Numerical Methods in Mechanics and <u>Environmental Flows</u>

NOV 17, 2017

STUDENTS…

Outline for Environmental Flows

Oct 13
- Introduction
- Delft3D Introduction and Assignment 1: Spin-up?

Oct 20
- Box models and solution methods
- Delft3D Assignment 2 - Boundary conditions; initial conditions

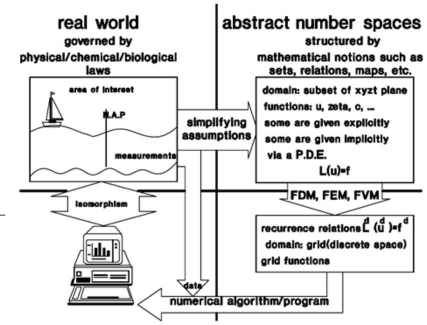
Oct 27
- Solution methods: vertical layers / transport processes
- Delft3D Assignment 3 – stratification (wind-driven flows)

Nov 3
- Transport processes in flows (2)
- Delft3d Project

Nov 10
- Transport processes in flows (3) – Practical issues
- Term project
- Delft3d assignment 4 – model extents (estuarine stratification as an example)

Nov 17
- Presentation of term assignment (4 groups)
- Revision / past exam questions

What have we covered in Part 2?



Started with a broad look into what are environmental flows as a concept and what do they add

Introduced box models and the use of a mass-spring system equivalent

Took a detailed look at approximations applied to environmental flow models

Looked at issues related to, components required for and concepts related to numerical solutions of environmental flows

What have we covered in Part 2?

Then we moved to flow transport processes, introduced the concept of diffusion and its importance

- Scales
- Noted the issues that grids can cause to diffusion

Introduced the concept of Reynolds Decomposition

Quick introduction to turbulence and basic turbulence models

Physical process of stratification and basic theory to estimate mixing possibility

$$Ri = \frac{\alpha g h \Delta T}{\Delta U^2}$$

Looked at estuarine mixing concepts and what causes mixing in estuaries

Looked at lake mixing and what causes mixing in lakes

And

Throughout that process you have been using an environmental flow modeling software to obtain solutions that tie-in those ideas

- Assignment 1: We looked at initial conditions, the concept of spin-up and the use of a mass-spring system to explain what you saw
- Assignment 2: Imposing correct boundary conditions but obtaining a wrong solution?

Then we moved to looking at mixing processes:

- Assignment 3: First, what happens when your physical assumptions are violated? Then how do we deal with stratification in a lake?
- Assignment 4: Looking at estuarine mixing processes and exploring the issue of appropriate model domain size

Now **YOU** get to put that all together and tell us what impact can occur because of adding or subtracting things in the environment

Group and Term Project

Assignment	Group	Presentation Sequence	Group Members
1	1	2	Cherie, Li Zhi, Wen Hao, Pui Yee, Yanan
2	4	3	Henry, Zhengping, Jiaqin, Zhuohong, Zhen Ni
3	3	1	Ruifeng, Tianyao, Xiaoqing, Xiaoxiao, Zhang Yi
4	2	4	Yingxuan, Dennis, Lee Lian, Jun Kai, Xiao Chen, Ying Yan

Reservoir with pollutant

Reservoir with following conditions:

- $V = 2x10^5 m^3$; $Q_{u/s} = 9x10^4 m^3/yr$; $Q_{evap} = 1x10^4 m^3/yr$; Assume steady state; upstream c = 6 mg/l; c decays at K = 0.12/year

Find c

- What is budget?
- What is then c?

What if now upstream c = 0 due to changes in management?

- What is budget?
- How long does it take to drop to 50%
- How long does it take to reach 0.1 mg/l?
- What is the main cause of the improvement in c?

Past year questions:

2016 Q3;

Question 3 [25 marks]

A reservoir with a volume of 7.5 x 10^6 m^3 has inflows that come from a canal, the only outflow from the reservoir is to water treatment plants. Evaporation occurs over the surface of the reservoir.

