CE5312 Project

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Negative Disturbance

Questions (a) & (b): To determine the water depth and velocity downstream at t=1.5hr and t=4hr by numerical method, we first discretised x-t plane into rectangular grids. We tried different values of Δx, which were 100m, 500m and 1km. To satisfy the criterion of stability, i.e.,$\Delta t \leq 0.9(U+c)_{max}$, the corresponding Δt were 8s, 40s and 60s respectively. A distance of 180km downstream was studied (this should be long enough to cover the disturbance based on simple calculations). At t=0, we had $U=U_0$=1m/s, $h=h_0$=10m and $c=c_0=\sqrt{gh_0}$=9.9m/s for every point along x. Next, we would take Δx=100m and Δt=8s as an example to roughly elaborate the simulations. The other two cases were with similar setting up.

i) At t=8s, we first found the reference points at the earlier time level, i.e. t=0 based on the two functions: $x_L=x_0-(U_0+c_0)\Delta t$ for the upstream reference point, and $x_R=x_0-(U_0-c_0)\Delta t$ for the downstream reference point.

ii) Flow conditions for each reference point were obtained by the following functions:

$$U_L = \frac{x_L - x_o}{x_W - x_o}(U_W - U_o) + U_o, \quad c_L = \frac{x_L - x_o}{x_W - x_o}(c_W - c_o) + c_o$$

$$U_R = \frac{x_R - x_o}{x_E - x_o}(U_E - U_o) + U_o, \quad c_R = \frac{x_R - x_o}{x_E - x_o}(c_E - c_o) + c_o$$

$$h = c^2/g$$

iii) Flow conditions for each point at t=8s were obtained based on the flow conditions at their reference points. Functions used are as below.

$$U_p = \frac{U_R + U_L}{2} + (c_L - c_R) + \Delta t g(S_0 - S_{f,O})$$

$$c_p = \frac{U_L - U_R}{4} + \frac{c_L + c_R}{2}$$

$$S_f = n^2 U^2 / h^{4/3}$$

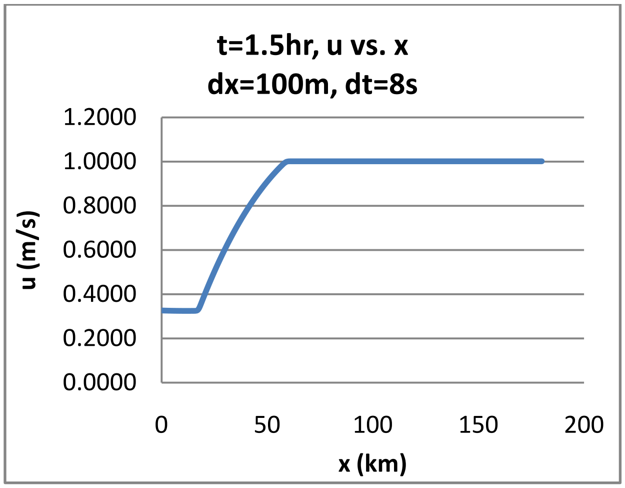
iv) For boundary nodes (first point), the following functions were used.

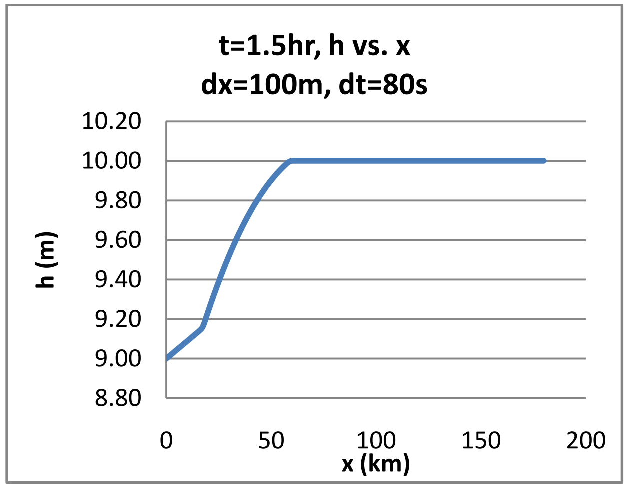
$$U_R = \frac{x_R - x_o}{x_E - x_o}(U_E - U_o) + U_o, \quad c_R = \frac{x_R - x_o}{x_E - x_o}(c_E - c_o) + c_o$$

