Potential Flow

$$Q_x = -\frac{\partial \Phi}{\partial x} \qquad Q_y = -\frac{\partial \Phi}{\partial y}$$

- Only confined (or constant transmissivity)

$$\Phi = kHh$$

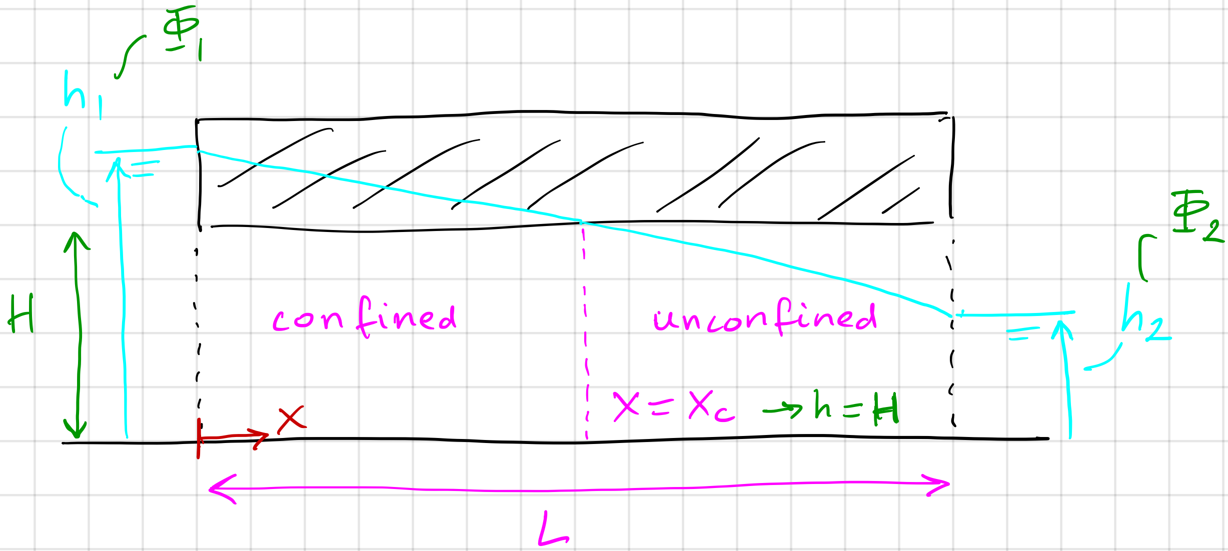
- Only unconfined

$$\Phi = \frac{1}{2}kh^2$$

- Combined confined / unconfined

$$\Phi = kHh - \frac{1}{2}kH^2 \quad \text{(confined)}$$

$$\Phi = \frac{1}{2}kh^2 \quad \text{(unconfined)}$$



$$\frac{d^2\Phi}{dx^2} = -N = 0$$

$$\Phi = Ax + B \rightarrow \Phi = \frac{\Phi_2 - \Phi_1}{L}x + \Phi_1$$

$H = 18$ m $\quad h_1 = 20$ m $\quad h_2 = 14$ m

$L = 2$ km $\quad k = 10$ m/d

a) Q_x ? $\quad$ b) x_c ?

