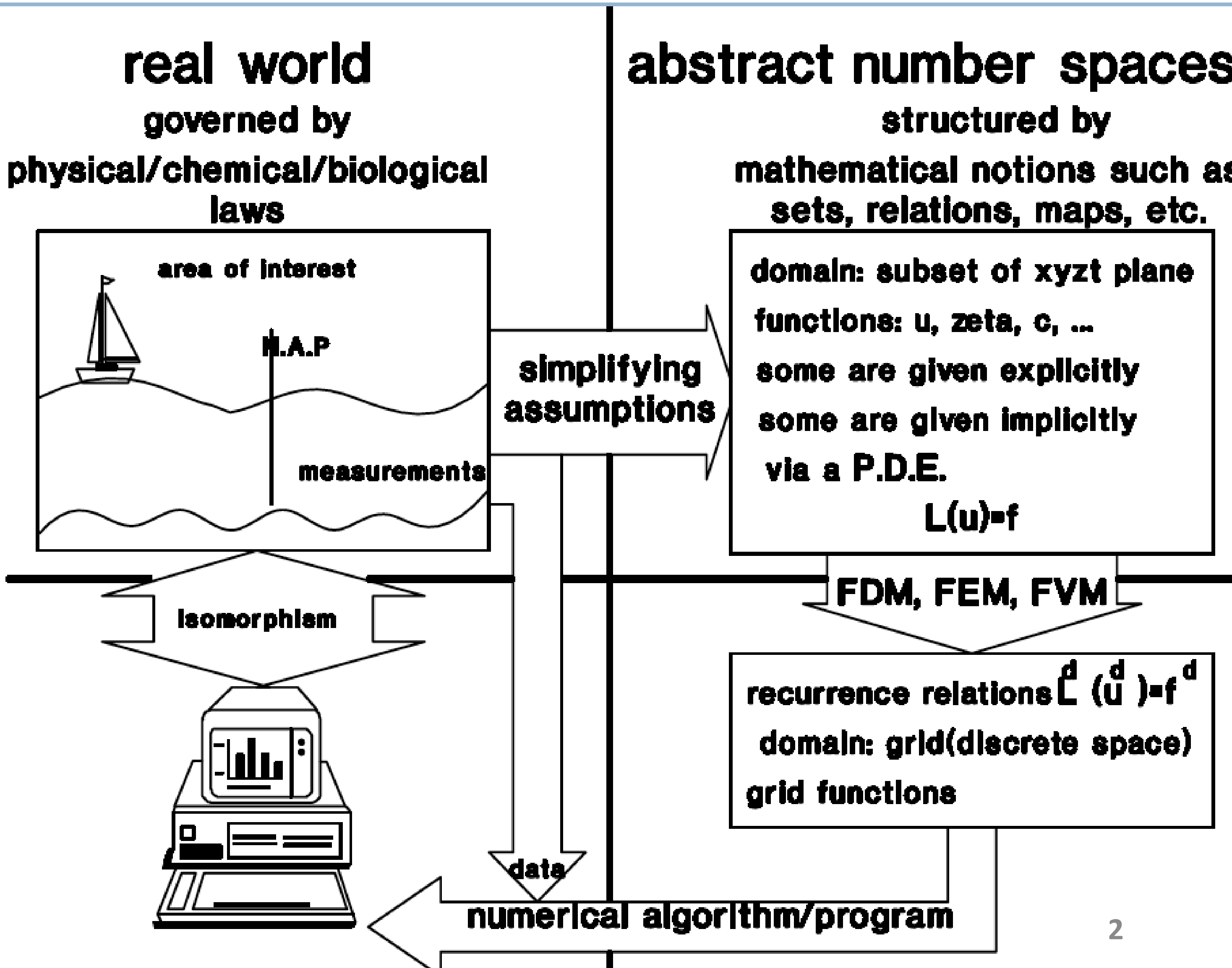
COMPUTER DETERMINISTIC MODELS

How do we CALCULATE?

What are we doing in this part?



Numerical Solutions

Numerical Solution

- What do we normally solve? →

Conservation equations, for example:

$$\Delta V = -\frac{\partial Q}{\partial x}\Delta x\,\Delta t + q_{lat}\Delta x\,\Delta t \qquad \sum F = \rho\,\Delta x\,\frac{\partial A_t u}{\partial t}$$

- What are these equations called?
- Want do we need to correctly solve these equations?
  - A discretization method to approximate the equations with a system of algebraic equations
  - A solver to solve the system of algebraic equations (direct/iterative)

Numerical Solvers

- Components
  - linearized equations
  - equation solvers
- Practical issues with them:
  - efficiency
  - accuracy
  - stability
  - robustness
  - conservation

Numerical Solutions – Accurate?