$$U_P - 2c_P = (U_R - 2c_R) + \Delta t g(S_0 - S_{f,O})$$

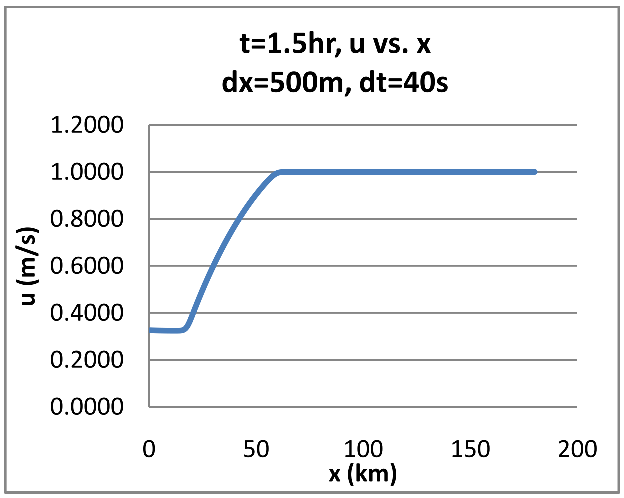
Repeat step (i) to (iv) to obtain the flow conditions for each point at each time step. After the simulation, data were collected and the relative graphs were drawn and shown as below.

