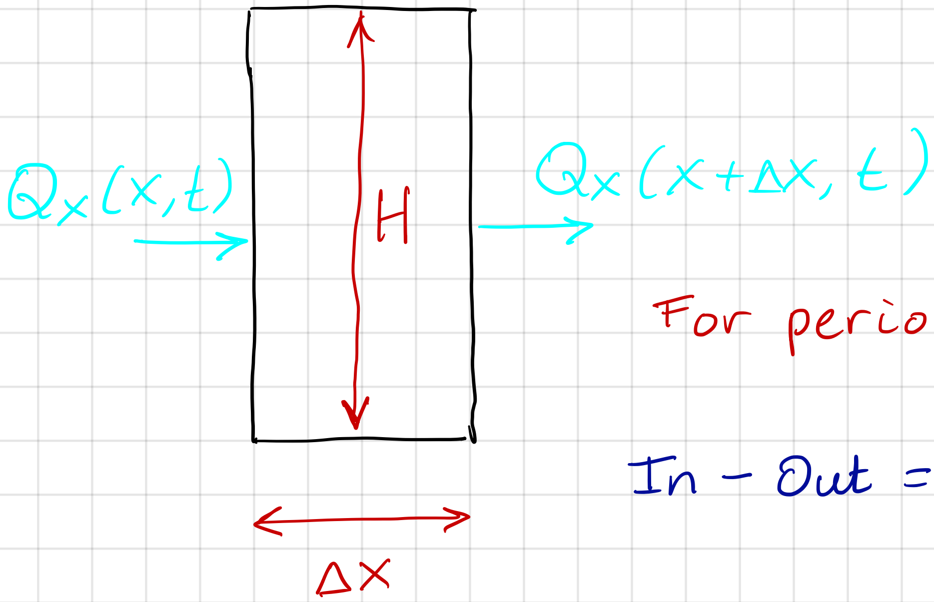
$$\Delta V = S \Delta h A$$

S : Storage coefficient : $10^{-3} - 10^{-5}$

Specific storage $S_s \sim 10^{-5} \left(\frac{1}{m}\right)$

$$S = S_s H \qquad H : \text{aquifer thickness}$$

Elastic storage



For period Δt

In − Out = Increase in storage

$$\frac{Q_x(x,t)\cancel{\Delta t} - Q_x(x+\Delta x,t)\cancel{\Delta t}}{\Delta x\,\cancel{\Delta t}} = \frac{S\,\Delta h\,\cancel{\Delta x}}{\cancel{\Delta x}\,\Delta t}$$

$$-\frac{\partial Q_x}{\partial x} = S\frac{\partial h}{\partial t} \qquad Q_x = -\frac{\partial \Phi}{\partial x}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} = S\frac{\partial h}{\partial t}}$$

Confined

$$\Phi = kHh$$

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{S}{kH}\frac{\partial (h\,kH)}{\partial t}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} = \frac{S}{kH}\frac{\partial \Phi}{\partial t}}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} = \frac{S}{kH} \frac{\partial \Phi}{\partial t}}$$

$D = \frac{kH}{S}$ Aquifer diffusivity $[m^2/d]$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} = \frac{1}{D} \frac{\partial \Phi}{\partial t}}$$

diffusion equation

$$D = \frac{kH}{S} = \frac{kH}{S_s H} = \frac{k}{S_s}$$

Unconfined

$$\frac{\partial^2 \Phi}{\partial x^2} = \frac{S_y \bar{h} \partial h k}{k \bar{h} \partial t}$$

Φ

$$\Phi = k \bar{h} h$$

$$= \bar{T} h$$

$$= \frac{S_y}{k \bar{h}} \frac{\partial \Phi}{\partial t}$$

$$= \frac{S_y}{\bar{T}} \frac{\partial \Phi}{\partial t}$$

$$D = \frac{k \bar{h}}{S_y}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} = \frac{1}{D} \frac{\partial \Phi}{\partial t}}$$

