A monthly interception equation based on the statistical characteristics of daily rainfall

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[1] This paper presents a simple analytical equation for monthly interception on the basis of the combination of a daily threshold model with the probability distribution of daily rainfall. In this paper, interception has a wider definition than merely canopy interception. It is the part of the rainfall that evaporates after it has been stored on the wetted surface, which includes the canopy, the understory, the bottom vegetation, the litter layer, the soil, and the hard surface. Interception is defined as the process of evaporation from intercepted rainfall. It is shown that this process has a typical timescale of 1 day. Monthly interception models can be improved by taking the statistical characteristics of daily rainfall into account. These characteristics appear to be less variable in space than the rainfall itself. With the statistical characteristics of daily rainfall obtained at a few locations where reliable records are available (for example, airports) monthly models can be improved and applied to larger areas (20–200 km). The equation can be regionalized, making use of the Markov property of daily rainfall. The equation obtained for monthly interception is similar to Budyko's curve.

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1. Introduction

[2] In water resources assessment, interception is often disregarded or modeled through highly empirical relations. Interception is an important flux for water resources planning. It is the so-called unproductive evaporation as opposed to transpiration, which produces biomass. Particularly in sub-Saharan Africa, options for shifting nonproductive to productive evaporation offer opportunities to improve rainfed agriculture for smallholder farmers [e.g., *Falkenmark and Rockström*, 2004; *Falkenmark and Lannerstad*, 2005]. This makes interception an important hydrological flux to be modeled. In water resources modeling, the monthly timescale is the most appropriate scale to assess water availability for different water using activities; it is the right compromise between data availability, modeling complexity and the required temporal resolution for planning decisions. Particularly in data-scarce environments data with a high temporal density are difficult to get and often unreliable.

[3] Here interception is defined as the evaporative flux that feeds back the moisture from rainfall during and shortly after a rainfall event before it runs off or infiltrates. This is a wider definition than merely canopy interception and includes interception by the understory, the forest floor, the surface vegetation, the wet surface and even the upper layer of the soil after wetting by rainfall. For hydrological modeling this is also the appropriate definition because in this way it is defined as the flux that accounts for the immediate feedback of moisture to the atmosphere before subsequent processes such as infiltration and surface runoff take place.

[4] To account for this interception flux at a monthly timescale there exist mostly empirical equations. Several monthly water balance models have a flux they call "initial loss": the part of the rainfall that will not be available for infiltration or runoff. This "initial loss" does not necessarily have the physical connotation of interception. The initial loss is often calibrated as the closure of the water balance using equations that have no relation with the variability of rainfall over the month. Take, for example, the widely used water balance model CROPWAT [*Clarke et al.*, 1998] of the Food and Agricultural Organization (FAO) (based on work of *FAO* [1977, 1979], obtainable from http://www.fao.org/AG/AGL/AGLW/cropwat.stm), which is used to determine irrigation requirements. CROPWAT uses as effective rainfall the rainfall minus the "initial loss". For that purpose the user can choose between two empirical relations, either the FAO/AGWL equation, or the USDA equation [*FAO*, 1977]. The FAO/AGWL equation reads:

$$I_m = \min(0.2P_m + 24, 0.4P_m + 10, P_m) \qquad (1)$$

where I_m is the monthly interception and P_m is the monthly rainfall. The USDA equation reads:

$$I_m = \max\left(\frac{0.2}{125}P_m^2, 0.9P_m - 125\right) \qquad (2)$$

[5] In the USDA equation the initial loss is proportional to the square of the monthly rainfall. The proportion of the rainfall that is considered initial loss increases with rainfall (see Figure 1). In the FAO/AGWL equation the proportion

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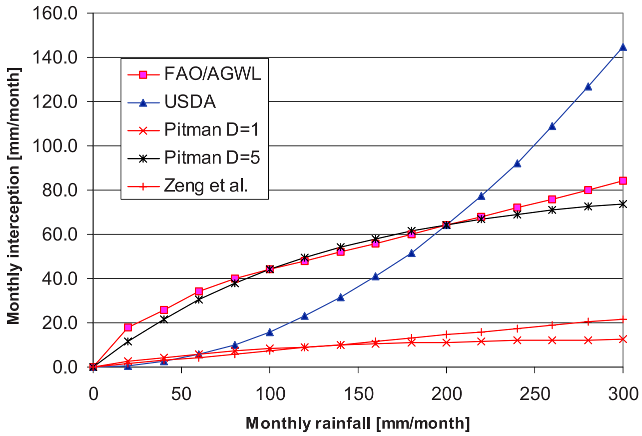

Figure 1. Different monthly interception models according to *Pitman* [1973], with two daily thresholds of 1 and 5 m/d; USDA, FAO/AGWL and *Zeng et al.* [2000].

of rainfall that is considered an initial loss decreases with increasing rainfall. This is more realistic. However, the initial loss in the FAO/AGWL equation is merely a function of monthly rainfall and does not account for the variability of rainfall within the month.

[6] *Pitman* [1973] developed a method particularly for Southern Africa. He used daily records of several hundreds of rainfall stations as input to a daily interception model. He then fitted monthly empirical models to the monthly interception totals. His model has been widely used in Southern Africa. The calibration parameters, at the time, were considered suitable for the whole of South Africa. Pitman's equation uses a daily interception threshold D (mm/d) and reads:

$$I_m = 13.08D^{1.14}\left(1 - \exp\left(P_m\left(0.00099D^{0.75} - 0.011\right)\right)\right) \qquad (3)$$

The coefficients in this equation have strange dimensions and there is no physical reasoning behind the equation. Pitman's empirical model required daily records from many stations in South Africa to derive. Application at other locations would require similar efforts.

[7] *Zeng et al.* [2000] derived a monthly interception equation for canopy interception based on a continuous evaporation model, according to *Rutter et al.* [1971]. It maintains a running water balance for canopy storage. As long as the canopy storage is less than the maximum value (here 0.6 mm), the evaporation is assumed to be proportional to the storage. However, these derivations use seasonally varying parameters to upscale interception to the monthly level. This means that this approach does not take account of the dependency of interarrival times on the daily cycle, or that convective storms often occur at the end of the day. As a consequence interception becomes more or less proportional to monthly rainfall. For canopy interception, where the evaporation rate is more important than the storage capacity, this is probably correct, but for the wider definition of interception this is not.

