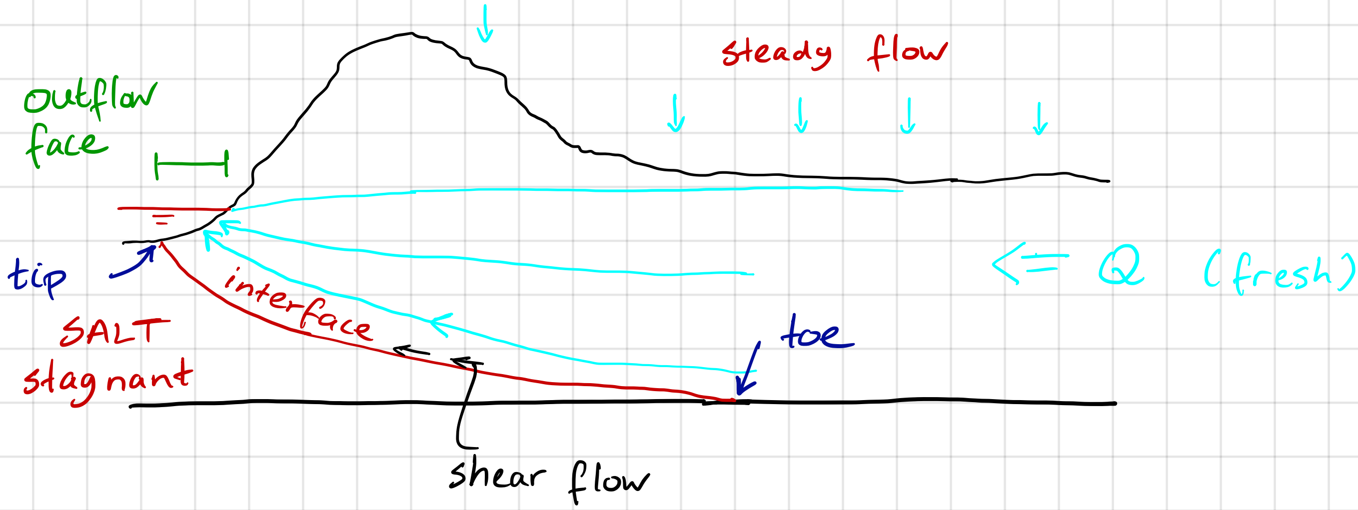
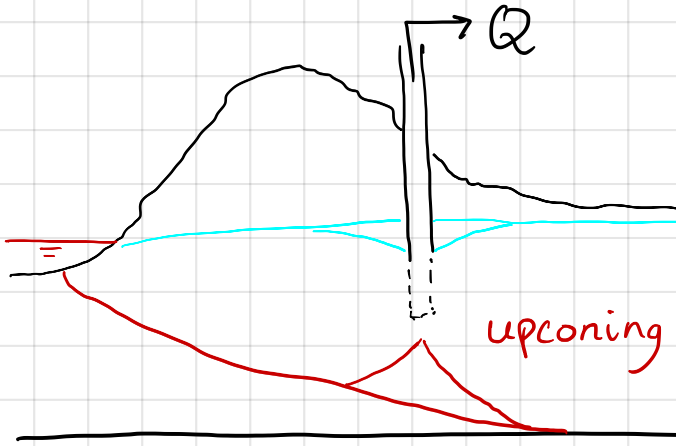
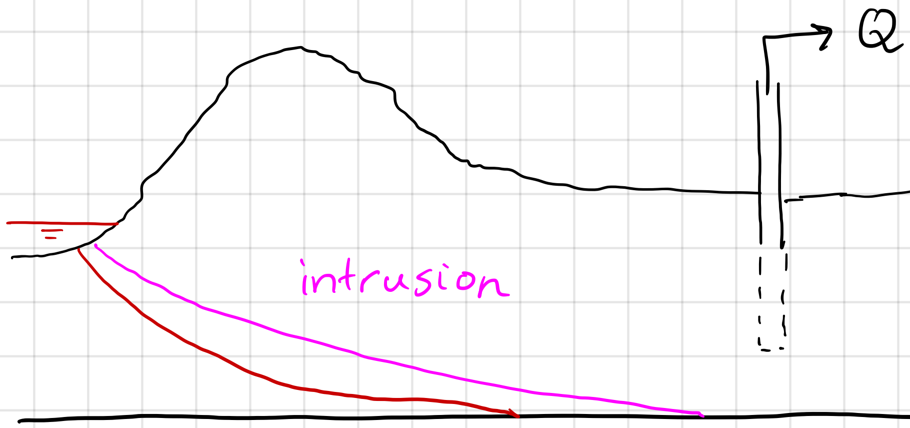
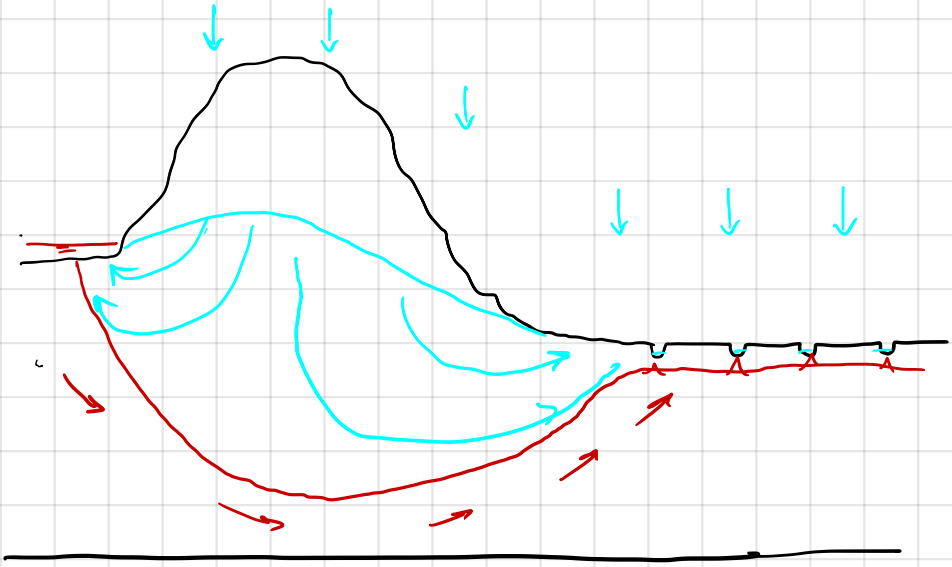
SALTWATER INTRUSION IN COASTAL AQUIFERS

