NATIONAL UNIVERSITY OF SINGAPORE

CE5311 – ENVIRONMENTAL MODELLING WITH COMPUTERS

(Semester I: AY2014/2015)

Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name**.
2. This assessment paper contains **FOUR** questions and comprises **FIVE** printed pages.
3. Answer **ALL** questions. All questions CARRY equal marks.
4. Please start each question on a new page.
5. This is an “OPEN BOOK” assessment.
6. ALL CLASS NOTES MAY BE BROUGHT IN.
7. Electronic calculator is permitted for this exam.

<u>Question 1</u> [25 marks]

The linear advection equation,

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} = 0$$

is defined on $x \subset [0, \infty)$ and $t \subset [0, \infty)$.

The initial condition is given as

$$\phi_{50}{}^{0} = 1$$

$$\phi_i{}^{0} = 0 \quad \text{for all } i \neq 5$$

ϕ is advected at a speed 2 m/s.

(a) Let $\Delta x = 0.5$ and $\Delta t = 0.2$, and check if the CFL condition is satisfied. (5 marks)

(b) Discretize with (5 marks)

i. Forward Euler in time and backward difference in space.

ii. Forward Euler in time and central difference in space.

(c) Plot ϕ at $t = \Delta t$ and $t = 2\Delta t$ for both (i) and (ii). (10 marks)

(d) Compare the differences between (i) and (ii). (5 marks)

<u>Question 2</u> [25 marks]

The diffusion equation is

$$\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2}$$

(a) Discretize with Crank-Nicholson in time and central difference in space (10 marks)

(b) Perform stability analysis for the discretization scheme in (a) (15 marks)

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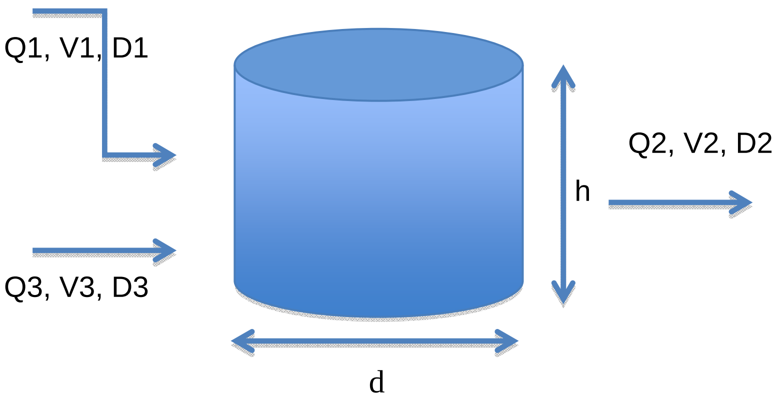
<u>Question 3</u> [25 marks]

A reservoir has the inputs and output as shown in Figure 1



a) Formulate an ordinary differential equation that describes the water level, *h*, as a function of time. (5 marks)

b) If D1=60 cm, D2 = 75 cm, Q3 = 10 liters/ s, determine V2 if V1 = 0.12 m/s and the water level is constant. (10 marks)

c) Now assume that Q3 = 0; Q1 = Q2 = 0.05 m^3/s, d = 160 m, h = 10m. In addition assume that initially the water in the reservoir and the outgoing and incoming water is clean. But at t= t_0, the inflow begins to contain a pollutant with a concentration of 35 mg/l, estimate the time (in days) it will take for the mean concentration in the reservoir to increase to 15 mg/l (assuming complete mixing).

(10 marks)

<u>Question 4</u> [25 marks]

There are some basic issues in numerical modeling of flows.

a. Boundary conditions are a basic issue

   i. What is the purpose of open boundaries for a numerical flow model? (4 marks)

   ii. Given that they are artificial in most cases, give three criteria for good open boundary conditions? (6 marks)

   iii. Where can one get values to specify at open boundaries? (5 marks)

b. Related to boundary conditions and numerical modeling is the issue of spin-up.

For a simple 1D harbor channel with the setup below:

boundary condition: $\zeta(0,t) = A\cos\left(\frac{2\pi}{T}t\right)$

boundary condition: $u(L,t) = 0$

where h = 20.0m and the friction is 1/2500 (1/s), L = 4000m and T = 40,000s.

   i. What is spin-up? (5 marks)

   ii. What is the time-scale for total spin-up time? (5 marks)

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