[8] Figure 1 shows the relationships of USDA, FAO/AGWL, *Zeng et al.* [2000], and *Pitman* [1973] with interception thresholds of 1 and 5 mm/d. In our approach, we have used the following considerations for improving monthly interception models. First, we shall demonstrate that the interception process has a typical timescale of one day (see section 3). Hence it is important to use the statistical characteristics of daily rainfall to determine monthly interception. Second, a daily threshold for interception is considered a proper physical description for the amount of water that can evaporate from the canopy, forest floor and soil surface and which may vary in space. Thirdly, we wanted a method that only uses a limited number of rainfall stations with reliable daily rainfall records to derive statistical parameters that are representative for larger areas ($\sim$200 $\times$ 200 km), not only to save analysis work, but also because long good daily rainfall records are difficult to obtain in poorly gauged countries. In Zimbabwe, for instance, there is a large number of stations with monthly records, but there are three stations (Harare, Bulawayo, and Masvingo) with reliable daily records of long duration.

[9] Although in water resources modeling the monthly timescale is the appropriate scale, interception is governed by processes at much smaller timescales which monthly models do not account for. Hence a simple analytical equation is derived for monthly interception that makes use of the daily characteristics of the interception process and which can be readily used in water resources modeling. The equation is derived in section 5 by a combination of a daily threshold model (section 3) and the statistical distribution of daily rainfall over a month (section 4). The Zimbabwe case (section 2) serves to illustrate the method and to demonstrate that the equation can be regionalized reliably.

2. Description of the Study Site

[10] The analytical equation has been derived for circumstances in Southern Africa and has been tested on data from Zimbabwe. Zimbabwe lies roughly between longitude 26° and 32°E and between latitude 16° and 22°S (see Figure 2). It covers an area of 390,000 km^2 and has an average rainfall of about 700 mm/yr. The details of the three key stations used in this paper (Harare, Masvingo, and Bulawayo) are presented in Table 1.

[11] The rainfall in Zimbabwe is strongly related to the seasonal fluctuations of the Intertropical Convergence Zone (ITCZ), the zone where the airstreams originating from both hemispheres meet. The ITCZ is not one permanent, globe-encircling region, where airstreams are always convergent, but a complex, ever changing band of growing and disintegrating convergences (see description by *Buckle* [1996]). The position, width and depth vary geographically, seasonally and even daily. The ITCZ moves with the sun, southward at the beginning of the rainy summer season and northward at the end. As a consequence, the rainy season in the north starts earlier and finishes later than in the south.

[12] The convergence within the ITCZ induces convection, which is the movement of air upward due to the heating at the ground level. Convection accounts for perhaps 90% of the Zimbabwean rainfall, although not all of this is related to the ITCZ [*Torrance*, 1981]. Convection can result in the generation of huge cumulonimbus clouds from which energy and rainfall are released during thunderstorms. The upward movement of air is compensated by the downward movement of air (subsidence) in areas around it. Convection thus only occupies a maximum of 10% of the

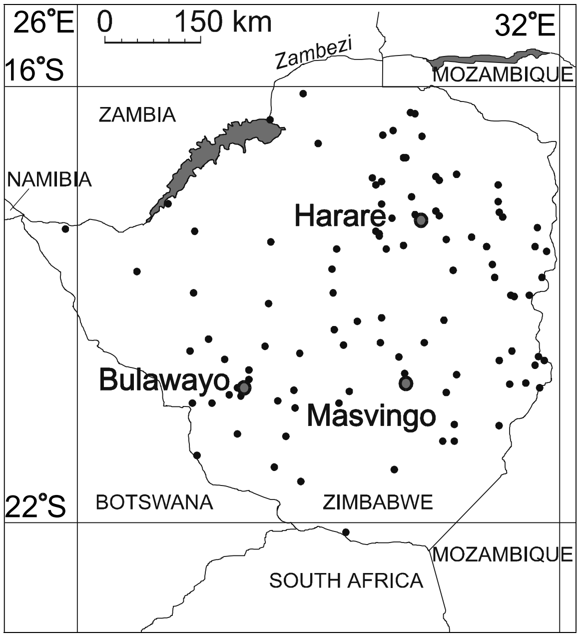

Figure 2. Map of Zimbabwe. Data from reliable rainfall stations in Harare, Masvingo and Bulawayo have been used for deriving the statistics of daily rainfall. The other dots show about a hundred rainfall stations for which only monthly rainfall data were readily available.

area, with dimensions in the order of 5 to 10 km square. Therefore daily rainfall has a very small spatial scale. Interception changes the energy balance locally and thus induces convection, causing persistence in the occurrence of rainy days [*Taylor et al.*, 1997].

[13] Apart from the ITCZ, Indian Ocean cyclones influence Zimbabwe rainfall, most violently shown by the cyclone Eline that struck Mozambique, southern Zimbabwe and northern South Africa in the beginning of 2000. Indian Ocean cyclones are frequent phenomena, but normally do not penetrate so far inland. The cyclone season is usually from December to April. In the Mozambique Channel most storms curve back southward and dissipate in higher latitudes [*Buckle*, 1996]. The influence on the weather in Zimbabwe varies, depending on the month, strength and precise track. Increased rain is usual within a 100 km radius of the cyclone, but further away the effects are sometimes totally the reverse. *Buckle* [1996] and *Matarira and Jury* [1992] mention the example of cyclones that curve south near Madagascar and that cause an influx of dry, upper air into Zimbabwe and Mozambique, yielding persisting dry spells. *Torrance* [1981] mentions the example that the remains of a cyclone cause a deep low-pressure area, which brings strong winds that enhance orographic rainfall.

3. Interception as a Threshold Process

[14] Interception is the process that feeds back rainwater to the atmosphere after it has been stored on the wetted surface. In most of the literature interception is limited to canopy interception, but here a wider definition of the wetted surface is used. The wetted surface is assumed to consist of the canopy, the understory, the bottom vegetation, the hard surface and the top layer of the soil. The interception process is defined as

$$I = \frac{dS_I}{dt} + E_I \tag{4}$$

where I is the interception process, S_I is the interception storage, and E_I is the evaporation from interception. The timescale of the interception process is in the order of 1 day. This follows from scaling. The maximum evaporation from interception equals the potential evaporation. Depending on the climate this is in the order of 2–5 mm/d. The maximum storage capacity is in the order of several millimeters, depending on the land cover. Scaling the interception storage and the evaporation to their maximum values, results in a timescale in the order of 1 day. As a result, if we consider daily rainfall, the interception I is about equal to the evaporation flux E_I.

