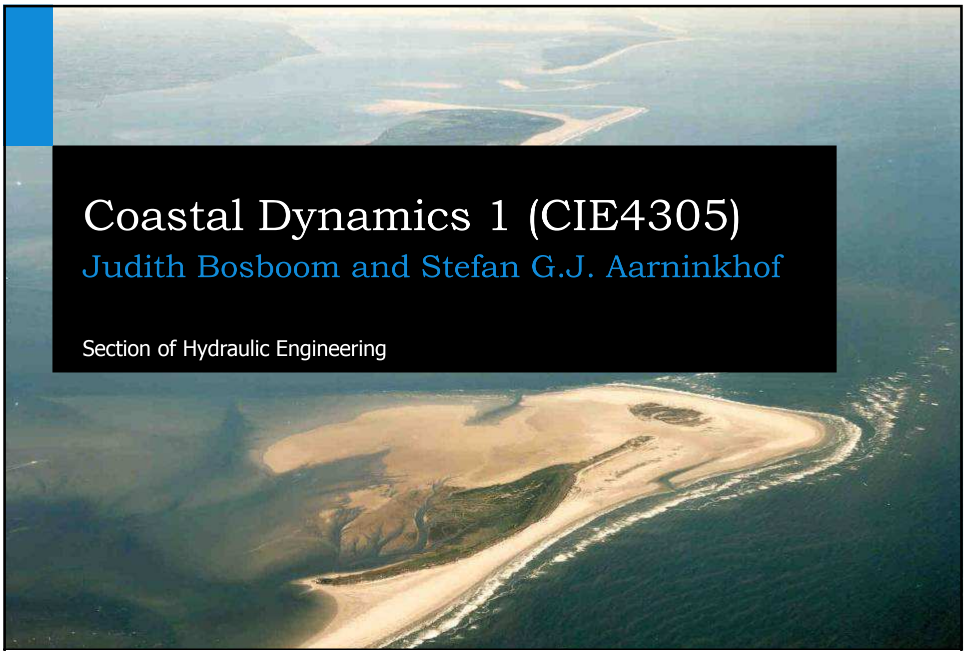


Coastal Dynamics 1 (CIE4305)

Judith Bosboom and Stefan G.J. Aarninkhof

Section of Hydraulic Engineering

3.

Oceanic wind waves and tide

Coastal Dynamics 1

Contents

Coastal Dynamics 1

Maple TA Chapter 1+2

Summary Data	Instructions (to be signed before start)	Chapter 1+2 - Stage A	Chapter 1+2 - Stage B	Total
# Students	196	196	196	196
# Attempts	195	195	188	195
# Attempts/Student	0.99	0.99	0.96	0.99
Mean	1	15.62	11.42 **82%**	-
Median	1	16	12	-
Total Points	1.0	17.0	14.0	32.0

See instructions on Brightspace for accessing gradebook

3. Oceanic wind waves and tide

Chapter 3 of lecture notes

A. Introduction
B. (Short-term) statistics of wind waves
C. Linear propagation
D. Generation of the tide
E. Tidal propagation
F. Tidal analysis and prediction

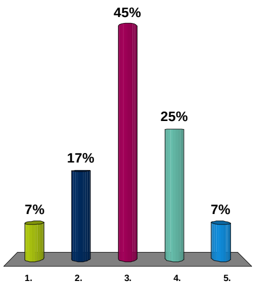
Ocean waves

(partly) Introduction Hydraulic Engineering & Open channel flow

Perusall experiences

Did you do the reading assignment (pages 81-110)?

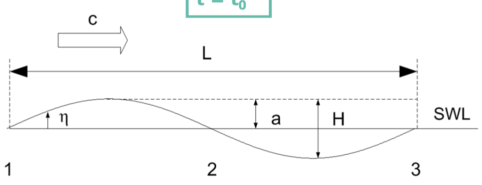
1. Yes, I read the pages and made annotations
2. I read all pages, but made no annotations
3. No, I did not read the pages at all
4. I started reading, but did not finish
5. Something else



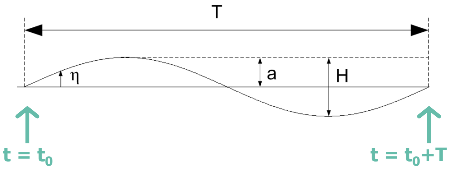
3-A Introduction

Sine wave form (linear wave)

$t = t_0$



$L = cT$ Location 1, 2 or 3?



$$\eta(x,t) = \frac{H}{2}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{L}x\right)$$

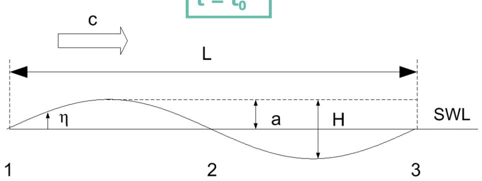
$$\eta(x,t) = a\sin\left(\omega t - kx\right) = a\sin S(x,t)$$

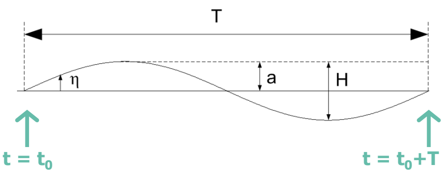
amplitude radial frequency wave number phase function

3-A Introduction

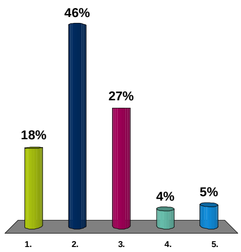
At which location was the time-series measured?

$t = t_0$ Location 1, 2 or 3?





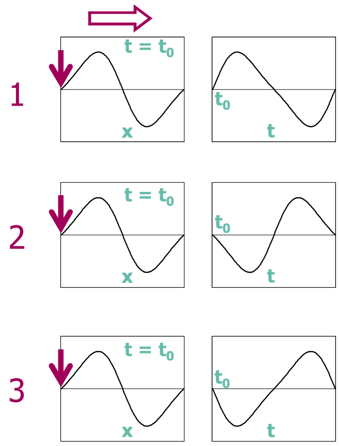
1. Location 1
2. ✓ Location 2
3. Location 3
4. Somewhere else
5. Abstain

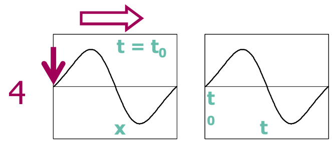


3-A Introduction

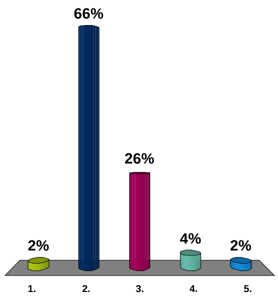
The right panel is measured at the location of the red arrow

What is the right combination?





1. Combination 1
2. Combination 2
3. Combination 3
4. Combination 4
5. Abstain



3. Oceanic wind waves and tide

Chapter 3 of lecture notes

A. Introduction
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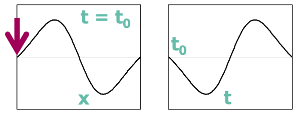
3-B Short-term statistics

Wave height parameters

2



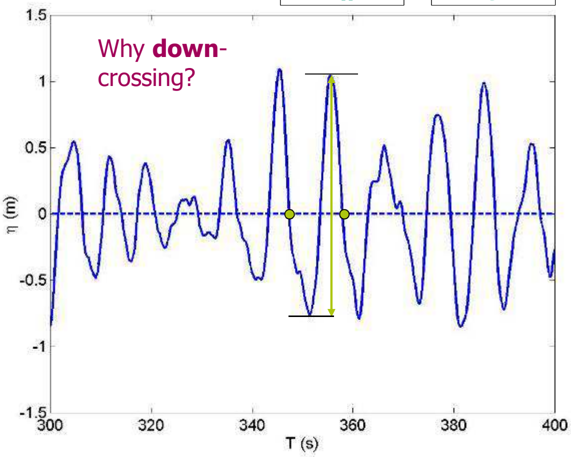
$$H_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} H_i^2}$$

$$H_{1/3} = \frac{1}{N/3}\sum_{j=1}^{N/3} H_j$$

$$\overline{T_0} = \frac{1}{N}\sum_{i=1}^{N} T_i$$

$$T_{1/3} = \frac{1}{N/3}\sum_{j=1}^{N/3} T_j$$

j



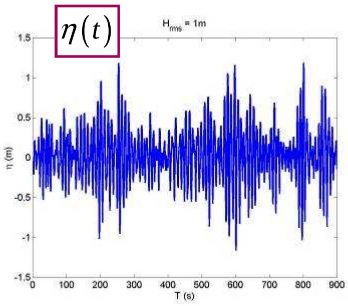
Zero-down crossings and wave height

3-B Short-term statistics

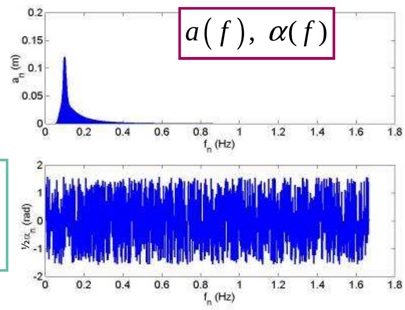
Spectral analysis of wind waves

- elevation η is summation of many sinusoids with 'own' a and period $T = 1/f$ (and direction)
- individual sine components can be determined by Fourier analysis

$$\eta(t) = \sum_n a_n \cos(2\pi f_n t + \alpha_n)$$



Fast Fourier Transform (FFT)