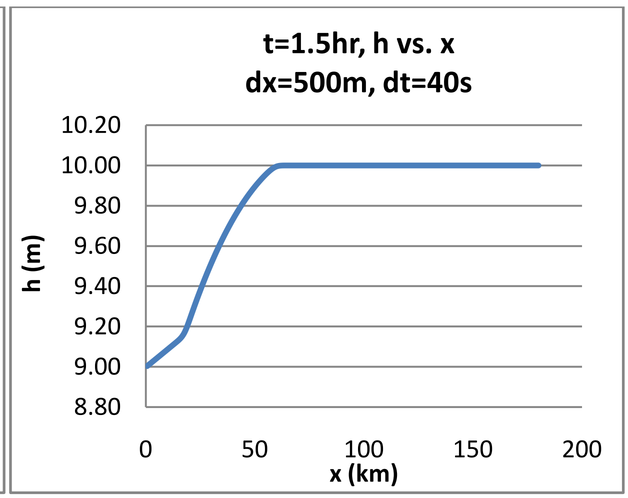
t=1.5 hr, Δx=100m, Δt=8s



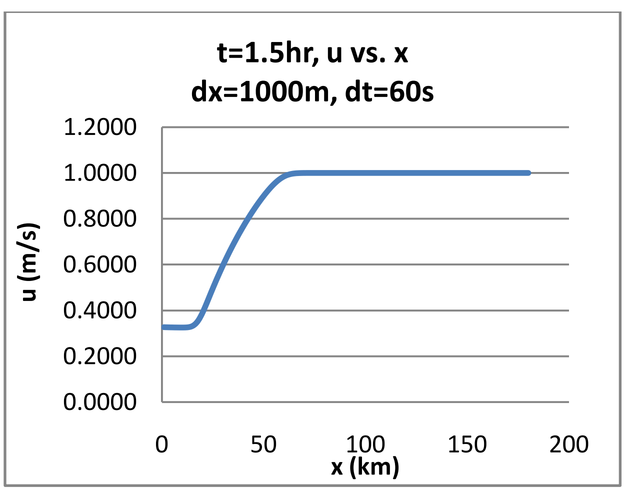


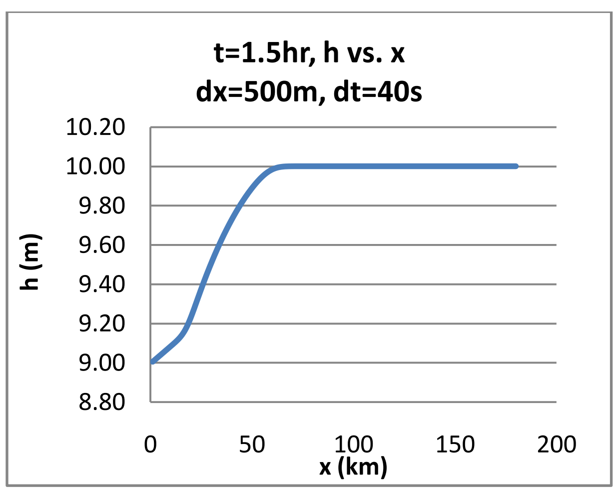
t=1.5 hr, Δx=500m, Δt=40s





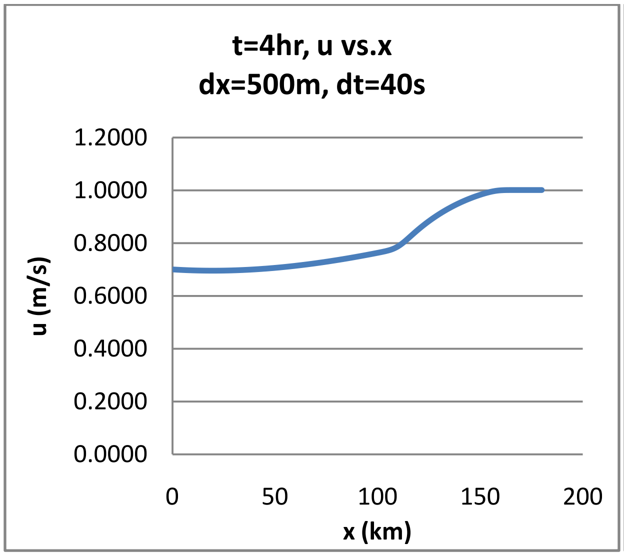
t=1.5 hr, Δx=1000m, Δt=60s

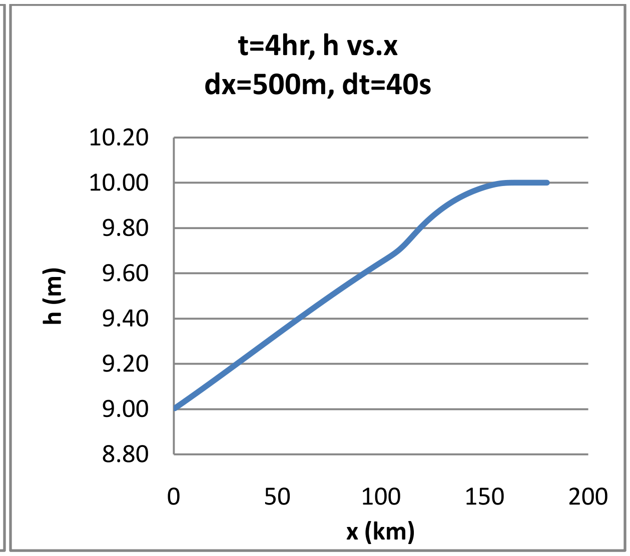




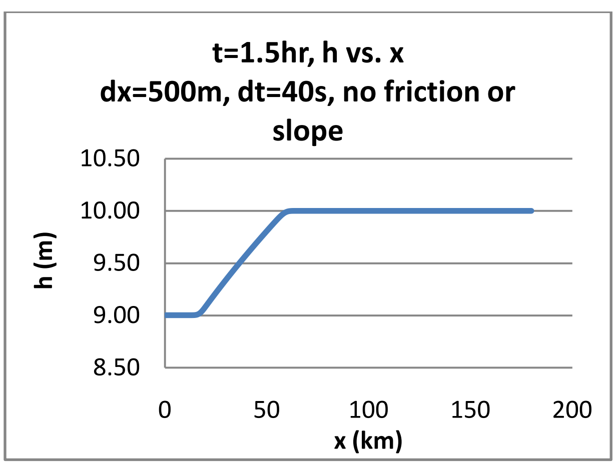
From the above results, the influence of different choices of Δx and Δt seems to be not significant. That may be because all sets of Δx and Δt were complied with the stability criterion. We also tried one more set of data, Δx=100m and Δt=40s, which was not complied with the stability criterion. The result started to turn unreasonable at the 7th iteration (t=280s), when the depth (h) hit $8.55*10^5$ metres at x=300m. Therefore, we may conclude that, as long as the stability criterion for Δx and Δt was met, the simulation would give reasonable results. For the rest of simulations, Δx and Δt were all set to be 500m and 40s.

