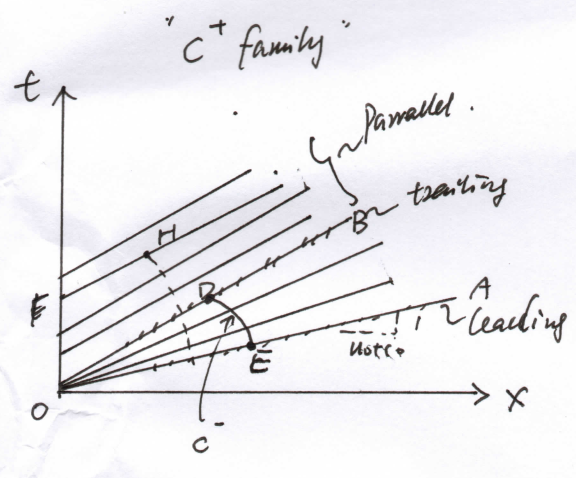
Quiz 2 solution.

Q1



(a) The leading edge C^+ is a straight line, and:

$u_0 = 1\,m/s$

$C_0 = \sqrt{g h_0} = \sqrt{9.8 \cdot 3} = 5.42\,m/s$

$\left.\frac{dx}{dt}\right|_0 = u_0 + C_0 = 6.42\,m/s.$ #

There are infinity # of C^+ started at $(0,0)$, because water depth suddenly changes from $h_0 = 3m$ to $h_1 = 2.8\,m$ at $t=0$. The slope of these C^+ issued at $(0,0)$ can be obtained by considering a C^- started from an arbitrary point E on the leading C^+ (OA). Consider points D and E.

$$u - 2c = u_0 - 2c_0$$

$$\Rightarrow \frac{dx}{dt} = u + c = u_0 - 2c_0 + 3c \quad \text{and} \quad u = u_0 - 2c_0 + 2c$$

Apparently, the minimum $\frac{dx}{dt}$ is achieved for $C_{min} = \sqrt{g h_1} = 5.24\,m/s$
This is the trailing edge of the disturbance. We denote this C^+ as OB.

For ~~any~~ all C^+ issued later on the t-axis, e.g. FH, we can show that they are // to OB. Again we connect FH with OA by a C^-

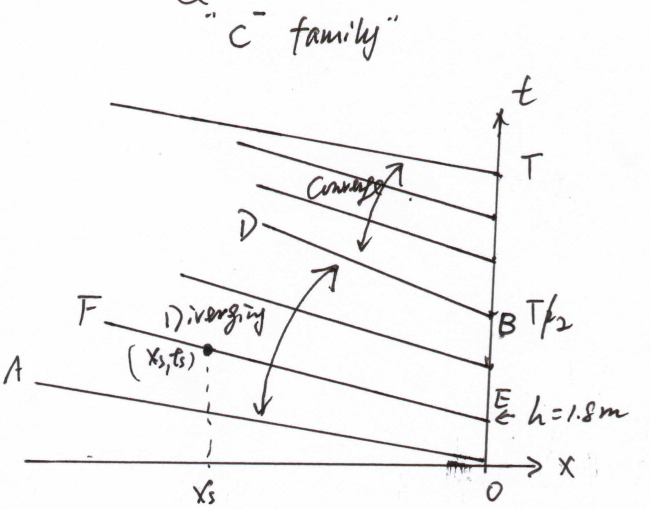
$$u - 2c = u_0 - 2c_0 \quad \Rightarrow \left.\frac{dx}{dt}\right|_{FH} = u_0 - 2c_0 + 3C$$

since $C = \sqrt{g h_1}$ for all C^+ issued later on the t-axis, FH has the same slope as OB, and they are parallel.

The traveling speed of the trailing edge is

$$\left.\frac{dx}{dt}\right|_{OB} = u_0 - 2c_0 + 3C_{min} = 5.88\,m/s$$ #

Q2



Since the water depth first decreases and the increases. We have negative surge before $T/2$ and positive surge after $T/2$.

The C^- family can be drawn as in the figure.

Point B is the moment of minimum ~~wa~~ water depth at $x=0$. The C^- through point B seperates the positive ~~and~~ the negative surges.

(a) $t = 1000s$, the positive surge yet starts, so we have a simple negative surge problem.

The C^- with $h = 1.8\,m$ (denoted as EF) starts at $\tau = \frac{2-1.8}{0.5} \cdot 1000 = 400s$.