[15] The total evaporation over the land surface can be split up into four components:

$$E = E_I + E_T + E_S + E_O \tag{5}$$

where E is the total evaporation, E_T is the transpiration, E_S is the soil evaporation, and E_O is the open water evaporation. In our definition, the split between E_I and E_S requires clarification. If the wetting of the soil remains purely superficial, implying that no contact is made with the soil moisture in the root zone and that the soil is dry directly under the wetted surface, then the soil wetting is part of the interception process. If connectivity is made with the moisture profile in the unsaturated zone, then the evaporation from the soil draws on the soil moisture stock and is

Table 1. Details of the Rainfall Stations From Which Data Have Been Used

	Harare	Masvingo	Bulawayo
Mean annual rainfall, mm/yr	790	640	630
Latitude	17°55′S	20°04′S	20°09′S
Longitude	31°06′	30°52′	28°37′
Altitude, m	1470	1100	1340
Mean number of thunderstorm days	73	54	63
Data period	1959/1960 to 1992/1993	1951/1952 to 1996/1997	Jan 1930 to 1994/1995
Missing data	1982/1983, 1989/1990	Feb 1959 to Nov 1965, 1978/1979, 1987/1988, 1988/1989	Feb 1932, Apr 1939, Apr 1942, Apr 1947, Jan 1956, Apr 1987, Mar 1990, Apr 1991

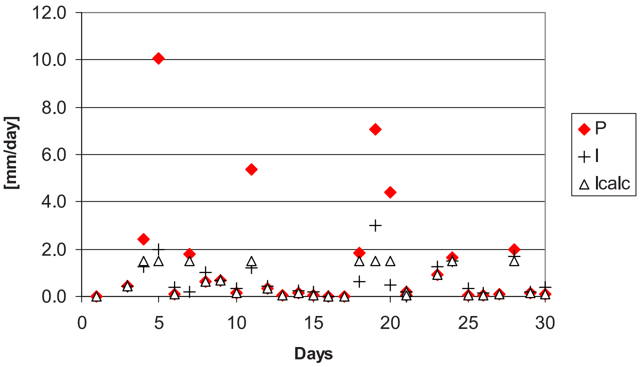

Figure 3. Rainfall and forest floor interception in the Huewelerbach catchment in Luxembourg. The diamonds indicate daily throughfall (P), the + indicates observed forest floor interception (I) and the triangles calculated forest floor interception (I_{calc}) with equation (6).

soil evaporation. Of course this is a fuzzy split, which is difficult to quantify. However in hydrological modeling this split can be parameterized and quantified by calibration. The inclusion of part of the soil evaporation in the interception process implies that there is less infiltration to the unsaturated zone.

[16] Most of the literature on interception [e.g., *Calder*, 1990] refers to canopy interception and not to the evaporation from the entire wetted surface (canopy, understory, forest floor, litter and bare soil). However, for hydrological modeling it is important to consider all forms of interception that prevent water from runoff or infiltration. The main difference between canopy interception and forest floor interception is that the canopy generally has a small storage capacity compared to the amount of energy available for evaporation, whereas the forest floor has a large capacity compared to the potential evaporation. The canopy evaporation rate appears to be more important than the canopy storage capacity [*Schellekens et al.*, 1999]. The timescale of the canopy interception process is therefore less than a day, while forest floor interception has a larger timescale. In other words, the canopy interception is moisture constrained, whereas the forest floor interception is energy constrained. The combined process has a timescale of approximately one day (as will be shown below).

[17] Interception at a daily timescale can be considered a threshold process [*Savenije*, 2004].

$$I_d = \min(P_d, D) \tag{6}$$

where I_d is the daily interception, P_d is the daily rainfall and D is the daily interception threshold, all expressed in mm/d. Only the amount of rainfall that exceeds the threshold takes part in subsequent processes such as infiltration and surface runoff. During and shortly after a rainfall event, the amount of water stored on the surface is evaporated. The amount of water evaporated from this store during a rain day is constrained by the amount of rainfall and the daily interception threshold D. This threshold has a physical meaning: it either represents the interception capacity of the entire moist surface (consisting of the vegetation cover, litter, soil and bare surface) or it represents the potential evaporation from a wet surface on a rain day, whichever is largest. If the interception capacity is larger than the potential evaporation, then some of the moisture is carried over to the next day, but here the whole amount is accounted for on the rain day itself.

[18] The concept of interception as a threshold process works well in Southern Africa, as was demonstrated by *Winsemius et al.* [2006], who applied the concept in different hydrological models in the Zambezi catchment. A further justification of equation (6) is provided from observations.

[19] *Gerrits et al.* [2006] developed a new device to measure forest floor interception. It consists of two basins located above each other that are continuously weighed. The upper basin is filled with litter, and the lower basin is a collector that is drained by a valve. Figure 3 presents the results form the first experiment during the month of November 2004 in a beech forest in the Huewelerbach catchment in Luxembourg. The figure presents the daily throughfall and the measured and computed forest floor interception by equation (6). For the computed interception a threshold of 1.5 mm/d is used. The threshold has been adjusted until the computed and measured monthly interception was equal. We can see that the computed interception is generally close to the measured interception but that deviations occur on days with heavy rainfall. During and after days with larger amounts of rainfall (days 4–8, days 18–20) there is an effect of carryover storage. Figure 4 presents a parity plot between the computed and observed interception. There is a maximum to the computed interception of D, and we see that at that point, there is a range of observed values (between 0 and 3) whereby the lower values are the results of a shortage of atmospheric energy during the rain day, while the higher values result from carryover to a next day with higher potential evaporation. What is clearly visible, however, is that as long as the rainfall is below the threshold value, all rainfall is converted into interception. The scatter can be explained by the heterogeneity of throughfall and the fact that the recording rain gauge is placed near but not on top of the device.

