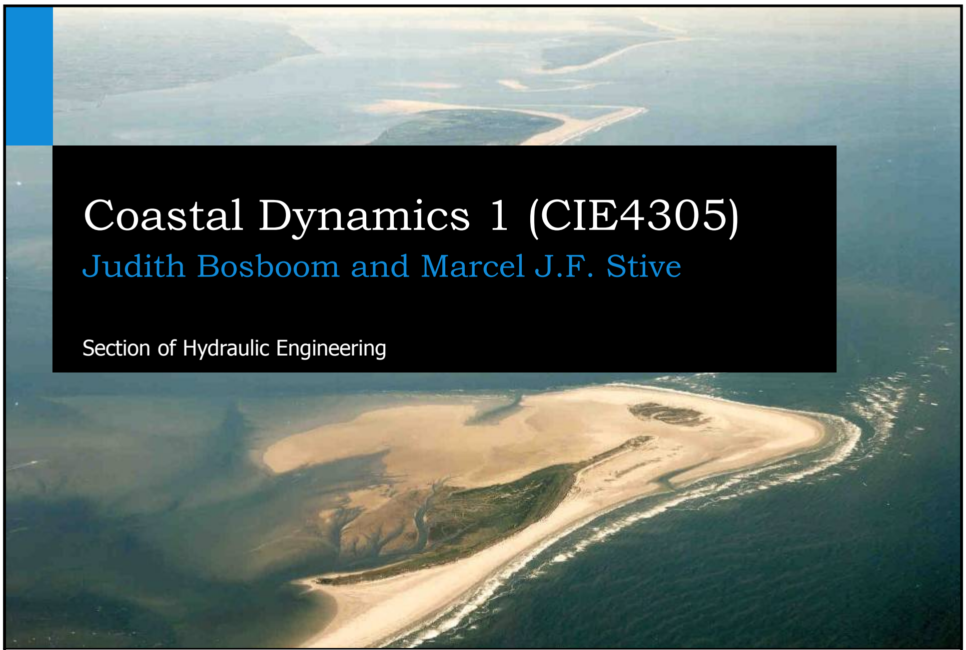


Coastal Dynamics 1 (CIE4305)

Judith Bosboom and Marcel J.F. Stive

Section of Hydraulic Engineering

5.

Coastal hydrodynamics – part I

Coastal Dynamics 1

Maple TA up to Chapter 4

Summary Data	Chapter 1+2 - Stage A	Chapter 1+2 - Stage B	Chapter 3 - Stage A	Chapter 3 - Stage B	Chapter 4 - Stage A	Chapter 4 - Stage B
# Students	209	209	209	209	209	209
# Attempts	197	188	192	190	184	181
Mean	92%	82%	93%	80%	96%	90%
Total Points	17.0	14.0	32.0	31.0	14.0	9.0

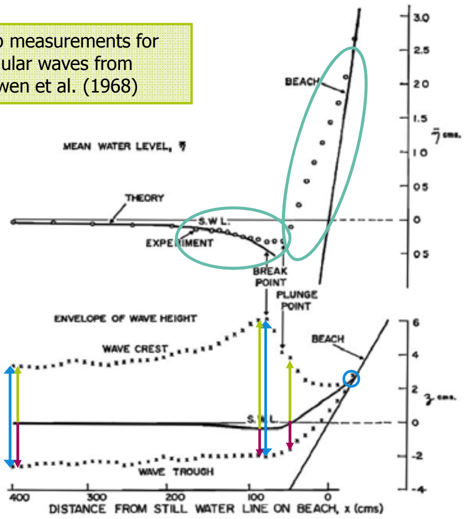
Coastal Dynamics 1

Contents

5-A Introduction

Wave transformation in shoaling and breaking zone

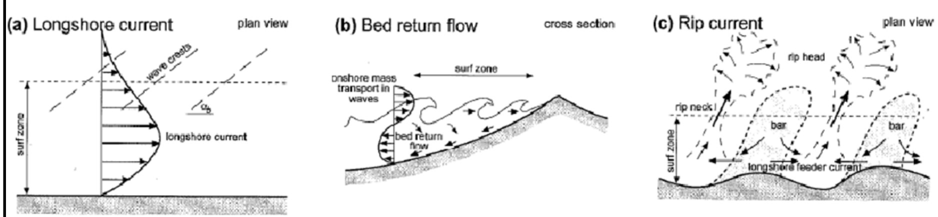
Lab measurements for regular waves from Bowen et al. (1968)



- Set-down in shoaling zone
- Set-up in surf zone
- Wave height increase in shoaling zone and decrease in surf zone
- Shoaling waves become more skewed => net sediment transport
- Pitching forward and wave-breaking

5-A Introduction

Wave-induced currents



- Alongshore wave force in surf zone => cross-shore gradient in radiation shear stress: $\frac{dS_{yx}}{dx}$
- ~1 m/s

- Large mass-flux in breaking waves gives undertow
- ~0.1-0.5 m/s

- Mass-flux and alongshore set-up differences
- ~ 0.5-1 m/s

5-A Introduction

Small rip currents carry sediment-laden water seaward through the surf zone

1-B The beach a river of sand

Horizontal circulation currents

South-Holland Coast with Delfland groynes.
These groynes perpendicular to the coast act like brakes with regard to longshore transport of sand. That is their positive function. They also have a negative function since they initiate rip currents perpendicular to the coast causing considerable off-shore sand transport. The resulting offshore sand plumes can be observed from the air.

Photo 1985 - © & courtesy Ir. Bureau Svašek b.v.

Coastal hydrodynamics – part I

Chapter 5 of lecture notes

A. Introduction
B. Wave height transformation
C. Bed friction
D. Wave asymmetry and skewness
E. Momentum and wave forces
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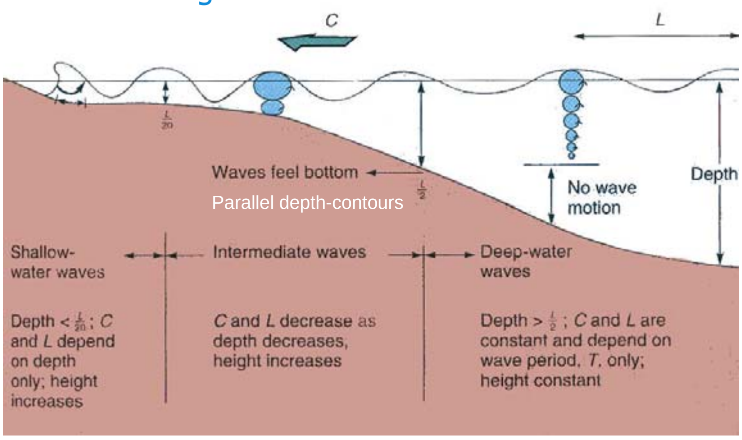
I. Wind-induced set-up and currents

5-B Wave height transformation

Waves start feeling the bottom in intermediate water



$c=\sqrt{g(h+\eta)}$ $c=\sqrt{gh}$ $c=\frac{gT}{2\pi}\tanh kh$ $c=\frac{gT}{2\pi}$

See Brightspace for animation wave orbital motion

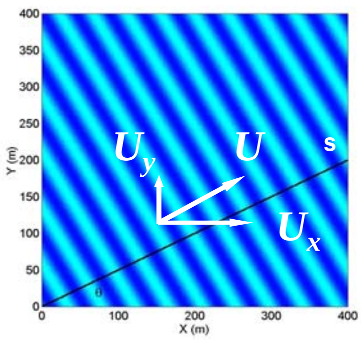
5-B Wave height transformation

Refraction as a result of depth-changes (changes in θ and associated changes in H)

5-B Wave height transformation

Energy conservation

- Waves transport energy in their propagation direction
- Consider energy E integrated over depth (J/m^2)
- Energy E is a scalar, energy flux U is a vector quantity



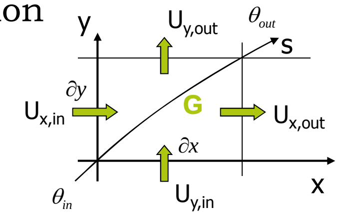
Neglect input of energy due to wind, but consider dissipation D (W/m^2). Assume stationary conditions.
What is the energy balance?

$$U_x = U \cos\theta$$
$$U_y = U \sin\theta$$

Assumption: one value for θ can be used

5-B Wave height transformation

Energy balance in 2 horizontal dimensions



NB. Compare from Ch3: $\boxed{\frac{\partial \kappa}{\partial t} + \frac{\partial \kappa \bar{u}}{\partial x} + \frac{\partial \kappa \bar{v}}{\partial y} = P}$

$$\frac{\partial E}{\partial t} + \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + D = 0$$

= **zero** for steady situation

$$\frac{d}{ds} E\, nc$$

Dependent on water depth!

or

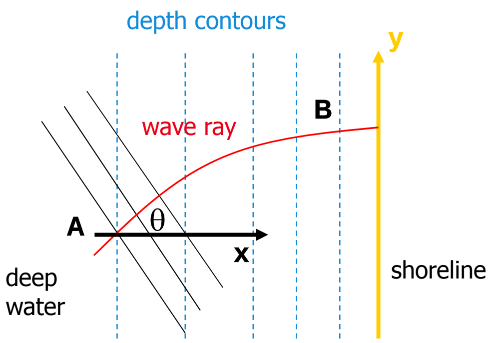
$$\frac{\partial}{\partial x} Enc\cos\theta + \frac{\partial}{\partial y} Enc\sin\theta$$

So in addition to group velocity wave angle has to be known!

- **wave breaking** (depth-induced and white-capping)
- bottom friction
- interaction with vegetation,......

5-B Wave height transformation

Shoaling and refraction on an alongshore uniform coast



Change in the wave angle is given by Snell's law:

$$\boxed{\frac{\sin\theta}{c} = \text{constant}}$$

How does the wave height change from A to B?

