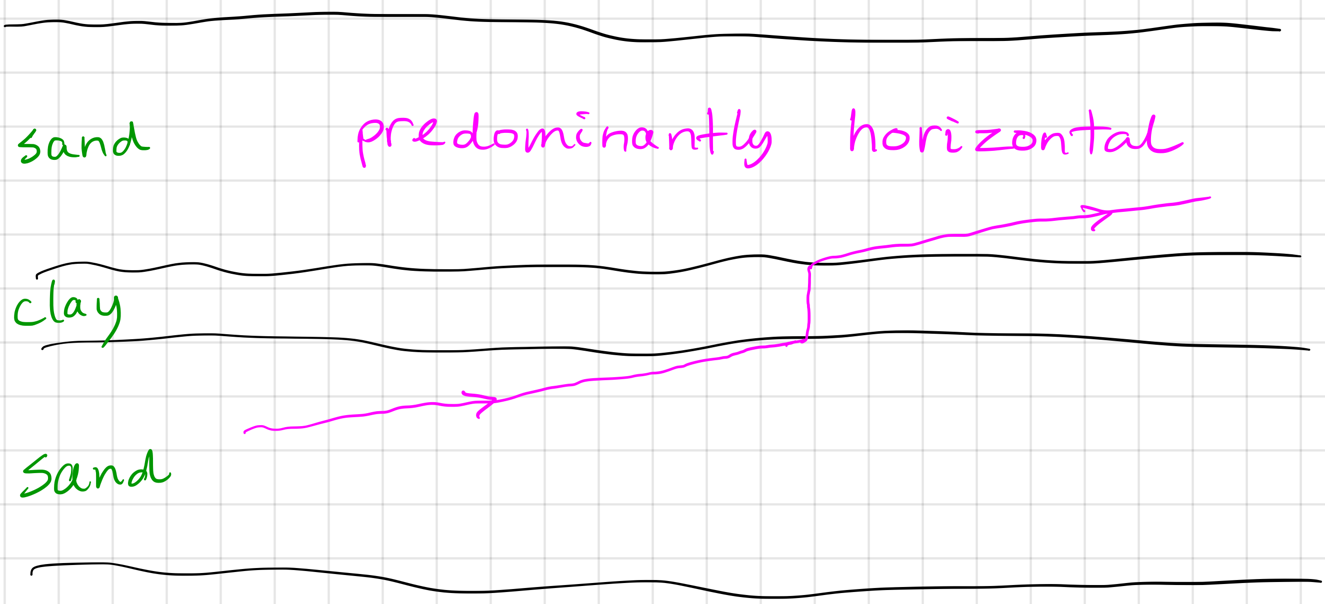
Principles of Geohydrology

Permeable layers → Aquifer (sand)