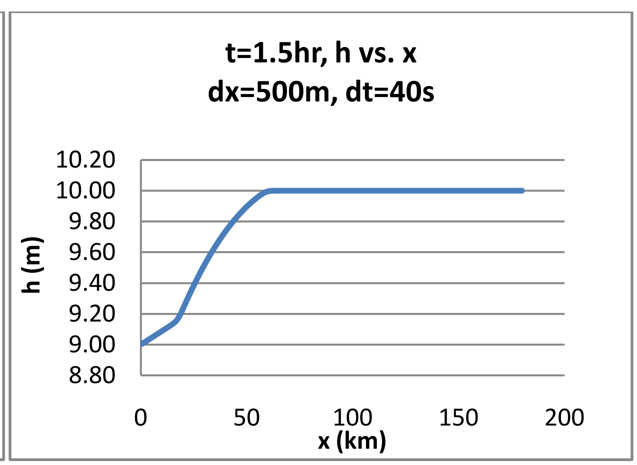
Same method was used to predict the water depth and velocity at t=4hr.



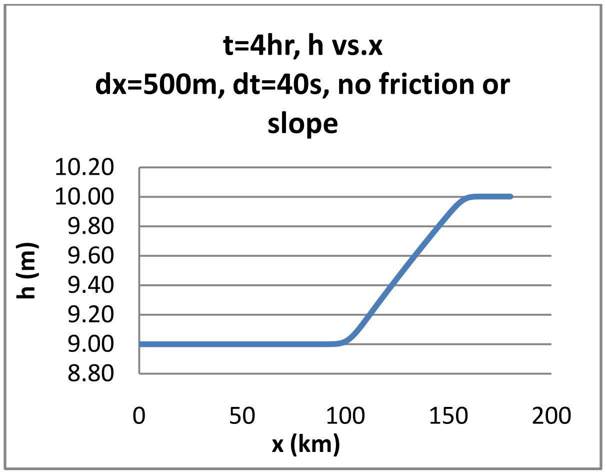


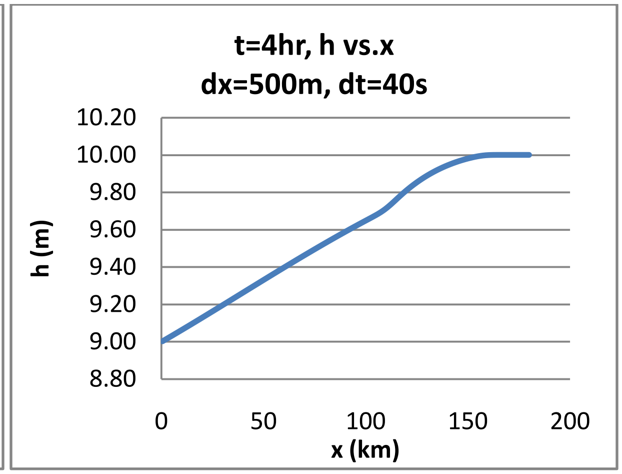
Question (c): Bottom slope and bottom frictions were neglected in the numerical prediction. When t=1.5hr, by comparing the two graphs below, we find that the results for the locations closer to the source of disturbance (within 50km distance) are quite different. For the area with no more influence caused by the source of disturbance, h remains flat without friction or slope, whereas h increases at the other graph. This difference implies that, the slope and friction definitely have impact on the depth of flow. However, the location where the flow has not yet been disturbed (i.e., h is 10m) is roughly the same. This shows that the propagating velocity at the leading edge may not have explicit relation with slope and friction.



