STEADY OPEN-CHANNEL FLOW 3

~~RIVER MECHANICS~~ (OPEN-CHANNEL HYDRAULICS)

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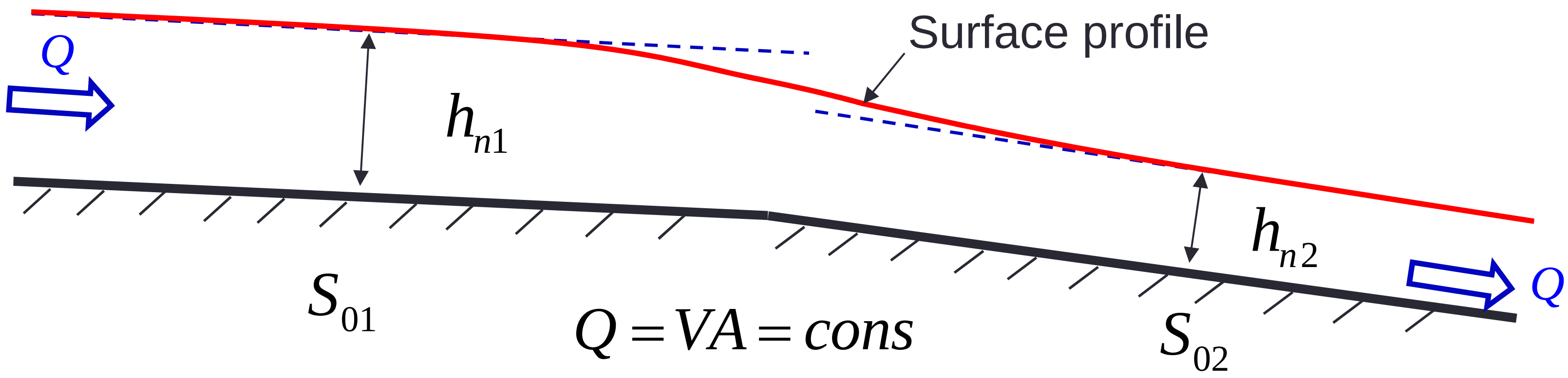
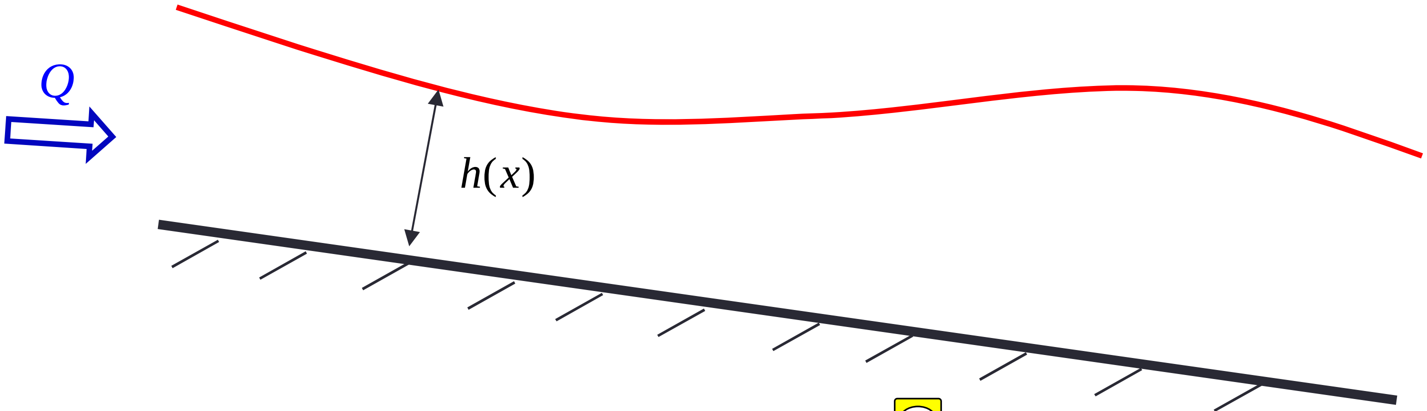
Gradually varied flow



Water depth varies slowly in the streamwise direction

Governing equation



$$\boxed{\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2}}$$

$$S_f = \frac{\tau_b}{\rho g R_h} = \begin{cases} \dfrac{f}{8g}\dfrac{P}{A^3}Q^2\left(Darcy - Weisbach\right) \\ \dfrac{1}{C^2}\dfrac{P}{A^3}Q^2\left(Chezy?s\ Equation\right) \\ n^2\dfrac{P^{4/3}}{A^{10/3}}Q^2\left(Manning's\ Equation\right) \end{cases} = F(h)$$

$$Fr^2 = \frac{Q^2 b_s}{gA^3} = f(h)$$

Mild and steep slope

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

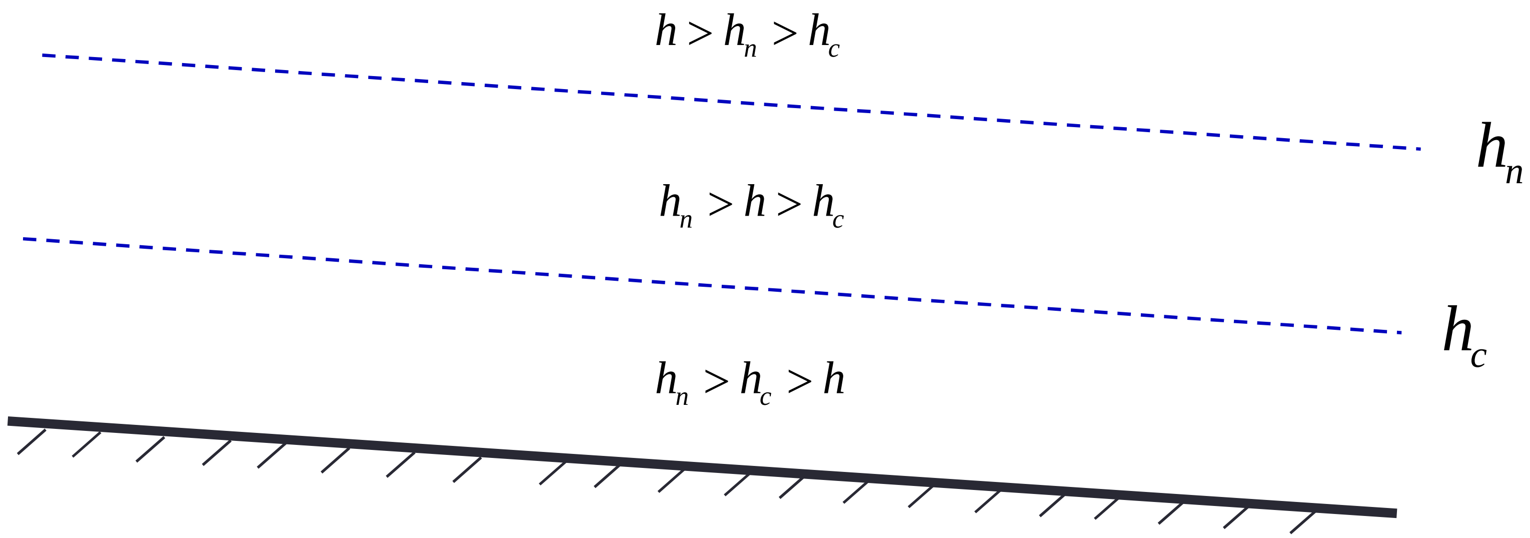
$$S_o - S_f \begin{cases} > 0, \text{ if } h > h_n \\ \leq 0, \text{ if } h \leq h_n \end{cases}$$

$$1 - Fr^2 \begin{cases} > 0, \text{ if } Fr < 1 \Leftrightarrow h > h_c \Leftrightarrow \text{subcritical} \\ \leq 0, \text{ if } Fr \geq 1 \Leftrightarrow h \leq h_c \Leftrightarrow \text{supercritical} \end{cases}$$

$$\begin{cases} h_n > h_c, \text{ mild slope} \\ h_n < h_c \text{ steep slope} \end{cases}$$

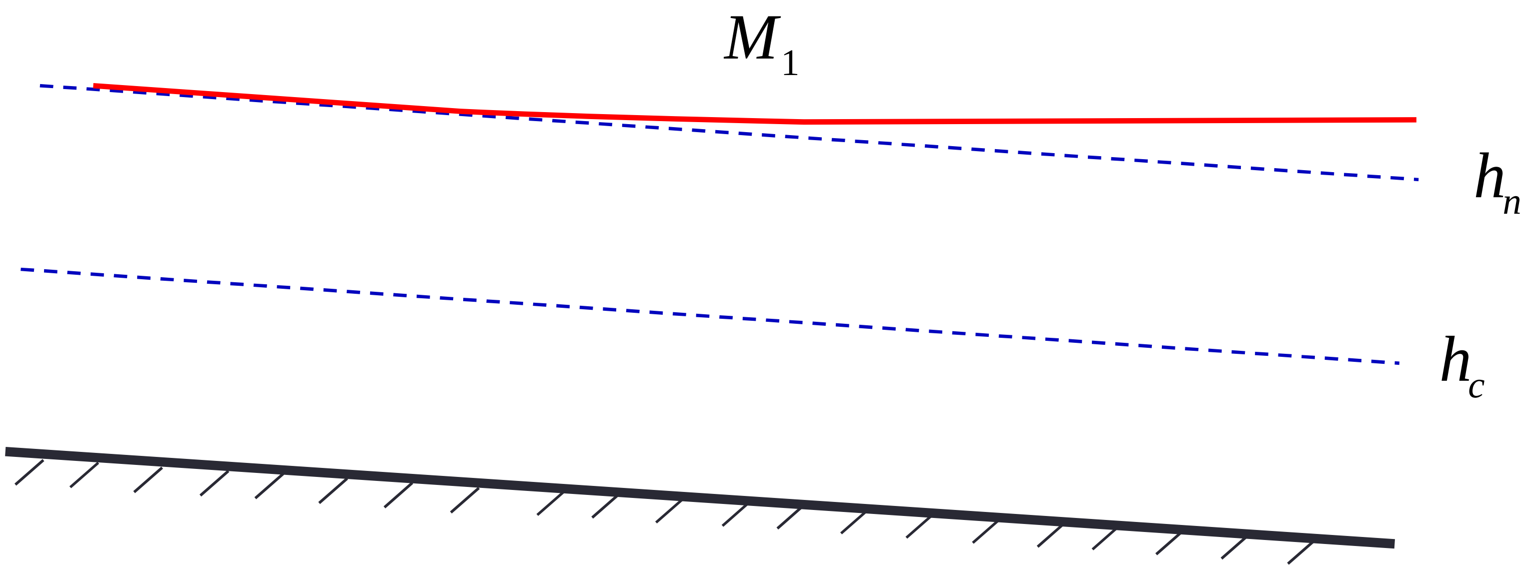
"If normal flow is $\begin{Bmatrix} \text{subcritical} \\ \text{supercritical} \end{Bmatrix}$ then $Fr_n \begin{Bmatrix} < \\ > \end{Bmatrix} 1$ and the slope, S_o is referred to as $\begin{Bmatrix} \text{mild} \\ \text{steep} \end{Bmatrix}$"