[20] Because of the daily timescale, monthly interception depends on the rainfall distribution over a month. A month with a high number of rain days generates more interception

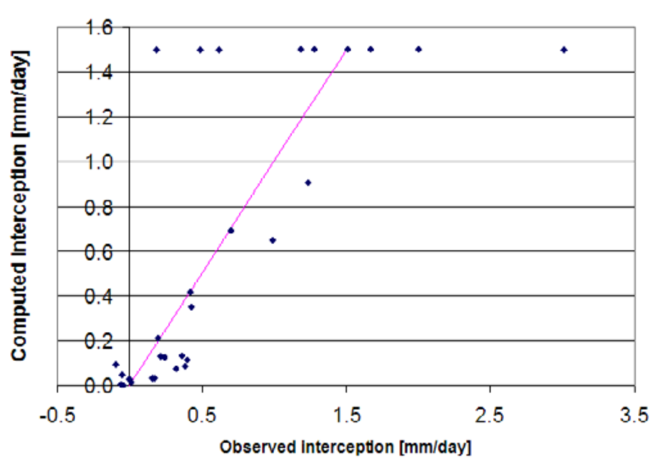

Figure 4. Parity plot between observed and computed forest floor interception at the Huewelerbach catchment in Luxembourg. The drawn line represents the line of perfect agreement.

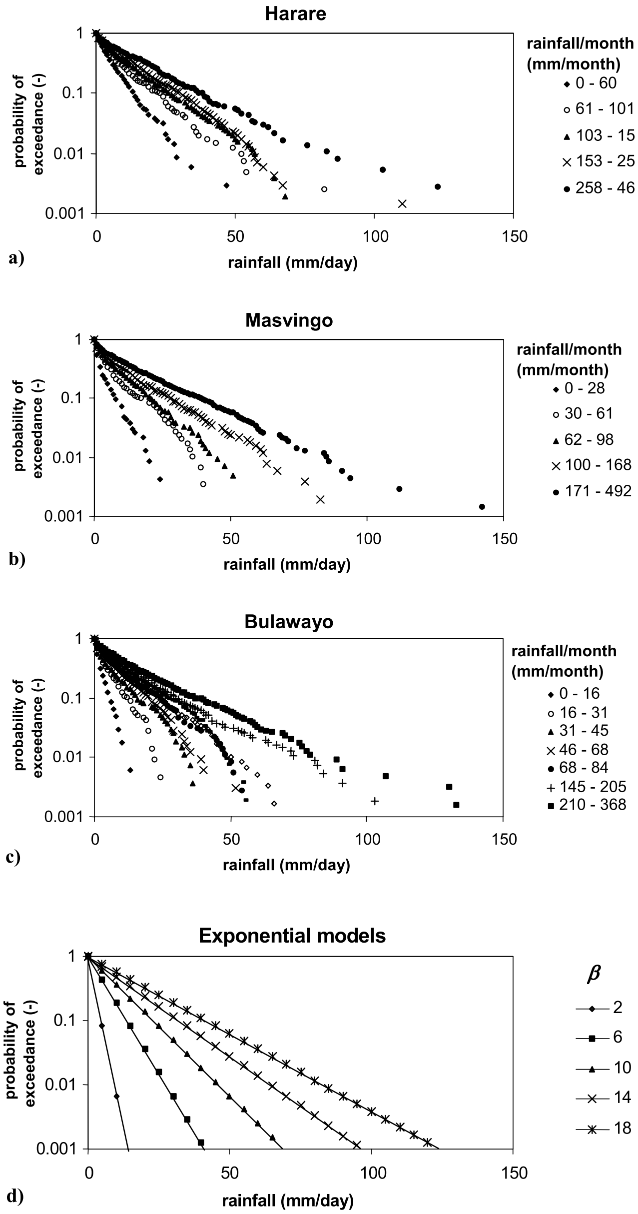

Figure 5. Probability of exceedance of rainfall amounts on rain days for Harare (a), Masvingo (b) and Bulawayo (c) for different rainfall classes. Figure 5d shows the probability of exceedence of daily rainfall using the exponential distribution for different values of β.

than a month with the same amount of rainfall but with just a few intensive rainfall events. However, existing monthly interception models hardly use the probability distribution of daily rainfall for a given monthly rainfall amount.

4. Probability Distribution of Rainfall on a Rain Day

[21] For every class of monthly rainfall P_m the probability density function of daily rainfall P_r is in good agreement with an exponential function:

$$\mathrm{f}(P_r) = \frac{1}{\beta}\exp\left(-\frac{P_r}{\beta}\right) \tag{7}$$

where P_r is the amount of rainfall on a rain day (mm/d) and β is the scaling parameter of the distribution (mm/d).

[22] This scaling parameter represents the expected rainfall on a rain day, which is a function of the monthly rainfall amount (see below). It follows from integration that the probability of exceedance $\mathrm{F}(P_r)$ is also an exponential function and that the mean μ and standard deviation σ are equal to the scaling parameter ($\mu = \sigma = \beta$).

[23] Several authors confirmed that the exponential distribution is a good approximation of the underlying rainfall processes at point scale [e.g., *Sivapalan and Blöschl*, 1998; *Todorovic and Woolhiser*, 1975; *Woolhiser et al.*, 1993]. In stochastic models, the scaling parameter β is often considered a function of the season. In these models exponential, Gamma or Weibull distributions are used to fit the data. Gamma and Weibull distributions are also of the 'exponential type'. The Gamma and Weibull distributions for rainfall on a rain day read respectively:

$$\mathrm{f}_\Gamma(P_r) = \frac{1}{\beta^k}\frac{P_r^{k-1}\exp\left(-\frac{P_r}{\beta}\right)}{\Gamma(k)} \tag{8}$$

$$\mathrm{f}_W(P_r) = \frac{k}{\beta^k}P_r^{k-1}\exp\left(-\left(\frac{P_r}{\beta}\right)^k\right) \tag{9}$$

[24] The calibration parameter k often makes these distributions similar to an exponential function. If $k = 1$ the Gamma distribution is an exponential function. So if k is close to 1 the Gamma distribution resembles an exponential function. Also the Weibull distribution equals an exponential distribution if $k = 1$. By calibration *Stern and Coe* [1982] found for Tanzania a Gamma distribution with $k = 0.76$. *Clarke* [1998] found for the Amazon in Brazil, a Gamma distribution with $k = 0.77$. Data of Zimbabwe show that by grouping daily rainfall records into monthly rainfall classes, the resulting distributions are close to exponential functions. In Figure 5 the probability of exceedance of daily rainfall is plotted for different monthly classes for Harare, the capital of Zimbabwe. The classes have been determined in a way that each contains an equal number of months. Because wet months have more rainfall events then dry months, the classes with higher rainfall contain more dots.