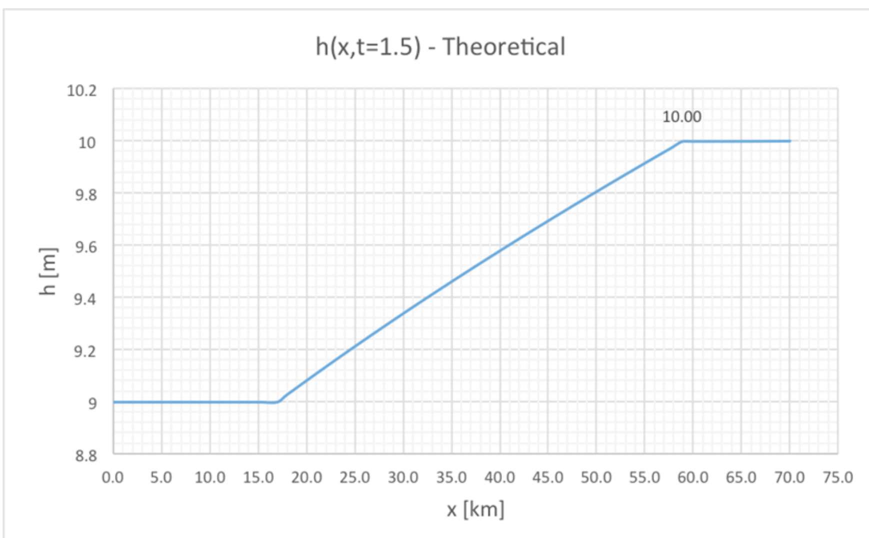


At t=4hr, the two graphs also show the similar trend.

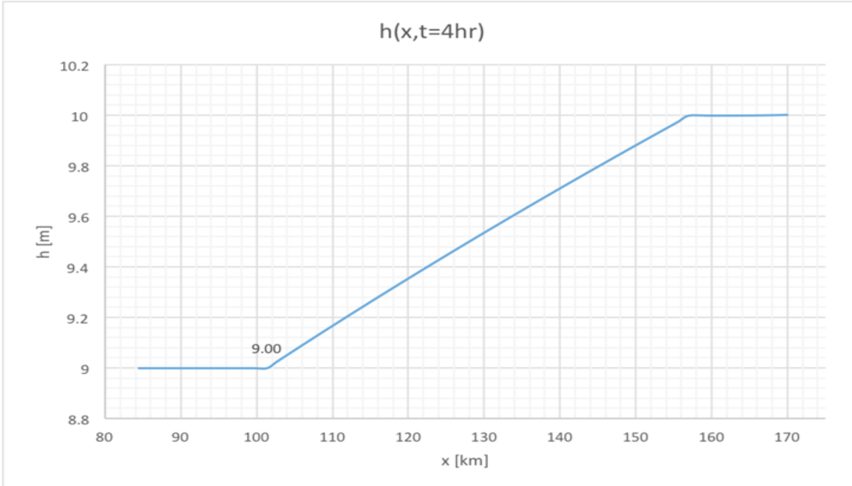




We also did the theoretical calculation, which was treated as a simple wave problem with S_f=0 and S_0=0. As it was a negative disturbance moving downstream, the characteristics would be C+ characteristics, meaning *U+2c=constant* along $dx/dt=U+c$. To find U(x_s,t_s), there was a need to find U (0,τ), and for each τ, to find the fitting x_s using $dx(0,\tau)/\ dt=\ x_s/(t_s-\tau)$. The graph for the surface profile was drawn as follow.



The graphs received for t=1.5hr, numerically and theoretically, both show the negative surge at approximately the same x coordinates and the same distance between the leading edge and the trailing edge.

