STEADY OPEN-CHANNEL FLOW 1

~~RIVER MECHANICS~~ OPEN-CHANNEL HYDRAULICS (CE5312)

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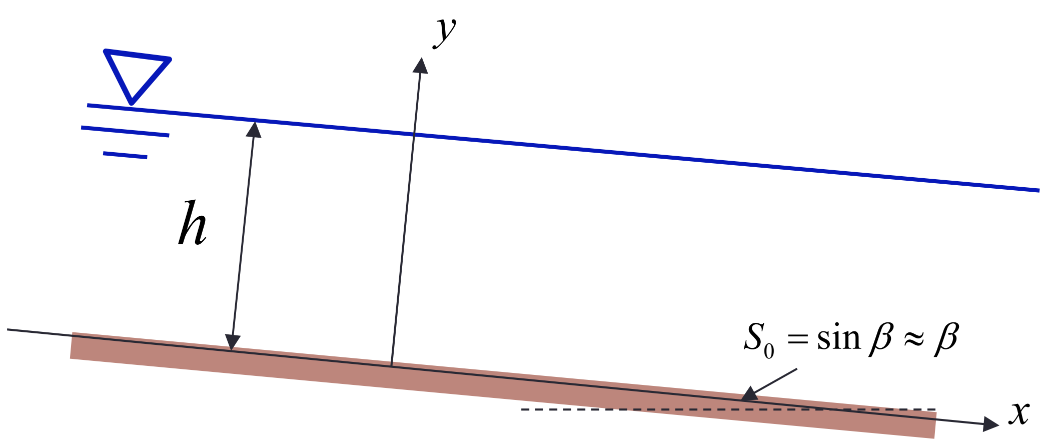
Some terminologies



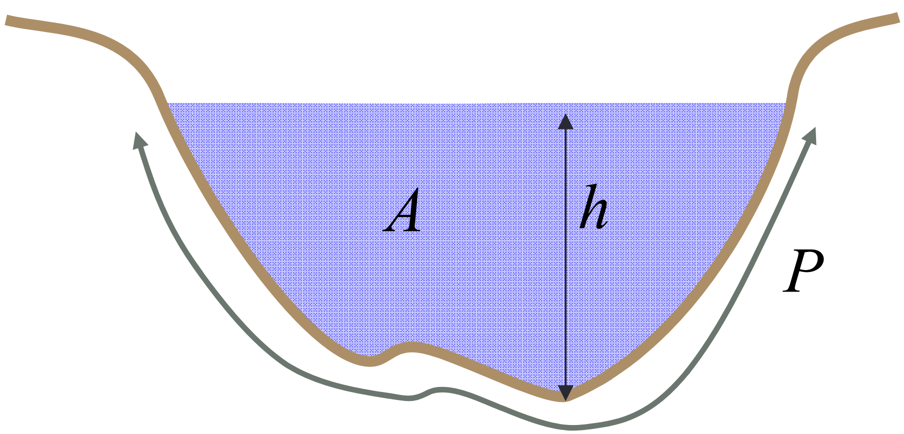
x : streamwise direction

y : direction $\perp$ bottom

h : water depth

A : Cross-section area

P : Wetted perimeter $[L]$



Prismatic channel: constant cross-section

$$A = f(h)$$

$$P = f(h)$$

Non-prismatic channel

$$A = f(x, h)$$

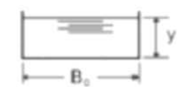
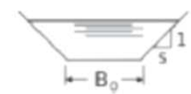
$$P = f(x, h)$$

Hydraulic radius

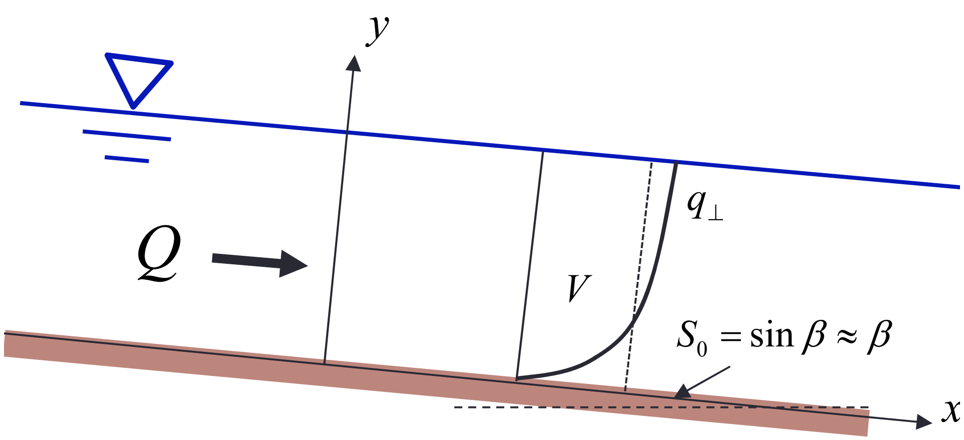
$$\boxed{R_h = \frac{A}{P}: \text{ hydraulic radius}}$$

A characteristic length scale

Table 1-1. Properties of typical channel cross sections

Section	Area, A	Wetted Perimeter, P	Hydraulic radius, R	Top width, B	Hydraulic depth, D	
Rectangular	$B_o y$	$B_o + 2y$	$\frac{B_o y}{B_o + 2y}$	B_o	y	
Trapezoidal	$(B_o + sy)y$	$B_o + 2y\sqrt{1+s^2}$	$\frac{(B_o + sy)y}{B_o + 2y\sqrt{1+s^2}}$	$B_o + 2sy$	$\frac{(B_o + sy)y}{B_o + 2sy}$	
Triangular	sy^2	$2y\sqrt{1+s^2}$	$\frac{sy}{2\sqrt{1+s^2}}$	$2sy$	$0.5y$	
Circular	$\frac{1}{8}(\theta - \sin\theta)D_o^2$	$\frac{1}{2}\theta D_o$	$\frac{1}{4}\left(1 - \frac{\sin\theta}{\theta}\right)D_o$	$D_o \sin\frac{1}{2}\theta$	$\left(\frac{\theta - \sin\theta}{\sin\frac{1}{2}\theta}\right)\frac{D_o}{8}$	

