Quiz-1 Q1

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(1) This is a simple normal flow calculation. The maximum flow occurs when the water depth in the channel is H.

$$P = \frac{(B+\sqrt{2}H)H}{B+2\sqrt{2}H} = 1.53\,m. \qquad Q = \frac{A}{n} R_h^{2/3} \sqrt{S} = 72\,m^3/s.$$

(2) This minimum flow probably has very shallow water depth, and therefore can be treated as a very wide rectangular cross-section.

$$R_h \approx h_m. \qquad h_m = \left(\frac{V_m \cdot n}{\sqrt{S}}\right)^{3/2} = 0.048\,m.$$

$$\text{check: } R_h = \frac{(B+2\sqrt{2}h_m)\cdot h_m}{B+2\sqrt{2}h_m} \approx 0.048\,m \quad \checkmark$$

The minimum discharge:

$$Q_m \doteq h_m B \cdot V_m \approx 0.145\,m^3/s.$$

(3) To minimize the cross-sectional area, we want the maximum discharge to occur when the trench is full.

$$Q_m = A R_h^{2/3} \sqrt{S_0}/n = A^{5/3}/P^{2/3} \sqrt{S_0}/n.$$

$$\Rightarrow \frac{n Q_m}{\sqrt{S_0}} P^{2/3} = A^{5/3} \qquad \text{Denote } \frac{nQ_m}{\sqrt{S_0}} = \alpha \Rightarrow A^{5/3} - \alpha(b+2h)^{2/3} = 0$$

$$\Rightarrow A^{5/3} - \alpha\left(\frac{A}{h}+2h\right)^{2/3} = 0 \qquad (1)$$

This suggests that A is a function of h, so the minimum A satisfies:

$$\frac{\partial A}{\partial h} = 0.$$

$$\frac{\partial (1)}{\partial h} \Rightarrow \frac{2}{3}\alpha\left(\frac{A}{h}+2h\right)^{-\frac{1}{3}}\left(-\frac{A}{h^2}+2+\frac{\partial A}{\partial h}\frac{1}{h}\right) = 0$$

since $\partial A/\partial h = 0$

$$-\frac{A}{h^2}+2 = 0 \Rightarrow A = 2h^2 \Rightarrow \underline{b = \frac{A}{h} = 2h} \quad \#$$

Thus: $b = 2h$ is the optimal shape of the trench, which minimize A.

For this shape $A = 2h^2 \quad R_h = \frac{A}{P} = \frac{2h^2}{4h} = \frac{h}{2}$

Thus: $Q_m = \frac{2h^2}{n}\cdot\left(\frac{h}{2}\right)^{2/3}\sqrt{S_0} \Rightarrow h = 0.33\,m$

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Quiz-1 Q2

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(1) We first obtain the discharge per unit width by considering the normal flow in the upper channel. Since the channel is very wide and has a rectangular cross section:

$$q = \frac{\sqrt{S_1}}{n_1} h_{n1}^{5/3} = 7.97\ m^2/s$$

Thus, we can get the critical water depth: $h_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = 1.86\ m < h_{n1}$ (mild slope).

Since the exit of the lake is not affected by the downstream flow, and the upper channel is mild, we should have normal flow at the exit. Thus:

$$H = \frac{V_{n1}^2}{2g} + h_{n1} = 4.2\ m \qquad \left(V_{n1} = \frac{q}{h_{n1}} = 1.99\ m/s \right)$$

(2) We first calculate the normal depth of the lower channel:

$$h_{n2} = \left(\frac{q\, n_2}{\sqrt{S_2}}\right)^{3/5} = 1.41\ m < h_c$$

So the lower channel is steep, and the normal flow is supercritical. Since normal flow is established before the gate, we shall have a hydraulic jump from hn2 to its conjugate:

$$h_{n2,c} = \frac{1}{2} h_{n2} \left(-1 + \sqrt{1 + 8 Fr_2^2}\right) = 2.4\ m$$

$$\text{where} \quad Fr_2 = \frac{V_{n2}}{\sqrt{g h_{n2}}} = \frac{q}{\sqrt{g h_{n2}}\, h_{n2}} = 1.51.$$

The flow after the gate goes to a vena contracta, where $h_2 = C_v h_g = 0.6m < h_{n2}$

Thus, an S3 profile after the vena contracta brings the water depth back to hn2. At the transition point, since this is a mild-to-steep transition, we have critical water depth.