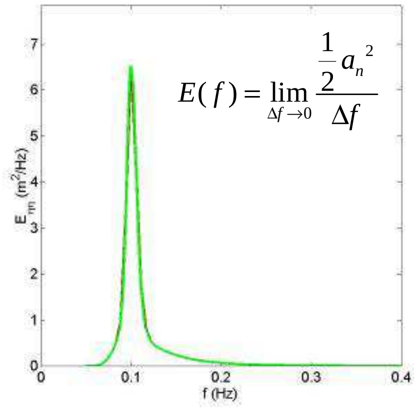


3-B Short-term statistics

Spectral variance / energy

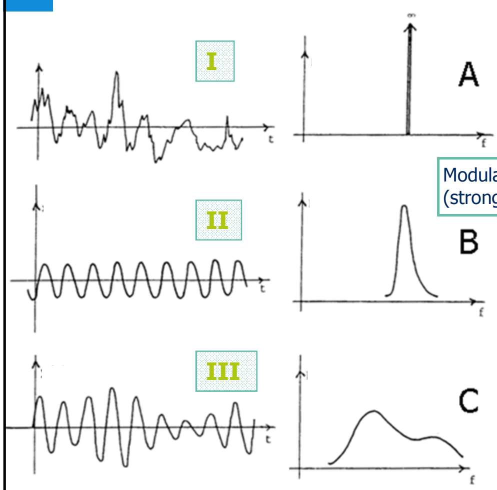
- Variance one harmonic: $\frac{1}{2}a_n^2$
- Spectral variance density $E(f)$ shows the contribution of each frequency to total variance
- Total variance: $\langle \eta^2 \rangle = \sum_n \frac{1}{2} a_n^2$

$$\langle \eta^2 \rangle = \int_0^\infty E(f)\, df = \sigma^2 = m_0$$



3-B Short-term statistics

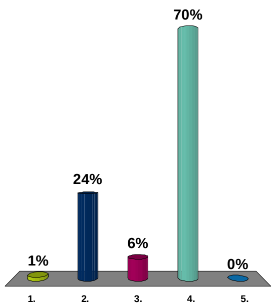
What is the corresponding spectrum?



Sine wave (II)

Modulated harmonic (strong wave grouping, III)

Irregular waves with secondary maxima and minima (I)



1. I-A; II-B; III-C
2. I-B; II-A; III-C
3. I-C; II-B; III-A
4. ✓ None of the above
5. Abstain

3-B Short-term statistics

Wave heights are Rayleigh-distributed

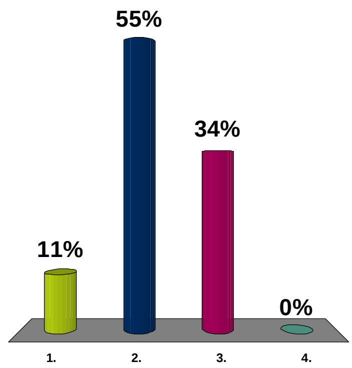
All characteristic wave heights can be derived from each other or from m_0

Description	Notation	$H/\sqrt{m_0}$	H/H_s
RMS height	H_{rms}	$2\sqrt{2}$	0.707
Mean height	$\bar{H}$	$\sqrt{2\pi}$	0.63
Significant height	$H_s = H_{1/3}$	4.004	1

3-B Short-term statistics

Which wave height represents the total energy content of an irregular wave field?

1. The mean wave height
2. ✓ The root-mean-square wave height
3. The significant wave height
4. Another wave height measure



3-B Short-term statistics

H_{rms} is the wave height representing the total energy content

$$E = \rho g \sigma^2 = \rho g \langle \eta^2 \rangle$$

$\boxed{\text{Regular (sine) wave}}$ $E = 1/2 \rho g a^2 = 1/8 \rho g H^2$

$\boxed{\text{Irregular}}$ $E = \rho g m_0 = 1/8 \rho g H_{repr}^2$

$$\Rightarrow H_{repr}^2 = 8 m_0$$

$$\Leftrightarrow H_{repr} = 2\sqrt{2}\sqrt{m_0} = H_{rms}$$

3. Oceanic wind waves and tide

Chapter 3 of lecture notes

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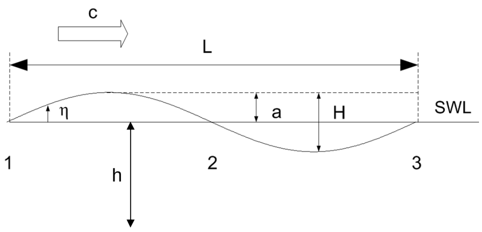
3-C Linear propagation

Review of Linear Wave Theory

Key assumptions:

- H/L or ak<<1 for kh large (deep water)
  - Not too steep
- H<<h or a<<h for kh small (shallow water)
  - Not too big

See Appendix B (p.531) for more



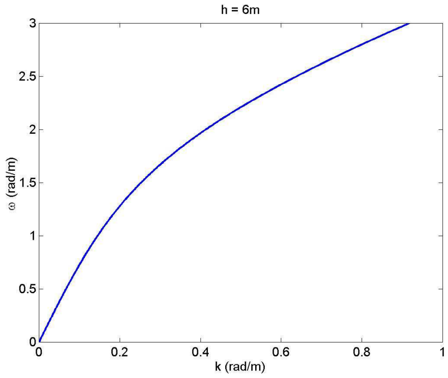
3-C Linear propagation

Dispersion relation for linear waves (no current)

Wave frequency is related to wave number via the dispersion relationship:

$$\boxed{\omega = \sqrt{gk \tanh kh}}$$

$$\omega = \frac{2\pi}{T} \qquad k = \frac{2\pi}{L}$$

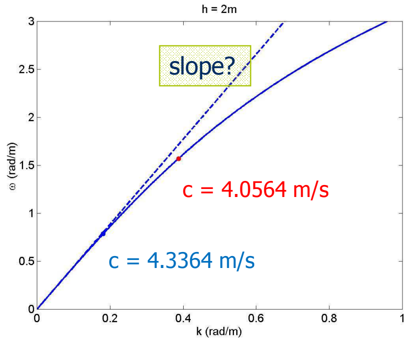


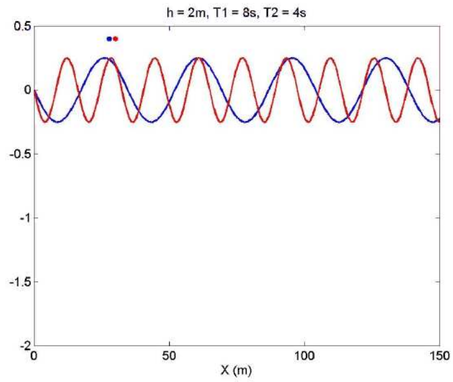
function of local water depth and restoring force g

3-C Linear propagation

Phase speed is the speed at which a particular phase of the wave (e.g. the crest) advances

$$\boxed{c = \frac{L}{T} = \frac{\omega}{k}}$$





Water waves are dispersive =>
phase velocity of longer waves is larger

3-C Linear propagation

See Appendix B

Deep and shallow water limits to dispersion relation

$$\omega^2 = gk \tanh kh$$

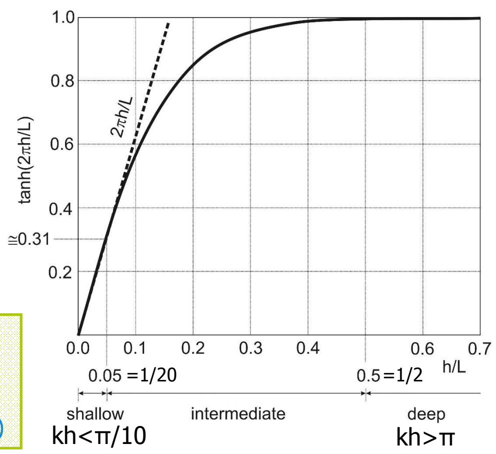
$\tanh kh \approx kh$

⇩

$$\omega^2 = gk^2 h$$

$$c = \sqrt{gh}$$

shallow water:
speed depends
on water depth
(non-dispersive)



$\tanh kh \approx 1$

⇩

$$\omega^2 = gk$$

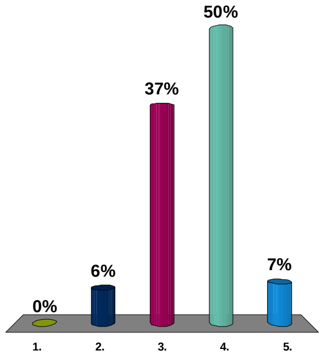
$$c = \frac{g}{\omega} = \frac{gT}{2\pi} = 1.56T = c_0$$

deep water:
speed depends
on frequency
(dispersive)

3-C Linear propagation

Consider a tsunami wave train with L = 500 km propagating in the deep ocean (h = 6 km). What is the propagation speed?

1. 2.3 km/hour
2. 8.25 km/hour
3. 243 km/hour
4. ✓ 873 km/hour
5. I do not know



And what if the question had been:
Consider a tsunami wave train with T = 34 min propagating in the deep ocean (h = 6 km). What is the propagation speed?

