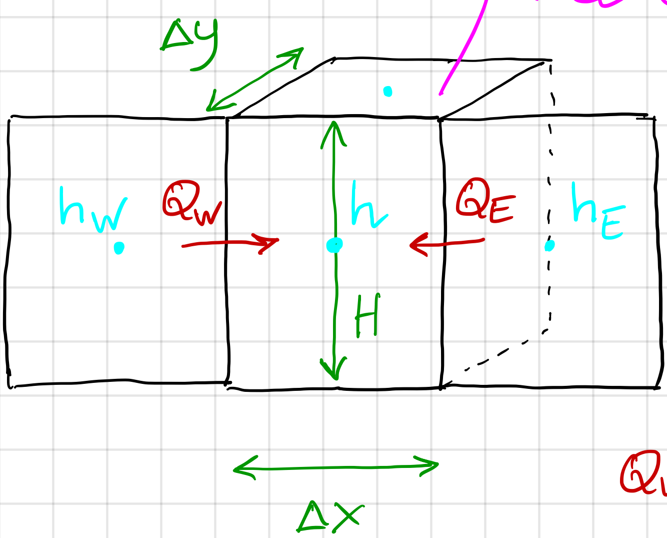
# Finite Difference Method

MODFLOW

Approximate Method

Compute :
- Heads at center of cells
- Flow across sides of cells

$\rightarrow Q$ (sink: positive for taking water out)




$Q_W, Q_E, Q$ are discharges ($m^3/d$)

In − Out = 0

$$Q_W + Q_E - Q = 0$$

$$Q_W = \frac{\cancel{\Delta y} H k (h_W - h)}{\cancel{\Delta x}}$$

square cells $\Delta x = \Delta y$

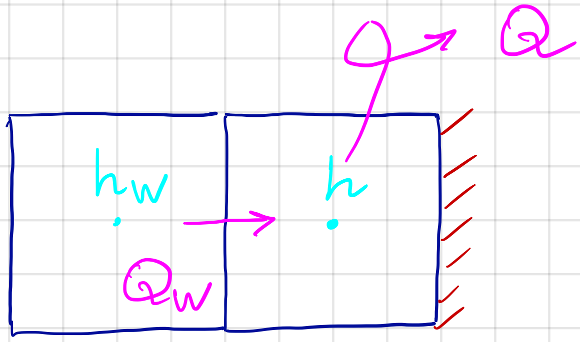
$$Q_W = T(h_W - h)$$

$$Q_E = T(h_E - h)$$

$$\cancel{T}(h_W - h) + \cancel{T}(h_E - h) - \frac{Q}{T} = 0$$

$$2h = h_W + h_E - \frac{Q}{T}$$

$$\boxed{h = \frac{h_W + h_E}{2} - \frac{Q}{2T}}$$



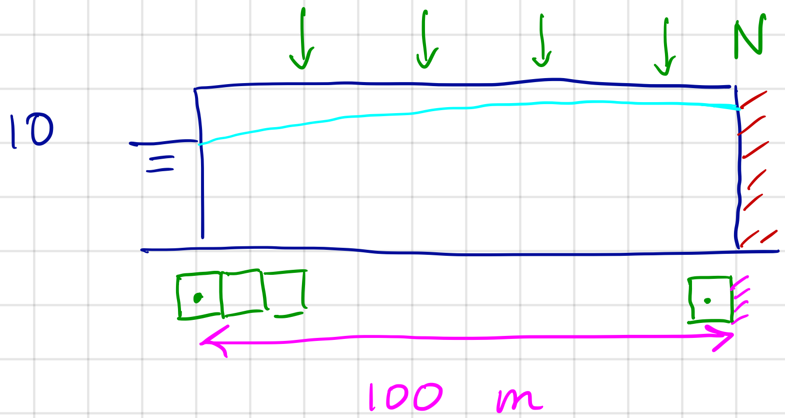

$$Q_w = T(h_w - h)$$

In − Out = 0

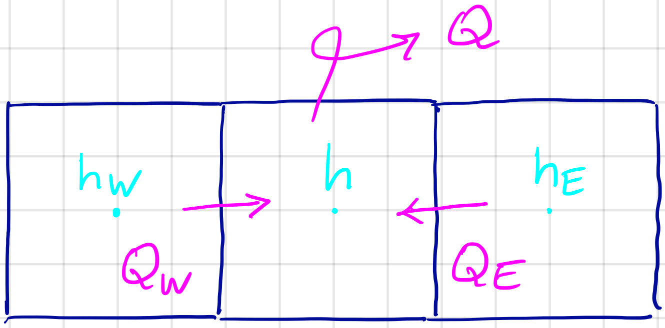
$$Q_w - Q = 0$$

$$T(h_w - h) - Q = 0$$

$$h = h_w - \frac{Q}{T}$$




11 cells $\rightarrow$ $\Delta x = \dfrac{L}{10.5}$




Square cells:

$$Q_W = T(h_W - h)$$

$$Q_E = T(h_E - h)$$

TRANSIENT

In − Out = Increase in storage    $\Delta t$ time period

$$(Q_W + Q_E - Q)\,\Delta t = S\big(h(t+\Delta t) - h(t)\big)(\Delta x)^2$$

$$\big(T(h_W - h) + T(h_E - h) - Q\big)\,\Delta t = S(\Delta x)^2\big(h(t+\Delta t) - h(t)\big)$$

Explicit → $h, h_W, h_E$ at $t = t$

Implicit → $h, h_W, h_E$ at $t + \Delta t$

Crank-Nicholson → $h, h_W, h_E$ average at $t$ & $t+\Delta t$

Now: Explicit

$$\left(\cancel{T}(h_W - h) + \cancel{T}(h_E - h) - \frac{Q}{T}\right)\cancel{\Delta t} = \underbrace{\frac{S(\Delta x)^2}{T\,\Delta t}}_{\frac{1}{\alpha}}\big(h(t+\Delta t) - h\big)$$

$$h(t+\Delta t) = h + \alpha\left(h_W + h_E - 2h - \frac{Q}{T}\right)$$

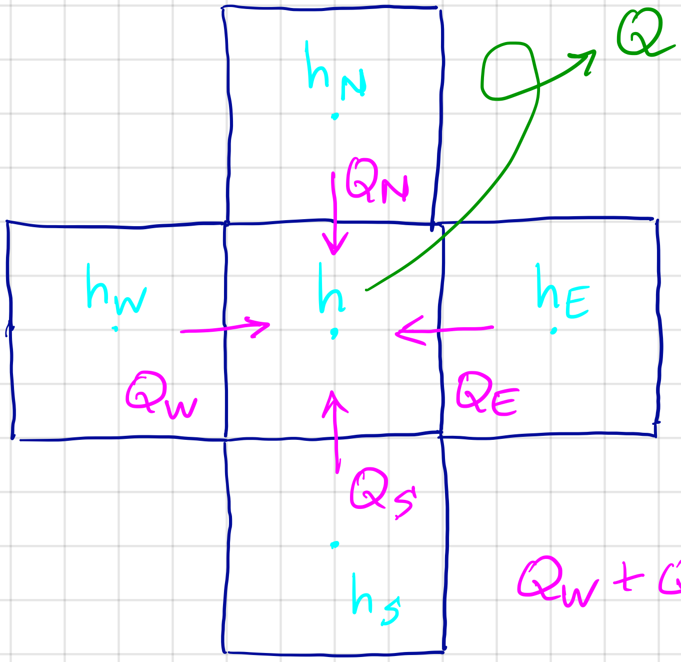
$$\alpha = \frac{T\Delta t}{S(\Delta x)^2}$$