5-B Wave height transformation

Interaction with variable bathymetry

outside surf zone

$$\boxed{\cancel{\frac{\partial E}{\partial t}} + \frac{\partial}{\partial x}(Ec_g \cos\theta) + \cancel{\frac{\partial}{\partial y}(Ec_g \sin\theta)} + \cancel{D} = 0} \quad \Rightarrow$$

steady alongshore uniform

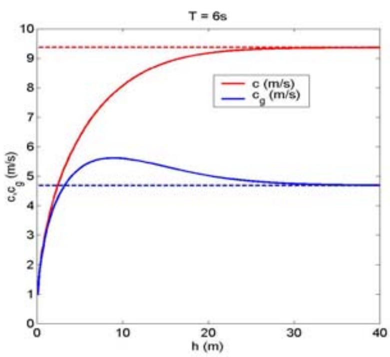
$$Ec_g \cos\theta = \text{constant} \quad \Rightarrow \quad E_A c_{g,A} \cos\theta_A = E_B c_{g,B} \cos\theta_B \quad \Rightarrow$$

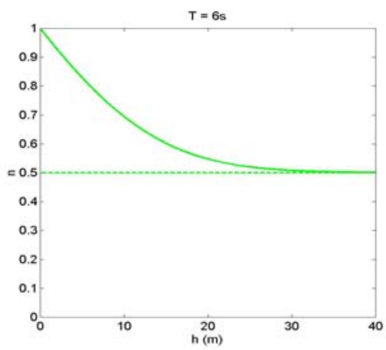
$$E_B = E_A \frac{c_{g,A}}{c_{g,B}} \frac{\cos\theta_A}{\cos\theta_B} \quad \Rightarrow \quad H_B = H_A \sqrt{\frac{c_{g,A}}{c_{g,B}}} \sqrt{\frac{\cos\theta_A}{\cos\theta_B}}$$

5-B Wave height transformation

Depth-induced changes in group velocity

The group velocity is a function of depth $c_g = \left[\frac{1}{2} + \frac{kh}{\sinh 2kh}\right]\frac{\omega}{k} = nc$



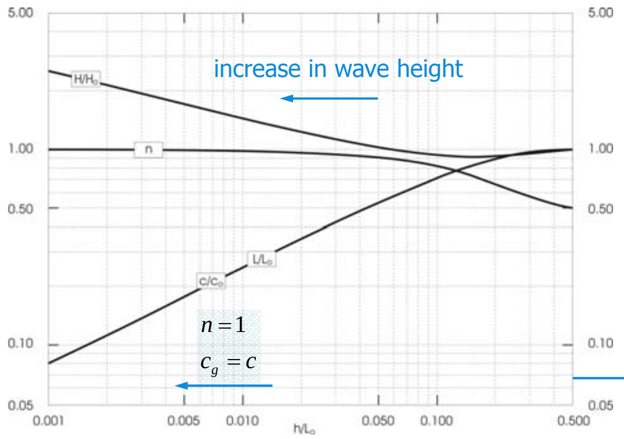


5-B Wave height transformation

Shoaling factor $\sqrt{c_{g,A}/c_{g,B}}$

Starting at deep water:

$$H(x) = H_0 \sqrt{\frac{c_{g,0}}{c_g(x)}} = K_s H_0$$



$$c_g = nc = \left[\frac{1}{2} + \frac{kh}{\sinh 2kh}\right]\frac{\omega}{k}$$

$$n = \frac{1}{2}$$

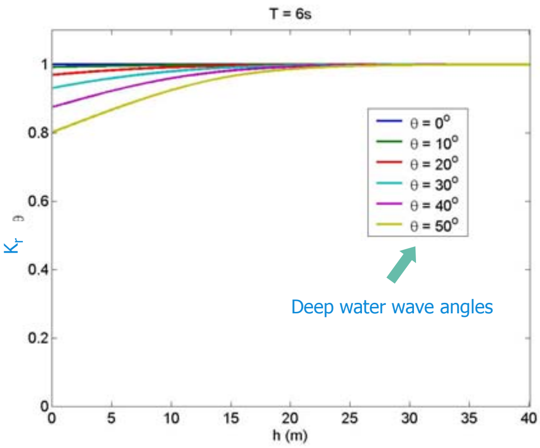
$$c_g = \frac{1}{2}c$$

5-B Wave height transformation

Refraction factor

$$K_r(x) = \sqrt{\frac{\cos\theta_0}{\cos\theta(x)}}$$

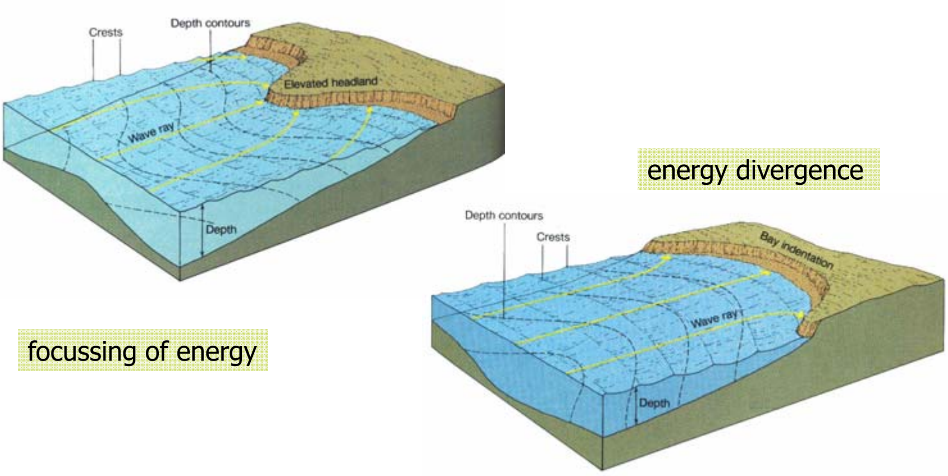
Where θ(x) is obtained with Snell's law!



Uniform coast:
effect of K_r is to reduce the increase in wave height due to shoaling

5-B Wave height transformation

Wave refraction as a result of along-crest phase speed variation (due to depth variations)

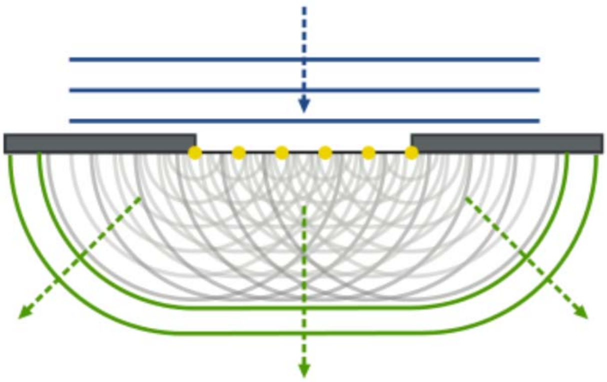


5-B Wave height transformation

Wave diffraction

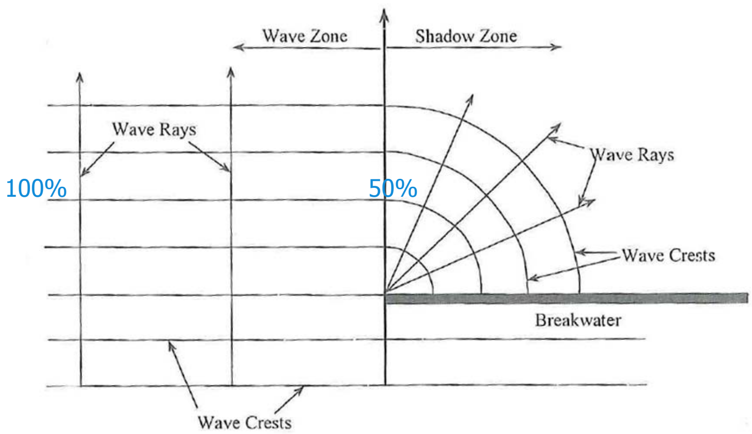
5-B Wave height transformation

Huygens-Fresnel principle



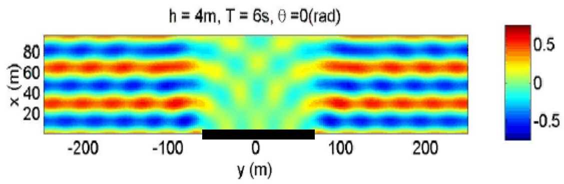
5-B Wave height transformation

Along crest transfer of energy

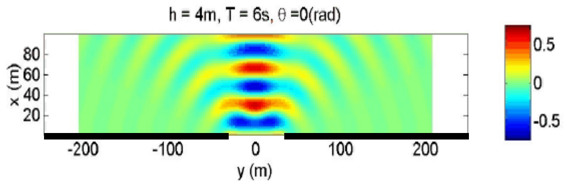


5-B Wave height transformation

Detached breakwater: combination of two diffraction patterns

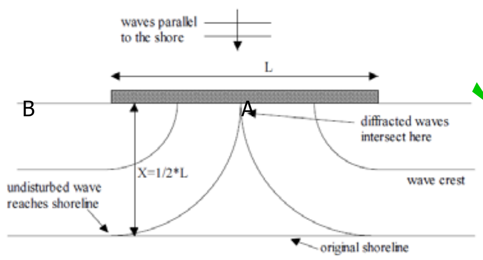


Gap, harbour

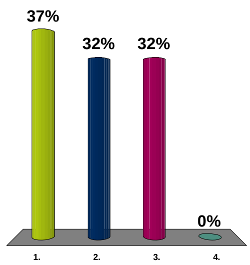


5-B Wave height transformation

At location **A** in the lee of the breakwater, I expect the wave height to be equal to:

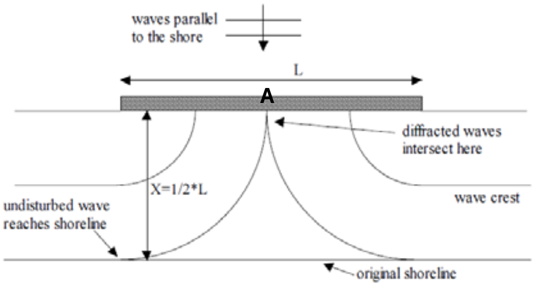


1. The wave height at B
2. Half the wave height at B
3. ✔ Zero
4. Abstain



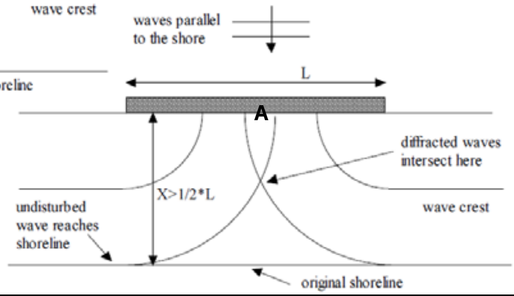
5-B Wave height transformation

Based on diffraction pattern: L/X > 2 implies H = 0 in centreline of the structure



L/X=2

Based on non-interfering diffraction patterns:
- L/X> 2 tombolo
- L/X< 2 salient (see below)



Coastal hydrodynamics – part I

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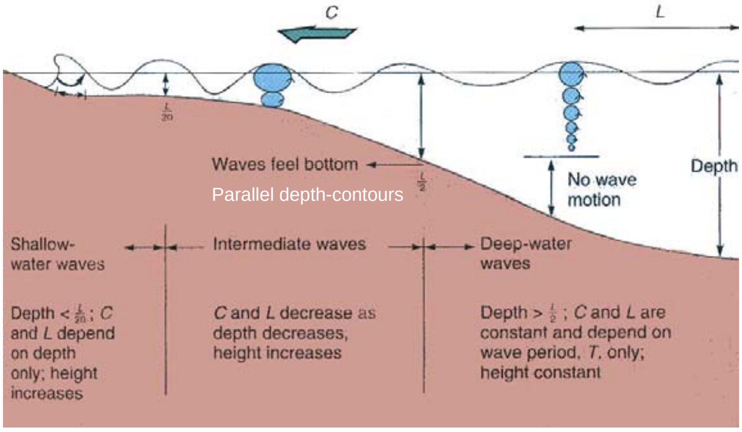
5-B Wave height transformation

Waves start feeling the bottom in intermediate water



$c = \sqrt{g(h+\eta)}$ $\quad c = \sqrt{g h}$ $\quad c = \frac{gT}{2\pi} \tanh kh$ $\quad c = \frac{gT}{2\pi}$

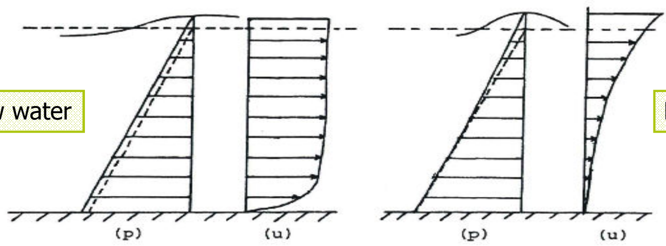
See Brightspace for animation wave orbital motion

5-C Bed friction

See Brightspace for animation wave orbital motion

Pressure (p) and velocity (u) in long and short waves

Shallow water



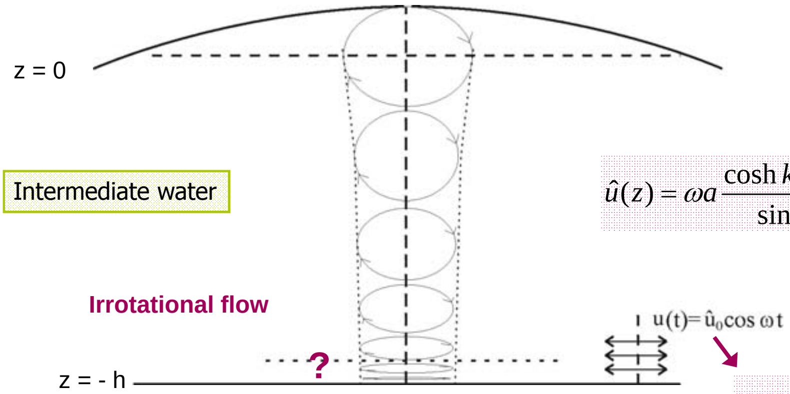
Deep water

- p hydrostatic $\quad p = p_0 + p_{wave} = -\rho g z + \rho g \eta$
- u uniform $\quad \hat{u} = \frac{\omega a}{kh} = c\frac{a}{h} = \sqrt{gh}\,\frac{H}{2h}$
- significant bed resistance

- p non-hydrostatic
- u non-uniform
- no bed resistance

5-C Bed friction

Wave orbital velocities



$$\hat{u}(z) = \omega a \frac{\cosh k(h+z)}{\sinh kh}$$

$$\hat{u}_0 = \frac{\omega a}{\sinh kh}$$

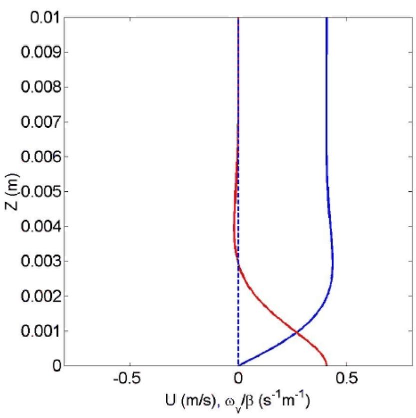
See Brightspace for animation wave orbital motion

5-C Bed friction

Bottom-induced boundary layer

- Wave boundary layer is transition from "normal" orbital motion to bed
- At bed: flow sticks to the bed (no-slip condition) due to viscosity and **turbulence** generated at the boundary
- Large velocity gradients and hence **large shear stresses**
- **Limited thickness** of order 1-10 cm (for short period waves)

Free stream velocity



5-C Bed friction

Shear stresses in wave boundary layer

Viscous stresses (in absence of turbulence):

$$\tau_{viscous}(z) = \rho \nu \frac{\partial u}{\partial z} \qquad \nu \sim 10^{-6} m^2/s$$

Turbulence stresses analogous:

$$\tau_{turbulence}(z) = \rho \overline{u'w'} = \rho \nu_T \frac{\partial u}{\partial z} \qquad \nu_T \sim 10^{-1} - 10^{-2} m^2/s$$ in nearshore zone

ν_T from various sources

- wave boundary layer
- wave breaking
- slope or wind driven current

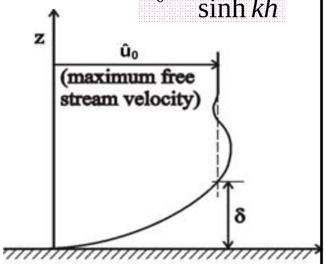
5-C Bed friction

Practical aspects of wave boundary layer

1. Orbital motion incurs bed shear stress
   => can set sediment into motion
2. Friction in the boundary layer
   => dissipation of energy from the flow above (D_f)
3. Wave force pushing the flow forward
   (Longuet-Higgins streaming: $U_\delta = \frac{3}{4}\frac{\hat{u}_0^2}{c}$)
   => net onshore directed sediment transport

Practical approach: relate all relevant quantities to free stream velocity

$$\hat{u}_0 = \frac{\omega a}{\sinh kh}$$



the thinner the boundary layer, the larger the velocity gradients and hence the stresses

5-C Bed friction

Quadratic friction law (instead of detailed modelling of wave boundary layer)

Uniform depth-averaged current $\vec{U}$

$$\tau_b = \rho c_f \left|\vec{U}\right| \vec{U}$$

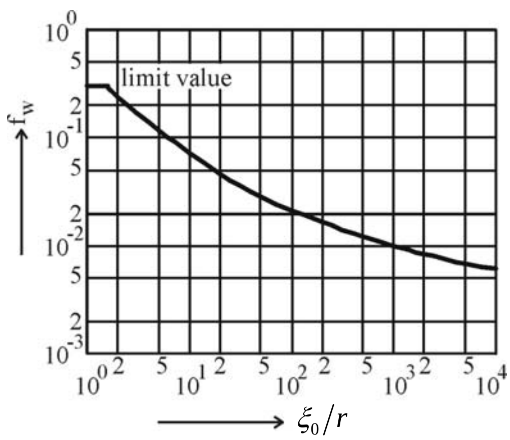
c_f depends on bed material and bed forms

for slope driven current: $c_f = \dfrac{g}{C^2}$

Quadratic friction law for waves? What would that look like for f.i. a single harmonic wave with $u_0 = \hat{u}_0 \cos \omega t$?

5-C Bed friction

Wave friction factor as a function of particle excursion amplitude over bed roughness: $\dfrac{\xi_0}{r} = \dfrac{\hat{u}_0}{\omega r}$



Magnitude wave versus current friction factor?

often $\boxed{\tfrac{1}{2} f_w >> c_{f,\,current}}$

See example 5-1

5-C Bed friction

Bed shear stress in combined wave-current flow

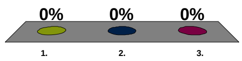
$$\tau_{\max} \overset{?}{=} \max\left(\tau_c + \tau_w\right) = \tau_c + \hat{\tau}_w$$

$$\tau_{\mathrm{m}} \overset{?}{=} \left\langle \tau_c + \tau_w \right\rangle = \tau_c$$

Do you agree?