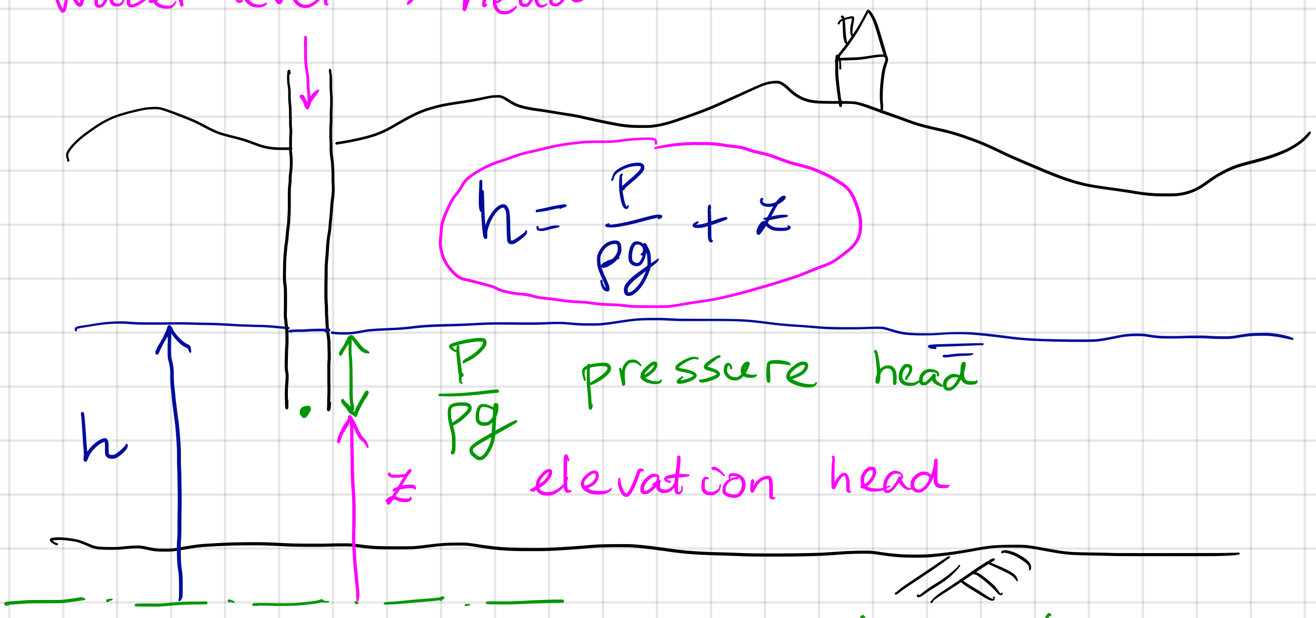
Low permeable layer → Aquitard

Leaky layer

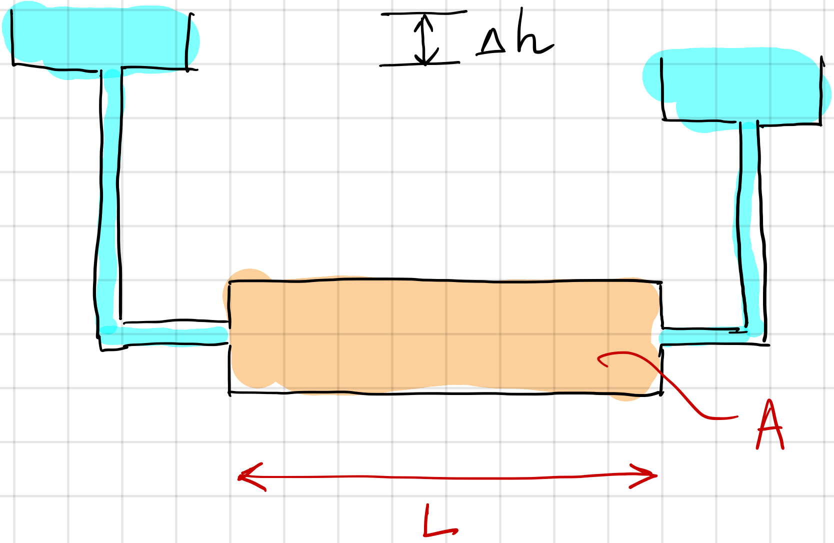
(clay)



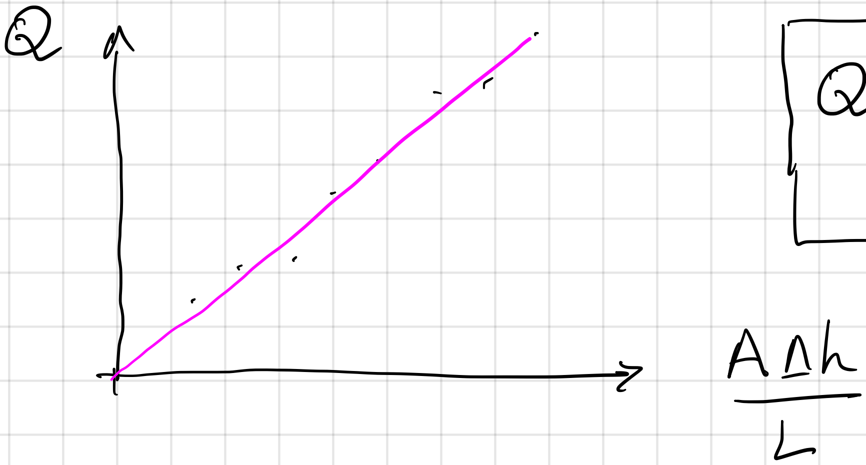
water level → head



$$h = \frac{P}{\rho g} + z$$



$$Q \sim \Delta h$$
$$Q \sim A$$
$$Q \sim \frac{1}{L}$$



$$\boxed{Q = K \frac{A \Delta h}{L}}$$

Darcy's Law

K : hydraulic conductivity

$$K = \frac{QL}{A\Delta h} \qquad \frac{\cancel{L^3}}{T} \frac{L}{\cancel{L^2}\cancel{L}} = \frac{L}{T}$$

sand : $K = 0.1 \frac{m}{d} - 100\ m/d$

fine coarse

clay $K < 0.1 \frac{m}{d}$

gravel $K > 100\ m/d$

$$K = \frac{k \rho g}{\mu}$$

k : intrinsic perm.
ρ : density
μ : dynamic viscosity

Specific discharge $\quad q = \frac{Q}{A} \quad \frac{m}{d}$

discharge per unit area

average velocity $\quad v = \frac{q}{n} \quad$ n : porosity

0.3 (sand)

