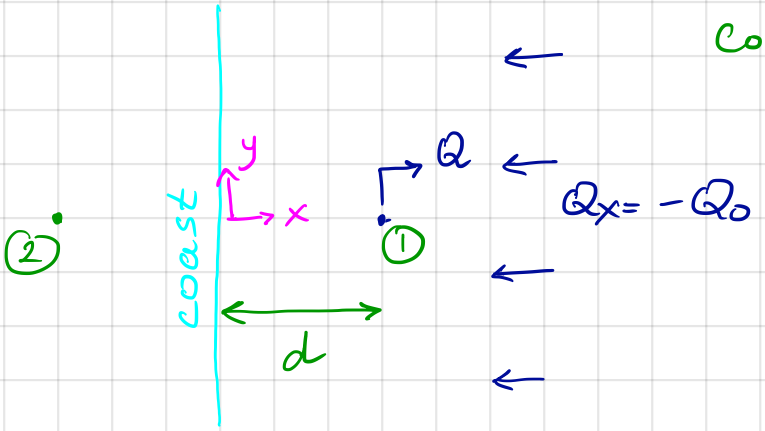
Confined interface



a) $\Phi(x,y)$

b) Q such that that toe is at $x = \frac{d}{2}$

a) $\Phi = \frac{Q}{2\pi} \ln \frac{r_1}{r_2} + Q_0 x$ $\qquad r_1 = \sqrt{(x-d)^2 + y^2}$

$r_2 = \sqrt{(x+d)^2 + y^2}$

b) $\Phi = \frac{1}{2} k\alpha \left(h - \frac{D}{\alpha}\right)^2$ $\qquad h_{toe} = \frac{D+H}{\alpha}$

$\Phi_{toe} = \frac{\frac{1}{2} k H^2}{\alpha}$

$\Phi(x = \frac{d}{2}, y = 0) = \Phi_{toe}$ $\qquad r_1 = \frac{d}{2} \qquad r_2 = \frac{3d}{2}$

$\frac{Q}{2\pi} \ln \frac{d/\cancel{2}}{3d/\cancel{2}} + Q_0 \frac{d}{2} = \Phi_{toe} \rightarrow Q = \frac{2\pi\left(\Phi_{toe} - Q_0 d/2\right)}{\ln(1/3)}$

Given: $k = 20 \ m/d \qquad H = 20 \ m \qquad D = 10 \ m \qquad Q = 200 \ m^3/d$

$\rho_s = 1020 \ kg/m^3 \qquad Q_0 = 0.2 \ m^2/d \qquad d = 1000 \ m$

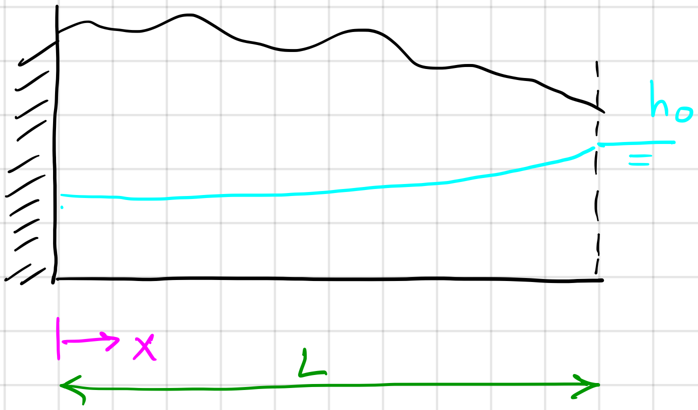
c) $h(d/2, 0)$ $\qquad$ d) depth of interface at $(\frac{d}{2}, 0)$ $\qquad$ e) thickness freshwater zone $(\frac{d}{2}, 0)$