The slope is: $\left.\frac{dx}{dt}\right|_{EF} = u - c = u_0 + 2c_0 - 3c = 1 + 2\cdot\sqrt{9.8\cdot 2} - 3\cdot\sqrt{9.8\cdot 1.8}$
$= -2.74\,m/s.$

so $x_s = (t_s - \tau)\left(\left.\frac{dx}{dt}\right|_{EF}\right) = (1000-400)\cdot(-2.74) = \underline{1644\ m}$ #

(b) For the positive surge, the initial flow condition is the flow condition along BD:

$h_0' = 1.5\,m.$ $\quad c_0' = \sqrt{9.8\cdot 1.5} = 3.83\,m/s$

$u_0' = u_0 + 2c_0 - 2c_0' = 2.2\,m/s$

$\left.dx/dt\right|_{BD} = u_0' - c_0' = -1.63\,m/s$

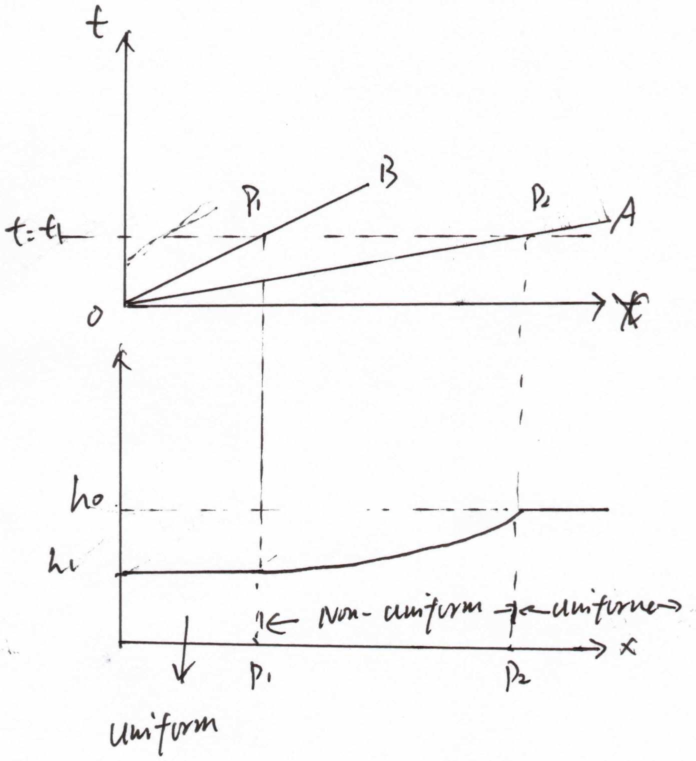
The ~~rate~~ of change of h at $x=0$: $\frac{dh}{d\tau} = \frac{0.5m}{1000s} = 5\times10^{-4}\,m/s.$

location of surge incipent:

$$x = \frac{-(u_0'-c_0')^2}{\frac{3}{2}\sqrt{g/h_0'}\left.\frac{\partial h}{\partial \tau}\right|_{\tau=T/2}} = \frac{-1.63^2}{\frac{3}{2}\sqrt{\frac{9.8}{1.5}}\cdot 5\times10^{-4}} = \underline{1.386\times10^3\,m.}$$ #

Time of surge incipent:

$$t = \frac{x}{u_0'-c_0'} + T/2 = 1850.3\,s$$ #



Based on the C^+ family we can determine the surface profile at any ~~state~~ instance, $t=t_1$

As shown in the figure, we have two uniform-flow regions.

To the right of P_2:

$$u = u_0 = 1 m/s$$

$$h = h_0 = 3 m.$$

To the left of P_1:

$$h = h_1 = 2.8m$$

$$u = u_0 - 2c_0 + 2c_1 = \cancel{0.8 m/s}\ 0.64 m/s.$$

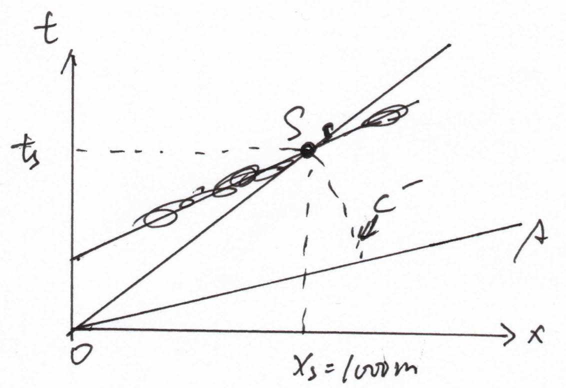
(b)

$$h_s = h_0 - 0.1 m = 2.9 m.$$

$$c_s = \sqrt{g h_s} = 5.33 m/s.$$

Since $h_s \neq h_1$. The point S on the x-t plane is on a C^+ through the origin. ~~It's slope is:~~

$$\cancel{\frac{dx}{dt} = u}$$



The local flow at S satisfies.

$$u_s - 2c_s = u_0 - 2c_0$$

(consider a C^- connect S to OA).

$$u_s = 2c_s + u_0 - 2c_0 = 0.82 m/s.$$

#