NATIONAL UNIVERSITY OF SINGAPORE

CE6003 – NUMERICAL METHODS IN ENGINEERING MECHANICS

(Semester 1: AY2016/2017)

Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name**.
2. This assessment paper contains **FOUR** questions and comprises **FIVE** printed pages.
3. Answer **ALL** questions. The questions DO NOT carry equal marks.
4. Please start each question on a new page.
5. This is a "CLOSED BOOK" assessment.
6. Students are allowed to bring in ONE A4-sized sheet of reference notes on both sides.

<u>Question 1</u> [30 marks]

The constitutive equation of a material is given as

$$\sigma_{ij} = 2\mu\left(\varepsilon_{ij}^{dev} - \varepsilon_{ij}^{p}\right) + K\varepsilon_{kk}\delta_{ij}$$

where σ_{ij} = stress tensor, ε_{ij} = strain tensor, ε_{ij}^{p} = plastic strain tensor, μ = shear modulus and K = bulk modulus. The superscript 'dev' denotes the deviatoric part of a tensor.

The equivalent stress is defined as $\sigma_{eq} = \sqrt{\frac{3}{2}\sigma_{ij}^{dev}\sigma_{ij}^{dev}}$.

The plasticity yield function is given as

$$F = \sigma_{eq} + 0.1\sigma_{kk} - H\lambda^{n} = 0$$

where λ = equivalent plastic strain, H and n are material parameters.

You are told that the deformation at the current time-step t is undergoing a plasticity increment. The plasticity kinematic fields at the previous time-step (t-1) are given as

$$\boldsymbol{\varepsilon}^{p\,(t-1)} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & -0.01 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad \lambda^{(t-1)} = 0.01 \ .$$

(a) Given that $\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\dfrac{\partial F}{\partial \sigma_{ij}^{dev}}$, find $\dot{\lambda}$ in terms of $\dot{\varepsilon}_{ij}^{p}$.

[6 marks]

(b) To solve for the current plastic strain, derive the 1D non-linear function with $\Delta\lambda$ as the unknown variable.

[12 marks]

(c) The strain field at current time-step t is $\boldsymbol{\varepsilon} = \begin{bmatrix} 0.03 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, and the material parameters are $\mu = 100$, $K = 100$, $n = 0.25$ and $H = 10$.

Use the Newton-Raphson iteration method to solve for the current stress σ_{ij} . Use $\Delta\lambda = 0$ as a starting value in the Newton-Raphson iteration. Assume that the solution has converged after **ONE** iteration.

[12 marks]

<u>Question 2</u> [20 marks]

Consider $\underline{\underline{C}} = \begin{bmatrix} 3.5 & -1.5 & -1 \\ -1.5 & 3.5 & 1 \\ -1 & 1 & 4 \end{bmatrix}$. For each mode i, its eigenvalue is λ_i, with the corresponding eigenvector $\underline{\phi}^{(i)}$, normalized such that $\underline{\phi}^{(i)T}\,\underline{\phi}^{(i)} = 1$. The smallest eigenvalue is given as $\lambda_1 = 2$.

Note that $\underline{\underline{C}}^{-1} = \begin{bmatrix} 0.3611 & 0.1389 & 0.0556 \\ 0.1389 & 0.3611 & -0.0556 \\ 0.0556 & -0.0556 & 0.2778 \end{bmatrix}$.

Find λ_2 and $\underline{\phi}^{(2)}$ using the Gram-Schmidt Orthogonalization method. Let the starting vector be $\underline{x}_1 = [1 \quad 1 \quad 1]^T$. Assume that the solution converges after **ONE** iteration.

[20 marks]

<u>Question 3</u> [20 marks]

Consider a 1D problem with discretization as shown in Figure 1. The standard step-size is of h units, except for OP, which has a step-size of ah units.

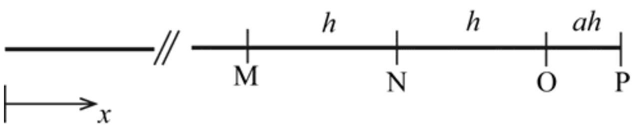


Figure 1

The field variable is denoted as U. At the boundary grid point P, impose a Neumann boundary condition involving the term $\left.\dfrac{\partial U}{\partial x}\right|_P$.

Utilizing **EITHER** the Forward, Backward **OR** Central Difference Method (**not** combinations of different schemes), derive the difference equation for $\left.\dfrac{\partial U}{\partial x}\right|_P$ such that the truncation error is $O(h^2)$. Use three consecutive grid points in your derivation, and indicate the leading truncation error in your solution.