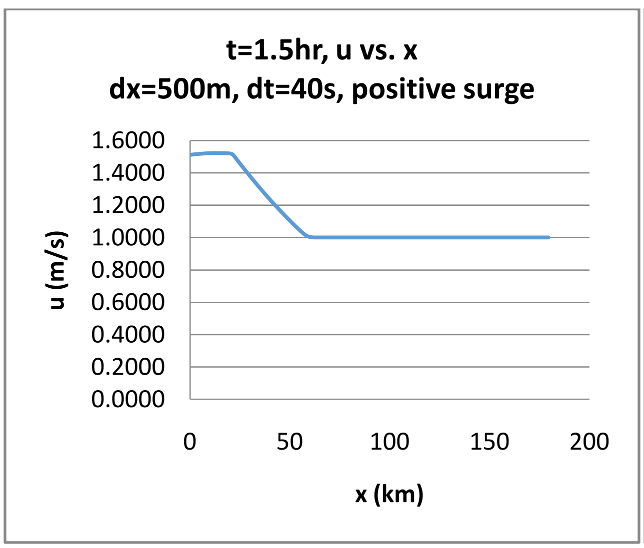


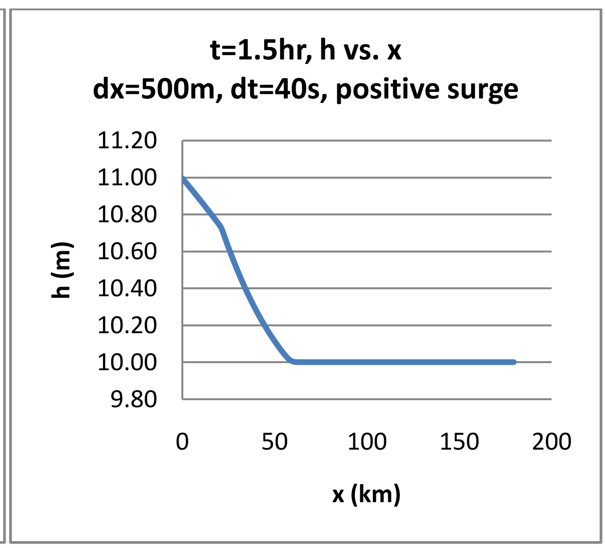
The graphs for t=4hr also have the same similarities. In the manually calculated graph, the change in slope is more pronounced. The graphs in both cases seem much alike, which may be due to the linearity assumptions made for both methods.

Positive Disturbance

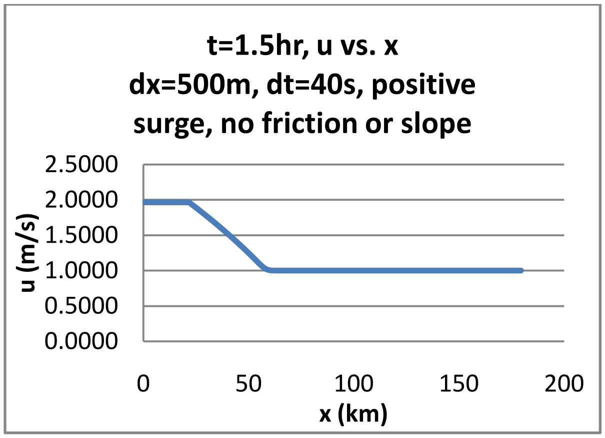
Question (d) & (e). Now the case changed to positive surge. We used the same programme for Q(a) with a different temporal variation of h at x=0.

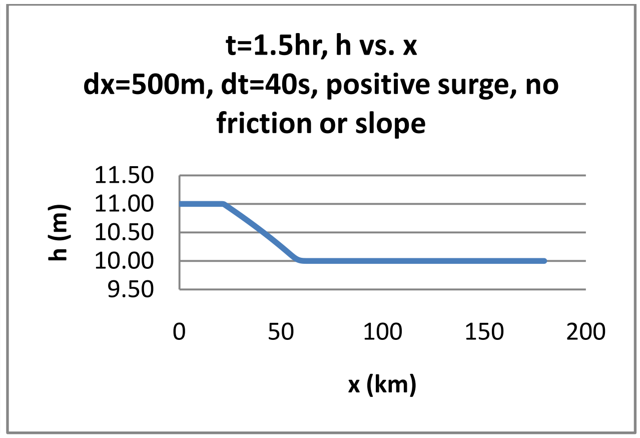
t=1.5 hrs, Δx=500m, Δt=40s, with bottom slope and bottom friction.