1. Yes
2. No
3. Abstain

Response Counter



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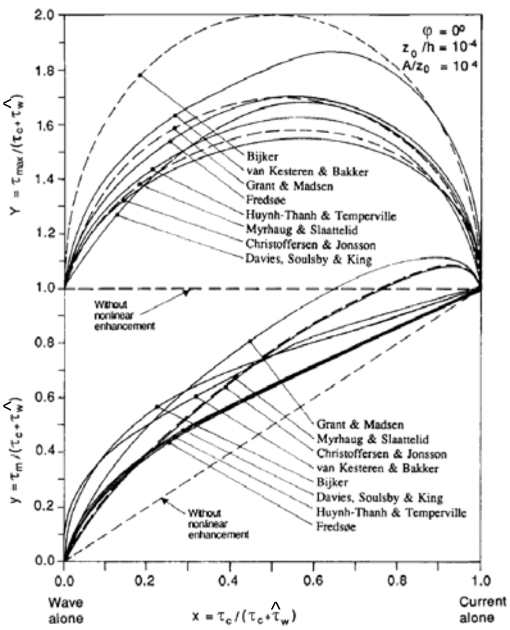
5-C Bed friction

Non-linear enhancement of bed shear stress in wave-current motion:

$$\tau_{\max} > \tau_c + \hat{\tau}_w$$

$$\tau_{\mathrm{m}} > \tau_c$$



From Soulsby et al. (1993)

5-C Bed friction

Relevance to sediment transport

- Shear stress sets sediment grains in motion and associated turbulence keeps them suspended in the water column
- High shear stresses under waves (or wave-current motion) in the nearshore
- But: a sinusoidal wave will just move the sand back and forth
- Hence the popular saying: waves stir up sediment, currents transport it
- or…..transport due to wave skewness

Coastal hydrodynamics – part I

Chapter 5 of lecture notes

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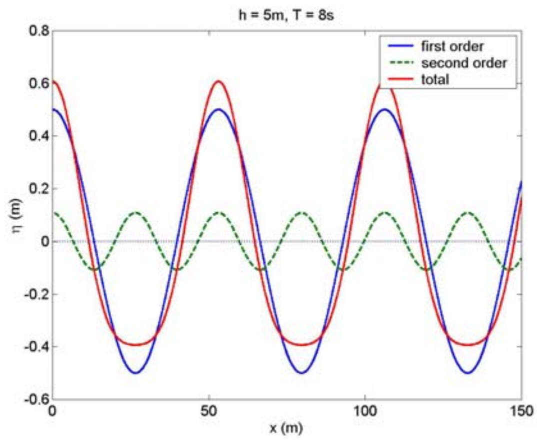
5-D Wave skewness and asymmetry

Non-linearities become important in shallow water

- In shallow water: a/h is not small anymore
- Stokes' higher order terms correct for the non-linear surface elevation

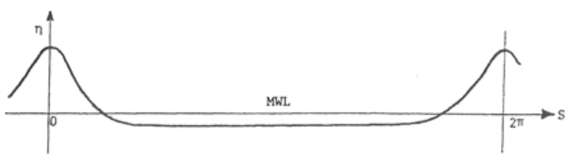


5-D Wave skewness and asymmetry

Skewness is asymmetry about the horizontal axis

Shoaling waves:

- long, flat troughs
- narrow, peaked crests



skewness

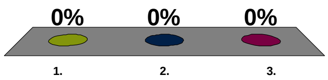
$$\langle \eta^3 \rangle > 0$$

5-D Wave skewness and asymmetry

Which of the following statements is **true**?
The skewness of a irregular deep water wave field is:

1. $\langle \eta^3 \rangle < 0$
2. $\langle \eta^3 \rangle = 0$
3. $\langle \eta^3 \rangle > 0$

Response Counter



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5-D Wave skewness and asymmetry

Sediment transport due to wave skewness

- Wave skewness in shoaling waves
  - higher on-shore velocities at the crest
  - lower off-shore velocities at the trough
- Near-bed sediment concentration

$$c_s(t) \approx A|\tau_b(t)|$$
$$\tau_b(t) \approx \rho c_{f,w} |u_0(t)| u_0(t) \quad \Rightarrow \quad c_s(t) \approx B u_0^{\,2}$$

- Sediment transport $S(t) \approx u_0 c_s \approx B u_0^3 \quad \Rightarrow \quad \langle S \rangle = B \langle u_0^3 \rangle$

for a sine wave: $\langle S \rangle = 0$

for a positively skewed signal: $\langle S \rangle > 0$

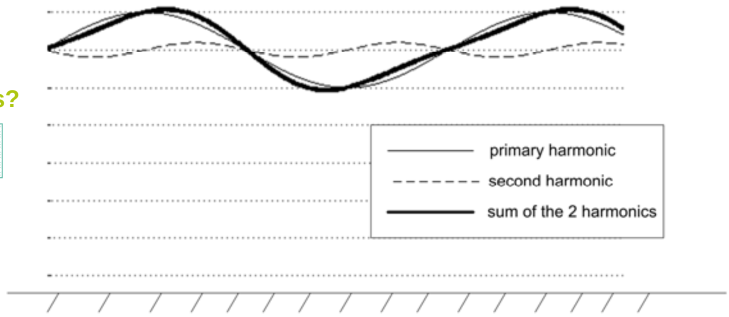
5-D Wave skewness and asymmetry

Asymmetry about the vertical axis

- Traditional non-linear shallow water theories: $c = \sqrt{g(h+\eta)}$
- Pitched forward shape and eventually wave breaking

Skewness?

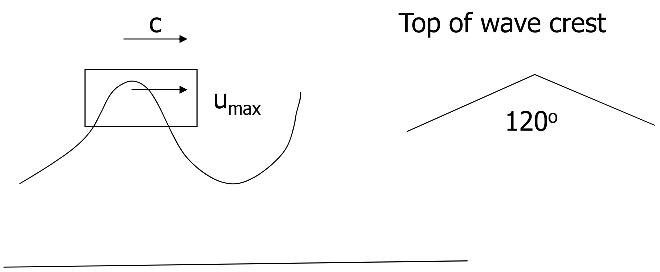
$\langle \eta^3 \rangle = 0$



Sawtooth-like wave with a steep face (out of phase second harmonic)

5-D Wave skewness and asymmetry

Maximum wave height



Miche, 1944

$$u_{max} = c \Rightarrow H_{max} \cong 0.14L \tanh\frac{2\pi h}{L}$$

$H_{max} \cong 0.14L$ (deep water)

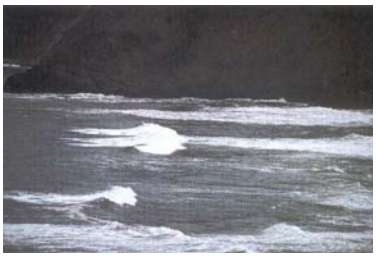
$H_{max} \cong 0.88h$ (shallow water)

Breaker parameter or index $\Rightarrow \gamma = \left[\frac{H}{h}\right]_{max} = \frac{H_b}{h_b} \approx 0.88$

5-D Wave skewness and asymmetry

A plunger loses energy more quickly than a spiller

plunger



spiller

Influence of relative bottom slope

5-D Wave skewness and asymmetry

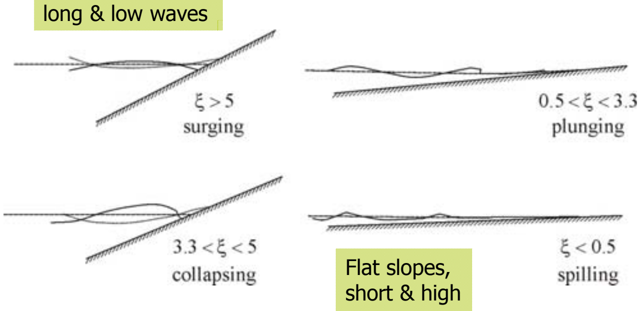
Breaker types described by Iribarren number ξ

bottom slope

$$\xi = \frac{\tan\alpha}{\sqrt{H_0 / L_0}}$$

wave steepness

Steep slopes, long & low waves



Flat slopes, short & high waves

$\gamma \approx 0.6 \sim 0.8$ for spilling breakers

$\gamma \approx 0.8 \sim 1.2$ for plunging breakers

5-D Wave skewness and asymmetry

Waves break when depth is the order of the wave height

Regular waves

- Breaker index $\gamma = \dfrac{H_b}{h_b} \approx 0.8$
  - Solitary wave theory: $\gamma = 0.78$ Miche: $\gamma = 0.88$

Irregular waves

- $\dfrac{H_{s,b}}{h_b}$ for which largest waves are breaking is half the value of breaker indices

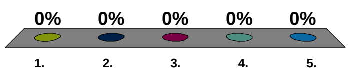
Simple dissipation model

- $\gamma = \dfrac{H}{h}$ is constant throughout the breaker zone

5-A & 5-B

Which of the following statements is **wrong**?

1. Due to refraction, wave crests tend to become parallel to the shallow water depth contours
2. At the Dutch coast we would expect spilling breakers most of the time
3. For the same free stream velocity shorter waves result in larger bed shear stresses
4. For waves propagating into intermediate water the phase velocity first slightly increases and then decreases
5. Diffraction implies along-crest transfer of energy



Coastal hydrodynamics – part I

Chapter 5 of lecture notes

5-E Momentum and wave forces

Waves carry mass and momentum

- Momentum = mass transport or mass flux:

$$\rho\vec{u} = (\rho u, \rho v, \rho w)$$

Vector quantity

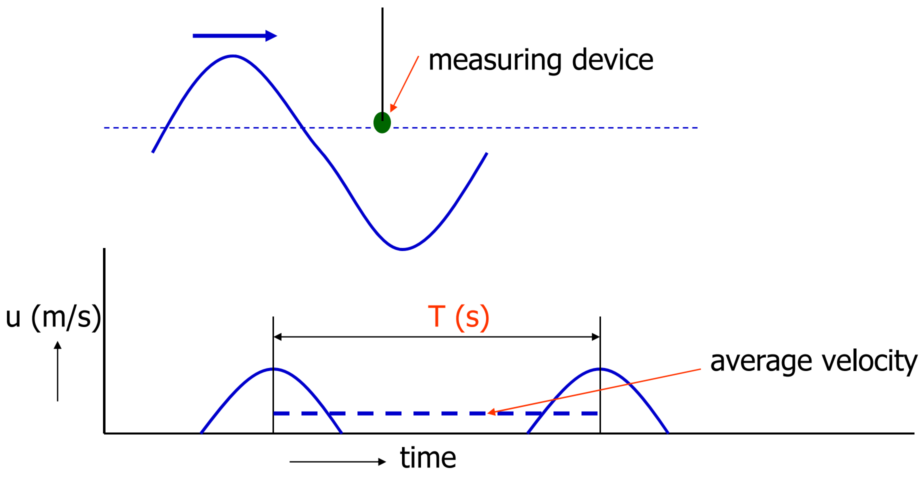
- Net flux of mass associated with wave propagation:

$$q = \overline{\int_{-h_0}^{\eta} \rho u dz}$$

Horizontal orbital velocity in wave propagation direction

5-E Momentum and wave forces

Time-averaged mass flux only non-zero above trough level



5-E Momentum and wave forces

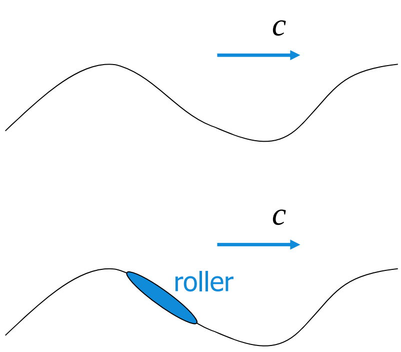
Mass flux in breaking and non-breaking waves

$$q_{non-breaking} = \overline{\int_0^{a\cos\omega t} \rho \frac{a\omega}{\tanh kh} \cos\omega t \, dz} =$$

$$\overline{a\cos\omega t \, \rho \frac{a\omega}{\tanh kh} \cos\omega t} =$$

$$\frac{\rho a^2 \omega}{2\tanh kh} = \frac{\rho g a^2}{2c} = \frac{E}{c}$$

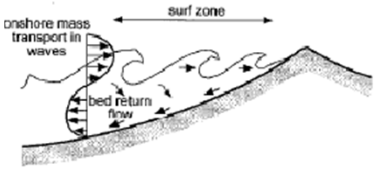
also known as Stokes' drift



$$q_{drift} = q_{non-breaking} + q_{roller} = \frac{E}{c} + \frac{\alpha E_r}{c}$$

5-E Momentum and wave forces

In a closed system (flume, coastline) continuity requires zero net mass transport



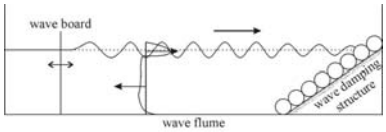
breaking: undertow

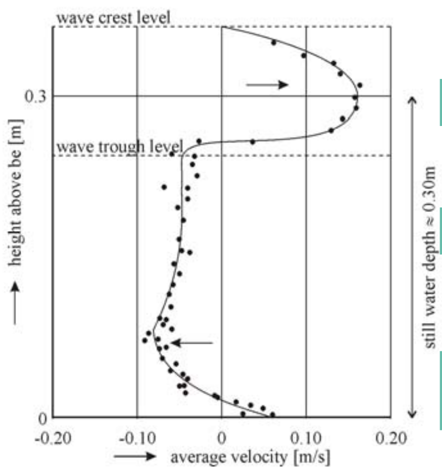
undertow velocity

$$U_{\text{below trough}} = -\frac{q_{drift,x}}{\rho h} = -\frac{q_{drift}\cos\theta}{\rho h}$$

5-E Momentum and wave forces

Mean wave-induced flow (non-breaking waves!)





mass flux in propagating waves

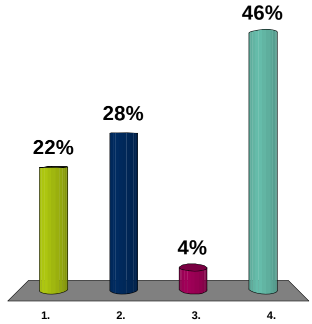
compensation or return current

LH streaming or boundary layer streaming

5-E Momentum and wave forces

Consider waves approaching the coast at an arbitrary angle. Which of the following statements is **not valid**?

1. In wave propagation direction, there is a time-averaged flow above wave trough level.
2. The mass flux is larger at the breaking point than at deeper water.
3. In the surf zone, the undertow compensates for the onshore mass flux above wave trough level.
4. Continuity requires a net flow under wave trough level against wave propagation direction.



Coastal hydrodynamics – part I

Dates student presentations, exam info and Ch9/10 stage B

	1	2	3	4	5	6	7	8	9	10
Content	Intro Ch2	Ch3 Ch4	Ch5-I	Ch5-II Ch6	Ch7 Ch8	Ch8 Ch9	Ch9 Ch10	Trial exam Apr4		Exam Apr18
Maple deadline	Ch1+2	Ch3 Ch4	Ch5-I	Ch5-II Ch6	Ch7	Ch8		Ch9,10 Apr3		

Moved to Thu April 4

- Student presentations Thu March 22 (7+8!)
- Volunteers can enroll on Brightspace (to be announced)

Exam info March 26

Ch8 stage A +B in class Tue March 27

5-E Momentum and wave forces

Effect of waves on the mean water motion and levels

- 2D momentum balance in x-direction (overbar denotes depth-averaging):

$$\boxed{\frac{\partial(\rho\bar{u}h)}{\partial t}+\frac{\partial(\rho\bar{u}h)\bar{u}}{\partial x}+\frac{\partial(\rho\bar{u}h)\bar{v}}{\partial y}=-\int_{-h_0}^{\eta}\frac{\partial p}{\partial x}dz-\tau_b}$$

- Velocity and pressure consist of mean and oscillatory component:

$$\bar{u}=U+\tilde{u} \qquad \bar{v}=V+\tilde{v} \qquad p=p_o+p_{wave}$$

- Now we average over the wave motion:

$$\cancel{\frac{\partial(\rho U h)}{\partial t}}+\cancel{\frac{\partial(\rho U h)U}{\partial x}}+\cancel{\frac{\partial(\rho U h)V}{\partial y}}=-\rho g h\frac{\partial\bar{\eta}}{\partial x}-\bar{\tau}_b\underbrace{-\frac{\partial\overline{(\rho\tilde{u}h)\tilde{u}}}{\partial x}-\overline{\int_{-h_0}^{\eta}\frac{\partial p_{wave}}{\partial x}dz}-\frac{\partial\overline{(\rho\tilde{u}h)\tilde{v}}}{\partial y}}_{\text{Residual terms: wave forces}}$$

- Neglect LHS (the mean flow is considered to be steady and slowly varying in space).

5-E Momentum and wave forces

Gradients in wave momentum flux impact the mean water motion and levels

$$\boxed{\cancel{\frac{\partial(\rho U h)}{\partial t}}+\cancel{\frac{\partial(\rho U h)U}{\partial x}}+\cancel{\frac{\partial(\rho U h)V}{\partial y}}=-\rho g h\frac{\partial\bar{\eta}}{\partial x}-\bar{\tau}_b\underbrace{-\frac{\partial\overline{(\rho\tilde{u}h)\tilde{u}}}{\partial x}-\overline{\int_{-h_0}^{\eta}\frac{\partial p_{wave}}{\partial x}dz}-\frac{\partial\overline{(\rho\tilde{u}h)\tilde{v}}}{\partial y}}_{\text{Wave forces } F_x \text{ (gradients in wave momentum flux)}}}$$

$$F_x=-\frac{\partial}{\partial x}\left[\overline{(\rho\tilde{u}h)\tilde{u}}+\overline{\int_{-h_0}^{\eta}p_{wave}dz}\right]-\frac{\partial}{\partial y}\overline{(\rho\tilde{u}h)\tilde{v}}$$

(u_x, u_y): depth- and time-dependent orbital motion in x- resp. y-direction

However: orbital motion generally depth-dependent

$$F_x=-\frac{\partial}{\partial x}\underbrace{\left[\overline{\int_{-h_0}^{\eta}(\rho u_x)u_x dz}+\overline{\int_{-h_0}^{\eta}p_{wave}dz}\right]}_{S_{xx}}-\frac{\partial}{\partial y}\underbrace{\left[\overline{\int_{-h_0}^{\eta}(\rho u_x)u_y dz}\right]}_{S_{xy}}$$

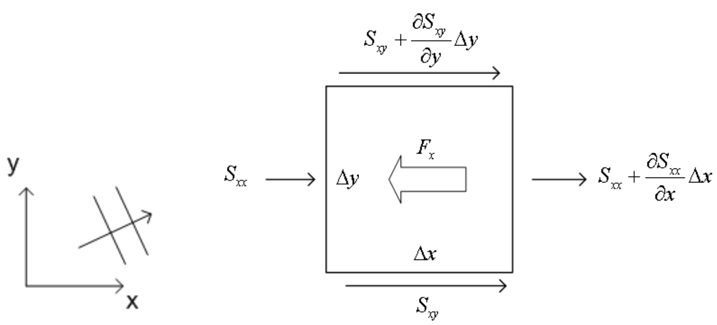
- The excess momentum flux (wave-averaged and depth-integrated) due to the presence of waves is called radiation stress S_{ij}
- Wave forces are due to wave-induced horizontal changes in momentum flux

5-E Momentum and wave forces

Net-wave averaged force in x-direction

$$F_x = -\left(\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}\right)$$

x-momentum ← S_{xy} → transported in y-direction (it acts on a plane with y is the normal)

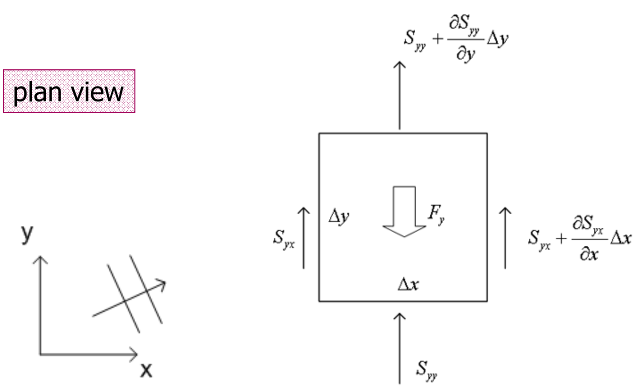


plan view

5-E Momentum and wave forces

Net-wave averaged force in y-direction

plan view



$$F_y = -\left(\frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x}\right)$$

5-E Momentum and wave forces

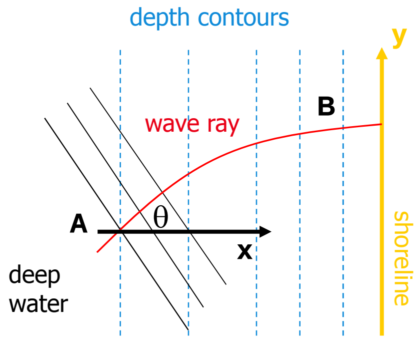
Wave forces can result in mean water level changes and drive a flow