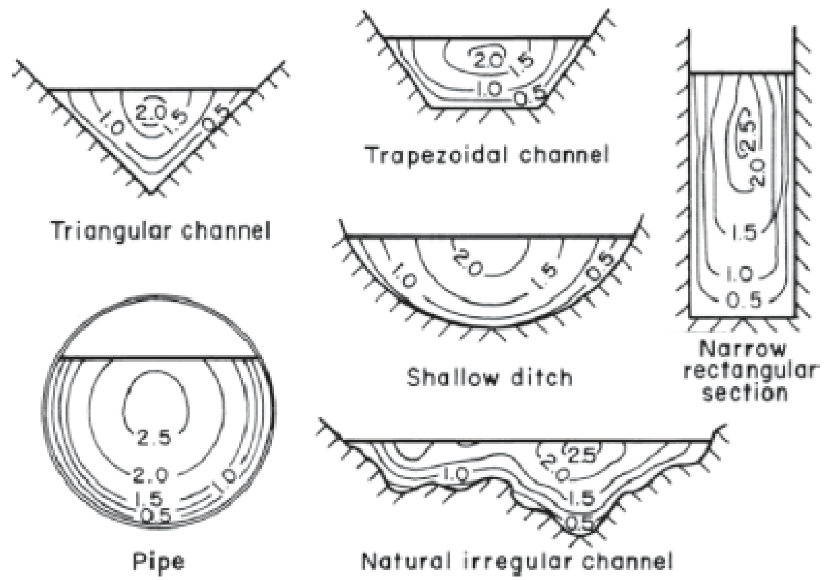
Velocity distribution in open channels



$$Q = \int_A q_\perp dA: \text{ Discharge } [L^3 / T]$$

$$V = \frac{Q}{A}: \text{ average velocity}$$

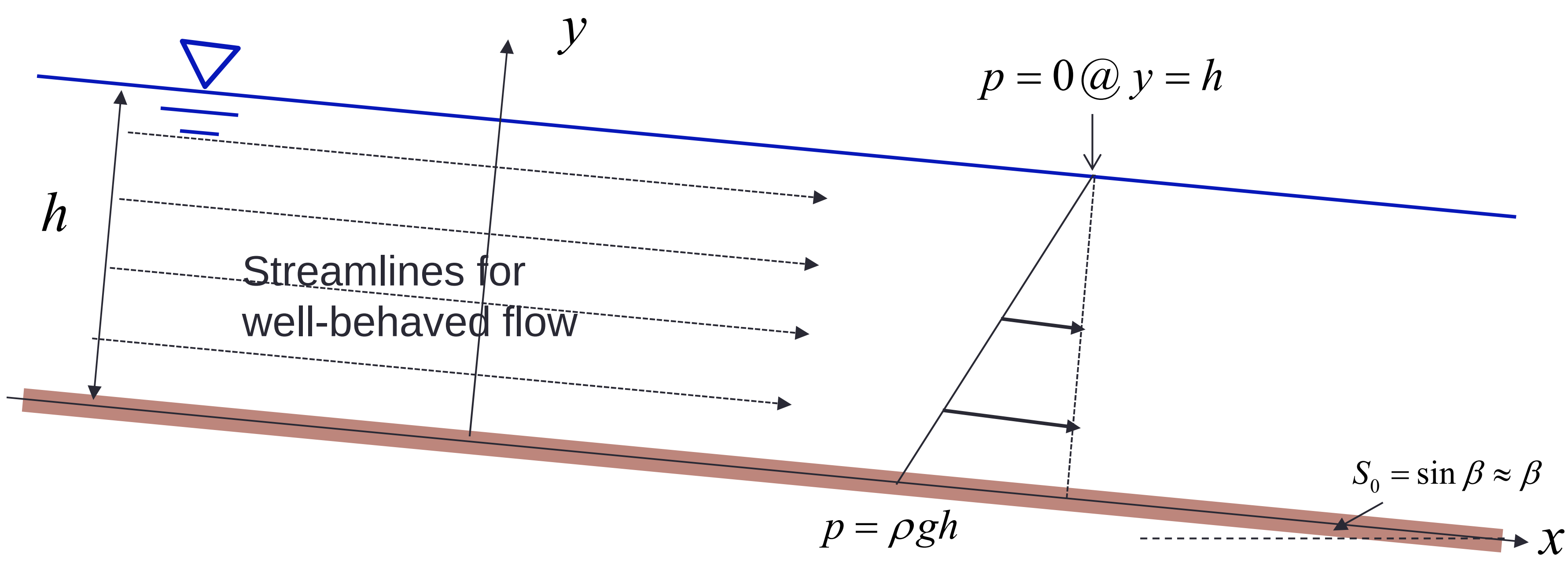
Velocity reduces to zero at boundaries



Velocity is not uniformly distributed, but usually we are interested in the total discharge (or the average velocity), not the spatial variation

The spatial variation is not dramatic, so we can treat the flow as well-behaved or spatially uniform