3-C Linear propagation

Any disturbance in the wave field travels at the **group velocity** (not the phase velocity!)

- wave front
- wave group
- wave energy

→ TRAVEL AT GROUP SPEED

3-C Linear propagation

Group velocity is smaller than phase velocity except for very long waves

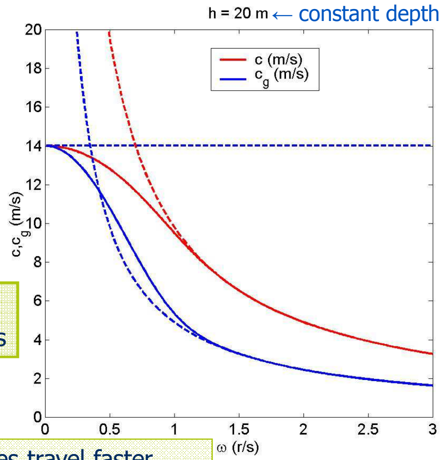
$$c_g = nc$$

$$n = \frac{1}{2} + \frac{kh}{\sinh 2kh}$$

"shallow" long waves

$n = 1$

h = 20 m ← constant depth



"deep" short waves

$n = \frac{1}{2}$

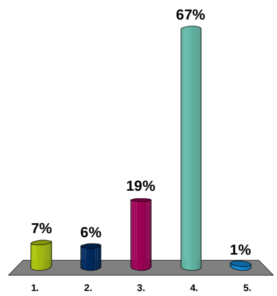
The fronts of longer period waves travel faster

3-C Linear propagation

Consider swell generated at Norway travelling towards the Dutch coast (say Hoek van Holland - HvH) corresponding to a distance of 1100 km. The wave period is 8 seconds. Assume the water depth is everywhere more than 50 m.

How long does it take for the swell to arrive in HvH?

1. ± 10 hours
2. ± 20 hours
3. ± 25 hours
4. ✓ ± 50 hours
5. I do not know



3-C Linear propagation

Travel time

Consider swell generated at Norway travelling towards the Dutch coast (say HvH) corresponding to a distance of 1100 km. The wave period is 8 seconds. Assume the water depth is everywhere at least 50 m.

Q: How long does it take before the swell arrives in HvH?

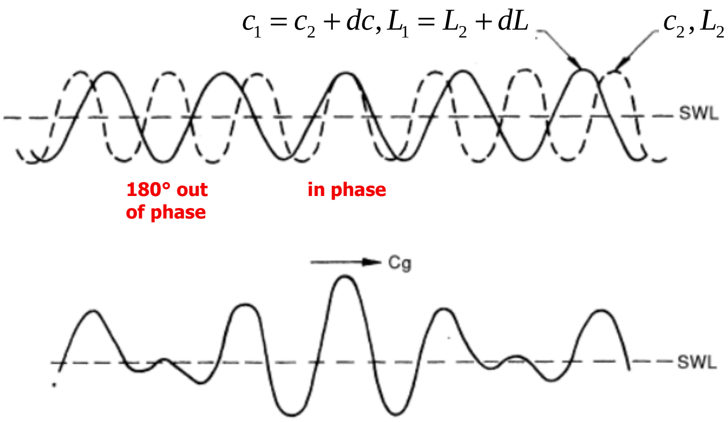
Since we have deep water conditions (h/L>0.5), the group velocity is given by:

$$c_g = \frac{1}{2}c \quad = \frac{1}{2}\frac{g}{\omega} = \frac{1}{2}\frac{gT}{2\pi} = \frac{9.81\,(\mathrm{m/s^2})\cdot 8\,(\mathrm{s})}{2\cdot 2\pi} = 6.25 \ \ \mathrm{m/s}$$

$$\Delta t = \frac{1100\,(\mathrm{km})}{6.25\,(\mathrm{m/s})} = 176\times 10^3\,(\mathrm{s}) \approx 49 \text{ hours}$$

3-C Linear propagation

Add two harmonics with slightly different periods



3-C Linear propagation

Add two harmonics with slightly different periods (2)

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\eta(x,t) = a\sin\left(\omega_2 t - k_2 x\right) + a\sin\left(\omega_1 t - k_1 x\right) =$$

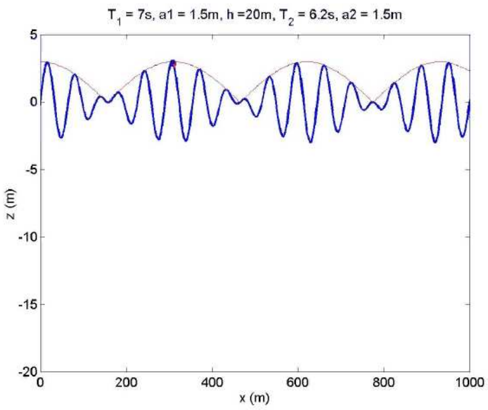
$$= 2a\sin\left(\tfrac{\omega_2+\omega_1}{2}t - \tfrac{k_2+k_1}{2}x\right)\cos\left(\tfrac{\omega_2-\omega_1}{2}t - \tfrac{k_2-k_1}{2}x\right) =$$

$$= 2a\underbrace{\sin\left(\bar{\omega}t - \bar{k}x\right)}_{\text{Carrier wave}}\underbrace{\cos\left(\tfrac{\Delta\omega}{2}t - \tfrac{\Delta k}{2}x\right)}_{\text{Slowly varying amplitude}}$$

Try this with the tool for summation of harmonics on Brightspace (Content Chapter 5 part II)

3-C Linear propagation

Wave grouping of waves with slightly different frequency



c_1 = 10.3 m/s

c_2 = 9.4 m/s

c_g = 5.7 m/s

$$k_{group} = \Delta k = k_2 - k_1 \Rightarrow \quad L_{group} = \frac{2\pi}{\Delta k}$$

$$\omega_{group} = \Delta\omega = \omega_2 - \omega_1 \Rightarrow \quad T_{group} = \frac{2\pi}{\Delta\omega}$$

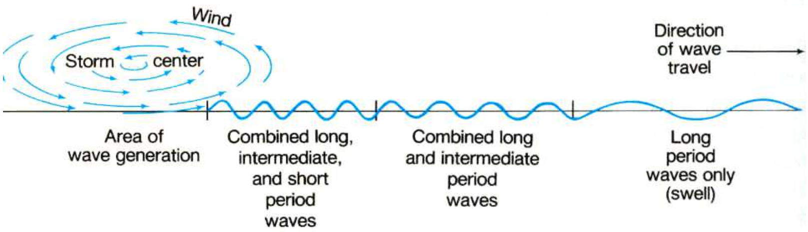
$$c_g = \frac{\Delta\omega}{\Delta k} = \frac{L_{group}}{T_{group}}$$

<u>Exam question 2010:</u>

How would you compute the period of a wave group consisting of two regular wave trains?

3-C Linear propagation

Transformation of sea waves into longer, faster and lower swell waves outside area of wave generation



- Waves of different lengths will disperse (spread out) because they propagate at different speeds
- Dissipation processes favour shorter waves and hence filter them out
- Propagation in various directions

Exam question 2010:

Why are swell waves lower and longer than storm waves?

3. Oceanic wind waves and tide

Chapter 3 of lecture notes

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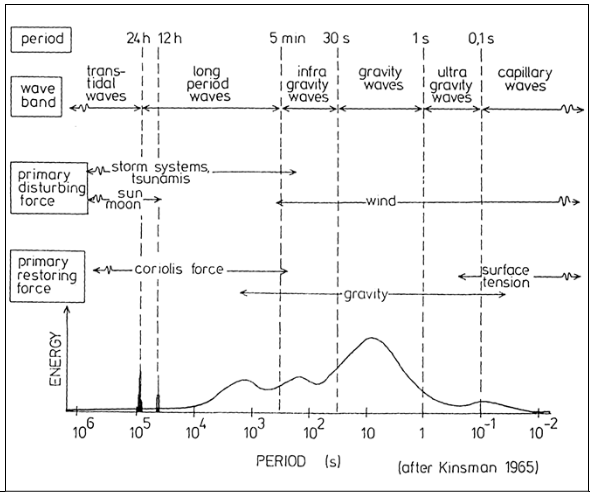
3-A Introduction

Classifications based on disturbing force, restoring force and wave period

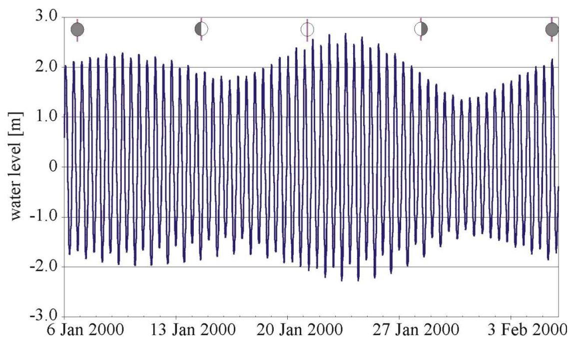
Relative amounts of energy as a function of wave period



