

## Coastal Dynamics 1

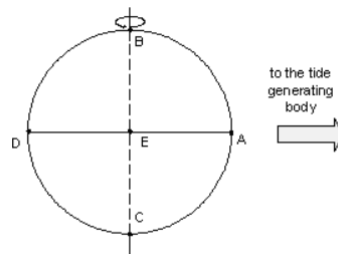
Student lectures: Thursday March 22 15:45-17:30

- Differential Pull (Maple TA question)  
Maud van Delden
- Wave asymmetry and skewness (Maple TA question)  
Geerten van der Zalm
- Velocity moments in the nearshore (Maple TA question)  
Charles Feys
- Dutch Basins (Trial Exam question)  
Florian Grossmann and Inés Báez Rivero

# Differential Pull

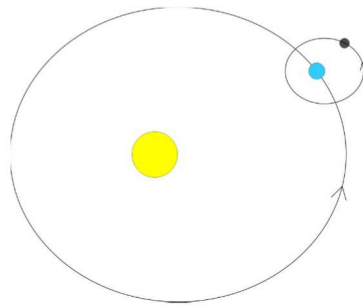
## Maple Question

- At location A in the below picture, the attractive force of the tide-generating body on 1 kg of mass is equal to  $a$  (in  $\text{m/s}^2$ ). At location D, the attractive force of the tide-generating body on 1 kg of mass is equal to  $d$  (in  $\text{m/s}^2$ ). What is at location A the approximate difference between the gravitational acceleration and the centripetal acceleration?



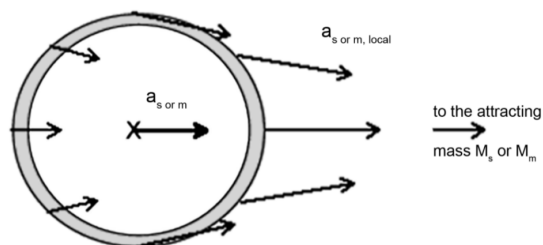
## Centripetal acceleration

- Works on the whole earth as a point mass
- Calculated with:  $a = G \frac{M}{r^2}$



## Gravitational acceleration

- Works on arbitrary point on earth
- Parallel to attracting mass
- Calculated with:  $a = G \frac{M}{r^2}$

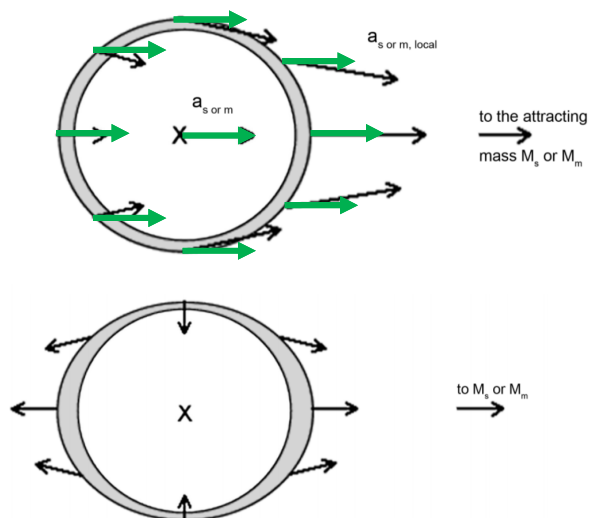


## Resulting acceleration

- Differential pull =  $F_{\text{gravitational}} - F_{\text{centripetal}}$   

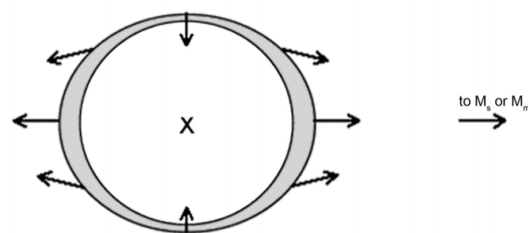
$$\Delta a = a_g - a_{\text{cent}} = G \frac{M}{(d-r)^2} - G \frac{M}{r^2}$$
- $\Delta a_s = 0.515 \times 10^{-7} g$
- $\Delta a_m = 1.13 \times 10^{-7} g$

## Resulting acceleration



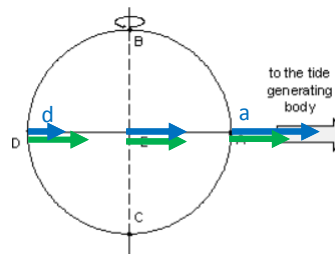
## Resulting acceleration

- Force normal to the surface is small compared to  $g$
- Tangential force perpendicular to surface is of importance  $\rightarrow$  water level gradient



## Maple Question

- At location A in the below picture, the **attractive force** of the tide-generating body **on 1 kg of mass** is equal to  $a$  (in  $\text{m/s}^2$ ). At location D, the **attractive force** of the tide-generating body **on 1 kg of mass** is equal to  $d$  (in  $\text{m/s}^2$ ). What is at location A the approximate **difference between the gravitational acceleration and the centripetal acceleration**?
- Attractive force on 1 kg of mass
- $= F_g/m = a_g$
- Difference between the gravitational acceleration and the centripetal acceleration = **differential pull**
- $= \Delta a = a_g - a_{\text{cent}}$



## Maple Question

- $\Delta a = a_g - a_{\text{cent}}$ 
  - $\Delta a_A = a_{g,A} - a_{\text{cent}}$  eq(1)
  - $\Delta a_D = a_{g,D} - a_{\text{cent}}$  eq(2)
- $\Delta a_A = -\Delta a_D$  eq(3)

From eq(3) and eq(2)

$$-\Delta a_A = a_{g,D} - a_{\text{cent}}$$

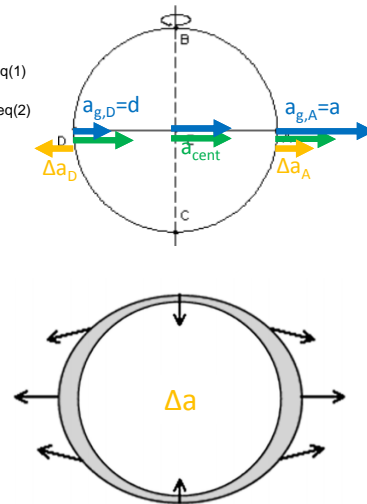
$$a_{\text{cent}} = a_{g,D} + \Delta a_A$$

In eq(1)

$$\Delta a_A = a_{g,A} - a_{g,D} - \Delta a_A$$

$$\Delta a_A = \frac{a_{g,A} - a_{g,D}}{2}$$

$$\text{Fill in: } \Delta a_A = \frac{a - d}{2}$$



## Differential Pull

# Wave asymmetry and skewness

Maple chapter 5, part 1

Geerten

1

Consider a horizontal wave-orbital velocity signal given by

$$u(t) = u_1 \cos(\omega t) + u_2 \cos(2\omega t - \varphi).$$

$u_1$  is 4 times larger than  $u_2$ .

*Paragraph 5.3 & 5.4*

A signal characteristic for breaking waves is obtained for:

- A)  $\Phi = 0$
- B)  $\Phi = \pi$
- C)  $\Phi = \pi/2$
- D)  $\Phi = 3\pi/2$

A signal characteristic for shoaling waves is obtained for:

- A)  $\Phi = 0$
- B)  $\Phi = \pi$
- C)  $\Phi = \pi/2$
- D)  $\Phi = 3\pi/2$

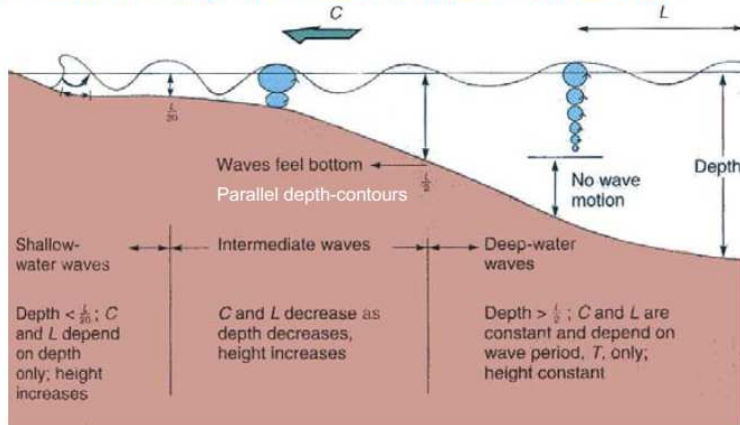
2

Consider a horizontal wave-orbital velocity signal given by

$$u(t) = u_1 \cos(\omega t) + u_2 \cos(2\omega t - \varphi).$$

$u_1$  is 2 times larger than  $u_2$ . What is  $\varphi$  for a shoaling and breaking wave?

