NATIONAL UNIVERSITY OF SINGAPORE

CE5311 ENVIRONMENTAL MODELLING WITH COMPUTERS

(Semester 1: AY2015/2016)

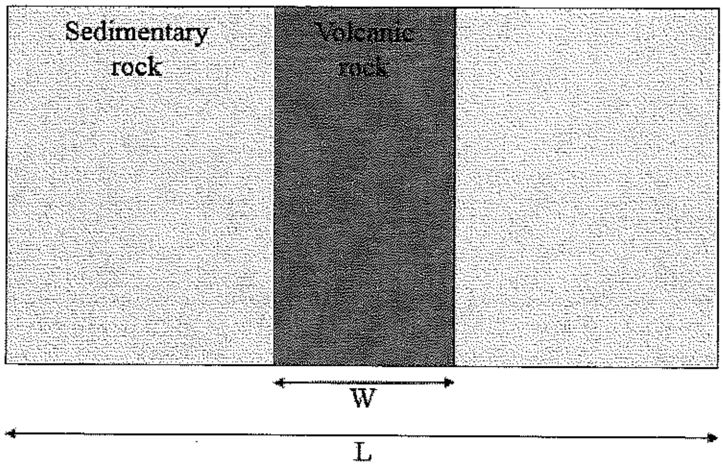
Time Allowed: 2.5 Hours

INSTRUCTIONS TO CANDIDATES

1. Please write your student number only. **Do not write your name.**
2. This assessment paper contains **FOUR** questions and comprises **FIVE** printed pages.
3. Answer **ALL** questions. All questions carry equal marks.
4. Please start each question on a new page.
5. This is an "OPEN BOOK" assessment.
6. **ALL LECTURE NOTES AND TUTORIAL SOLUTIONS ARE ALLOWED**
7. **ELECTRONIC CALCULATORS ARE ALLOWED**

Question 1 [25 marks]

A hot volcanic rock intrudes between cooler sedimentary rocks. The setup of the thermal cooling model is shown below. A group of scientists are interested to find out how long it takes to cool the volcanic rock.



The temperature evolution versus time $T(x,t)$ is described with,

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

where κ is the thermal diffusivity.

The sedimentary rock has a temperature of 300 °C and the volcanic rock has a temperature of 1200 °C. The temperature at the boundaries of the sedimentary rock remains constant. The width of the volcanic rock is 5 m.

Assume variations in the x-direction only, and that properties in the other directions are constant.

(a) What are the initial and boundary conditions for this problem?

[10 marks]

(b) Discretize the PDE with (i) Forward in Time and Center in Space (FTCS) and (ii) Backward in Time and Center in Space (BTCS).

[10 marks]

(c) Identify the scheme in (b) that is explicit/implicit, and explain which scheme is preferred.

[5 marks]

<u>Question 2</u> [25 marks]

(a) The one-dimensional heat equation is

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

where κ is the thermal diffusivity. Discretize with Forward in Time, Center in Space (FTCS) scheme, and perform stability analysis using Hirt's method or von Neumann analysis.

[10 marks]

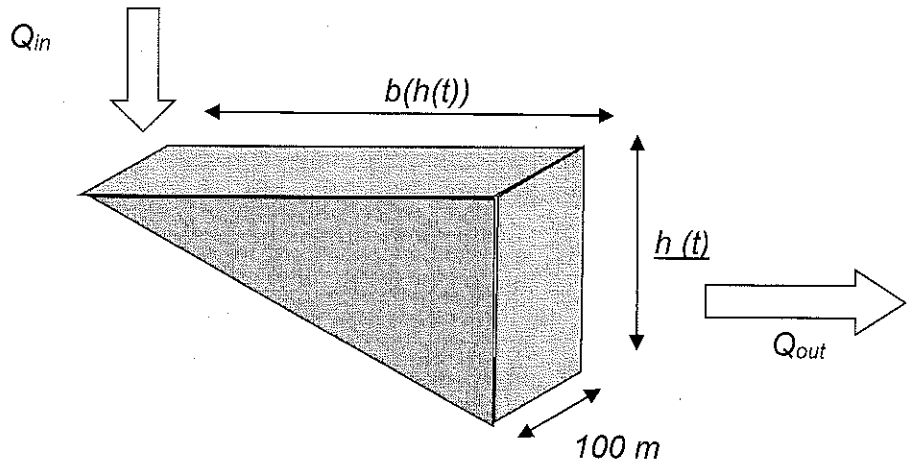
(b) The two-dimensional heat equation is