$$q = \frac{Q}{A} = \frac{KA}{A}\frac{\Delta h}{L} = k\frac{\Delta h}{L}$$

$$\left.\begin{aligned} q_x &= -k\frac{\partial h}{\partial x} \\ q_y &= -k\frac{\partial h}{\partial y} \\ q_z &= -k\frac{\partial h}{\partial z} \end{aligned}\right\} \quad \vec{q} = -k\vec{\nabla} h$$

$$\vec{q} = \begin{bmatrix} k_h & 0 & 0 \\ 0 & k_h & 0 \\ 0 & 0 & k_v \end{bmatrix} \vec{\nabla} h$$

$k_v < k_h$

$$= \underline{\underline{K}}\,\vec{\nabla} h$$

What is a reasonable v

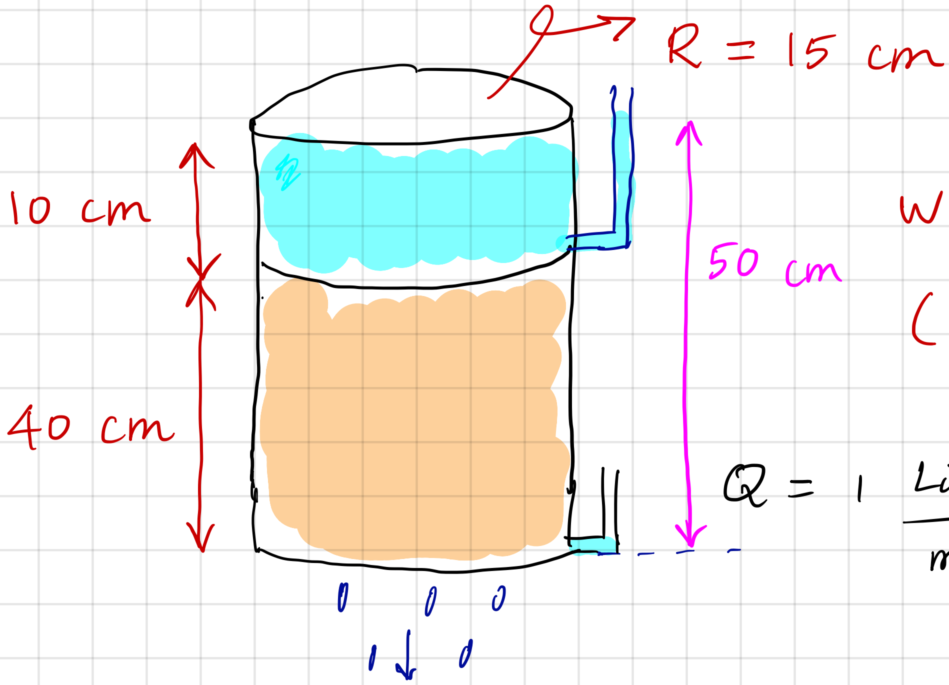
bigger $\quad 10\ m/d \quad 1\ m/d \quad 0.1\ m/d \quad$ smaller

Sand $\quad k = 10 \ m/d$

$$\frac{\partial h}{\partial x} = \frac{-1}{1000}$$

$$q_x = -k \frac{\partial h}{\partial x} = -10 \cdot \frac{-1}{1000} = 0.01 \ m/d$$

$$v_x = \frac{q_x}{n} = \frac{0.01}{0.3} \approx 0.03 \ m/d \simeq 1 \ \frac{m}{month}$$



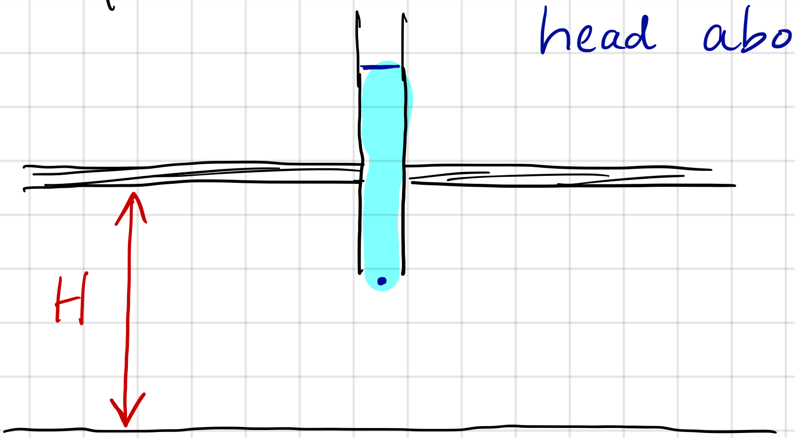
What is k ?