Waves start feeling the bottom in intermediate water



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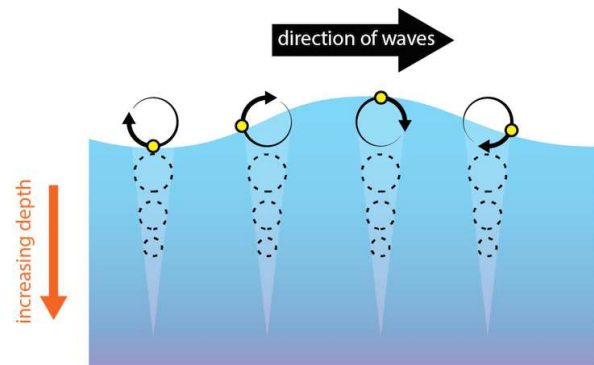
Consider a horizontal wave-orbital velocity signal given by

$$u(t) = u_1 \cos(\omega t) + u_2 \cos(2\omega t - \varphi).$$

$u_1$  is 4 times larger than  $u_2$ .

Wave orbital velocity

- Dealing with a progressive wave



Source:

<https://manoa.hawaii.edu/exploringourfluide>

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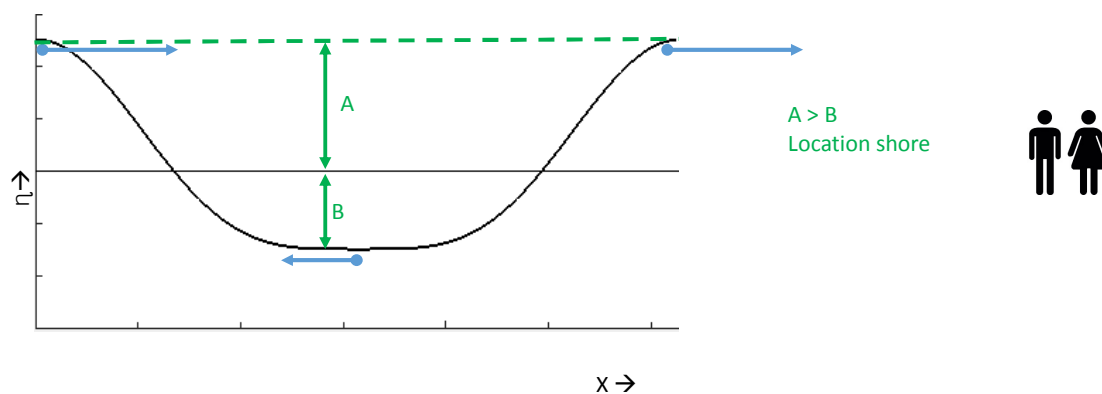


Consider a horizontal wave-orbital velocity signal given by  
 $u(t) = u_1 \cos(\omega t) + u_2 \cos(2\omega t - \varphi)$ .  
 $u_1$  is 4 times larger than  $u_2$ .

- We are dealing with a **propagating** wave
  - No friction
  - No reflection

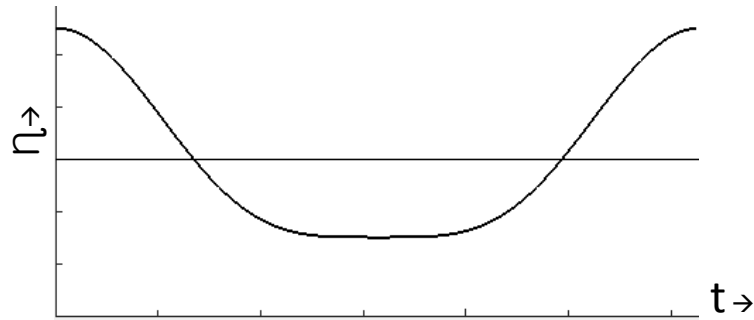
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Shoaling wave – surface elevation in space



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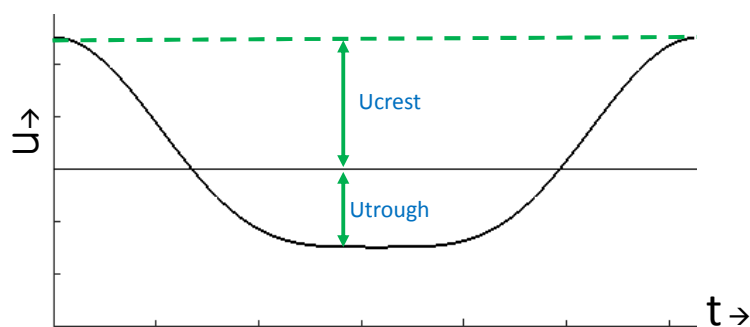
## Shoaling wave – surface elevation in time



- Where are highest & lowest velocities
- How to connect these lines

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## Shoaling wave - velocities in time


 $U_{trough} < U_{crest}$ 

- How would we model this mathematically?
- <https://www.desmos.com/calculator/wwh2hc3n2w>

$$u(t) = 2u_2 \cos(\omega t) + u_2 \cos(2\omega t - \varphi).$$

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Consider a horizontal wave-orbital velocity signal given by

$$u(t) = u_1 \cos(\omega t) + u_2 \cos(2\omega t - \varphi).$$

Paragraph 5.3 & 5.4

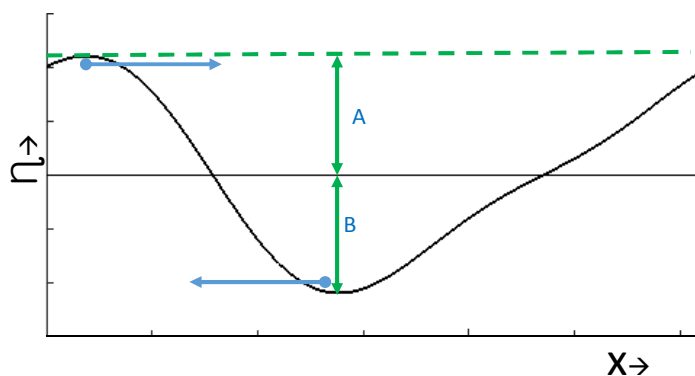
$u_1$  is 4 times larger than  $u_2$ .

A signal characteristic for shoaling waves is obtained for:

- A)  $\Phi = 0$
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- D)  $\Phi = 3\pi/2$

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## Breaking wave – surface elevation in space



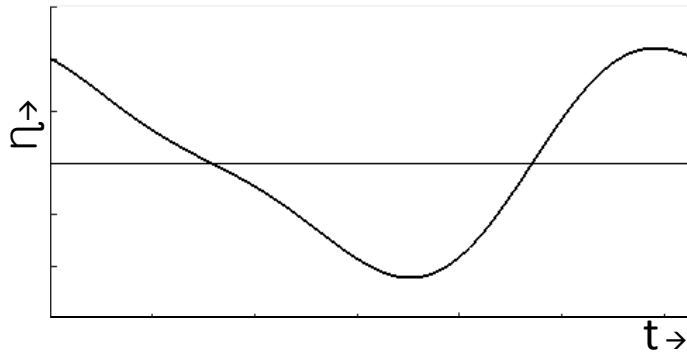
$A = B$   
Location shore



- How will this look in time?

