Practice Problems CIE4420

Question 1

Consider an extraction well and an injection well with equal but opposite discharge Q. The extraction well is located at $(x, y) = (-d, 0)$ and the injection well is located at $(x, y) = (d, 0)$; the radius of both wells is r_w. The wells have been pumping for a very long time in a confined aquifer; the top of the confined aquifer is at $z = z_t$. Far away from the two wells, the head is equal to h_0. The head at $(x_1, y_1) = (2d, 0)$ is measured and is equal to h_1. The storage coefficient of the aquifer is S.
Questions:
a) Determine an expression for the transmissivity T of the confined aquifer based on the measurement of h_1.
b) Determine an expression for the maximum discharge Q of both wells such that the head at the extraction well remains confined.

At time $t = 0$, the injection well breaks down and it takes a week to fix the well. In the mean time, the extraction well keeps pumping with discharge Q.
Questions:
c) Determine an expression for the head in the aquifer in the week that the injection well is broken.
d) Determine an expression for the head in the aquifer for the time after the injection well has been fixed and starts injecting again with a discharge Q.

Question 2

Consider steady one-dimensional flow between two long canals. The water level in the canal on the left is h_L and the water level in the canal on the right is h_R (see Figure). The two canals are a distance L apart and a coordinate system is chosen such that $x = 0$ at the left canal. The transmissivity of the aquifer may be approximated as constant and equal to T. There is a uniform infiltration rate N between the two canals. Given: $h_L = 5$ m, $h_R = 5.6$ m, $L = 600$ m, $T = 200$ m^2/d, $N = 0.002$ m/d.

Questions:

a) Determine an expression for the head as a function of x between the two canals.

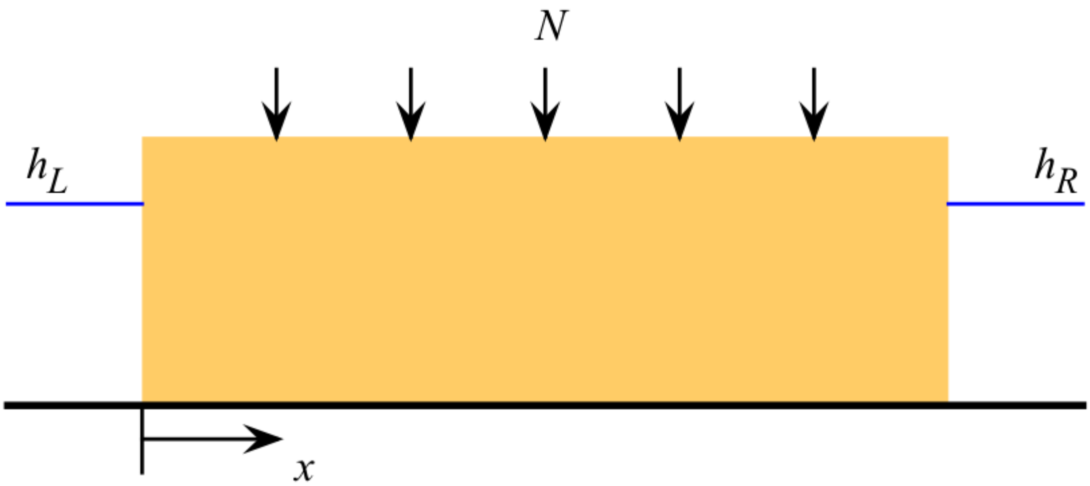
b) Determine the head at $x = L/3$ and $x = 2L/3$.

Approximate flow in the aquifer with the finite difference method using just 4 cells with $\Delta x = \Delta y = 200.0$ m, where the y direction is normal to the plane of flow.

c) Give a finite difference equation for the head in cell 2.

d) Give a finite difference equation for the head in cell 3.

e) Determine the finite difference solution for the head in cells 2 and 3 by solving the two equations you derived under c) and d) and compare your answer to the exact solution you determined under part b).