[25] Figure 5 demonstrates that exponential functions represent the probability of exceedance of daily rainfall ($\mathrm{F}(P_r) = \exp(-P_r/\beta)$) quite well: Figure 5a for Harare, Figure 5b for Masvingo, and Figure 5c for Bulawayo. Figure 5d shows the theoretical lines for different values of β, where the scaling parameter increases with monthly rainfall amounts.

[26] The characteristics of an exponential equation imply that the scaling parameter is the expected amount of rainfall on a rain day. Hence it is the ratio of the monthly rainfall to the number of rain days in a month:

$$\beta = \frac{P_m}{n_r} \tag{10}$$

where P_m is the observed rainfall in a month (mm/month), and n_r is the number of rain days per month (days/month).

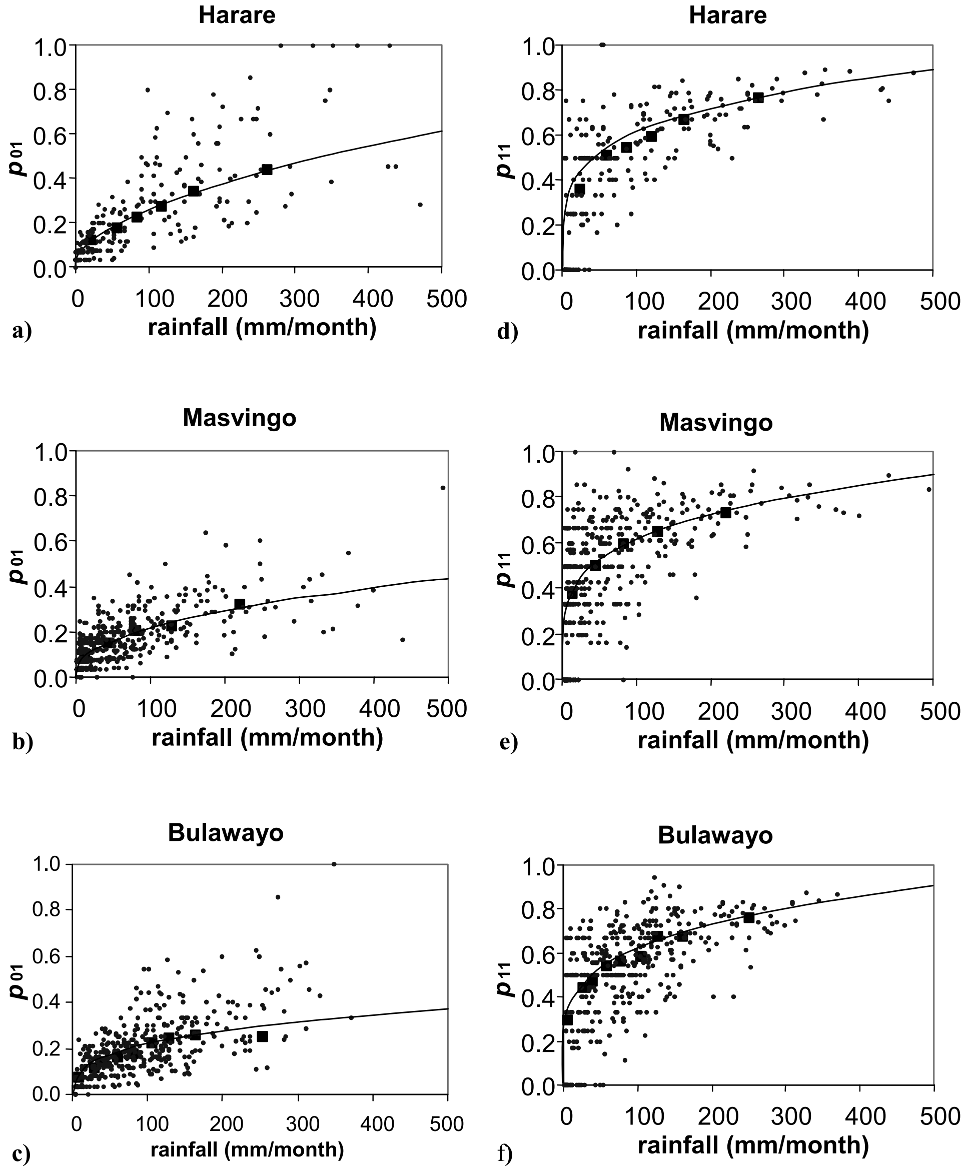

Figure 6. Markov probabilities for the occurrence of a rain day after a dry day (p_{01}) and the occurrence of a rain day after a rain day (p_{11}) for Harare, (a and d); Masvingo, (b and e); and Bulawayo, (c and f). Each dot represents an individual month. Each square represents the average of a class of monthly rainfall.

This number is also a function of the monthly rainfall, making β a mere function of the monthly rainfall.

[27] The number of rain days n_r can be obtained as a function of monthly rainfall using the Markov property of the rainfall. In many areas around the world, the occurrence of rain days can be modeled as a Markov process where the occurrence of rainfall on a certain day only depends on the occurrence of rainfall on the previous day. *Gabriel and Neumann* [1962] apparently were the first to use Markov processes to describe the climate in Tel Aviv. In a Markov process, p_{11} signifies the probability of a rain day occurring after a rain day and p_{01} the probability of a rain day after a dry day.