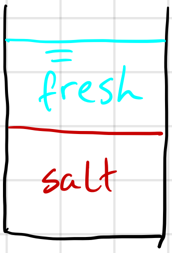


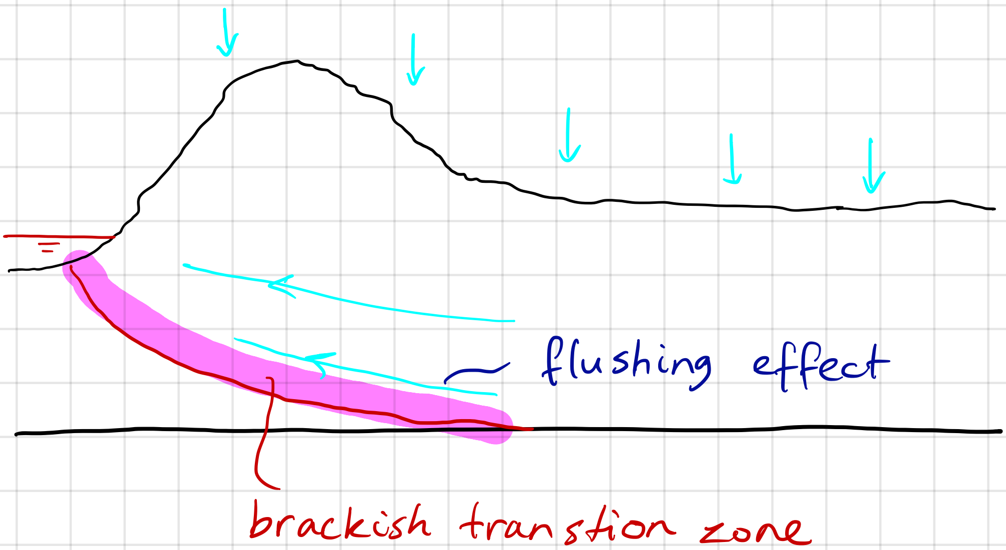


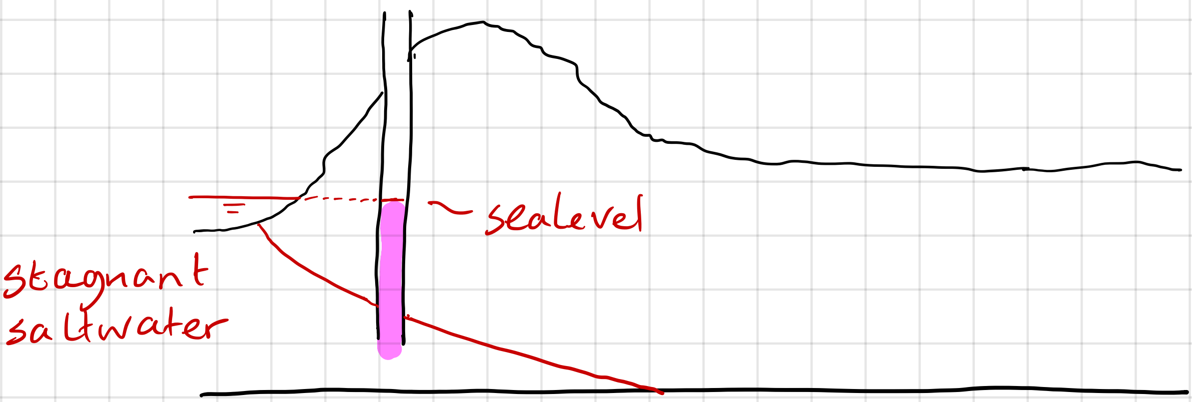
Managed Aquifer Recharge (MAR)






fresh
salt


flushing effect
brackish transtion zone

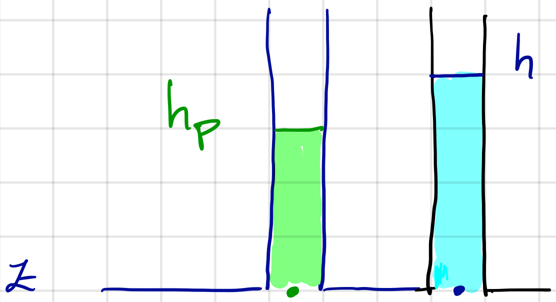

sealevel
stagnant
saltwater

freshwater head $$h = \frac{P}{\rho_f g} + z$$ (most useful)

saltwater head $$h_s = \frac{P}{\rho_s g} + z$$ ρ_s: saltwater density

pointwater head $$h_p = \frac{P}{\rho_p g} + z$$ (measured on the field)

ρ_p: density of water at filter of observation well



pressure: $P_p = P$

$$P_p = P$$

$$(h_p - z)\frac{\rho_p}{\rho_f}\cancel{g} = (h - z)\cancel{\rho_f}\cancel{g}$$

$$\boxed{h = z + \frac{\rho_p}{\rho_f}(h_p - z)}$$

h (no subscript) is freshwater head

Darcy's law in terms pressure

$$q_x = -\frac{k}{\mu}\frac{\partial P}{\partial x} \qquad q_y = -\frac{k}{\mu}\frac{\partial P}{\partial y}$$

$$q_z = -\frac{k}{\mu}\frac{\partial P}{\partial z} - \frac{k}{\mu}\rho_w g$$