Question 3

Consider transient flow in a confined aquifer. A well with discharge Q turns on at time $t = 0$. The transmissivity of the aquifer is T and the storage coefficient is S. Assume that the head in the aquifer is equal to zero before the well starts pumping. Given: $Q = 500$ m^3/d.

Questions:

a) Give an expression for the head as a function of radial distance r and time t.

b) Give an expression for the head as a function of radial distance r and time t for large time using that for $u \ll 1$ the exponential integral may be approximated as $\mathrm{E}_1(u) \approx -\ln(u) - \gamma$ where $\gamma \approx 0.577$.

c) Give an expression for $\partial h/\partial t$ for large time.

In an observation well at $r = 10$ m from the well, the head at time $t_1 = 2$ days is $h_1 = -2.95$ m, and the head at time $t_2 = 2.2$ days is $h_2 = -2.975$ m.

Questions:

d) Estimate the transmissivity using the given head measurements assuming equations for large time are valid. Hint: Use your answer from part c).

e) The well is turned off after $t_3 = 3$ days of pumping. Give an expression for the head as a function of the radial distance r and time t that is valid after the well is turned off (so for $t > t_3$).

Question 4

Consider part of a finite difference model of a confined aquifer as shown below, where heads are given in meters. The hydraulic conductivity of the aquifer is $k = 10$ m/d, the thickness is $H = 10$ m, and the porosity is $n = 0.3$. All cells are square with $\Delta x = \Delta y = 100$ m.

Questions:

a) Compute how much water flows from cell 5 to cell 6.

b) Compute the average velocity for water flowing from cell 5 to cell 6.

c) There is areal leakage q_z in cell 6. Compute the specific discharge q_z (positive for upward leakage) through the top of cell 6.

d) The vertical leakage through the top of cell 6 comes through a leaky layer at the top of the confined aquifer. The resistance of the leaky layer is 2000 days. Compute the water level above the leaky layer.

e) Compute the head in cell 8 if it is given that there are no sinks or sources in cell 8 and that the right side of cell 8 is impermeable.

h_1=20.6	h_2=20.2	h_3=20.1	h_4=19.8
h_5=20.8	h_6=20.5	h_7=20.1	h_8
h_9=20.9	h_{10}=20.7	h_{11}=20.4	h_{12}=20.1

Question 5

Consider one-dimensional flow between a polder and a lake separated by a dike (see Figure). The lake extends to infinity to the left and the polder extends to infinity to the right. The aquifer has transmissivty $kH = 500\,\mathrm{m}^2/\mathrm{d}$ and thickness $H = 50\,\mathrm{m}$. The water level in the lake is $h_L = 0\,\mathrm{m}$ and the water level maintained in the polder is $h_p = -5\,\mathrm{m}$. The resistance of the aquitard on the lake bottom is c_L and the resistance of the aquitard below the polder is c_p; both values are unknown and will be determined in this question. Two piezometers are installed. One piezometer is located in the middle of the dike (at $x = 0$) and has head $h_0 = -2\,\mathrm{m}$; the other is installed at $d = 400\,\mathrm{m}$ from the dike in the polder and has a head $h_1 = -3.5\,\mathrm{m}$.

Questions:

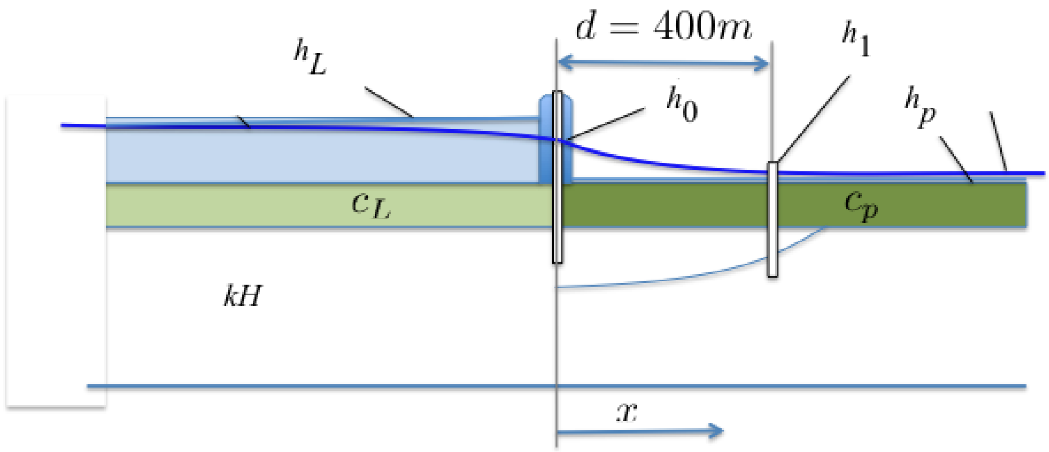
a) State all boundary conditions of the problem.

b) Derive an expression for the head below the lake using that $h(x = 0) = h_0$.

c) Derive an expression for the head below the polder using that $h(x = 0) = h_0$.

d) Compute the λ-value below the polder using that $h(x = d) = h_1$.

e) Compute the λ-value below the lake.



Question 6

A piezometer nest is located in a dune area near the North-Sea coast. The elevation of the top (z_t) and bottom (z_b) of the piezometer screens, the head h in the piezometers, and the chloride concentrations are given in the table. The density of fresh water is $\rho_f = 1000\,\text{kg/m}^3$ and that of seawater with chloride concentration of $18\,\text{g/L}$ is $\rho_s = 1022\,\text{kg/m}^3$

nr	z_t (m)	z_b (m)	h (m)	Cl^- (g/L)
1	+3	+2	5.01	0.03
2	-15	-16	4.98	0.04
3	-26	-27	1.20	0.06
4	-45	-46	1.18	0.06
5	-70	-71	0.53	0.07
6	-85	-86	0.51	0.11
7	-101	-102	0.49	0.12
8	-120	-121	-2.89	17.9
9	-145	-146	-2.89	18.0

Questions:

a) Indicate between which piezometer screens there is a confining bed.

b) Determine the approximate elevation of the interface at the piezometer nest.

At the piezometer nest, the freshwater discharge in the aquifer where the interface is located is $Q_x = 0.52\,\text{m}^2/\text{d}$ towards the sea; this aquifer is confined with an impermeable top at $z = -60$ m and a hydraulic conductivity $k = 25\,\text{m/d}$.

Questions:

c) What is the head gradient at the piezometer nest?

d) What is the gradient of the interface at the piezometer nest?

Question 7

Consider steady flow in a vertical cross-section between a canal on the left side and an impermeable rock outcrop on the right side. Both the bottom of the canal and the rock outcrop are approximated as vertical. Areal infiltration onto the aquifer is equal to N. The water level in the canal is h_0, measured with respect to the bottom of the aquifer. You may approximate the saturated thickness H of the aquifer as constant and equal to h_0 everywhere. The distance between the canal and the rock outcrop is L (see Figure).

Questions:

a) Derive an equation for the head as a function of position in the aquifer.

b) Derive an equation for the discharge vector Q_x as a function of position in the aquifer.