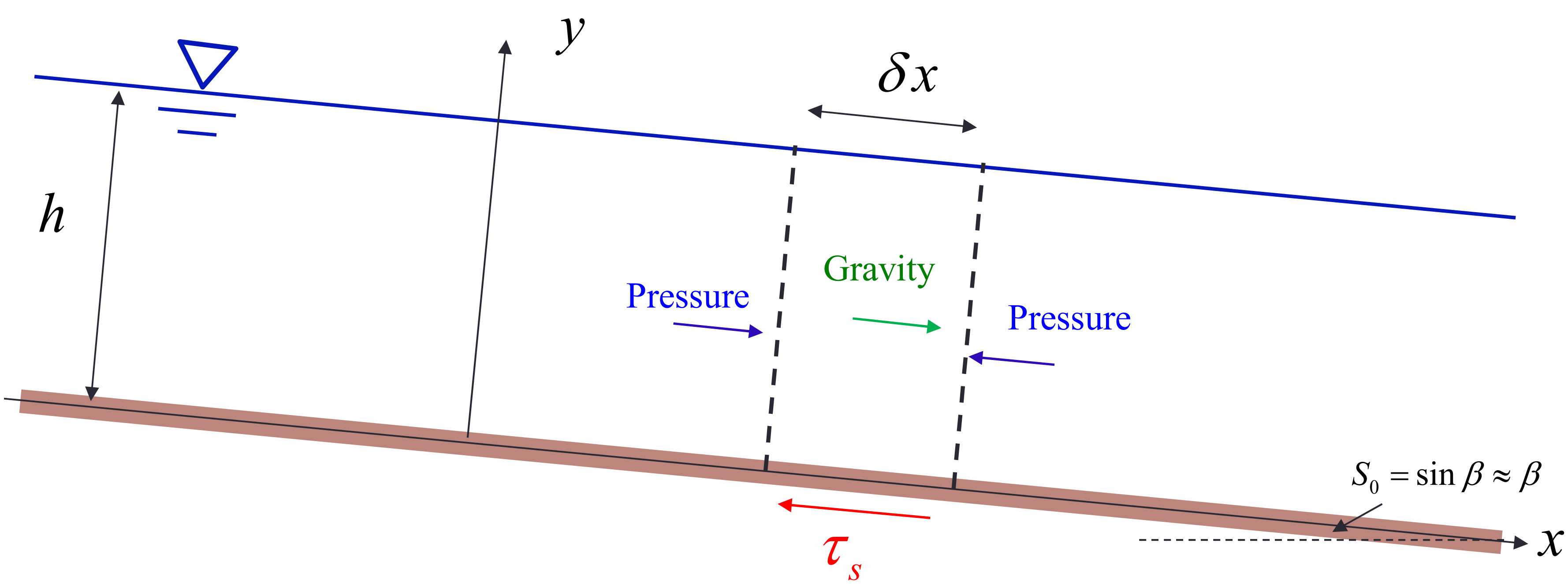
Pressure distribution



Bernoulli equation perpendicular to streamlines:

$$\left.\begin{aligned} &\frac{dp}{dy} = -\rho g \cos\beta \\ &p = 0, @\, y = h \end{aligned}\right\} \Rightarrow p = \rho g \cos\beta\left(h - y\right) \Rightarrow p \approx \rho g\left(h - y\right)$$

Basic hydraulic formula



$$\boxed{\tau_s = \rho g R_h S_o}$$

Interpretation: for steady uniform flow, gravity is balanced by bottom resistance

How do we relate bottom shear stress to flow?

Let us consider what we have obtained for PIPE FLOW.

Factors related to pipe-wall shear stress?

- Flow intensity $V = \frac{Q}{A}$
- Pipe geometry
  D (diameter)
  k_s (surface roughness)
- Fluid Property
  ν (viscosity)
  ρ (density)

	k_s [mm]
Riveted steel	3
Concrete	0.3-3
Wood	0.3
Cast iron	0.26
Galvanized iron	0.15
Wrought iron	0.046
Drawn tubing	0.0015

$$\tau_s \sim F(D, \rho, \nu, V, k_s) \Rightarrow \boxed{\tau_s = \frac{1}{8} f \rho V^2}$$

Dimensional analysis

$$\boxed{f = f\left(\frac{k_s}{D}, \mathrm{Re} = \frac{DV}{\nu}\right)}$$

Darcy-Weisbach friction factor

Friction factor for turbulent pipe flows

Rough pipes:

$$\tau_s = \frac{1}{8} f \rho V^2, \; f = f(\frac{k_s}{D}, \mathrm{Re})$$

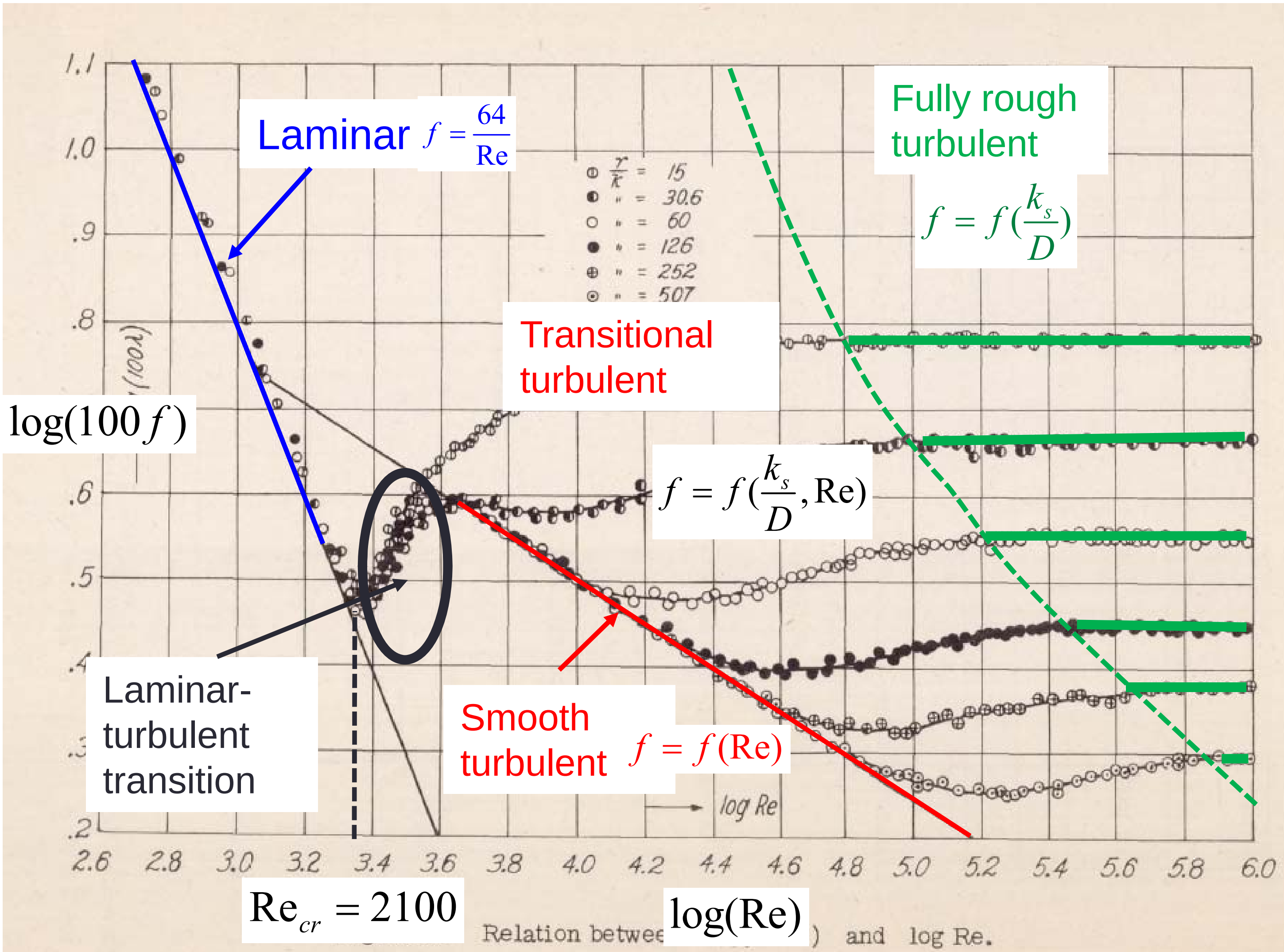
No simple analytical way to get $f(k_s/D, \mathrm{Re})$

So we do experiments!

