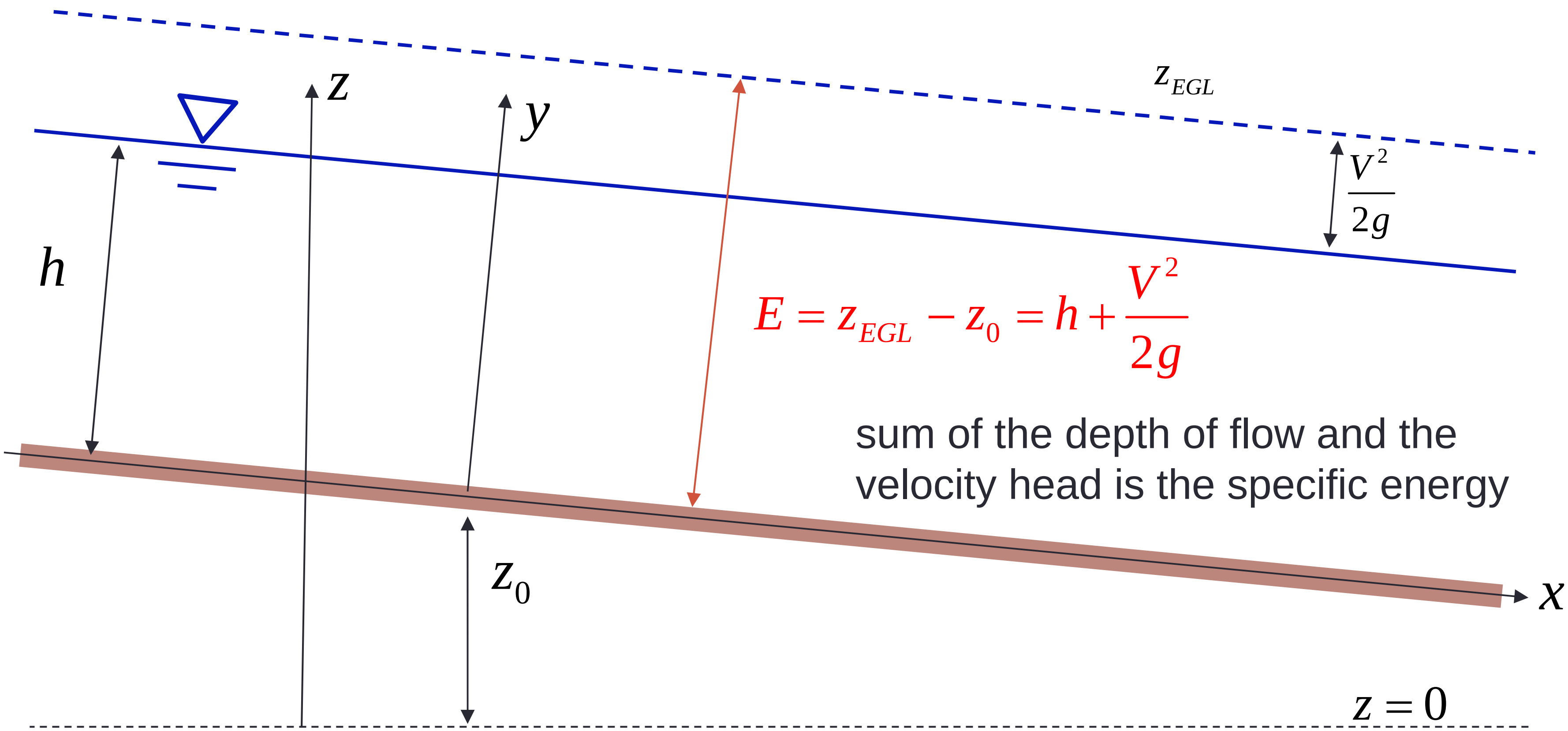
STEADY OPEN-CHANNEL FLOW B

~~RIVER MECHANICS~~ (OPEN-CHANNEL HYDRAULICS)

Dr. Yuan Jing,
Department of Civil and Environmental Engineering
Office: E2-05-20
Phone: 65162160
Email: ceeyuan@nus.edu.sg

Specific energy

- Total energy requires definition of z=0, which can be defined arbitrarily
- More convenient to refer to the EGL elevation above the channel bottom



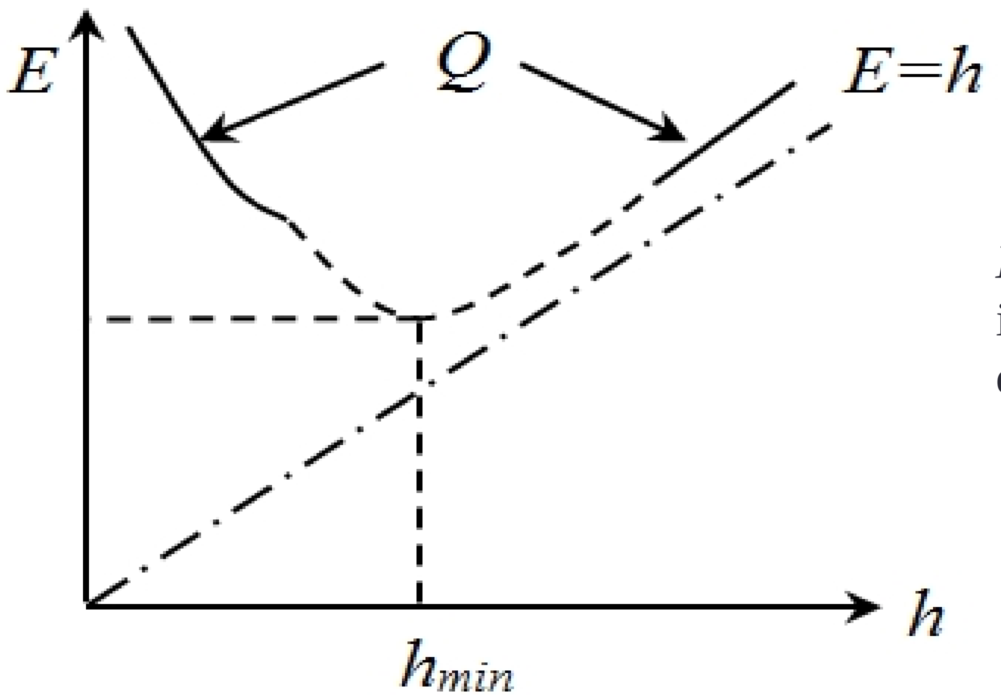
$$E = z_{EGL} - z_0 = h + \frac{V^2}{2g}$$

sum of the depth of flow and the velocity head is the specific energy

How specific head varies with depth (for a given Q)?

$$E = h + \frac{V^2}{2g} = h + \frac{Q^2}{2gA(h)^2}$$

$$h \to \infty,\ A \to \infty \quad \Rightarrow \quad E \approx h \to \infty$$
$$h \to 0,\ A \to 0 \quad \Rightarrow \quad E \approx Q^2/(2gA^2) \to \infty$$

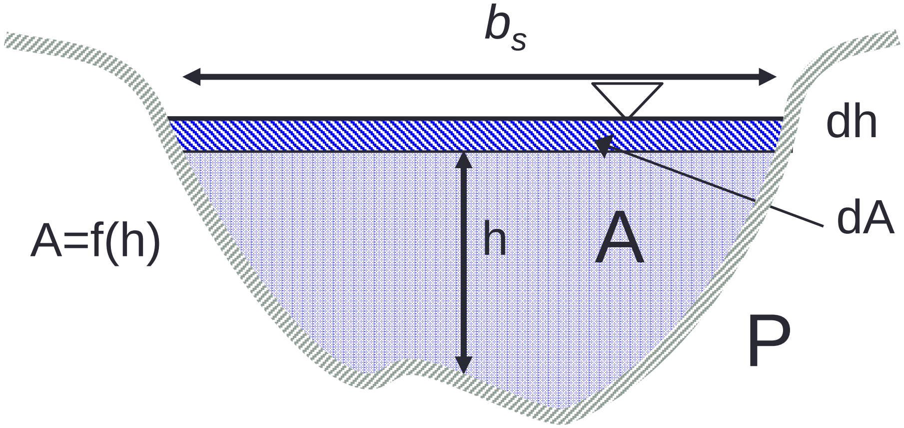


E goes to infinity as *h* approaches infinity or zero, so there must be a depth gives minimum *E*.

Critical Flow

The depth gives minimum specific energy for a given total discharge:

$$\frac{\partial E}{\partial h} = 0$$

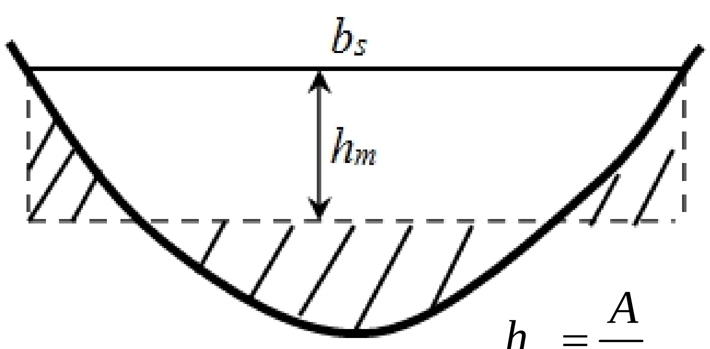


Condition for critical flow: Froude number equals to 1

$$\boxed{Fr = \frac{V}{\sqrt{gh_m}} = 1 \Leftrightarrow \frac{Q^2 b_s(h)}{gA(h)^3} = 1}$$

$$\frac{A(h)^3}{b_s(h)} = \frac{Q^2}{g}$$

Use this to solve for **critical depth** h_c



$$h_m = \frac{A}{b_s}$$

Mean depth

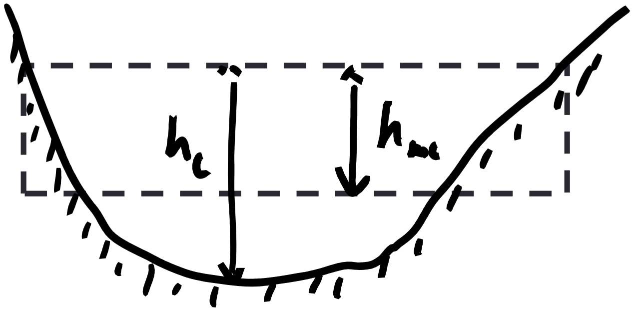
Critical flow (continued)

The velocity head for critical flow:

$$\boxed{\frac{Q^2}{2gA_c^{\,2}} = \frac{1}{2} h_{mc}}$$

specific energy:

$$\boxed{E_c = h_c + \frac{1}{2} h_{mc}}$$

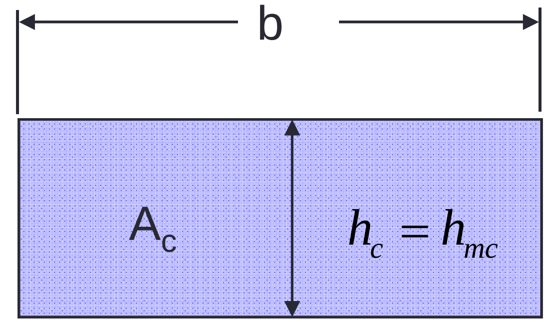


h_c: critical water depth
hmc: mean water depth for critical condition.