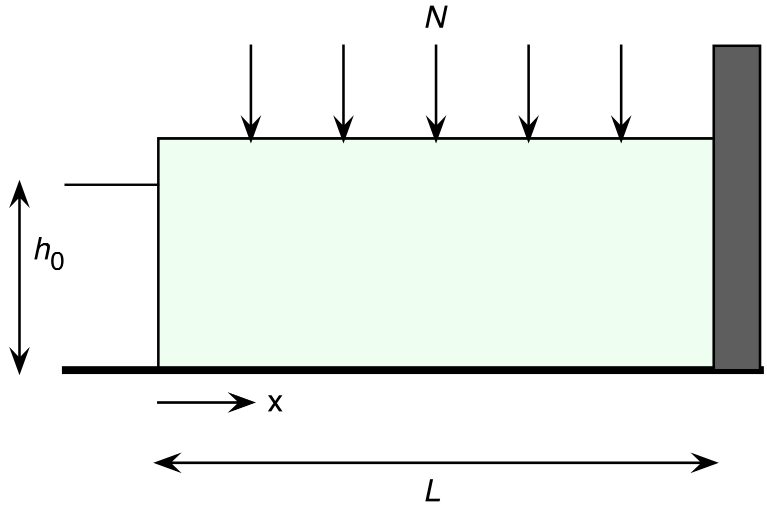
Upon further investigation, it turns out that there is a mud layer at the bottom and sides of the canal, which provides a resistance to outflow into the canal. Outflow Q_{out} from the aquifer into the canal may be written as $Q_{\text{out}} = H(h(x=0) - h_0)/c$, where $h(x=0)$ is the head in the aquifer right next to the mud layer, and c is the resistance to outflow of the mud layer.

Questions:

c) Derive an equation for the head as a function of position in the aquifer for the situation with the mud layer.

d) Derive an expression for the difference in head at the rock outcrop for the situation with and without the mud layer.

e) Compute the hydraulic conductivity if the head at the rock outcrop is measured to be $h(x = L) = h_r$, and it is given that $L = 2000$ m, $N = 1$ mm/d, $c = 4$ days, $h_0 = 20$ m, $h_r = 25.4$ m.



Question 8

Consider steady confined flow to two wells in a uniform flow field. Well 1 is located at $(x, y) = (-d, 0)$ and well 2 is located at $(x, y) = (d, 0)$. Both wells have the same discharge Q. The uniform flow is in the x direction and the magnitude is Q_{x0}. A figure with streamlines is shown below. Given: $Q = 160$ m^3/d, $d = 50$ m, and $Q_{x0} = 1$ m^2/d.

Questions:

a) Clearly draw the dividing streamlines of well 1 and well 2 in the figure.

b) Clearly indicate the approximate location of the stagnation points.

c) What is the distance, far upstream of well 1, between the dividing streamlines that separate water flowing to well 1 and water flowing to well 2?

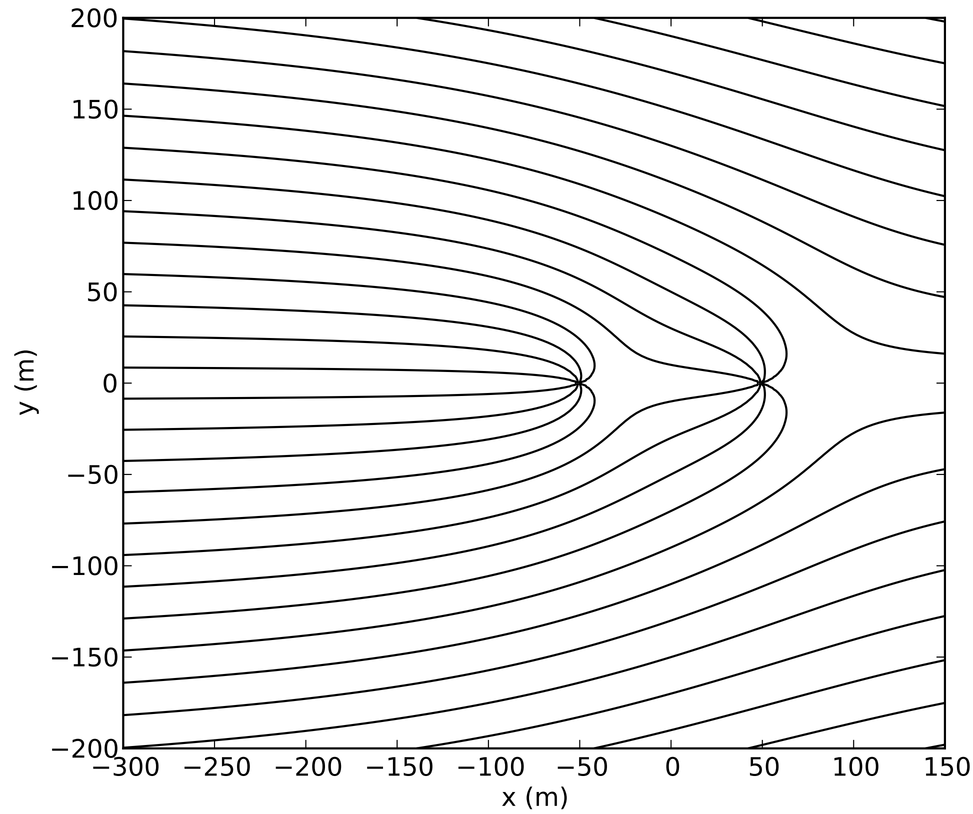
d) What is the distance between the dividing streamlines of well 2 far upstream of well 2?

