Q1. To assess the impact of boundary conditions, initial conditions and physical and numerical parameters by modeling the natural (Eigen) and forced behavior of a harbor.

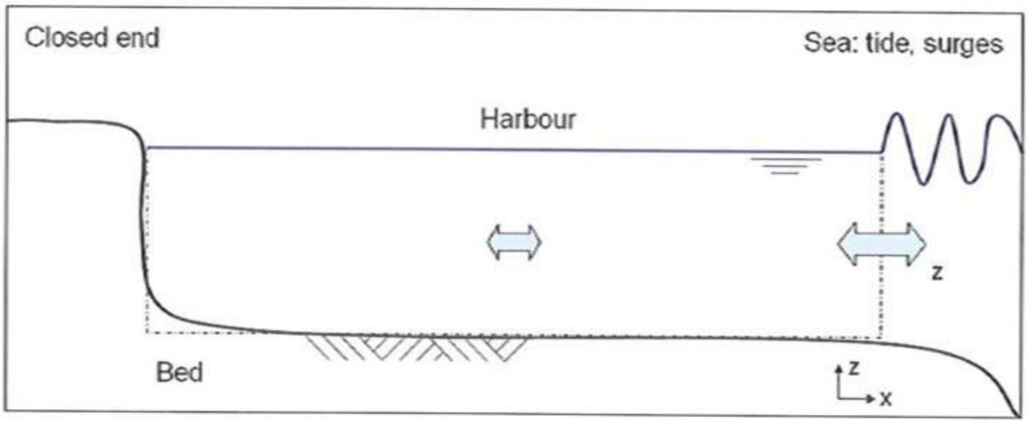


Fig.1 sketch of harbour problem

To consider this assignment as a natural solution, we introduce mass spring system to analyse.

$$\frac{d^2x}{dt^2} + \frac{A}{m}\frac{dx}{dt} + \frac{K}{m}x = \frac{f}{m}\cos(\omega t + \varphi_0) \tag{1.1}$$

Where A is damping ratio, K is spring constant, f is frequency and m is mass.

Boundary condition,

Natural solution

In a mass spring system, boundary condition of this case may come from right hand side of eq.(1.1) corresponding to sea in real solutions. It affects amplitude of this oscillating process and phase of the graph.

Real solution

In this case, closed end lies in the left side boundary which interprets no flow and stationary water in the boundary.

Water flows into sea eventually in the right side. Sea waves propagate towards upstream due to its forcing type, amplitude and phase etc. when hit the closed end, water level goes up and down as sea wave in the whole process shown in fig.2.

Initial condition,

Natural solution

Initial condition comes from LHS in eq.(1.1) x_0, once x_0 matches boundary condition which shows above, there will no spin-up time. Hence, that depends where you put your mass.

Real solution

We set up initial water level as initial condition. When initial condition dismatches boundary condition water level becomes oscillating in the previous time and this is called “spin-up time”. To avoid this, we are supposed to calibrate initial condition or boundary condition.

Physical and numerical parameters

Natural solution

Physical parameters are mass, frequency, damping ration and spring constant. These stuffs affect eigenvalue of this solution as shown in eq.(1.2).

$$\lambda_{1,2} = \frac{A}{2m} \pm \sqrt{\frac{A^2}{4m^2} - \frac{K}{m}} \qquad (1.2)$$

Once eigenvalue is small than 1, result decays with time. Otherwise, it will blow up. Generally, this natural solution won't blow up because mass is greatly larger than amplitude so that eigenvalue will not be bigger than 1.

In numerical parameters, time step is the significant one. While in this case, we found time step has no relationship with eigenvalue. That proves stability of the solution doesn't depend on time step. On the other hand, smaller time step could get more accurate result due to exact solution.

Real solution

Physical parameters consist of roughness, viscosity etc. roughness affects the velocity and discharge of the flow and viscosity also determines Reynold's number and flow types according to the definition. Lower viscosity contributes to higher Reynold's number, flow may transform from laminar to turbulent flow.

Numerical parameters consist of time step. To analyze stability about varying time step, we introduce courant number as follows

$$C = c\frac{\Delta t}{\Delta x} \leq 1$$

Where, C=courant number

Once time step satisfies that inequation, we can get a stable solution while it is applicable for explicit shallow water. In this case, this instability is due to the physical process.