General

$$F_x = -\left(\frac{\partial S_{xx}}{\partial x} + \frac{\partial S_{xy}}{\partial y}\right)$$

$$F_y = -\left(\frac{\partial S_{yy}}{\partial y} + \frac{\partial S_{yx}}{\partial x}\right)$$

Alongshore uniform



Set-up and set-down

$$F_x = -\frac{dS_{xx}}{dx}$$

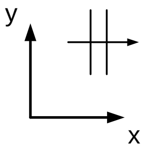
$$F_y = -\frac{dS_{yx}}{dx}$$

Longshore current

5-E Momentum and wave forces

Depth-integrated and time-averaged horizontal momentum transfer due to waves **in propagation direction x** is:

$$S_{xx} = \int_{z=-h_0}^{\eta} \left[(\rho u)u + p_{\text{wave}}\right] dz$$



$$u_x = u$$

$$u_y = 0$$

Transfer of momentum with particle velocity $u_x = u$

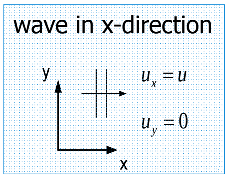
Transfer of momentum by the wave-induced pressure (due to oscillating pressure above wave trough level and vertical fluid motion)

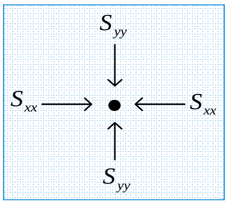
to 2nd order:

$$nE \quad + \quad (n-\frac{1}{2})E \quad = \quad (2n-\frac{1}{2})E$$

5-E Momentum and wave forces

Normal stresses for 1D-configuration





fluid particle momentum + pressure

$$S_{xx} = \overline{\int_{-h_0}^{\eta} (\rho u_x) u_x dz} + \overline{\int_{-h_0}^{\eta} p_{wave} dz}$$

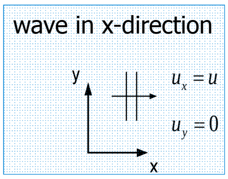
$$\boxed{S_{xx} = nE + (n - \tfrac{1}{2})E}$$

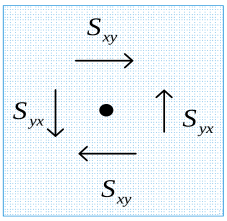
$$S_{yy} = \overline{\int_{-h_0}^{\eta} (\rho u_y) u_y dz} + \overline{\int_{-h_0}^{\eta} p_{wave} dz}$$

$$\boxed{S_{yy} = 0 + (n - \tfrac{1}{2})E}$$

5-E Momentum and wave forces

Shear stresses for 1D-configuration





fluid particle momentum pressure

$$S_{xy} = \overline{\int_{-h_0}^{\eta} (\rho u_x) u_y dz}$$

$$\boxed{S_{xy} = 0 + 0}$$

$$S_{yx} = \overline{\int_{-h_0}^{\eta} (\rho u_y) u_x dz}$$

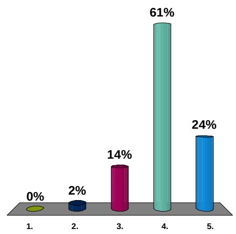
$$\boxed{S_{yx} = 0 + 0}$$

5-E Momentum and wave forces

For which of the following combinations is Syx ≠ 0?

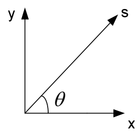
A. Normally incident waves
B. Obliquely incident waves
C. x-axis in wave propagation direction, y-axis along wave crests
D. x-axis perpendicular to the coast, y-axis along the coast

1. AC only
2. AD only
3. BC only
4. ✓ BD only
5. BC and BD



5-E Momentum and wave forces

General expressions for $S_{xx}, S_{xy}, S_{yx}, S_{yy}$

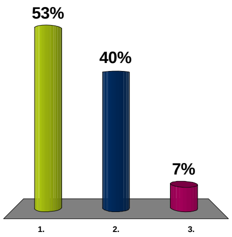


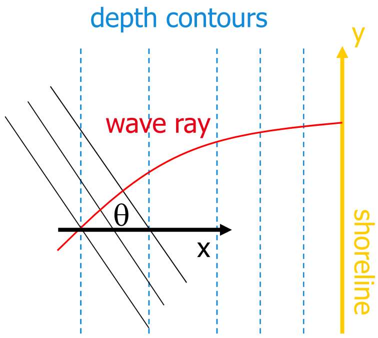
	General	$\theta = 0$	$\theta = 0$ shallow water (n=1)
S_{xx}	$(n - \frac{1}{2} + n\cos^2\theta)E$	$(2n - \frac{1}{2})E$	$\frac{3}{2}E$
S_{yy}	$(n - \frac{1}{2} + n\sin^2\theta)E$	$(n - \frac{1}{2})E$	$\frac{1}{2}E$
$S_{xy} = S_{yx}$	$n\cos\theta\sin\theta E$	0	0

5-E Momentum and wave forces

The wave force that drives the longshore current is equal to the alongshore gradient of the radiation shear stress

1. True
2. False ✔
3. Abstain





Set-up and set-down

$$F_x = -\frac{dS_{xx}}{dx}$$

$$F_y = -\frac{dS_{yx}}{dx}$$

Longshore current

Coastal hydrodynamics – part I

Chapter 5 of lecture notes

A. Introduction
B. Wave height transformation
C. Bed friction
D. Wave skewness and asymmetry
E. Momentum and wave forces
F. Set-up and set down (and undertow)
G. Longshore current
H. 3D effects
I. Wind-induced set-up and currents

5-F Set-up and set-down

Changes in the shore-normal transfer of x-momentum are equivalent to a force in cross-shore direction

$$F_x = -\frac{dS_{xx}}{dx} = -\frac{d}{dx}\left[\left(n - \frac{1}{2} + n\cos^2\theta\right)E\right]$$

Alongshore uniform coast

In nearshore for normally incident waves:

- Increase in radiation stress in shoaling zone: $\frac{dS_{xx}}{dx} > 0$
  (offshore directed wave force)
- Decrease in radiation stress in the surf zone: $\frac{dS_{xx}}{dx} < 0$
  (onshore directed wave force)

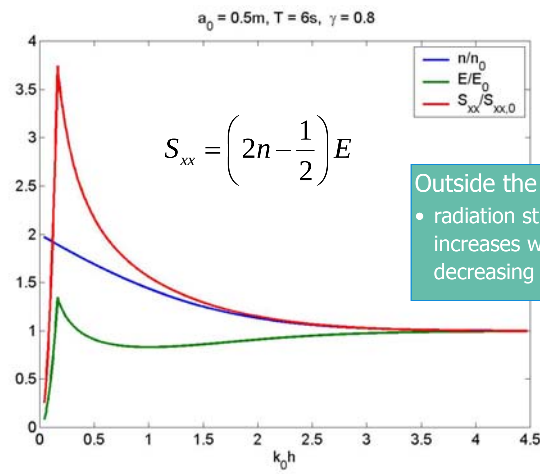
5-F Set-up and set-down

Normally incident waves (no refraction)

Suppose breaking wave height is fraction of depth

$$E_{max} = \frac{1}{8}\rho g(\gamma h)^2$$

Inside the surf zone:
- radiation stress decreases with decreasing depth



Outside the surf zone:
- radiation stress increases with decreasing depth

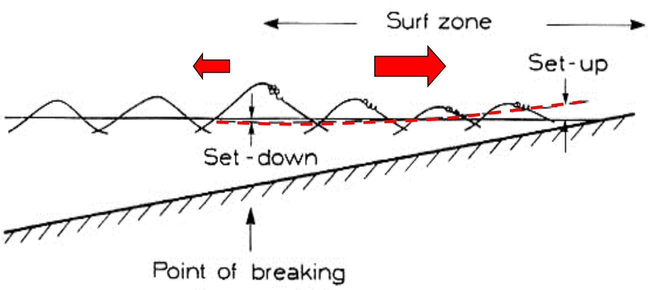
5-F Set-up and set-down

The cross-shore wave forces are balanced by a water level gradient (because coast imposes zero-flux condition)

- Set-down in shoaling zone
- Set-up in in the surf zone

Order of magnitude?

$$F_x = -\frac{dS_{xx}}{dx} = \rho g h \frac{d\bar{\eta}}{dx}$$

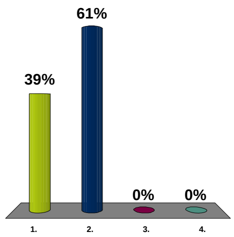


This differential equation can be solved using the energy balance.

5-F Set-up and set-down

Consider a wave with a wave height at breaking of H_b = 2 m. What order of magnitude of set-up do you expect?