(in $\frac{m}{day}$)

$Q = 1 \frac{\text{Liter}}{\text{min}}$

$$k = \frac{QL}{A \Delta h} = \frac{1 \cdot 10^{-3} \cdot 60 \cdot 24 \cdot 0.4}{\pi (0.15)^2 \cdot 0.5} = 16 \frac{m}{d}$$

Confined



head above top of aquifer

Dupuit

Discharge vector $\vec{Q} = \vec{\underline{q}} H$

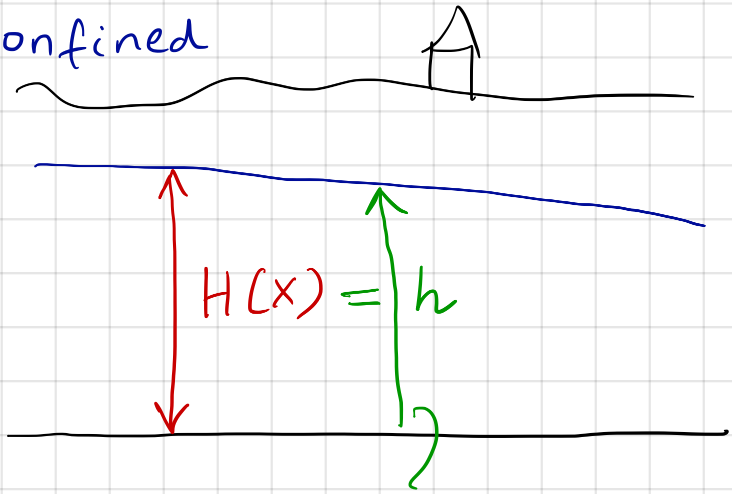
$Q_x = \underline{q}_x H \qquad Q_y = \underline{q}_y H$

Transmissivity : $T = kH \quad (m^2/d)$

$Q_x = -kH \frac{\partial h}{\partial x} = -T \frac{\partial h}{\partial x} \qquad Q_y = -T \frac{\partial h}{\partial y}$

Dupuit: Neglect the resistance to vertical flow

Unconfined

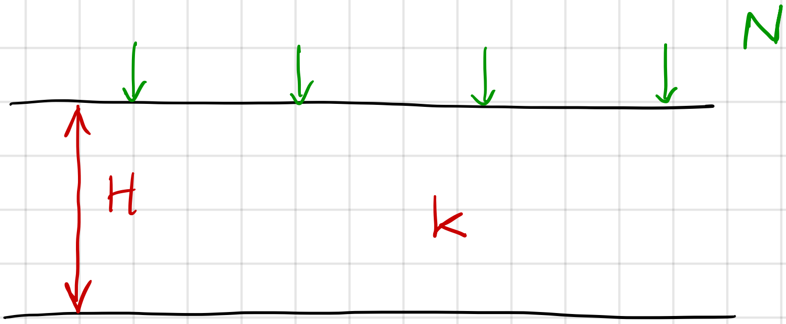


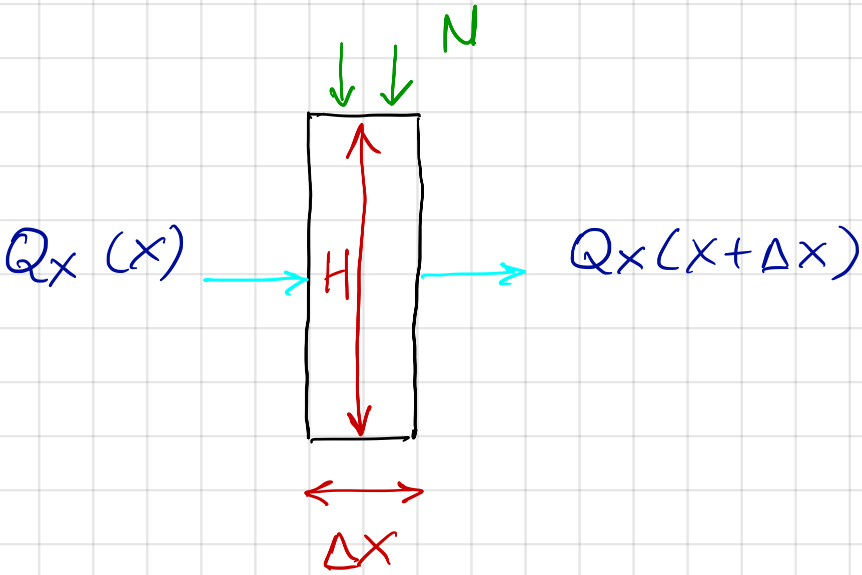
$$Q_x = H \underline{q}_x = h \underline{q}_x = -k h \frac{\partial h}{\partial x}$$