a) $$Q_x = -\frac{d\Phi}{dx} = \frac{\Phi_1 - \Phi_2}{L}$$

$$\Phi_1 = kHh_1 - \frac{1}{2}kH^2 = 1980 \text{ m}^3/\text{d}$$

$$\Phi_2 = \frac{1}{2}kh_2^2 = 980 \text{ m}^3/\text{d}$$

$$Q_x = 0.5 \text{ m}^2/\text{d}$$

b) $x = x_c \rightarrow h = H \rightarrow \Phi = \frac{1}{2}kH^2 = \Phi_c$

$$x_c = \frac{(\Phi_c - \Phi_1)L}{\Phi_2 - \Phi_1} = 720 \text{ m}$$

c) $h(x=500)$? $\quad$ d) $h(x=1500)$ $\quad$ e) Q_x ? everywhere

saturated thickness 18 m

$$\Phi = \frac{\Phi_2 - \Phi_1}{L} x + \Phi_1 = -0.5x + 1980$$

$$\Phi_c = \frac{1}{2} k H^2 = 1620$$

c) $\Phi(500) = 1730 > \Phi_c$ $\qquad \Phi = kHh - \frac{1}{2} k H^2$

$$h = \frac{\Phi}{kH} + \frac{1}{2} H = 18.61 \text{ m.}$$

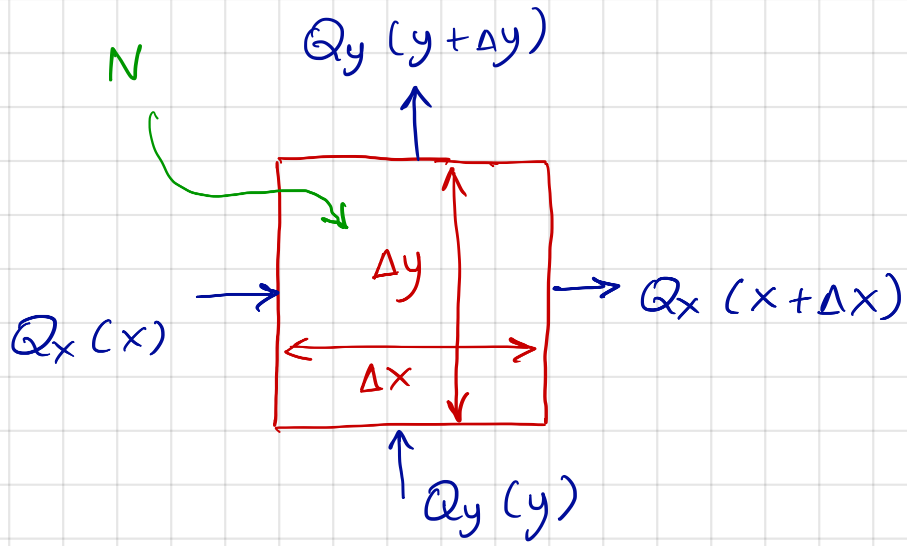
d) $\Phi(1500) = 1230 < \Phi_c$ $\qquad \Phi = \frac{1}{2} k h^2$

$$h = \sqrt{\frac{2\Phi}{k}} = 15.68 \text{ m}$$

e) $$Q_x = \frac{\Phi_1 - \Phi_2}{L} = \frac{k\tilde{H} h_1 - k\tilde{H} h_2}{L} = 0.54 \text{ m}^2/\text{d}$$

less than

10 % off !



$$\text{Out} - \text{In} = 0$$

$$\frac{Q_x(x+\Delta x)\cancel{\Delta y} - Q_x(x)\cancel{\Delta y}}{\Delta x \cancel{\Delta y}} + \frac{Q_y(y+\Delta y)\cancel{\Delta x} - Q_y(y)\cancel{\Delta x}}{\cancel{\Delta x}\Delta y} - \frac{N\cancel{\Delta x}\cancel{\Delta y}}{\cancel{\Delta x}\cancel{\Delta y}} = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - N = 0$$

$$Q_x = -\frac{\partial \Phi}{\partial x} \qquad Q_y = -\frac{\partial \Phi}{\partial y}$$

$$\boxed{\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = -N}$$ Poisson's Eq.

$N = 0$ $$\boxed{\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0}$$ Laplace's DEQ

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

$\Phi_1(x,y)$ $\Phi_2(x,y)$ are solutions to DEQ

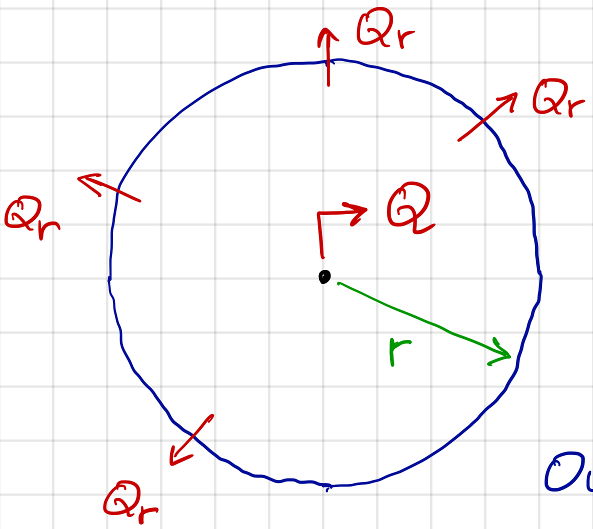
$$\Phi = A\Phi_1 + B\Phi_2$$

$$\frac{\partial^2(A\Phi_1 + B\Phi_2)}{\partial x^2} + \frac{\partial^2(A\Phi_1 + B\Phi_2)}{\partial y^2} \stackrel{?}{=} 0$$