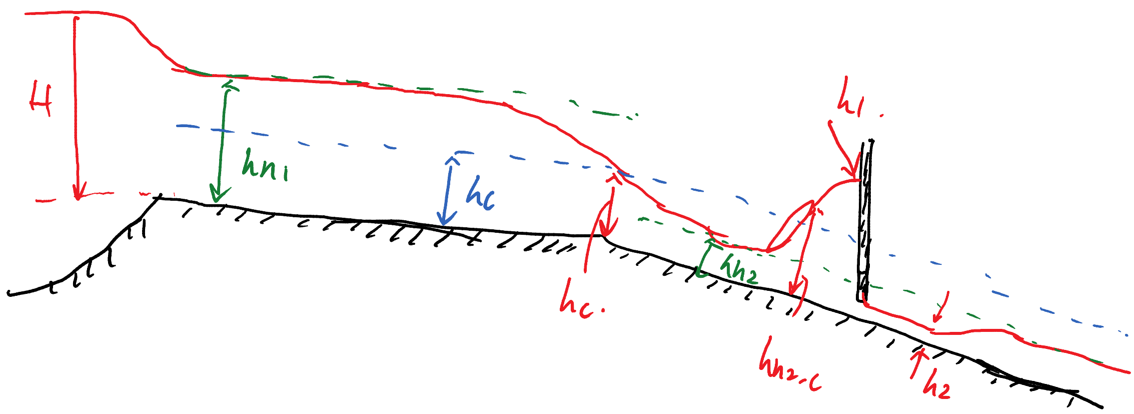
To get the water depth h1 before the sluice gate, we consider energy equation:

$$\frac{q^2}{2g h_1^2} + h_1 = \frac{q^2}{2g h_2^2} + h_2 \Rightarrow \frac{q^2 (h_1 + h_2)}{2g h_1^2 h_2^2} = 1$$

Re-arrange this equation:

$$h_1^2 - \frac{q^2}{2g h_2^2} h_1 - \frac{q^2}{2g h_2^2} h_2 = 0 \Rightarrow h_1 = 9.6\ m.$$

The surface profile is:



Quiz-1 Q2(cont'd)

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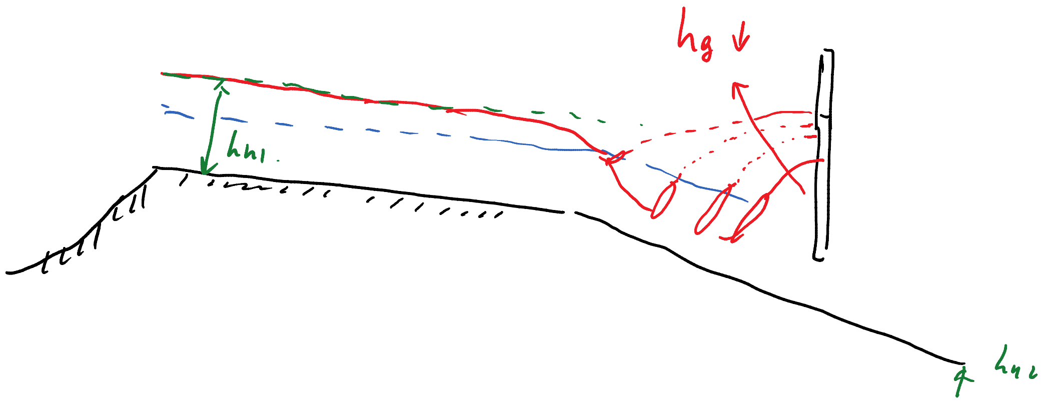
(3) If the gate is further closed, since the discharge remains the same, you will see h1 increases, and therefore the S1 profile leading to h1 will start earlier. This will push the hydraulic jump upstream. Eventually, S1 will start right from the transition point, and no hydraulic jump will occur. (1).