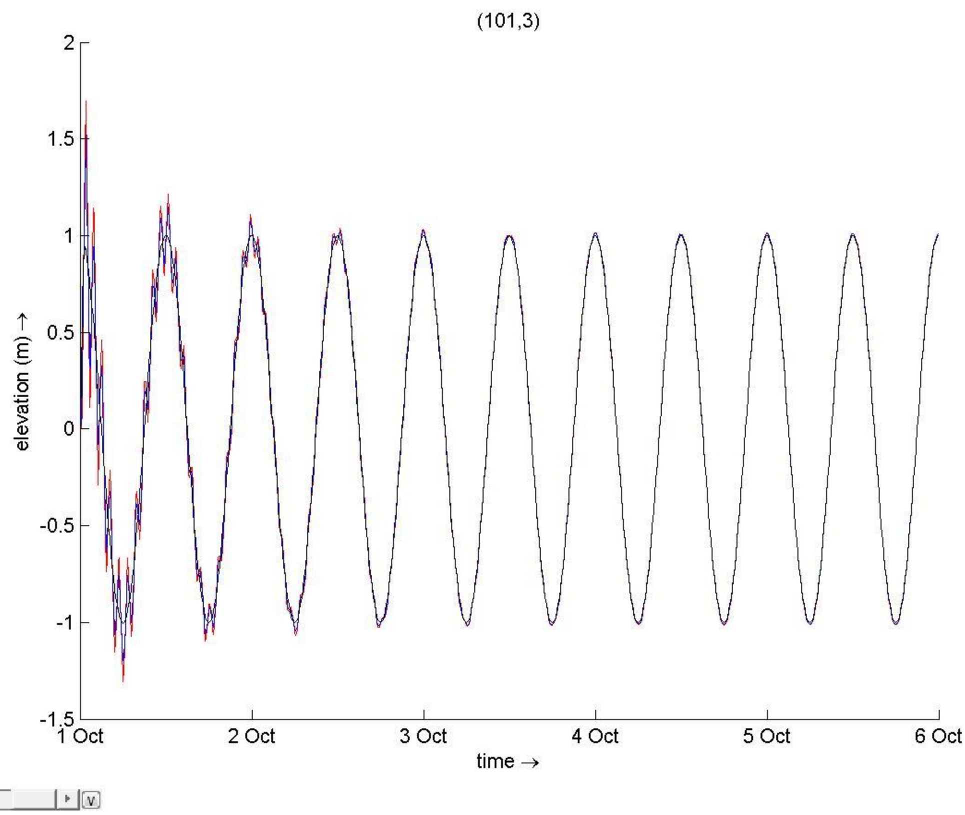


Fig.2 water level changes with time

Noted: red line is at observation (2,3) blue line is at (51,3) and black line is at (103,3) of all graphs.

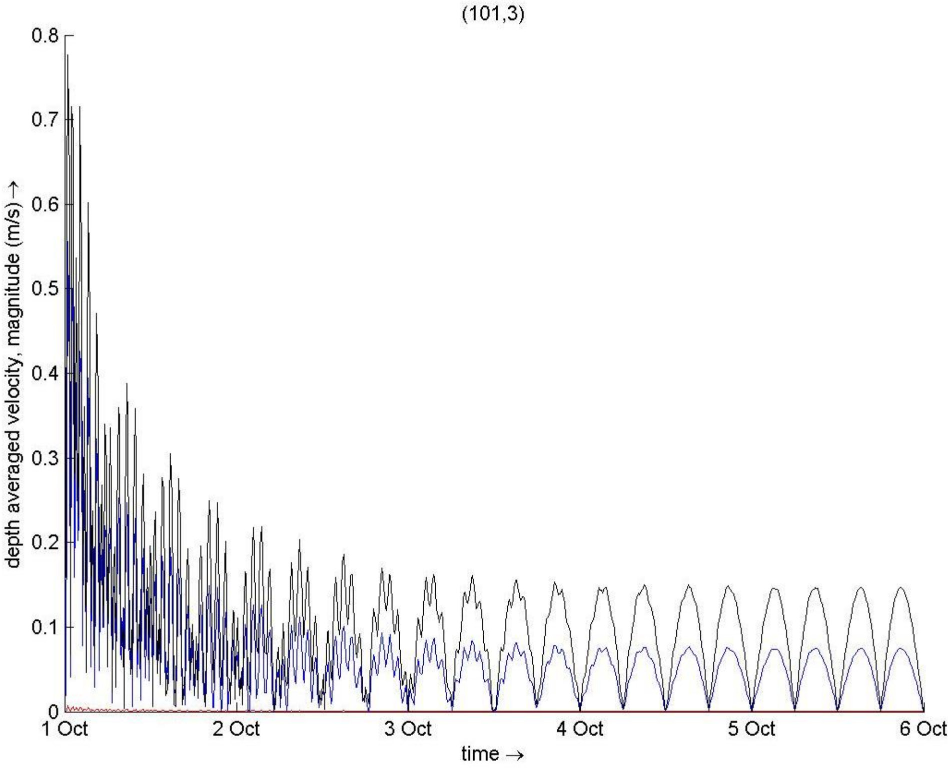
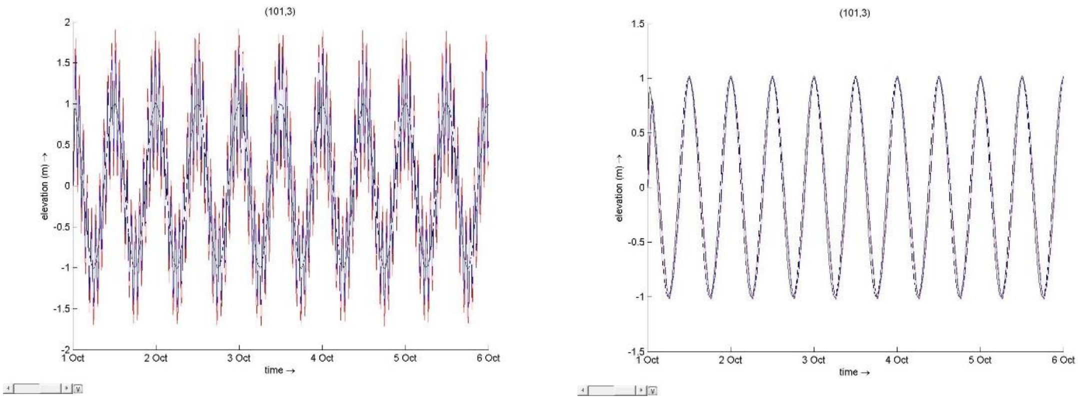