$$A\frac{\partial^2 \Phi_1}{\partial x^2} + B\frac{\partial^2 \Phi_2}{\partial x^2} + A\frac{\partial^2 \Phi_1}{\partial y^2} + B\frac{\partial^2 \Phi_2}{\partial y^2} \stackrel{?}{=} 0$$

$$A\underbrace{\left(\frac{\partial^2 \Phi_1}{\partial x^2} + \frac{\partial^2 \Phi_1}{\partial y^2}\right)}_{0} + B\underbrace{\left(\frac{\partial^2 \Phi_2}{\partial x^2} + \frac{\partial^2 \Phi_2}{\partial y^2}\right)}_{0} = 0$$

QED



Q : Discharge of well

$\frac{m^3}{d}$ Positive when taking water out

$$Out - In = 0$$

$$Q + Q_r 2\pi r = 0$$

$$Q_r = -\frac{d\Phi}{dr}$$

$$Q - \frac{d\Phi}{dr} 2\pi r = 0$$

$$\frac{d\Phi}{dr} = \frac{Q}{2\pi} \frac{1}{r}$$

$$\boxed{\Phi = \frac{Q}{2\pi} \ln r + C}$$

$$\Phi = \frac{Q}{2\pi} \ln r + C$$

$r = R$ $\quad h = h_0$

$\downarrow$

$$\Phi(r=R) = \frac{Q}{2\pi} \ln R + C = \Phi_0$$

$\Phi = \Phi_0$ — kHh_0 | $\frac{1}{2}kh_0^2$

or combined

$$C = \Phi_0 - \frac{Q}{2\pi} \ln R$$

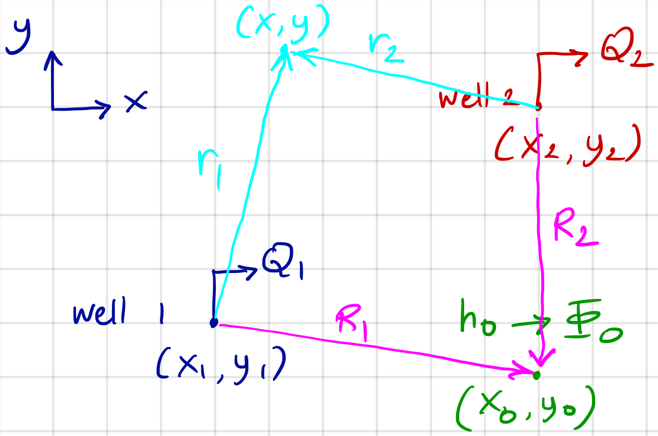
$$\Phi = \frac{Q}{2\pi} \ln r - \frac{Q}{2\pi} \ln R + \Phi_0$$

$$\boxed{\Phi = \frac{Q}{2\pi} \ln \frac{r}{R} + \Phi_0}$$

@ well $\rightarrow$ $r = r_w$ radius of well

Cone of depression

Drawdown at the well



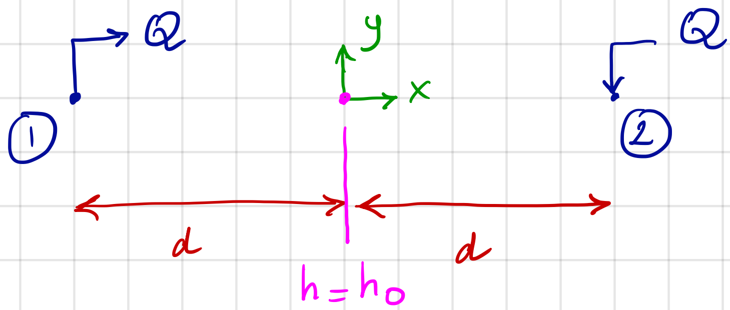
$$r_1 = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$r_2 = \sqrt{(x-x_2)^2 + (y-y_2)^2}$$

$$\Phi = \frac{Q_1}{2\pi} \ln r_1 + \frac{Q_2}{2\pi} \ln r_2 + C$$

$$\Phi = \frac{Q_1}{2\pi} \ln R_1 + \frac{Q_2}{2\pi} \ln R_2 + C = \Phi_0 \quad \rightarrow \text{solve for } C$$

$$\boxed{\Phi = \frac{Q_1}{2\pi} \ln \frac{r_1}{R_1} + \frac{Q_2}{2\pi} \ln \frac{r_2}{R_2} + \Phi_0}$$