Surface profile for mild slope



M1 profile: backwater profile $h > h_n > h_c$

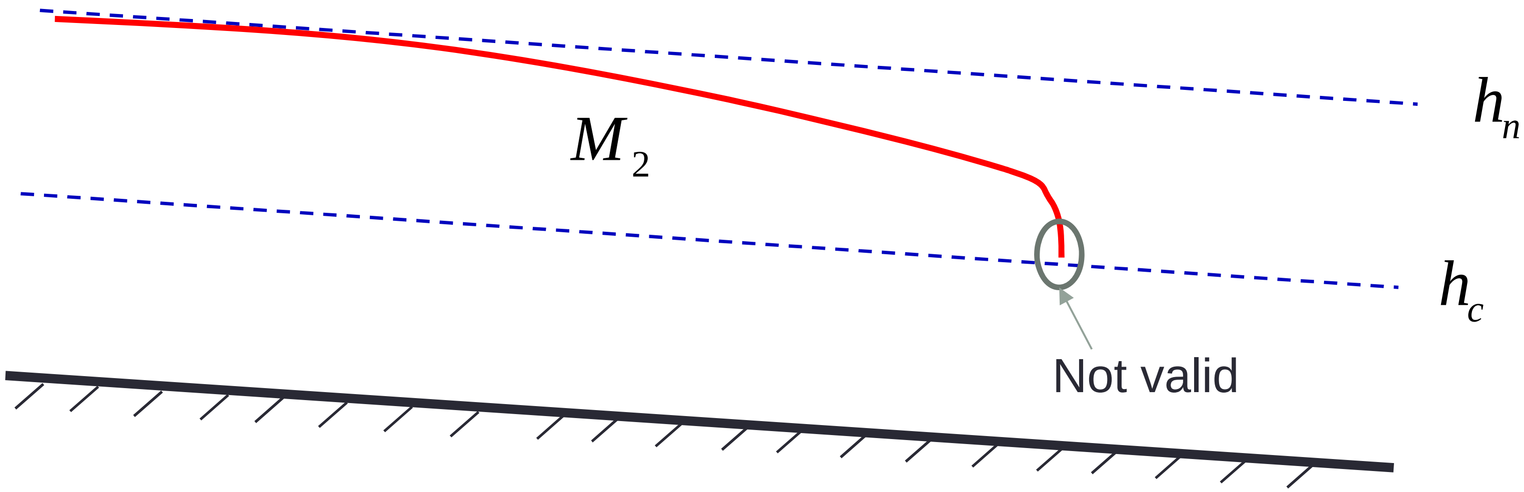
$$\frac{\mathrm{d}h}{dx} = \frac{S_o - S_f}{1 - Fr^2} = \frac{+}{+} > 0$$



Limiting behavior:

- Becomes horizontal in the downstream direction
- Asymptotically converge to normal depth in the upstream direction

M2 profile: draw-down profile $h_n > h > h_c$ $$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2} = \frac{-}{+} < 0$$

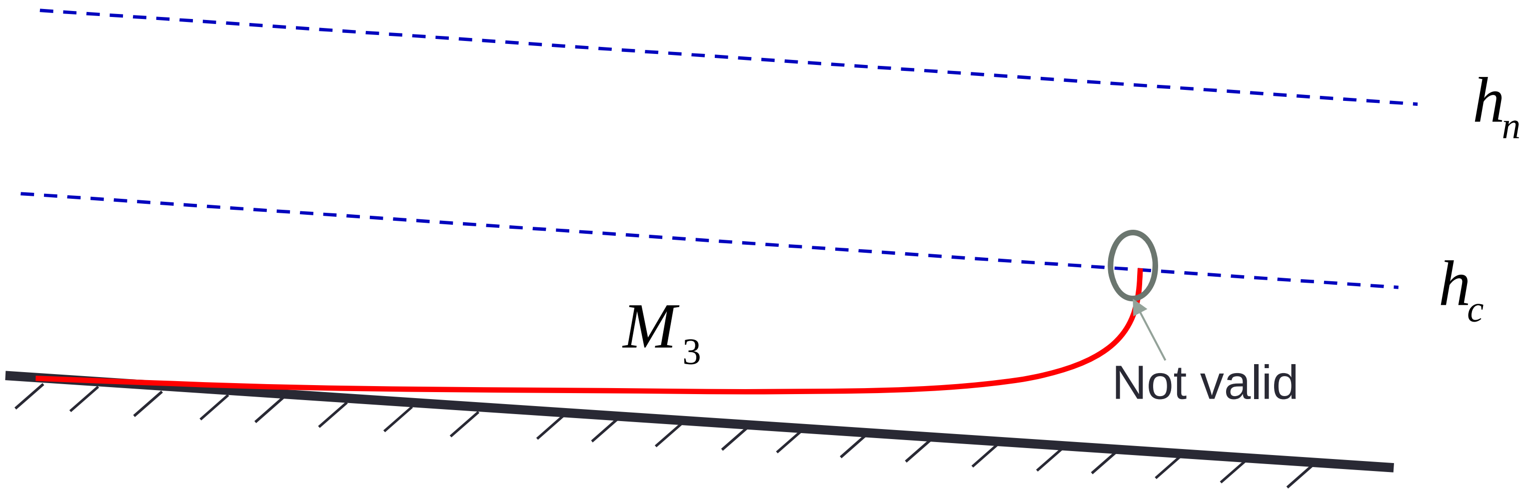


Limiting behavior:

- Abruptly change to critical depth in the downstream direction
- Asymptotically converge to normal depth in the upstream direction

M3 profile: $h_n > h_c > h$

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2} = \frac{-}{-} > 0$$



Limiting behavior:

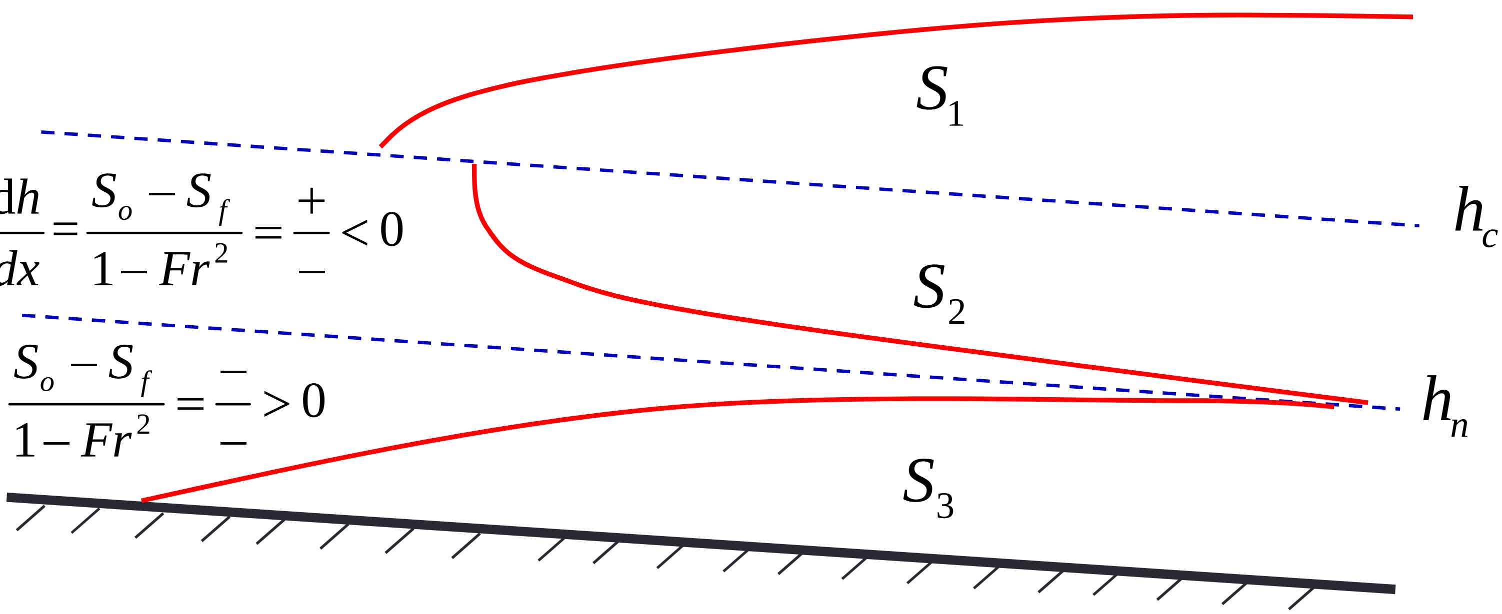
- Abruptly change to critical depth in the downstream direction
- Depth linearly decrease in the upstream direction

Surface profile for steep slope

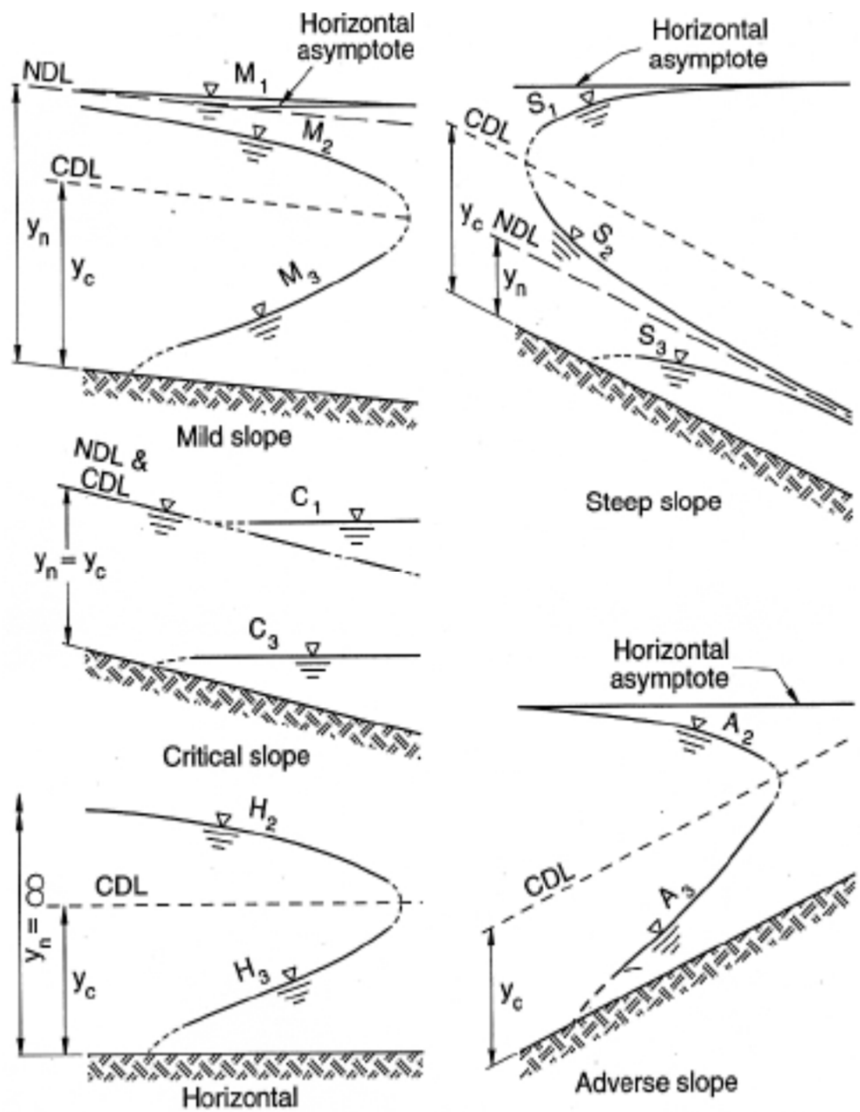
$$\frac{\mathrm{d}h}{dx}=\frac{S_o-S_f}{1-Fr^2}=\frac{+}{+}>0$$