- What do you have to keep in mind?
  - Numerical results will always be approximate. Where do these approximations come from?
    - Equations (e.g. shallow water, Boussinesq, turbulence etc.)
    - Discretization process
    - Solution method (typically iterative although direct solvers exist)
  - How do we overcome these approximations?
    - Data for our models
    - More accurate interpolation schemes or smaller regions
    - Have we completed enough iterations
  - This means → KNOW what your ERROR ESTIMATES are

General Issues with Numerical solutions

- Conservation
  - Equations are conservation laws!
  - Schemes should respect these laws.
  - Becomes a constraint on the solution.
  - Typically guaranteed for FV but not for other methods.
- Boundedness
  - Solutions should lie within proper bounds.
- Realizability
  - Not necessarily a numerical issue
  - Physical model we use should be physically realistic as well

Mathematical issues with numerical solutions

- Consistency
  - Truncation error $\longrightarrow$ 0 as the time step / grid spacing $\longrightarrow$ 0
  - Even if consistent, solution may not become exact when step sizes are small. For this to occur the solution must be stable.
- Stability
  - Scheme is considered stable when errors from solution process are not magnified in the solution process
  - Mostly investigated with linear problems and constant coefficients without boundary conditions using von Neumann method
  - Typically most schemes require some restrictions on time step or grid size (CFL criteria)
- Convergence
  - Discretized solution $\longrightarrow$ exact solution of the differential equation as the grid spacing $\longrightarrow$ 0.
  - Lax equivalence theorem is used for linear initial value problems
  - However for real problems, we usually do a grid-independent study

Solution Methods (Iterative)

- Solve matrix from discretized equations
  - Guess solution
  - Improve by following a procedure until convergence criteria is satisfied.
  - Typically split matrices into Lower, Upper, Diagonals
- Basic methods: Jacobi, Gauss-Seidel
- Overrelaxation: SOR, SLOR, Red-black point relaxation
- Other Methods: Stone's, ADI, Multigrid

Discretization Method

- Various methods exist to discretize the equations
  - Finite Difference
  - Finite Volume
  - Finite Element (similar to FV in many ways; typically use a triangular element in 2D
  - And others:
    - Spectral methods
    - Boundary element
    - Vorticity-based
    - Lattice Boltzmann
    - And many more

Finite Difference Overview

- Oldest method
  - Advantages: Easy to implement
  - Disadvantages: Restricted to structured grids, doesn't conserve mass without special treatment
- Methodology
  - Requires a structured mesh (i,j,k)
  - Discretize domain of interest into grid points
  - Discretize equations –obtain derivatives from Taylor series expansions
  - Each grid point has an algebraic equation that must be solved
  - Solve resulting linear algebraic equations

Finite Volume Overview

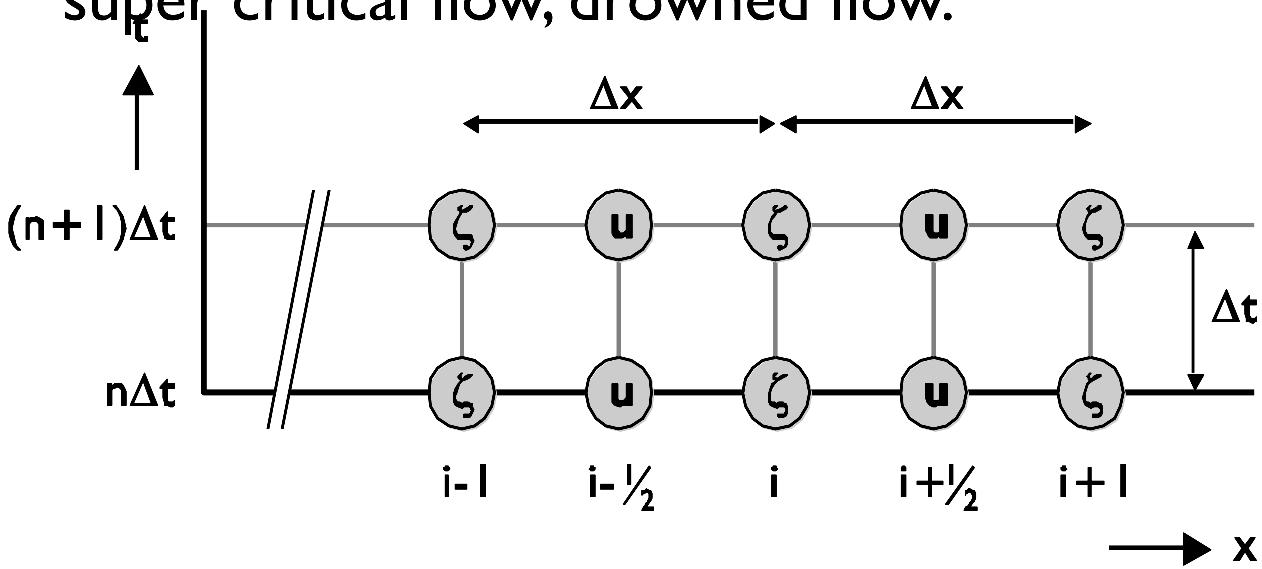
- Advantages
  - Mass, momentum and energy are conserved in the formulation
  - Well developed iterative solvers exist
- Disadvantages
  - False diffusion depending on numerical scheme
- Methodology:
  - Solve integral equations (not differential)
  - Divide domain into CVs, assign computational node to CV centroid
  - Interpolate to obtain values at surfaces (f of node values)
  - Approximate surface and volume integrals
  - Each grid point has an algebraic equation but with neighboring values as well
  - Solve set of Linear Algebraic Equations

Finite Element Method

- More well known in structural analysis
- Developed for fluid flow solution in 1970s
- Similar in approach to Finite Volume but uses weights as multiplier to equations. Results in non-linear algebraic equations
- Advantages
  - Accurate for coarse grids
  - Works well for viscous free surface problems
- Disadvantages
  - Slow for large problems
  - Weak for solving turbulent flows

Staggered grid approach

- Water levels are calculated at grid points
- Discharges or velocities are calculated at grid sections
- stable, robust, accurate: can handle dry-bed, sub- and super critical flow, drowned flow.