Confined $K = 10\ m/d$ $r_w = 0.2\ m$ $d = 100\ m$

$H = 20\ m$ $Q = 500\ m^3/d$ $h_0 = 40\ m$

a) $\Phi(x,y)$ b) h at well 1

$$\Phi = \frac{Q}{2\pi}\ln r_1 - \frac{Q}{2\pi}\ln r_2 + C$$

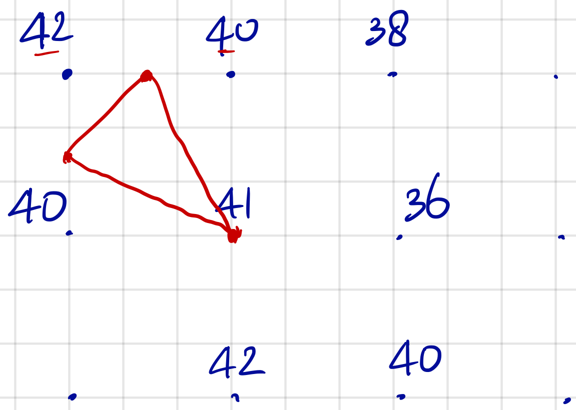
$r_1 = r_2 = d \rightarrow \Phi = \Phi_0 = kHh_0$

$$\Phi = \frac{Q}{2\pi}\ln\frac{r_1}{r_2} + \Phi_0 \qquad r_1 = \sqrt{(x+d)^2 + y^2}$$

$$r_2 = \sqrt{(x-d)^2 + y^2}$$

b) $r_1 = r_w$ $r_2 = 2d$

Streamfunction : $\Psi = \frac{Q}{2\pi}\theta_1 - \frac{Q}{2\pi}\theta_2$


42
40
38
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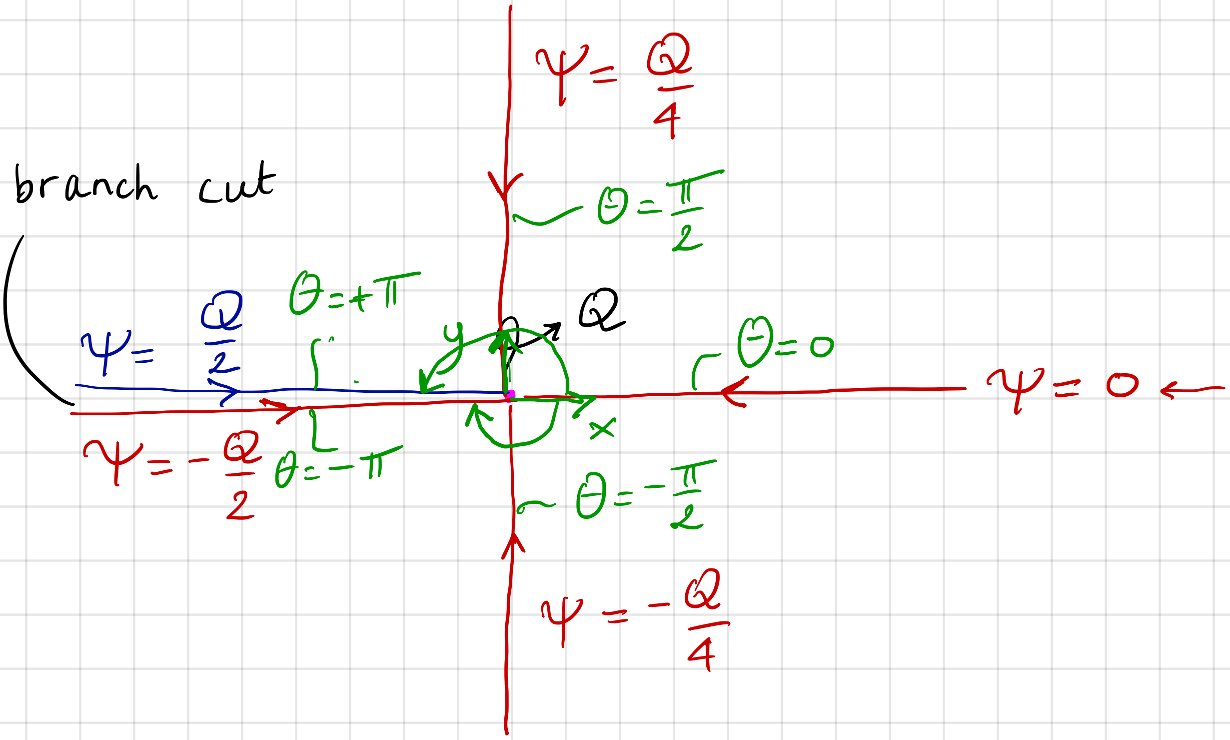
41
36
42
40