Stable solution $\alpha < 0.5$

$$\frac{T\Delta t}{S(\Delta x)^2} < 0.5$$

$$\Delta t < \frac{0.5 \cdot S \cdot (\Delta x)^2}{T}$$

# 2D Steady flow




SQUARE CELLS

$Q_W = T(h_W - h)$

$Q_S = T(h_S - h)$

$Q_E = T(h_E - h)$

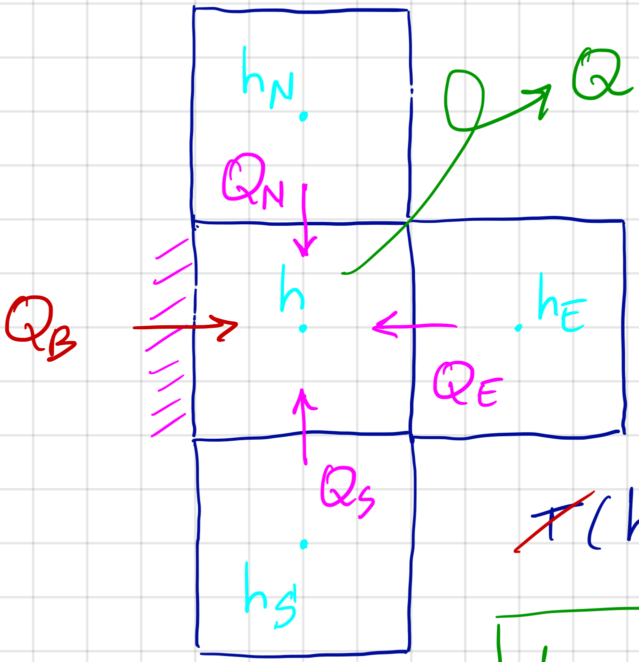
$Q_N = T(h_N - h)$

In − Out = 0

$$Q_W + Q_S + Q_E + Q_N - Q = 0$$

$$\cancel{T}(h_W - h) + \cancel{T}(h_S - h) + \cancel{T}(h_E - h) + \cancel{T}(h_N - h) - \frac{Q}{T} = 0$$

$$\boxed{h = \frac{(h_W + h_S + h_E + h_N)}{4} - \frac{Q}{4T}}$$

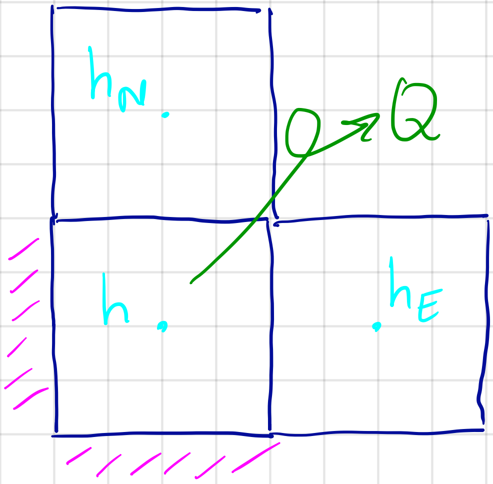



$$\text{In} - \text{Out} = 0$$

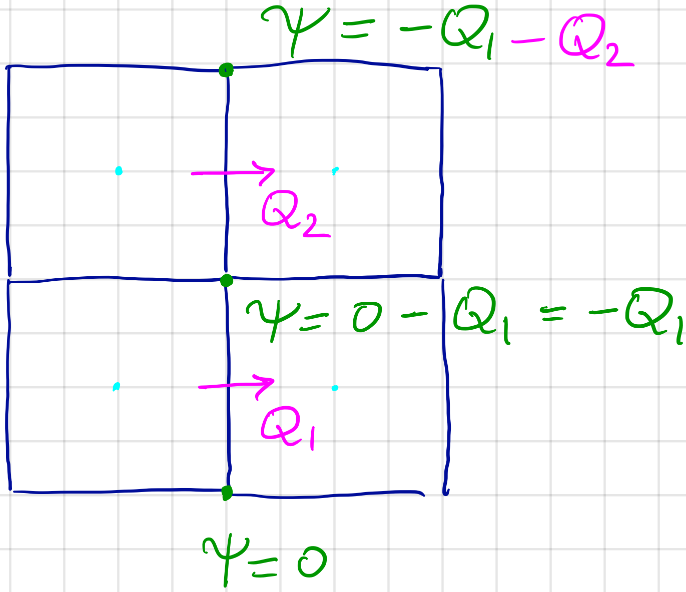
$$Q_S + Q_E + Q_N - Q\,{}^{+Q_B} = 0$$

$$\cancel{T}(h_S - h) + \cancel{T}(h_E - h) + \cancel{T}(h_N - h) - \frac{(Q - Q_B)}{T} = 0$$

$$\boxed{h = \frac{h_S + h_E + h_N}{3} - \frac{(Q - Q_B)}{3T}}$$




$$h = \frac{h_E + h_N}{2} - \frac{Q}{2T}$$


$\psi = -Q_1 - Q_2$
$Q_2$
$\psi = 0 - Q_1 = -Q_1$
$Q_1$
$\psi = 0$