3-D Generation of the tide

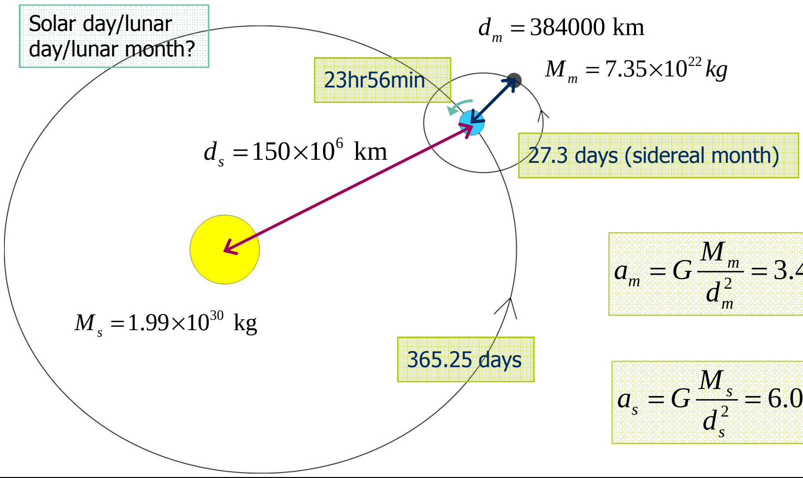
For practical purposes we can see the tide as a sinusoidal semi-diurnal water level variation modified with:

- a fortnightly spring- and neap-tide amplitude variation
- a daily inequality that varies with latitude and with the monthly and annual cycle



3-D Generation of the tide

Attractional forces are proportional to the masses and inversely proportional to the square of the distance

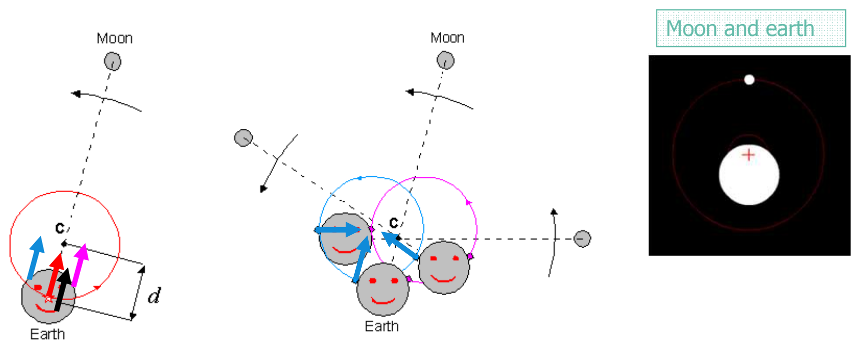


$$a_m = G\frac{M_m}{d_m^2} = 3.4\times10^{-6} g$$

$$a_s = G\frac{M_s}{d_s^2} = 6.0\times10^{-4} g$$

3-D Generation of the tide

Gravitational attraction provides centripetal acceleration

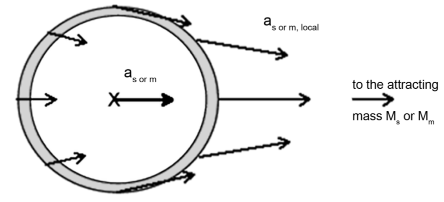


- The "circular" motion requires inward centripetal acceleration (to change direction)
- Magnitude and direction of centripetal acceleration is the same for every point on earth
- The centripetal acceleration is provided by the gravitational attraction of the moon

Fig from CT5517

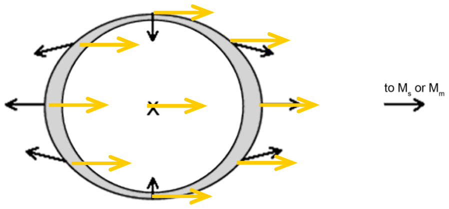
3-D Generation of the tide

The **differential** pull generates the tide



Gravitational attraction (black)

Used for:

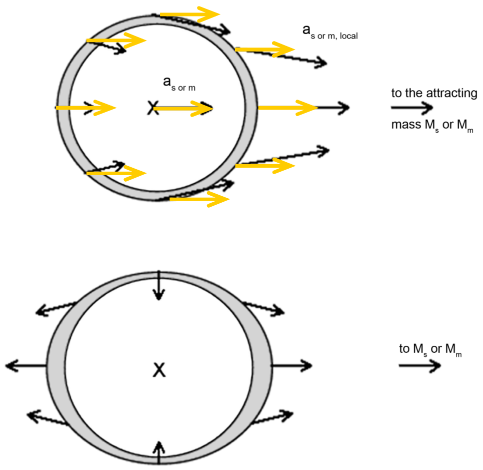


Centripetal acceleration (yellow) + Differential pull (black)

3-D Generation of the tide

The **differential** pull generates the tide

On the **near** side of the earth:



$$\Delta a_s = G\frac{M_s}{(d_s-R)^2} - G\frac{M_s}{d_s^2} - \approx$$

$$2G\frac{M_s R}{d_s^3} = a_s\frac{2R}{d_s} =$$

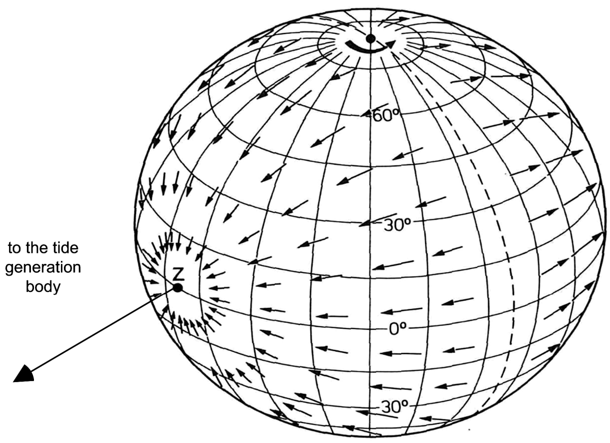
$$0.515\times10^{-7}\,g$$

$$\Delta a_m \approx a_m\frac{2R}{d_m} = 1.13\times10^{-7}\,g$$

$$\Delta a_s = 0.46\Delta a_m$$

$$\frac{\Delta a_m}{\Delta a_m+\Delta a_s} = 69\%$$

3-D Generation of the tide
The tangential components shift water around



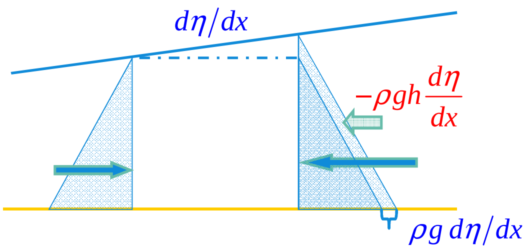
3-D Generation of the tide
Equilibrium theory of tides

Approximately the same for sloping bed, see Fig 5-31

- immediate response of the oceans to the tidal forces
  - no inertia
  - earth is entirely covered by an ocean of uniform depth
  - negligible friction and Coriolis

⇨ Tidal forces

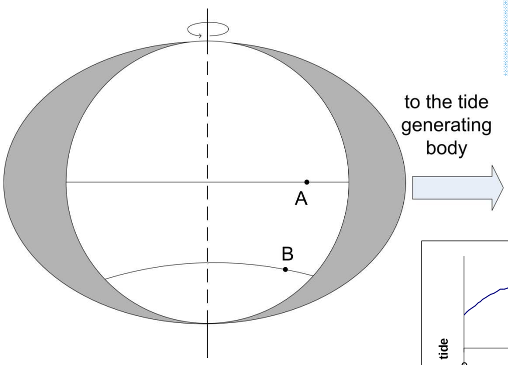
Pressure force uniformly distributed over depth

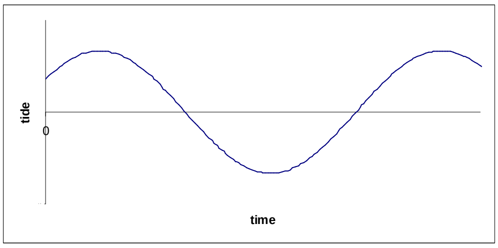


3-D Generation of the tide

Two high and low waters a day of equal height

$$\eta = a\cos(\omega t - \beta) = a\cos\left(\frac{2\pi t}{T} - \beta\right)$$

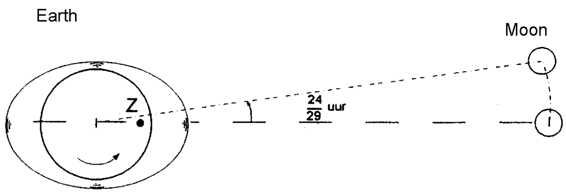




3-D Generation of the tide

Period of the lunar tide = 12 hrs 25 min

- Solar day is time for earth to return to same position w.r.t. sun as seen from earth (24 hrs)
- Lunar day is longer (24 hrs 50 min) because
  - Moon takes 29.5 days to return to same position relative to the sun as seen by an observer on earth (lunar month)
  - Extra time = 24/29 hr = 50 min



The earth must rotate a bit more than a full rotation to "catch up" with the lunar bulge

3-D Generation of the tide

Principal solar and lunar tidal components

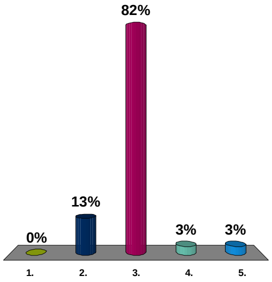
- Principal lunar tide M2
  - period 12 hrs 25 min = 12.42 hrs (half lunar day)
  - equilibrium amplitude 0.24 m
- Principal solar tide S2:
  - period of 12 hrs (half solar day)

What is the equilibrium amplitude of the solar tide S2?