- For each k_s/D, we can get a curve for $f(\mathrm{Re})$

Figure 1 Johann Nikuradse around 1925 at KWI, Göttingen

Nikuradse's experiment: rough pipes



Moody diagram: the $f = f(\frac{k_s}{D}, \mathrm{Re})$ for turbulent pipe flows

$\frac{\varepsilon}{D} = 0.0001$

$Re = 10^6$

$f = ?$

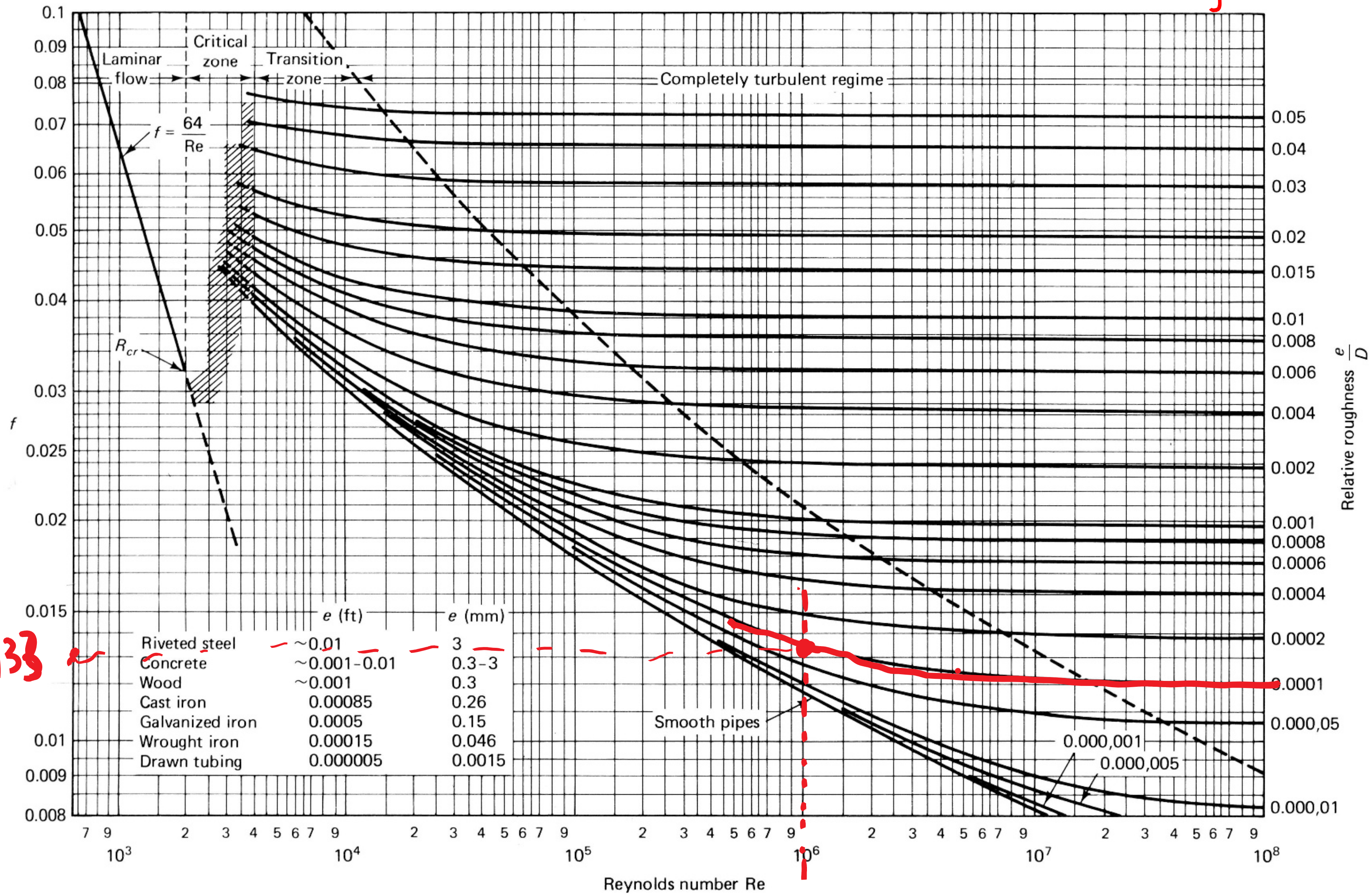


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

$f \approx 0.0133$

How to use Moody for open-channel flow?

