Homework 5: Finite Difference Method

Finite difference problem 1

Consider one-dimensional flow in a confined aquifer between two fully penetrating canals with water level h_1 on the left and h_2 on the right (see Figure). The transmissivity of the aquifer is T and there is a uniform recharge N. The distance between the two canals is L. Given: $N = 2$ mm/d, $T = 20$ m^2/d, $L = 200$ m, $h_1 = 10$ m, $h_2 = 10.5$ m.

Question:

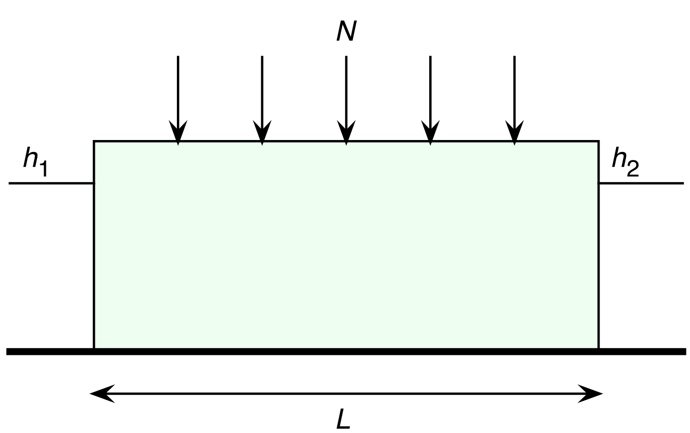
a) Use the finite difference method with a cell size of 10 meter to compute the head halfway between the two canals.

b) Use your spreadsheet model to compute the head halfway between the two canals and compare this finite difference answer to the analytic solution.

c) Use your spreadsheet model to compute the flow into the left canal and the flow into the right canal. Is the water balance met? Explain any discrepancies between your answer and the analytic solution for this problem.

d) Modify your spreadsheet model so that the left half of the model has an infiltration rate of 1 mm/d, and the right half an infiltration rate of 3 mm/d.

e) Use your spreadsheet model to compute the head halfway between the two canals and the flow into the left canal and the flow into the right canal. Can you derive the exact solution to determine whether this answer is correct? (you may want to do the latter solution at home).



Finite difference problem 2

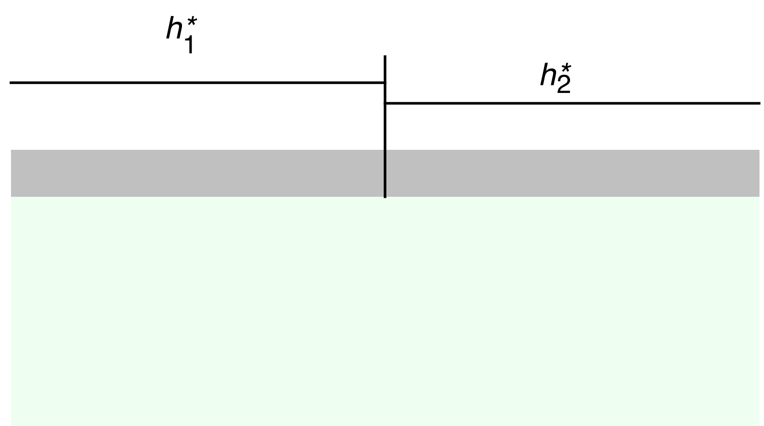
Consider one-dimensional flow between two polders with different polder levels h_1^* and h_2^* as shown in the Figure (Note: this is one-dimensional flow, so only one row of cells). The resistance of the clay layer that separates the polder from the aquifer is $c = 400$ days and the transmissivity of the aquifer is $T = 100$ m^2/d. Given: $h_1^* = 20$ m, and $h_2^* = 19m$.

Questions:

a) Derive a finite difference equation for the head in the aquifer.

b) Use your finite difference equation in a spreadsheet to compute the head in the aquifer below the polder. Use an appropriate cell size and use an appropriate number of cells (think about how far your model extends to the left and right). Explain how you select both the cell size and the number of cells.

c) Compute the total amount of water that flows from the left polder to the right polder and compare your answer to the exact solution (if the finite difference solution is not very close to your analytic solution, you are doing something wrong). Does you finite difference answer get better when you use a smaller cell size?



Finite difference problem 3

Consider one-dimensional transient flow in a semi-infinite unconfined aquifer ($x \geq o$). The aquifer is bounded in the west by a fully penetrating canal at $x = 0$. Initially the water level in the canal and the head everywhere in the aquifer are equal to $h_0 = 10$ m. At time $t = 0$, the water level in the canal is raised by $\Delta h = 1$ m. Approximate the transmissivity of the unconfined aquifer as constant and equal to kH. Use cell sizes of $\Delta x = 10$ m.

Given: $kH = 10$ m^2/d, $S_y = 0.1$.

Questions:

a). Use an implicit finite difference scheme to compute the head at 50 m from the canal after 10 days and after 20 days and compare your answer to the exact solution (which was given in Hydrology I). Report your time step and the relative error with the exact solution.

b). After 20 days, the water level in the canal is dropped back to $h_0 = 10$ m. Compute the head in the aquifer at 50 m from the canal 10 days after the water level has dropped back to $h_0 = 10$ m and compare your answer to the exact solution (which was given in Hydrology I).

Finite difference problem 4

Consider a well pumping near a straight river with head $h_0 = 20$ m as shown in the figure. The distance between the well and the river is $d = 100$ m. In absence of the well there is no flow in the aquifer. The aquifer may be approximated as confined with transmissivity $T = 100$ m^2/d. The discharge of the well is $Q = 500$ m^3/d.

Questions:

a) Make a conceptual model of the aquifer. Explain how large you make the model and what boundary conditions you specify along the boundaries of the model.

b) Create a contour plot of the head in the aquifer using the finite difference method. Use cell sizes of 20 meter.

c) Create a contour plot of the streamfunction in the aquifer using the finite difference method.

d) Use your spreadsheet model to compute the head along the horizontal line between the river and the well. Compute the head at distances of 20, 40, 60, and 80 meters from the river, and at the well. Compare your answers to the exact solution obtained with image wells. Explain any differences.