Table 2. Coefficients of Power Functions Describing the Markov Probabilities for Rainfall Stations in Different Parts of the World

	$p_{01} = q\, P_m^{\ r}$		$p_{11} = u\, P_m^{\ v}$	
Location	q	r	u	v
Harare	0.020	0.55	0.20	0.24
Masvingo	0.030	0.43	0.20	0.24
Bulawayo	0.044	0.34	0.20	0.24
Peters Gate (SA)	0.094	0.33	0.034	0.40
Hyderabad (India)	0.092	0.38	0.024	0.53
Indianapolis (Indiana, USA)	0.129	0.30	0.045	0.42
Kansas (Missouri, USA)	0.129	0.27	0.061	0.30
Sheridan (Wyoming, USA)	0.216	0.22	0.084	0.30
Tallahassee (Florida, USA)	0.127	0.29	0.017	0.55

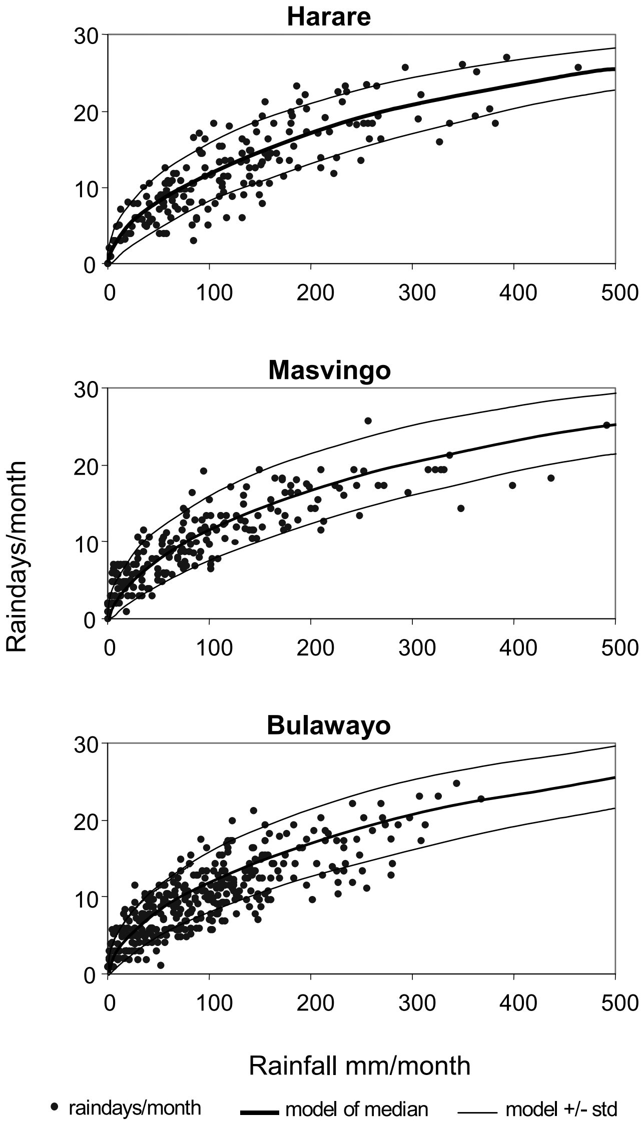

Figure 7. Expected number of rain days per month (n_r) as a function of monthly rainfall amounts, showing the theoretical line of equation (12) (thick lines) and observed number of rain days (dots). The thin lines indicate a band of ± one standard deviation.

[28] *Gabriel and Neumann* [1962] developed an exact solution for the probability of n_r rain days in a time span of n days.

$$\mathrm{P}(n_r|n) = \frac{p_{01}}{1 - p_{11} + p_{01}} \mathrm{P}(n_r|n, 1) + \frac{1 - p_{11}}{1 - p_{11} + p_{01}} \mathrm{P}(n_r|n, 0) \tag{11}$$

where $\mathrm{P}(n_r|n,1)$ is the probability of having exactly n_r rain days in a period of n days following a rain day; and $\mathrm{P}(n_r|n,0)$ is the probability of having exactly n_r rain days in a period of n days following a dry day. *De Groen* [2002] showed that for a large number of days the expected number of rain days in this period converges to

$$n_r = n \frac{p_{01}}{1 - p_{11} + p_{01}} \tag{12}$$

[29] With n as the number of days in a month (30 days is enough for Equation 12 to be valid), n_r is the expected number of rain days in a month. Applied to 70 years of historic data in Zimbabwe, *De Groen* [2002] showed that the transition probabilities can be expressed as simple power functions of monthly rainfall. The relationship $p_{11} = 0.20\ P_m^{0.24}$ is the same for the three main rainfall stations in Zimbabwe, each about 300 km apart. The relationships for p_{01} are different: $p_{01} = 0.020\ P_m^{0.55}$ for Harare, $0.030\ P_m^{0.43}$ for Masvingo and $0.044\ P_m^{0.34}$ for Bulawayo (see Figure 6). Although there is considerable scatter around the mean for individual months, the expected values for monthly rainfall classes (indicated by the squares) are consistent. The latter is relevant for the expected number of rain days in a month.

[30] Similar equations for p_{01} and p_{11} have been obtained in other parts of the world (see Table 2). *Zucchini et al.* [1992] derived relations for Peters Gate, close to Cape Town, South Africa; *Stern and Coe* [1982] derived relations for Hyderabad, India; and *Woolhiser and Pegram* [1979] derived relations for Indianapolis, Kansas City, Sheridan, and Tallahassee in the United States. One may question the applicability of the power functions, since they do not have an upper limit. From a theoretical point of view an equation of the logistic form would be more correct ($p = 1/(1 + x\ P_m^y)$), having an upper limit of unity. This has not been done for reasons of simplicity and because the probability p remains well below unity in all practical cases.

[31] Figure 7 shows the relation between the expected number of rain days for a given amount of monthly rainfall (thick lines) and the observed number of rain days (the dots) for Harare, Masvingo, and Bulawayo. The bands around the theoretical line indicate ±1 standard deviation.

[32] Combination of equations (7), (10), and (12) yields a probability density function of daily rainfall based on monthly rainfall records and the Markov probabilities of daily rainfall. In the next Section, this probability density functions is used to derive an equation for monthly interception based on a daily threshold model.