(a) Derive the time varying storage budget for the reservoir assuming steady flow. Then simplify it assuming everything is steady-state

[4 marks]

(b) Use the steady-state equation derived in (a) to determine the evaporation rate (cm/year) if reservoir surface area is 1.5 x 10^6 m^2; canal supplies 2.0 x 10^6m m^3/year and outflow to water treatment plants is 9.5 x10^5 m^3/year.

[6 marks]

The water in the canal brings in a pollutant.

(c) Derive the time varying budget for the pollutant in term of its concentration, c, in the water. Then simplify the budget assuming that it is steady-state.

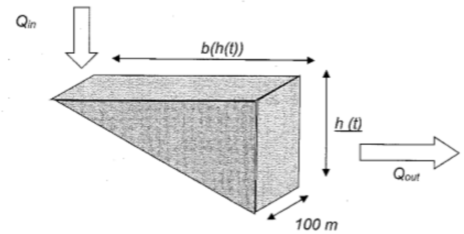
[4 marks]

(d) If the concentration of the pollutant as it exits to the water treatment plants is 8 g/m^3 and the concentration of the pollutant as it enters the canal is 5 g/m^3. Using the equation from (e), and appropriate simplifications, determine if the reservoir is a source or a sink and the amount contributed to (if sink) or comes from the reservoir (if source) in kg/day (assuming 365 days in a year)

[11 marks]

2015 Q3;

Question 3 [25 marks]



Consider the triangular wedge shaped reservoir above.
You are given $h(0) = 5$ m, $b(h(0)) = 6\ m$, $V(h(t)) = (A(h(t)) \times 100)$m3, $Q_{in} = 0.991$ m^3/s, $Q_{out}\ (h(t)) = 0.1\sqrt{2gh}$.
$h(t)$ is the water level with respect to the bottom of the reservoir, $V(h(t))$ is the volume of water in the reservoir, $A\ (h(t))$ is the triangular cross section of the reservoir and $Q_{in,out}$ are the incoming and outgoing discharges.

The reservoir contains a pollutant with concentration of $c(t)$.

a) Formulate the ordinary differential equation that describes the water level in the reservoir as a function of time including all the necessary variables above.

[6 marks]

b) What is the steady state solution for the equation in part (a) to 1 decimal place?

[3 marks]

c) Now consider a pollutant that will enter this reservoir. Formulate an ordinary differential equation that will describe the concentration of the pollutant in the reservoir as a function of time, $c(t)$.

[6 marks]

d) How many kg of the pollutant is there in the reservoir initially if the initial concentration at t = 0 is 0.2 kg /m^3. What will happen to this mass of pollutants in time

[5 marks]

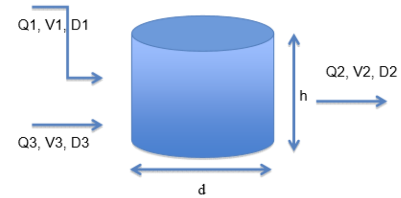
e) Now consider what will happen to the pollutants in the reservoir if modifications are made upstream so that the incoming discharge has no pollutants. How long will it take for the pollutants in the reservoir to flush out?

[5 marks]

2014 Q3:

Question 3 [25 marks]

A reservoir has the inputs and output as shown in Figure 1



a) Formulate an ordinary differential equation that describes the water level, h, as a function of time. (5 marks)

b) If D1=60 cm, D2 = 75 cm, Q3 = 10 liters/ s, determine V2 if V1 = 0.12 m/s and the water level is constant. (10 marks)

c) Now assume that Q3 = 0; Q1 = Q2 = 0.05 m^3/s, d = 160 m, h = 10m. In addition assume that initially the water in the reservoir and the outgoing and incoming water is clean. But at t= t_0, the inflow begins to contain a pollutant with a concentration of 35 mg/l, estimate the time (in days) it will take for the mean concentration in the reservoir to increase to 15 mg/l (assuming complete mixing).

(10 marks)

Similar yet…

The rest are conceptual questions

Question 4 [25 marks]

There are some basic issues in numerical modeling of flows.

a. Boundary conditions are a basic issue

i. What is the purpose of open boundaries for a numerical flow model? (4 marks)

ii. Given that they are artificial in most cases, give three criteria for good open boundary conditions? (6 marks)

iii. Where can one get values to specify at open boundaries? (5 marks)

b. Related to boundary conditions and numerical modeling is the issue of spin-up.