Critical Flow: Rectangular channel

arbitrary cross-section

$$\frac{Q^2}{2gA_c^{\;2}} = \frac{1}{2} h_{mc} \qquad E_c = h_c + \frac{1}{2} h_{mc}$$



Depth is also the mean depth

Rectangular cross-section

$$\frac{(Q/b)^2}{2gh_c^{\;2}} = \frac{1}{2} h_c \qquad E_c = \frac{3}{2} h_c$$

Velocity head is ½ of critical depth for rectangular channel.

$$\boxed{\begin{gathered} h_c = \left(\frac{(Q/b)^2}{g} \right)^{1/3} \\ \frac{Q}{b} = V_c = \sqrt{g h_c^{\;3}} \end{gathered}}$$

You only need to know the critical water depth to get total discharge, and vice versa (Basic idea for flow control/measurement)

Critical depth vs normal depth

Normal depth is given by the balancing between gravity and bottom resistance, so it requires knowledge of slope and bottom roughness condition in addition to total discharge and channel geometry

$$h_n = f(Q, Geometry, S_0, bottom)$$

Critical depth is given by the minimizing specific energy, so it has nothing to do with bottom resistance, and thus only requires knowledge of channel geometry

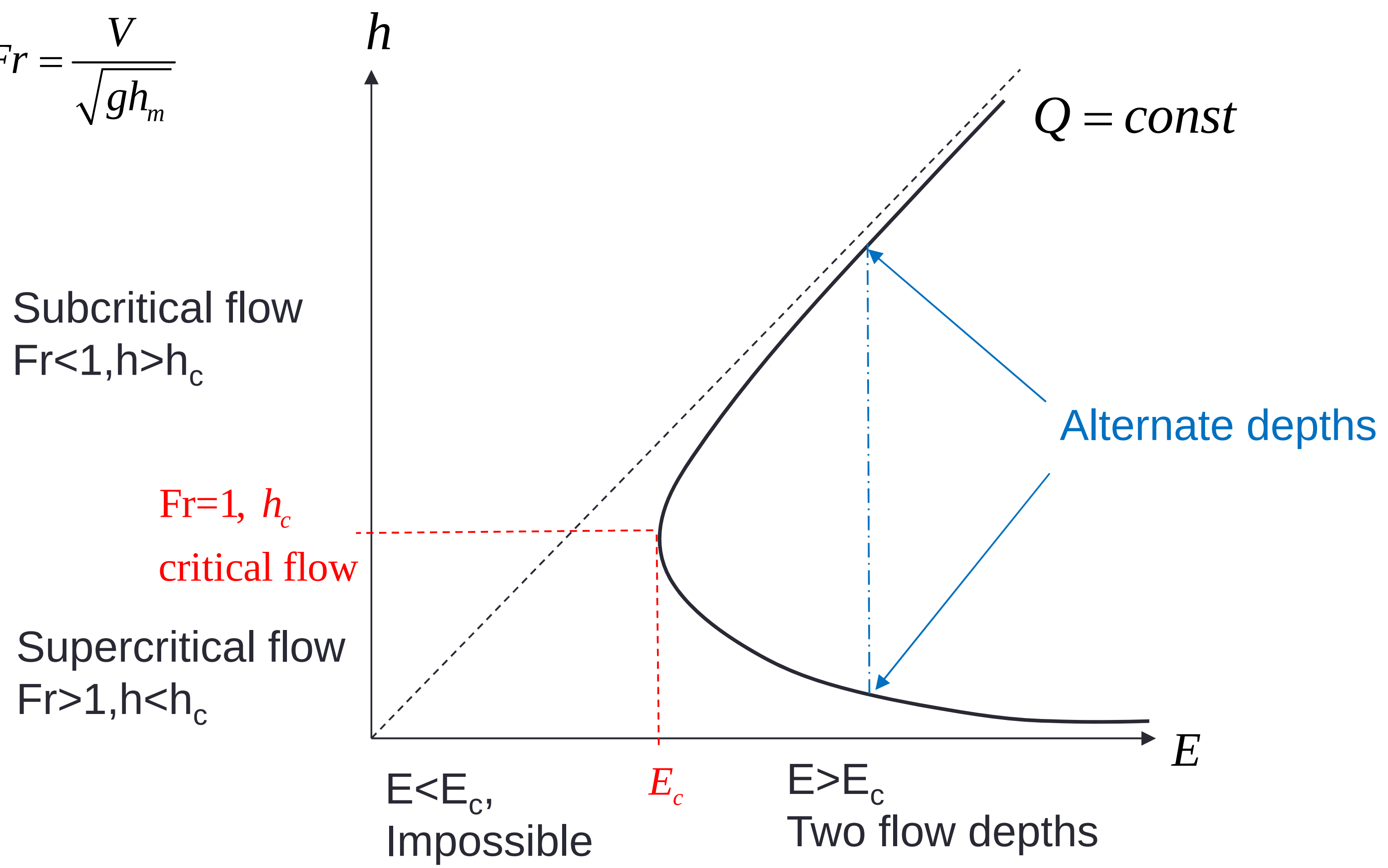
$$h_c = f(Q, Geometry)$$

Specific-head-depth diagram

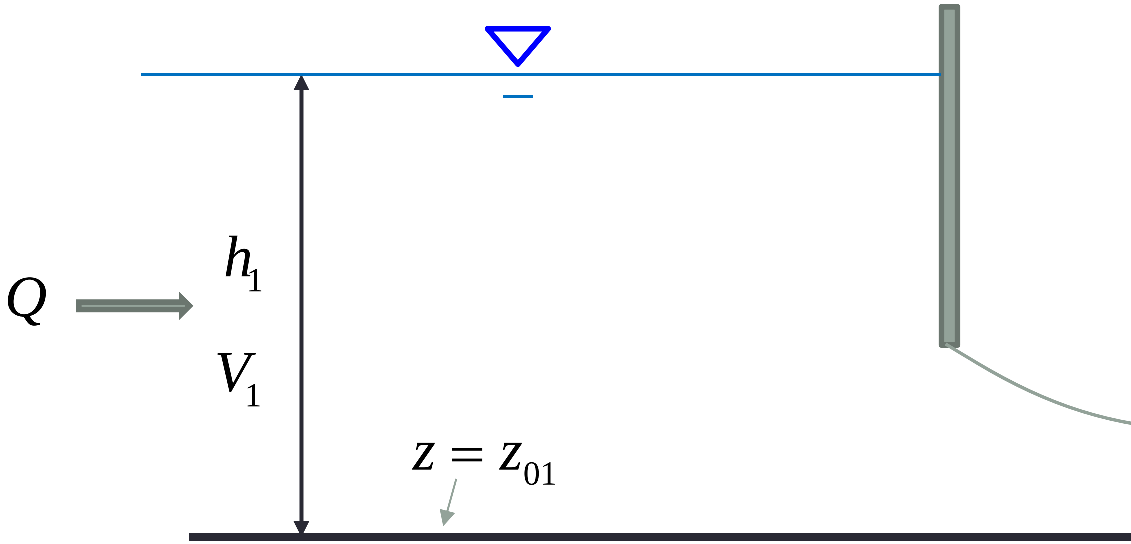
$$E = h + \frac{V^2}{2g}$$

$$Fr = \frac{V}{\sqrt{g h_m}}$$

- Subcritical: slow flow with deep water depth, *h* dominates in *E*.
- Supercritical: fast flow with shallow water depth, $V^2/2g$ dominates in *E*.



Example of alternate depth: sluice gate

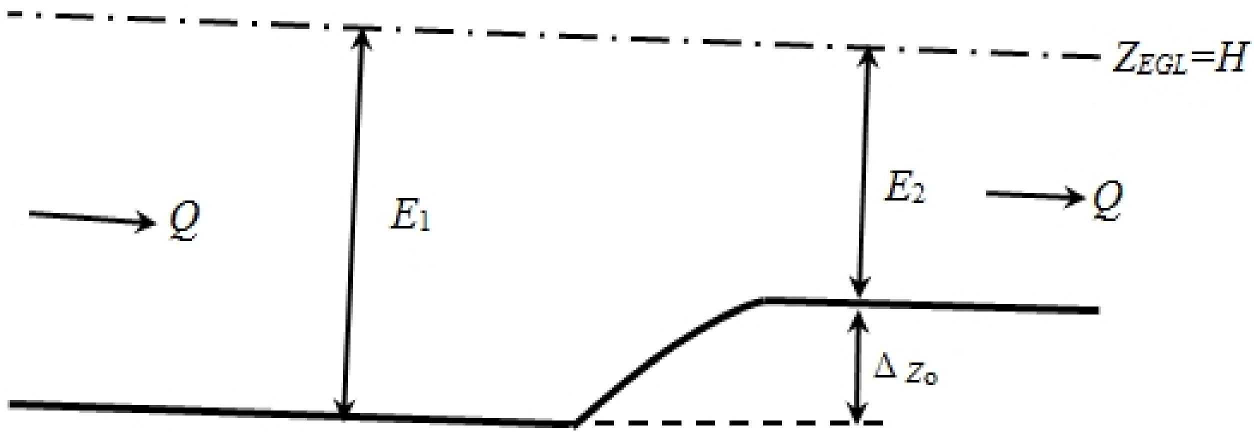


$$\left.\begin{array}{l} H_1 = H_2 \\ z_{01} = z_{02} \end{array}\right\} \Rightarrow E_1 = H_1 - z_{01} = H_2 - z_{02} = E_2$$

h_1 and h_2 are alternate depths.