Stream function Ψ (Psi)

Constant along a streamline

$\Delta Q = \Psi_1 - \Psi_0$

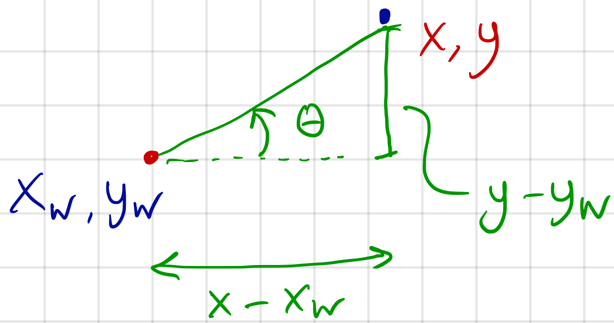
Difference between two stream function values (is ΔQ)



$$\boxed{\Psi = \frac{Q}{2\pi}\theta}$$

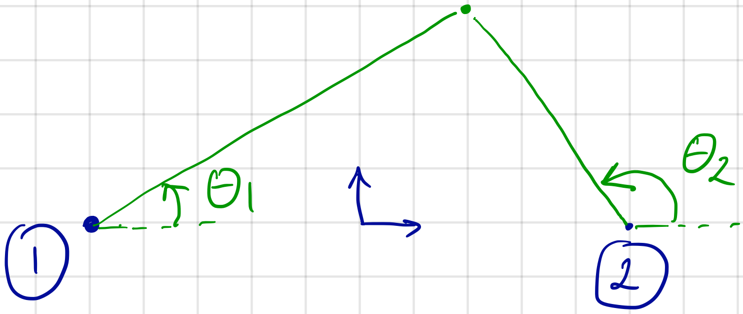
θ : angle with respect to positive x-axis from the well

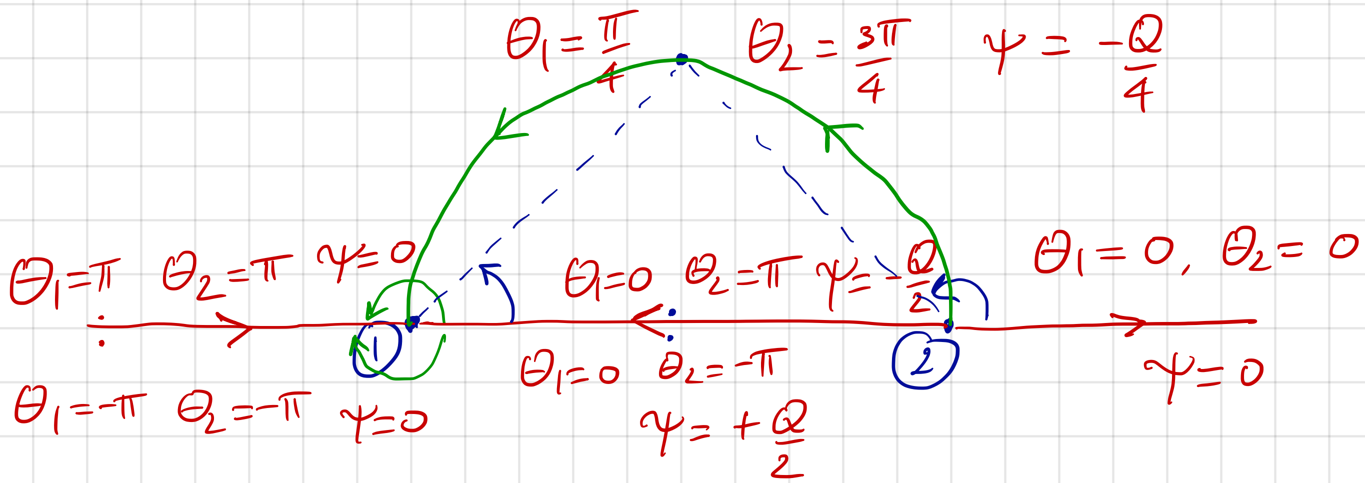
Stream function increases to your right when looking in the direction of flow



