## 1. Brief background

Consider a bi-material composite bar, subjected to a compressive loading at one end. The composite bar has different Young's modulus and density in the two parts, but with the same cross-sectional area as shown in in Fig 1. The boundary and loading conditions are such that the bar deformation is uniform in the traverse and out-of-plane directions. Since the variation of deformation occurs only in the longitudinal direction ($x$), it reduces to a 1D problem, with $u(t) = 0$ at $x = L_A + L_B$. The governing equation is given by

$$u_{,tt} = c^2 u_{,xx}$$

where $c^2 = E/\rho$ is the wave speed. Note that $c$ can be different in the two bars.

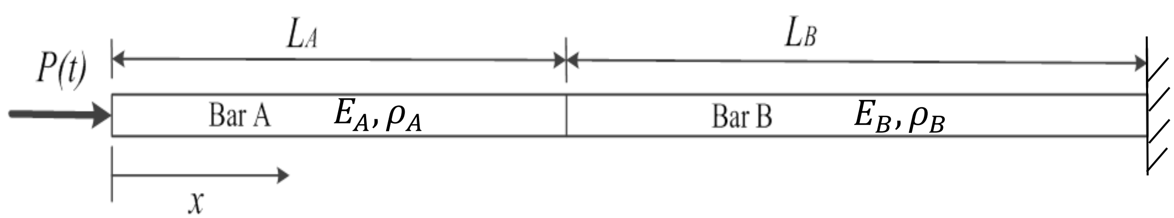



*Figure 1: A composite bar subjected to a compressive load.*

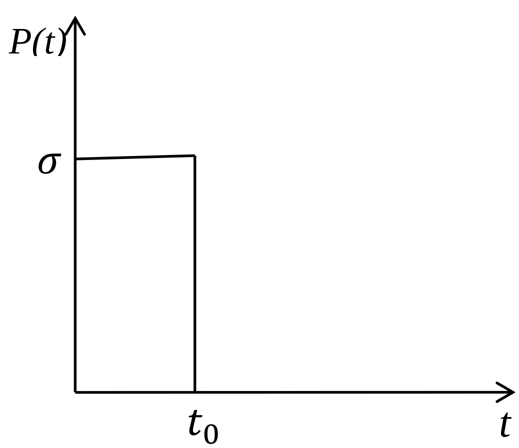



*Figure 2: Time history of applied load*

The load is applied as a compressive stress boundary condition as shown in Fig. 2, which generates an incident compressive stress wave. The stress wave propagates through bar A with the speed $c_A$ towards the material interface. At the material interface, a portion of the incident stress will be reflected and the other portion transmitted across the interface. The magnitudes of the reflected stress and transmitted stress are given by

$$\sigma_r = \frac{\sqrt{E_B\rho_B} - \sqrt{E_A\rho_A}}{\sqrt{E_B\rho_B} + \sqrt{E_A\rho_A}}\sigma_i \quad , \quad \sigma_t = \frac{2\sqrt{E_B\rho_B}}{\sqrt{E_B\rho_B} + \sqrt{E_A\rho_A}}\sigma_i$$

where $\sigma_i$ is the incident stress, $\sigma_r$ and $\sigma_t$ are the reflected and transmitted stresses respectively.

## 2. Problem parameters:

Material A: concrete

$$E_A = 30GPa \quad \rho_A = 2400kg/m^3 \;\; L_A = 0.1m \;\; c_A = \sqrt{\frac{E_A}{\rho_A}} = 3535.5m/s$$

Material B: steel

$$E_B = 210GPa \quad \rho_B = 7800kg/m^3 \;\; L_B = 0.2m \;\; c_B = \sqrt{\frac{E_B}{\rho_B}} = 5188.7m/s$$

Boundary condition at *x*=0:

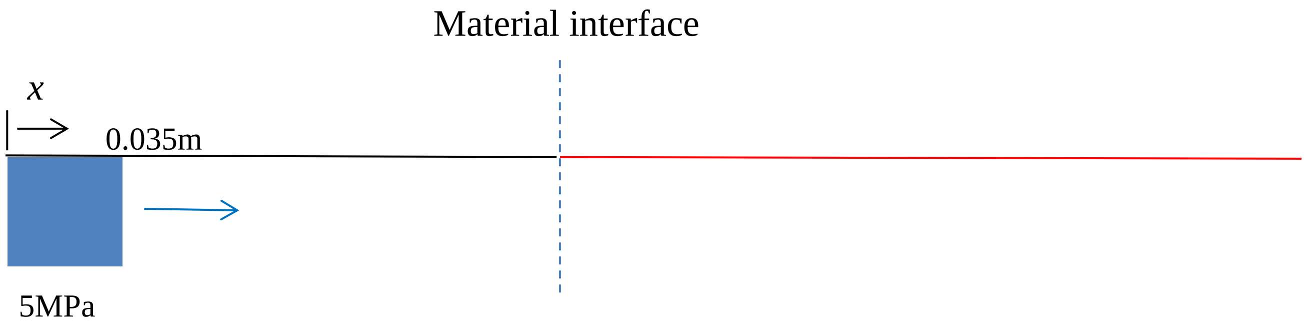
$\sigma = 5MPa \quad t_0 = 10^{-5}s$

$\sigma_r = \frac{\sqrt{E_B\rho_B} - \sqrt{E_A\rho_A}}{\sqrt{E_B\rho_B} + \sqrt{E_A\rho_A}}\sigma_i = 3.25$ MPa , $\sigma_t = \frac{2\sqrt{E_B\rho_B}}{\sqrt{E_B\rho_B} + \sqrt{E_A\rho_A}}\sigma_i = 8.25$ MPa

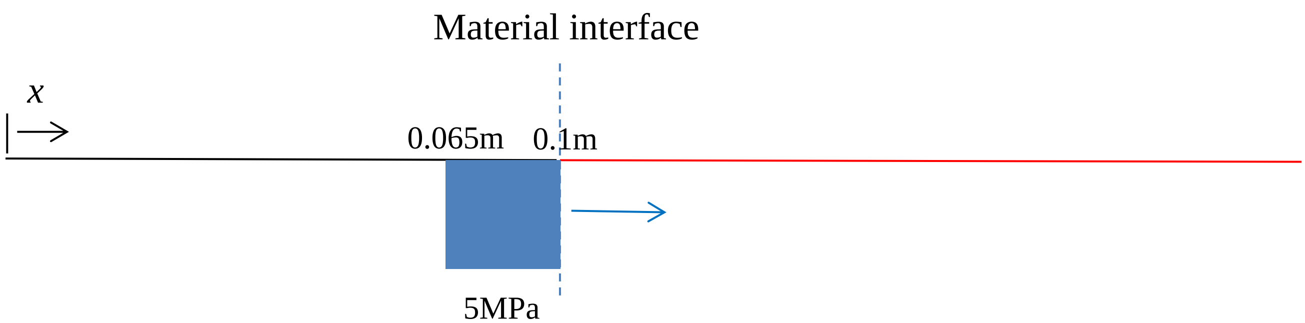
Note: In material *k*, the stress is given as $\boldsymbol{\sigma(x,t) = E_k\, u_{,x}(x,t)}$ with $\boldsymbol{k = A, B}$.