Given: V, h, ν, k_s

Find : f, τ_s

$h \dashrightarrow R_h(h)$

$D \dashrightarrow 4R_h(h)$

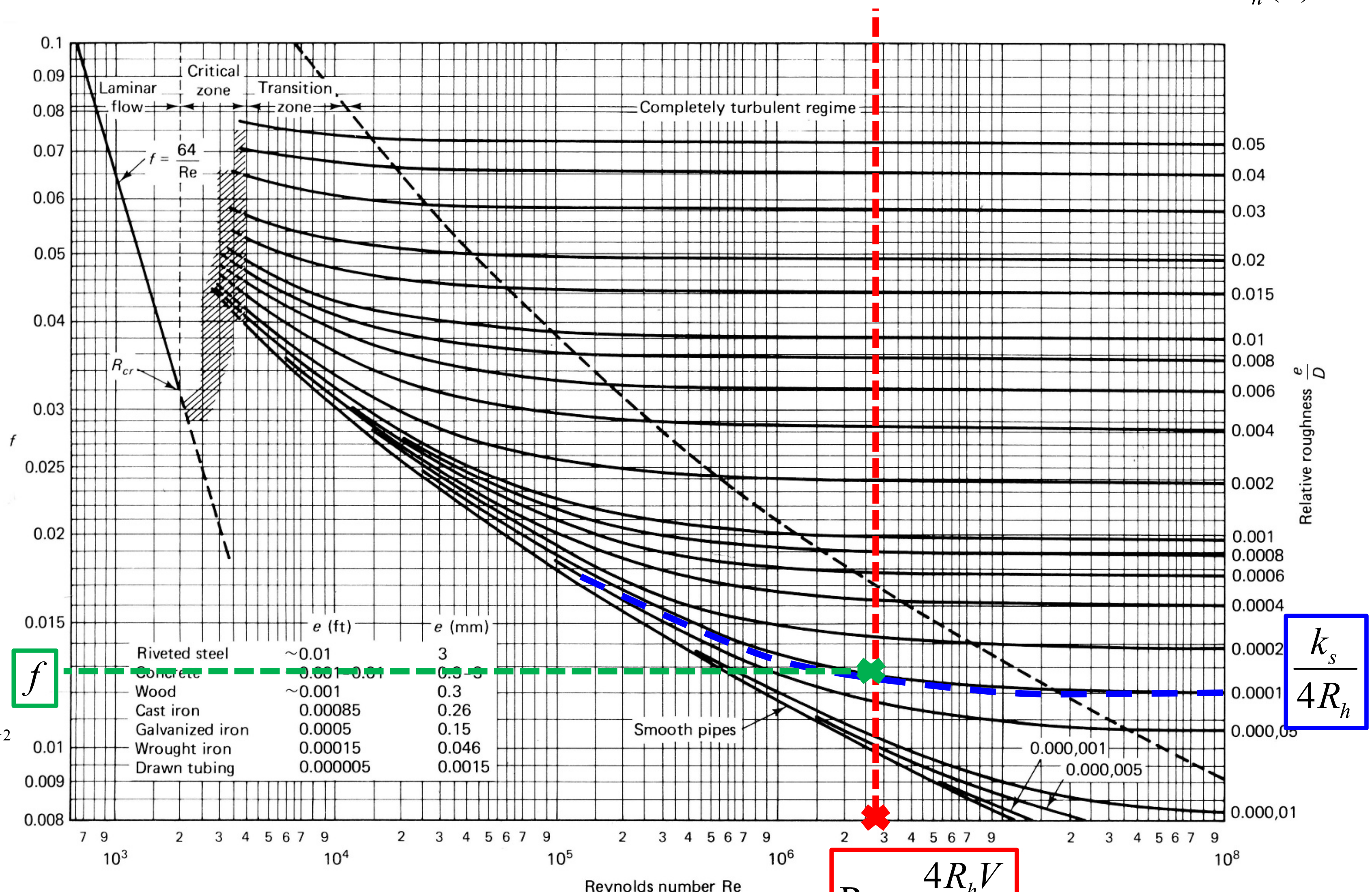


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. A*

$\boxed{f}$

$\tau_s = \frac{1}{8} f \rho V^2$

$\boxed{\frac{k_s}{4R_h}}$

$\boxed{\mathrm{Re} = \frac{4R_h V}{\nu}}$

Use Moody diagram:

$$\text{Given: } \rho, \nu, k_s, h, S_0 \qquad h --> R_h(h), A(h) \qquad \tau_s = \rho g R_h S_0$$