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} + \kappa \frac{\partial^2 T}{\partial y^2}$$

Similarly, discretize with Forward in Time, Center in Space (FTCS) scheme, and perform stability analysis using Hirt's method or von Neumann analysis.

[15 marks]

<u>Question 3</u> [25 marks]



Consider the triangular wedge shaped reservoir above.
You are given $h(0) = 5$ m, $b(h(0)) = 6\ m$, $V(h(t)) = (A(h(t)) \times 100)$m3, $Q_{in} = 0.991\ m^3/s$, $Q_{out}\ (h(t)) = 0.1\sqrt{2gh}$.
$h(t)$ is the water level with respect to the bottom of the reservoir, $V(h(t))$ is the volume of water in the reservoir, $A\ (h(t))$ is the triangular cross section of the reservoir and $Q_{in,out}$ are the incoming and outgoing discharges.

The reservoir contains a pollutant with concentration of $c(t)$.

a) Formulate the ordinary differential equation that describes the water level in the reservoir as a function of time including all the necessary variables above.

[6 marks]

b) What is the steady state solution for the equation in part (a) to 1 decimal place?

[3 marks]

c) Now consider a pollutant that will enter this reservoir. Formulate an ordinary differential equation that will describe the concentration of the pollutant in the reservoir as a function of time, $c(t)$.

[6 marks]

d) How many kg of the pollutant is there in the reservoir initially if the initial concentration at t = 0 is 0.2 kg /m^3. What will happen to this mass of pollutants in time

[5 marks]

e) Now consider what will happen to the pollutants in the reservoir if modifications are made upstream so that the incoming discharge has no pollutants. How long will it take for the pollutants in the reservoir to flush out?

[5 marks]

Question 4 **[25 marks]**

To model environmental flows, differential equations are typically used.

a) To ensure only a single solution to these time-dependent differential equations, what does one need to have to limit the solution in the physical and temporal space?

[4 marks]

Environmental flows have certain concepts that are important when modelling.

b) One of them is dispersion. What is dispersion in a transport equation?

[4 marks]

c) Why is the magnitude of the dispersion coefficient in 1D transport equation normally larger than the magnitude of the dispersion coefficient in a 3D transport equation?

[4 marks]

The equation below is a form of the transport equation:

$$\frac{\partial C}{\partial t} - \frac{\partial}{\partial x}\left(K(x)\frac{\partial C}{\partial x} \right) = 0$$

d) What type of transport is described by the simplified form of the transport equation above if K is a space varying, positive coefficient?

[3 marks]

e) If the equation above is used for transport in a closed basin (x=0,L), give the boundary conditions at the two ends x=0, and x=L

[4 marks]

The equation below is a 2D transport equation, where x is the longitudinal coordinate and z is the vertical coordinate.

$$\frac{\partial C}{\partial t} - \left[\frac{\partial}{\partial x}\left(D(x)\frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial z}\left(D(z)\frac{\partial C}{\partial z} \right) \right] = 0$$

f) If $D(x)$ and $D(z)$ are positive coefficients that vary in the longitudinal and vertical directions respectively, which value is typically larger in practice?

[3 marks]

g) These equations can then be discretized to be solved numerically. One of the issues that arises when carrying out a numerical solution is spin-up. What is spin-up in your own words?

[3 marks]

- END OF PAPER -