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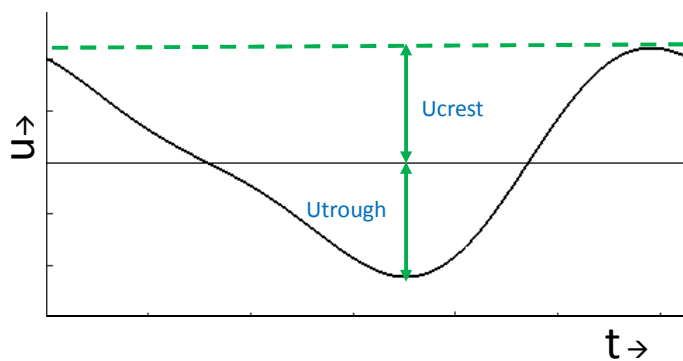
## Breaking wave – surface elevation in time



- Where are the highest and lowest velocities?
- How to connect these lines?

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## Breaking wave – velocities in time



$$U_{crest} = U_{trough}$$

- How would we model this mathematically?
- <https://www.desmos.com/calculator/9f4sepulnr>

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Consider a horizontal wave-orbital velocity signal given by

$$u(t) = u_1 \cos(\omega t) + u_2 \cos(2\omega t - \phi).$$

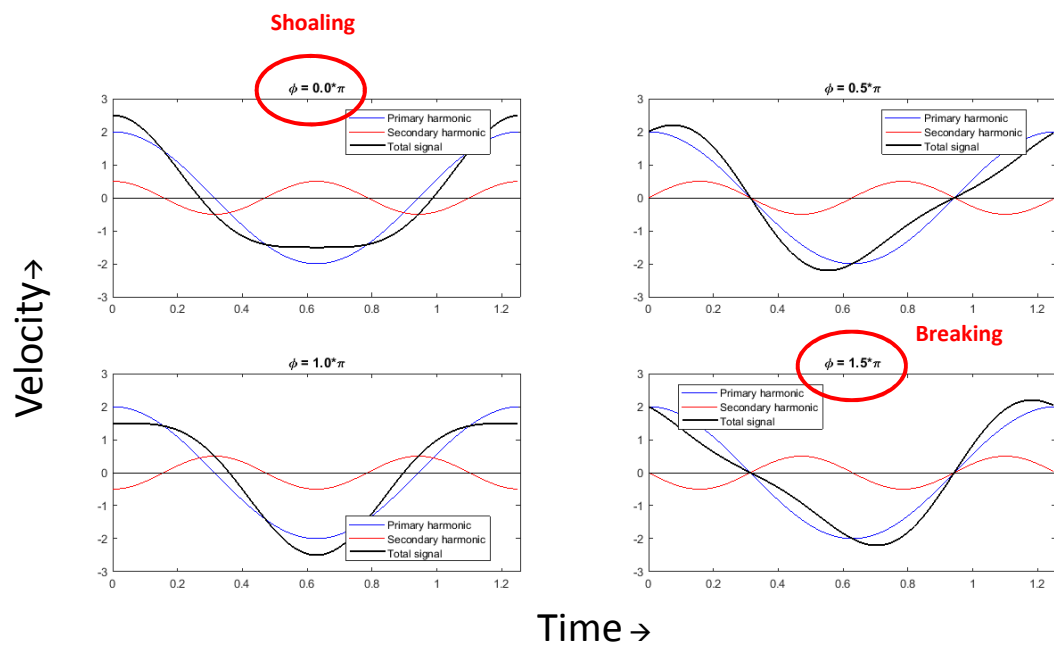
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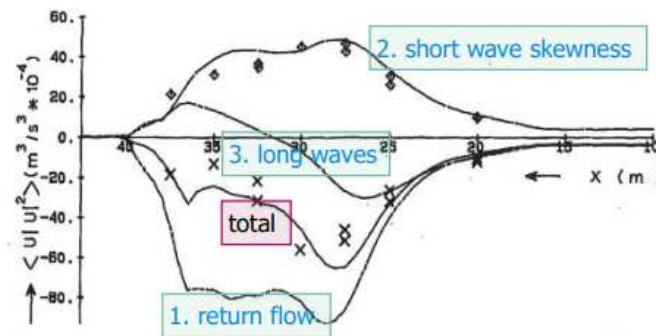


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## Back to cross shore profile

- Change in u-signal over profile
- What are the implications for the sediment transport?



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## For further study

- Theory on pages 172-179 (chapter 5.3 & 5.4) and 332-335 (chapter 7.5.2)
- Try the summations of harmonics yourself with the link on brightspace

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# Cross-shore transport components

Charles FEYS

## Question (Maple TA 7B)

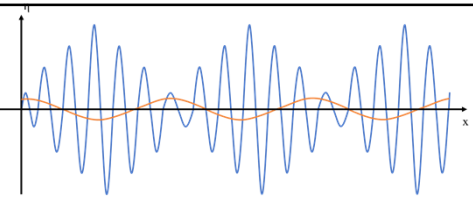
- Third odd velocity moment:

$$\underbrace{\langle u|u|^2 \rangle}_1 = 3 \underbrace{\langle \bar{u}|u_{hi}|^2 \rangle}_2 + \underbrace{\langle u_{hi}|u_{hi}|^2 \rangle}_3 + 3 \underbrace{\langle u_{lo}|u_{hi}|^2 \rangle}_4 + \dots$$

- What term(s) are never negative on the shore face?
- Term 4 changes sign from outside the surf zone towards near the shore. At the location of changing sign, the maximum onshore long wave velocity occurs simultaneously with:

- The smallest short wave in the group;
- The largest short wave in the group; or
- The average short wave in the group?

## Setting the scene

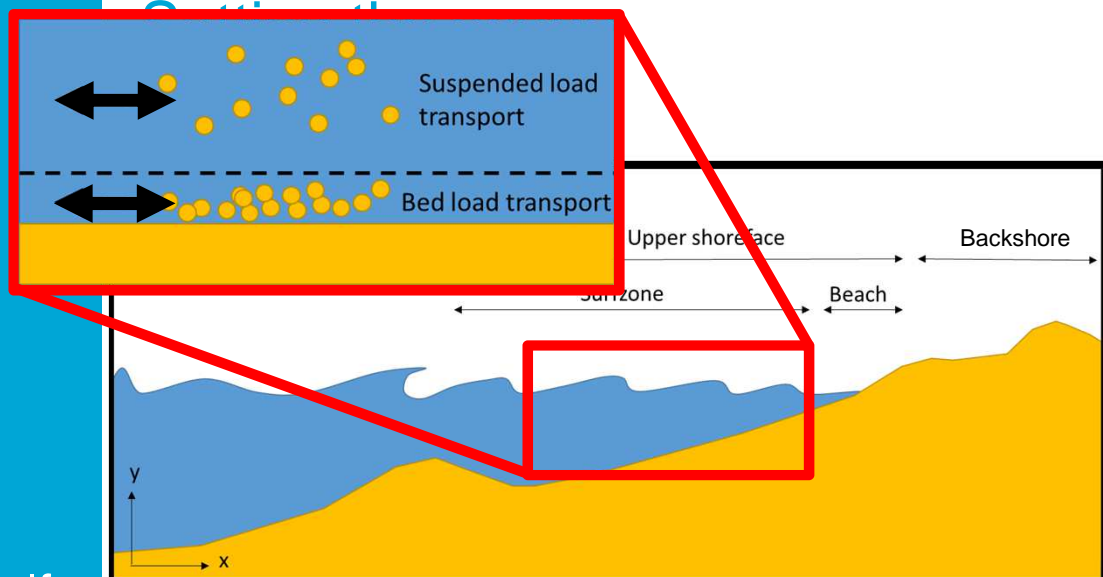
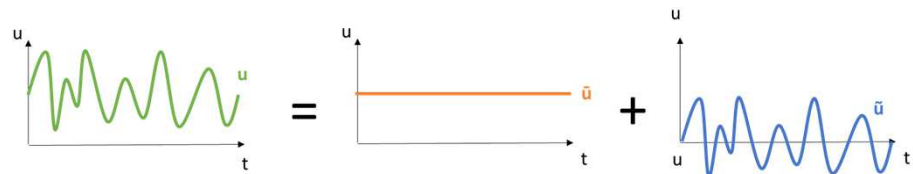


- $u = \bar{u} + \tilde{u}$

- $u$  = Total velocity
- $\bar{u}$  = Mean velocity
- $\tilde{u}$  = fluctuating part