$$\frac{\mathrm{d}h}{dx}=\frac{S_o-S_f}{1-Fr^2}=\frac{+}{-}<0$$

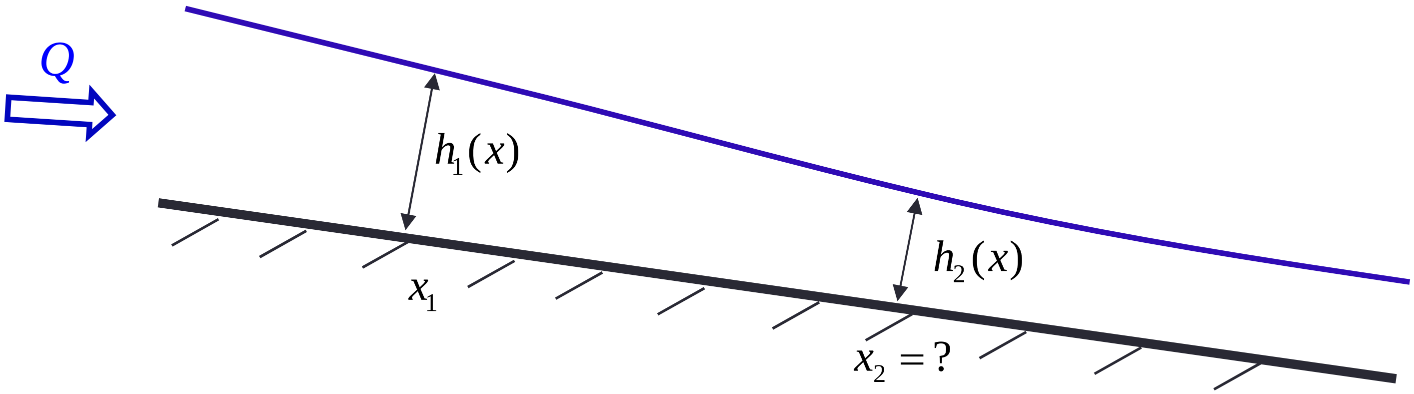
$$\frac{\mathrm{d}h}{dx}=\frac{S_o-S_f}{1-Fr^2}=\frac{-}{-}>0$$



Summary of surface profile:



Estimate of channel length affected by gradually varied flow



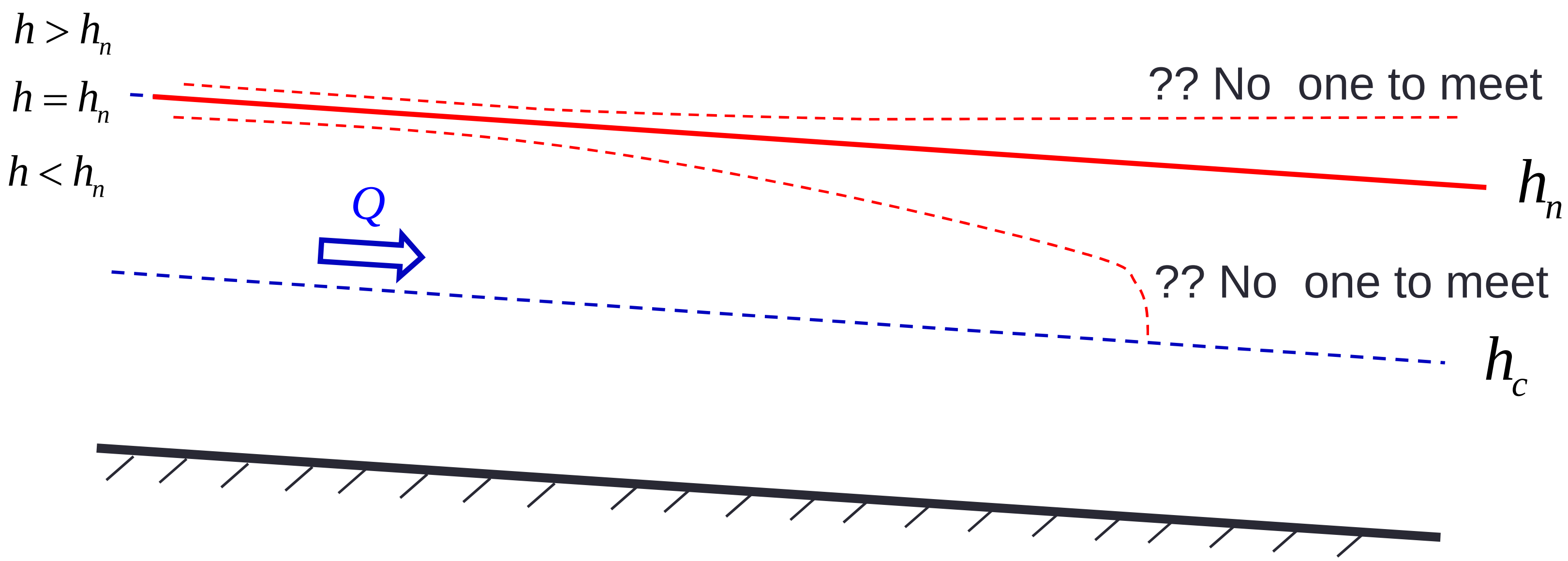
$$x_2 = x_1 + \frac{\left(1 - \overline{Fr^2}\right)}{\left(S_o - \overline{S}_f\right)}\left(h_2 - h_1\right)$$

Control points

$$\frac{dh}{dx} = \frac{S_o - S_f}{1 - Fr^2}$$

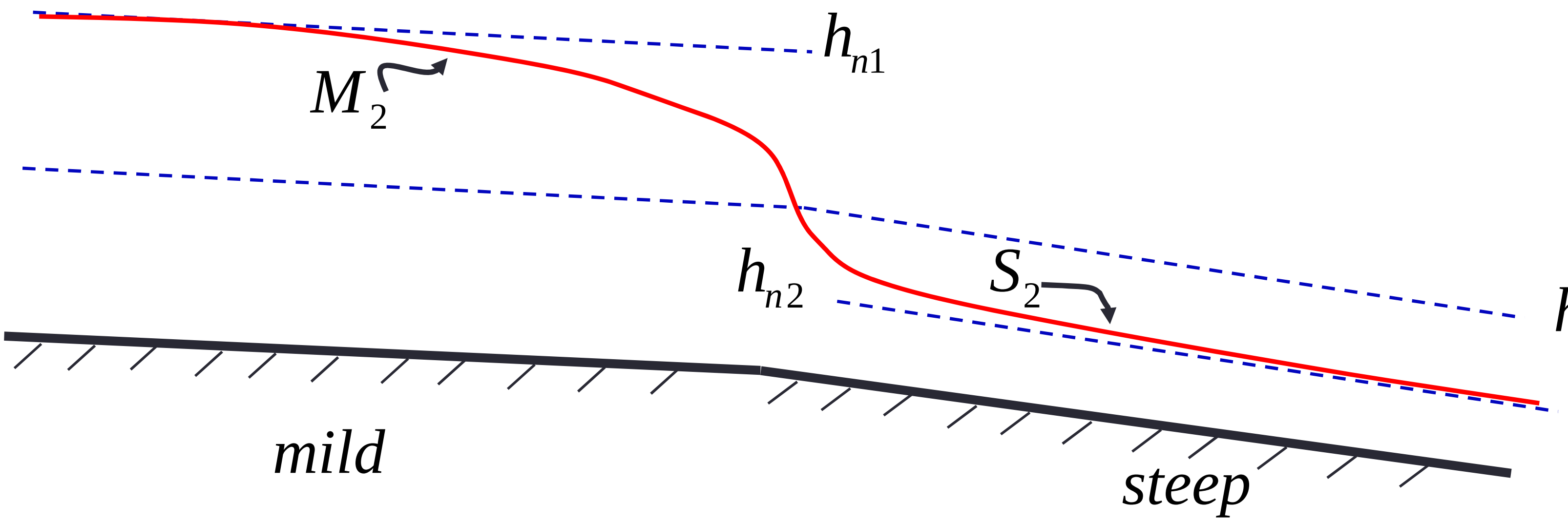
To solve a 1st-order ordinary differential equation, you need boundary conditions, i.e. a given water depth and flow condition at a given point. These points are control points.

Entrance condition for mild slope



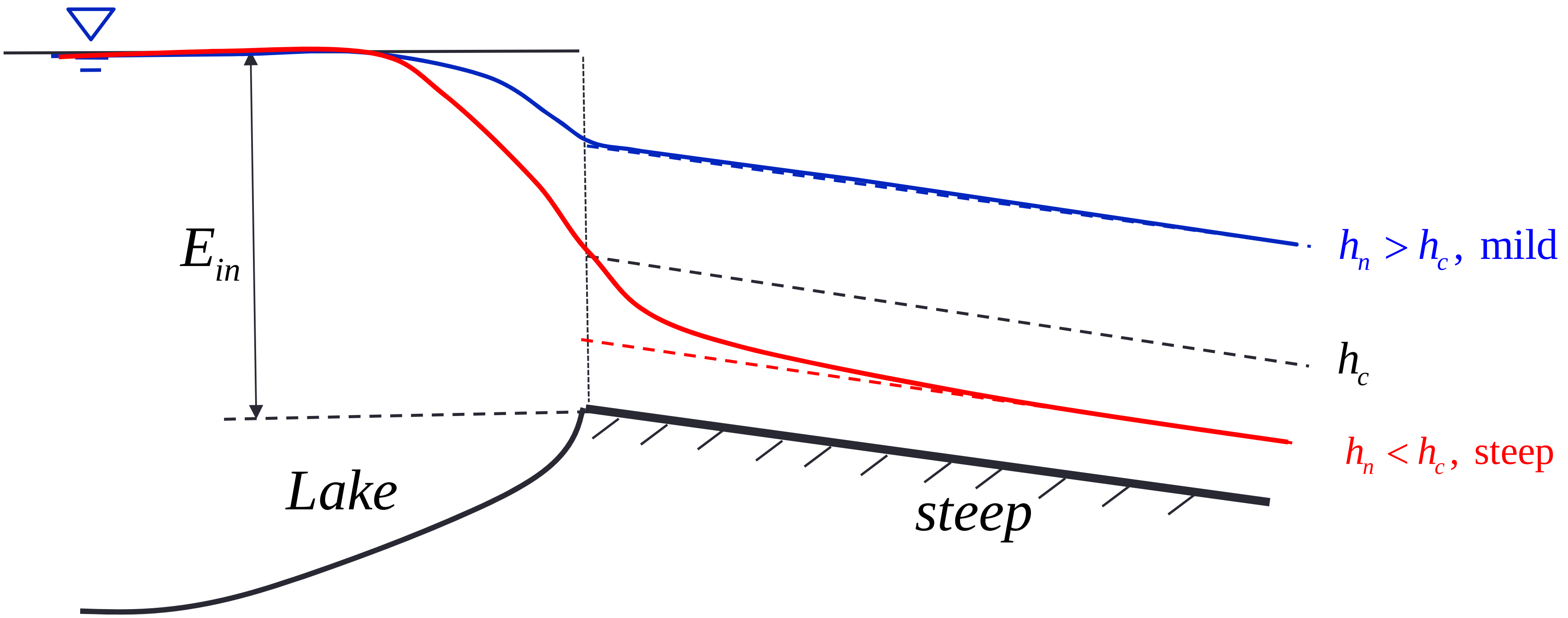
In the absence of a downstream control a subcritical flow entering a mildly sloping channel must do so at normal depth. We call this <u>Normal Depth Control</u>

Transition from mild slope to steep slope



If the slope of a channel changes from mild to steep, the flow must pass through critical flow at the location of the change in slope. We call this Critical Depth Control