Transient well

$t = t_0 \longrightarrow h = h_0 \longrightarrow \Phi = \Phi_0$

$t \geq t_0 \longrightarrow$ well with discharge Q

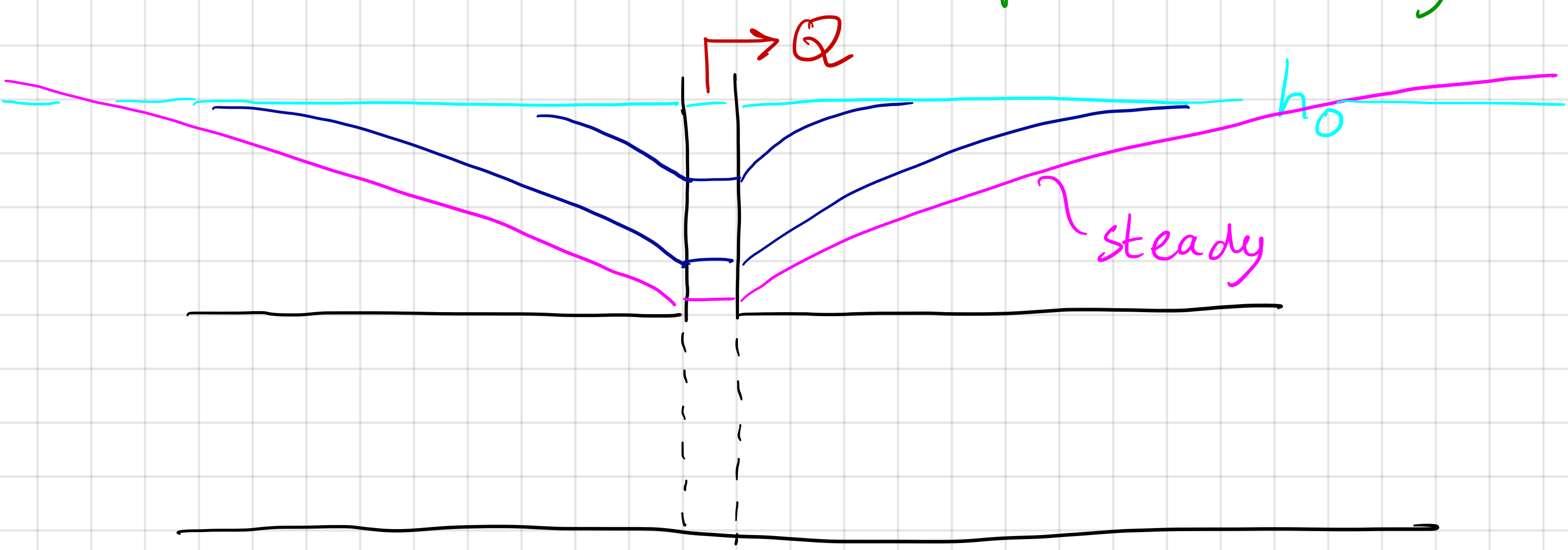
$r \to \infty \longrightarrow h = h_0$

$$\Phi(r,t) = \Phi_0 - \frac{Q}{4\pi} E_1(u)$$

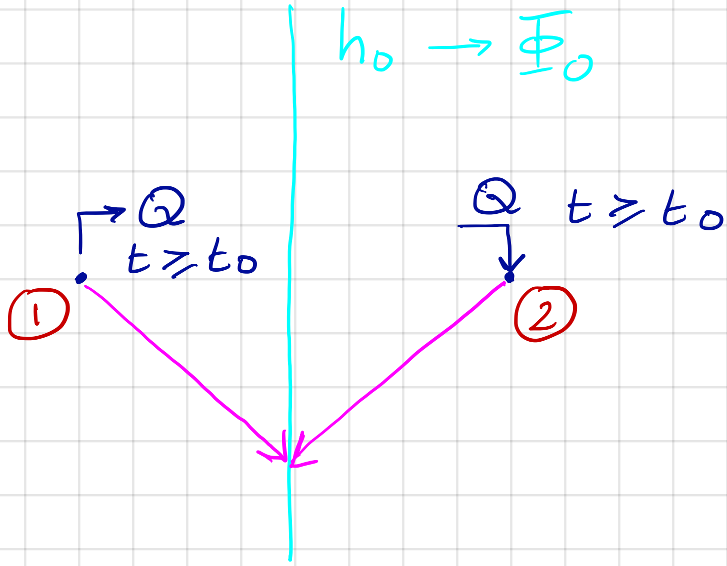
Theis solution

$$u = \frac{r^2 S}{4T(t-t_0)} \qquad E_1(u) = \int_u^\infty \frac{e^{-s}}{s}\, ds$$