Application of specific energy: flow over a hump

Assume no head loss

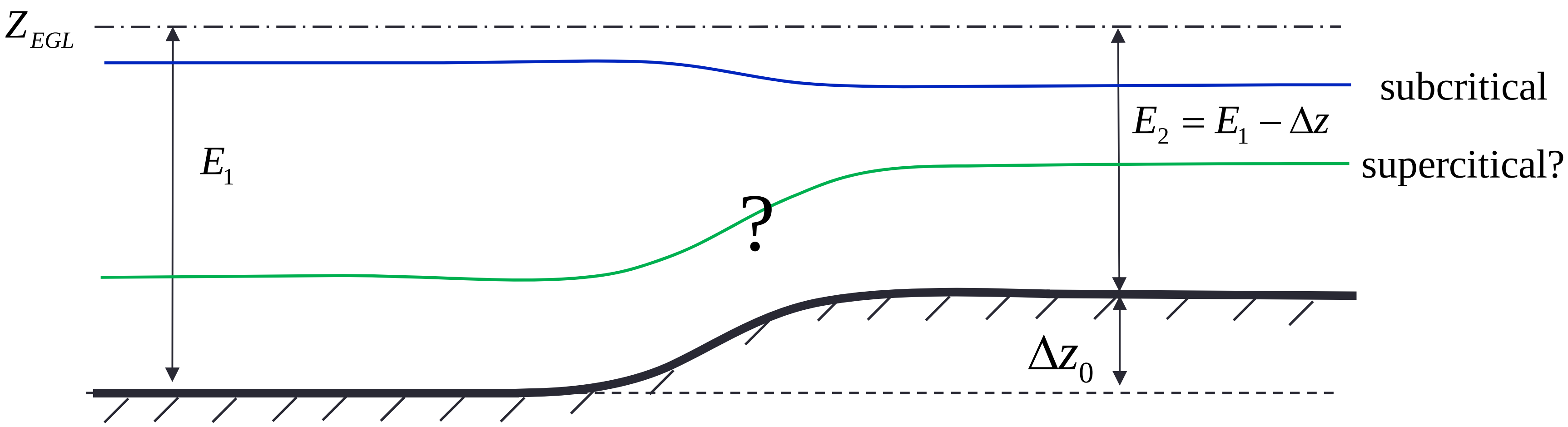


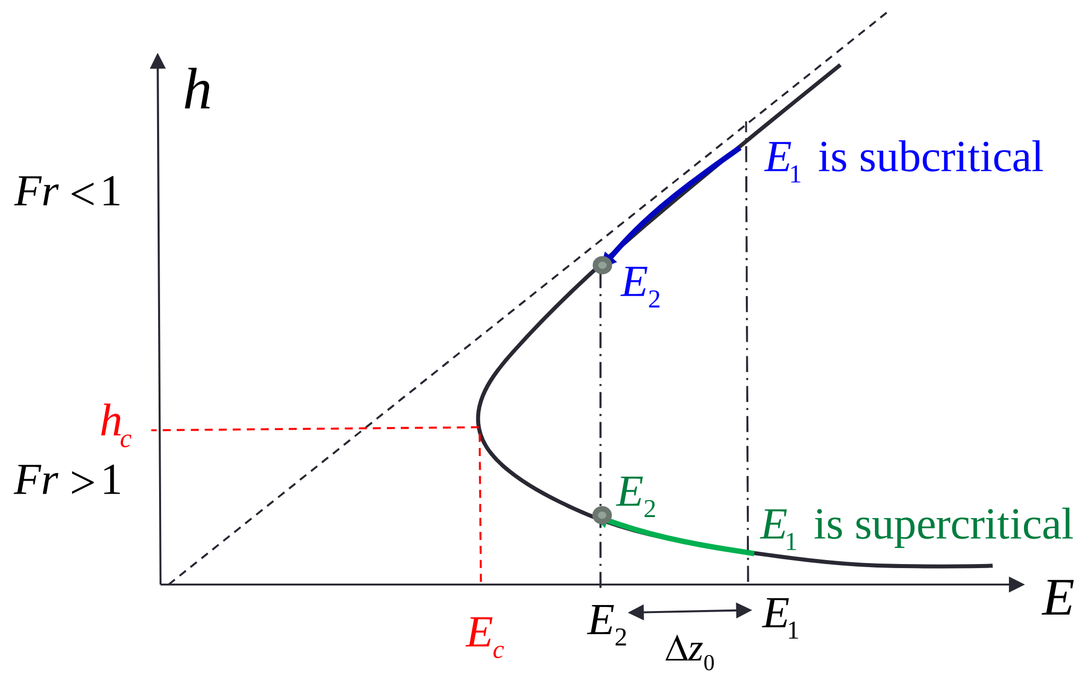
$$E_1 = h_1 + \frac{Q^2}{2gA_1^2}$$

$$E_2 = h_2 + \frac{Q^2}{2gA_2^2} = E_1 - \Delta z_0$$

Task: for given Q and h_1, solve for h_2.