Entrance from a lake: very long channel



Steep slope: Critical flow at entrance
Then a S2 profile to normal depth

$$E_{in} = h_c + \frac{Q^2}{2gA_c^2} \quad \& \quad Fr_{in}{}^2 = \frac{Q^2 b_{sc}}{gA_c^3} = 1$$

Mild slope: Normal flow at entrance

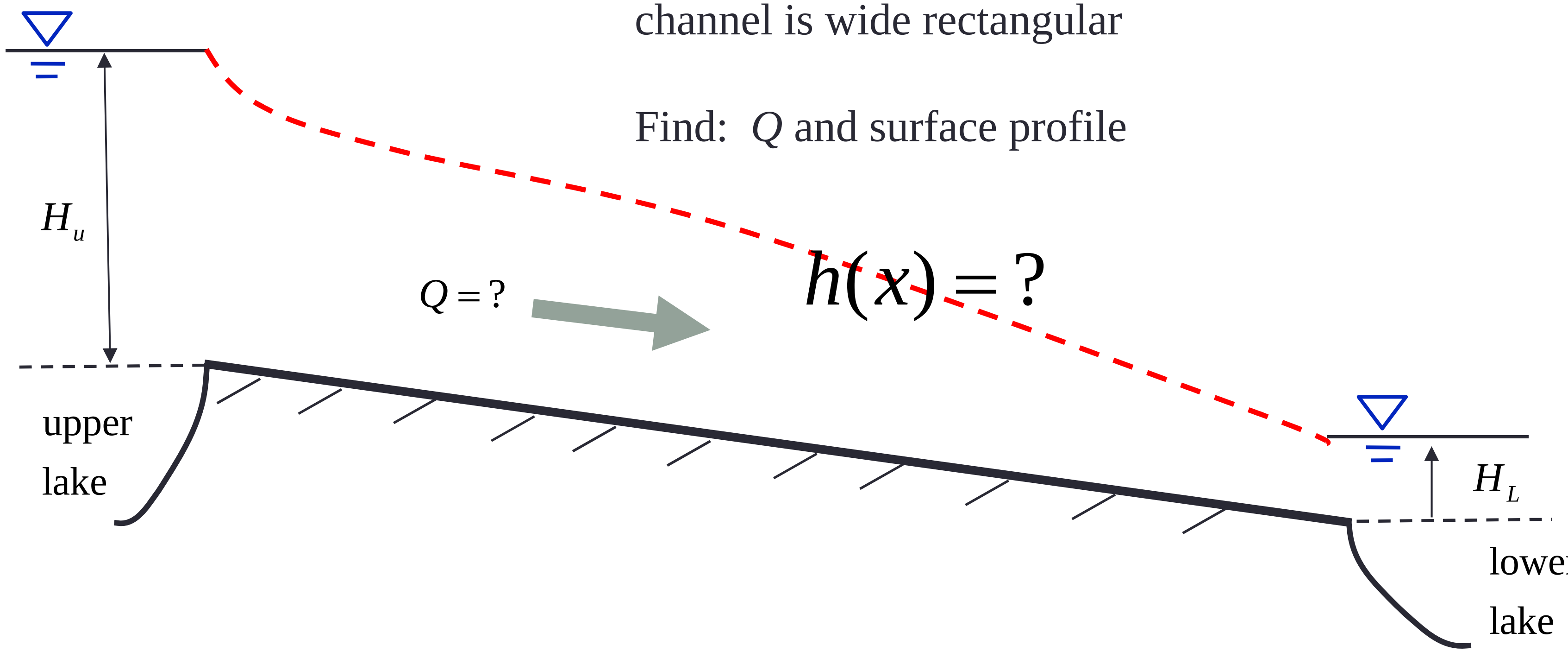
$$E_{in} = h_n + \frac{Q^2}{2gA_n^2} \quad \& \quad Q = \frac{1}{n}\frac{A_n^{5/3}}{P_n^{2/3}}\sqrt{S_o}$$

Let’s apply what we have learned to a simple problem

Two lake problem

Given H_u and H_L, S_0, bottom condition (Manning's n), channel is wide rectangular

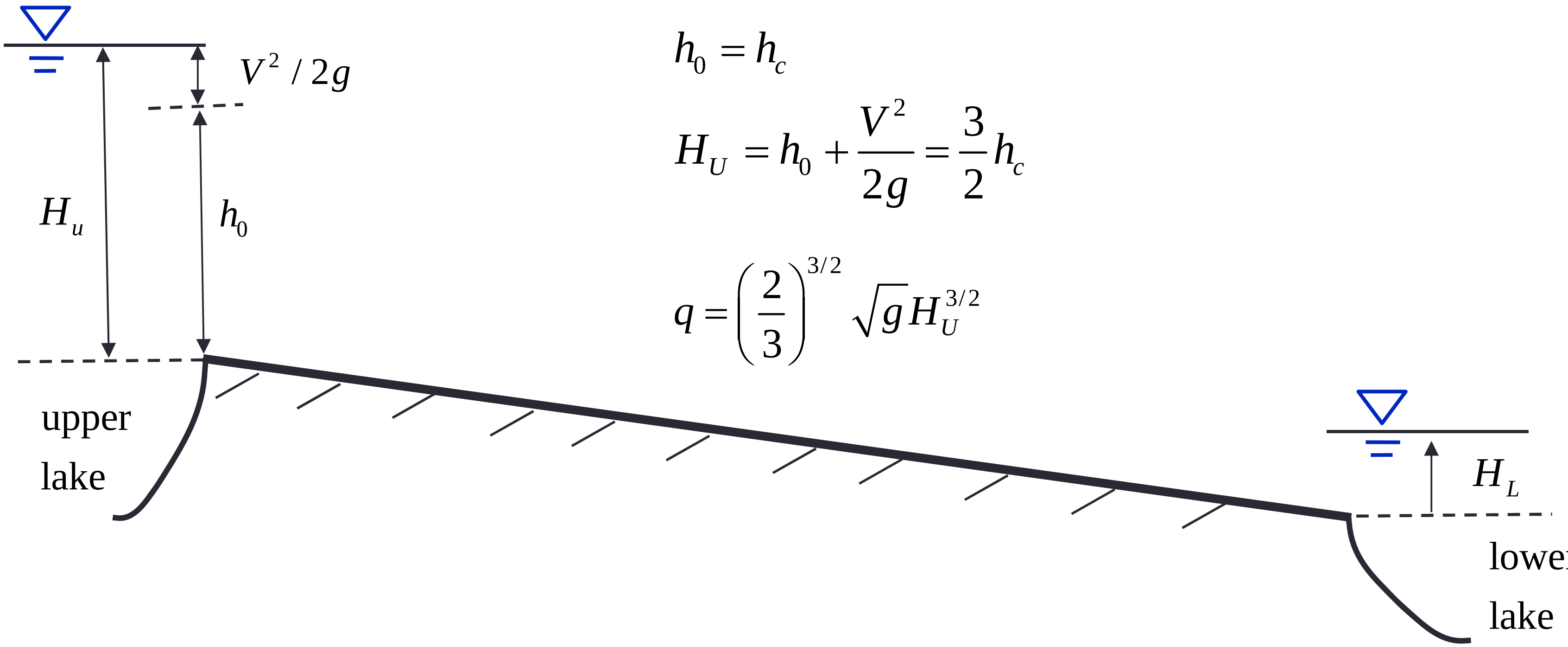
Find: Q and surface profile



Don't know Q, so cannot get h_c and h_n!

Strategy: assume steep channel and see if calculation confirms this!

Entrance control: critical flow for steep channel:



$$h_0 = h_c$$

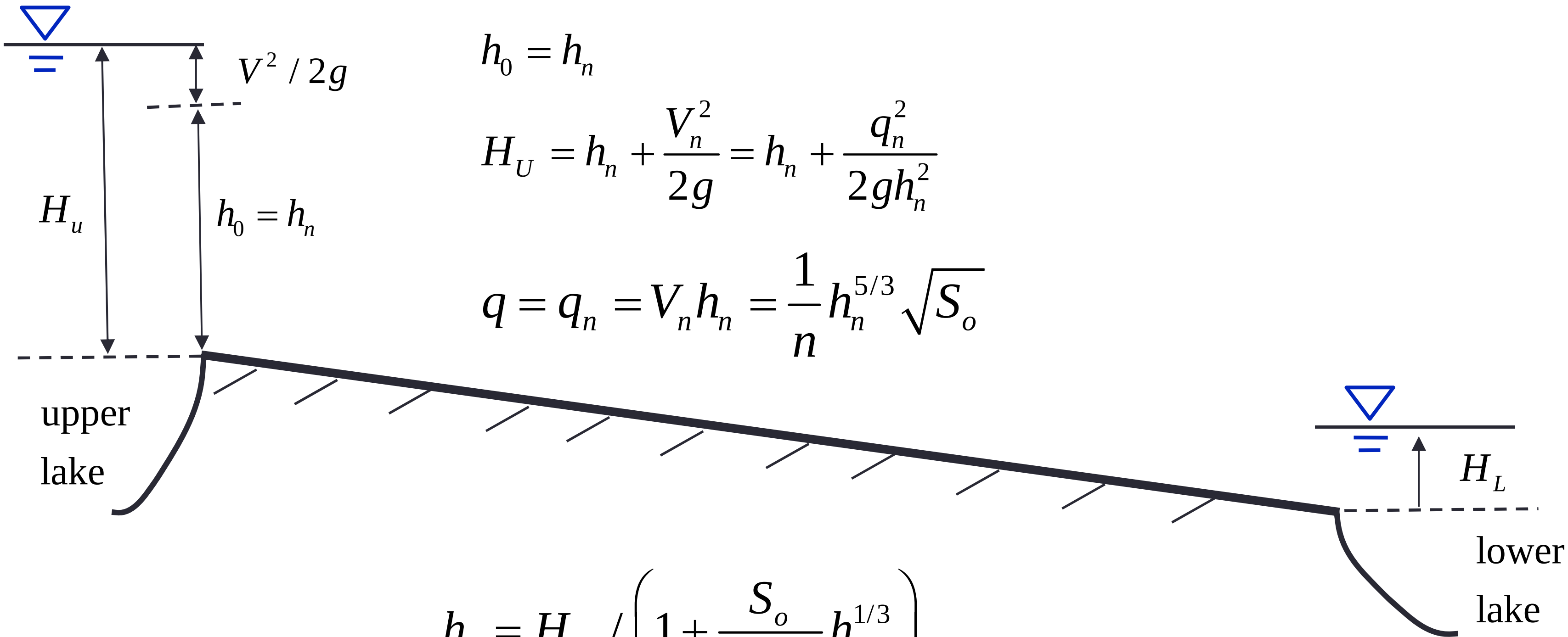
$$H_U = h_0 + \frac{V^2}{2g} = \frac{3}{2} h_c$$

$$q = \left(\frac{2}{3}\right)^{3/2} \sqrt{g} H_U^{3/2}$$

Check normal flow

$$h_n = \left(\frac{nq}{\sqrt{S_o}}\right)^{3/5} \begin{cases} < h_c & \text{slope is steep (assumption is correct, done)} \\ > h_c & \text{slope is mild (assumption is wrong, re-do calculation with mild slope)} \end{cases}$$