$$\theta = \arctan\left(\frac{y - y_w}{x - x_w}\right)$$

Streamfunction : $\psi = \frac{Q}{2\pi}\theta_1 - \frac{Q}{2\pi}\theta_2$



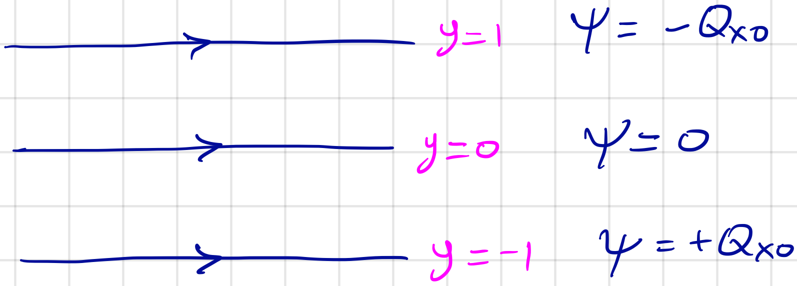


Uniform flow : $\Phi = -Q_{x0}x - Q_{y0}y + C$

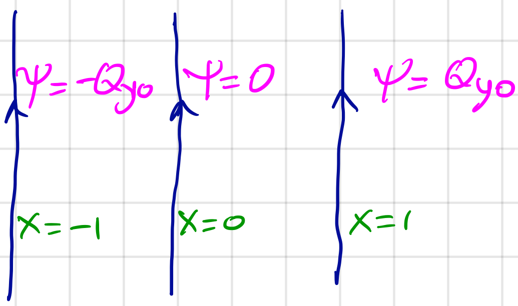
$Q_x = -\dfrac{\partial \Phi}{\partial x} = Q_{x0}$ $\quad$ $Q_y = -\dfrac{\partial \Phi}{\partial y} = Q_{y0}$