ρ_w density of water.

Horizontal components

$$h = \frac{P}{\rho_f g} + z \qquad \frac{\partial h}{\partial x} = \frac{1}{\rho_f g}\frac{\partial P}{\partial x}$$

$$q_x = -\underbrace{\frac{k\rho_f g}{\mu}}\frac{\partial h}{\partial x} \qquad q_y = -\frac{k\rho_f g}{\mu}\frac{\partial h}{\partial y}$$

$$K = \frac{k\rho_f g}{\mu}$$ hydraulic conductivity

$$\boxed{q_x = -K\frac{\partial h}{\partial x} \qquad q_y = -K\frac{\partial h}{\partial y}}$$

h : freshwater head

freshwater : $\rho_f = 1000\ kg/m^3$

sea water : $\rho_s \sim 1025\ kg/m^3$

saturated salt solution (brine) : $\rho_{ss} \sim 1200\ kg/m^3$

h : freshwater head

$$q_x = -k \frac{\partial h}{\partial x} \qquad q_y = -k \frac{\partial h}{\partial y}$$

Problem $k = 20 \ m/d$

$x_1 = 0 \ m \quad z_1 = -40 \ m \quad h_{p1} = 0.5 \ m \quad \rho_1 = 1010 \ kg/m^3$

$x_2 = 100 \ m \quad z_2 = -40 \ m \quad h_{p2} = 0.4 \ m \quad \rho_2 = 1012 \ kg/m^3$

What is q_x ?