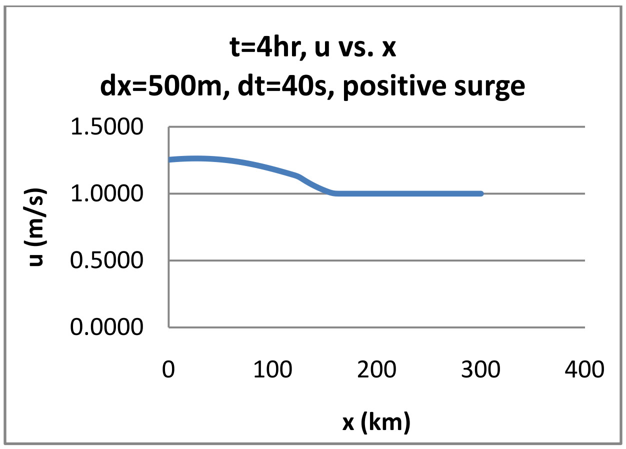


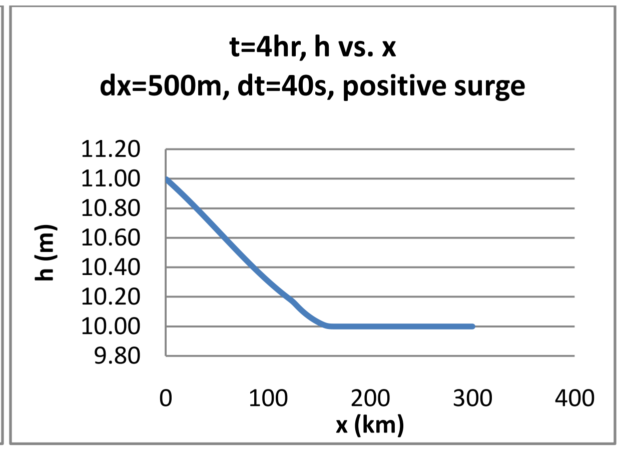
t=1.5 hrs, Δx=500m, Δt=40s, neglecting bottom slope and bottom friction.



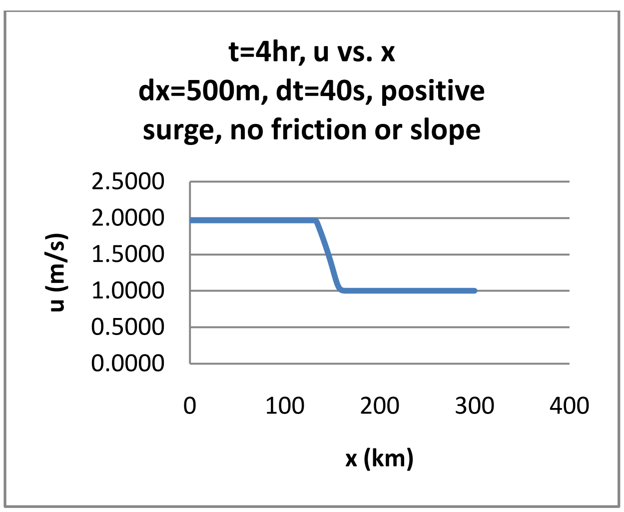


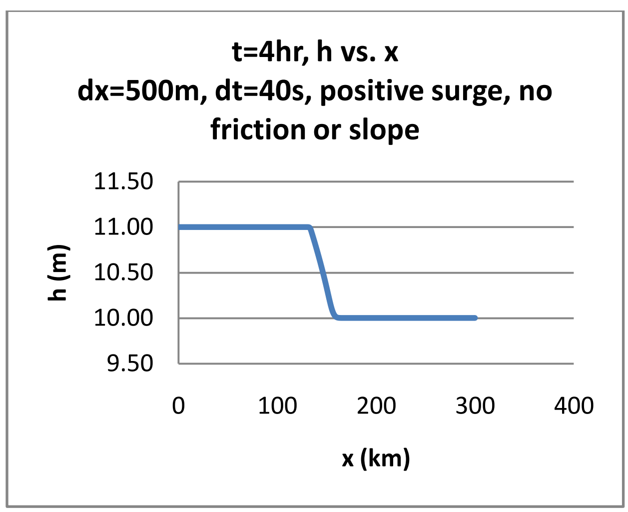
t=4 hrs, Δx=500m, Δt=40s, with bottom slope and bottom friction.