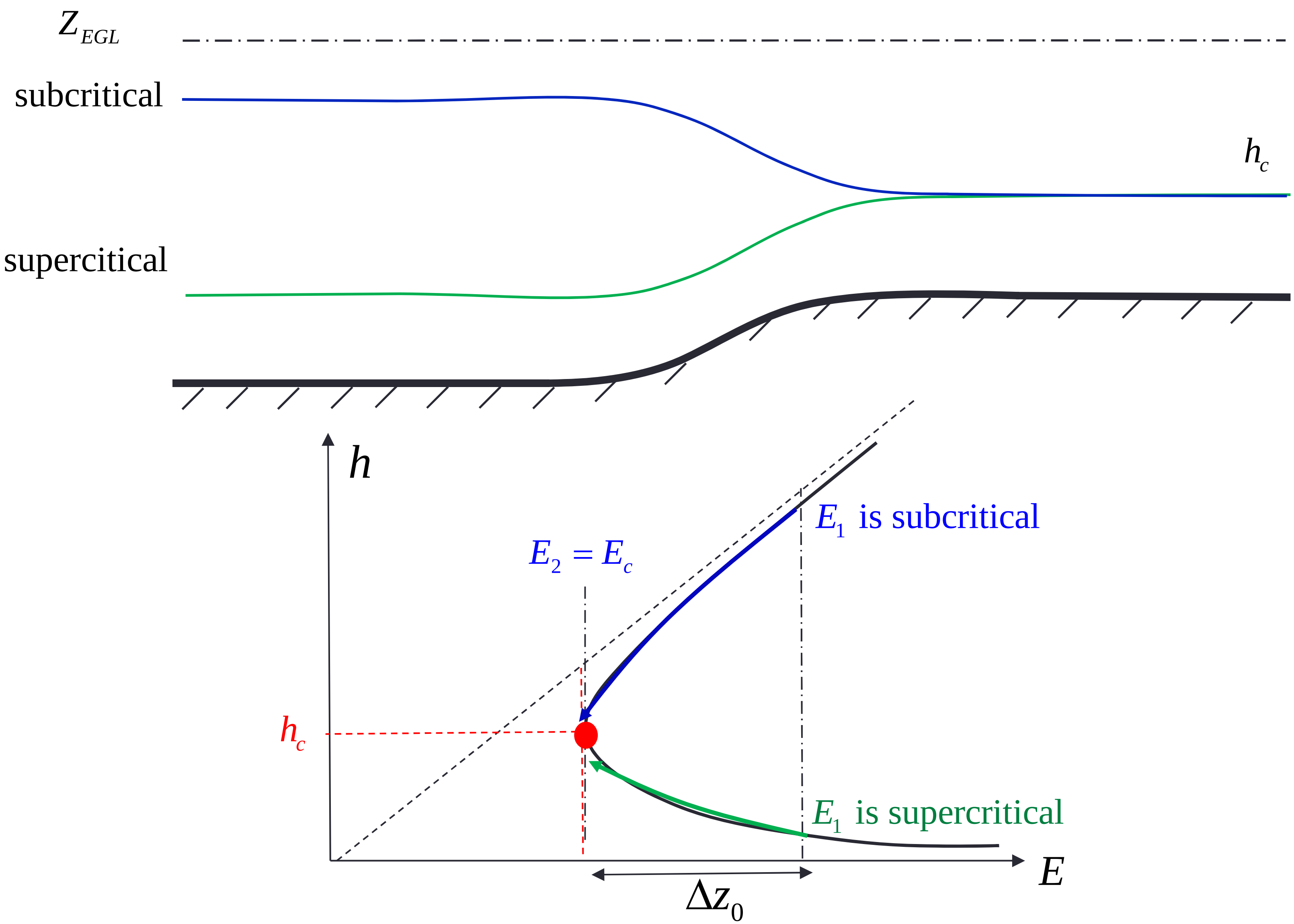
Flow a over hump: enough energy



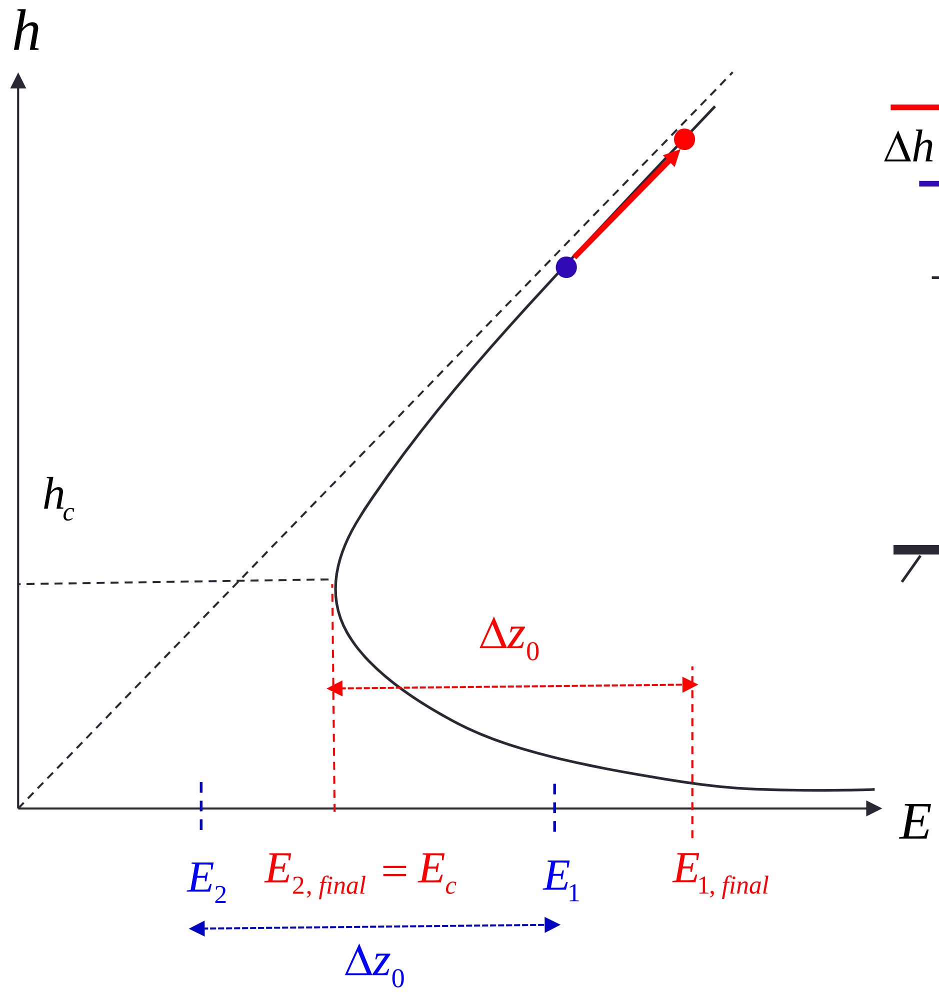


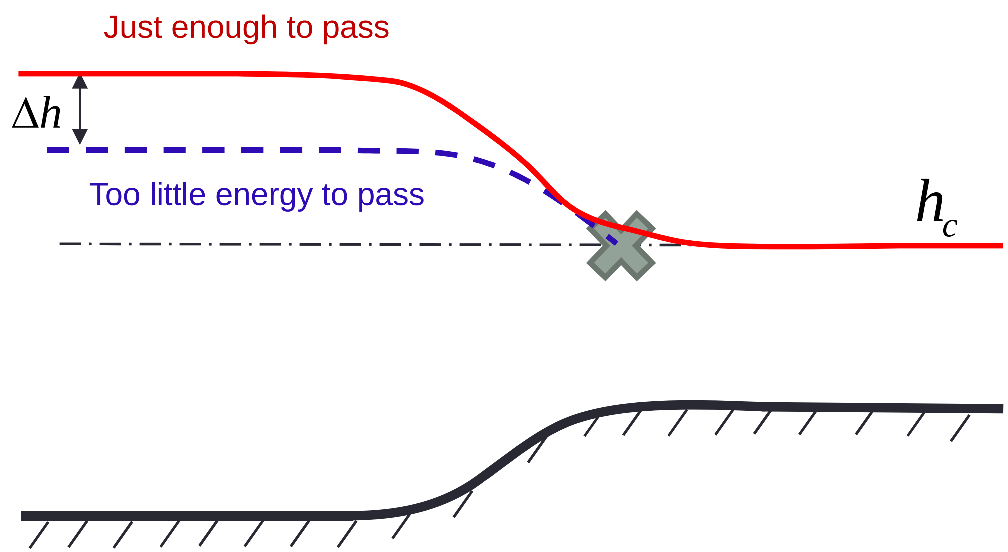
Two solutions for downstream are possible!

Flow a over hump: just enough energy



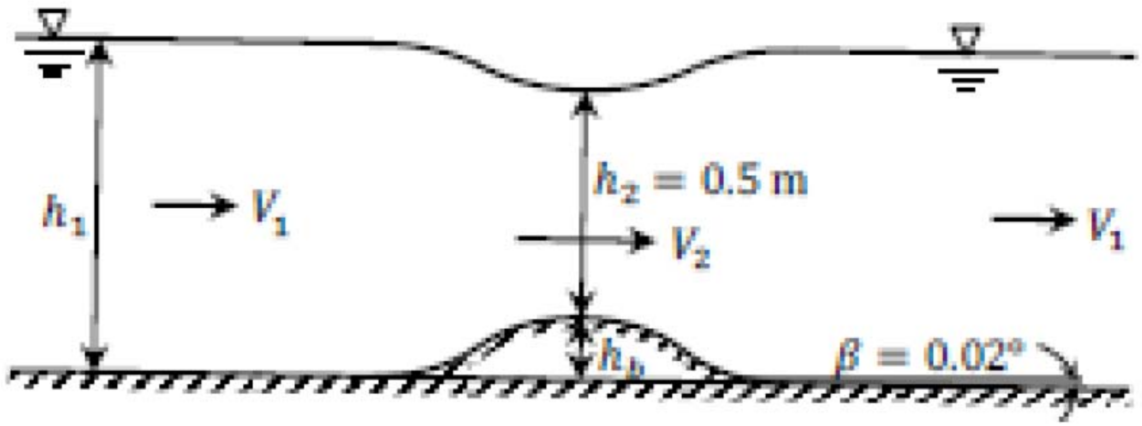
Flow a over hump: not enough energy





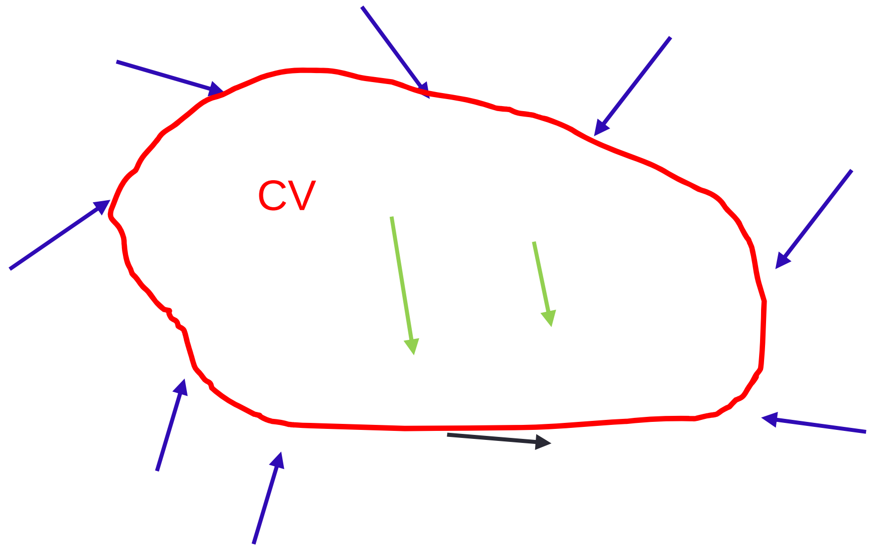
- For subcritical flow, water will pile up before the hump to increase its specific energy, and the channel is choked.
- For supercritical, the situation is different and more complicated.

Water is flowing down a wide rectangular brick channel (n=0.015) when it hits a bump in the bottom, causing the flow to attain critical velocity over the top of the bump. As a result, the water surface depresses slightly.



- Find the velocity over the bump and the flowrate per meter width of the channel.
- Find the height of the bump.

General momentum principle

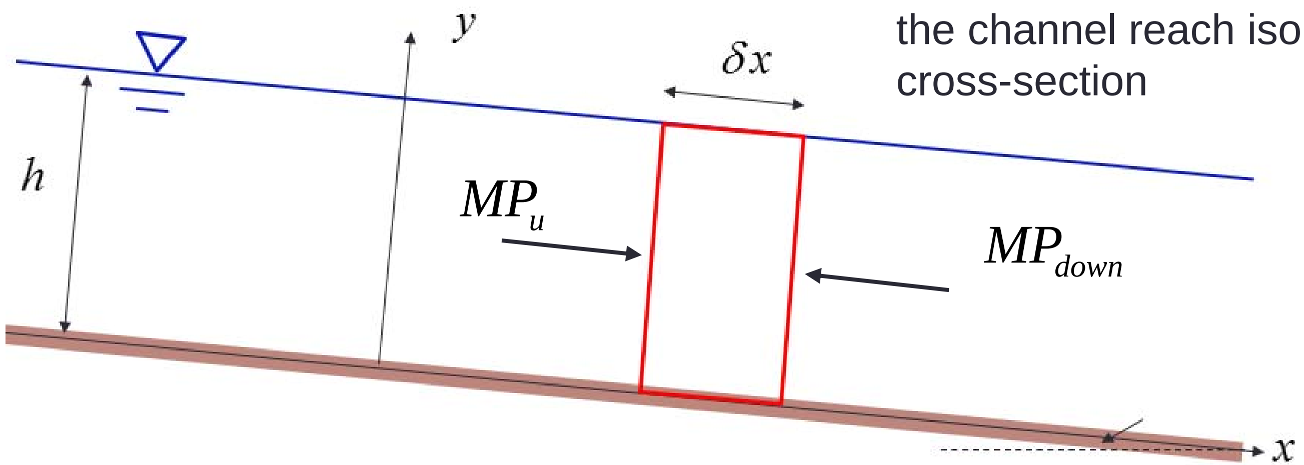


$$\boxed{\frac{\partial}{\partial t}\int_{CV}\rho\vec{q}\,dV = -\int_{CS}\rho\vec{q}(\vec{q}\cdot\vec{n})\,dA + \sum \text{surface forces} + \sum \text{body forces}}$$

Rate of change of momentum within control volume

Net Momentum exchange across control surface

In O-C flow, we often define a CV as the channel reach isolated by two cross-section



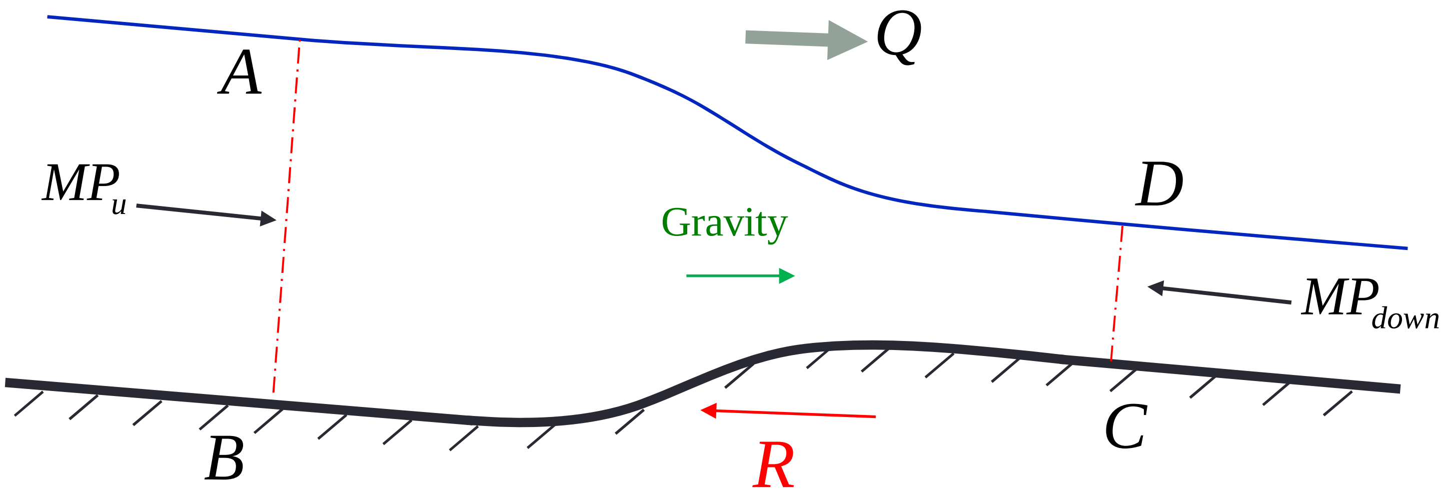
$$\boxed{\frac{\partial}{\partial t}\int_{CV} \rho \vec{q} dV = -\int_{CS} \rho \vec{q}(\vec{q}\cdot\vec{n})dA + \sum \text{surface forces} + \sum \text{body forces}}$$