e) Give an expression for the stream function as a function of x and y.

f) What is the value of the stream function at $(x, y) = (2d, 0)$?

g) What is the value of the stream function at $(x, y) = (0, d)$?

h) Compute the discharge of well 2 such that well 2 doesn't capture any water from the point $(x, y) = (0, d)$. The discharge of well 1 remains at $Q = 160$ m^3/d.



Question 9

A thick sandy unconfined coastal aquifer extends from a rocky mountain range to the sea (see Figure). The length of the aquifer is $L = 4000\,\mathrm{m}$ and the areal recharge is $N = 0.25\,\mathrm{mm/d}$. The runoff from the mountain front on the left side is $Q_{x0} = 1\,\mathrm{m^2/d}$. The hydraulic conductivity of the aquifer is $k = 20\,\mathrm{m/d}$, and the density of the seawater is $\rho_s = 1025\,\mathrm{kg/m^3}$. The freshwater is separated from the saltwater by an interface and the saltwater is at rest.

Questions:

a) Determine an expression for the discharge in the aquifer as a function of x.

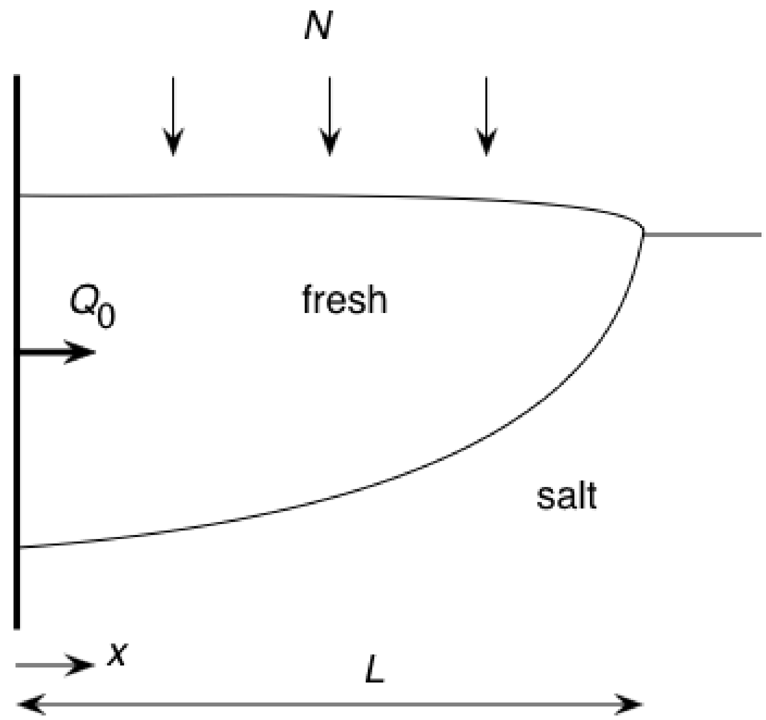
b) Determine an expression for the discharge potential in the aquifer as a function of x.

c) Compute the head at the mountain range.

d) Compute the thickness of the freshwater lens at the mountain range.

e) Assume the sea level rises by 1 meter. How will the head at the mountain range change?

f) Assume the sea level rises by 1 meter. How will the thickness of the freshwater lens change?