Wrong answer :

$$q_x \approx \frac{k(h_{p1} - h_{p2})}{\Delta x} = \frac{20(0.5 - 0.4)}{100} = 0.002 \ m/d$$

Correct answer :

$$h_1 = \frac{\rho_1}{\rho_f}(h_{p1} - z_1) + z_1 = 0.905 \ m$$

$$h_2 = \frac{\rho_2}{\rho_f}(h_{p2} - z_2) + z_2 = 0.885 \ m$$

$$q_x \approx k \frac{(h_1 - h_2)}{\Delta x} = 0.004 \ m/d$$

$$q_z = -\frac{k}{\mu}\frac{\partial P}{\partial z} - \frac{k}{\mu}\rho_w g$$

$$h = \frac{P}{\rho_f g} + z \qquad \frac{\partial h}{\partial z} = \frac{1}{\rho_f g}\frac{\partial P}{\partial z} + 1$$

$$\frac{\partial P}{\partial z} = \rho_f g \frac{\partial h}{\partial z} - \rho_f g$$

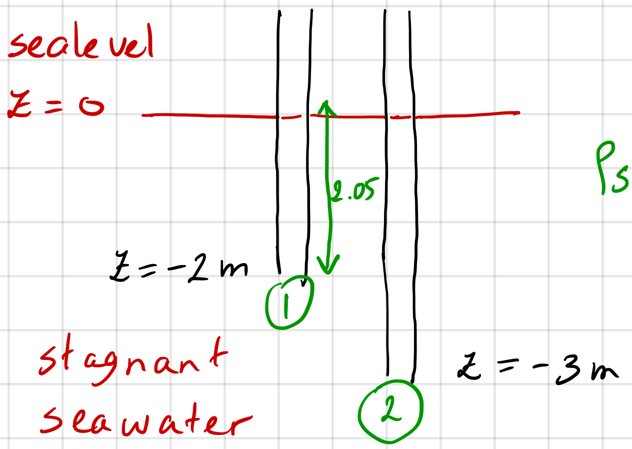
$$q_z = \underbrace{\frac{-k\rho_f g}{\mu}}_{k}\frac{\partial h}{\partial z} + \frac{k\rho_f g}{\mu} - \frac{k}{\mu}\rho_w g \frac{\rho_f}{\rho_f}$$

$$q_z = -k\frac{\partial h}{\partial z} + \frac{\rho_f}{\rho_f}k - \frac{\rho_w}{\rho_f}k$$

$$\boxed{q_z = -k\frac{\partial h}{\partial z} - \frac{(\rho_w - \rho_f)}{\rho_f}k}$$

$$\nu = \frac{\rho_w - \rho_f}{\rho_f}$$ dimensionless density difference

$$\boxed{q_z = -k\frac{\partial h}{\partial z} - \frac{(\rho_w - \rho_f)}{\rho_f} k}$$



$\rho_s = 1025 \frac{kg}{m^3}$

$$h = \frac{\rho_p}{\rho_f}(h_p - z) + z$$

$z = -2m \rightarrow \quad h = \frac{1025}{1000}(0 - -2) - 2 = 0.05 \text{ m}$

$z = -3m \rightarrow \quad h = \frac{1025}{1000}(0 - -3) - 3 = 0.075 \text{ m}$

$$q_z \simeq -k\frac{(h_1 - h_2)}{\Delta z} - k\frac{(\rho_s - \rho_f)}{\rho_f}$$

$$= -k\frac{(0.05 - 0.075)}{1} - k\frac{(1025 - 1000)}{1000}$$

$$= +k \cdot 0.025 - k \cdot 0.025 = 0$$

$$q_z = -k\frac{\partial h}{\partial z} - k\frac{(\rho_w - \rho_f)}{\rho_f}$$

$$q_z \simeq -k\frac{(h(z+\Delta z) - h(z))}{\Delta z} - k\frac{(\bar{\rho} - \rho_f)}{\rho_f}$$

Problem $\quad z = -20\ m \quad h_{p1} = 1\ m \quad \rho_1 = 1005\ kg/m^3$

$k = 10\ m/d \quad z = -25\ m \quad h_{p2} = 1.05\ m \quad \rho_2 = 1015\ kg/m^3$

What is q_z?