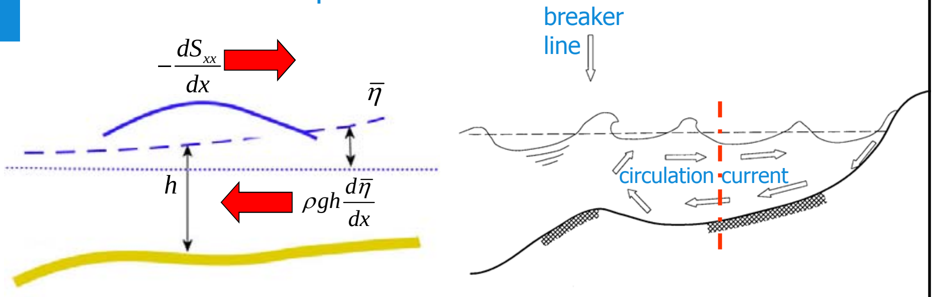
1. Set-up ≈ 0.1 m
2. ✓ Set-up ≈ 0.5 m
3. Set-up ≈ 2 m
4. Abstain

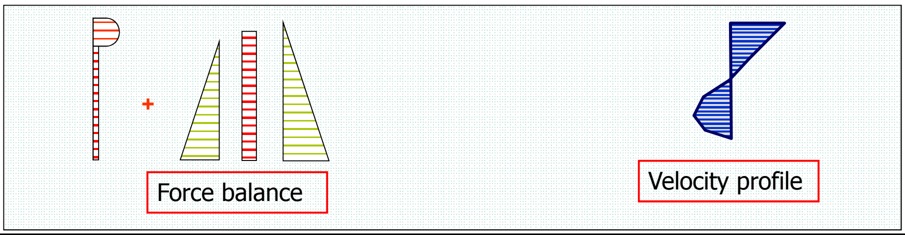


$$\bar{\eta}_{\text{shore}} = \frac{5}{16}\gamma H_b \approx 0.25 H_b$$

5-F Set-up and set-down

Undertow vertical profile





Exam question june 2010 (1)

17(75)

3. Momentum balance equations [17 points – 41 minutes]

Consider the following balance equation for an alongshore uniform coast:

$$-\frac{dS_{xx}}{dx} = \rho g h \frac{d\bar{\eta}}{dx} \qquad (3)$$

The x-direction is in cross-shore direction (positive onshore). In this equation the term $\bar{\eta}$ is the mean water level, h is the water depth and $S_{xx} = \left(2n - \frac{1}{2}\right)E$ is a radiation stress (with E is the wave energy and n is the ratio between group and phase velocity). Assume normally incident regular waves.

a. **[3]** Make a sketch of the cross-shore distribution of S_{xx} from deep water to the water line, in which you indicate the width of the surf zone. Explain your reasoning.

b. **[4]** Discuss the physical meaning of the left-hand-side (LHS) and right-hand-side (RHS) of Equation (3).

c. **[3]** Explain and sketch the cross-shore mean water level variation from deep water to the water line corresponding to your answer to a). Use the same horizontal scale as in answer a).

Exam question june 2010 (continued)

So far we have only discussed depth-averaged quantities.

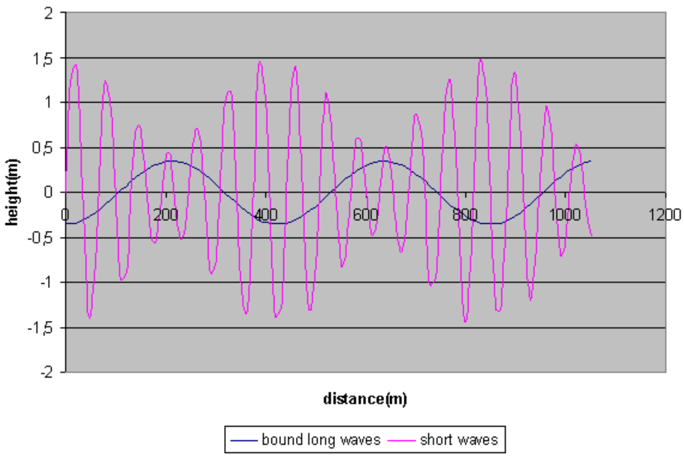
d. **[4]** Indicate how the depth-variation of the terms in Equation (3) results in a secondary current pattern in the surf zone. Also sketch a cross-shore profile and indicate the circulation pattern by means of arrows.

Now consider a geostrophic balance equation. An example of such an equation is the cross-shore momentum equation for a Kelvin wave.

e. **[3]** Explain the main correspondence and the main difference between Equation (3) and a geostrophic balance equation.

5-F Set-up and set-down

Time-varying set-down on the wave group scale => bound long wave



Coastal hydrodynamics – part I

Chapter 5 of lecture notes

5-G Longshore current

Alongshore wave force non-zero only in surf zone

$$F_y = -\frac{dS_{yx}}{dx} = -\frac{d}{dx}\left[\underbrace{Enc\cos\theta}_{U_x}\ \underbrace{\frac{\sin\theta}{c}}_{\text{constant}}\right]$$

- Cross-shore energy flux U_x
- Follows from energy balance:

$$\frac{dU_x}{dx} + D = 0$$

Snell's law: constant

$$\boxed{F_y = -\frac{\sin\theta_0}{c_0}\frac{d}{dx}U_x = \frac{\sin\theta_0}{c_0}D}$$

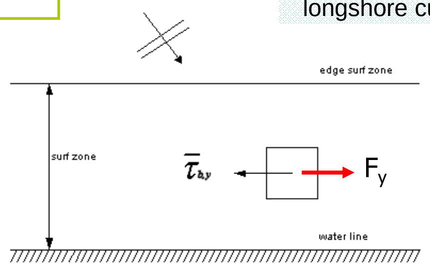
Wave force is **non-zero only in surf zone** where the wave energy flux is no longer conserved

5-G Longshore current

The alongshore wave force is balanced by a bed shear stress associated with a current

$$\boxed{-\frac{dS_{yx}}{dx} = \frac{\sin\theta_0}{c_0} D = \bar{\tau}_{b,y}}$$

Outside the surf zone: no driving of the longshore current



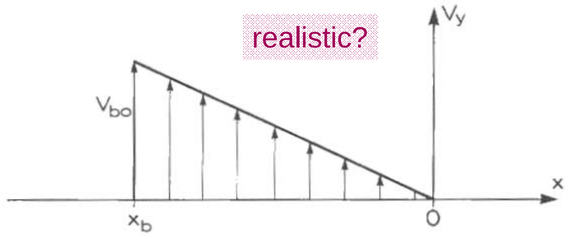
5-G Longshore current

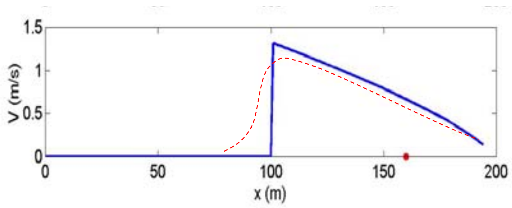
The velocity distribution (neglecting lateral dispersion of momentum by turbulence)

realistic?

$$V_b = \frac{5}{16}\pi \frac{H_b}{c_f} g \frac{\sin\theta_0}{c_0} \tan\alpha$$

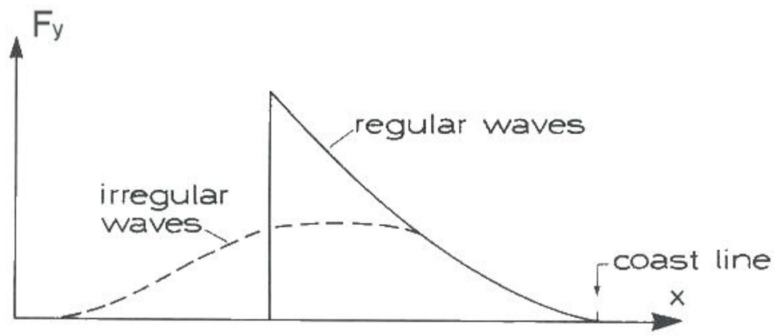
Longuet-Higgins (1970)





5-G Longshore current

Distribution of the longshore driving force for regular and irregular waves



See Fredsøe and Deigaard (1992)

5-G Longshore current

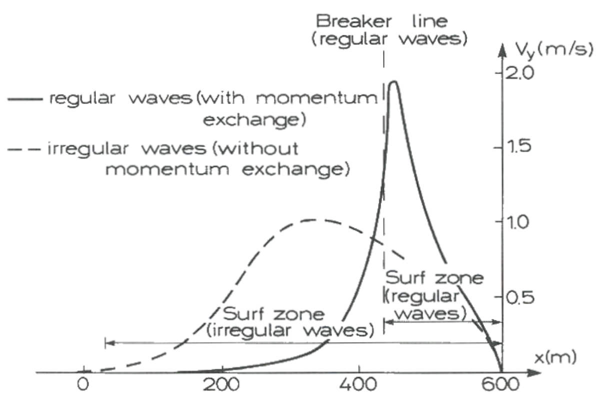
Effect momentum exchange versus wave irregularity

Constant beach slope

$\tan\alpha = 0.01$

Deep water conditions

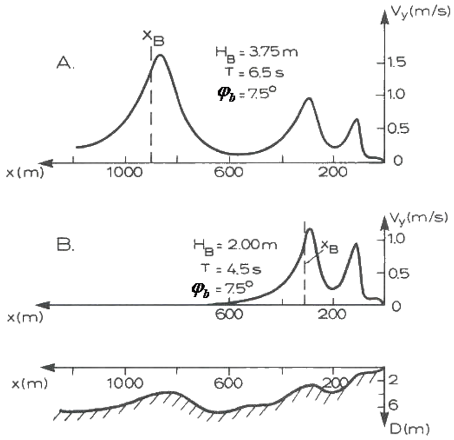
$H_{rms} = 1.2 \text{ m} \quad T = 7.5 \text{ s} \quad \varphi_0 = 45^\circ$



See Fredsøe and Deigaard (1992)

5-G Longshore current

Barred profile (three bars)



See Fredsøe and Deigaard (1992)

Coastal hydrodynamics – part I

Chapter 5 of lecture notes

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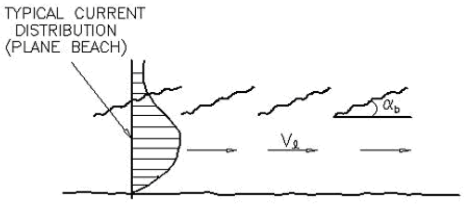
5-H 3D effects

Alongshore uniform barred beach



large angle of incidence α_b:
longshore current + undertow

2D situation

5-H 3D effects

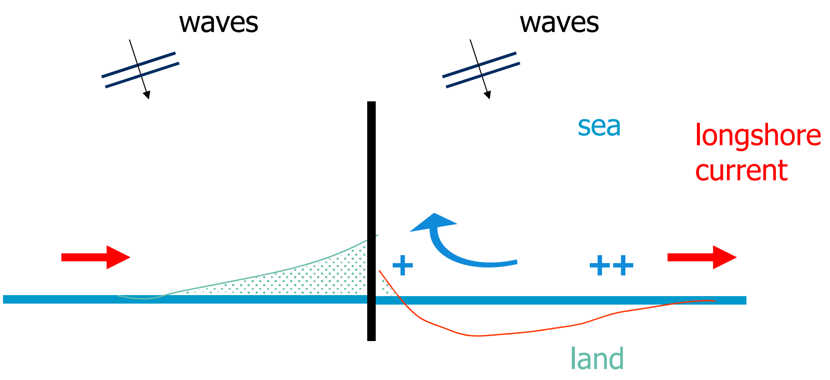
Alongshore variations in wave forces drive 3D current patterns