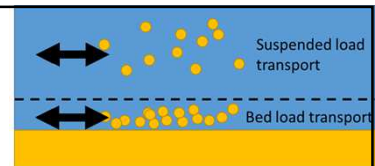
- $\tilde{u} = u_{lo} + u_{hi}$

- $u_{lo}$  = low-frequency motion at wave-group scale
- $u_{hi}$  = oscillary motion at short wave scale





## Velocity moments



- $\langle S_b \rangle \propto \langle u|u|^2 \rangle$  for bed load transport
- $\langle S_s \rangle \propto \langle u|u|^3 \rangle$  for suspended load transport

- $\langle u|u|^2 \rangle \rightarrow$  direction and magnitude
- $\langle |u|^3 \rangle \rightarrow$  only magnitude

## Third odd velocity moment

- $\langle u|u|^2 \rangle = 3\langle \bar{u}|u_{hi}|^2 \rangle + \langle u_{hi}|u_{hi}|^2 \rangle + 3\langle u_{lo}|u_{hi}|^2 \rangle + \dots$

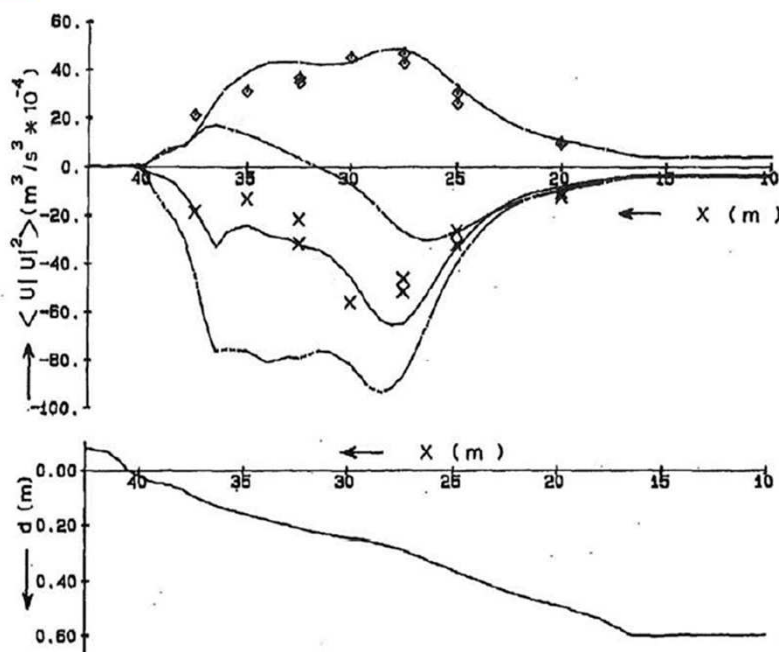
$$\begin{aligned} \langle u|u|^2 \rangle = & \langle \bar{u}|\bar{u}|^2 \rangle + 2\langle \bar{u}|\bar{u}u_{lo}| \rangle + \langle \bar{u}|\bar{u}^2u_{lo}| \rangle + 2\langle \bar{u}|\bar{u}u_{hi}| \rangle + \langle \bar{u}|\bar{u}^2u_{hi}| \rangle + \langle \bar{u}|u_{lo}|^2 \rangle + 2\langle u_{lo}|\bar{u}u_{lo}| \rangle \\ & + \langle \bar{u}|u_{hi}|^2 \rangle + 2\langle u_{hi}|\bar{u}u_{hi}| \rangle + 2\langle \bar{u}|u_{lo}u_{hi}| \rangle + 2\langle u_{lo}|\bar{u}u_{hi}| \rangle + 2\langle u_{hi}|\bar{u}u_{lo}| \rangle + \langle u_{lo}|u_{lo}|^2 \rangle \\ & + 2\langle u_{lo}|u_{lo}u_{hi}| \rangle + \langle u_{hi}|u_{lo}|^2 \rangle + \langle u_{lo}|u_{hi}|^2 \rangle + 2\langle u_{hi}|u_{lo}u_{hi}| \rangle + \langle u_{hi}|u_{hi}|^2 \rangle \end{aligned}$$

- Assume that  $\bar{u} \ll u_{lo} \ll u_{hi}$

- $|u_{hi}|^2$   
Short waves stir up most sediment

## Third odd velocity moment

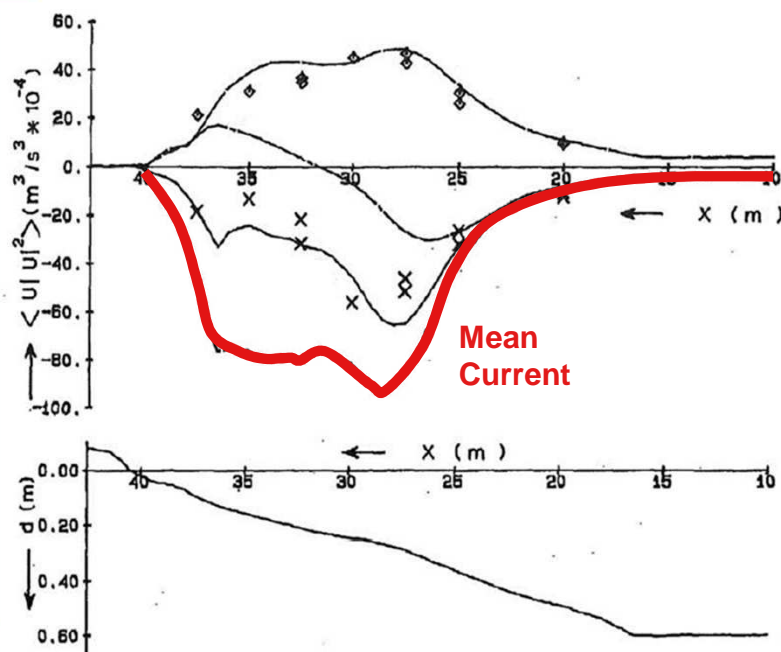
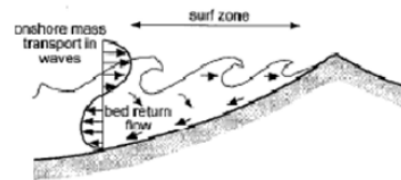
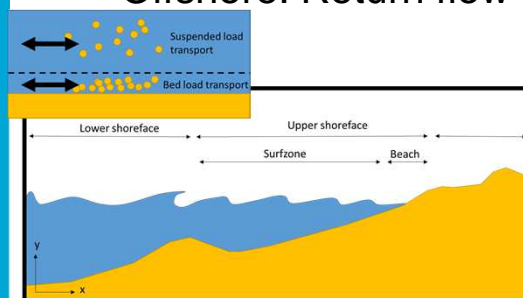
- $\underbrace{\langle u|u|^2 \rangle}_1 = \underbrace{3\langle \bar{u}|u_{hi}|^2 \rangle}_2 + \underbrace{\langle u_{hi}|u_{hi}|^2 \rangle}_3 + \underbrace{3\langle u_{lo}|u_{hi}|^2 \rangle}_4 + \dots$
- $\bar{u}$  represents the mean current
- $u_{hi}$  represents the the short-waves
- $u_{lo}$  represents the long waves



$$\langle u|u|^2 \rangle = 3\langle \bar{u}|u_{hi}|^2 \rangle + \langle u_{hi}|u_{hi}|^2 \rangle + 3\langle u_{lo}|u_{hi}|^2 \rangle + \dots$$