Wrong answer: $q_z = k\frac{(h_{p2} - h_{p1})}{\Delta z} = \frac{10(1.05-1)}{5} = 0.1\ \frac{m}{d}$

Correct Answer:

$$h_1 = \frac{\rho_1}{\rho_f}(h_{p1} - z_1) + z_1 = 1.105\ m$$

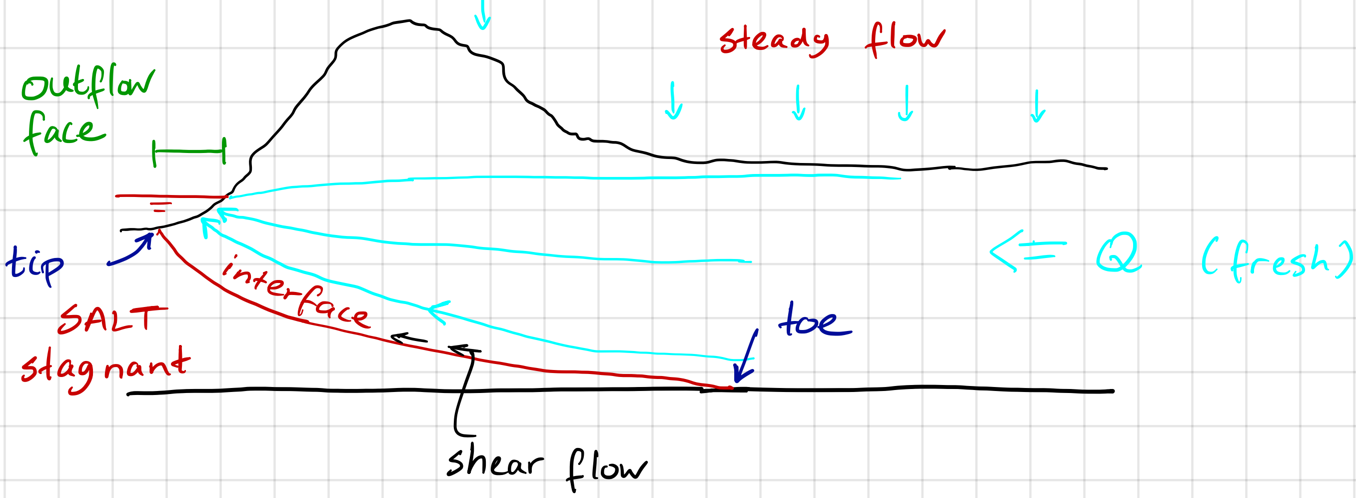
$$h_2 = \frac{\rho_2}{\rho_f}(h_{p2} - z_2) + z_2 = 1.44\ m$$

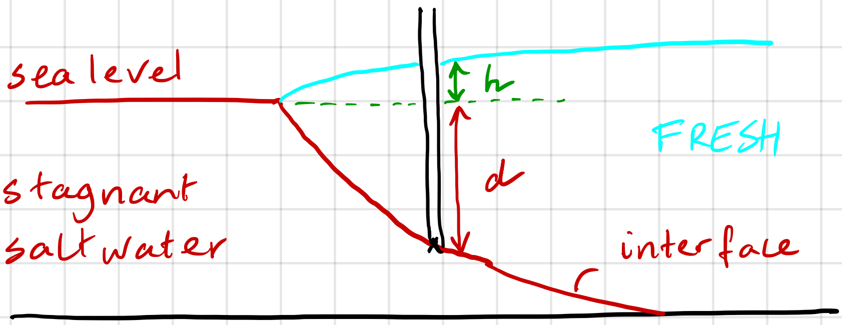
$$q_z \simeq \frac{k(h_2 - h_1)}{\Delta z} - \frac{k(\bar{\rho} - \rho_f)}{\rho_f}$$