if measured w.r.t. bottom of aquifer

often the saturated thickness can be approximated as constant

Confined





$$OUT - IN = 0$$

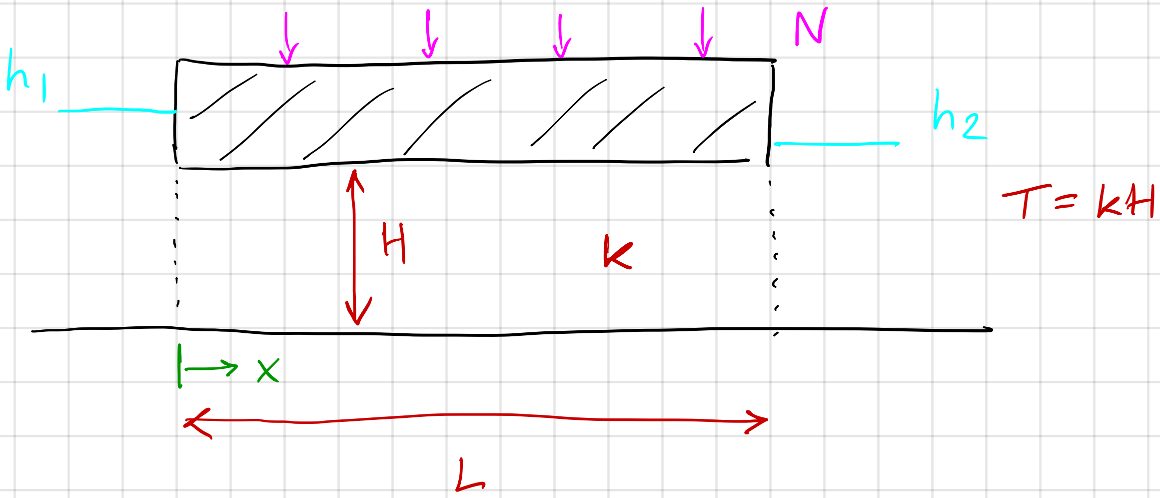
$$\frac{Q_x(x+\Delta x) - Q_x(x)}{\Delta x} - \frac{N\cancel{\Delta x}}{\cancel{\Delta x}} = 0$$

$$\frac{dQ_x}{dx} - N = 0 \qquad Q_x = -T\frac{dh}{dx}$$

if constant

$$\frac{d}{dx}\left(-T\frac{dh}{dx}\right) - N = 0$$

$$\boxed{\frac{d^2h}{dx^2} = -\frac{N}{T}}$$



$$\frac{d^2h}{dx^2} = -\frac{N}{T} \qquad \begin{matrix} x=0 & h=h_1 \\ x=L & h=h_2 \end{matrix}$$

$$\frac{dh}{dx} = -\frac{N}{T}x + A$$

$$h = -\frac{N}{2T}x^2 + Ax + B$$

$$x=0 \qquad h(x=0) = \boxed{B = h_1}$$

$$x=L \qquad h(x=L) = -\frac{N}{2T}L^2 + AL + h_1 = h_2$$

$$\boxed{A = \frac{h_2 - h_1}{L} + \frac{NL}{2T}}$$

$$h = -\frac{N}{2T}x^2 + \frac{NLx}{2T} + \frac{h_2 - h_1}{L}x + h_1$$

$$\boxed{h = -\frac{N}{2T}(x^2 - xL) + \frac{h_2 - h_1}{L}x + h_1}$$

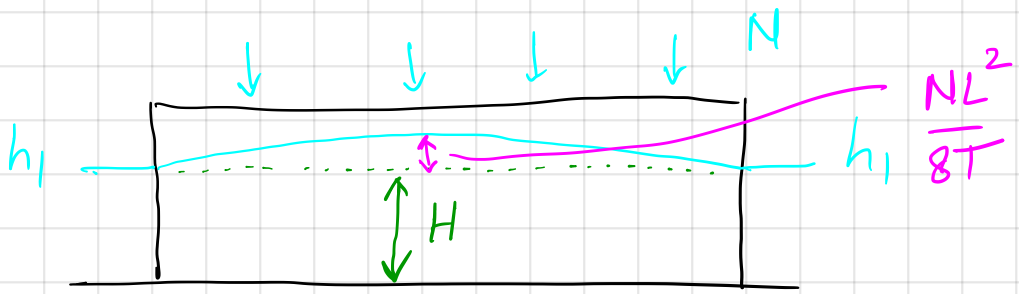
$$\boxed{h = \frac{-N}{2T}\left(x^2 - xL\right) + \frac{h_2 - h_1}{L}x + h_1}$$