**3. Reference solution**

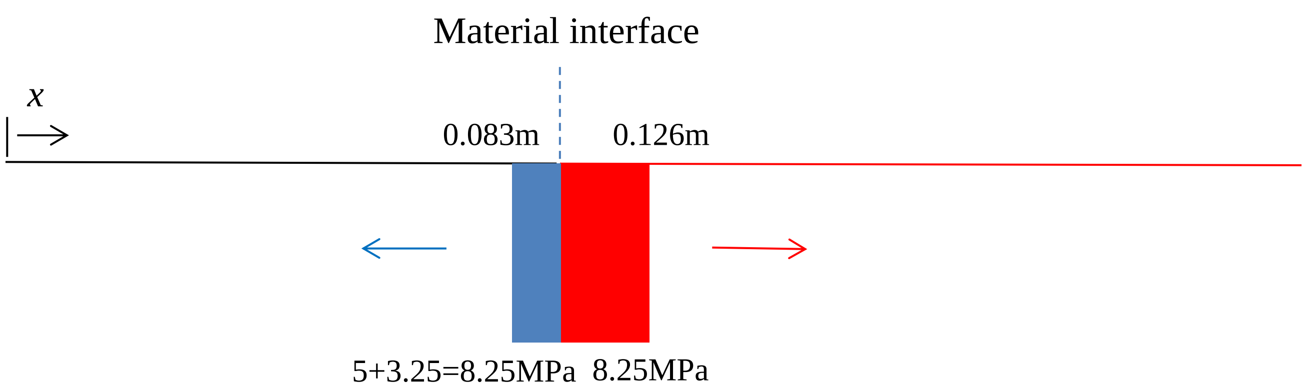
At $t = 1 \times 10^{-5} s$



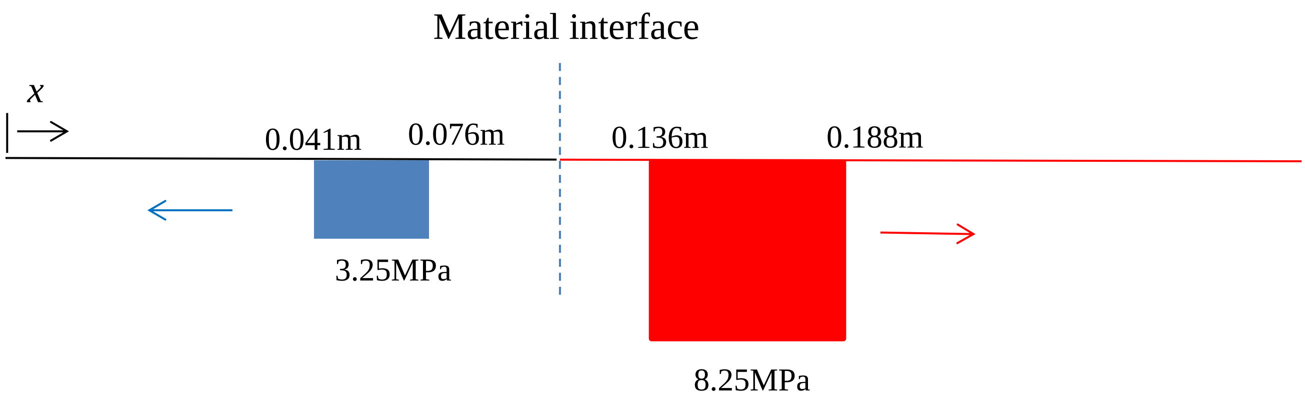

At $t = 2.8 \times 10^{-5} s$




At $t = 3.3 \times 10^{-5} s$



At $t = 4.5 \times 10^{-5} s$




4. **Mini-project**

Solve the problem in Section 1, using the parameters provided in Section 2, with the stated numerical schemes.

**Group A:** Solve the problem *in time* using the Backward Difference Method. Use a weighted average of a suitable difference scheme at time (n) and (n-1) for discretization in space.

**Group B:** Solve the problem *in time* using the Linear Acceleration Method (special case of Newmark method). You can use any scheme for the discretization in space.

Some suggested questions to explore are provided below. Do whatever you can within the given time, and report accordingly.

- Specifically for this problem, is there any critical time step to adopt for each numerical scheme? If so, how do you determine the critical time step?
- How does the spatial discretization size affect the critical time step, if any?
- Discuss on the stability and accuracy of the numerical schemes.