When evaluating a numerical modeling solution you should be able to

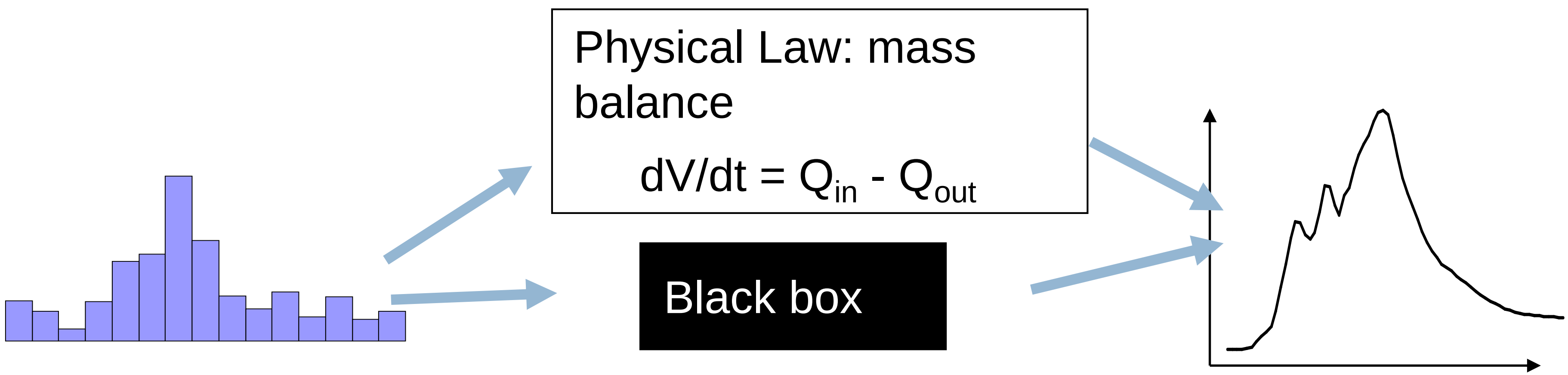
- Be aware of the 3 Primary Error Sources
  - Modeling errors → Difference between actual conditions and the exact mathematical model solution
    - Mainly arises due to assumptions made in deriving equations (Turbulence, Shallow water etc.)
    - Simplification of geometry; boundary conditions etc. (Unfortunately not known a priori, can only be known after comparing to data!)
  - Discretization Errors → Errors due to differences between exact mathematical solution and that derived from exact solution of discretized equations
    - On a particular grid, methods of the same order may produce differences in solution up to an order of magnitude depending on the discretization method
  - Iteration / Convergence Errors → Differences between the exact discretized solution and that obtained from an iterative solution method
    - Easiest to control
    - Normally we stop when the difference between successive iterations are less than a pre-selected value (typically normalized)
- Unfortunately sometimes these errors can cancel each other

Modeling Rainfall into Runoff

Part 1

Hydrological modeling

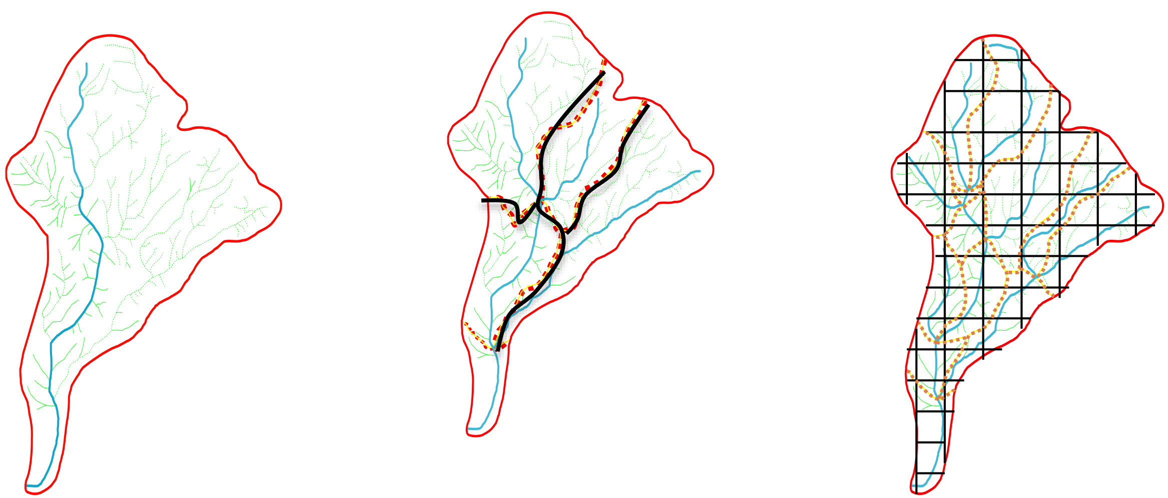
- Often the terminology physically based versus non-physically based is used:
  - Physically-based models: physical equations are used
  - Non-physically based: empirical, statistical models (neural networks, unit hydrograph)