$$\text{Find: } \quad Q$$

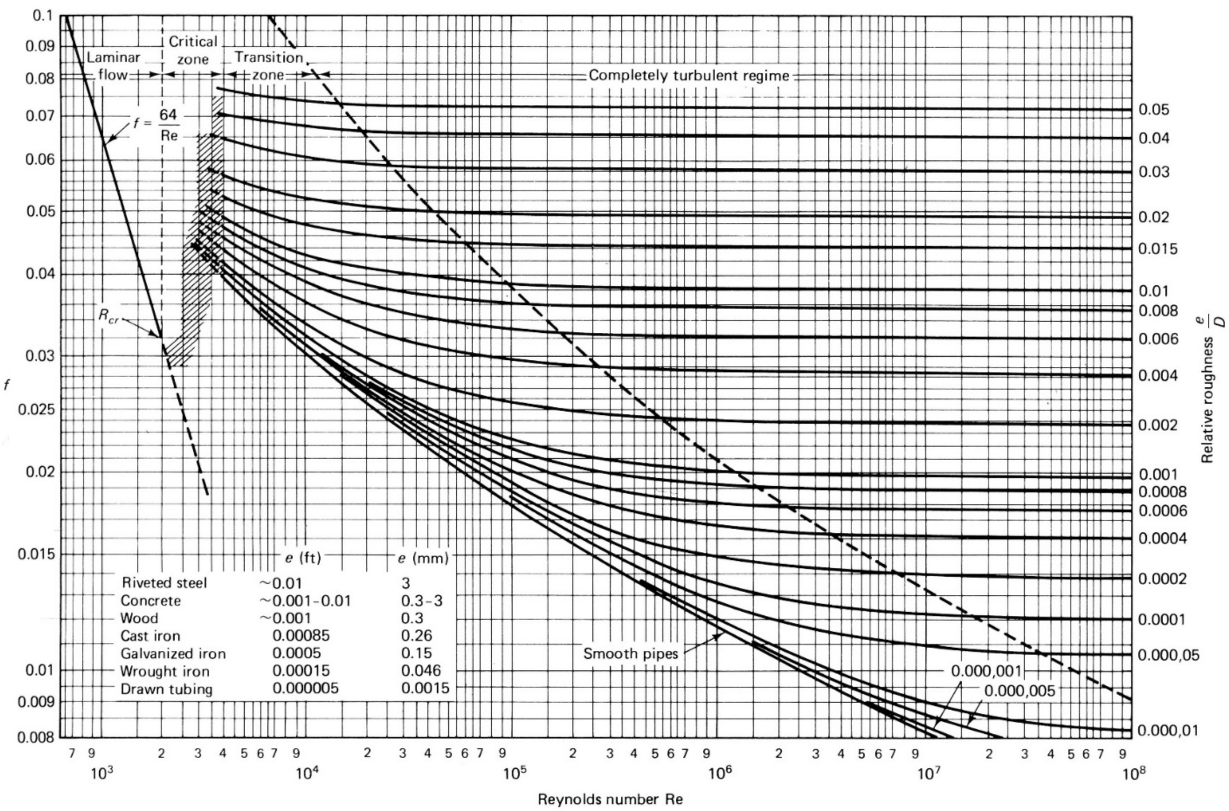


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

$$\text{Guess, } f^{(1)}$$

$$V^{(1)} = \sqrt{\frac{8\tau_s}{f^{(1)}\rho}}$$

$$\mathrm{Re}^{(1)} = \frac{4R_h V^{(1)}}{\nu}$$

$$f^{(2)} = Moody(\mathrm{Re}^{(1)}, \frac{k_s}{4R_h})$$

$$V^{(2)} = \sqrt{\frac{8\tau_s}{f^{(2)}\rho}}$$

$$\cdots$$

$$\text{until } V^{(n+1)} = V^{(n)}$$

$$Q = VA(h)$$

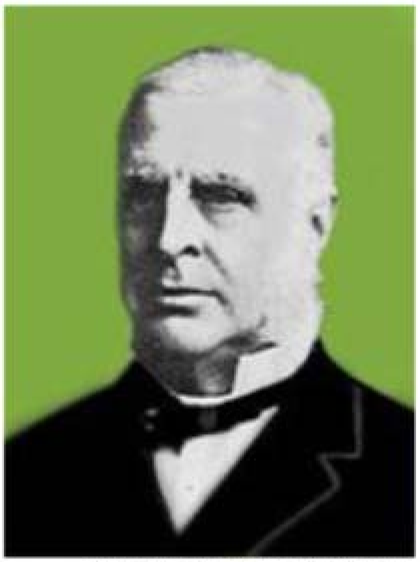
Chezy Equation

French engineer
1718-1798

$$V = C\sqrt{R_h}\sqrt{S_o}$$

$$C = \text{Chezy's Coefficient} \sim [L^{1/2} / T]$$

Manning Equation



ROBERT MANNING

Irish accountant!
1816-1897

$$V = \frac{1}{n} R_h^{2/3} \sqrt{S_o}$$

n = Manning's coefficient

$\sim[L^{-1/3} \cdot T]$

Inconsistent but widely used

Table 4-1. Typical values* of Manning n

Material	n
Metals	
Steel	0.012
Cast iron	0.013
Corrugated metal	0.025
Non-metals	
Lucite	0.009
Glass	0.010
Cement	0.011
Concrete	0.013
Wood	0.012
Clay	0.013
Brickwork	0.013
Gunite	0.019
Masonary	0.025
Rock cuts	0.035
Natural streams	
Clean and straight	0.030
Bottom: gravel, cobbles and boulders	0.040
Bottom: cobbles with large boulders	0.050

Chezy Vs. Manning Vs. Darcy-Weisbach

$$\boxed{\frac{1}{8} f \rho V^2 = \rho g R_h S_o}$$

D.W.

$$C = \sqrt{\frac{8g}{f}}$$

$$n = 0.038 \cdot k_s^{1/6} \text{ [SI unit]}$$

$$\boxed{V = C\sqrt{R_h}\sqrt{S_o}}$$

Chezy

$$C = \frac{1}{n} R_h^{1/6}$$

$$\boxed{V = \frac{1}{n} R_h^{2/3} \sqrt{S_o}}$$

Manning

- For smooth bottom, we use D.W. or Chezy, NOT Manning

Discharge for steady uniform flow

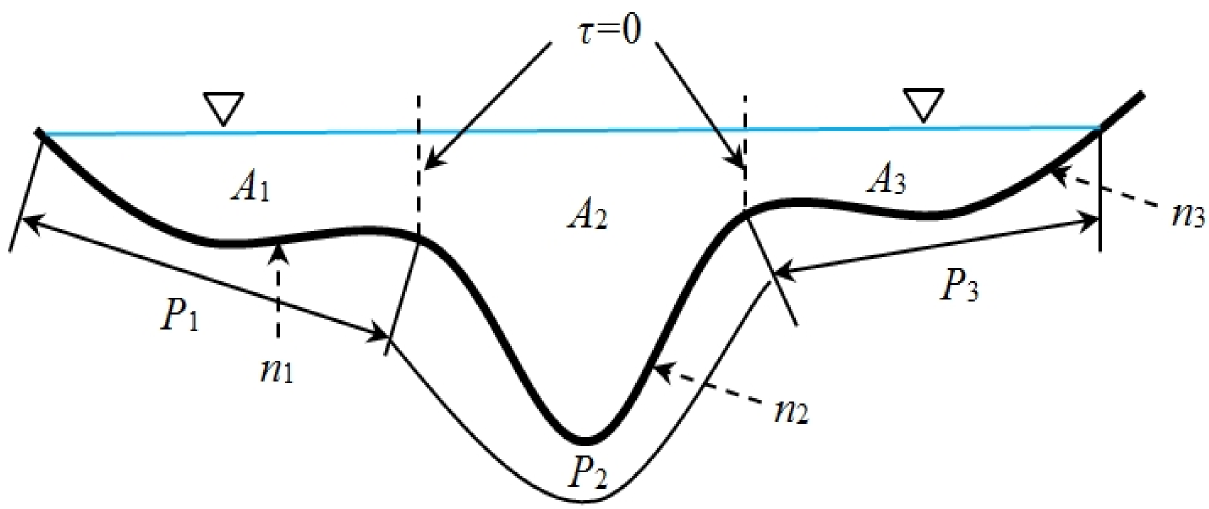
$$Manning: \ Q = \frac{1}{n}\frac{A(h)^{5/3}}{P(h)^{2/3}}\sqrt{S_o}$$

$$Q = f(h, S_0, bottom)$$

For steady uniform flow, the discharge is a function of depth, slope and bottom condition

- Depth and slope determine gravity, which is the driving force
- Bottom condition determines the bottom resistance, which is the balancing force

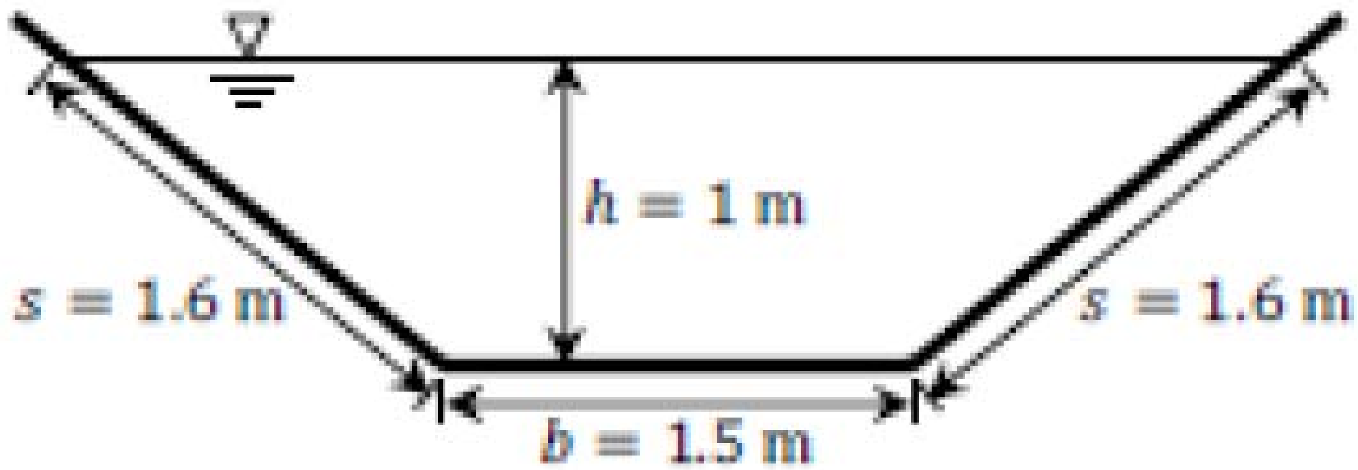
Compound channel



$$Q = \sum Q_n = \sum \frac{1}{n_n} A_n \left(\frac{A_n}{P_n} \right)^{2/3} \sqrt{S_o}$$