$$\Phi = \frac{1}{2} k\alpha \left(h - \frac{D}{\alpha}\right)^2$$

c) $\Phi(\frac{d}{2}, 0) = 65 \ \frac{m^3}{d} < \Phi_{toe} \rightarrow h = \sqrt{\frac{2\Phi}{k\alpha}} + \frac{D}{\alpha} = 0.56 \ m$

d) $d = \alpha h \qquad \alpha = \frac{\rho_f}{\rho_s - \rho_f} = 50 \qquad d = \alpha h = 28 \ m$

e) thickness : $d - D = 18 \ m.$



Steady flow

Net recharge:

$$N = -\frac{Ex}{L}$$

Approximate constant T

a) Q_x at river

b) $h(x)$

c) $Q_x(x)$

d) $h(x=0)$ given $L = 2000$ m, $T = 200$ m²/d, $h_0 = 10$ m, $E = 1$ mm/d

a) $$Q_x(x=L) = -\frac{EL}{2}$$

b) $$\frac{d^2h}{dx^2} = \frac{-N}{T} = \frac{Ex}{TL}$$

$x=0$: $Q_x = 0 = -T\frac{dh}{dx}$

$x=L$: $h = h_0$

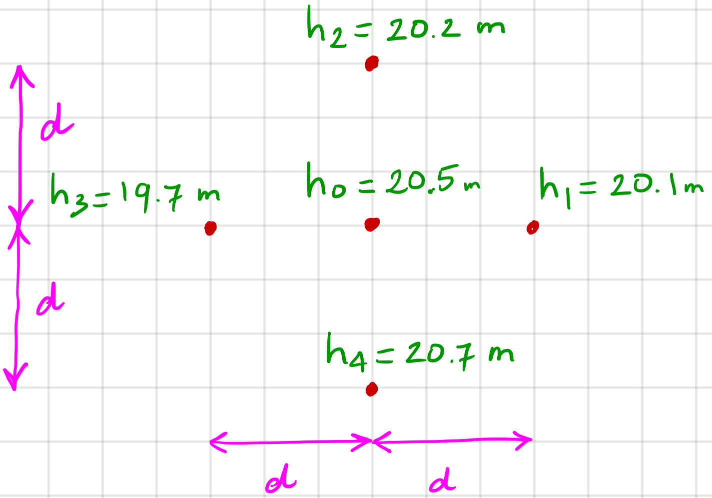
$$\frac{dh}{dx} = \frac{Ex^2}{2TL} + A \rightarrow A = 0$$

$$h = \frac{Ex^3}{6TL} + Ax + B \qquad \frac{EL^3}{6TL} + B = h_0$$

$$h = \frac{E}{6TL}(x^3 - L^3) + h_0$$

c) $$Q_x = -T\frac{dh}{dx} = -\frac{Ex^2}{2L}$$

d) $h(x=0) = 6.7$ m



$d = 100$ m

Transmissivity T

Recharge N uniform

a) FD eq. for h_0

b) Compute $\frac{N}{T}$

a) $$h_0 = \frac{h_1 + h_2 + h_3 + h_4}{4} - \frac{Q}{4T} \qquad Q = -Nd^2$$

b) $$\frac{N}{T} = \frac{4h_0 - h_1 - h_2 - h_3 - h_4}{d^2} = 1.3 \cdot 10^{-4} \ \frac{1}{m}$$

$h_{deep} = 24.5$ m $\qquad c = 1000$ d

c) N_0? $\qquad$ d) FD eq for h_0 $\qquad$ e) T?

c) $$N = \frac{h_{deep} - h_0}{c} = \frac{4}{1000} = 0.004 \ m/d$$

d) $$h_0 = \frac{h_1 + h_2 + h_3 + h_4}{4} + \frac{h_{deep} - h_0}{c} \frac{d^2}{4T}$$

$h_0 = \ldots$

e) $$\frac{N}{T} = 1.3 \cdot 10^{-4} \qquad T = \frac{N}{1.3 \cdot 10^{-4}} = 31 \ \frac{m^2}{d}$$