$$= \frac{10(1.44 - 1.05)}{5} - 10 \cdot \frac{(1010 - 1000)}{1000}$$

$$= 0.57\ m/d$$

STEADY INTERFACE FLOW





At interface: $P_f = P_s$

$$(h+d)\rho_f \cancel{g} = d\rho_s \cancel{g}$$

$$\rho_f h = d(\rho_s - \rho_f)$$

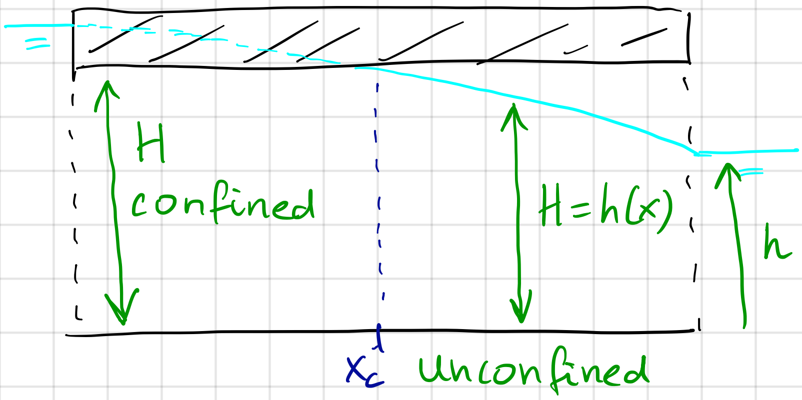
$$\boxed{d = \frac{\rho_f}{\rho_s - \rho_f} h}$$

Badon-Ghyben Herzberg equation

$$d = \alpha h \qquad \alpha = \frac{\rho_f}{\rho_s - \rho_f}$$

seawater $\rho_s = 1025 \frac{kg}{m^3}$

$$\alpha = \frac{1000}{1025 - 1000} = 40$$



confined: $\Phi = kHh - \frac{1}{2}kH^2$

$\Phi = Ax + B$

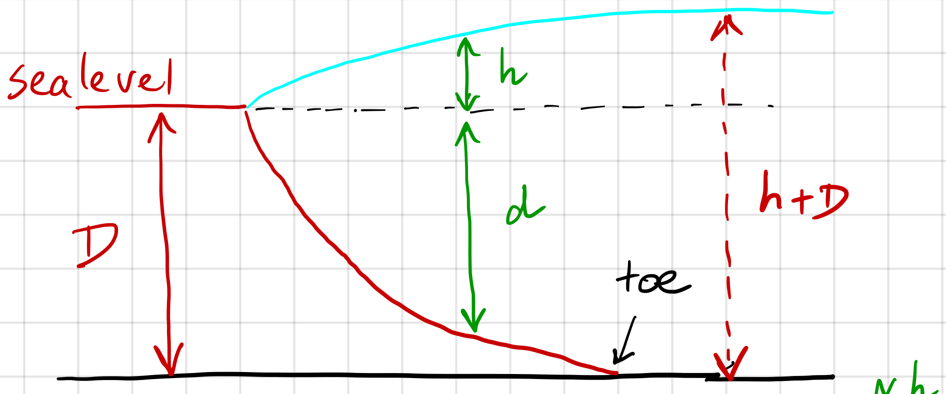
unconfined: $\Phi = \frac{1}{2}kh^2$

@ x_c $\longrightarrow$ $h = H$

$\Phi = kHH - \frac{1}{2}kH^2 = \frac{1}{2}kH^2$

$\Phi = \frac{1}{2}kH^2$

EQUAL



$h = \dfrac{d}{\alpha}$

at toe: $h_{toe} = \dfrac{D}{\alpha}$

Interface part: $Q_x = -k(h+d)\dfrac{dh}{dx}$

$= -k(h+\alpha h)\dfrac{dh}{dx}$

$= -k(1+\alpha)\, h \dfrac{dh}{dx} = -\dfrac{d\Phi}{dx}$

unconfined interface flow

$$\boxed{\Phi = \tfrac{1}{2}k(1+\alpha)h^2} \qquad Q_x = -\frac{d\Phi}{dx}$$

Regular unconfined flow:

$$Q_x = -k(h+D)\frac{d(h+D)}{dx} = -\frac{d\Phi}{dx}$$

$$\boxed{\Phi = \tfrac{1}{2}k(h+D)^2 + C_u}$$

at toe! $h = \dfrac{D}{\alpha}$

$$\Phi = \tfrac{1}{2}k(1+\alpha)\frac{D^2}{\alpha^2} = \tfrac{1}{2}k\left(\frac{D}{\alpha}+D\right)^2 + C_u$$

$$\Phi = \frac{1}{2} k (1+\alpha) \frac{D^2}{\alpha^2} = \frac{1}{2} k \left(\frac{D}{\alpha} + D \right)^2 + C_u$$

$$= \frac{1}{2} k \frac{D^2}{\cancel{\alpha^2}} \left(\frac{1}{\cancel{\alpha}} + \cancel{\alpha} \right)^2 + C_u$$

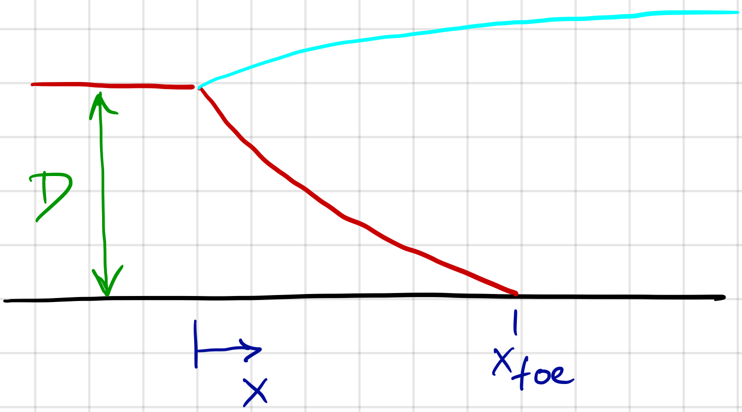
$$C_u = \frac{1}{2} k \frac{D^2}{\alpha^2} \left[1 + \alpha - (1+\alpha)^2 \right]$$

$$= \frac{1}{2} k \frac{D^2}{\alpha^2} \left[\cancel{1} + \alpha - (\cancel{1} + 2\alpha + \alpha^2) \right]$$

$$= \frac{1}{2} k \frac{D^2}{\alpha^2} \left[-\alpha^2 - \alpha \right]$$

$$= -\frac{1}{2} k D^2 \frac{(\alpha^2 + \alpha)}{\alpha^2}$$

$$\boxed{C_u = -\frac{1}{2} k D^2 \frac{(\alpha + 1)}{\alpha}}$$



$k = 10\ m/d$

$D = 20\ m$

$\rho_s = 1025\ kg/m^3$

$$\Phi = Q_o x + \Phi_o \longrightarrow Q_x = -\frac{d\Phi}{dx} = -Q_o$$

$$x = 0 \rightarrow h = 0 \longrightarrow \Phi = \Phi_o = 0$$

a) Where is x_{toe}? if $Q_o = 0.4\ m^2/d$

b) $h(x = 200) = 0.4\ m \longrightarrow$ What is Q_o?

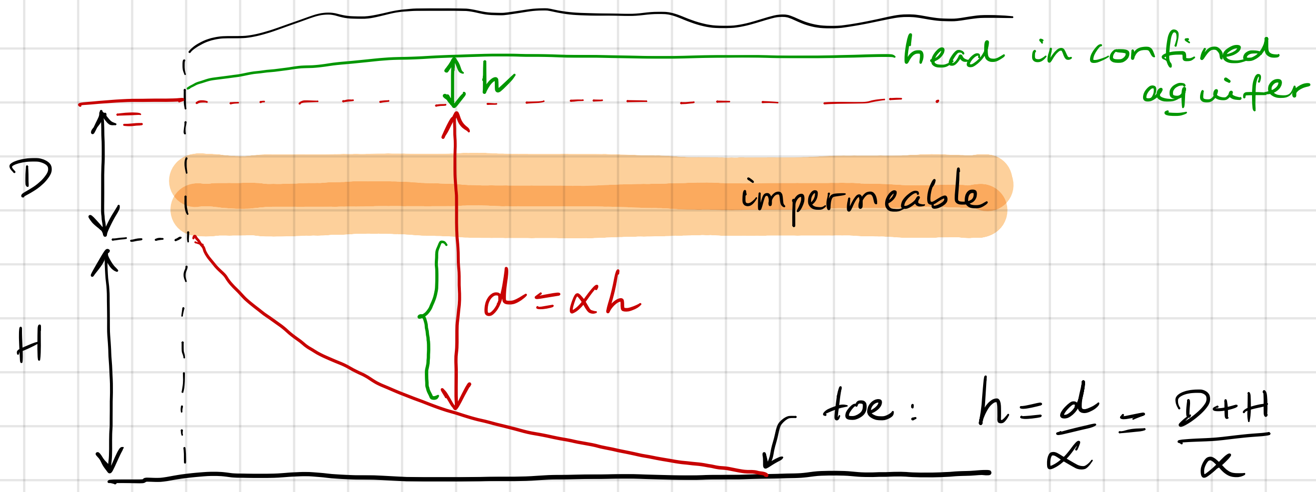
$$Q_o x_{toe} = \frac{1}{2} k (1+\alpha) \frac{D^2}{\alpha^2} \longrightarrow x_{toe} = 128\ m$$