5. Equation for Monthly Interception

[33] The amount of rainfall that is intercepted on a rain day is the amount that remains under the threshold D. Integration of equation (6), using the probability distribution of equation (7) yields the average amount of rainfall caught under the threshold:

$$\overline{I_r} = \int_0^D P_r\, \mathrm{f}(P_r) \mathrm{d}P_r + \int_D^\infty D\, \mathrm{f}(P_r) \mathrm{d}P_r \tag{13}$$

[34] The first term represents the rain days where rainfall is less than the threshold (on those days the total rainfall is intercepted), the second term represents rain days where rainfall exceeds the threshold (on those days

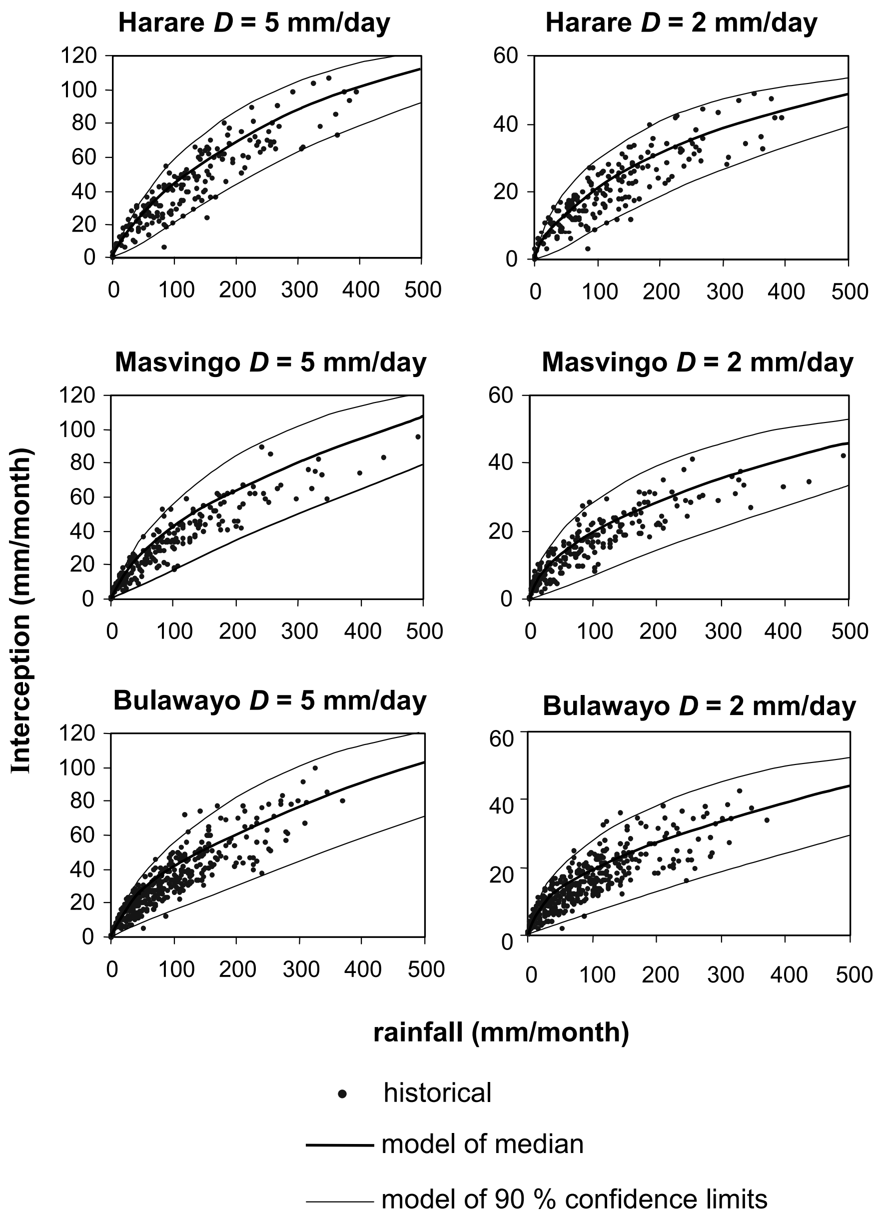

Figure 8. Relationship between monthly rainfall and monthly interception. The thick lines represent the computed interception with equation (16). D is the assumed daily threshold. The dots have been computed with equation (6), using the daily time series of Harare, Masvingo, and Bulawayo. The thin lines form the boundary of the 90% confidence band. They represent the equations for 95% and 5% probability of exceedance.

an amount of D mm/d is intercepted). Elaboration of equation (11) yields

$$\begin{aligned}\overline{I_r} &= \int_0^D \frac{P_r}{\beta} \exp\left(-\frac{P_r}{\beta}\right) \mathrm{d}P_r + D(1 - \mathrm{F}(D)) \\ &= \left[-P_r \exp\left(-\frac{P_r}{\beta}\right)\right]_0^D - \left[\beta \exp\left(-\frac{P_r}{\beta}\right)\right]_0^D + \left[D \exp\left(-\frac{D}{\beta}\right)\right] \\ &= \left[-D \exp\left(-\frac{D}{\beta}\right) - \beta \exp\left(-\frac{D}{\beta}\right) + \beta\right] + \left[D \exp\left(-\frac{D}{\beta}\right)\right] \\ &= \beta\left(1 - \exp\left(-\frac{D}{\beta}\right)\right) \end{aligned} \tag{14}$$

[35] Combination with equation (10) and realizing that the monthly interception (I_m) is n_r times the mean daily interception, yields the sought equation for the monthly interception as a function of monthly rainfall (P_m), the daily interception threshold (D) and the expected amount of rainfall on a rain day (β):

$$I_m = P_m\left(1 - \exp\left(\frac{-D}{\beta}\right)\right) \tag{15}$$

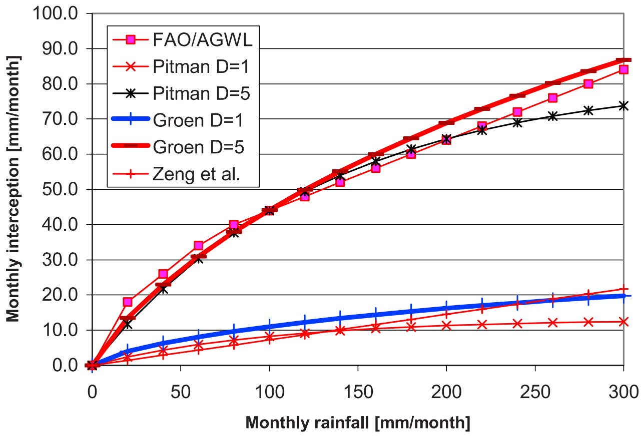

Figure 9. Comparison between the new equation for monthly interception (indicated by thick lines) for two values of the daily threshold ($D = 1$ and $D = 5$) and existing models by Pitman (thin lines with x and *), FAO (thin line with square) and Zeng et al. (thin line with +).