Entrance control: normal flow for mild channel:



$$h_0 = h_n$$

$$H_U = h_n + \frac{V_n^2}{2g} = h_n + \frac{q_n^2}{2gh_n^2}$$

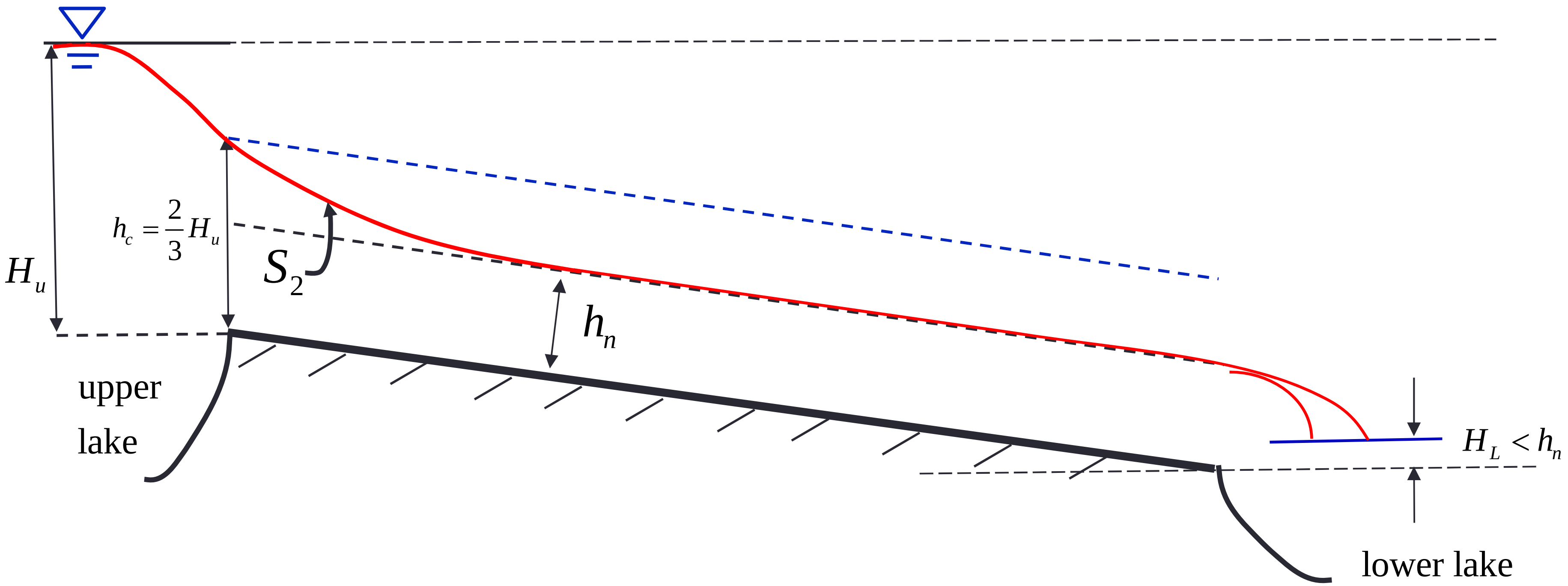
$$q = q_n = V_n h_n = \frac{1}{n} h_n^{5/3} \sqrt{S_o}$$

$$h_n = H_U \Big/ \left(1 + \frac{S_o}{2gn^2} h_n^{1/3} \right)$$

$$h_c = \left(\frac{q_n^2}{g} \right)^{1/3}$$

Two lake problem: downstream of steep slope

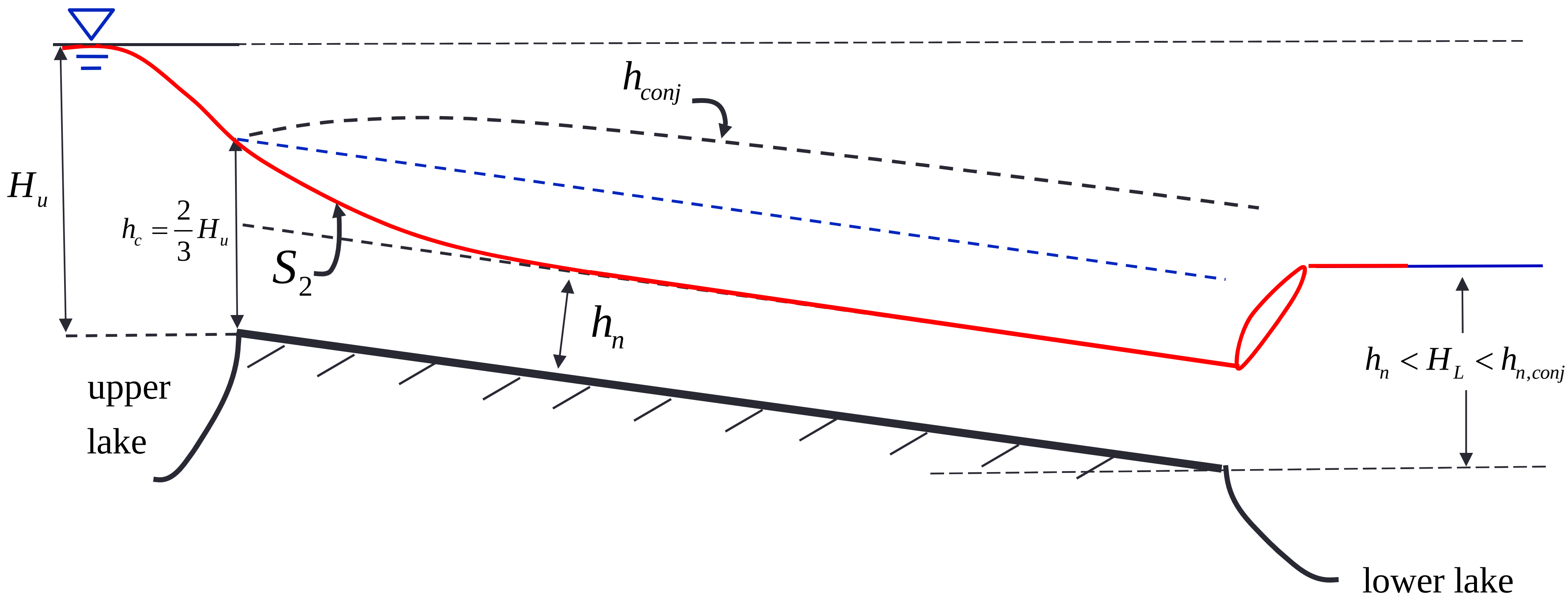
$$H_L < h_n$$



- Free outflow at the exit to lower lake
- No downstream effect over the entire channel (make sense because supercritical flow all the way)

Two lake problem: downstream of steep slope

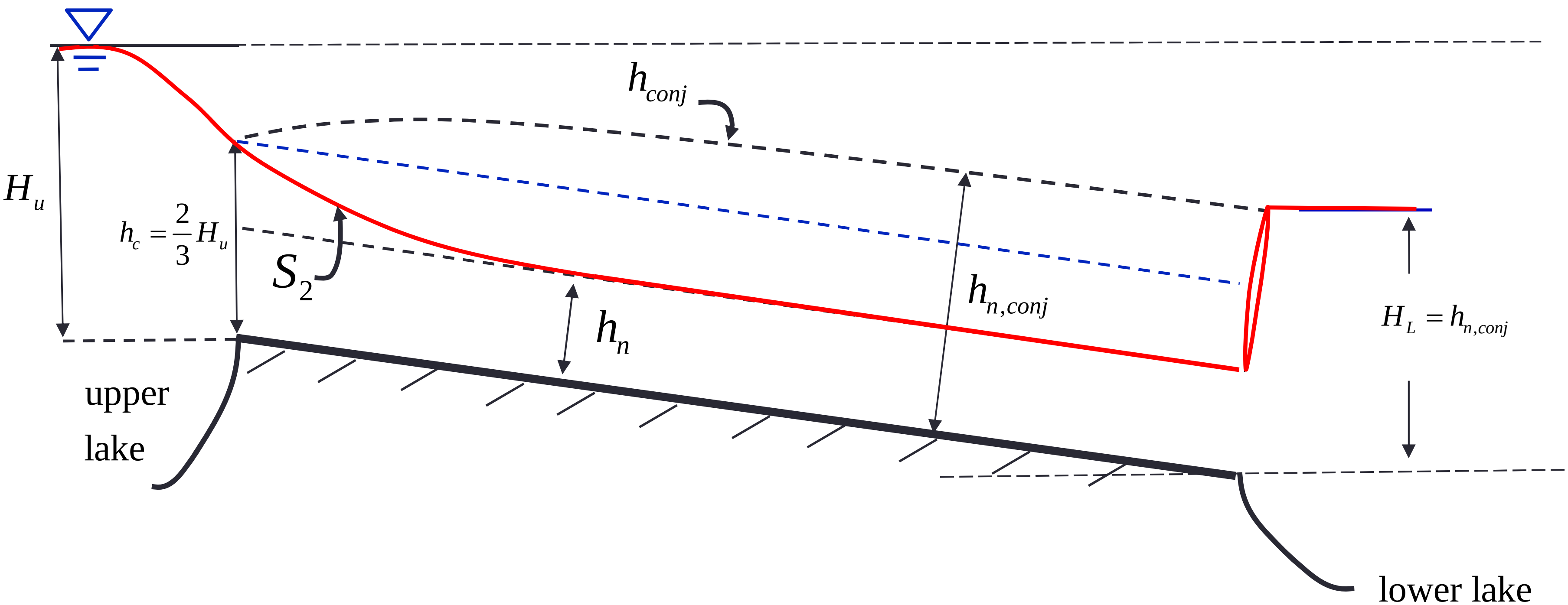
$$h_n < H_L < h_{n,conj}$$



- A partial hydraulic jump at the exit to lower lake
- No downstream effect over the entire channel

Two lake problem: downstream of steep slope

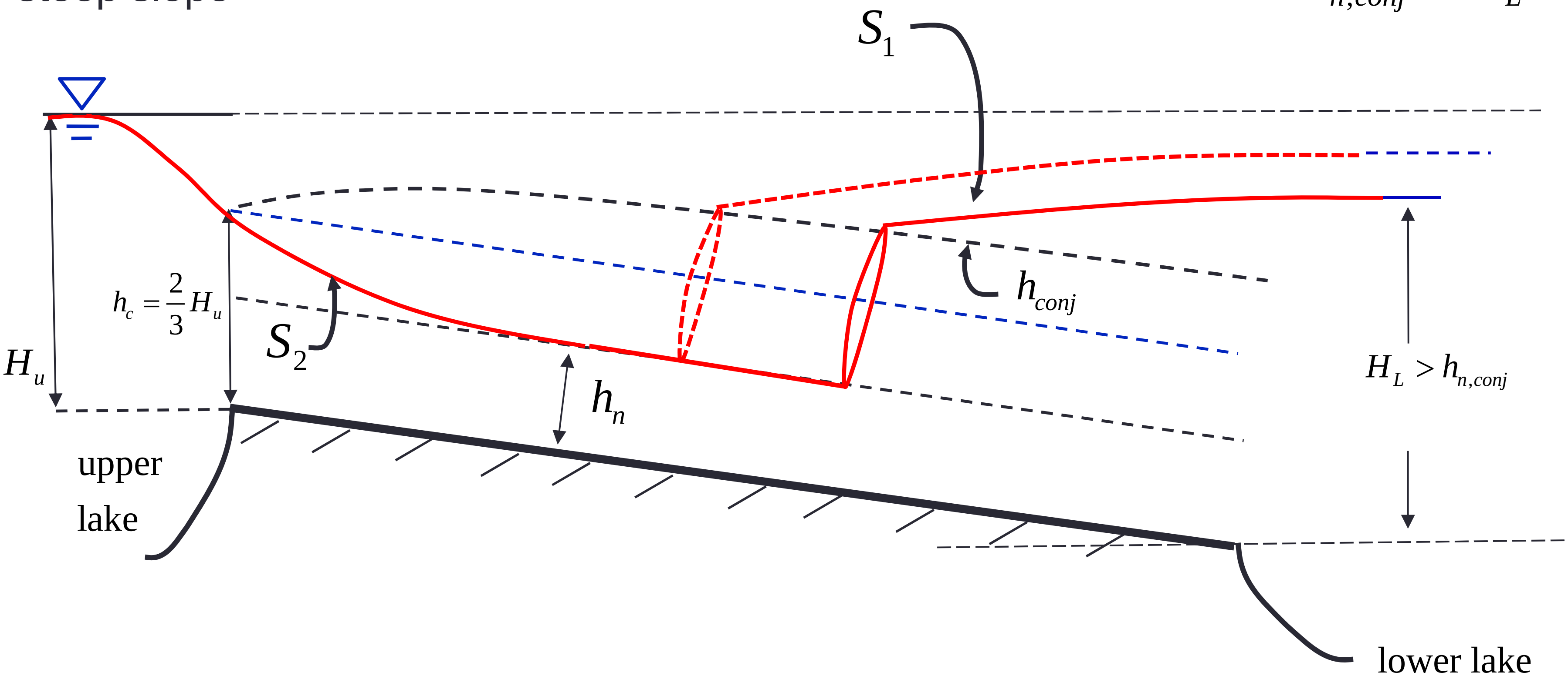
$$H_L = h_{n,conj}$$



- A full unassisted hydraulic jump at the exit to lower lake
- No downstream effect over the entire channel