Practice

Water flows in a long straight channel with a trapezoidal cross-section. The bottom drops $1.6\ \mathrm{m}$ per km. The horizontal bottom is finished concrete (n=0.012) and the sides are clay lined ($n = 0.026$). Find Q



Normal depth

$$Q = f(h, S_0, bottom) \Rightarrow h = F(Q, S_0, bottom)$$

The depth for steady uniform flow with a given discharge in an open channel with specified slope and bottom condition.

Normal depth calculation: Manning

$$Manning: \ Q = \frac{1}{n}\frac{A(h)^{5/3}}{P(h)^{2/3}}\sqrt{S_o}$$

$$\Downarrow$$

$$K(h) = \frac{A(h)^{5/3}}{P(h)^{2/3}} = \frac{nQ}{\sqrt{S_o}}$$

Normal depth calculation: D.W.

Given: k_s, Q, S_0

Find: h_n

$$V = \sqrt{\frac{8g}{f}}\sqrt{R_h}\sqrt{S_0} \Rightarrow \frac{fQ^2}{8gS_0} = R_h A^2$$

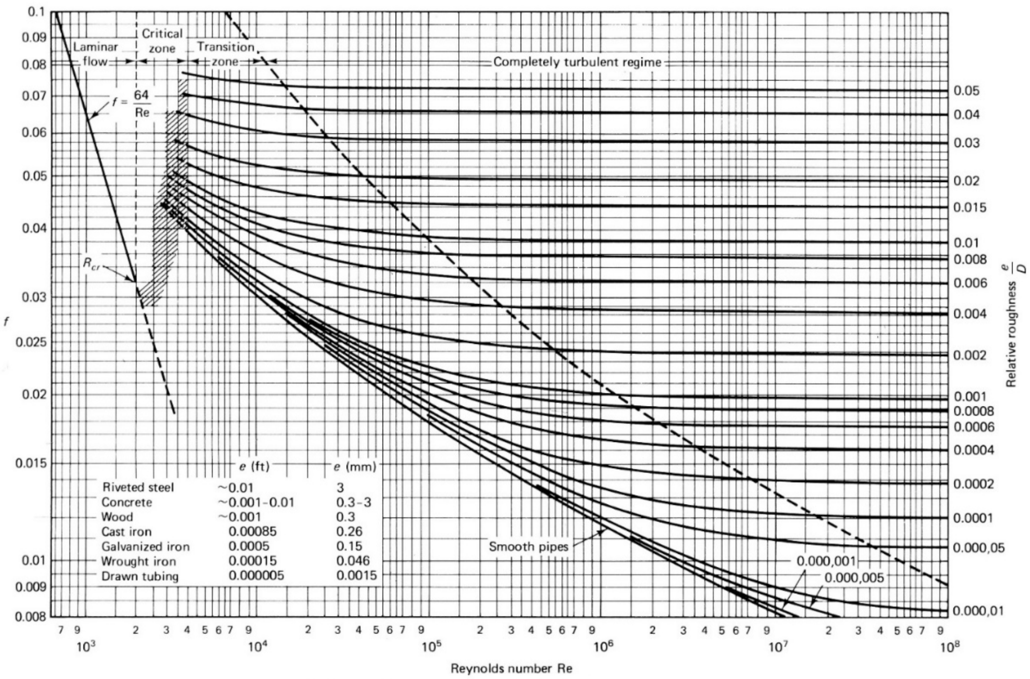


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

Guess, $f^{(1)}$

$$\frac{f^{(1)}Q^2}{8gS_0} = R_h^{(1)}\left(A^{(1)}\right)^2 \Rightarrow h^{(1)}, R_h^{(1)}, V^{(1)} = \frac{Q}{A^{(1)}}$$

$$f^{(2)} = Moody\left(\frac{4R_h^{(1)}V^{(1)}}{\nu}, \frac{k_s}{4R_h^{(1)}}\right)$$