- Eddy formation in the lee side of structures
- 3D current patterns around shoals
- Creation of rip currents

alongshore variations in wave height (or angle)=>
variations in (alongshore and) cross-shore wave forces =>
alongshore variations in set-up =>
3D current patterns

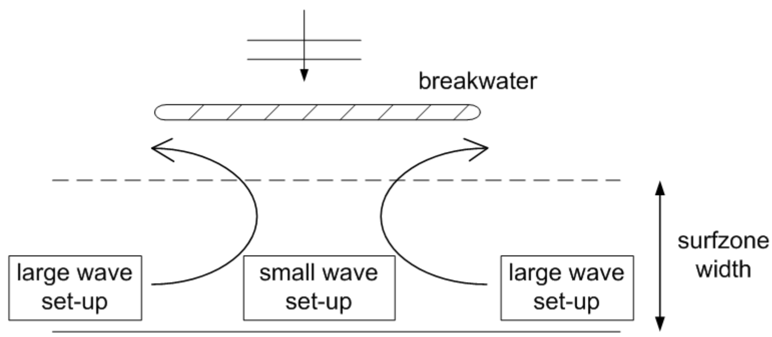
5-H 3D effects

Current pattern in lee side of groin



5-H 3D effects

Current patterns behind detached **emerged** breakwater



Variations in wave height along a coastline create alongshore set-up differences and hence 3D current patterns

5-H 3D effects

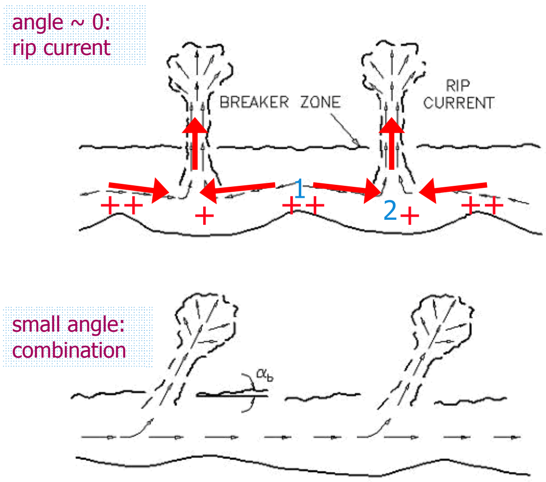
Rhythmic bar

Transverse bar

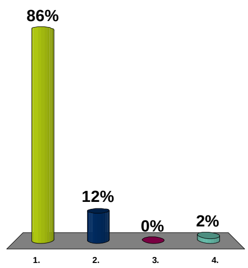
5-H 3D effects

Near-shore circulation patterns for smaller wave angles α_b

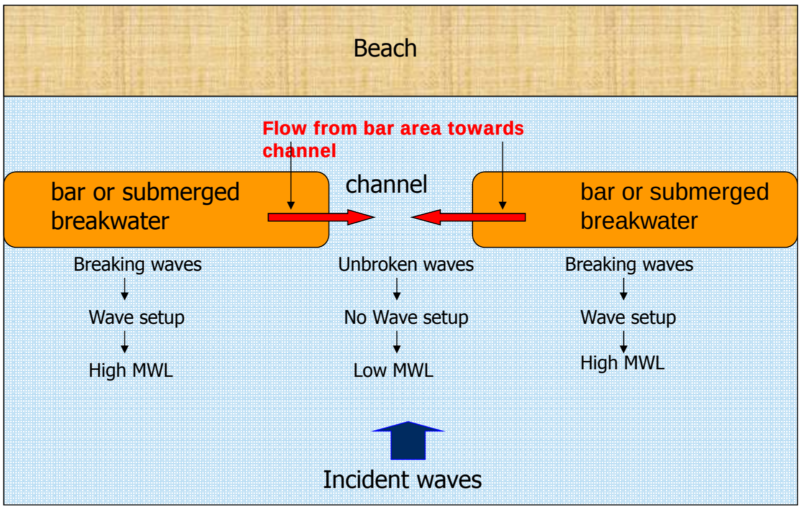
The set-up is larger at:



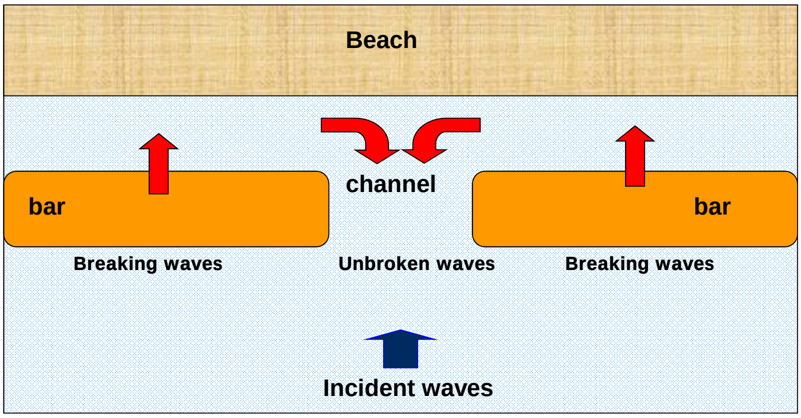
1. Location 1
2. Location 2
3. Same at both locations
4. No idea



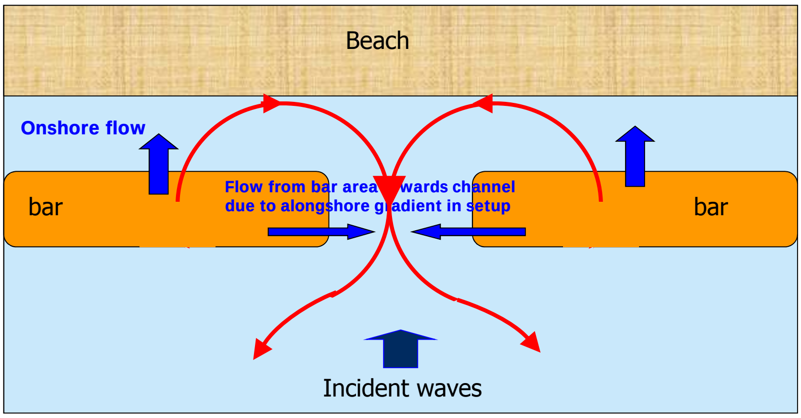
5-H 3D effects
Rip current generation (1): set-up differences



5-H 3D effects
Rip current generation (2): net onshore flow



5-H 3D effects



Resultant flow field

Under highly energetic wave conditions the rip cells do not close =>
Mechanism of sediment loss from nearshore zone

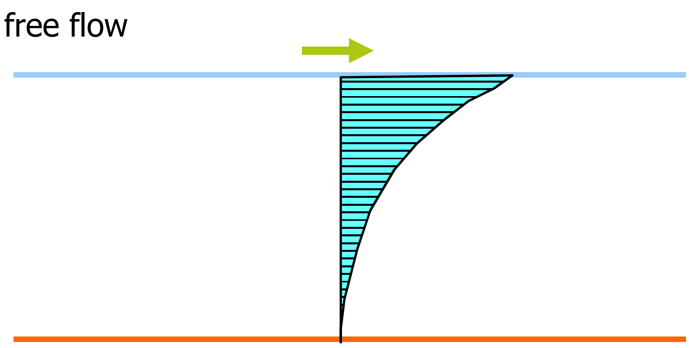
Coastal hydrodynamics – part A

Chapter 5 of lecture notes

A. Introduction
B. Wave height transformation
C. Bed friction
D. Wave skewness and asymmetry
E. Momentum and wave forces
F. Set-up and set down (and undertow)
G. Longshore current
H. 3D effects
I. Wind-induced set-up and currents

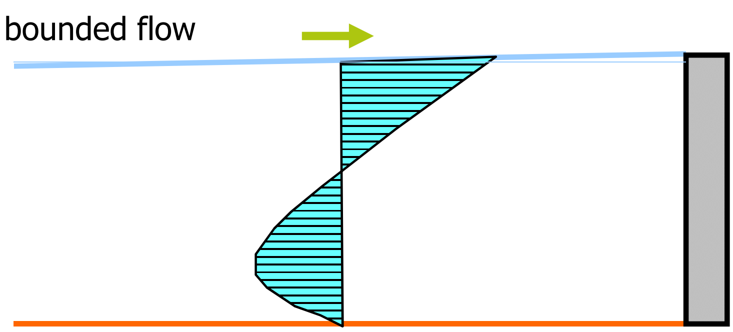
5-I Wind-induced set-up and currents

Wind-driven current profile: no closed boundary



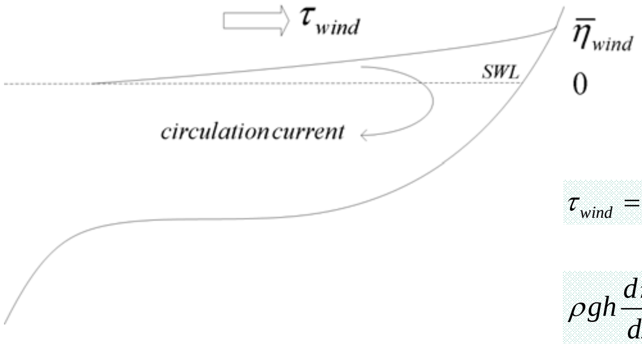
5-I Wind-induced set-up and currents

Wind-driven current profile: closed boundary



5-I Wind-induced set-up and currents

Wind set-up balancing wind shear stress in the case of onshore wind on a shelf



$$\tau_{wind} = C_d \rho_a W^2$$

$$\rho g h \frac{d\overline{\eta}}{dx} = \tau_{wind,x}$$