Hydrological modeling

- 3 kind of models
  - Lumped models
  - Semi-distributed models
  - Distributed models

Lumped model | Semi-distributed model | Fully Distributed model



What is a distributed physically based model?

- These models consider more than just the mass balance
- Additional effects are taken into account:
  - Presence of gravity: momentum balance equation
    - Allows for the conservatoin of Mass and Momentum
    - Darcy's law for infiltration is a momentum balance
  - Topography: needed for calculating the gravity component along the flow direction?
    - Need for putting in surface levels, invert and cross sections of streams/ channels
  - Diffusive processes
    - Water moving through unsaturated soil (Richards' equation)
    - Water flowing in a river: Saint Venant
  - Vegetation processes (evapotranspiration)
  - Energy balance

Distributed Flow routing in channels

- St. Venant equations
  - Continuity equation

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

  - Momentum Equation

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

Local acceleration term

Convective acceleration term

Pressure force term

Gravity force term

Friction force term

Kinematic Wave

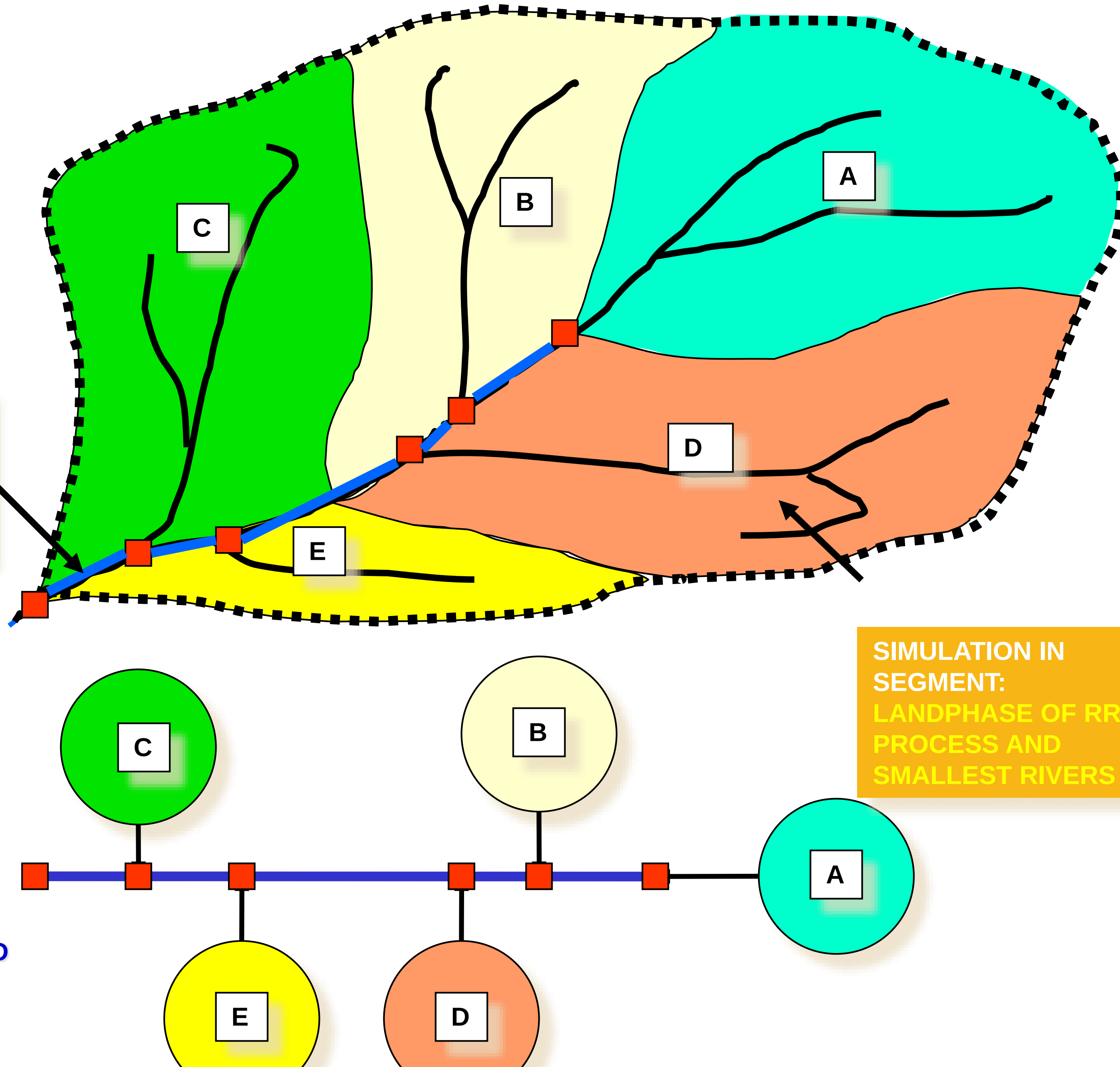
Diffusion Wave

Dynamic Wave

Pro's and Con's of distributed models

- Advantages:
  - physical processes are captured in the best possible manner
  - Future scenarios can be modeled (e.g. climate change) because the model is not calibrated statically
- Disadvantages:
  - Highly detailed information needed
  - Many parameters (soil, vegetation, channel flow etc.)
  - Parameter uncertainty issues (see e.g. GLUE procedure)