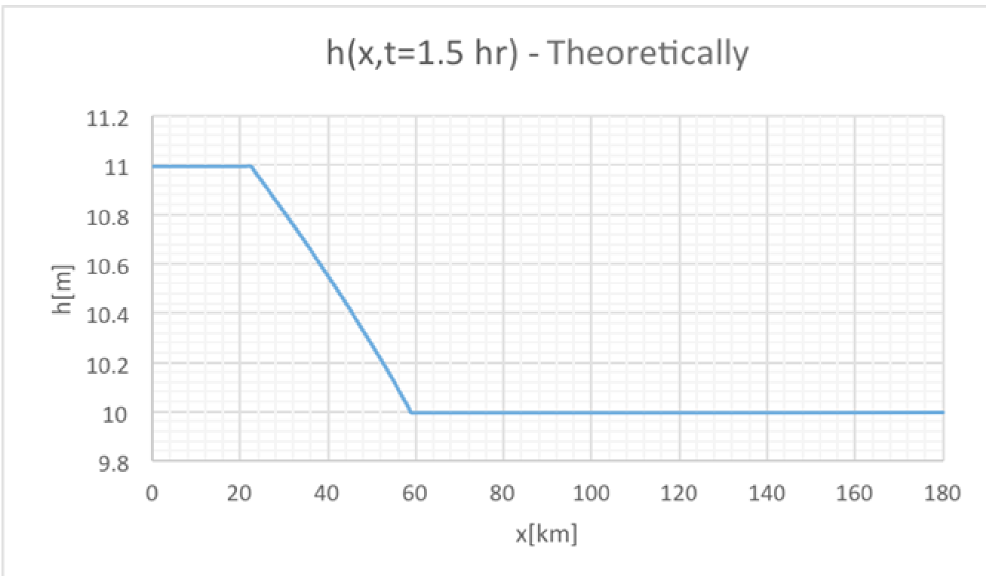
t=4 hrs, Δx=500m, Δt=40s, neglecting bottom slope and bottom friction.

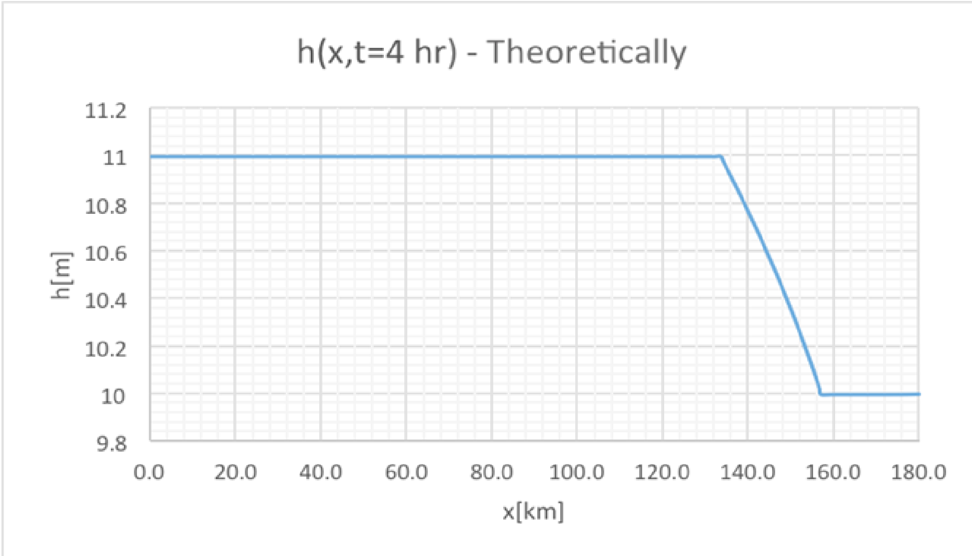




We can see that, for both t=1.5hr and t=4hr, the area before the trailing edge has a constant depth of 11m in the case of no bottom slope or friction. Compared to this, the depth at the same location and same time keeps dropping in the presence of bottom slope and friction. This difference is similar to the case of the negative disturbance discussed above. The average velocity before the trailing edge also behaves different in cases with and without bottom slope and friction, which is similar to the difference in depth. However, the distance between the leading edge and the trailing edge is roughly the same at the same time for both cases with and without bottom slope and friction.

A theoretical calculation, which was treated as a simple wave problem with no friction or bottom slope, was also done for comparison. As for this positive disturbance, it was also moving downstream. Thus, in this case, the C+ characteristic would also fit. The x_s coordinates was found using the equation $t_s = \tau + \frac{x_s}{U_0 - 2c_0 + h(0,\tau)}$. The graphs are displayed as below.





Comparing the manual graphs with the program-manufactured graphs, the shift in the slopes is sharper in the manual graphs. The surge is located at the same x coordinates at both graphs, and the distance between the leading edge and the trailing edge is almost the same in both.