NATIONAL UNIVERSITY OF SINGAPORE

CE6003 – NUMERICAL METHODS IN ENGINEERING MECHANICS

(Semester I: AY2014/2015)

Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name**.

2. This assessment paper contains **FIVE** questions and comprises **FIVE** printed pages.

3. Answer **ALL** questions. The questions DO NOT carry equal marks.

4. Please start each question on a new page.

5. This is an "OPEN NOTES" assessment. Students are allowed to bring in reference notes on ONE sheet of A4-size paper (double-sided).

<u>Question 1</u>

The eigen-solutions to $\underline{\underline{C}} = \begin{bmatrix} 4.4 & 0 & -1.2 \\ 0 & -1 & 0 \\ -1.2 & 0 & 2.6 \end{bmatrix}$ are denoted as $(\lambda_i \ , \underline{\phi_i})$. For each mode i, the eigenvalue is λ_i with corresponding eigenvector $\underline{\phi_i}$, normalized such that $\underline{\phi_i^T} \ \underline{\phi_i} = 1$.

Note that $\underline{\underline{C}}^{-1} = \begin{bmatrix} 0.26 & 0 & 0.12 \\ 0 & -1 & 0 \\ 0.12 & 0 & 0.44 \end{bmatrix}$.

(a) Using the information above, determine whether $\underline{\tilde{\phi}} = \frac{1}{\sqrt{5}}\begin{bmatrix} 0 & 1 & 2 \end{bmatrix}^T$ is an eigenvector of $\underline{\underline{C}}$.

[8 marks]

(b) You are now given that $\boldsymbol{\varphi} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$ is an eigenvector of $\underline{\underline{C}}$. The other two normalized eigenvectors are to be found simultaneously with the subspace iteration method:

$$\mathbf{C}\,\overline{\mathbf{X}}_{k+1} = \mathbf{X}_k \quad , \quad \mathbf{X}_k = \begin{bmatrix} \mathbf{u}_k & \mathbf{v}_k \end{bmatrix}$$

where $\boldsymbol{\varphi}$, $\mathbf{u}_k$ and $\mathbf{v}_k$ are normalized vectors that are orthogonal to one another.

For a given matrix of trial vectors $\mathbf{X}_k$, you obtain $\overline{\mathbf{X}}_{k+1} = \begin{bmatrix} \overline{\mathbf{u}}_{k+1} & \overline{\mathbf{v}}_{k+1} \end{bmatrix}$ with $\overline{\mathbf{u}}_{k+1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$ and $\overline{\mathbf{v}}_{k+1} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$.

Determine the corresponding matrix of iteration vectors $\mathbf{X}_{k+1} = \begin{bmatrix} \mathbf{u}_{k+1} & \mathbf{v}_{k+1} \end{bmatrix}$.

[12 marks]

<u>Question 2</u>

In this question, consider the special case such that

- $\varepsilon_{i3} = \varepsilon_{3i} = \varepsilon_{i3}^{p} = \varepsilon_{3i}^{p} = 0$ (i = 1, 2, 3)
- σ_{ij}, ε_{ij} and ε_{ij}^{p} are symmetric and deviatoric

where σ_{ij} = stress tensor, ε_{ij} = strain tensor and ε_{ij}^{p} = plastic strain tensor.

The constitutive equation of a material is given by

$$\sigma_{ij} = 2\mu\left(\varepsilon_{ij} - \varepsilon_{ij}^{p}\right)$$

where μ is the shear modulus.

The plasticity relation is given by

$$\sigma_{ij} = H\left(\frac{p}{\varepsilon_0}\right)^{n} \varepsilon_{ij}^{p} \quad , \quad p = \sqrt{\varepsilon_{ij}^{p}\,\varepsilon_{ij}^{p}}$$

where H is a hardening modulus, ε_0 and n are material parameters.

(a) Show that $p = \sqrt{2\left(\varepsilon_{11}^{p}\right)^2 + 2\left(\varepsilon_{12}^{p}\right)^2}$.

[10 marks]

(b) Assume material parameters $\mu = 100$, $H = 10$, $\varepsilon_0 = 0.005$ and $n = 0.3$. For a given strain, the problem can be solved by considering only the 11 and 12 components of σ_{ij}, i.e., to satisfy functions *f* and *g* at all times

$$f = \sigma_{11} - H\left(\frac{p}{\varepsilon_0}\right)^{n} \varepsilon_{11}^{p} = 0$$

$$g = \sigma_{12} - H\left(\frac{p}{\varepsilon_0}\right)^{n} \varepsilon_{12}^{p} = 0$$

Given strain values $\varepsilon_{11} = 0.03$ and $\varepsilon_{12} = 0.02$, determine the corresponding values for ε_{11}^{p} and ε_{12}^{p} using the Newton Raphson method.