[20 marks]

Question 4 **[30 marks]**

The steady state heat flow problem is described with a Laplace problem $\nabla^2 U = 0$. The specimen geometry and boundary conditions are illustrated in Figure 2.

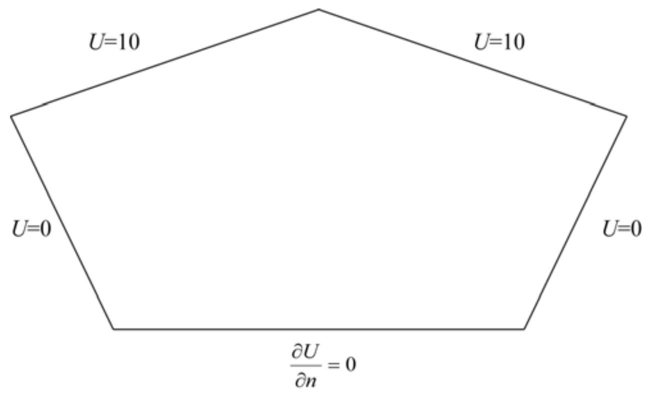
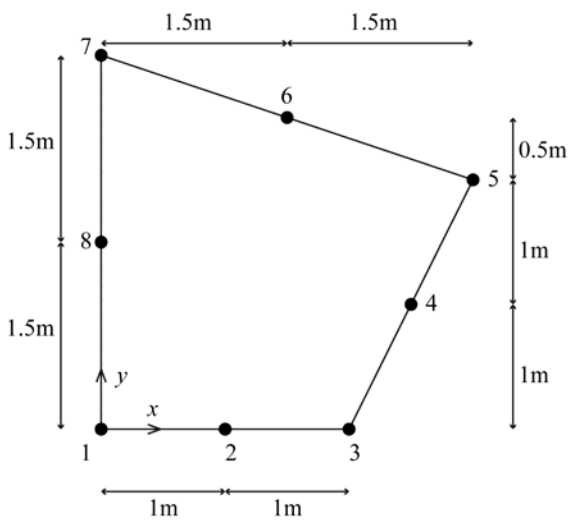


Figure 2

The problem can be reduced by considering half a specimen, as shown in Figure 3, which is next solved using the Boundary Element Method.



Element	Nodes
1	1, 2, 3
2	3, 4, 5
3	5, 6, 7
4	7, 8, 1

Figure 3

(a) Sketch Figure 3 in your answer book, and indicate the boundary conditions to impose at all external boundaries of the specimen. Explain why the boundary conditions are adopted.

[5 marks]

(b) The assembled system of equations is written as $\left[A\right]_{8\text{x}8}\left[\phi\right]_{8\text{x}1}=\left[B\right]_{8\text{x}8}\left[\dfrac{\partial\phi}{\partial n}\right]_{8\text{x}1}$.

Discuss how B_{54} and B_{55} are obtained numerically. You are to compute the Jacobian $J(\xi)$ of the elements involved.

[25 marks]

Wherever necessary,

- let n_G and n_L be the number of ordinary and Logarithmic Gauss points respectively;
- state clearly the coordinate transformation(s) required;
- write your equations in terms of f, g, N_1, N_2 and N_3 where

$$f(\xi)=\text{Num}/\text{Den}$$

$$\text{Num}=-\left(\sum_{k=1}^{3}N_k(\xi)\,x_k-x_p\right)n_x-\left(\sum_{k=1}^{3}N_k(\xi)\,y_k-y_p\right)n_y$$

$$\text{Den}=2\pi\left\{\left(x_p-\sum_{k=1}^{3}N_k(\xi)\,x_k\right)^2+\left(y_p-\sum_{k=1}^{3}N_k(\xi)\,y_k\right)^2\right\}$$

$$g(\xi)=\frac{-1}{2\pi}\ln\left\{\left(x_p-\sum_{k=1}^{3}N_k(\xi)\,x_k\right)^2+\left(y_p-\sum_{k=1}^{3}N_k(\xi)\,y_k\right)^2\right\}^{0.5}$$

$$N_1(\xi)=-\frac{\xi}{2}(1-\xi)\quad,\quad N_2(\xi)=(1+\xi)(1-\xi)\quad,\quad N_3(\xi)=\frac{\xi}{2}(1+\xi)$$

Note: **NO** marks will be awarded if you provide a generic algorithm.

- END OF PAPER -