Two lake problem: downstream of steep slope

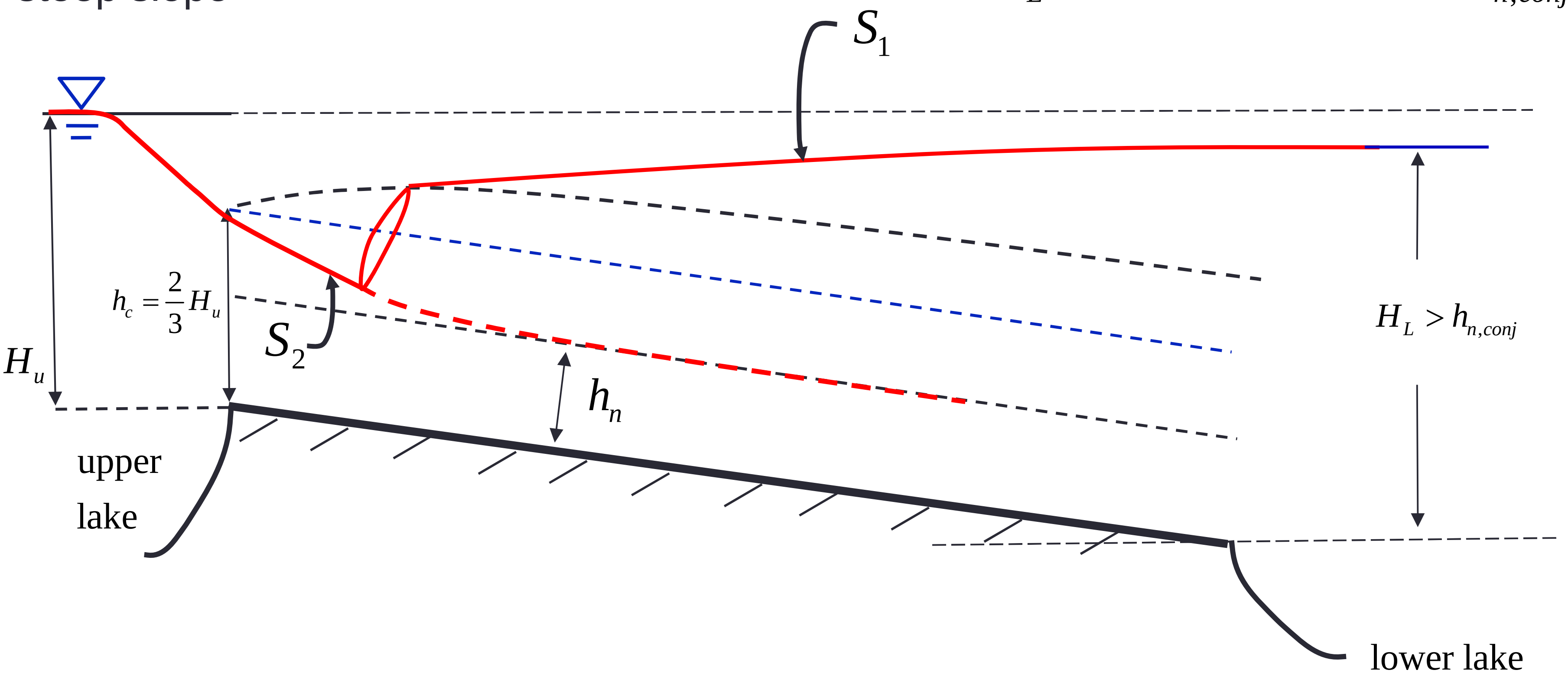
$$h_{n,conj} < H_L$$



- An S1 brings water depth from that of lower lake entrance to $h_{n,conj}$
- An Unassisted hydraulic jump from h_n to $h_{n,conj}$ within the channel
- Downstream controls the flow since hydraulic jump (where flow becomes subcritical)
- Jump location moves upstream as lower lake water level rises

Two lake problem: downstream of steep slope

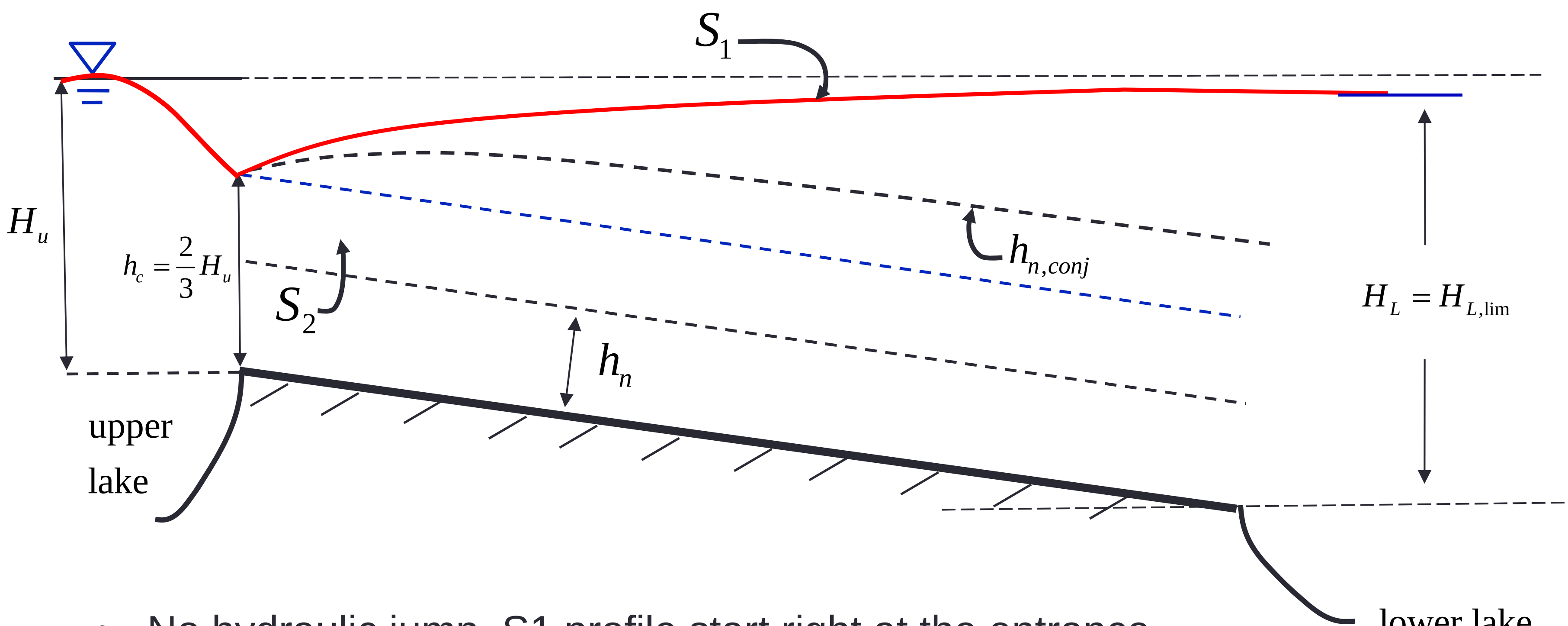
H_L further increases ($> h_{n,conj}$)



- Due to high lower lake level, the S1 profile extend beyond the region with uniform flow
- An Unassisted hydraulic jump occurs in the region with S2 profile
- Downstream controls the flow since hydraulic jump (where flow becomes subcritical)
- Jump location moves upstream as lower lake water level rises

Two lake problem: downstream of steep slope

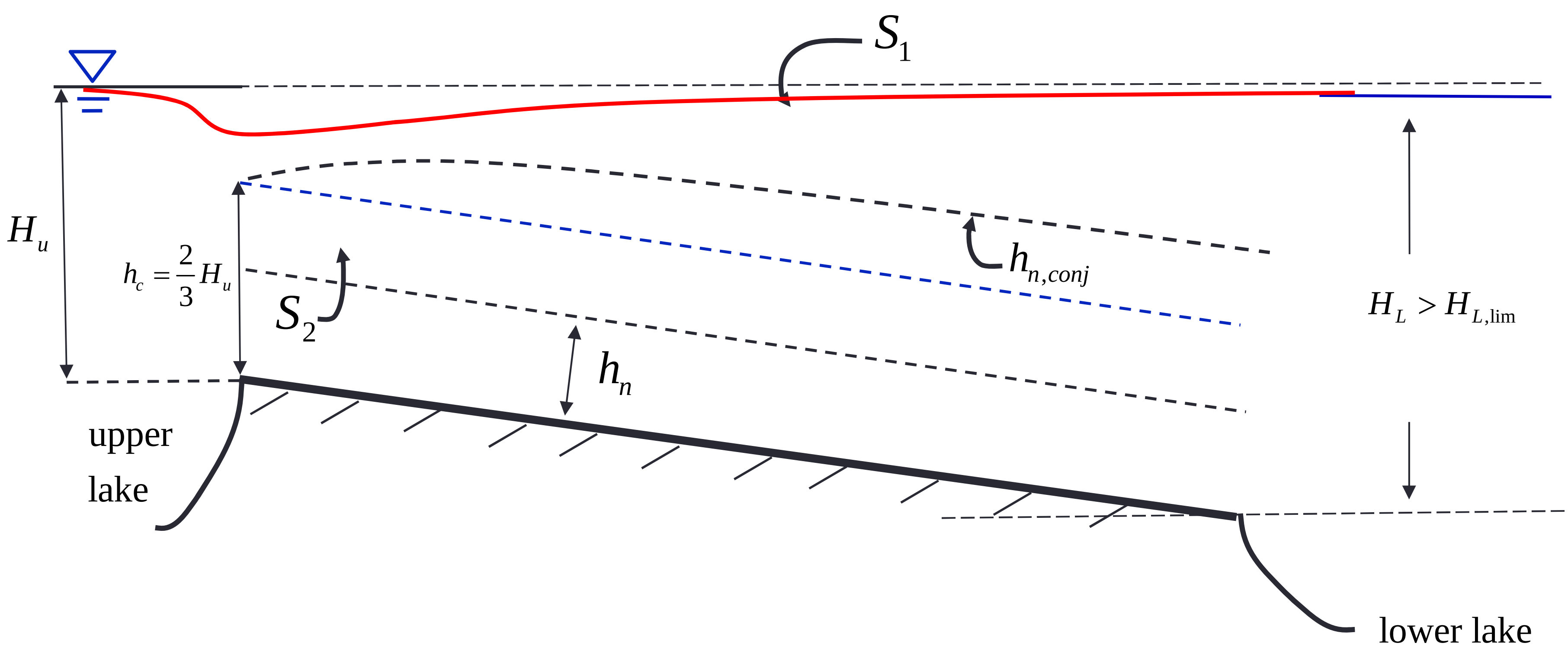
$$H_L = H_{L,\lim}$$



- No hydraulic jump, S1 profile start right at the entrance where critical flow occurs
- The entire channel is controlled by downstream!