$$Q_x = -T\frac{dh}{dx}$$

$$= -T\left(-\frac{N}{2T}\left(2x - L\right) + \frac{h_2 - h_1}{L}\right)$$

$$\boxed{Q_x = \frac{N}{2}\left(2x - L\right) + T\frac{\left(h_1 - h_2\right)}{L}}$$

$$h_1 = h_2 \longrightarrow h = \frac{-N}{2T}\left(x^2 - xL\right) + h_1$$



$$Q_x = \frac{N}{2}\left(2x - L\right)$$

$x=0$ $\quad Q_x = -\frac{NL}{2}$ $\qquad x = L$ $\quad Q_x = \frac{NL}{2}$

$x = \frac{L}{2}$ $\qquad h = \frac{-N}{2T}\left(\frac{L^2}{4} - \frac{L^2}{2}\right) + h_1$

$$= \frac{NL^2}{8T} + h_1$$

$$h = \frac{NL^2}{8T} + h_1$$

$N = 0.001 \, m/d$ $\quad K = 1 \, m/d$ $\quad H = 10 \, m$

$T = 10 \, \frac{m^2}{d}$ $\quad L = 100 \, m$

$$\frac{NL^2}{8T} = 0.125 \, m$$

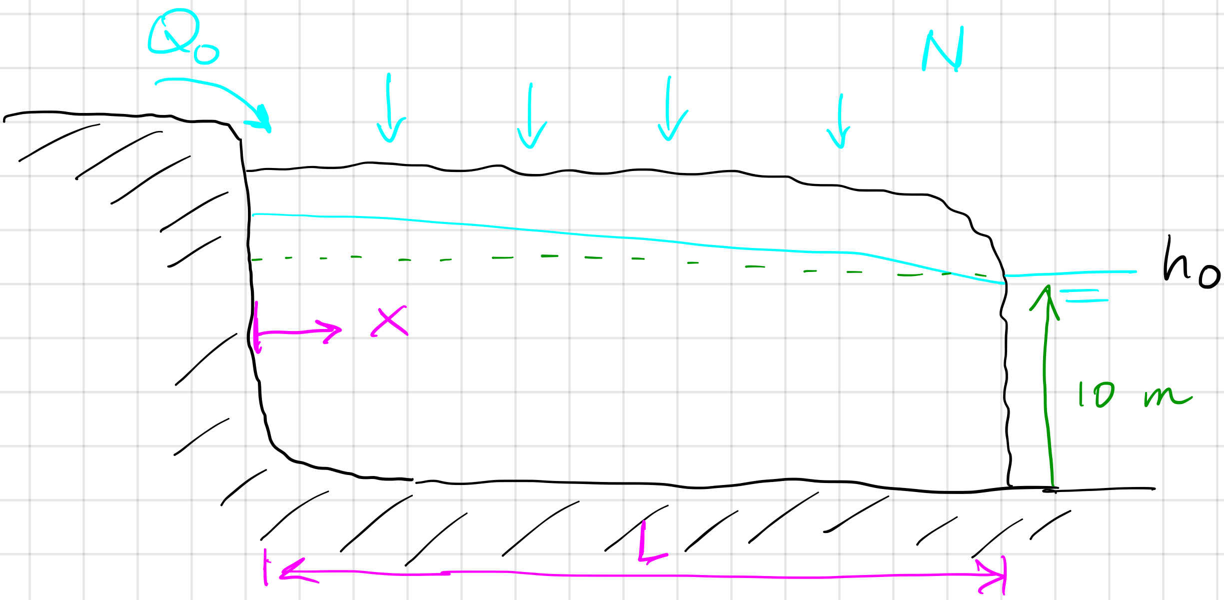
$$\boxed{Q_x = \frac{N}{2}(2x - L) + T\frac{(h_1 - h_2)}{L}}$$

What is h_1 such that $Q_x(x=0) = 0$

$$\frac{N}{2}(-L) + T\frac{(h_1 - h_2)}{L} = 0$$

$$\frac{T(h_1 - h_2)}{L} = \frac{NL}{2}$$

$$h_1 = \frac{NL^2}{2T} + h_2$$



$N = 1\ \frac{mm}{d}$ $\quad h_0 = 10\ m$ $\quad Q_0 = 0{,}05\ m^2/d$

$H = 10\ m$ $\quad L = 500\ m$ $\quad k = 5\ m/d$

a) $h(x) \rightarrow$ formula

b) $h(x=0)$

Solution

$$\frac{d^2h}{dx^2} = -\frac{N}{T} \qquad \begin{array}{ll} x=0 & Q_x = Q_0 \\ x=L & h = h_0 \end{array}$$