Fig.3 velocity changes with time

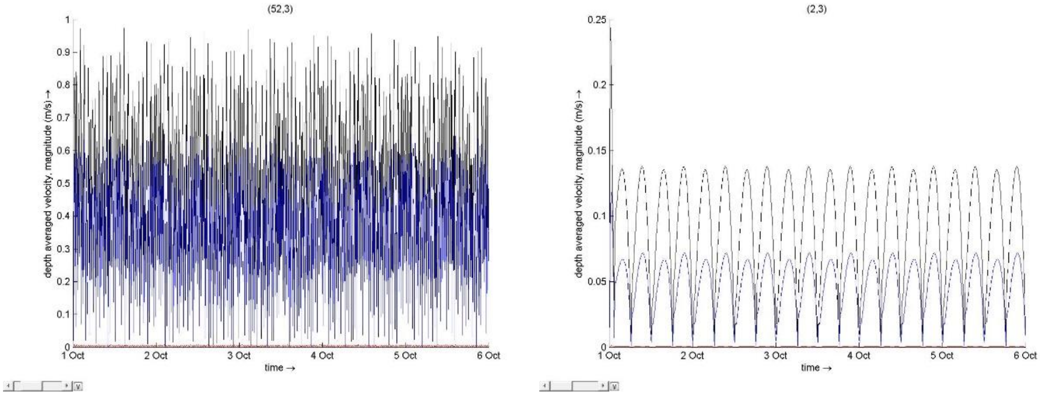
Q2. Assess the influence of bottom friction (roughness and depth)

Bottom friction is determined by roughness varying from different formulas. by comparison, we adjust coefficient form 65 to 10000 and 5 in Chezy's formula without changing other parameters.



(a) (b)

Fig.4 water level changes with time and Checy's coefficient (a)10000 (b)5



(a) (b)

Fig.5 average velocity changes with time and Checy's coefficient (a)10000 (b)5

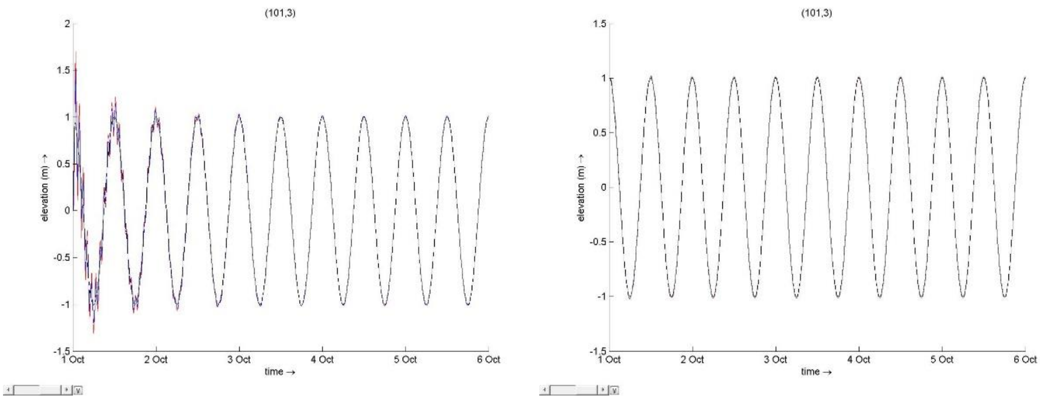
Checy's coefficient is proportional to $1/_n$, where n stands for roughness. Basically in (a) roughness is quite low and in (b) roughness is relatively high. As shown above, in lower roughness situation, water level and averaged velocity seem to be oscillating all the time and they become smooth in higher roughness.

