NATIONAL UNIVERSITY OF SINGAPORE

CE6003 – NUMERICAL METHODS IN ENGINEERING MECHANICS

(Semester 1: AY2015/2016)

Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name**.

2. This assessment paper contains **FOUR** questions and comprises **FIVE** printed pages.

3. Answer **ALL** questions. The questions DO NOT carry equal marks.

4. Please start each question on a new page.

5. This is an "OPEN NOTES" assessment. Students are allowed to bring in reference notes on ONE sheet of A4-size paper (doubled-sided).

<u>Question 1</u>

Note: The following relations may be useful for this question.

$$f\left(x+\Delta x\ ,\ y+\Delta y\right)=f(x,y)+\left[\ \Delta x\ f_x(x,y)+\Delta y\ f_y(x,y)\right]$$
$$+\tfrac{1}{2!}\left[\ (\Delta x)^2\ f_{xx}(x,y)+2\Delta x\Delta y\ f_{xy}(x,y)+(\Delta y)^2\ f_{yy}(x,y)\right]\ +\ \ldots$$

$$f'(t)=f_t(t,y)+f_y(t,y)\ f(t,y)$$

(a) A numerical scheme is given as

$$y_{i+1}=y_i+h\left[A\ f\left(t_{i+1/4}\ ,\ y_{i+1/4}\right)+B\ f\left(t_i\ ,\ y_i\right)\right]$$

where $t_{i+1}=t_i+h$ and $f\left(t_i\ ,\ y_i\right)=y_i'$.

Determine the coefficients A and B such that the global error of the numerical scheme is of O(h^2). [10 marks]

(b) The numerical scheme is now modified as a predictor-corrector method

$$y^*_{i+1/4}=y_i+\frac{h}{4}f\left(t_i\ ,\ y_i\right)$$
$$y_{i+1}=y_i+h\left[A\ f\left(t_{i+1/4}\ ,\ y^*_{i+1/4}\right)+B\ f\left(t_i\ ,\ y_i\right)\right]$$

Determine the coefficients A and B such that the global error of the modified numerical scheme is of O(h^2). [15 marks]

Question 2

The constitutive equation of a material is given by

$$\sigma_{ij} = 2\mu\left(\varepsilon_{ij}^{dev} - \varepsilon_{ij}^{p}\right) + K\varepsilon_{kk}\delta_{ij}$$

where σ_{ij} = stress tensor, ε_{ij} = strain tensor, ε_{ij}^{p} = plastic strain tensor, μ = shear modulus and K = bulk modulus. The superscript 'dev' denotes the deviatoric part of a tensor.

The equivalent stress is defined as $\sigma_{eq} = \sqrt{\frac{1}{2}\sigma_{ij}^{dev}\sigma_{ij}^{dev}}$.

During a plasticity deformation, the yield function is given as

$$F = \sigma_{eq} - (1-\omega)H\lambda^{n} = 0$$

where λ = equivalent plastic strain, H and n are material parameters.
The variable ω is a 'damage' variable defined as

$$\omega = 1 - e^{-\lambda}$$

Let the material parameters be $\mu = 100$, $K = 100$, $n = 0.3$ and $H = 10$. You are told that the deformation at the current time-step t is subjected to a plasticity increment. The kinematic fields are given as:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} 0.03 & 0.001 & 0 \\ 0 & 0.03 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad \boldsymbol{\varepsilon}^{p\,(t-1)} = \begin{bmatrix} 0.005 & 0 & 0 \\ 0 & -0.005 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , \quad \lambda^{(t-1)} = 0.008$$

where superscript (t-1) denotes values at the previous time-step.