Exponential Integral



Steady : $\Phi = \frac{Q}{2\pi} \ln r$



$$\Phi = -\frac{Q}{4\pi} E_1(u_1) + \frac{Q}{4\pi} E_1(u_2) + \Phi_0$$

$$u_1 = \frac{r_1^2 S}{4T(t-t_0)} \qquad u_2 = \frac{r_2^2 S}{4T(t-t_0)}$$

$$E_1(u) = -\gamma - \ln(u) + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \dots \quad (u<1$$

$\gamma \doteq 0.5772\dots$ Euler's constant

u small $\rightarrow$ $E_1(u) \simeq -\gamma - \ln(u)$

large time:

$$\Phi = -\frac{Q}{4\pi}\left(-\cancel{\gamma} - \ln(u_1)\right) + \frac{Q}{4\pi}\left(-\cancel{\gamma} - \ln(u_2)\right) + \Phi_0$$

$$= \frac{Q}{4\pi} \ln\left(\frac{u_1}{u_2}\right) + \Phi_0$$

$$= \frac{Q}{2\,\cancel{4}\pi} \ln\left(\frac{r_1^{\cancel{2}} \cancel{S}}{\cancel{4T(t-t_0)}} \; \frac{\cancel{4T(t-t_0)}}{r_2^{\cancel{2}} \cancel{S}}\right) + \Phi_0$$

$$\frac{Q}{2\pi} \ln\left(\frac{r_1}{r_2}\right) + \Phi_0$$

steady solution!

Drawdown: $h_0 - h(x,y,t)$

$t_0 \leq t \leq t_1$ $\quad Q = Q_0$

$t_1 \leq t \leq t_2$ $\quad Q = 0$

$t_2 \leq t$ $\quad Q = 0.5\,Q_0$

$t_0 \leq t \leq t_1$

$$\Phi = -\frac{Q}{4\pi} E_1(u_0) + \Phi_0$$

$$u_0 = \frac{r^2 S}{4T(t-t_0)}$$

$t_1 \leq t \leq t_2$

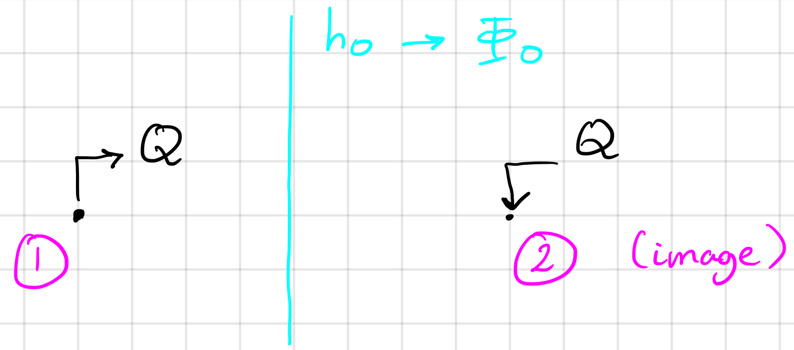
$$\Phi = -\frac{Q}{4\pi} E_1(u_0) + \frac{Q}{4\pi} E_1(u_1) + \Phi_0$$

$$u_1 = \frac{r^2 S}{4T(t-t_1)}$$

$t_2 \leq t$

$$\Phi = -\frac{Q}{4\pi} E_1(u_0) + \frac{Q}{4\pi} E_1(u_1) - \frac{0.5Q}{4\pi} E_1(u_2) + \Phi_0$$

$$u_2 = \frac{r^2 S}{4T(t-t_2)}$$



Steady :
$$\Phi = \frac{Q}{2\pi} \ln\left(\frac{r_1}{r_2}\right) + \Phi_0$$

Turn well off at $t = t_0$

$$\Phi = \frac{Q}{2\pi} \ln \frac{r_1}{r_2} + \Phi_0 + \frac{Q}{4\pi} E_1(u_1) - \frac{Q}{4\pi} E_1(u_2)$$

$$u_1 = \frac{r_1^2 S}{4\pi(t-t_0)} \qquad u_2 = \frac{r_2^2 S}{4\pi(t-t_0)}$$