Use initial trial values $\varepsilon_{11}^{p} = \varepsilon_{12}^{p} = 0.01$. You need to perform only <u>one</u> iteration.

[20 marks]

Note: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Question 3

A simple harmonic problem is given as

$$\ddot{u} + \omega^2 u = 0 \quad , \quad u(0) = u_0 \quad , \quad \dot{u}(0) = 0$$

(a) Discuss how the problem can be solved with the first order Euler explicit method.

[10 marks]

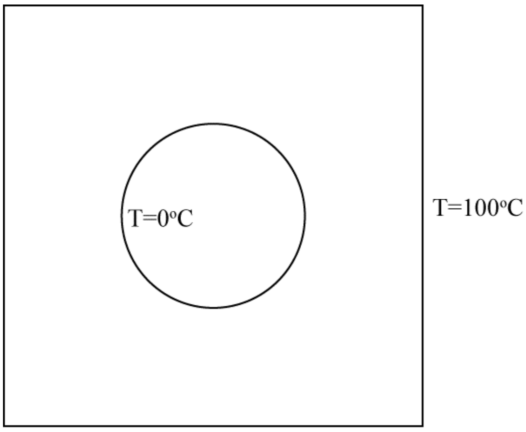
(b) Determine the condition(s) such that the numerical scheme is stable.

[10 marks]

Note: the first order Euler explicit method is given by $u_{i+1} = u_i + \Delta t \, \dot{u}_i$.

Question 4

A square plate with a circular hole in its center, shown in the figure below, maintains a temperature (T) of 0^oC and 100^oC at its inner and outer surfaces respectively.

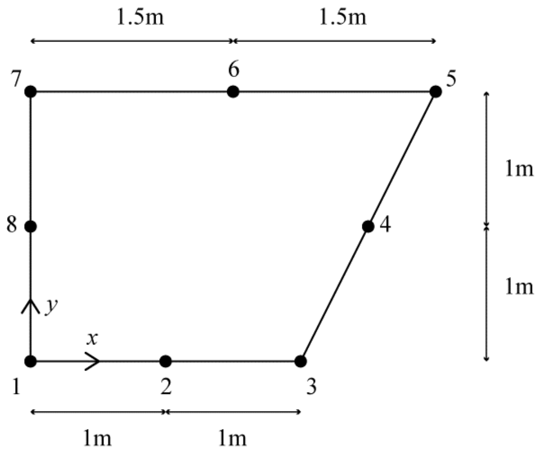


When solving for the profile of steady-state temperature numerically (e.g. Finite difference method, BEM), discuss how the problem can be reduced by making use of symmetry and the appropriate boundary conditions.

[10 marks]

Question 5

A Laplace problem $\nabla^2\phi = 0$ is solved using the boundary element method. The specimen is discretized as shown in the figure below:



Element	Nodes
1	1, 2, 3
2	3, 4, 5
3	5, 6, 7
4	7, 8, 1

The assembled system of equations is written as $[A]_{8x8}\,[\phi]_{8x1} = [B]_{8x8}\left[\dfrac{\partial\phi}{\partial n}\right]_{8x1}$.

Discuss how B_{44} and B_{45} are obtained numerically. You are to compute the Jacobian $J(\xi)$ of the elements involved.

[20 marks]

Wherever necessary,

- let n_G and n_L be the number of ordinary and logarithmic Gauss points respectively;
- state clearly the coordinate transformation(s) required;
- write your equations in terms of f, g, N_1, N_2 and N_3 where

$$f(\xi) = \text{Num}/\text{Den}$$

$$\text{Num} = -\left(\sum_{k=1}^{3} N_k(\xi)\, x_k - x_p\right)n_x - \left(\sum_{k=1}^{3} N_k(\xi)\, y_k - y_p\right)n_y$$

$$\text{Den} = 2\pi\left\{\left(x_p - \sum_{k=1}^{3} N_k(\xi)\, x_k\right)^2 + \left(y_p - \sum_{k=1}^{3} N_k(\xi)\, y_k\right)^2\right\}$$

$$g(\xi) = \frac{-1}{2\pi}\ln\left\{\left(x_p - \sum_{k=1}^{3} N_k(\xi)\, x_k\right)^2 + \left(y_p - \sum_{k=1}^{3} N_k(\xi)\, y_k\right)^2\right\}^{0.5}$$

$$N_1(\xi) = -\frac{\xi}{2}(1-\xi) \quad , \quad N_2(\xi) = (1+\xi)(1-\xi) \quad , \quad N_3(\xi) = \frac{\xi}{2}(1+\xi)$$

Note: <u>No</u> marks will be awarded if you provide the generic algorithm.

- END OF PAPER -