For a cross-section with well-behaved flow, the momentum exchange and surface force terms can be combined into a **hydraulic thrust**:

$$MP = -\int_{CS} \rho \vec{q}(\vec{q}\cdot\vec{n})dA + \sum \text{surface forces} = \left(\rho V^2 + p_{CG}\right)A$$

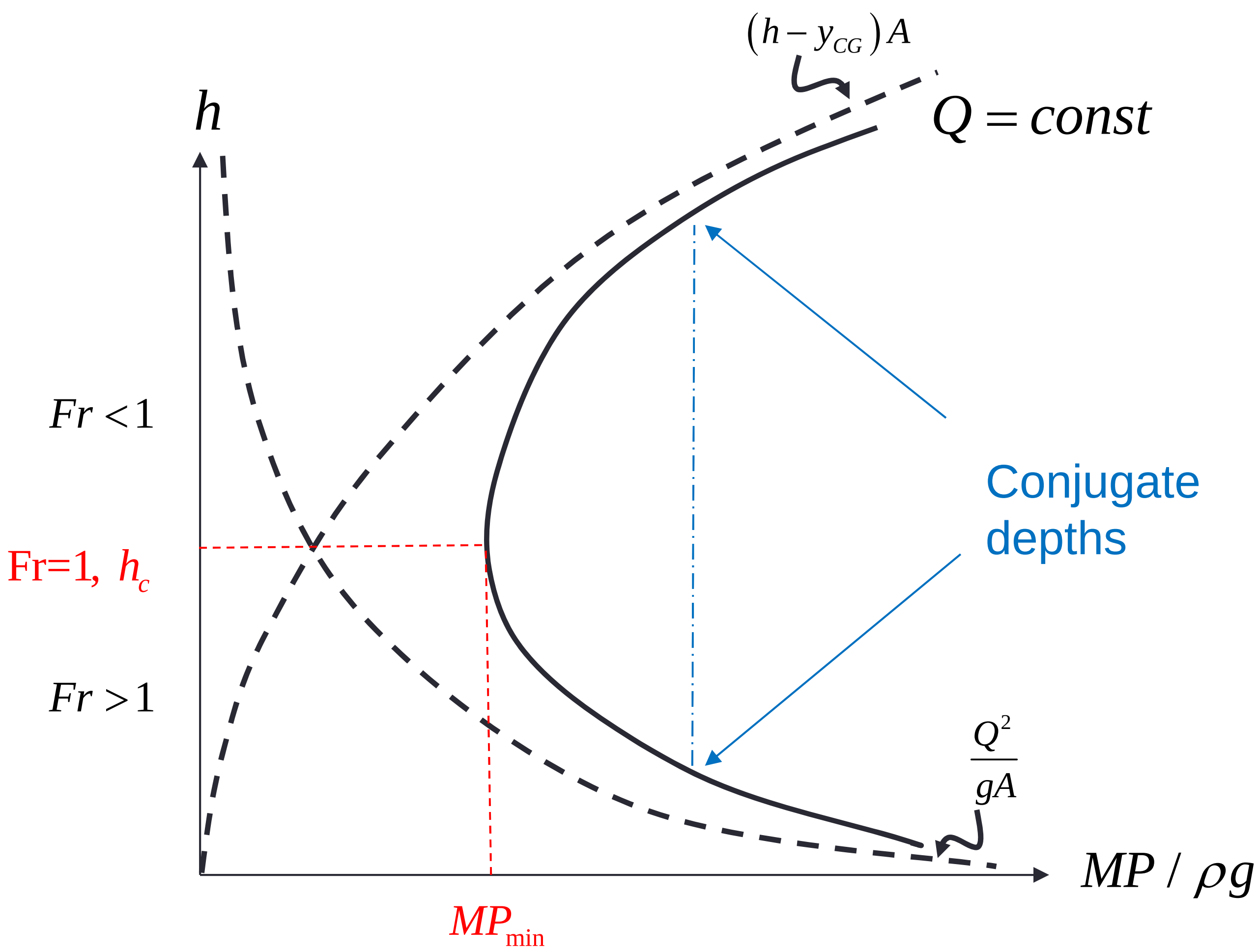
hydraulic thrust is a vector. It is always perpendicular to the control surface and **point into the control volume**

Apply momentum principle for flow over a hump



$$\boxed{MP_u - MP_d + \text{Gravity} - \text{R} = 0}$$

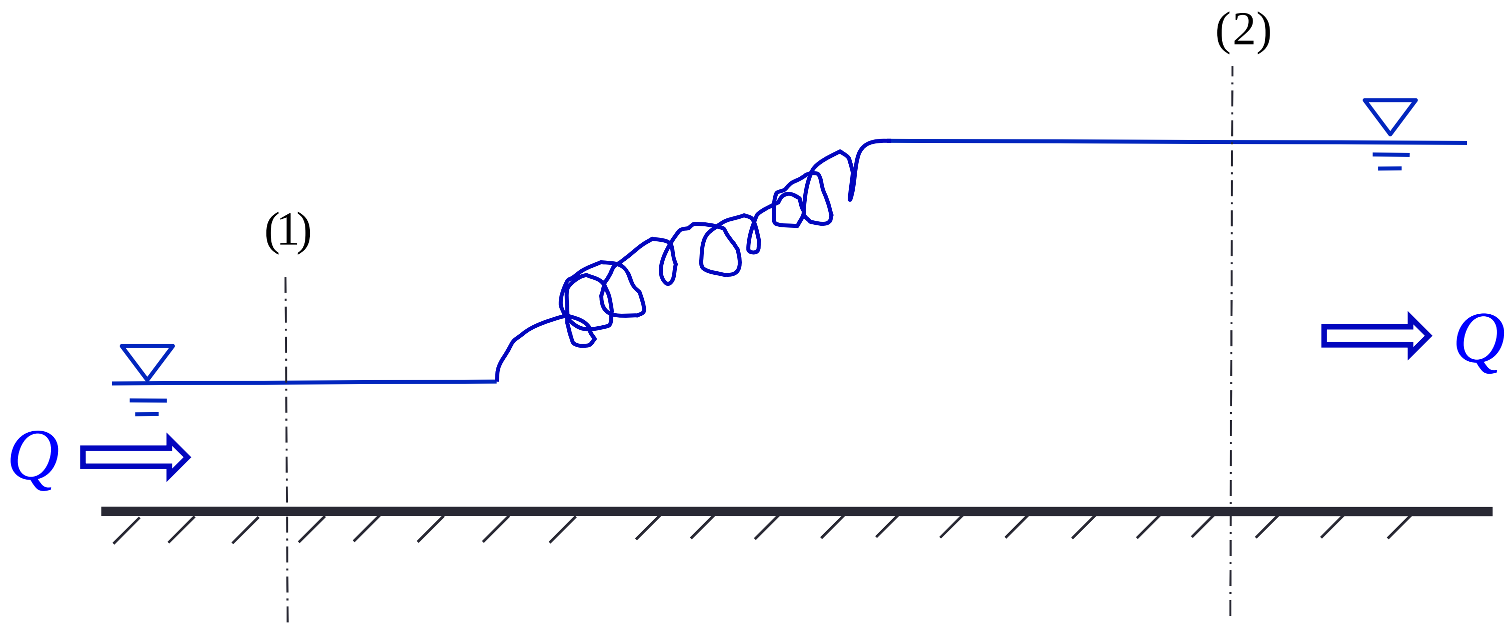
Thrust-depth diagram

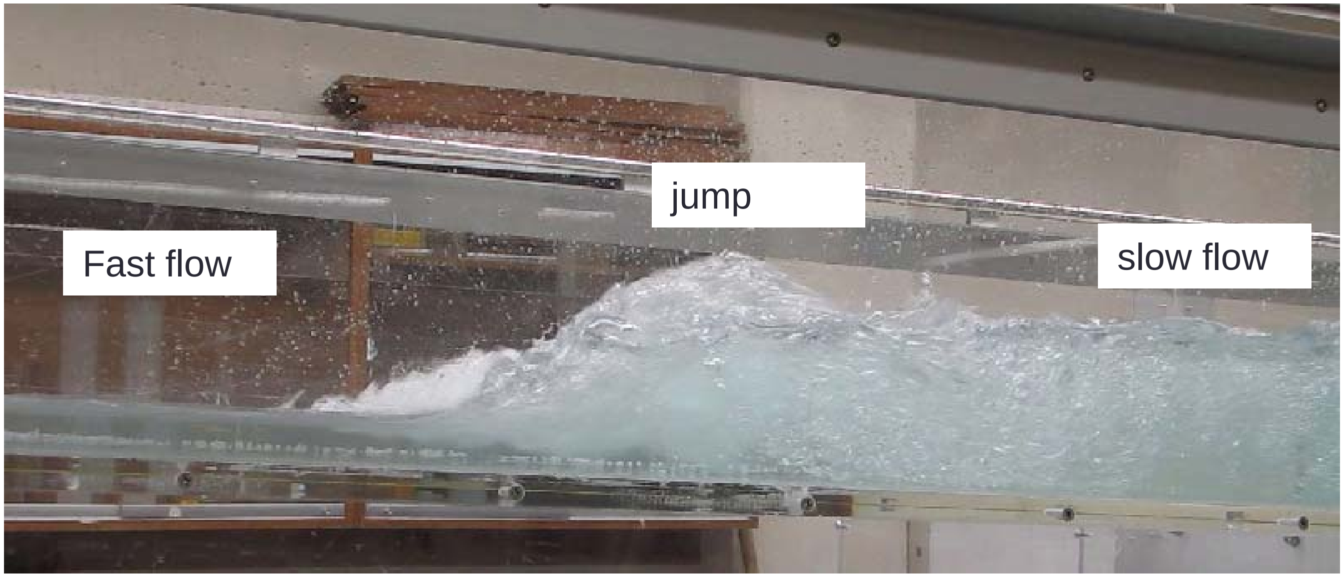


$$\frac{MP}{\rho g} = \frac{Q^2}{gA} + (h - y_{CG})A$$

Minimum thrust under a given discharge is achieved at critical flow condition!