$\psi = -Q_{x0}y + Q_{y0}x$

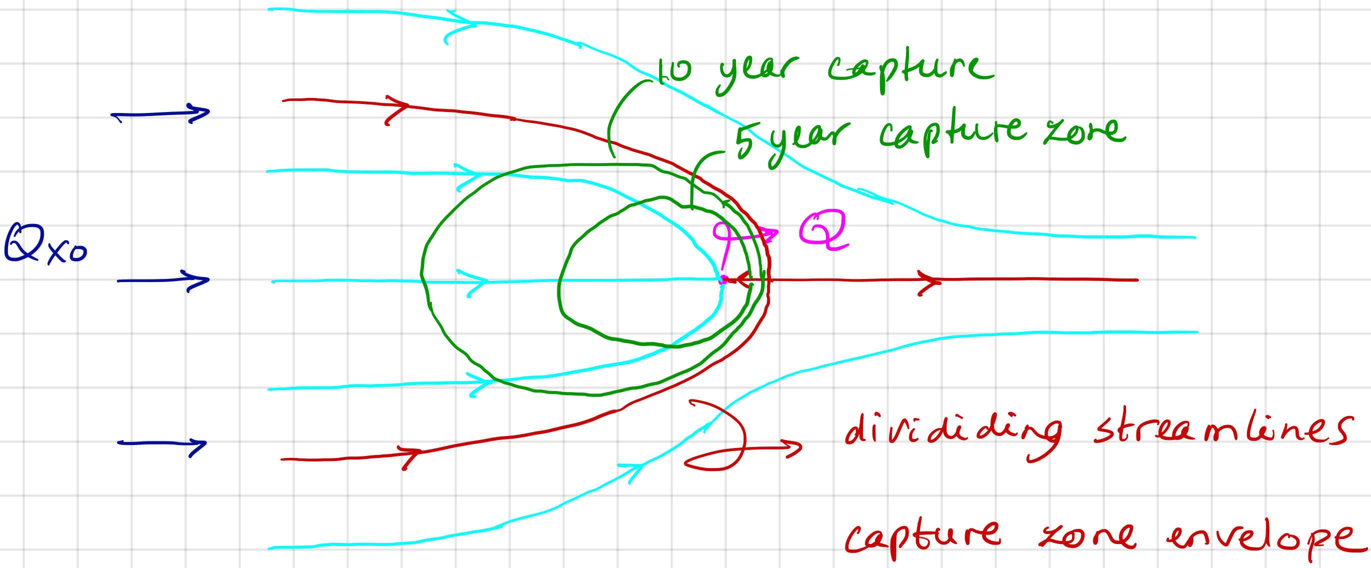
Case $Q_x = Q_{x0}$ $\quad$ $Q_y = 0$

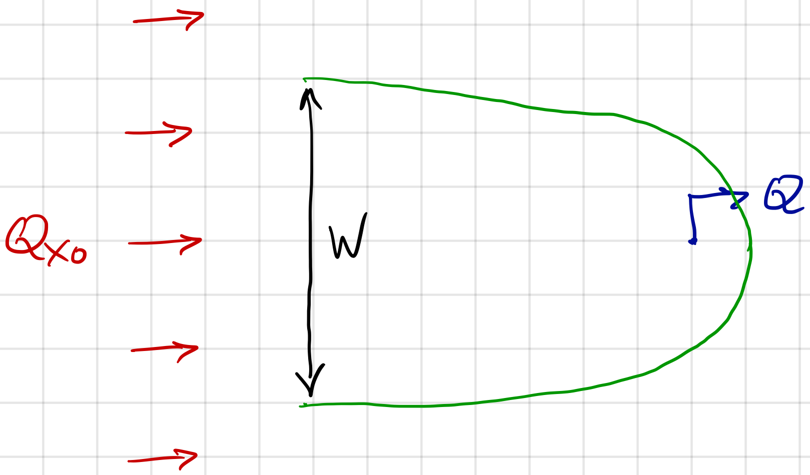


Case $Q_y = Q_{y0}$ $\quad$ $Q_x = 0$ $\longrightarrow$ $\psi = Q_{y0}x$



Well in uniform flow



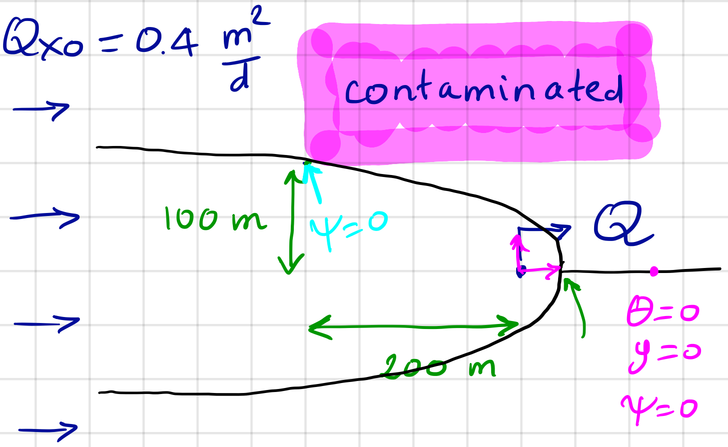


$$Q_{xo} W = Q$$

$$W = \frac{Q}{Q_{xo}}$$

FAR UPSTREAM

$Q_{xo} = 0.4 \, \frac{m^2}{d}$

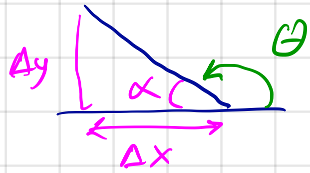


Question :