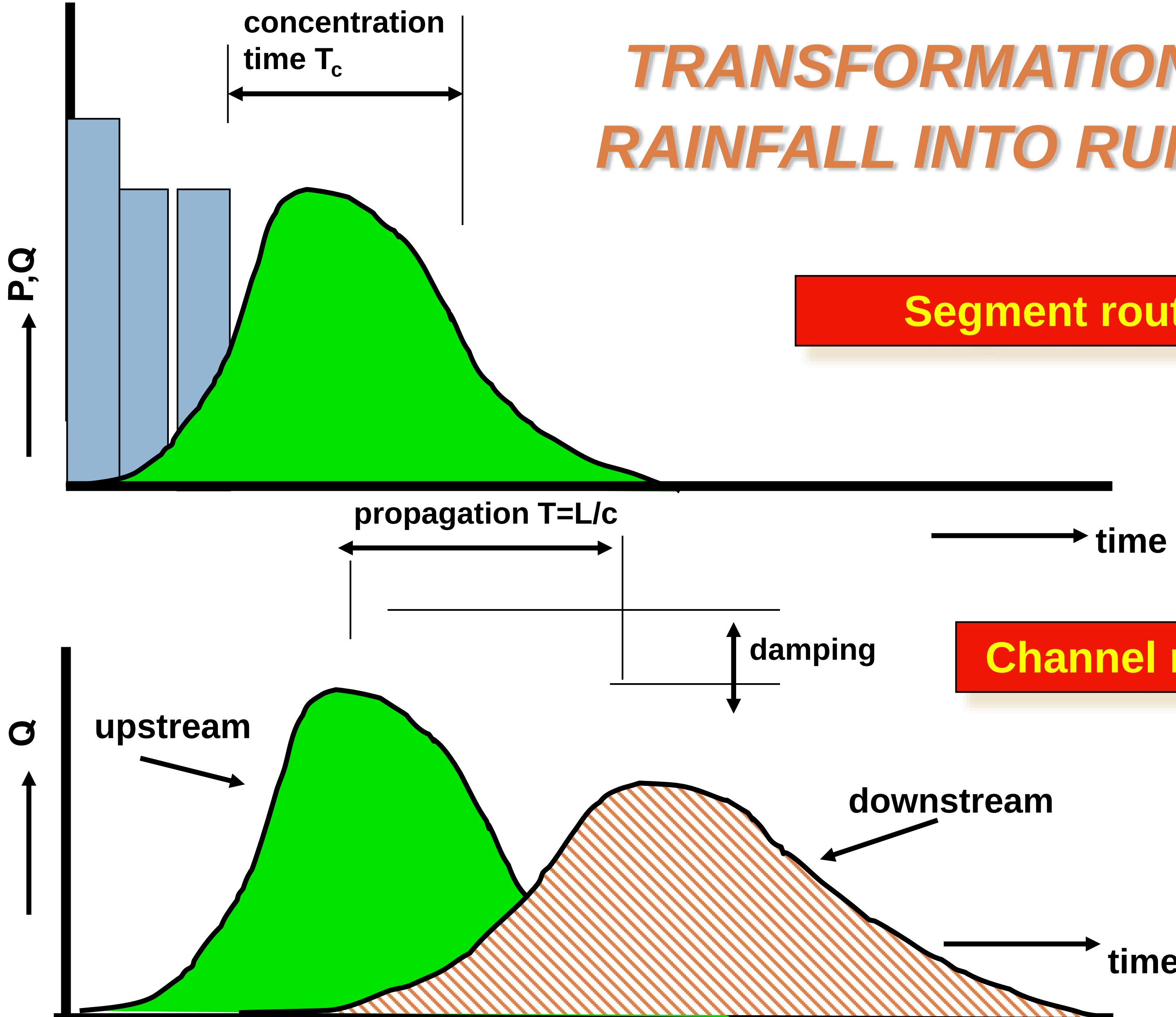
SIMULATION IN CHANNEL:
PROPAGATION AND ATTENUATION OF FLOW



SIMULATION IN SEGMENT:
LANDPHASE OF RR-PROCESS AND SMALLEST RIVERS

SEGMENTS CONNECTED BY CHANNEL AND RESERVOIR ELEMENTS

TRANSFORMATION OF RAINFALL INTO RUNOFF



Segment routing

Channel routing

Modeling Runoff and Routing

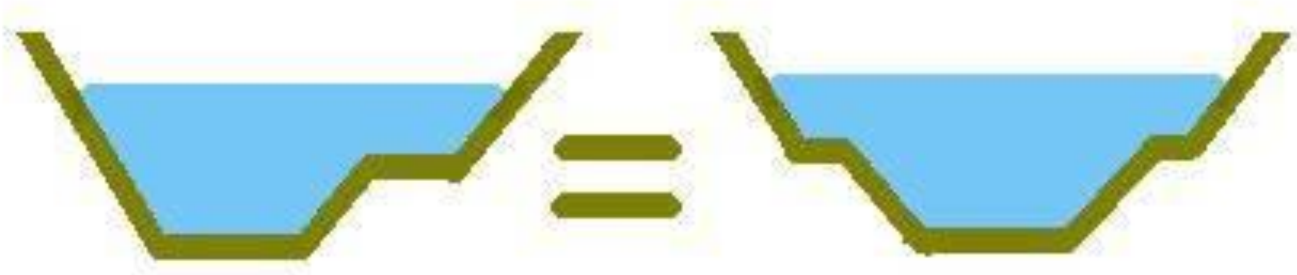
Part 2

1D modelling

Physics are always 3D in space and 1D in time, therefore:

The 1D model must be orientated along the river axis

- Processes that take place in the two remaining space-dimensions (vertical and transversal) are either parameterized or ignored; e.g.