Application of momentum Principle: Hydraulic jump



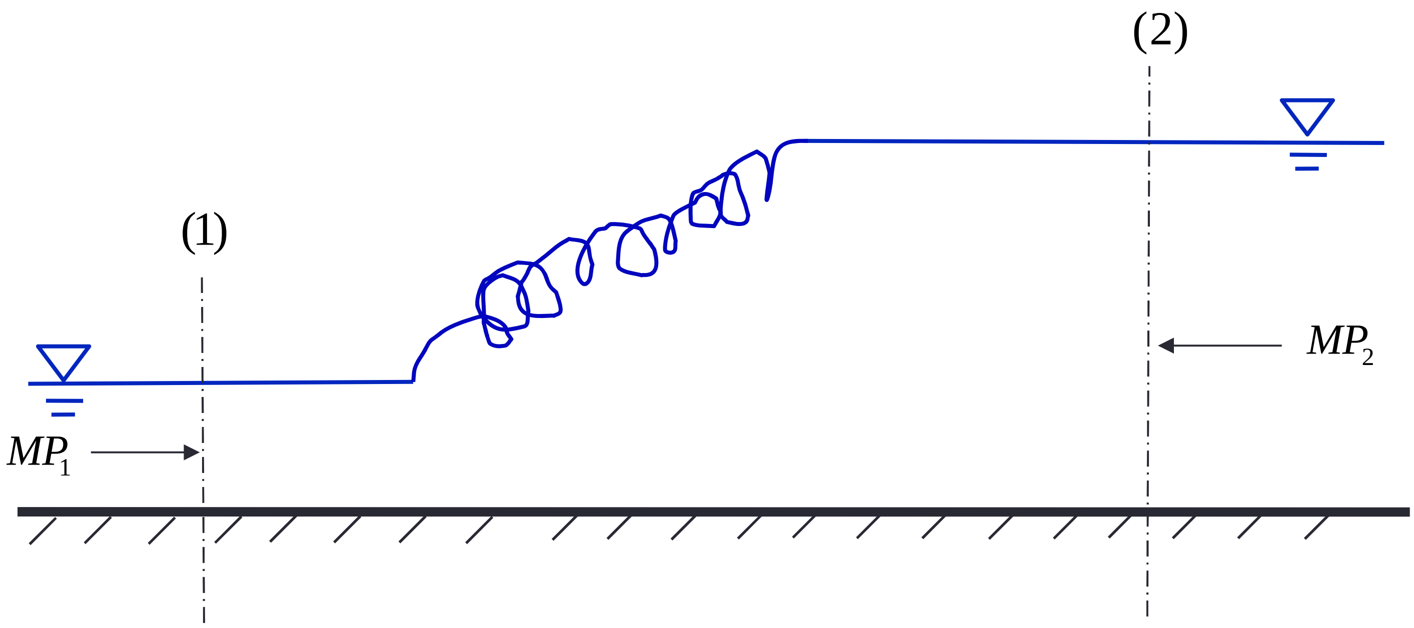


Usage of hydraulic jump

Dissipate flow kinematic energy to protect the channel from erosion

Recreation!

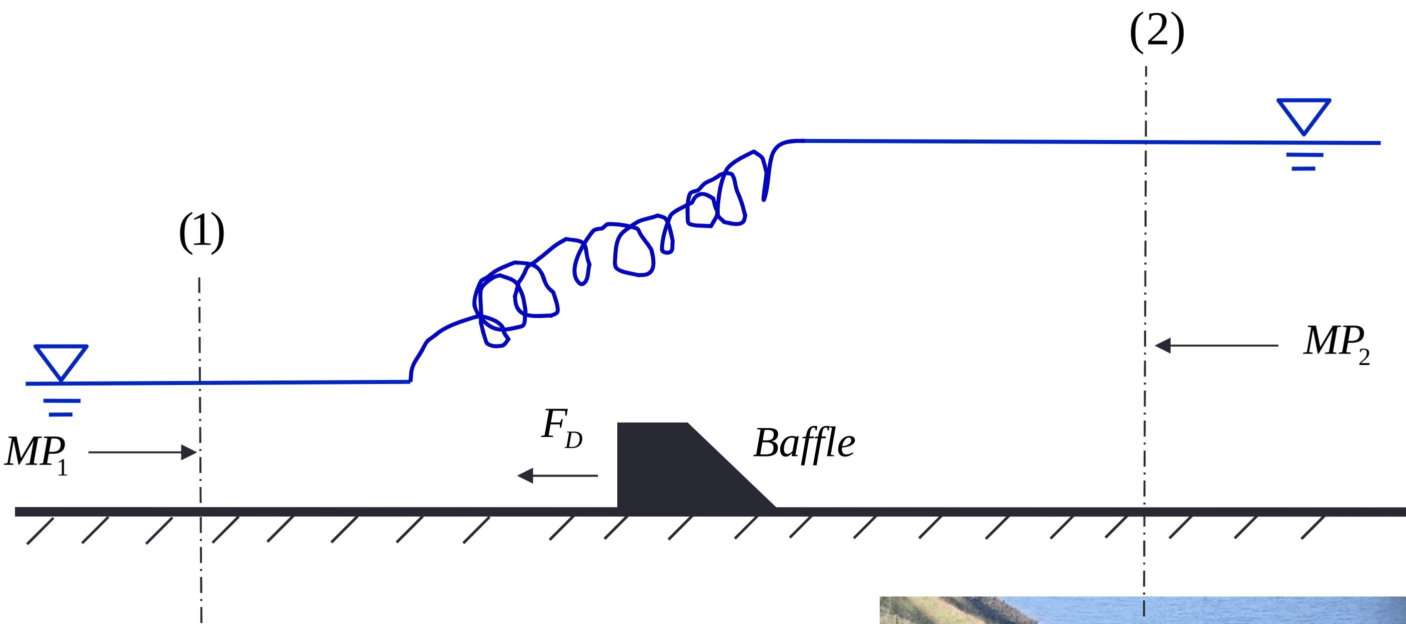
Unassisted hydraulic jump



$$MP_1 = MP_2$$

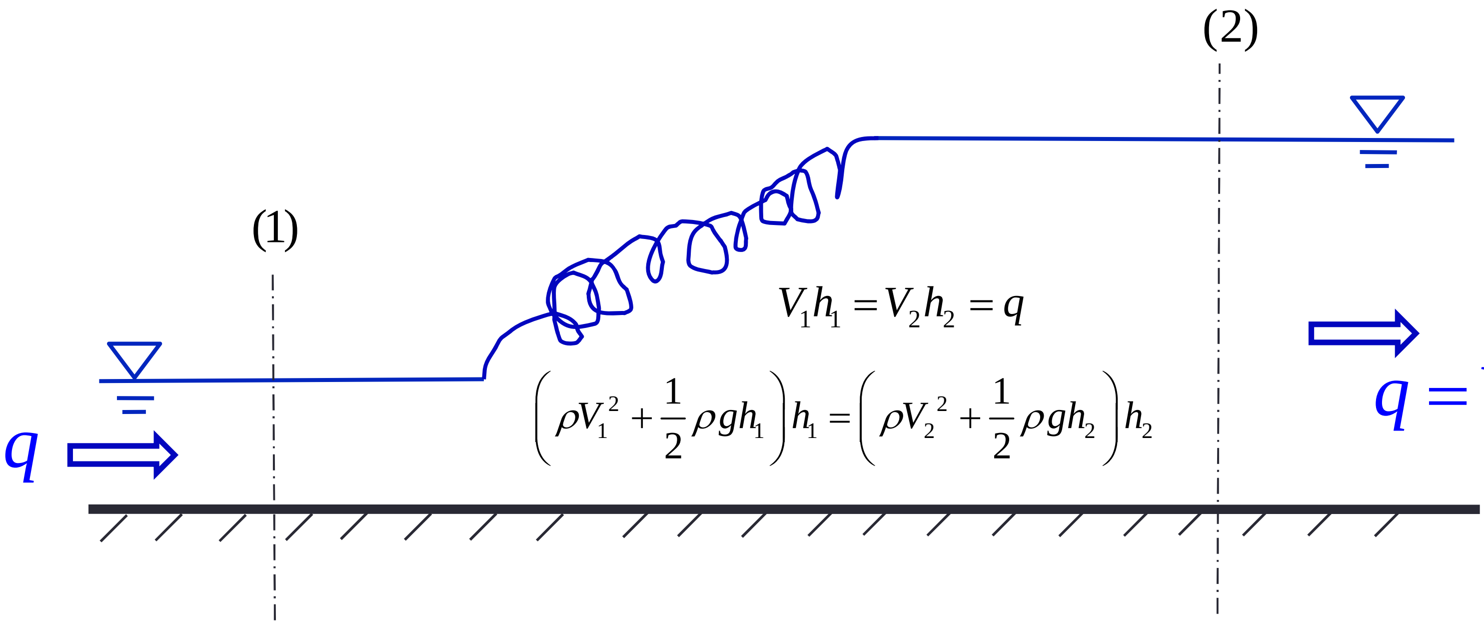
- The jump is very short, so we can neglect bottom resistance and gravity
- No force on the free surface
- Flow within the jump is steady