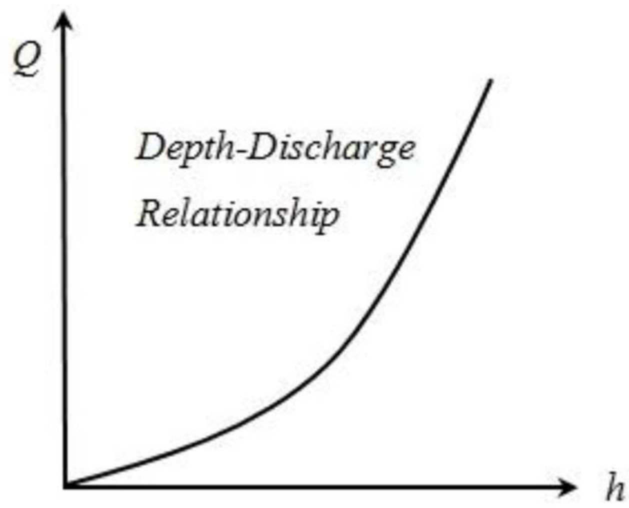
$$\cdots$$

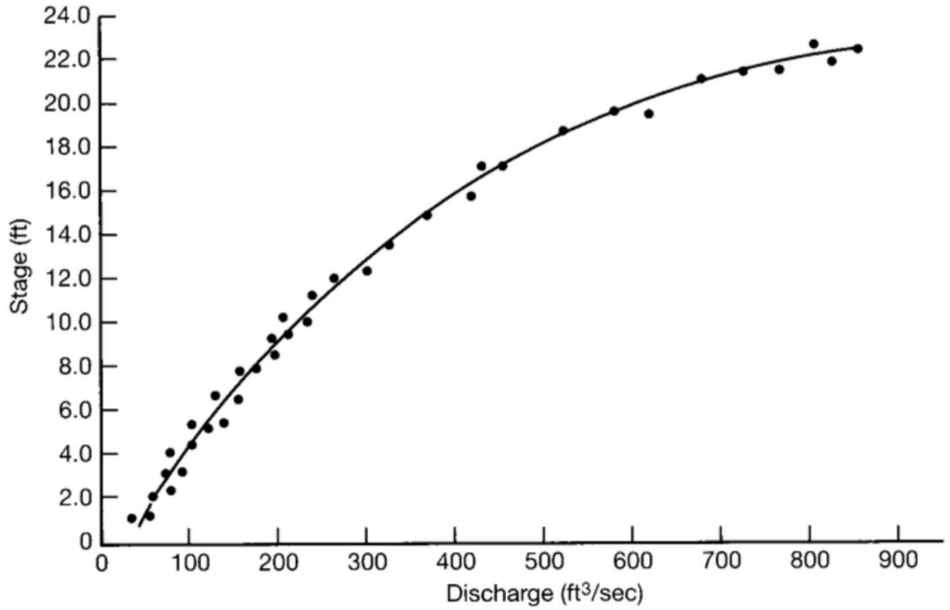
$$\text{until } f^{(n+1)} = f^{(n)}$$

Rating curve

$$K(h) = \frac{A(h)^{5/3}}{P(h)^{2/3}} = \frac{nQ}{\sqrt{S_o}} \quad \Rightarrow \quad Q = \frac{\sqrt{S_o}}{n} K(h)$$

Not a golden rule!





d L. L. Sanders (1998) A Manual of Field Hydrogeology

Normal depth In rectangular channel

Using Manning's equation:

$$\boxed{h_n = \left(\frac{Qn}{\sqrt{S_o} b} \right)^{3/5} \left(1 + 2h_n / b\right)^{2/5}}$$

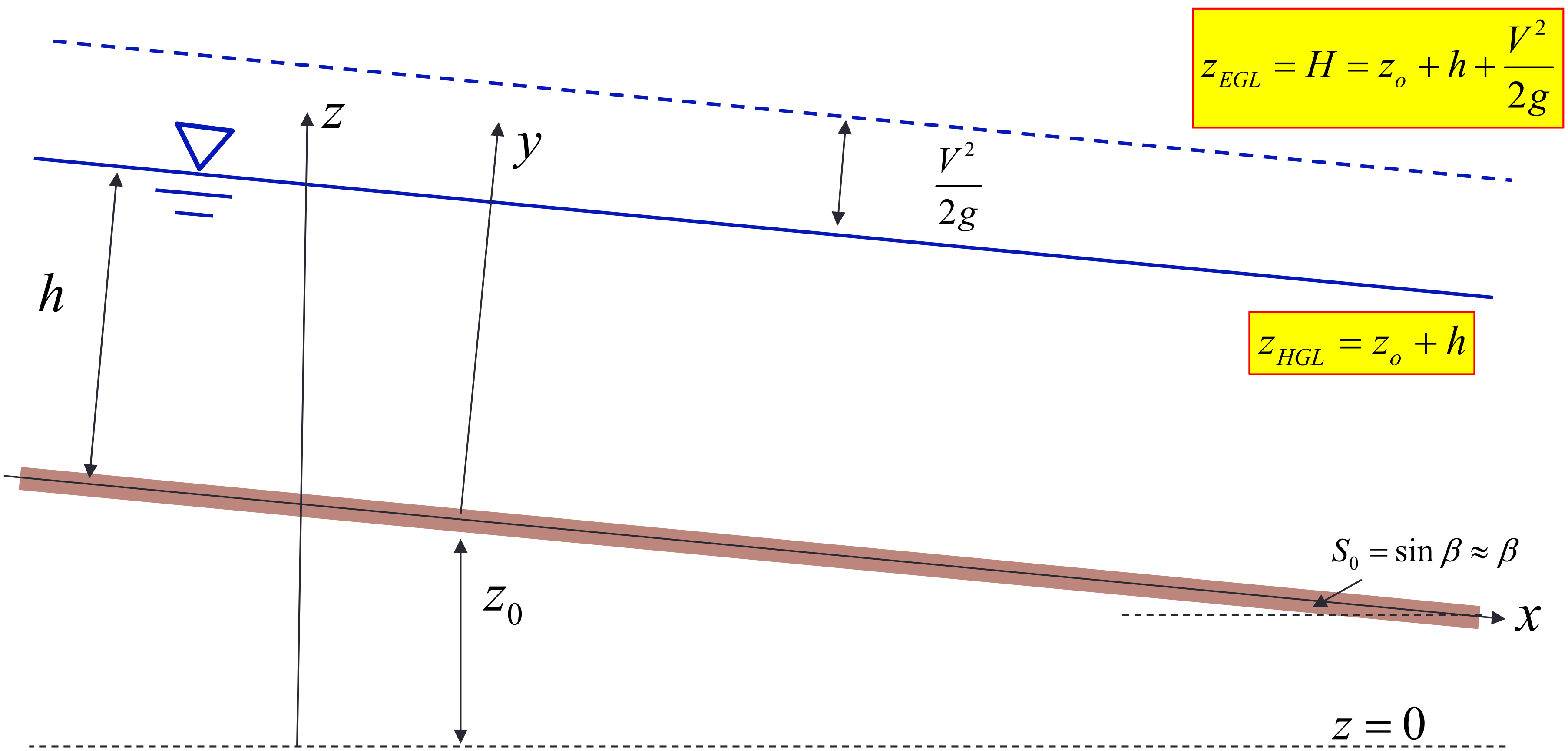
Iterative solution:

$$h^{(n+1)} = \left(\frac{Qn}{\sqrt{S_o} b} \right)^{3/5} \left(1 + 2h^{(n)} / b\right)^{2/5} \text{ , staring with } h^{(0)}=0$$

Energy grade line and hydraulic grade line

For steady uniform open-channel flow:

- Energy grade line is parallel to free surface, and with $V^2/2g$ above.
- Hydraulic grade line is identical to free surface



Energy equation for steady uniform flow

$$\boxed{\frac{\partial H}{\partial x} = -S_f}$$

change of total water head, $\partial H/\partial x$, is due to bottom flow resistance, S_f

It's actually equivalent to the Basic Hydraulic Formula.