Two lake problem: downstream of steep slope

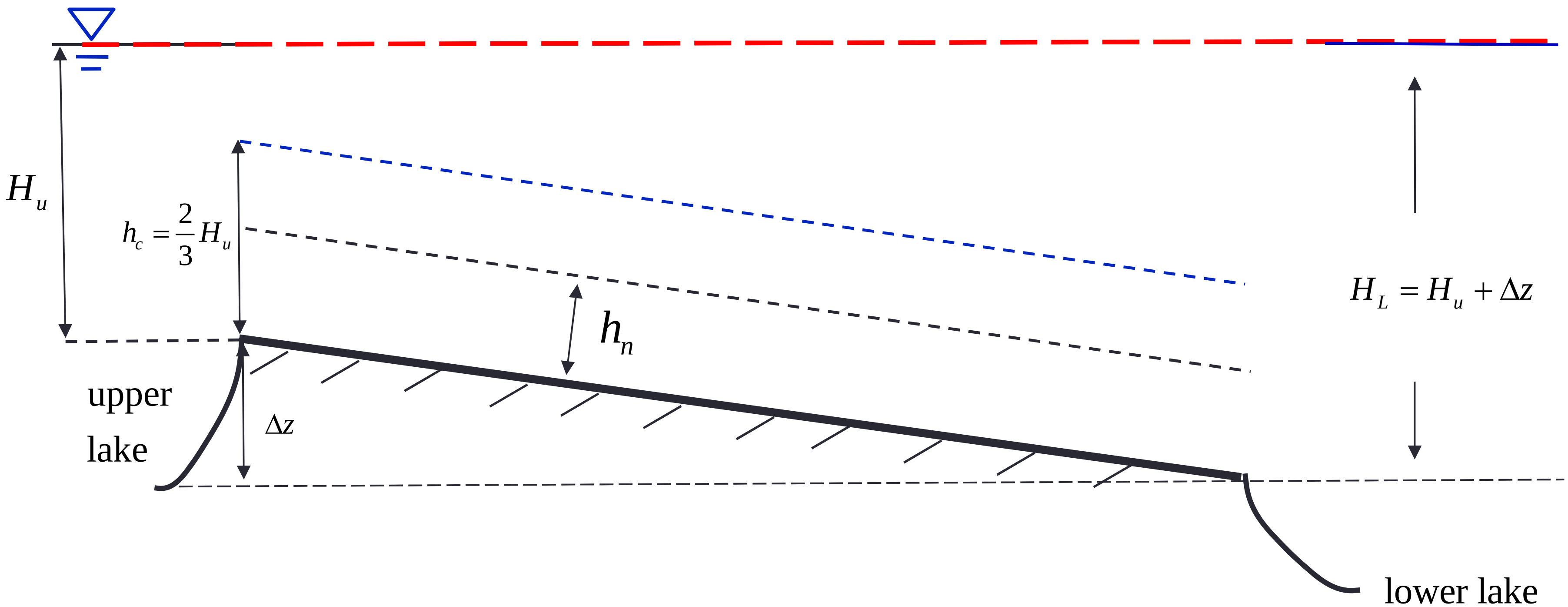
$$H_L > H_{\lim}$$



- Channel is drown, no critical flow at entrance

Two lake problem: downstream of steep slope

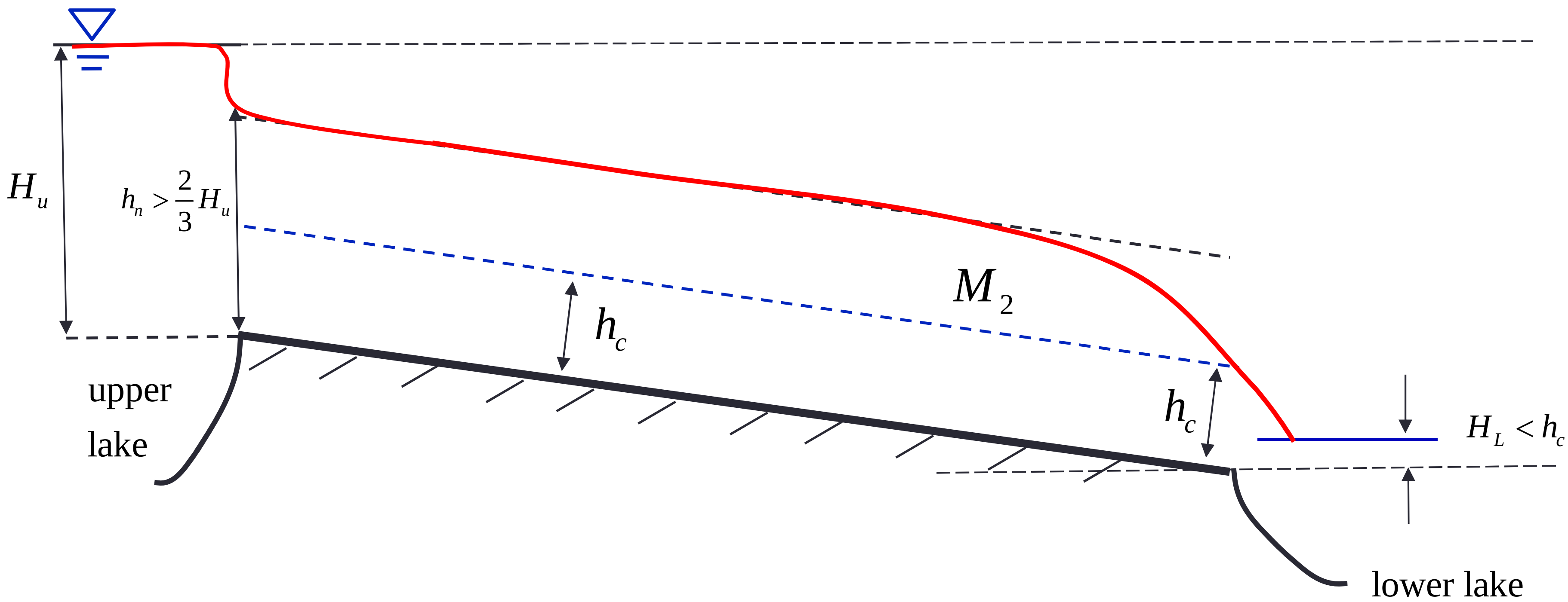
$$H_L = H_u + \Delta z$$



- Water level in the two lakes are the same
- there is no flow in the channel

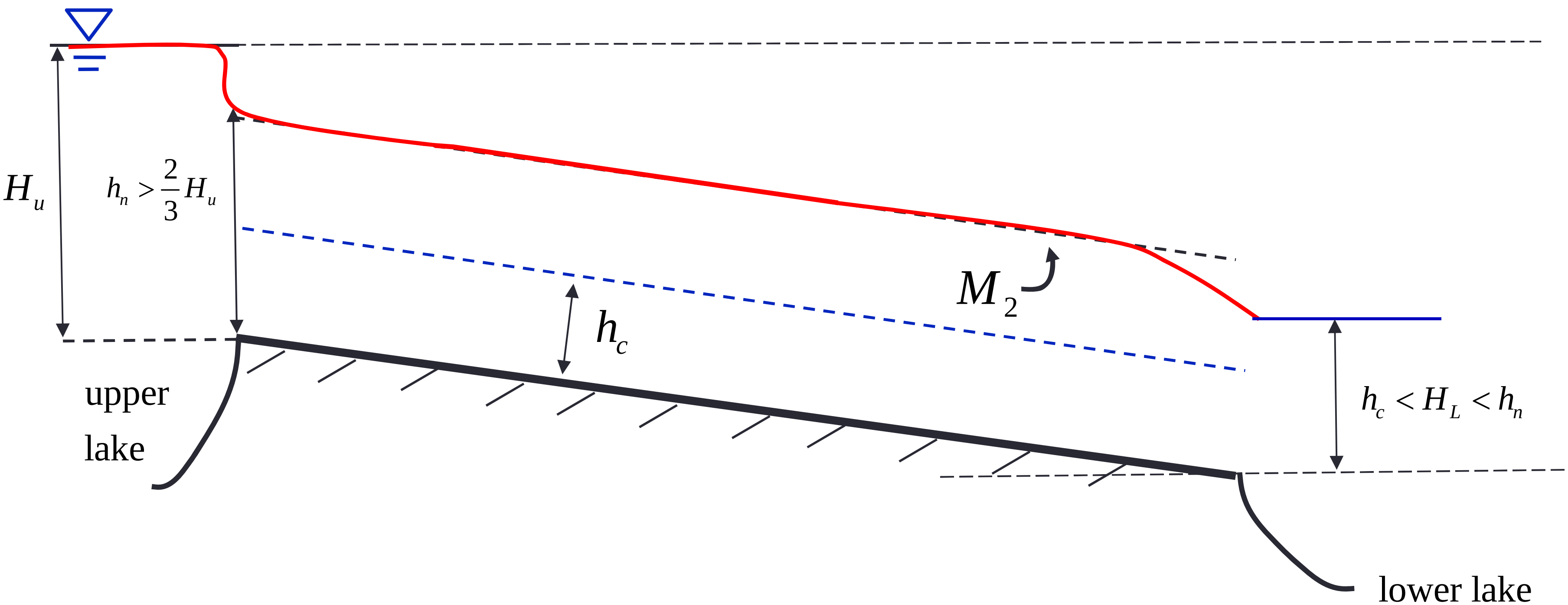
Two lake problem: downstream of mild slope