Assisted hydraulic jump



$$MP_1 = MP_2 + F_D$$

Jump condition: rectangular channel



$$V_1 h_1 = V_2 h_2 = q$$

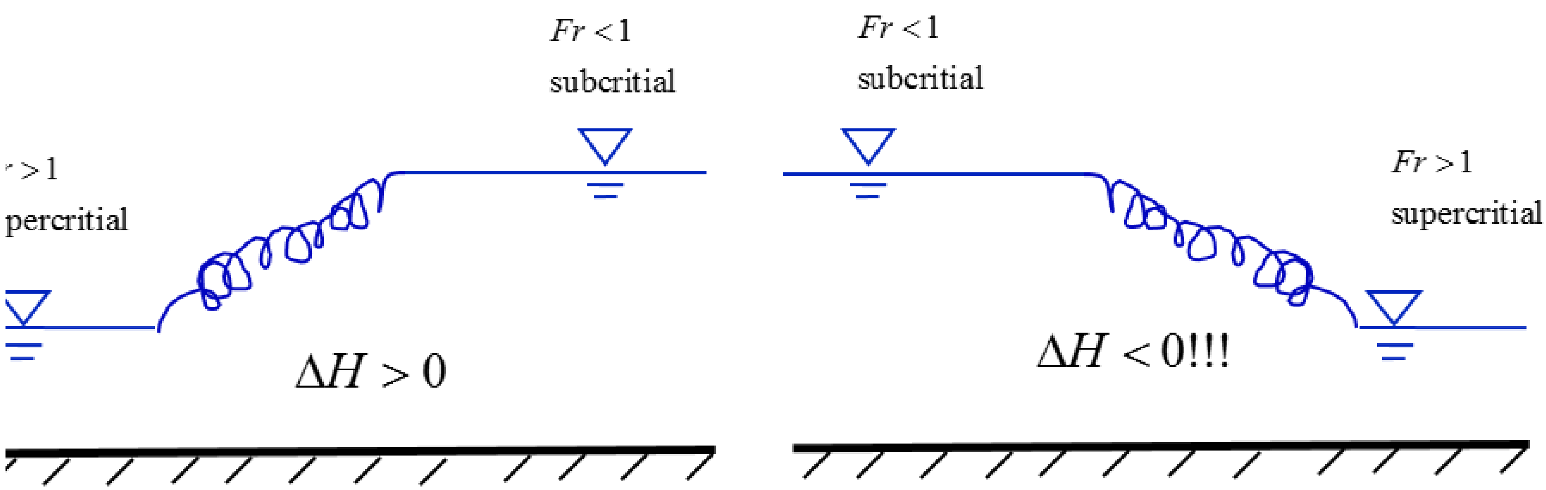
$$\left(\rho V_1^2 + \frac{1}{2}\rho g h_1\right) h_1 = \left(\rho V_2^{\,2} + \frac{1}{2}\rho g h_2\right) h_2$$

$$q = Vh$$

$$h_2 / h_1 = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

No jump from subcritical to supercritical

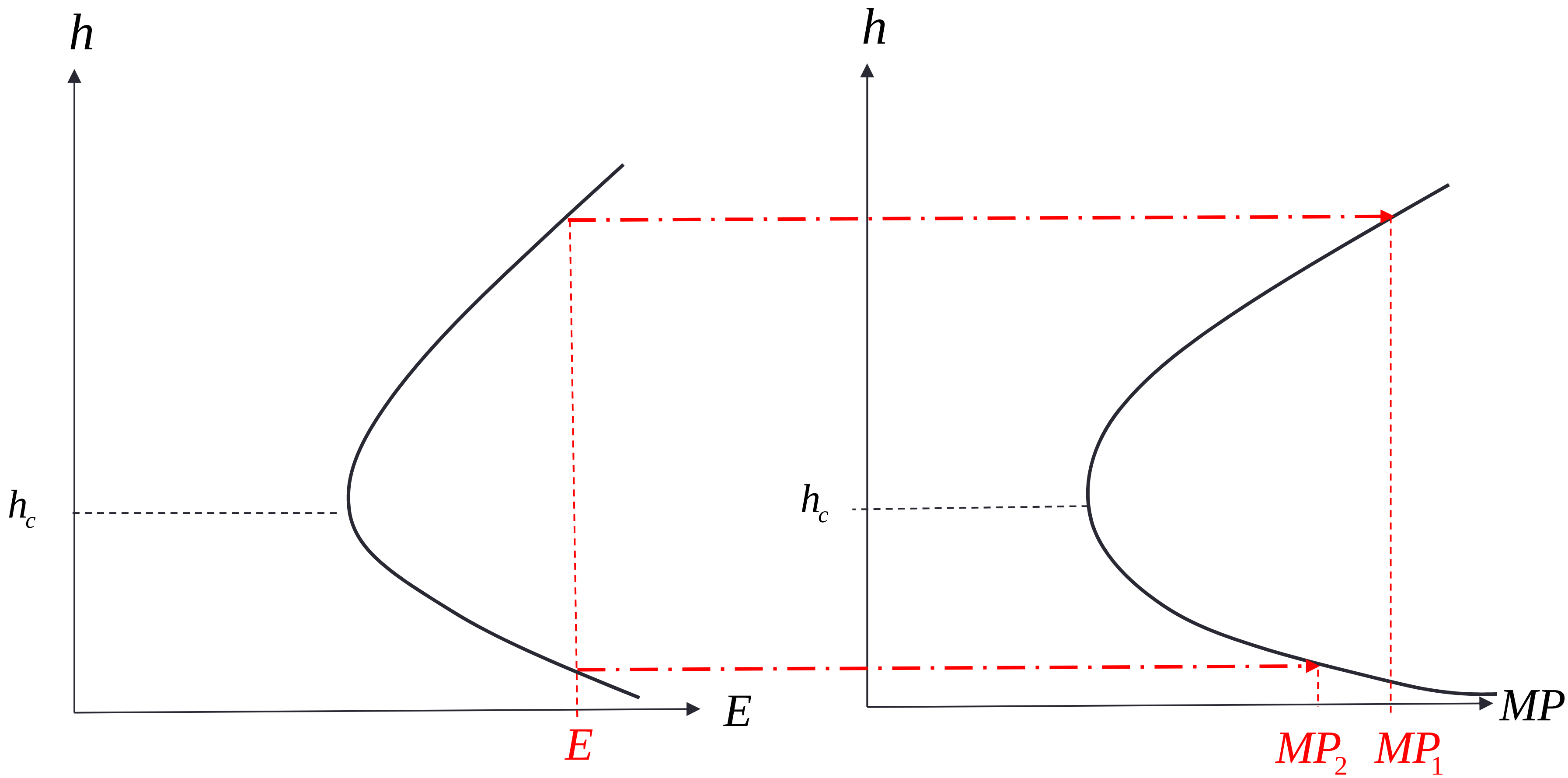
$$\Delta H_j = \frac{(h_2 - h_1)^3}{4h_1h_2}$$



Jump from supercritical to subcritical is possible

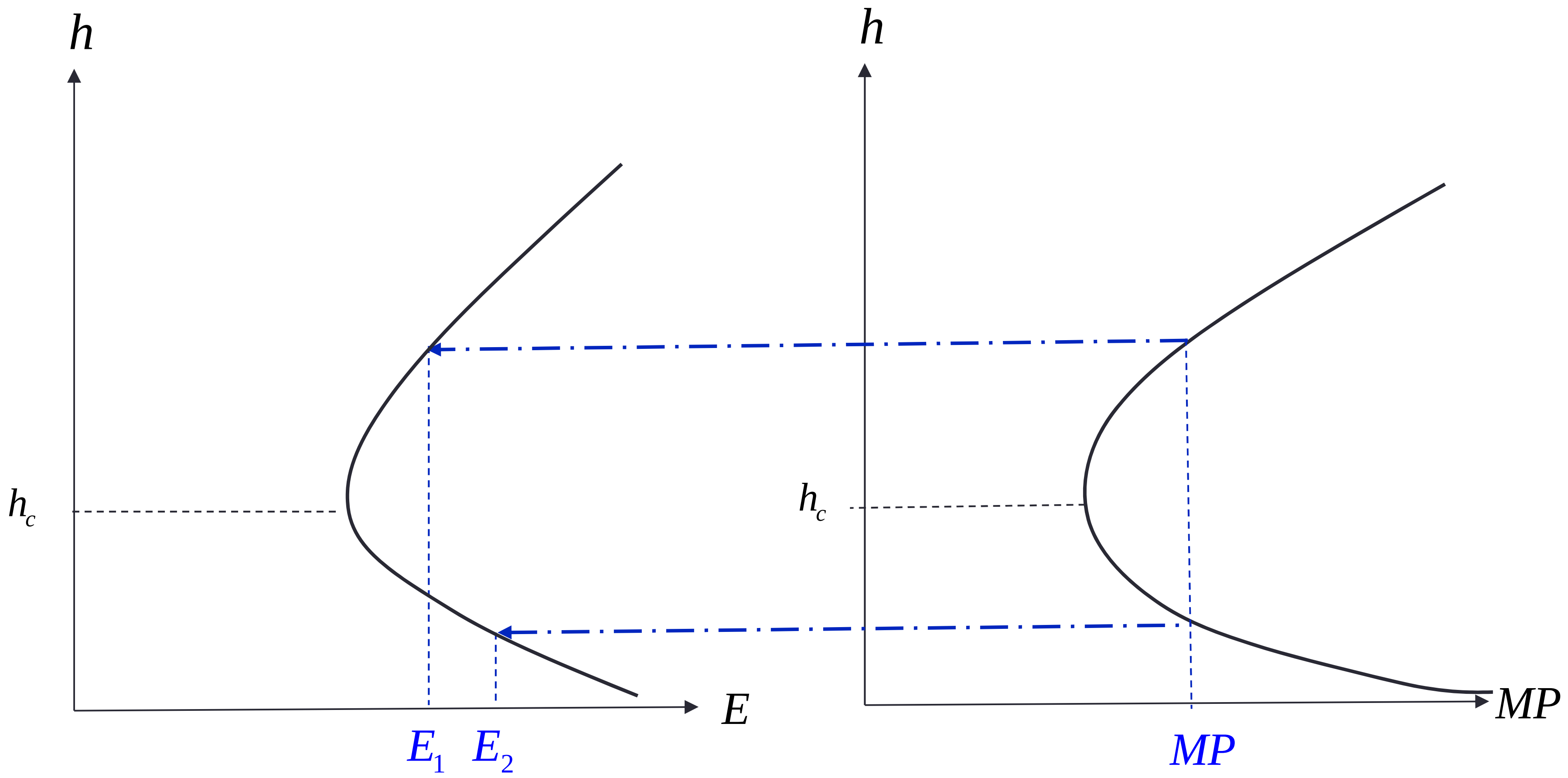
Jump from subcritical to supercritical is impossible

E-h and MP-h diagrams together



Same E, MP1>MP2
(flow over a sluice gate)

E-h and MP-h diagrams together

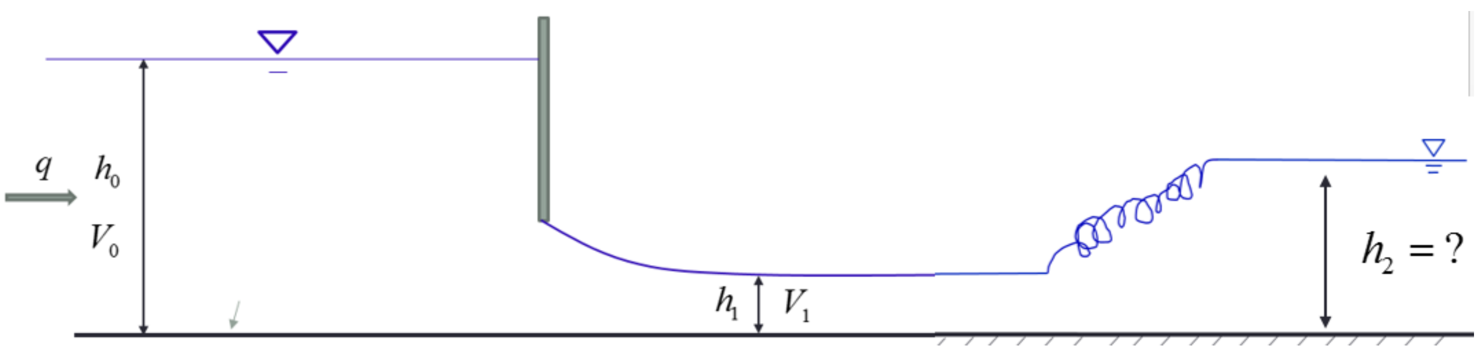


Same MP, E2>E1
(hydraulic jump)