Question 10

Consider one-dimensional steady flow between two long parallel canals (fig. (a) below). The water level in the canals is h_0 and they are a distance $2d$ apart. The origin of the coordinate system is chosen halfway the two canals. The average saturated thickness between the two canals is H and the hydraulic conductivity is k.

Questions

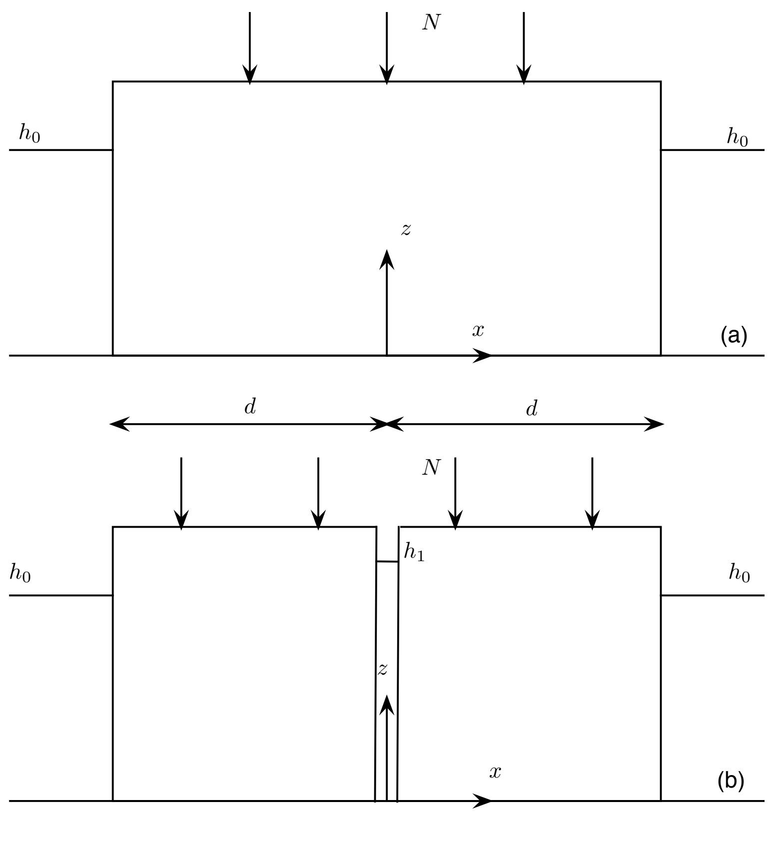
a) Give an expression for the head halfway the two canals.

b) Give an expression for the total flow into each canal per unit length of canal normal to the plane of flow.

To increase the heads in the aquifer, an irrigation canal is dug halfway the two existing canals (fig. (b) below). The head in the irrigation canal is maintained at h_1.

c) Give an expression for the head $h(x)$ to the right of the irrigation canal.

d) Derive an expression for the amount of water that needs to be added to the irrigation canal, per meter of canal normal to the plane of flow, to maintain the head h_1 in the canal.



