The BBQ sambal stingray is one of the most favourite dish in Malaysia and Singapore. Without proper regulations, overfishing of stingrays can happen.

(To the foreign students, you have to try this dish!)

## 1. Problem

The dynamics of stingrays in the Malaysia waters can be described as

$$\dot{P} = \frac{dP}{dt} = -rP\left(1 - \frac{P}{k}\right)\left(1 - \frac{P}{m}\right) - 2N$$

where $P$ is population measured in thousand tons, *t* = number of years*, r* is the growth rate, *k* is the population capacity in tons, *m* is a minimum value and *N* is the number of fishing licenses. (See Section 0 for a quick discussion on the IVP. Here, each fishing license is allowed to catch 2 thousand tons of stingrays per year.)

The current stingray population was estimated as $P_0 = 30$ at $t = 0$. (Do not dispute this number)

The population dynamics equation was postulated 10 years ago, with the following parameters adopted: $r = 4$, $k = 55$, $m = 10$. Over time, the population dynamics and/or modeling parameters may change due to many factors: e.g. climate change, pollution, illegal fishing …

The authority has engaged two consulting firms to recommend the appropriate number of licenses to issue for the next 5 years. The requirement is that $P \geq 1.5m$ at all times for the next 20 years.

The two consulting teams have the following hidden agenda:

**Team A** (funded by fishing industry)

- Try securing at least 10 licenses. The more the merrier!

**Team B** (funded by environmentalists)

- Try limiting to at most 10 licenses. Go green!

To convince the authority, your team needs to do a professional job, and take into account important factors that may affect the population dynamics model. Of course, depending on your hidden agenda, your team will "downplay" or "exaggerate" these factors (with supporting data/reports[1]).

**Note:**

This is a numerical methods project. You also need to discuss on the numerical approach adopted. Some questions to explore (may or may not attempt everything, depending on your time):

- Why was a particular method chosen?
- How does this method compare with an alternative method that is of higher-order accuracy / easier to implement?
- Comment on stability issues.
- How do you know that your solution is correct?
- Comment on accuracy.

**Grading:**

- Report – grading by lecturer
    - 'Professionalism' of study leading to recommendation
    - Technical discussions on numerical methods

- Presentation – grading by course mates (Authority committee members)
    - To what extent can you convince the committee members on the recommendation?
    - To what extent can you make the committee members understand the technical details in your numerical approach?

- Class participation

[1] Please do not spend too much time researching for very accurate statistics…
Something like this will be considered as sufficient: An online report stated X illegal fishing in Malaysia waters for past 5 years, so you subtract X/5 in the IVP.

## 2. Background information on population model

A simple population model is given as

$$\dot{P} = rP \tag{1}$$

where *P* is the population, *t* = number of years, *r* is the growth rate.

This implies that the growth rate is proportional to the population size, i.e. the larger the population, the higher the rate of population growth. In fact, the growth is exponential, for the solution is given by $P = P_0 e^{rt}$.

However, the population size cannot go unbounded. A factor $(1 - P/k)$ is thus introduced to (1) to give

$$\dot{P} = rP(1 - P/k) \tag{2}$$

where *k* is the population capacity.

With this factor, the population growth is such that

- When $P << k$, $P/K$ is negligible and $\dot{P}$ is fast.
- As $P$ increases but still smaller than *k* ($P < k$), growth rate $\dot{P}$ decreases but is still positive. Population is still increasing, but at a slower rate.
- Once $P > k$, growth rate becomes negative, i.e. population decreases.
- A steady state $\dot{P} = 0$ (no change in population) occurs when $P = k$. This is why *k* is known as the 'population capacity'.

Another issue to consider. In general, if the population size falls below a minimum threshold *m*, then, the reproduction rate is less than the death rate, assuming all other factors being constant. In other words, below a threshold population, the population growth rate becomes negative, and over time, the entire population will be wiped out. So, we can add another factor $-(1 - P/m)$ to (2).

$$\dot{P} = -rP(1 - P/k)(1 - P/m) \tag{3}$$

Note that now, below the threshold value ($P < m$), growth rate becomes negative. Population gets smaller and will eventually vanish.

Suppose that there are *N* hostile elements, and each element will remove 10 members from the population per year. This can be incorporated into the model as

$$\dot{P} = -rP\left(1 - \frac{P}{k}\right)\left(1 - \frac{P}{m}\right) - 10N \tag{4}$$

This is a simple population dynamics model that we will use as a reference.

Have fun!