For a simple 1D harbor channel with the setup below:

boundary condition: $\zeta(0,t) = A\cos\left(\frac{2\pi}{T}t\right)$ **x=0** ———— **x=L** **boundary condition:** $u(L,t) = 0$

where h = 20.0m and the friction is 1/2500 (1/s), L = 4000m and T = 40,000s.

i. What is spin-up? (5 marks)

ii. What is the time-scale for total spin-up time? (5 marks)

Question 4 [25 marks]

To model environmental flows, differential equations are typically used.

a) To ensure only a single solution to these time-dependent differential equations, what does one need to have to limit the solution in the physical and temporal space? [4 marks]

Environmental flows have certain concepts that are important when modelling.

b) One of them is dispersion. What is dispersion in a transport equation? [4 marks]

c) Why is the magnitude of the dispersion coefficient in 1D transport equation normally larger than the magnitude of the dispersion coefficient in a 3D transport equation? [4 marks]

The equation below is a form of the transport equation:

$$\frac{\partial C}{\partial t} - \frac{\partial}{\partial x}\left(K(x)\frac{\partial C}{\partial x}\right) = 0$$

d) What type of transport is described by the simplified form of the transport equation above if K is a space varying, positive coefficient? [3 marks]

e) If the equation above is used for transport in a closed basin (x=0,L), give the boundary conditions at the two ends x=0, and x=L [4 marks]

The equation below is a 2D transport equation, where x is the longitudinal coordinate and z is the vertical coordinate.

$$\frac{\partial C}{\partial t} - \left[\frac{\partial}{\partial x}\left(D(x)\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial z}\left(D(z)\frac{\partial C}{\partial z}\right)\right] = 0$$

f) If $D(x)$ and $D(z)$ are positive coefficients that vary in the longitudinal and vertical directions respectively, which value is typically larger in practice? [3 marks]

g) These equations can then be discretized to be solved numerically. One of the issues that arises when carrying out a numerical solution is spin-up. What is spin-up in your own words? [3 marks]

Question 4 [25 marks]

You are asked to assess the potential impact of pollutants after a spill in a canal. The width of the canal is 12 m, the depth is assumed to be uniform at 3.0 m. It is assumed that 100 L of a pollutant is accidentally spilled uniformly across the canal and that the pollutant has a relative density of 0.9 kg/L.

For part (a) to (d) you assume that the canal waters are still and that the diffusion coefficient is uniform, D = 2.0 m^2/s.

Thus use the solution to the 1D diffusion equation; $c(x,t) = \frac{M}{\sqrt{4\pi Dt}}\exp\left(-\frac{(x)^2}{4Dt}\right)$

(a) Determine the concentration of the pollutant at x = 0 at T = 2 hours, 3 hours, 4 hours and 8 hours. [6 marks]

(b) Determine the concentration of the pollutant at x = 200m at T= 2 hours, 3 hours, 4 hours and 8 hours. [4 marks]

(c) Show that the time t_{max} for the maximum concentration at any distance x is given by the relationship $t_{max} = \frac{(x)^2}{2D}$ and the corresponding maximum concentration $c_{max} = 0.2420\frac{M}{x}$ [4 marks]

(d) Find t_{max} and corresponding c_{max} at x = 200 m and at x = 300 m. [2 marks]

(e) If your initial assumption was wrong and the water in the canal was not still but flowing (u = 0.016 m/s), determine the concentration of the pollutant at x = 200 m at T = 1 hours, 2 hours, 3 hours and 8 hours using the solution, $c(x,t) = \frac{M}{\sqrt{4\pi Dt}}\exp\left(-\frac{(x-ut)^2}{4Dt}\right)$ [4 marks]

(f) Comparing your findings in part (b) and (e)

i) What is the impact of flow (advection) in the canal in terms of magnitude and timing of maximum concentration of pollution at x = 200 m? [2 marks]

ii) What non-dimensional number can be used to estimate if diffusion or the advection dominates? [2 marks]

iii) Calculate the non-dimensional number using appropriate scales at x = 200 m and state which, if any of the following transport processes dominate (advection, diffusion, neither). [1 mark]