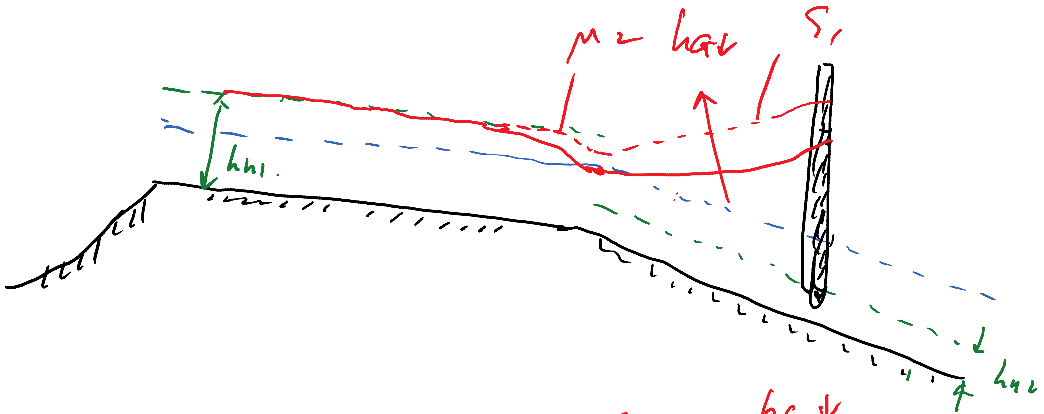
If we keep closing the gate, the water depth at the transition point will be larger than critical water depth, and the M2 profile's start point moves downstream. (2)

When the water depth at the transition point exceeds hn1, we will have M1 profile in the upper channel. (3)

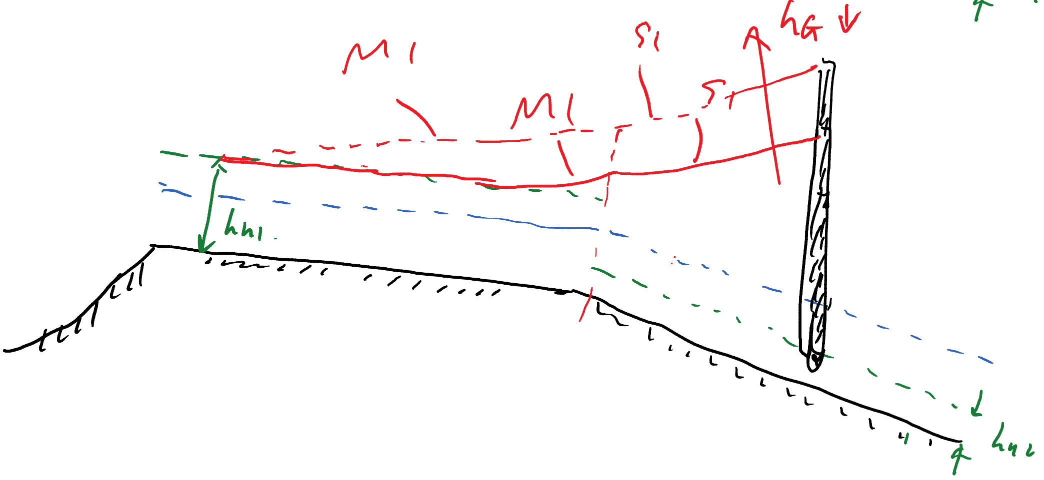
The start point of the M1 profile will move upstream as the gate opening reduces, and eventually the exit of the lake will be submerged. (3)



(1)



(2)



(3).