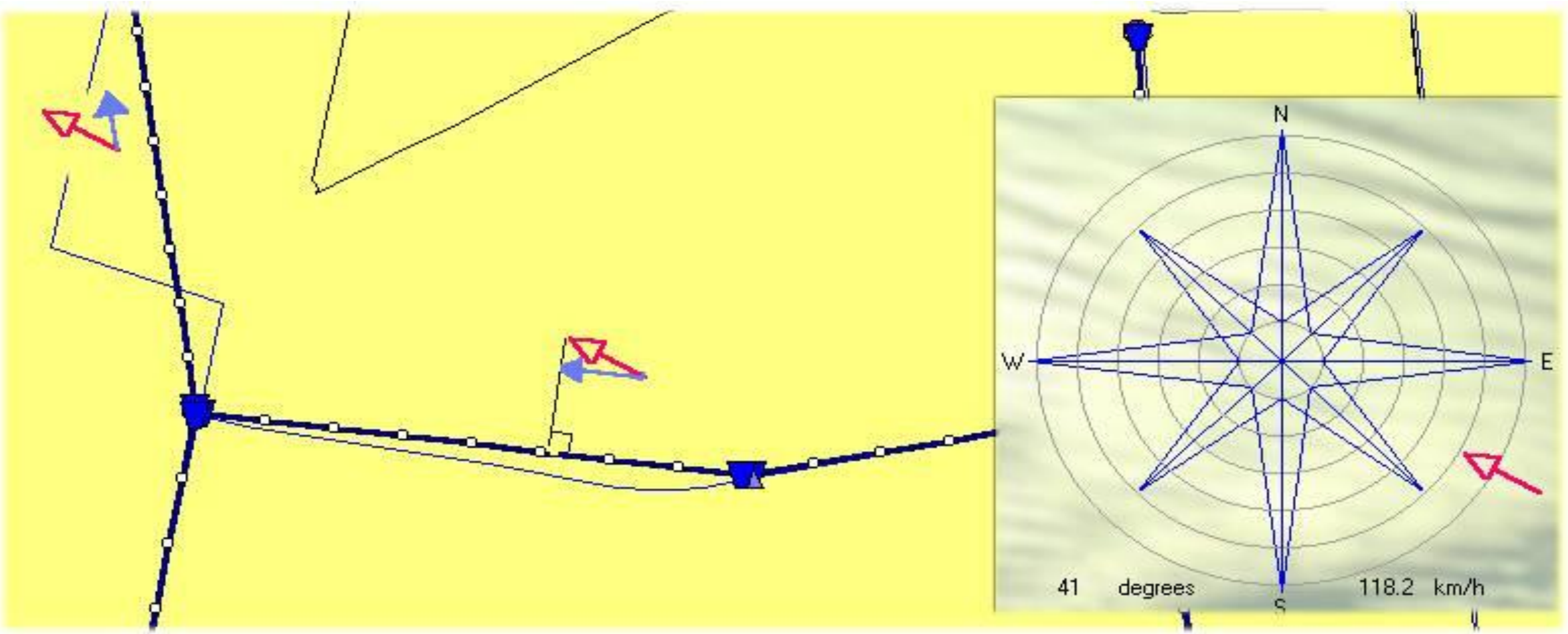


Other assumptions and simplifications

- Besides the first one that two- and three-dimensional effects can not be studied
- Hydrostatic pressure
- Velocity perpendicular to the main flow direction is very small
- Cross sections are changing gradually

1D modelling (3)

- Example 2:



Quick Aside on the Method of Characteristics

- Developed in pre-computer days.
- Makes use of the fact that we can use the hyperbolic nature of PDEs to transform PDE into 2 ODEs
- Gives us a way to
  - Visualize flow disturbances
  - Help understand numerical procedures required
    - Lets us know the Initial and Boundary conditions required!