$$h(x) = -\frac{N}{2T}(x^2-L^2) - \frac{Q_0}{T}(x-L) + h_0$$

$$h(x=0) = 13\ m$$

$$Q_x = -T\frac{dh}{dx} = -\frac{d(Th)}{dx} \qquad \Phi = Th$$

$$\boxed{Q_x = -\frac{d\Phi}{dx}}$$

Φ : discharge potential

minus gradient of potential gives the flow

$$\frac{dQ_x}{dx} = +N$$

$$-\frac{d^2\Phi}{dx^2} = N \longrightarrow \boxed{\frac{d^2\Phi}{dx^2} = -N}$$

$$\frac{d\Phi}{dx} = -Nx + A \longrightarrow Q_x = -\frac{d\Phi}{dx}$$

$$= Nx - A$$

$$\Phi = -\frac{N}{2}x^2 + Ax + B$$

Solve $\quad x = 0 \quad Q_x = Q_0$

$\quad x = L \quad \Phi = \Phi_0 = Th_0$

$$Q_x(x=0) = -A = Q_0 \longrightarrow A = -Q_0$$

$$\Phi(x=L) = -\frac{N}{2}L^2 - Q_0 L + B = \Phi_0$$

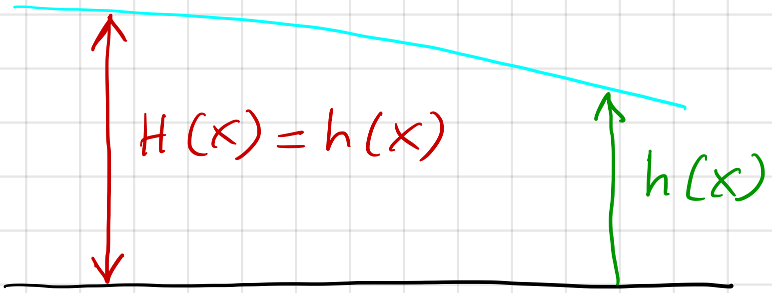
$$B = \Phi_0 + Q_0 L + \frac{NL^2}{2}$$

$$\Phi = -\frac{N}{2}(x^2 - L^2) - Q_0(x - L) + \Phi_0$$

$$\Phi = -\frac{N}{2}(x^2 - L^2) - Q_0(x - L) + \Phi_0$$

$$\Phi = T h \longrightarrow h = \frac{\Phi}{T}$$

Unconfined with $H(x) = h(x)$



h is measured w.r.t. the base

Aside

$$\frac{1}{2}\frac{d(h^2)}{dx} = \frac{\cancel{2}}{\cancel{2}} h \frac{dh}{dx}$$

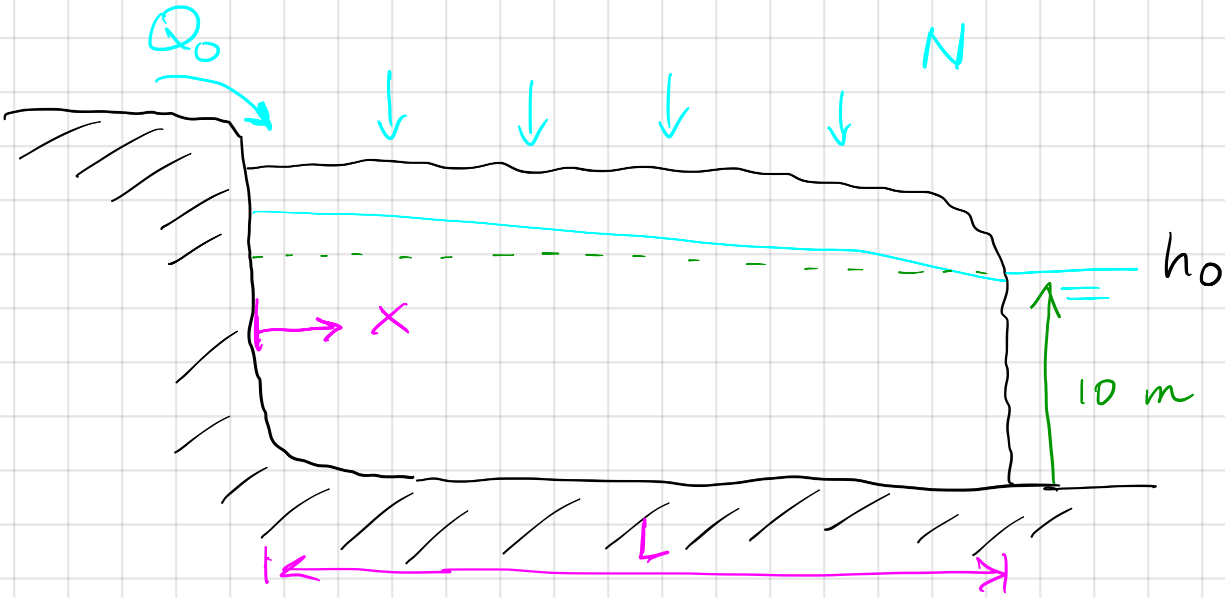
$$Q_x = -k h \frac{dh}{dx} = -\frac{d\Phi}{dx}$$