b) @ toe $\longrightarrow h = 0.5 \longrightarrow d = \alpha h = 20$

$x = 200$ $\quad \Phi = \frac{1}{2} k (\alpha+1) \underset{0.4}{h^2} = Q_o \underset{200}{x}$

$$Q_o = \frac{\frac{1}{2} k (\alpha+1) h^2}{x} = 0.164\ \frac{m^2}{d}$$

STEADY CONFINED INTERFACE FLOW



Confined interface flow

$$Q_x = -k(\overset{\alpha h}{d} - D)\frac{dh}{dx}$$

$$= -k(\alpha h - D)\frac{dh}{dx}$$

$$= -k\alpha\left(h - \frac{D}{\alpha}\right)\frac{d\left(h - \frac{D}{\alpha}\right)}{dx}$$

$$= -\frac{d\Phi}{dx}$$

$$\boxed{\Phi = \frac{1}{2}k\alpha\left(h - \frac{D}{\alpha}\right)^2}$$

confined interface flow

Regular confined flow

$$Q_x = -\frac{d\Phi}{dx}$$

$$\boxed{\Phi = kHh + C_c}$$

@ toe : $h = \dfrac{D+H}{\alpha}$

$$\frac{1}{2}k\alpha\left(\frac{\cancel{D}+H}{\alpha} - \cancel{\frac{D}{\alpha}}\right)^2 = kH\frac{(D+H)}{\alpha} + C_c$$

$$\frac{1}{2}k\cancel{\alpha}\frac{H^2}{\alpha^{\cancel{2}}} = kH\frac{(D+H)}{\alpha} + C_c$$

$$C_c = \frac{1}{2}\frac{kH^2}{\alpha} - \frac{kH(D+H)}{\alpha}$$

$$C_c = \frac{\frac{1}{2}kH^2}{\alpha} - \frac{kH(D+H)}{\alpha}$$

$$= -\frac{\frac{1}{2}kH^2}{\alpha} - \frac{kHD}{\alpha}$$

$$\boxed{C_c = -\frac{k(\frac{1}{2}H^2 + DH)}{\alpha}}$$

$$\Phi = kHh + C_c$$

Problem 1D uniform flow towards coast

$$Q_0 = 0.4 \frac{m^2}{d}$$

$k = 10\, m/d$ $H = 20\,m$ $D = 8\,m$ $\rho_s = 1025 \frac{kg}{m^3}$

a) head and interface at $x = 100$ m.

depth $\downarrow$ d

$$\Phi = Q_0 x + \xcancel{\Phi_0} \nearrow 0$$

$x = 0 \longrightarrow h = \frac{D}{\alpha}$

$$\Phi = \frac{1}{2}k\alpha\left(h - \frac{D}{\alpha}\right)^2 = 0$$

$$\Phi(x = 100) = 0.4 \cdot 100 = 40 \frac{m^3}{d} < \Phi_{toe}\ !$$

$$\Phi_{toe} = \frac{1}{2}k\frac{H^2}{\alpha} = 50\ m^3/d$$

$$\Phi = \frac{1}{2}k\alpha\left(h - \frac{D}{\alpha}\right)^2$$

$$h = \frac{D}{\alpha} + \sqrt{\frac{2\Phi}{k\alpha}}$$

$h = 0.65$ m

$d = \alpha h = 25.9$ m