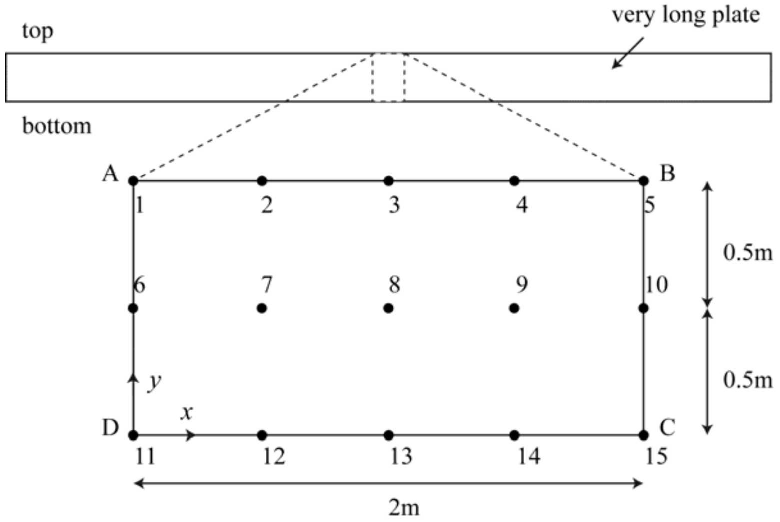
(a) Given that $\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}\dfrac{\partial F}{\partial \sigma_{ij}^{dev}}$, find $\dot{\lambda}$ in terms of $\dot{\varepsilon}_{ij}^{p}$. [7 marks]

(b) Derive the (1D) non-linear function with $\Delta\lambda$ as the unknown variable. Discuss how the Newton-Raphson iteration method is used to solve for the current σ_{ij}.

[15 marks]

(c) Determine the incremental plastic strain tensor $\Delta\boldsymbol{\varepsilon}^{p}$ at the current time-step t. Use $\Delta\lambda = 0$ as a starting value for the Newton-Raphson iteration. Assume that the solution has converged after <u>one</u> iteration. [10 marks]

Question 3



Consider a very long plate of thickness 1m. The temperature ϕ at its top surface has a sinusoidal profile, while its bottom surface is maintained at 0^{o}C. In this problem, we are interested in the temperature distribution of the plate interior, far away from the left and right boundaries. The problem can thus be solved by considering a plate section of one wavelength of the given sinusoidal profile, with the appropriate boundary conditions.

The plate section is shown in the Figure above as ABCD. The temperature profile along AB is given as $\phi(x,1) = 10\sin(\pi\ x)$. Along DC, temperature $\phi(x,0) = 0$. The nodes 1 – 15 are spaced equally apart.

The steady state problem is described by

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0\ .$$

Use the central difference method in x and y to find the temperatures at nodes 6 – 10.

Leave your solution in the following form:

$$[A]_{n\times n}[\phi]_{n\times 1} = [b]_{n\times 1}$$

where n is the <u>smallest</u> integer required to solve this problem. Do <u>not</u> solve the system of equations.

[23 marks]

<u>Question 4</u>

The eigen-solutions to $\underline{\underline{C}} = \begin{bmatrix} 5 & 5 & -3 \\ 5 & 13 & -5 \\ -3 & -5 & 5 \end{bmatrix}$ are denoted as $\left(\lambda_i \, , \underline{\phi_i}\right)$. For each mode i, the eigenvalue is λ_i with corresponding eigenvector $\underline{\phi_i}$, normalized such that $\underline{\phi_i^T} \, \underline{\phi_i} = 1$.

The eigenvector for the first mode is given as $\underline{\phi_1} = \left[1/\sqrt{2} \quad 0 \quad 1/\sqrt{2}\right]^T$.

Note that $\underline{\underline{C}}^{-1} = \begin{bmatrix} 0.3704 & -0.0926 & 0.1296 \\ -0.0926 & 0.1481 & 0.0926 \\ 0.1296 & 0.0926 & 0.3704 \end{bmatrix}$.

(a) Solve for $\underline{\phi_2}$ with the Gram-Schmidt Orthogonalization method. Let the starting vector $\underline{x_1} = \left[1 \quad 1 \quad 1\right]^T$. Assume that the solution converges after <u>one</u> iteration.

[13 marks]

(b) Based on your solution in (a), and without performing any numerical iterations, determine $\underline{\phi_3}$.

[7 marks]

- END OF PAPER -