Solutions

Solution question 1

a) $h = \frac{Q}{2\pi T}\ln(r_1/r_2) + h_0$. $\frac{Q}{2\pi T}\ln(3d/d) + h_0 = h_1 \to T = \frac{Q}{2\pi(h_1-h_0)}\ln(3)$
b) $h = z_t$ for $r_1 = r_w$ and $r_2 = 2d$. $\frac{Q}{2\pi T}\ln(r_w/2d) + h_0 = z_t \to Q = 2\pi T(z_t - h_0)/\ln(r_w/(2d))$
c) $h = \frac{Q}{2\pi T}\ln(r_1/r_2) + h_0 - \frac{Q}{4\pi T}\mathrm{E}_1(u_1)$ where $r_1 = \sqrt{(x+d)^2+y^2}$, $r_2 = \sqrt{(x-d)^2+y^2}$, $u_1 = r_2^2 S/(4Tt)$.
d) $h = \frac{Q}{2\pi T}\ln(r_1/r_2) + h_0 - \frac{Q}{4\pi T}\mathrm{E}_1(u_1) + \frac{Q}{4\pi T}\mathrm{E}_1(u_2)$ where $u_2 = r_2^2 S/(4T(t-7))$

Solution question 2

a) $h = -N/(2T)(x^2 - xL) + (h_R - h_L)x/L + h_L$
b) $h(x = 200) = 5.6$ m, $h(x = 400) = 5.8$ m.
c) $h_L - 2h_2 + h_3 = -N(\Delta x)^2/T \to -2h_2 + h_3 = -5.4$
d) $h_2 - 2h_3 + h_R = -N(\Delta x)^2/T \to h_2 - 2h_3 = -6$
e) $h_2 = 5.6$ m, $h_3 = 5.8$ m. They are the same!

Solution question 3

a) $h = -\frac{Q}{4\pi T}\mathrm{E}_1(u)$ with $u = \frac{r^2 S}{4Tt}$
b) $h = \frac{Q}{4\pi T}[\ln(r^2 S/(4Tt)) + \gamma]$
c) $\frac{\partial h}{\partial t} = -\frac{Q}{4\pi T}\frac{1}{t}$
d) $\frac{\partial h}{\partial t}(t = 2.1) \approx (h_2 - h_1)/(t_2 - t_1) = -0.125$ m/d.
$T = -\frac{Q}{4\pi t}/\frac{\partial h}{\partial t} \approx -500/(4 * \pi * 2.1)/(-0.125) \approx 150$ m^2/d.
e) $h = -\frac{Q}{4\pi T}[\mathrm{E}_1(u) - \mathrm{E}_1(u_3)]$ with $u_3 = \frac{r^2 S}{4T(t-t_3)}$

Solution question 4

a) $Q_{5\to 6} = kH(h_5 - h_6) = 30$ m^3/d.
b) $q = Q_{5\to 6}/H/\Delta y = 0.03$ m/d. $v = q/n = 0.1$ m/d.
c) $Q = kH(h_7 + h_2 + h_5 + h_{10}) - 4kHh_6 = -20$ m^3/d. $q_z = Q/(\Delta x)^2 = -0.002$ m/d.
d) $q_z = (h_6 - h_{\text{top}})/c = -0.002 \to h_{\text{top}} = 24.5$ m.
e) $h_8 = (h_4 + h_7 + h_{12})/3 = 20$ m.

Solution question 5

a) $x \to -\infty$, $h \to h_L$. $x \to \infty$, $h \to h_p$. $h(0^+) = h(0^-)$. $Q_x(0^+) = Q_x(0^-)$
b) $h = h_L + (h_0 - h_L)\exp(x/\lambda_L)$
c) $h = h_p + (h_0 - h_p)\exp(-x/\lambda_p)$
d) $h(x = d) = h_p + (h_0 - h_p)\exp(-d/\lambda_p) = h_1 \to \exp(-d/\lambda_p) = (h_1 - h_p)/(h_0 - h_p)$ so that

$\lambda_p = -d/\ln[(h_1 - h_p)/(h_0 - h_p)] = 580$ m.
e) $Q_x(0^+) = Q_x(0^-)$ to get $\lambda_L = -\lambda_p(h_0 - h_L)/(h_0 - h_p)$