3-D Generation of the tide

What is the equilibrium amplitude of the solar tide S2?

1. Same as equilibrium amplitude of M2 (= 0.24 m)
2. 69% of 0.24 m = 0.17 m
3. ✓ 0.46 x 0.24 m = 0.11 m
4. 6/3.4 x 0.24 m = 0.42 m
5. I do not know



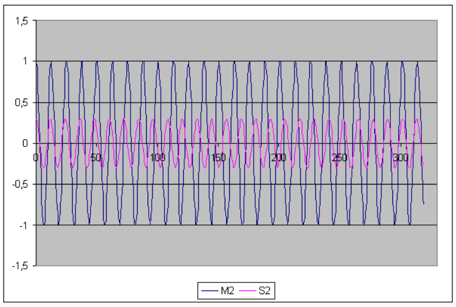
3-D Generation of the tide

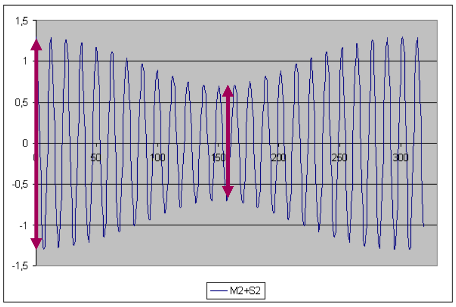
The beating of M2 and S2 results in spring-neap tide variability

Cf. Eq. 3.32

2.6 m

1.4 m



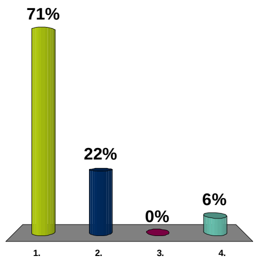


Compare short wave amplitude modulation (wave groups)

3-D Generation of the tide

How would you compute the period between two spring tides from the periods of the M2 and S2 tide?

1. $T = \dfrac{1}{T_{S2}^{-1} - T_{M2}^{-1}}$ ✓
2. $T = \dfrac{2\pi}{\omega_{S2}^{-1} - \omega_{M2}^{-1}}$
3. $T = \dfrac{2\pi}{\omega_{M2} - \omega_{S2}}$
4. Abstain



As with the wave grouping:

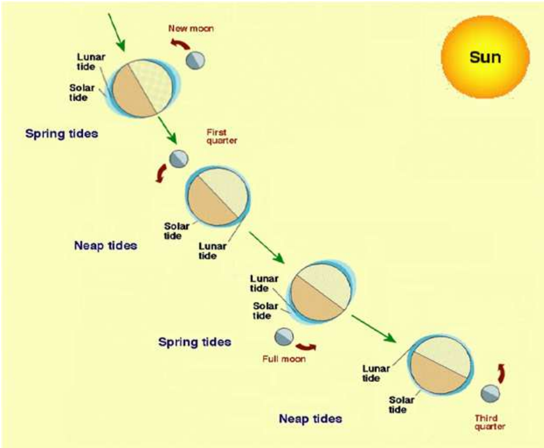
$$\boxed{\omega_{group} = \Delta\omega = \omega_2 - \omega_1 \Rightarrow T_{group} = \frac{2\pi}{\Delta\omega}}$$

$$\boxed{T = \frac{2\pi}{\omega_{S2} - \omega_{M2}} = \frac{1}{T_{S2}^{-1} - T_{M2}^{-1}} = \frac{1}{1/12 - 1/12.42}\text{ hrs} \approx 355\text{ hrs} \approx 14.8\text{ days}}$$

Exam question 2010

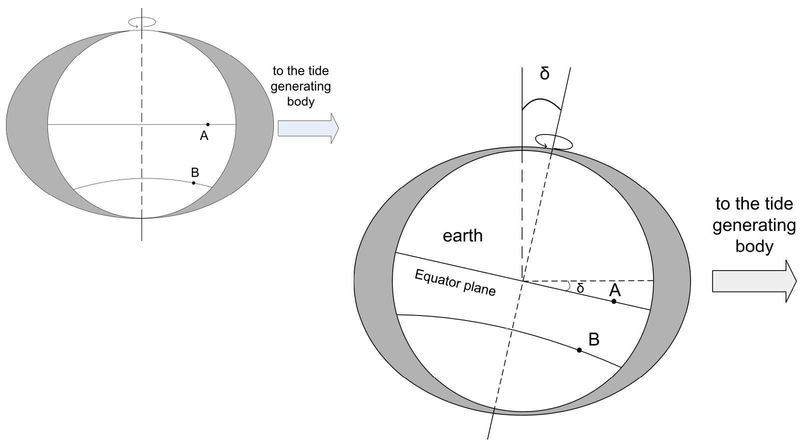
3-D Generation of the tide

Solar and lunar tide reinforce each other at spring tide and cancel each other at neap tide



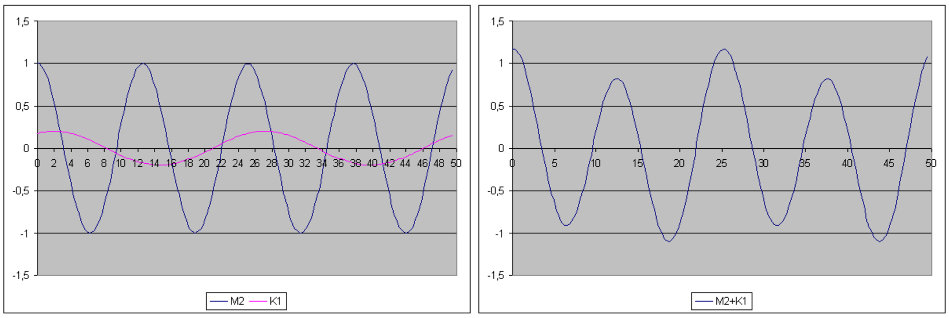
3-D Generation of the tide

Daily inequality



3-D Generation of the tide

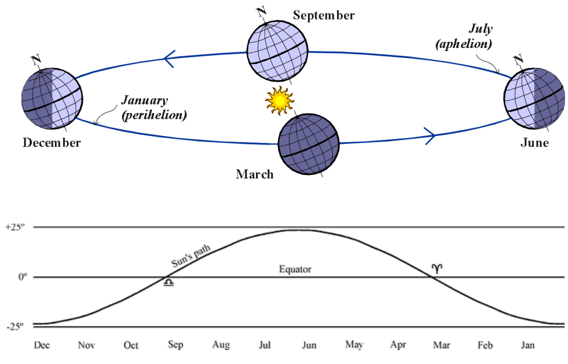
Declination of the earth axis introduces semi-diurnal and diurnal tidal constituents K1, K2, O1 and P1



Also: component with period of 18.6 years

3-D Generation of the tide

Declination varies with the annual and monthly cycle



Sun:
- Annual cycle
- Max 23.5° N/S

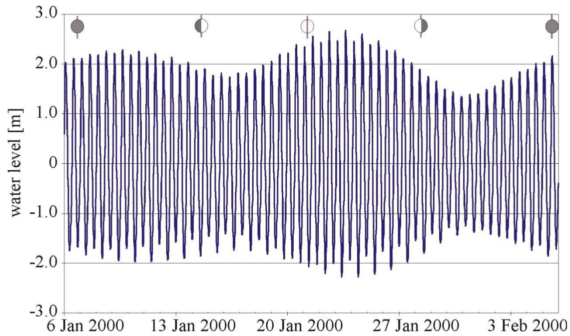
18.6 years cycle

Moon:
- Monthly cycle
- Max 23.5° +/-5° N/S

3-D Generation of the tide

For practical purposes we can see the tide as a sinusoidal semi-diurnal water level variation modified with:

- a fortnightly spring- and neap-tide amplitude variation
- a daily inequality that varies with latitude and with the monthly and annual cycle



3. Oceanic wind waves and tide

Chapter 3 of lecture notes

A. Introduction
B. (Short-term) statistics of wind waves
C. Linear propagation
D. Generation of the tide
E. Tidal propagation
F. Tidal analysis and prediction

3-E Tidal propagation

Dynamic theory of the tide

- Continents prevent the tidal wave from covering the circumference of the earth
- Land masses move water masses along with them, instead of moving through the tide

→
- Only around 65°S an equilibrium tide can exist (more or less)
- From about 65°S the tidal wave propagates northward into the oceans

3-E Tidal propagation

From about 65° the tidal wave propagates northward into the oceans

The further a location is away from the South Pole, the longer the time shift between the celestial event and its appearance in the form of the tide

What is the time between spring and neap tide in the Netherlands and the corresponding moon configurations?

$$c = \sqrt{gh} = \sqrt{9.81 \cdot 4000} \approx 200 \ {}^{\mathrm{m}}\!/\!_{\mathrm{s}} \approx 720 \ {}^{\mathrm{km}}\!/\!_{\mathrm{hr}}$$

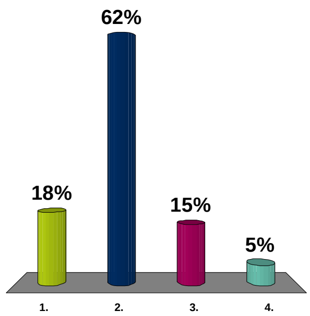
$$\text{From } 65^{\circ} S \text{ to } 53^{\circ} N \Rightarrow 118 \times 60 \times 1.8 \text{ km} \approx 12,750 \text{ km}$$

$$\Rightarrow \text{delay} = \frac{12,750}{720} = 18 \text{ hrs}$$

3-E Tidal propagation

In reality, the time between full moon and spring tide is about two days? This is because:

1. Since the tide does not travel in a straight line the travelled distance is twice as large
2. ✓ The tide needs another day or so to travel the shallow North Sea basin from Scotland to the Dutch coast
3. The water depths vary so much that the shallow water approximation is invalid
4. Something else



In reality ± 2 days! (shallow N-Sea basin!)