$$= -k \cdot \frac{1}{2} \frac{d(h^2)}{dx}$$

$$\boxed{Q_x = -\frac{d\Phi}{dx} \qquad \Phi = \frac{1}{2} k h^2}$$

unconfined flow

$$h = \sqrt{\frac{2\Phi}{k}}$$



Unconfined with $H(x) = h(x)$

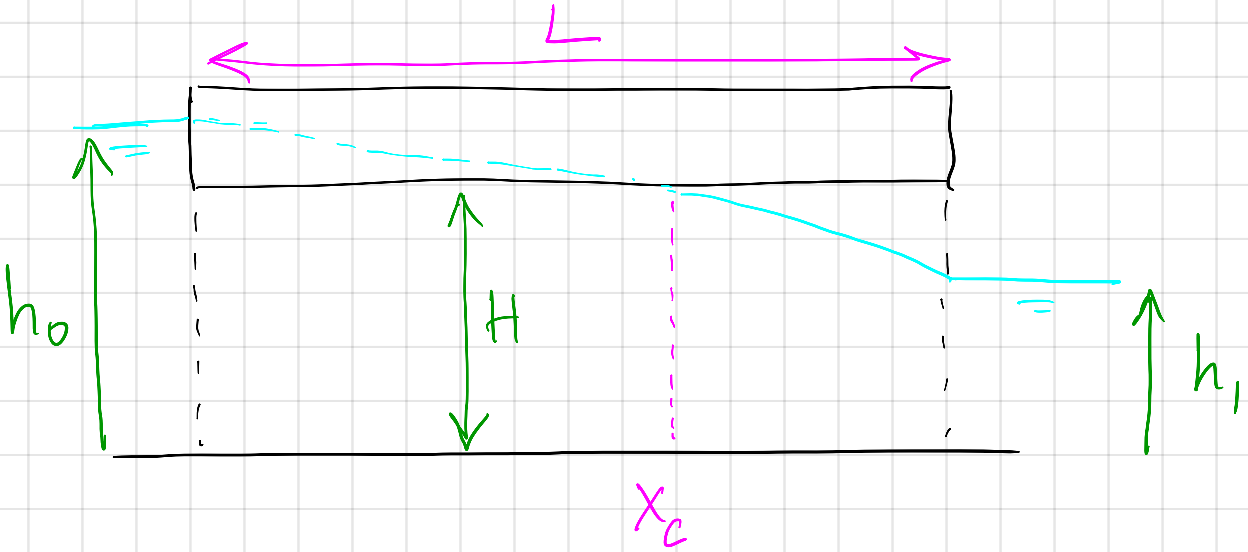
$x = 0 \qquad Q_x = Q_0$

$x = L \qquad \Phi = \Phi_0 = \frac{1}{2} k h_0^2$

$$\Phi = -\frac{N}{2}(x^2 - L^2) - Q_0(x - L) + \Phi_0$$

$$Q_x = -\frac{d\Phi}{dx} = Nx + Q_0$$

$$h = \sqrt{\frac{2\Phi}{k}}$$



$$x = x_c \longrightarrow h = H$$

confined : $\Phi = kHH = kH^2$

unconfined : $\Phi = \frac{1}{2}kH^2$

$$\Phi(x) = \Phi_0 + \frac{\Phi_1 - \Phi_0}{L}x$$

Confined : $Q_x = -kH\frac{dh}{dx} = -\frac{d}{dx}(kHh + C)$

$$\Phi = kHh + C$$

@ x_c : $kH^2 + C = \frac{1}{2}kH^2$

$$\boxed{C = -\frac{1}{2}kH^2}$$

Potential Flow

- Only confined (or constant transmissivity)

$$\Phi = kHh$$

- Only unconfined

$$\Phi = \frac{1}{2} k h^2$$

- Combined confined / unconfined

$$\Phi = kHh - \frac{1}{2} k H^2 \quad \text{(confined)}$$

$$\Phi = \frac{1}{2} k h^2 \quad \text{(unconfined)}$$