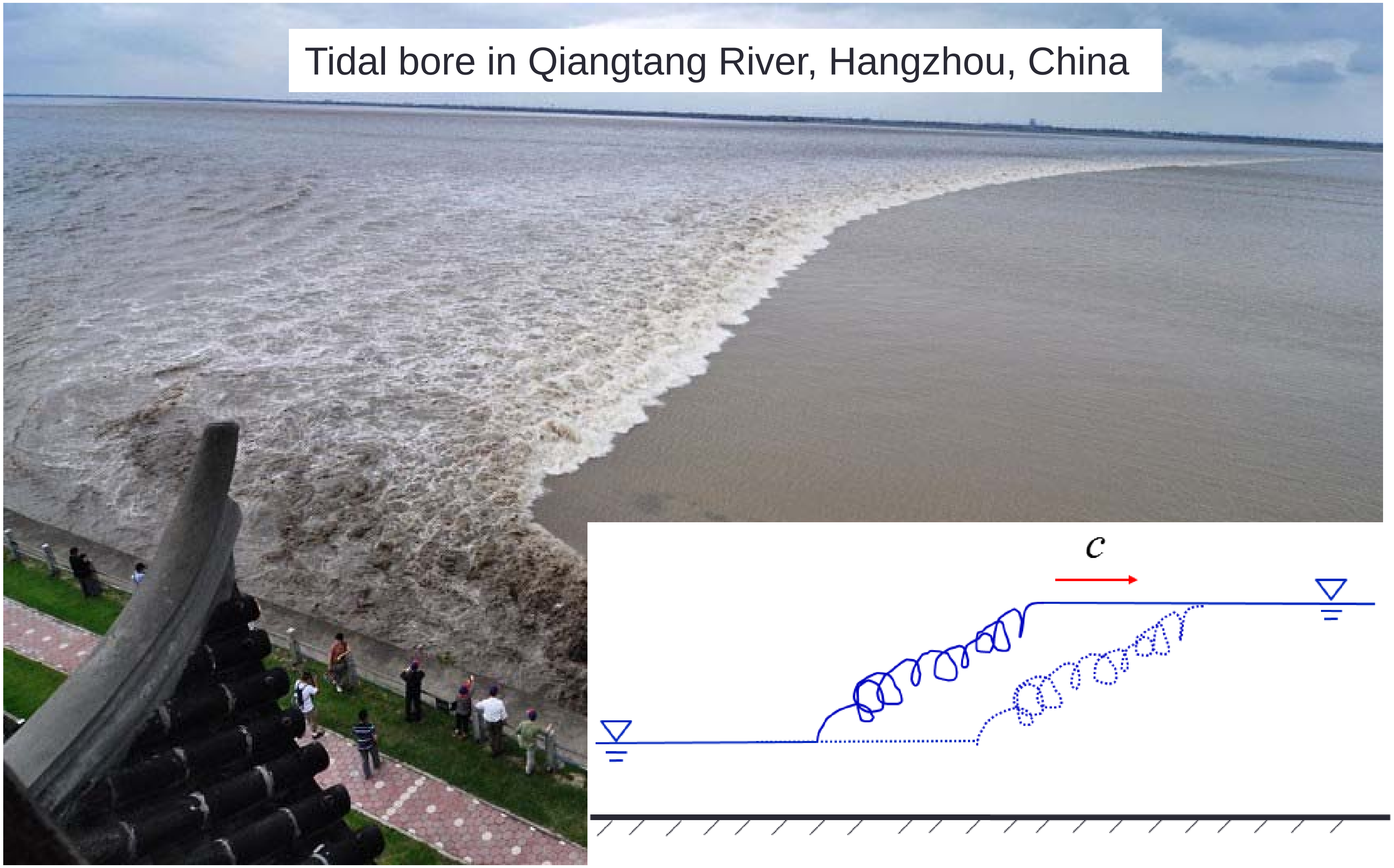
Practice

A sluice gate controls the flow in a prismatic rectangular channel. The flow becomes well-behaved shortly downstream from the gate, where the local water depth is h_1=1.7m. The water depth upstream from the gate is h_0=3m. If a hydraulic jump forms in the channel

- What will be the water depth after the jump?
- What is the head loss due to the hydraulic jump?

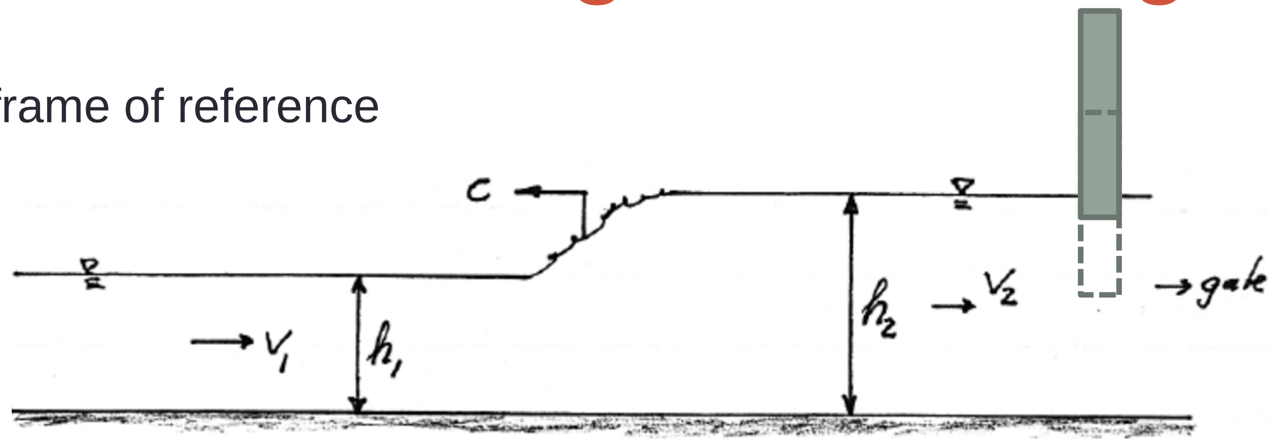


Moving hydraulic jump

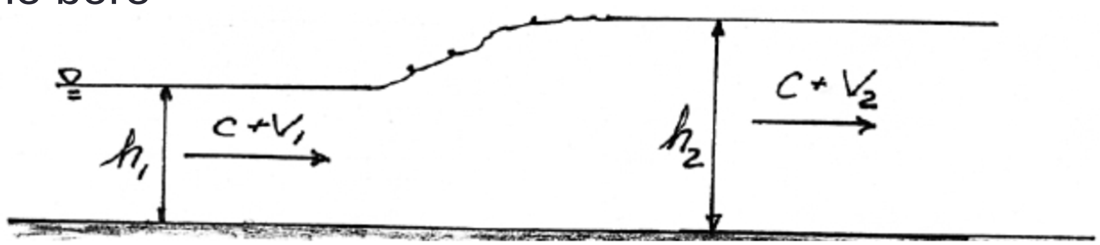


Bore due to closing of a sluice gate

Stationary frame of reference



Move with the bore



$$\left(C+V_1\right)h_1=\left(C+V_2\right)h_2$$

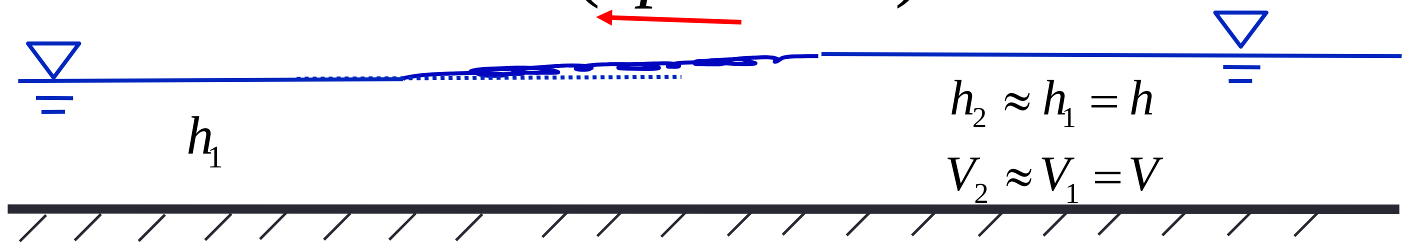
$$\left(\rho\left(C+V_1\right)^2+\frac{1}{2}\rho g h_1\right)h_1=\left(\rho\left(C+V_2\right)^2+\frac{1}{2}\rho g h_2\right)h_2$$

$$\boxed{\left(V_1+C\right)^2=\frac{1}{2}\frac{h_2}{h_1}\left(1+\frac{h_2}{h_1}\right)gh_1}$$

Jump condition

Very small bore

$c > 0(upstream)$



$$\left(V_1 + c\right)^2 = \frac{1}{2}\frac{h_2}{h_1}\left(1 + \frac{h_2}{h_1}\right) g h_1 \approx gh$$

$$c = -V \pm \sqrt{gh} = -V \pm c_0 = -\sqrt{gh}\left(1 \pm Fr\right)$$

$$c_0 = \sqrt{gh} \qquad Fr = \frac{V}{\sqrt{gh}} = \frac{V}{c_0}$$

$Fr < 1$

$$c = \begin{cases} \sqrt{gh}\left(1 - Fr\right) > 0 & \text{moving upstream} \\ -\sqrt{gh}\left(1 + Fr\right) < 0 & \text{moving downstream} \end{cases}$$

$Fr > 1$

$$c = \begin{cases} \sqrt{gh}\left(1 - Fr\right) < 0 & \text{moving downstream} \\ -\sqrt{gh}\left(1 + Fr\right) < 0 & \text{moving downstream} \end{cases}$$

The physical meaning of Froude number

$$Fr = \frac{V}{\sqrt{gh_m}} = \frac{\text{Fluid velocity}}{\text{Speed of small disturbance}}$$

- Fr <1 (subcritical flow): Changes in flow conditions are felt upstream of the location where changes occur. Subcritical flows are controlled by downstream conditions
- Fr >1 (Supercritical flow): Changes in flow conditions are felt only downstream of location where changes occur. Upstream is entirely unaware of what happens downstream. Supercritical flows are controlled by upstream conditions.