3-E Tidal propagation

Waves carry mass and momentum

- Depth-integrated (water) mass balance (ρ constant)

$$\boxed{\frac{\partial \kappa}{\partial t}+\frac{\partial \kappa \bar{u}}{\partial x}+\frac{\partial \kappa \bar{v}}{\partial y}=P} \Rightarrow \boxed{\frac{\partial \rho h}{\partial t}+\frac{\partial \rho h \bar{u}}{\partial x}+\frac{\partial \rho h \bar{v}}{\partial y}=0} \Rightarrow \boxed{\frac{\partial h}{\partial t}+\frac{\partial h \bar{u}}{\partial x}+\frac{\partial h \bar{v}}{\partial y}=0}$$

- If a particle with a certain mass ρ is moving, it has momentum
- Momentum is mass transport or mass flux: $\rho \vec{u}=(\rho u, \rho v, \rho w)$
- Momentum is a vector quantity:
  - its direction is the same as the direction of the velocity vector
- Depth-integrated momentum balance in x-direction:

$$\boxed{\frac{\partial(\rho \bar{u} h)}{\partial t}+\frac{\partial(\rho \bar{u} h)\bar{u}}{\partial x}+\frac{\partial(\rho \bar{u} h)\bar{v}}{\partial y}=P_x} \overset{\text{mass balance}}{\Rightarrow} \boxed{\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial x}+\bar{v}\frac{\partial \bar{u}}{\partial y}=\frac{P_x}{\rho h}}$$

?

Equation of motion

3-E Tidal propagation

Newton's second law: a momentum balance

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{\text{Local acceleration}} + \underbrace{\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y}}_{\text{(non-linear) advective acceleration terms}} = \underbrace{\frac{P_x}{\rho h}}_{\text{RHS ?}}$$

RHS ?

Production of momentum

= force, for instance pressure force

$$-g\frac{\partial \eta}{\partial x}$$

$F = ma$ (only for m=constant)

Force = mass x acceleration = mass x rate of change of velocity = rate of change of momentum

3-E Tidal propagation

Progressive **small amplitude** long wave

Acceleration (inertia) balances pressure gradient $\frac{\partial v}{\partial t} = -g\frac{\partial \eta}{\partial y}$

Continuity $\frac{\partial \eta}{\partial t} + h\frac{\partial v}{\partial y} = 0$ $\left(h = h_0 + \cancel{\eta}\right)$

This gives for a progressive (co)sine wave $\eta = a\cos(\omega t - ky)$:

$$\omega^2 = ghk^2 \Rightarrow c = \frac{\omega}{k} = \sqrt{gh}$$

$$v = \frac{gak}{\omega}\cos(\omega t - ky)$$

Velocity in phase with surface elevation with amplitude: $\hat{v} = a\sqrt{\frac{g}{h}}$

3-E Tidal propagation
Effect of depth restriction

$h = 4 \text{ km} \Rightarrow c = 200 \ {}^{\text{m}}\!/_{\text{s}}$ and with $T = 12.42 \text{ hrs} \Rightarrow L = 9000 \text{ km}$

Smaller water depths:

$h = 10 \text{ m} \Rightarrow c = 10 \ {}^{\text{m}}\!/_{\text{s}}$ and $L = 450 \text{ km}$ $\quad (c\downarrow, L\downarrow)$

$\Rightarrow$ Amplitudes increase

for instance

$h = 4 \text{ km}, \ a = 0.25 \text{ m}$

$h = 10 \text{ m}, \ a = 1 \text{ m}$

$\Rightarrow$ Velocity amplitudes increase even more:

$$\boxed{\hat{v} = a\sqrt{\frac{g}{h}}}$$

$\Rightarrow \hat{v} = 1.2 \ {}^{\text{cm}}\!/_{\text{s}}$

$\Rightarrow \hat{v} = 1 \ {}^{\text{m}}\!/_{\text{s}}$

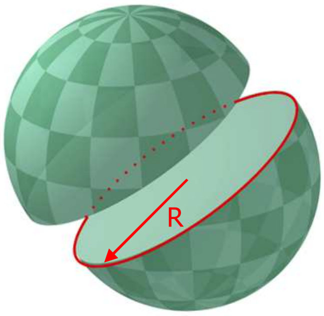
3-E Tidal propagation
The merry-go-round (Dutch: draaimolen)

https://www.youtube.com/watch?v=aeY9tY9vKgs

Only from 1:17 to 2:17

3-E Tidal propagation

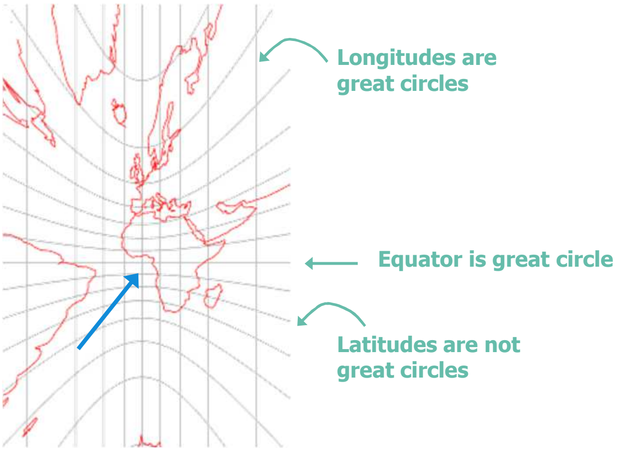
The shortest distance between two points on earth is along a great circle that divides the sphere in two equal hemispheres



What is the path travelled by swell waves?

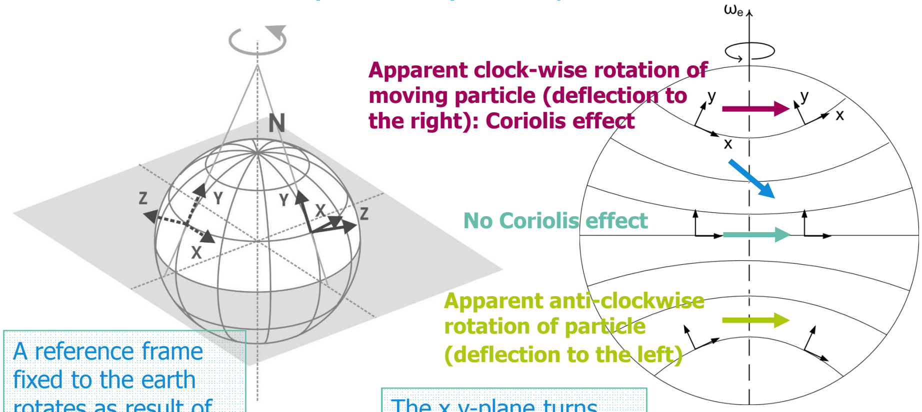
3-E Tidal propagation

A gnomonic map projects great circles as straight lines. Directions are preserved.



3-E Tidal propagation

Coriolis effect diverts a **moving** particle to the right (left) on the Northern (Southern) hemisphere



A reference frame fixed to the earth rotates as result of the earth's rotation: it is non-inertial

The x,y-plane turns anti-clockwise in NH and clockwise in SH

3-E Tidal propagation

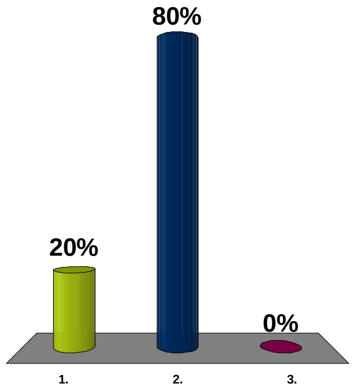
Coriolis force: a pseudo-force to make Newton's equations of motion valid in our non-inertial frame of reference

- Acceleration $a_c = f\,V = 2\,\omega_e \sin\varphi\,V$
  - ω_e = angular velocity of the earth (72.9 10^{-6} rad/s)
  - V = current velocity
  - φ = latitude
  - for not too large areas f can be considered constant
- to the right on N hemisphere
- to the left on S hemisphere

Winds, ocean currents, tides

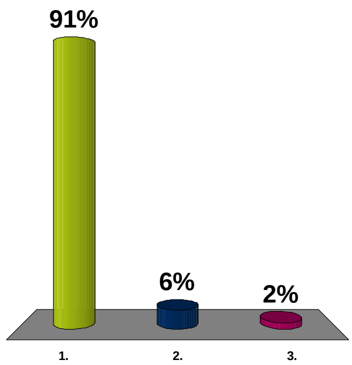
In an inertial (non-accelerating) frame of reference Coriolis forces must be taken into account:

1. Yes
2. No
3. Abstain



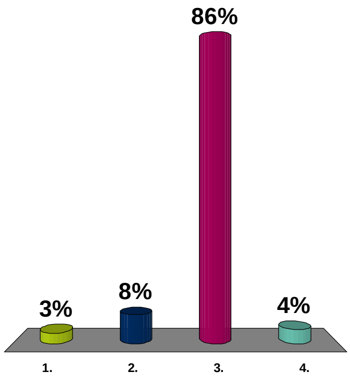
In the Southern Hemisphere the Coriolis acceleration is directed to

1. Port (left)
2. Starboard (right)
3. Abstain



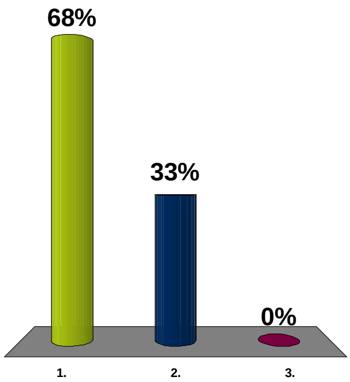
The Coriolis acceleration is maximum at:

1. The equator
2. Mid-latitudes
3. ✓ The poles
4. Abstain



Swell waves are deflected by Coriolis

1. Yes
2. ✓ No
3. Abstain

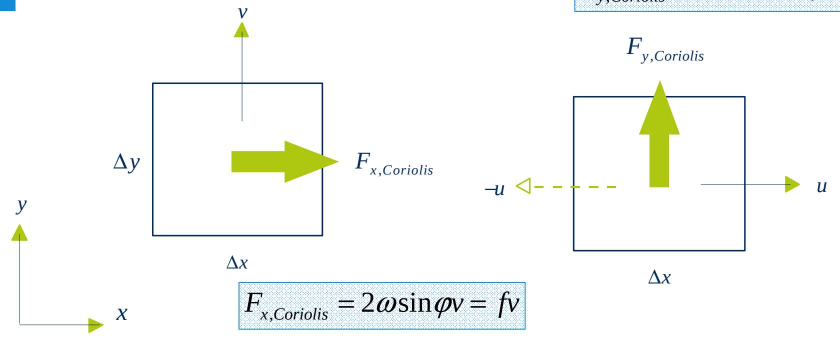


3-E Tidal propagation

The signs of the Coriolis terms

- NH: sin φ > 0
- SH: sin φ < 0

$$F_{y,Coriolis} = -2\omega u \sin\varphi = -fu$$



$$F_{x,Coriolis} = 2\omega \sin\varphi v = fv$$

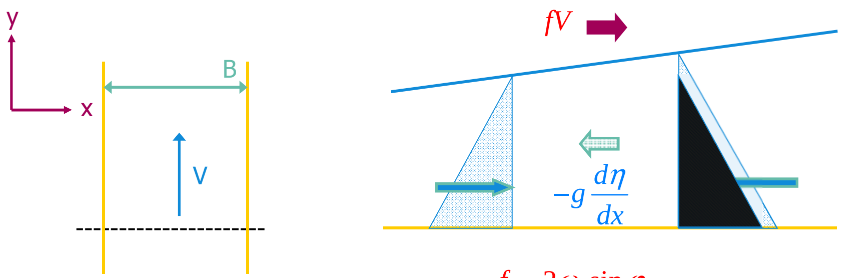
Momentum balance Eqs. without advective terms:

$$\frac{\partial u}{\partial t} = \sum F_x \qquad \frac{\partial v}{\partial t} = \sum F_y$$

- See for instance Eq. 3.41 and 3.42 in lecture notes
- Rossby number U/|f|L

3-E Tidal propagation

Coriolis effect on steady current in channel (NH)



$$f = 2\omega_e \sin\varphi$$

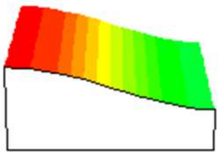
Example Strait of Florida

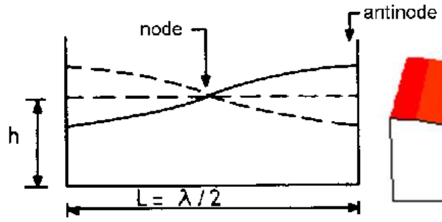
$$\left.\begin{array}{l} V \approx 1 \ \mathrm{m/s} \\ B \approx 80 \ \mathrm{km} \\ \varphi = 26^\circ \mathrm{N} \end{array}\right\}$$

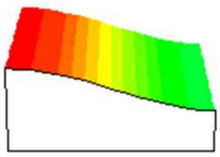
$$g\frac{d\eta}{dx} = fV \Leftrightarrow \Delta\eta = \frac{fV\Delta x}{g} = \frac{2\cdot 72.9\times10^{-6}\cdot \sin 26^\circ \cdot 1\cdot 80\times10^{3}}{9.81} = 0.52 \ \mathrm{m}$$

3-E Tidal propagation

Coriolis-effect (due to Earth's rotation) => tidal wave behaves as a rotary wave forming amphidromic system







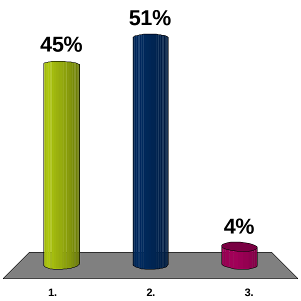
Water movement around an amphidromic point

Standing wave movement around a node in a closed body of water

3-E Tidal propagation

Coriolis deflection (to the right in the NH), results in a rotation around the amphidromic point that is anti-clockwise in the NH

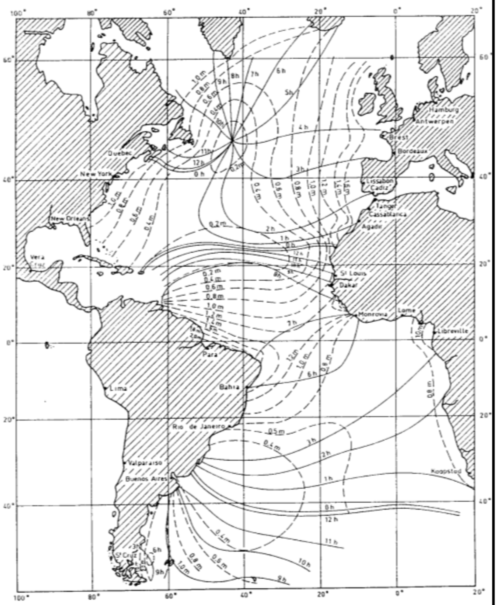
✔ 1. True
2. False
3. Abstain



3-E Tidal propagation

Atlantic Ocean

- **Co-phase or co-tidal lines** (lines of constant phase) connect all places with high water at the same time
- **Co-range lines** (lines of constant tidal range) connect all places with the same tidal range
- **Amphidromic points** have a zero tidal range. Co-range lines run around the amphidromic points and co-phase lines radiate away from amphidromic points



TUDelft

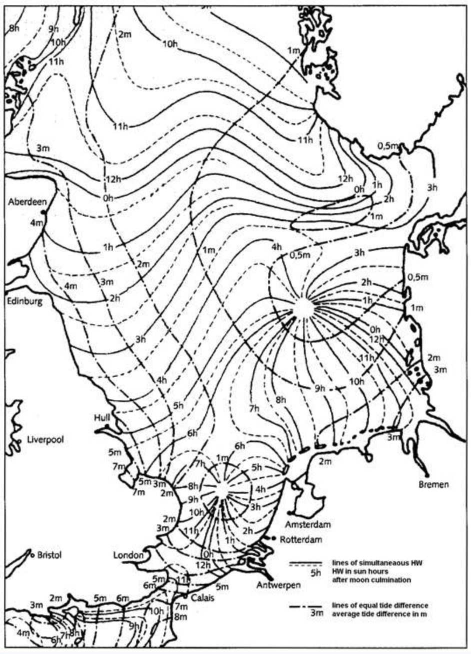
3-E Tidal propagation

Amphidromic system of shallow North Sea basin

Tidal wave enters from North and propagates around 2 amphidromic points

Along British coast:

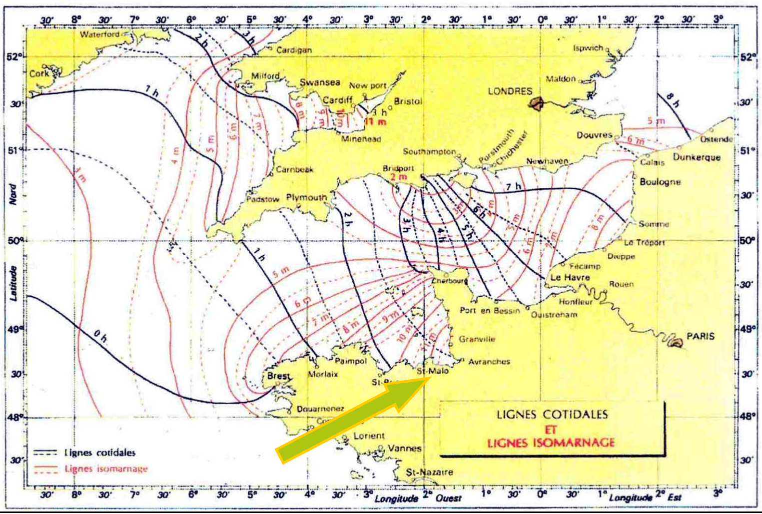
- far from amphidromic point)=>
- large amplification of tidal range



TUDelft

3-E Tidal propagation

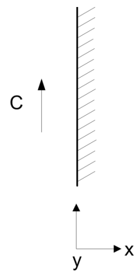
Large amplification of tidal range due to near-resonance



Near-resonance in Channel: tidal range 10m at St. Malo!