Model (Open) boundaries

- Location of boundaries:
  - model boundaries have to be located at points which are not influenced by system modifications made inside the computational domain
- Boundary data:
  - ONE boundary condition for every characteristic entering the computational domain
- Types of boundaries
  - Q(t)
  - h(t)
  - Q(h)

What to use in a 1D model?

Riverine area (friction dominated flow)

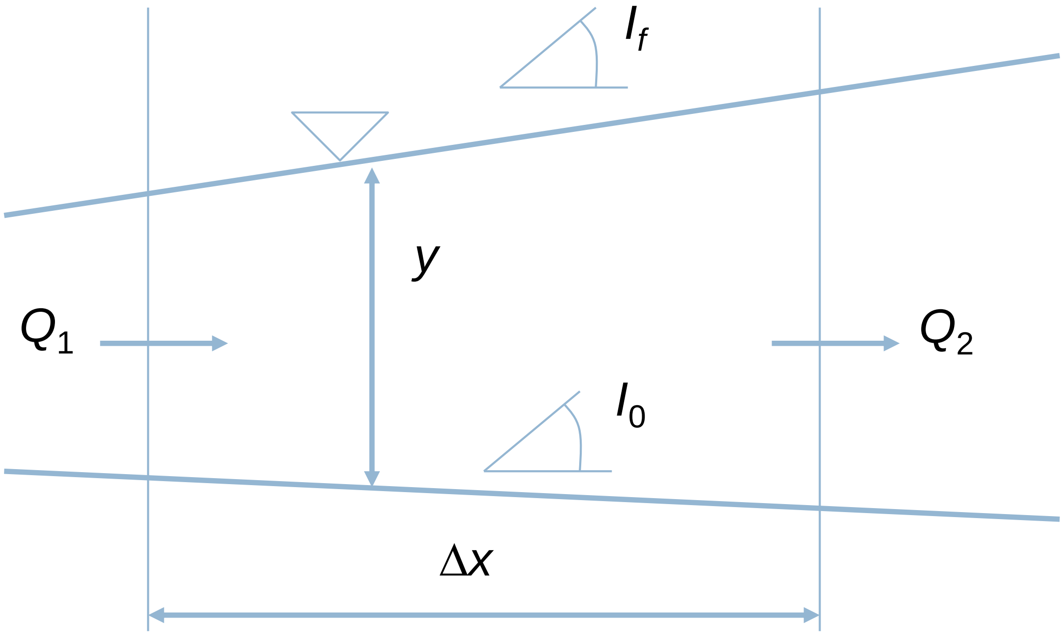
- Downstream
- Upstream

Tidal area

- Downstream
- Upstream

What do we solve?

- Conservation equations. Of what?



Q = discharge

y = water depth

I = slope

x= length

A longitudinal profile of a channel, between two sections

Conservation of Mass

Balance of inflow and outflow

$$\Delta V = -\frac{\partial Q}{\partial x}\Delta x\,\Delta t + q_{lat}\Delta x\,\Delta t$$

$$\frac{\partial A_t}{\partial t} + \frac{\partial Q}{\partial x} = q_{lat}$$

$$\Delta V = \frac{\partial A_t}{\partial t}\Delta x\,\Delta t$$

time depending change of water level

Conservation of Momentum

The second law of Newton:

$$\sum F = m \cdot a$$

$$\sum F = \rho \Delta x \frac{\partial A_t u}{\partial t}$$

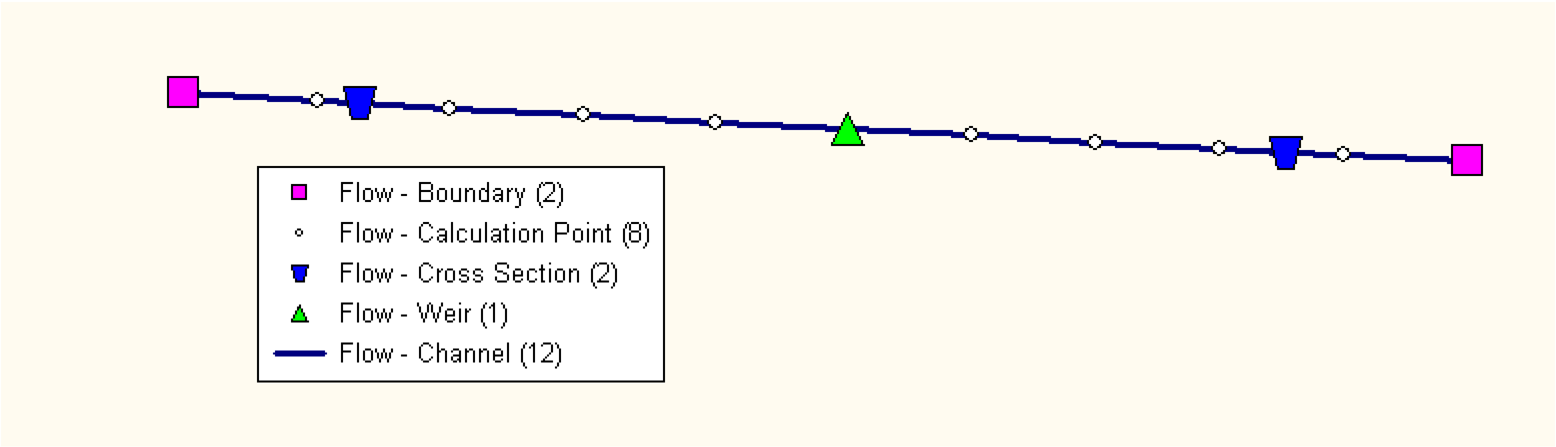
Forces: gravity, friction, momentum

$$F_g = -\rho g A_f \frac{\partial h}{\partial x} \Delta x$$

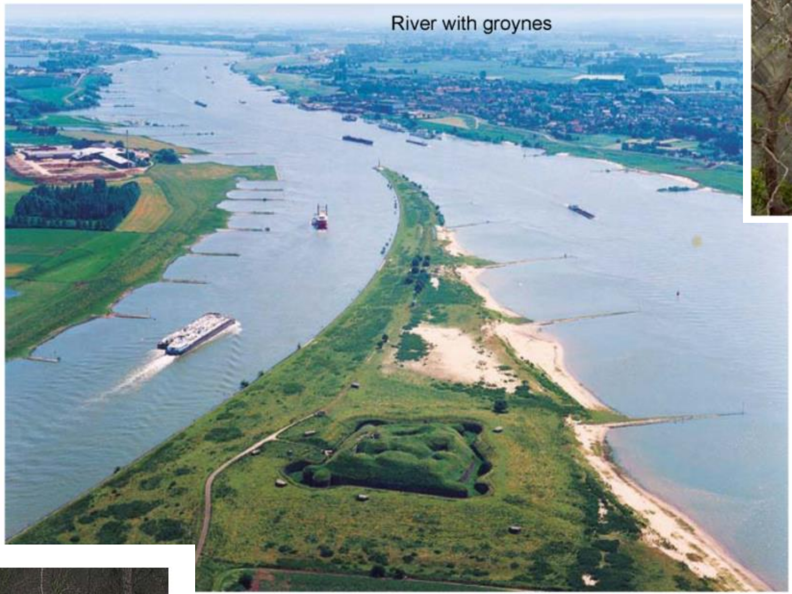
$$F_b = -\tau l_u \Delta x, \quad (\tau = \rho g r_{hy} I_f)$$

$$F_m = -\rho \frac{\partial A_f u^2}{\partial x} \Delta x$$

Example of simple 1-D network



Hydraulic Structures

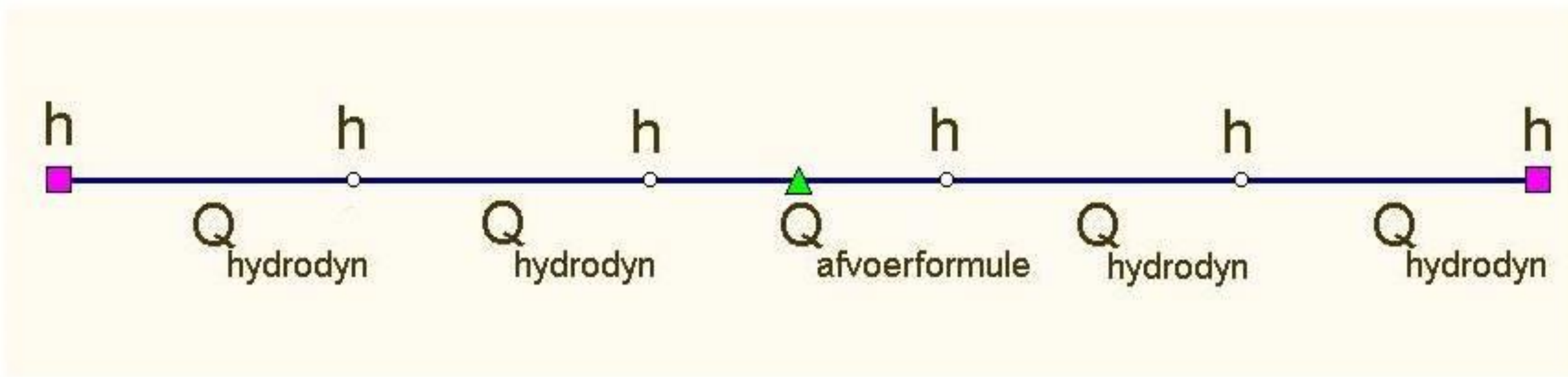


Hydraulic structures

- Type of structure (weir, orifice, general structure etc)
- Waterlevel or Energy level based structure equations
- Structure equation parameters based on laboratory results
- Database structure
- Extra resistance (modelling local losses)
- Control of structures (Controllers, Triggers and RTC)

Stuctures in the schematisation

- Reach segment containing a structure: hydrodynamic equations are replaced by the discharge formula for the structure:



note:

- The storage on such reach segment is still taken into account
- The structure's crest is never lower than the bed level