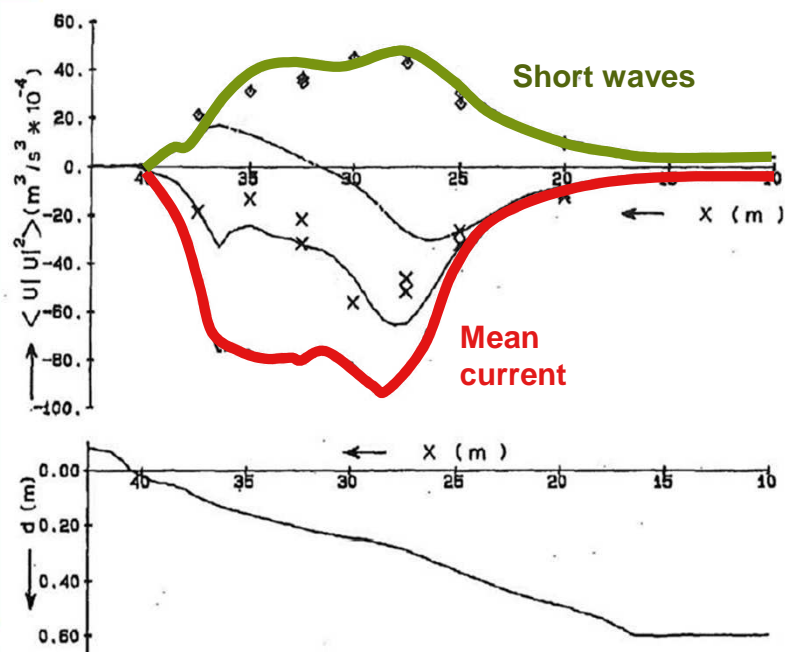
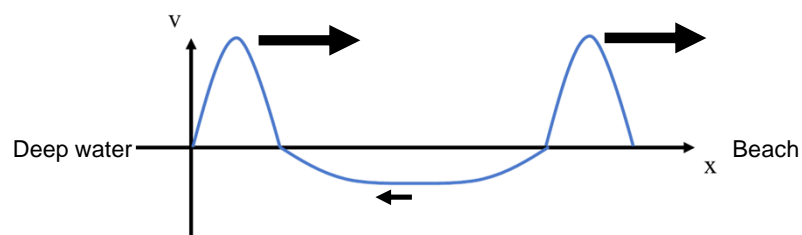
- Mean current

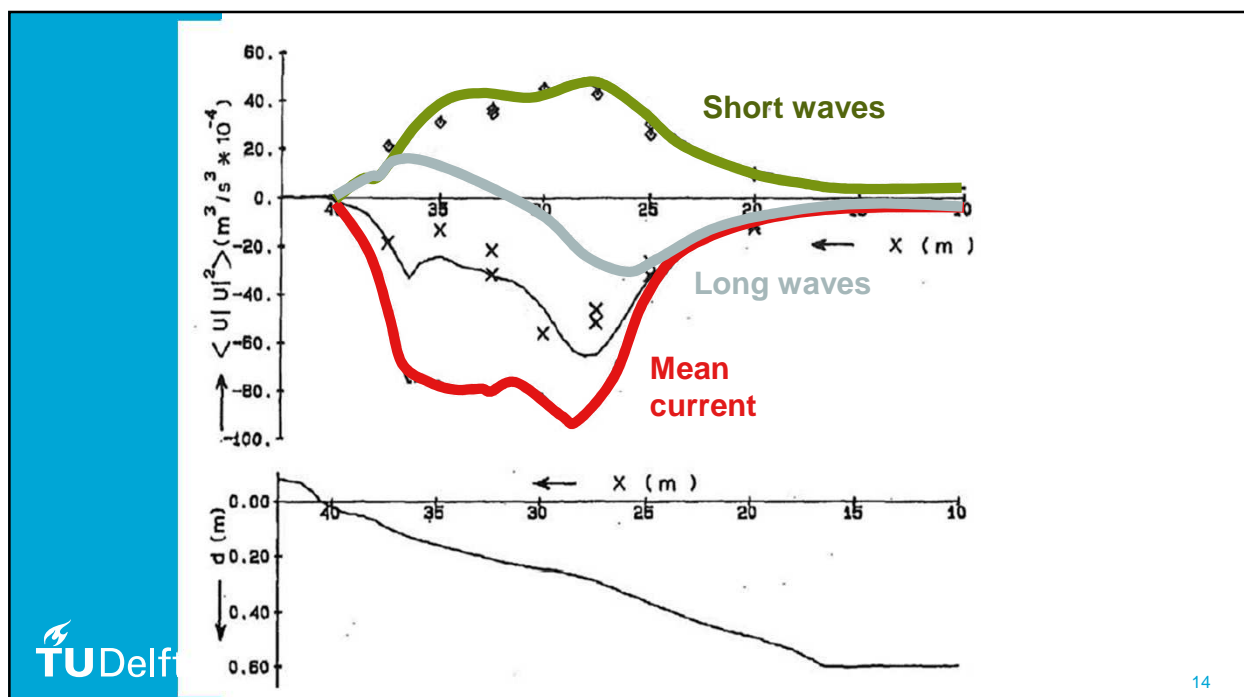
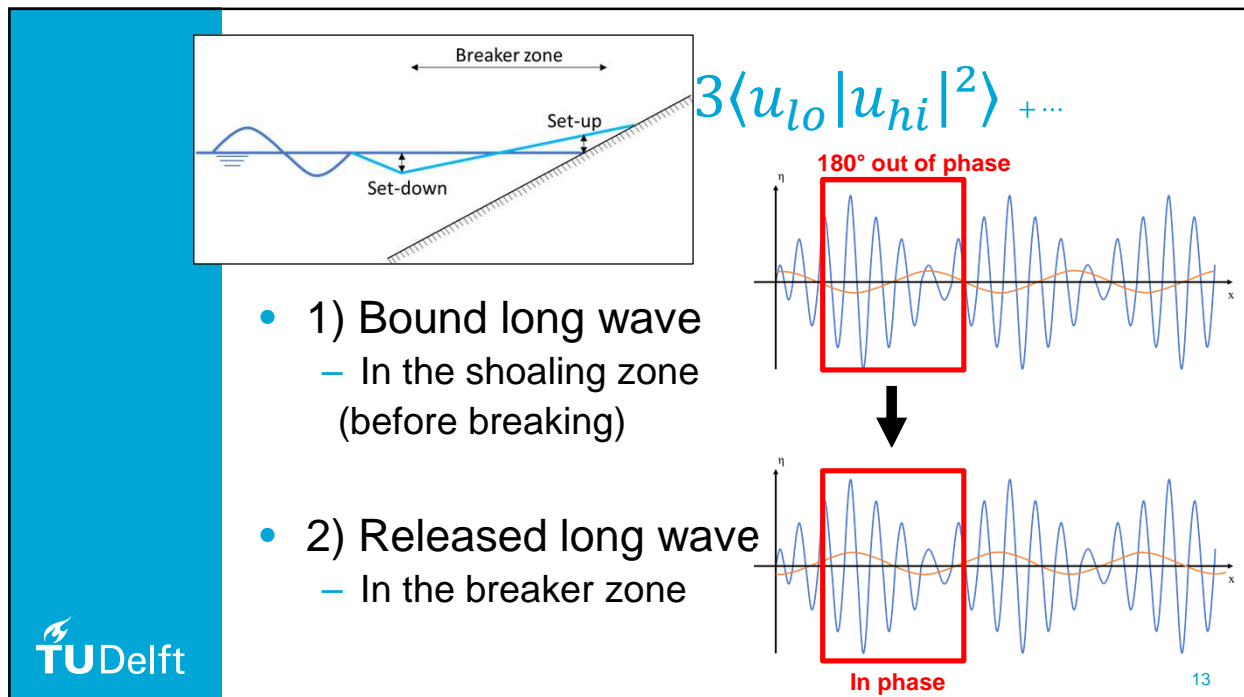
- Onshore: Longuet-Higgins streaming
- Offshore: Return flow



$$\langle u|u|^2 \rangle = 3\langle \bar{u}|u_{hi}|^2 \rangle + \langle u_{hi}|u_{hi}|^2 \rangle + 3\langle u_{lo}|u_{hi}|^2 \rangle + \dots$$

- Short waves
  - skewness





## Question

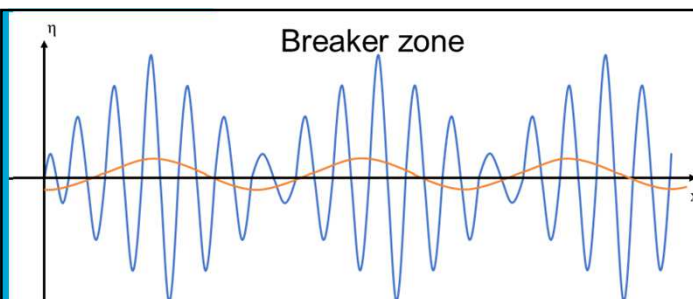
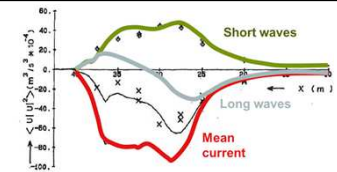
- Third odd velocity moment:

$$\underbrace{\langle u|u|^2 \rangle}_1 = 3 \underbrace{\langle \bar{u}|u_{hi}|^2 \rangle}_2 + \underbrace{\langle u_{hi}|u_{hi}|^2 \rangle}_3 + 3 \underbrace{\langle u_{lo}|u_{hi}|^2 \rangle}_4 + \dots$$

a). What term(s) are never negative on the shore face?

b). Term 4 changes sign from outside the surf zone towards near the shore. At the location of changing sign, the maximum onshore long wave velocity occurs simultaneously with:

- The smallest short wave in the group;
- The largest short wave in the group; or
- The average short wave in the group?