[36] Using equation (10) for the argument of equation (12) yields

$$I_m = P_m\left(1 - \exp\left(\frac{-Dn_r}{P_m}\right)\right) \tag{16}$$

[37] The expected number of rain days n_r is a function of the monthly rainfall as well, which will be explained further on, while the daily interception threshold may depend on soil characteristics, potential evaporation and vegetation cover. Hence D may vary spatially and over the season.

[38] Equation (16) has a striking similarity to the empirical equation for average annual evaporation $\bar{E}$ proposed by *Schreiber* [1904], as described by *Dooge* [1997] and by *Arora* [2002]. *Schreiber* [1904] proposed a purely empirical formula fitted to annual data for central Europe:

$$\overline{E} = \overline{P}\left(1 - \exp\left(\frac{-\overline{E}_p}{\overline{P}}\right)\right) \tag{17}$$

where E is the total evaporation, E_p is the potential evaporation and the overbars indicate annual averages. This kind of relation with its typical asymptotic behavior is commonly referred to as a Budyko curve, after *Budyko* [1948, 1951], who further elaborated Schreiber's equation. The ratio of annual potential evaporation to annual rainfall is called the aridity index after *Budyko* [1974]. Equations (16) and (17) have similar asymptotes. If the rainfall is low, all moisture is evaporated ($E = P$; $I_m = P_m$). If the rainfall is very large, then the annual evaporation approaches the potential evaporation E_p, just like Dn_r forms the upper limit for monthly interception.

[39] Figure 8 illustrates how the model performs in the three key stations of Zimbabwe. The dots have been computed by the daily threshold model, using the 70 years of historical daily rainfall records. The central lines have been computed with equation (16). The thin lines represent the confidence band of 90%.

[40] Figure 9 shows how the new equations compare to the existing empirical (Pitman, FAO) and theoretical [*Zeng et al.*, 2000] equations. The equation by Zeng et al. relates to canopy interception only and can best be compared with equations for a daily threshold of 1 mm/d. Our equation (16) is based on the Markov probabilities at Harare. This equation compares well to the fully empirical equations of Pitman and FAO. It also compares well to the equation of Zeng et al. for a reasonable threshold value of 1 mm/d for canopy interception [*Germer et al.*, 2006]. This makes the new equation very attractive, particularly for a data-scarce region such as Southern Africa.

[41] In equation (16) there are two key parameters: the number of rain days in a month and the daily interception threshold. The interception threshold is spatially heterogeneous, as it depends on the land cover. Land cover maps or Remote Sensing data may be used to determine distributed values of the daily threshold, but a deterministic method to derive D remains difficult. Alternatively, the daily threshold may be considered a constant for a certain subcatchment and be determined through calibration. This leaves the number of rain days as a key parameter to be determined from daily rainfall data.

[42] The relationship between the monthly rainfall and the number of rain days is spatially far more homogeneous (in Zimbabwe a correlation length of ~200 km) than the monthly rainfall itself (in Zimbabwe a correlation length of ~50 km with a correlation coefficient of 0.6–0.9). In many countries reliable long daily rainfall records are scarce. Moreover, correlation between daily rainfall series is low (in Zimbabwe a correlation length of ~50 km with a correlation coefficient of 0.2–0.5), in particular in semiarid areas where rainfall is mostly convective (thunderstorms). Figure 10 is an illustration of this. We see that the lines for the three key stations are not far apart and close to Pitman's curves. Similarly curves can be drawn for other stations in the region based on interpolation of the coefficients of the Markov probabilities.

[43] We therefore suggest using monthly rainfall records for water resources modeling, but to use daily records of a few well maintained rainfall stations to determine key model parameters that require the statistical properties of daily rainfall. The beauty of this approach is that the coefficients of the power functions can be interpolated

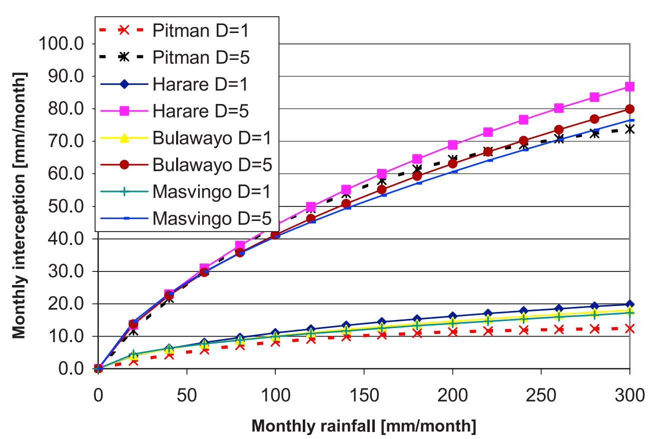

Figure 10. Equation (16) applied to the data of three key stations in Zimbabwe (Harare, Masvingo and Bulawayo) for two daily threshold values ($D = 1$ and $D = 5$) compared to the equations of Pitman (dashed lines).

reliably between the few locations where daily data are available. This results in a country-wide estimate of rain days per month as a function of monthly rainfall.

[44] Because n_r is a sole function of P_m, equation (16) provides a relationship with which the monthly interception can be computed directly from monthly rainfall records. One may, of course, use the observed number of rain days directly to determine monthly interception and skip the Markov analysis, but this number may not be available everywhere. Moreover, additional useful parameters can be determined using the Markov probabilities and monthly rainfall, such as: the lengths of dry and wet spells, the lengths of the longest dry and wet spells within the month, the number of wet and dry spells in the month, and the date of the first day with rainfall occurrence [*De Groen*, 2002]. These indicators are very useful for water resources analysis, particularly in semiarid regions.

6. Concluding Remarks

[45] This paper shows that by studying the statistical properties of daily rainfall records, a simple analytical equation for monthly interception can be derived that requires only a small number of reliable rainfall records, and in which the daily interception threshold D is a key parameter.

[46] The correct representation of interception at a monthly timescale is very important for water resources planning. Water resources planning models often use an empirical equation for interception (or initial loss) to determine the water needs for supplementary irrigation. It is much more efficient to use the method developed in this paper at a monthly timescale than to use daily models to simulate the water resources of a basin. Not only because of the efforts of processing the available data, but also because most networks of rain gauges in the world are not sufficiently dense and not sufficiently reliable to permit spatial interpolation of daily rainfall records.

[47] The fact that the obtained relationship resembles Budyko's curve, supports the physical basis of the new equation.

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