According to Checy's formula, $v = C\sqrt{RS}$, when Checy's coefficient becomes large as 10000, celerity of wave propagation becomes large as well. In wave theory, more waves could propagate in the same time duration. It looks like waves are squeezed in this case. While when coefficient becomes smaller, less waves propagate in the same time duration, so it looks more smooth intuitively.

Q3. Try to minimize and maximise the spin-up time by varying the initial conditions (water levels and velocities) and boundary type (water levels and velocities)

As mentioned before, spin-up time occurs when boundary condition dismatch initial condition. To avoid this, we are allowed to match both by adjusting one. In Fig.2, we see spin-up time takes around two and a half days.

In doing this, minimizing spin-up time, we set boundary condition as before (tide type:S2 and amplitude:1m) and then adjust initial condition water level to 1m according to the amplitude.

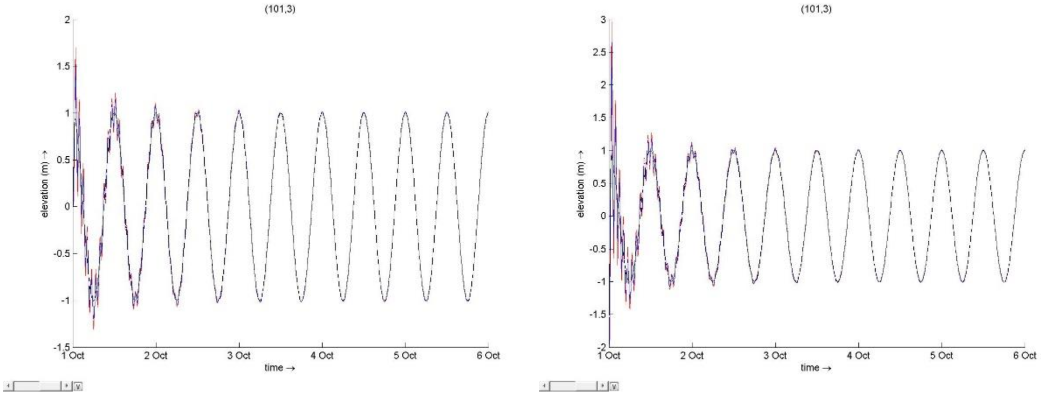


(a) (b)

Fig.6 water level changes with time and initial conditions (a)water level 0m (b)water level 1m

As we see above, water elevation becomes oscillating in the first two and half days in (a) while smooth in (b) by comparison. Typically we eliminate spin-up time by making initial condition match boundary condition.

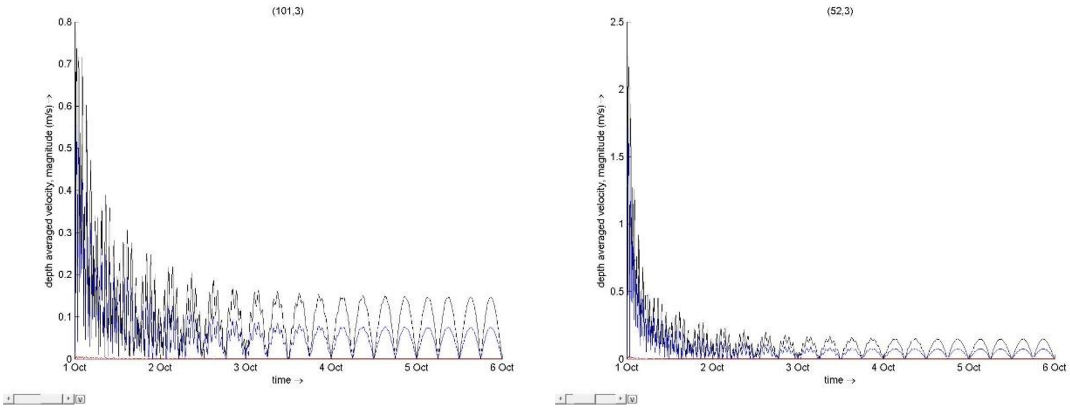
If we desire to maximize the spin-up time in this case, adjust initial condition(water level) to -2 meters with the same boundary condition.



(a) (b)

Fig.7 water level changes with time and initial conditions(a)water level 0m (b)water level-2m

Comparing these two figures, when initial condition deviates boundary condition more, the oscillation becomes more violent. While taken velocity into consideration, it affects it the most.



(a) (b)

Fig.8 velocity changes with time (a) water level 0m (b)water level -2m