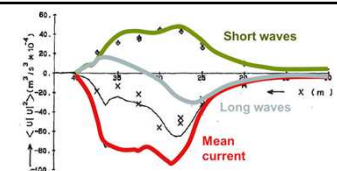


$$\underbrace{u_{hi}|^2}_3 + 3 \underbrace{\langle u_{lo}|u_{hi}|^2 \rangle}_4 + \dots$$

a). What term(s) are never negative on the shore face?

b). Term 4 changes sign from outside the surf zone towards near the shore. At the location of changing sign, the maximum onshore long wave velocity occurs simultaneously with:

- The smallest short wave in the group;
- The largest short wave in the group; or
- The average short wave in the group?



## Question

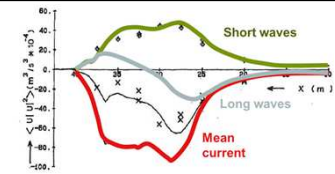
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a). What term(s) are never negative on the shore face?

**b). Term 4 changes sign from outside the surf zone towards near the shore. At the location of changing sign, the maximum onshore long wave velocity occurs simultaneously with:**

- A) The smallest short wave in the group;
- B) The largest short wave in the group; or
- ✓ C) The average short wave in the group?



## Thank you !

- Questions?

# TIDAL PROPAGATION INTO BASINS

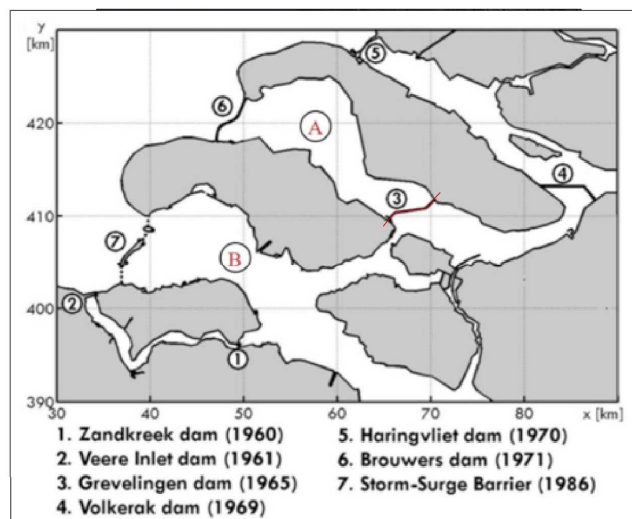
Coastal Dynamics 1

Florian Grossman  
Inés Báez Rivero



1

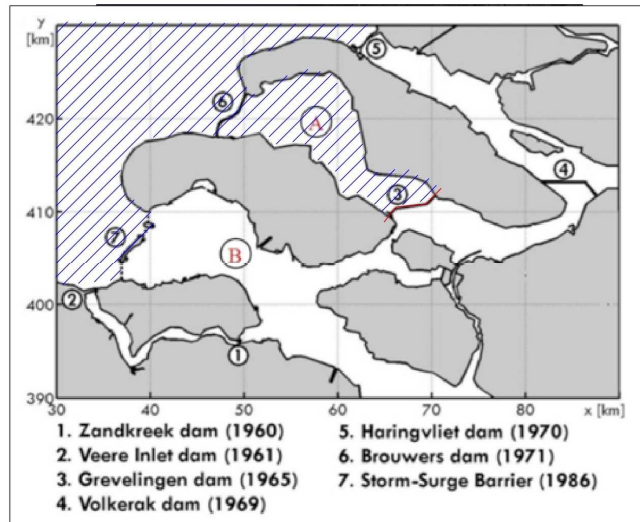
## Trial Exam – Section B



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## Trial Exam – Section B



## Question 8

*Consider the following aspects of waves: surface elevation, horizontal and vertical orbital motion, horizontal and vertical particle velocity. For each of these five aspects indicate whether it is related to the horizontal or vertical tide or to neither.*

## Question 8 – Definitions

- Vertical Tide: Vertical rise and fall of water
- Horizontal Tide: Horizontal movement back and forward, associated with the rise and fall of water
- Flood and Ebb: Horizontal in- and outflow
- Rising Period: Time it takes for the water to get from lowest to highest elevation
- Falling Period: Time it takes for the water to get from highest to lowest elevation



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## Question 8

*Consider the following aspects of waves: surface elevation, horizontal and vertical orbital motion, horizontal and vertical particle velocity. For each of these five aspects indicate whether it is related to the **horizontal** or **vertical tide** or to neither.*

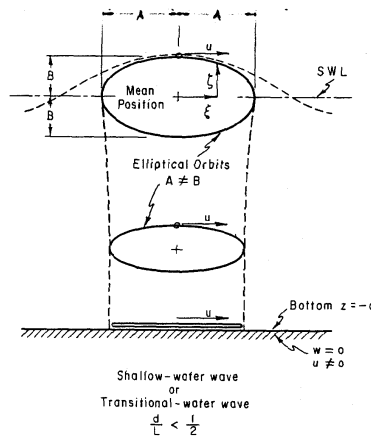
- 1.) Surface elevation: vertical tide
- 2.) Horizontal orbital motion: horizontal tide



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## Question 8 – Orbital Motion

Consider the following aspects of waves: surface elevation, horizontal and vertical orbital motion, horizontal and vertical particle velocity. For each of these five aspects indicate whether it is related to the horizontal or vertical tide or to neither.



## Question 8

Consider the following aspects of waves: surface elevation, horizontal and vertical orbital motion, horizontal and vertical particle velocity. For each of these five aspects indicate whether it is related to the **horizontal** or **vertical tide** or to neither.

- 1.) Surface elevation: vertical tide
- 2.) Horizontal orbital motion: horizontal tide
- 3.) Vertical orbital motion: neither
- 4.) Horizontal particle velocity: horizontal tide
- 5.) Vertical particle velocity: neither

## Question 9 – Part I

Assume that the horizontal and the vertical tide are described by the following equations:

$$\eta(x, t) = a * \cos(\omega t - kx)$$

$$u(x, t) = u * \cos(\omega t - kx - \varphi)$$

Where  $\varphi$  = phase of tidal velocity relative to tidal elevation

Indicate the value of  $\varphi$  (in radians) for:

1.) a **propagating tidal wave**

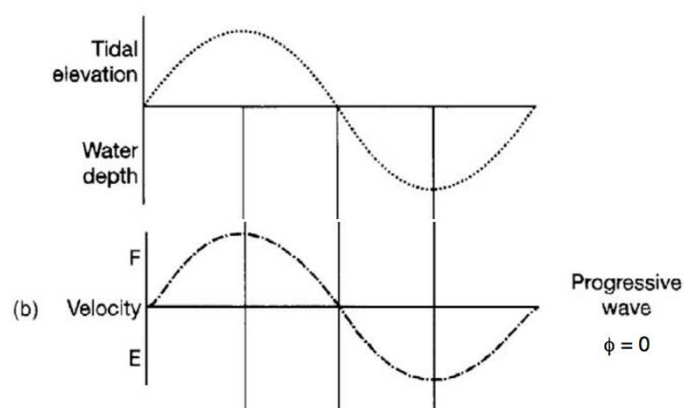
2.) a standing tidal wave

$$\frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x}$$



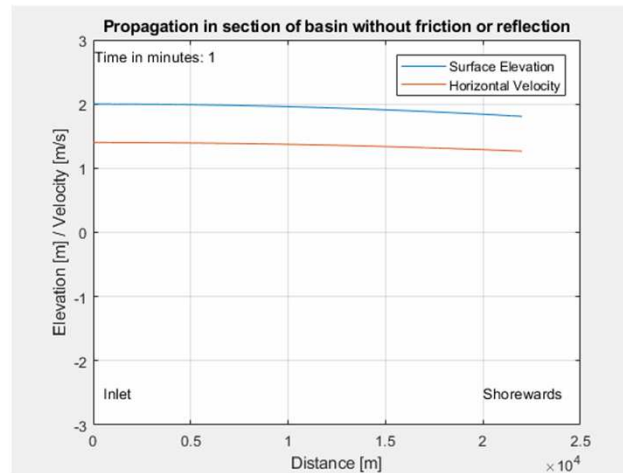
9

## Question 9 – Propagating tidal wave



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## Question 9 – Propagation in basin



## Question 9 – Part II

Assume that the horizontal and the vertical tide are described by the following equations:

$$\eta(x, t) = a * \cos(\omega t - kx)$$

$$u(x, t) = u * \cos(\omega t - kx - \varphi)$$

Where  $\varphi$  = phase of tidal velocity relative to tidal elevation

Indicate the value of  $\varphi$  (in radians) for:

1.) a propagating tidal wave

2.) a standing tidal wave

## Question 9 – Influence of reflection

No reflection

$$\eta(x, t) = a * \cos(\omega t - kx)$$

$$u(x, t) = u * \cos(\omega t - kx - \phi)$$

With reflection

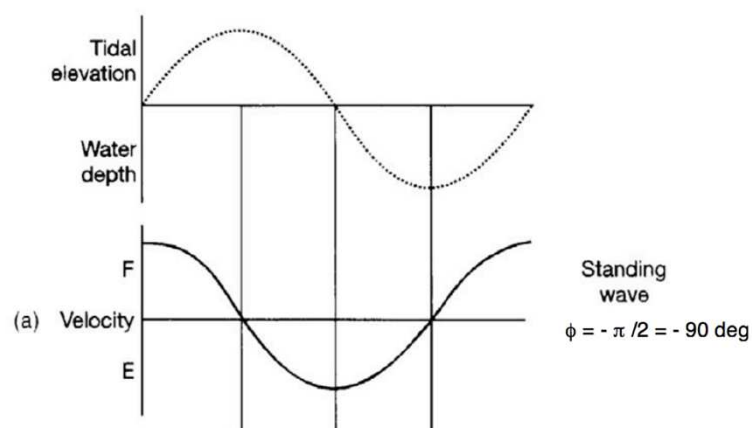
$$\eta(x, t) = a \frac{\cos(k(L_b - x))}{\cos(kL_b)} \cos(\omega t)$$

$$u(x, t) = -\frac{ac}{h} \frac{\sin(k(L_b - x))}{\cos(kL_b)} \sin(\omega t)$$



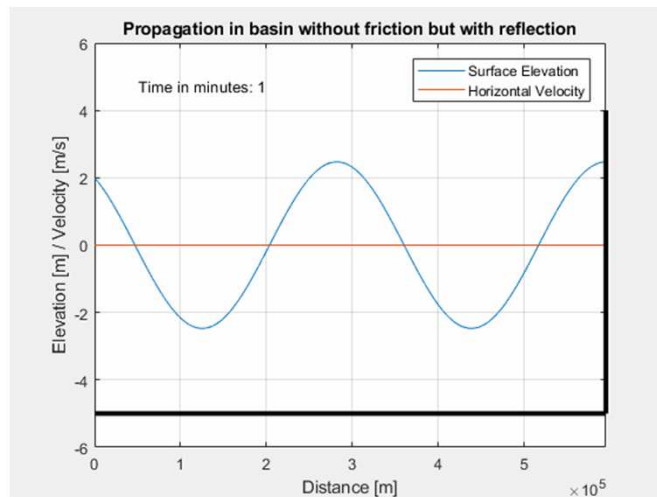
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## Question 9 – Standing tidal wave

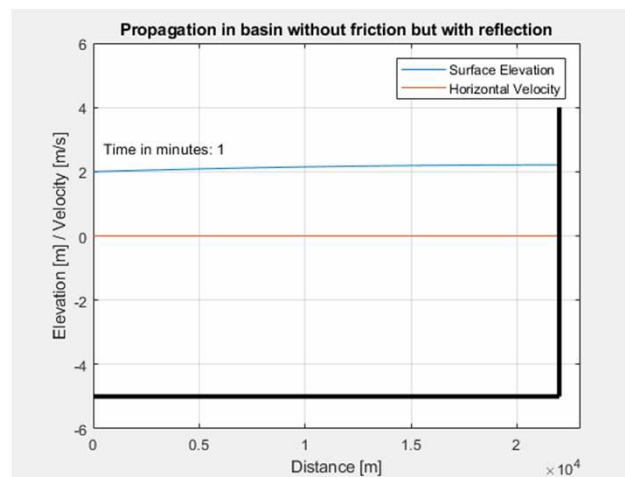


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## Question 9 – Long basin

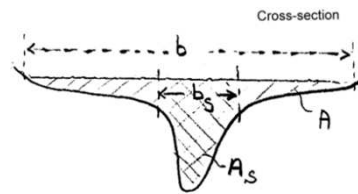


## Question 9 – Short basin



## Question 10

- What do you expect in the Grevelingen basin?
  - A propagating tidal wave
  - A standing tidal wave
- Assumptions made:
  - No tidal flats
  - Constant depth
    - $h = 5$  m
  - Tidal period?
  - Semidiurnal M2 tidal component
    - $T_{M2}$ : 12.42 hrs



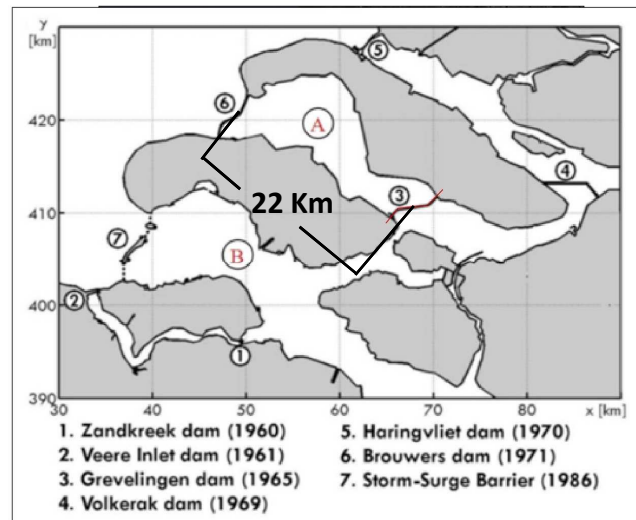
## Question 10

$$\frac{L_b}{L_T} = \frac{\text{Basin length}}{\text{Tidal wave length}}$$

- $h = 5$  m
- $T_{M2}$ : 12.42 hrs
- 1. How long is the basin =  $L_b$  ?
- 2. And the tidal wave length =  $L_T$  ?



## Question 10



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## Question 10

Tidal Wave Length =  $L = \lambda = c * T$

- Phase speed of a linear progressive wave in shallow water:

$$c = \sqrt{g * h} = \sqrt{9.81 \frac{m}{s^2} * 5m} = 7 m/s$$

- And we are assuming a Semidiurnal M2 component

$$T_{M2} = 12.42 * 3600 \text{ (sec)}$$

- $\therefore L_T = c_T * T_{M2} = 314143 m \cong 300 Km$

$$\frac{L_b}{L_T} = \frac{20}{300} < \frac{1}{10}$$

→ SHORT BASIN

↓  
Pumping mode →

**Standing Wave**  
with a constant  
tidal range over  
the basin

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## Question 11

How to estimate the tidal prism in the Grevelingen basin?