$$H_L < h_c$$



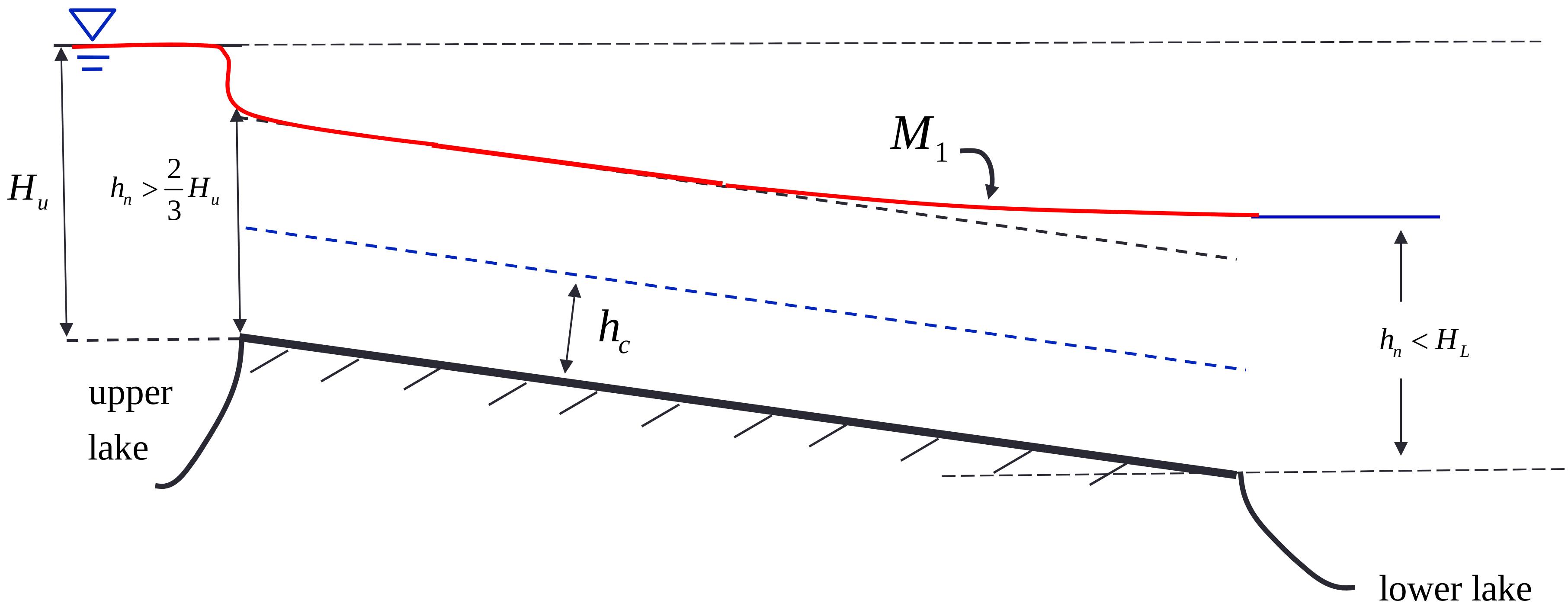
Two lake problem: downstream of mild slope

$$h_c < H_L < h_n$$



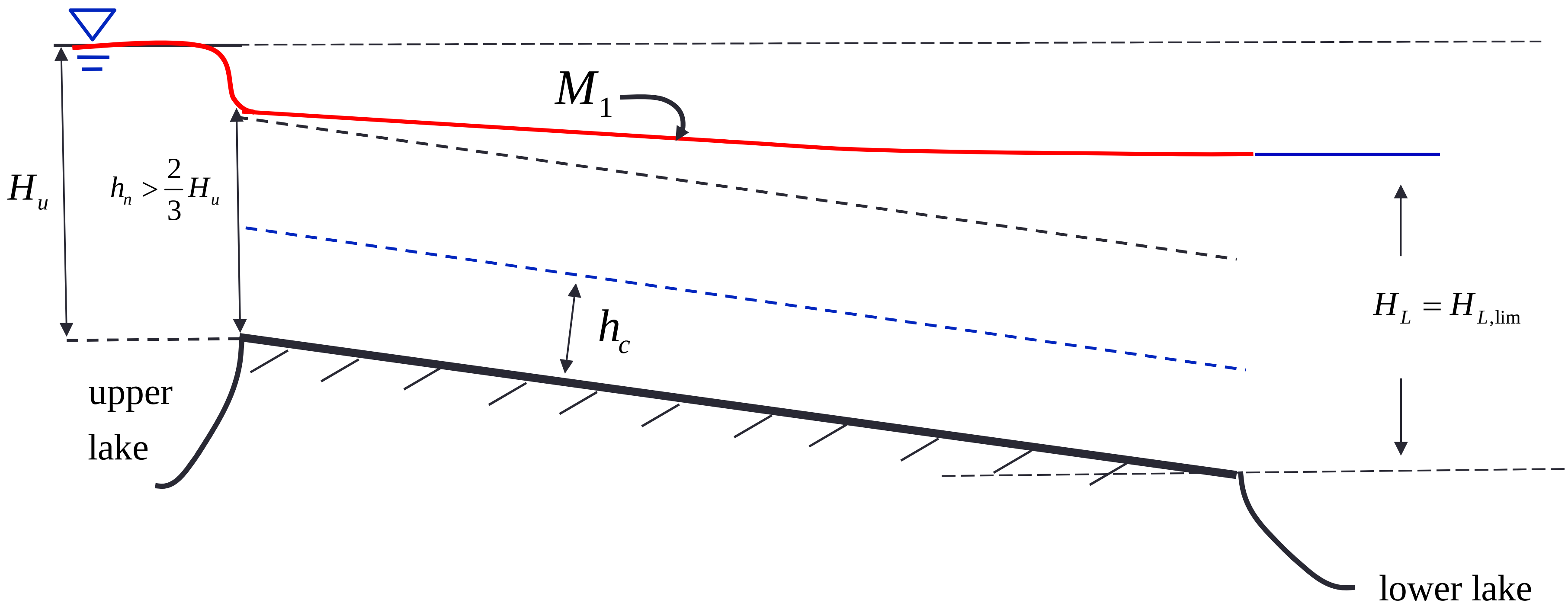
Two lake problem: downstream of mild slope

$$h_n < H_L$$



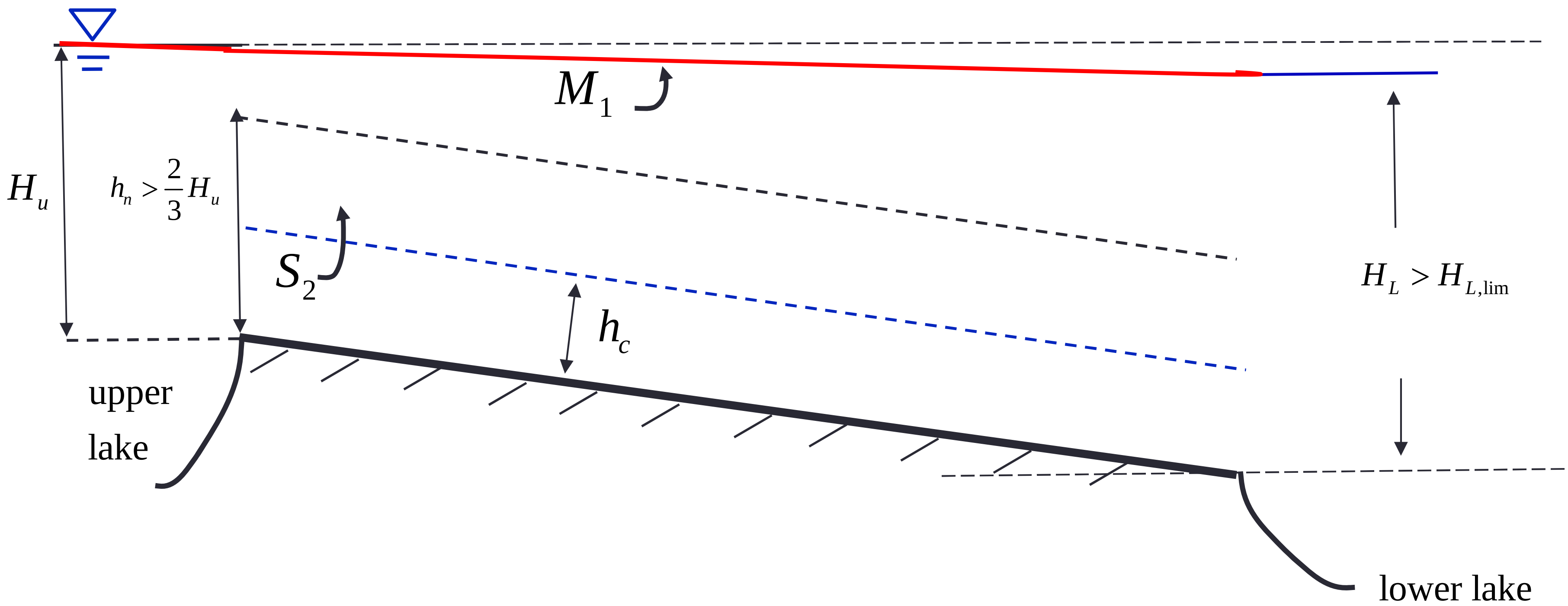
Two lake problem: downstream of mild slope

$$H_L = H_{L,\lim}$$



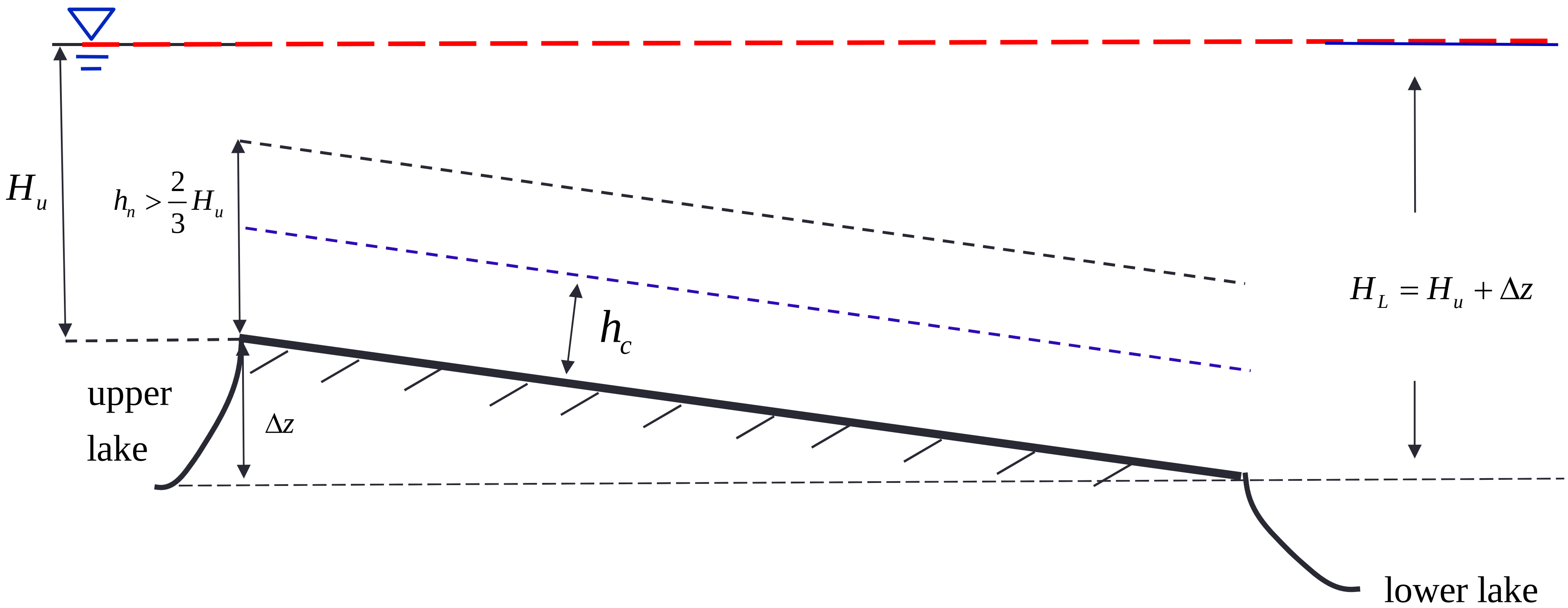
Two lake problem: downstream of mild slope

$$H_L > H_{L,\lim}$$



Two lake problem: downstream of mild slope

$$H_L = H_u + \Delta z$$



- Water level in the two lakes are the same
- there is no flow in the channel

General comments for determining surface profile

Rule 1: subcritical flow, $Fr<1$ or $h>h_c$, is always controlled from downstream location

Rule 2: supercritical flow, $Fr>1$ or $h < h_c$, is always having upstream control

Rule 3: in the absence of any control the only possible flow is normal, $h=h_n$.

Rule 4: gradually varied flow must follow surface profiles given by M1-3 or S1-3

Rule 5: transition from a supercritical to a subcritical flow possible only through a hydraulic jump.