3-E Tidal propagation

Coastally trapped Kelvin wave (no friction!)



Linearised equations of motion

Assume u≈0 (close to coastline) and f = constant

$$\cancel{\frac{\partial u}{\partial t}} - fv = -g\frac{\partial \eta}{\partial x}$$

Geostrophic in cross-shore

$$\frac{\partial v}{\partial t} + \cancel{fu} = -g\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + h\cancel{\frac{\partial u}{\partial x}} + h\frac{\partial v}{\partial y} = 0$$

Progressive wave alongshore

$$\eta = \hat{\eta}\cos(\omega t - ky) \text{ with } \hat{\eta} = e^{\left(\frac{fx}{c}\right)}\eta_0$$

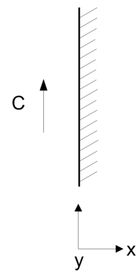
$$v = \sqrt{\frac{g}{h}}\,\hat{\eta}\cos(\omega t - ky)$$

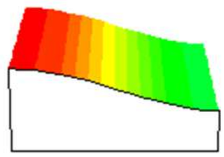
$$c = \sqrt{gh} \quad \text{(NH)}$$

3-E Tidal propagation

Coastally trapped Kelvin wave

- Neglect friction!
- Cross-shore momentum balance is geostrophic:
  - pressure (water level) gradient balances Coriolis force
  - compare with effect Coriolis on flow in confined channel
  - amplitude is maximum at the coast and then decays
- Alongshore momentum balance as for progressive shallow water waves:
  - inertia balances pressure gradient
  - alongshore velocity in phase with water level
  - in NH keeps the coast on starboard side

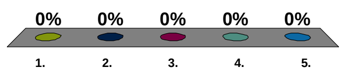




3-E tidal propagation

Which of the following statements is **true**?

1. A Kelvin wave rotates anti-clockwise in the Southern hemisphere
2. Co-phase or co-tidal lines encircle amphidromic points
3. The cross-shore tidal velocity is in phase with the water level
4. ✓ Amphidromic systems are resonance phenomena influenced by the size and shape of basins
5. The Coriolis force occurs in inertial (non-accelerating) reference frames



3. Oceanic wind waves and tide

Chapter 3 of lecture notes

A. Introduction
B. (Short-term) statistics of wind waves
C. Linear propagation
D. Generation of the tide
E. Tidal propagation
F. Tidal analysis and prediction

3-F Tidal analysis and prediction

Harmonic analysis and prediction of tide

- As for wind waves, but now discrete and known frequencies from astronomical forcing:

$$\eta_t = a_0 + \sum_{n=1}^{N} a_n \cos(\omega_n t - \alpha_n)$$

- Determine amplitudes and phases from observed water level records at certain location
- Be careful with:
  - nodal factor (to capture effect 18.6 year cycle)
  - astronomical argument / definition of time-origin

$$\eta_t = a_0 + \sum_{n=1}^{N} \underbrace{f_n}_{\text{Nodal factor}} a_n \cos(\omega_n t - \alpha_n + \underbrace{\beta_n}_{\text{Astronomical argument}})$$

3-F Tidal analysis and prediction

See Appendix C

Tidal levels

Tidal levels

LAT	Lowest Astronomical Tide
HAT	Highest Astronomical Tide
MLW	Mean Low Water
MHW	Mean High Water
MLWS	Mean Low Water Springs
MHWS	Mean High Water Springs
MLWN	Mean Low Water Neaps
MHWN	Mean High Water Neaps
MLLW	Mean Lower Low Water
MHHW	Mean Higher High Water

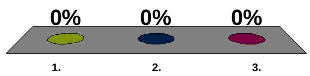
Tidal ranges

MHW-MLW	Normal tidal range
MHWS-MLWS	Spring tidal range
MHWN-MLWN	Neap tidal range

3-F Tidal analysis and prediction

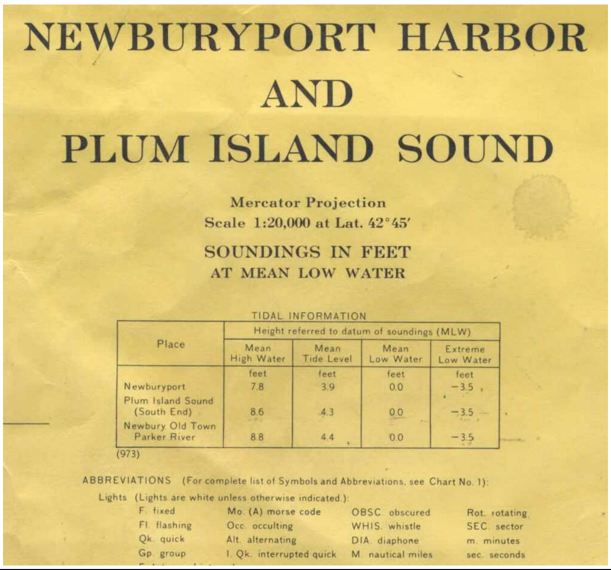
What is lower: MLWN or MLWS?

1. MLWN
2. ✓ MLWS
3. I don't know



3-F Tidal analysis and prediction

Tidal levels and chart datum



NEWBURYPORT HARBOR AND PLUM ISLAND SOUND

Mercator Projection
Scale 1:20,000 at Lat. 42°45′

SOUNDINGS IN FEET
AT MEAN LOW WATER

TIDAL INFORMATION

Place	Height referred to datum of soundings (MLW)			
	Mean High Water	Mean Tide Level	Mean Low Water	Extreme Low Water
	feet	feet	feet	feet
Newburyport	7.8	3.9	0.0	−3.5
Plum Island Sound (South End)	8.6	4.3	0.0	−3.5
Newbury Old Town Parker River	8.8	4.4	0.0	−3.5

(973)

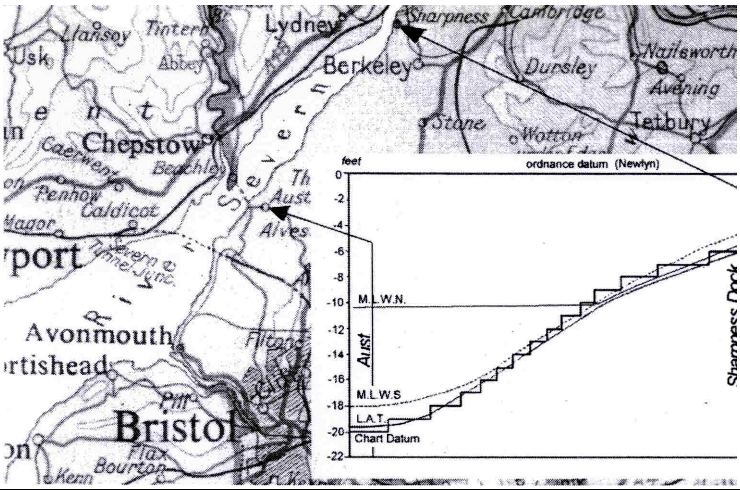
ABBREVIATIONS (For complete list of Symbols and Abbreviations, see Chart No. 1):

Lights (Lights are white unless otherwise indicated.):

F. fixed	Mo. (A) morse code	OBSC. obscured	Rot. rotating
Fl. flashing	Occ. occulting	WHIS. whistle	SEC. sector
Qk. quick	Alt. alternating	DIA. diaphone	m. minutes
Gp. group	I. Qk. interrupted quick	M. nautical miles	sec. seconds

3-F Tidal analysis and prediction

Chart datum is not necessarily horizontal!



Exam question april 2010

7(90)

4. Deltas [7]

a. **[2]** In what geotectonic setting have deltas typically developed? Explain your answer. (80 words).

b. **[1]** Delta formation implies a positive sediment budget. Which long-term effect can influence this negatively, so that delta formation is suppressed? (30 words).

Consider the below satellite image of the Danube delta.

c. **[2]** Historically the Danube delta has developed as a river-dominated delta. Explain how you would classify the present-day Danube delta according to Galloway's classification? (40 words).

d. **[2]** What is the dominant wave direction? Explain your reasoning in terms of sediment transport processes and the resulting morphological development. (70 words)

See Chapter 2

Exam question april 2010

8(90)

6. Tidal generation and propagation [8]

a. **[3]** Explain the period of the spring and neap tide cycle from the moon phases. (70 words)

Equivalently, the spring and neap tide cycle follows from adding up the principal lunar and solar semi-diurnal constituents of the tide (M2 and S2 respectively).

b. **[3]** Describe how to compute the period of the spring and neap tide cycle from the M2 period (12 hrs and 25 minutes) and S2 period (12 hrs). Note: you do not have to perform the actual computation! (70 words).

Consider a coastally trapped Kelvin wave in the absence of friction such that in cross-shore direction the momentum balance is geostrophic.

c. **[2]** Describe and sketch the amplitude variation along a co-phase line from the amphidromic point to the coast. (30 words).