1. What is tidal prism?

Is the amount of water that has to flow in and out with the tides.

$$P = \int_0^{\frac{1}{2}T} A_e u dt$$

○ Assumptions made:

1. There are limited tidal flats
2. The tidal range (R) is approximately constant in the basin  
→ SHORT BASIN



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## Question 11

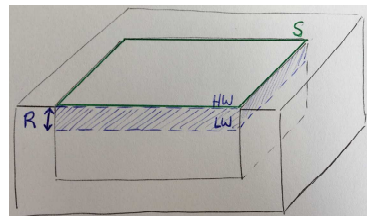
How to estimate the tidal prism in the Grevelingen basin?

→ SHORT BASIN

Tidal prism is the volume of water difference between mean high tide and low tide

- The surface area of the basin (averaged over tidal period) (S)
- The tidal range at the entrance (R)

$$P = R * S$$



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## Question 12

In the Grevelingen basin, the channel volume was too large with respect to its equilibrium condition.

*Explain the characteristics of the tidal velocity signal that can restore equilibrium.*

1. Channel volume is too large with respect to its equilibrium condition → Not enough sediments

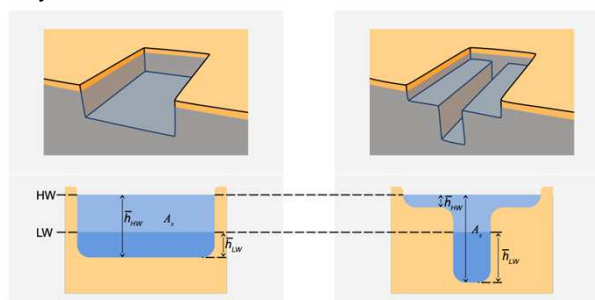
How to restore equilibrium? → Need sediment input



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## Question 12

Geometry of the basin:



$$\overline{h_{HW}} > \overline{h_{LW}}$$

Shorter rising period  
Shorter flood duration  
Stronger flood velocities



Import of sediments

$$\overline{h_{HW}} < \overline{h_{LW}}$$

Longer rising period  
Shorter ebb duration  
Stronger ebb velocities



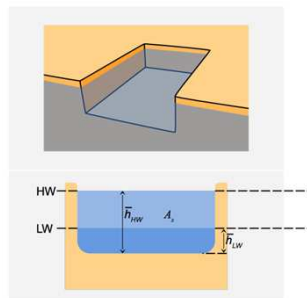
Export of sediments



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## Question 12

Geometry of the basin:



$$\overline{h_{HW}} > \overline{h_{LW}}$$

Shorter rising period  
Shorter flood duration  
Stronger flood velocities



Import of sediments

flood-dominant tidal asymmetry



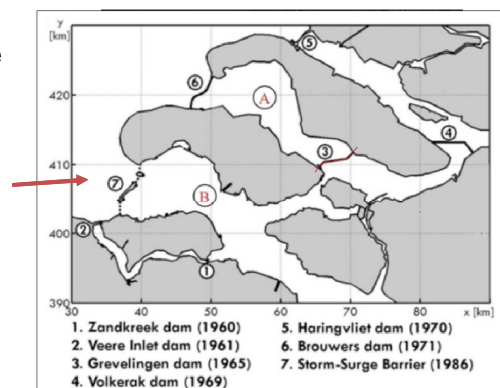
Equilibrium would be restored

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## Question 13

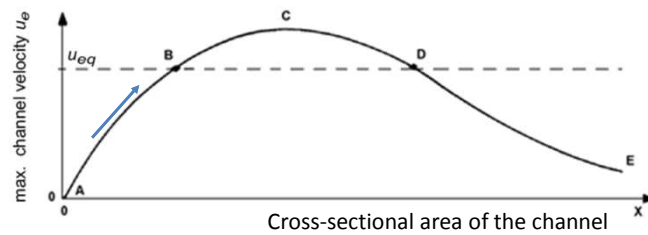
- The storm surge barrier **is not fully closed** under normal conditions.
- Limits the cross-sectional area of the tidal channel** at that location
- After the construction of the storm surge barrier, the **tidal range in the Oosterschelde basin decreased**.

➤ How can this decrease of the tidal range be explained?



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Escoffier's  
model for inlet  
stability



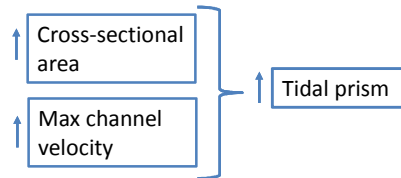
$$\hat{u}_e = \frac{\pi P}{A_e T}$$

Where

$\hat{u}_e$  is the entrance velocity

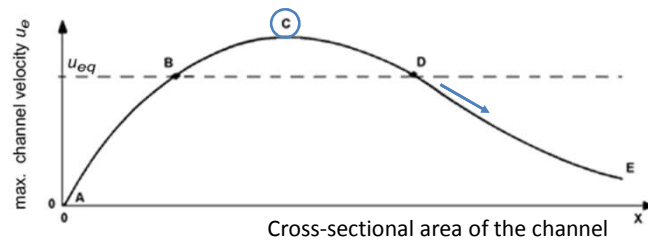
P is the tidal prism

$A_e$  is the entrance area



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Escoffier's  
model for inlet  
stability



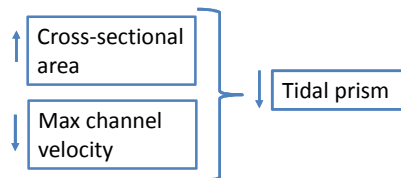
$$\hat{u}_e = \frac{\pi P}{A_e T}$$

Where

$\hat{u}_e$  is the entrance velocity

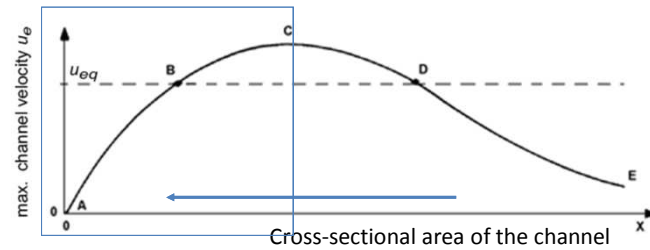
P is the tidal prism

$A_e$  is the entrance area



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Escoffier's  
model for inlet  
stability



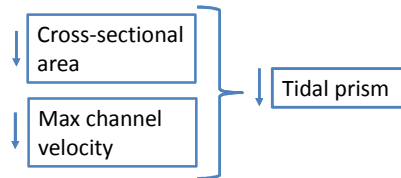
$$\hat{u}_e = \frac{\pi P}{A_e T}$$

Where

$\hat{u}_e$  is the entrance velocity

$P$  is the tidal prism

$A_e$  is the entrance area

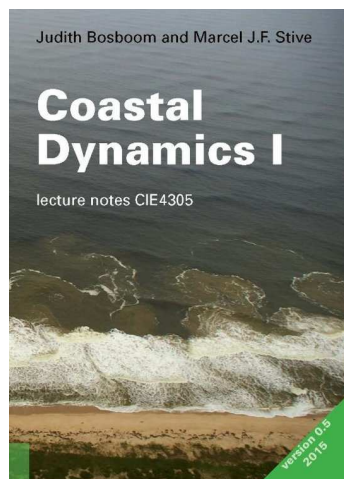


Apparently, the reduction of the cross-sectional was large enough to reduce the tidal prism and hence the tidal range



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## References & Further reading



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## ANY QUESTIONS?

## Question 14

- Assume that the tidal prism was reduced with 30% after building the storm surge barrier
  - $V_1$  is the outer delta volume after the construction of the storm surge barrier
  - $V_0$  is the volume before the construction.
  - What is a good estimate for the ratio  $V_1/V_0$ ?

Empirical relationship for ebb-tidal delta volume:

$$V_{od} = C_{od} * P^{1.23}$$

$$\frac{0.7^{1.23}}{1^{1.23}} = 0.645$$