Solution question 6

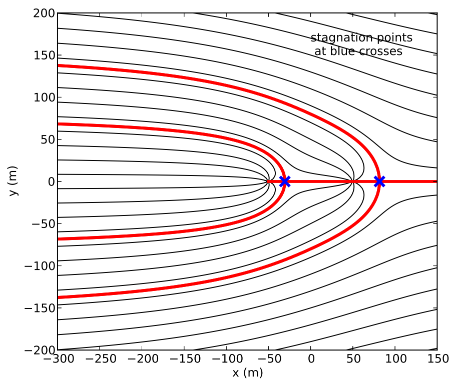
a) There are confining beds between piezometers 2 and 3, and between 4 and 5.
b) $z \approx 112$ m.
c) $H_f = -60 - (-112) = 52$ m. $Q_x = -kH_f \mathrm{d}h/\mathrm{d}x$, so that $\mathrm{d}h/\mathrm{d}x = -Q_x/(kH_f) = -0.0004$
d) $\alpha = \rho_f/(\rho_s - \rho_f) = 1/0.022 = 45.5$. Gradient of interface $\mathrm{d}\zeta/\mathrm{d}x = -\alpha \mathrm{d}h/\mathrm{d}x = 0.018$.

Solution question 7

a) $\Phi = -\frac{1}{2}Nx^2 + NLx + \Phi_0$, with $\Phi_0 = kHh_0$ and $h = \Phi/(kH)$
b) $Q_x = N(x - L)$
c) $h(x = 0) = NLc/H + h_0 = h^*$, so that $\Phi = -\frac{1}{2}Nx^2 + NLx + \Phi^*$ with $\Phi^* = kHh^*$
d) $\Delta h = NLc/H$, the same for any point in the aquifer.
e) $\Phi_r = \frac{1}{2}NL^2 + \Phi^*$ so that $h_r = \frac{1}{2kH}NL^2 + h^*$ and $k = NL^2/(2H(h_r - h^*) = 20$ m/d

Solution question 8

a) and b)



c) $w = Q/Q_{x0} = 160$ m
d) $w = 2Q/Q_{x0} = 320$ m
e) $\Psi = \frac{Q}{2\pi}\theta_1 + \frac{Q}{2\pi}\theta_2 - Q_{x0}y$
f) $\theta_1 = \theta_2 = 0$, $y = 0$, $\Psi(2d, 0) = 0$.
g) $\theta_1 = \pi/4$, $\theta_2 = 3\pi/4$, $y = d$, $\Psi(0, d) = Q/2 - Q_{x0}d = 160/2 - 1 \cdot 50 = 30$ m^3/d
h) $Q_2 = [-\frac{Q_1}{2\pi}\theta_1 + Q_{x0}d]/\frac{2\pi}{\theta_2} = 80$ m^3/d

Solution question 9

a) $Q_x = Q_0 + Nx$
b) $\Phi = -\frac{N}{2}(x^2 - L^2) - Q_0(x - L)$
c) $\Phi(x = 0) = NL^2/2 + Q_0L$ $h = \sqrt{2\Phi/[k(\alpha + 1)]} = 3.8$ m.
d) $H_f = (\alpha + 1)h = 157$ m.
e) Head rises by 1 m.
f) Thickness of the freshwater lens doesn't change.

Solution question 10

a) $h = -\frac{N}{2T}(x^2 - d^2) + h_0$ so that $h(0) = \frac{Nd^2}{2T} + h_0$, where $T = kH$.
b) Nd (from continuity of flow)
c) $h(x) = -\frac{N}{2T}(x^2 - xd) + \frac{h_0 - h_1}{d}x + h_1$
d) $Q_x = N(x - d/2) + T(h_1 - h_0)/d$, so $Q_x(0) = -Nd/2 + T(h_1 - h_0)/d$. Total discharge is twice this as water also flows to the left. $Q_{\text{tot}} = -Nd + 2T(h_1 - h_0)/d$