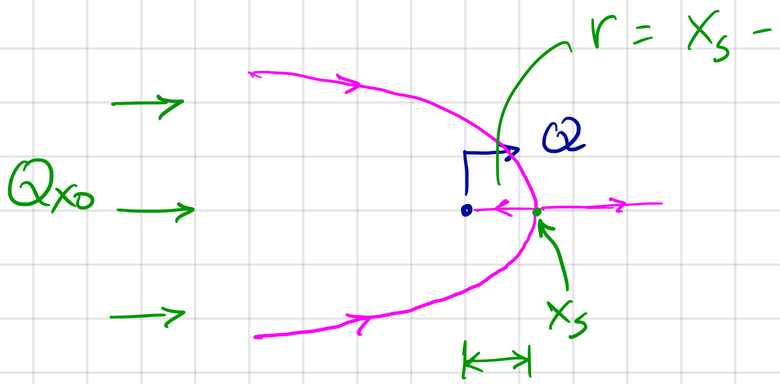
Max. Q such that no water comes from the contaminated area

$$\psi = \frac{Q}{2\pi}\theta - Q_{xo}\,y$$

$y = 100$ $\qquad$ $\underbrace{\theta = \operatorname{arctan2}(100, 200)}_{= 2.68}$



$\rightarrow Q = 93 \; m^3/d$



$$\Phi = \frac{Q}{2\pi}\ln(r) - Q_{x0}\,x + C$$

$$Q_x = -\frac{\partial \Phi}{\partial x}$$

$$Q_x = -\frac{\partial \Phi}{\partial x} = -\frac{Q}{2\pi}\,\frac{1}{r}\,\frac{\partial r}{\partial x} + Q_{x0}$$

$$r = \left[(x-x_w)^2 + (y-y_w)^2\right]^{1/2}$$

$$\frac{\partial r}{\partial x} = \cancel{\tfrac{1}{2}}\,\underbrace{\left[(x-x_w)^2 + (y-y_w)^2\right]^{-\frac{1}{2}}}_{\frac{1}{r}}\,\cancel{2}(x-x_w) = \frac{x-x_w}{r}$$

$$Q_x = -\frac{Q}{2\pi}\,\frac{1}{r}\,\frac{(x-x_w)}{r} + Q_{x0}$$

$$Q_x = -\frac{Q}{2\pi}\,\frac{(x-x_w)}{r^2} + Q_{x0}$$

Q_x & Q_y for a well

$$Q_x = -\frac{Q}{2\pi}\,\frac{x-x_w}{r^2} \qquad Q_y = -\frac{Q}{2\pi}\,\frac{y-y_w}{r^2}$$

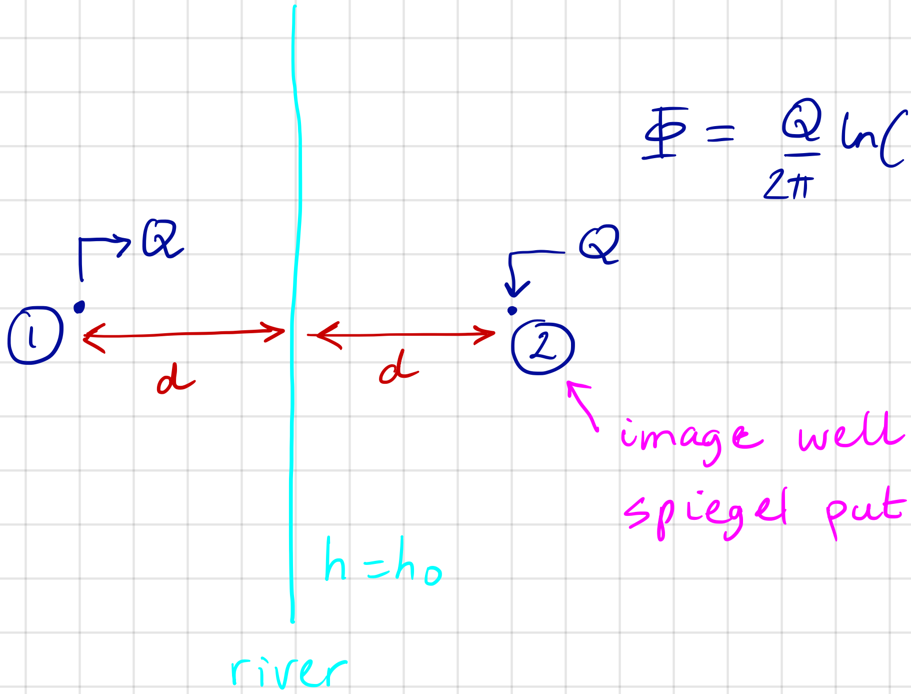
$$Q_x = \frac{-Q}{2\pi} \frac{(x_s - x_w)}{r^2} + Q_{x0} = 0$$

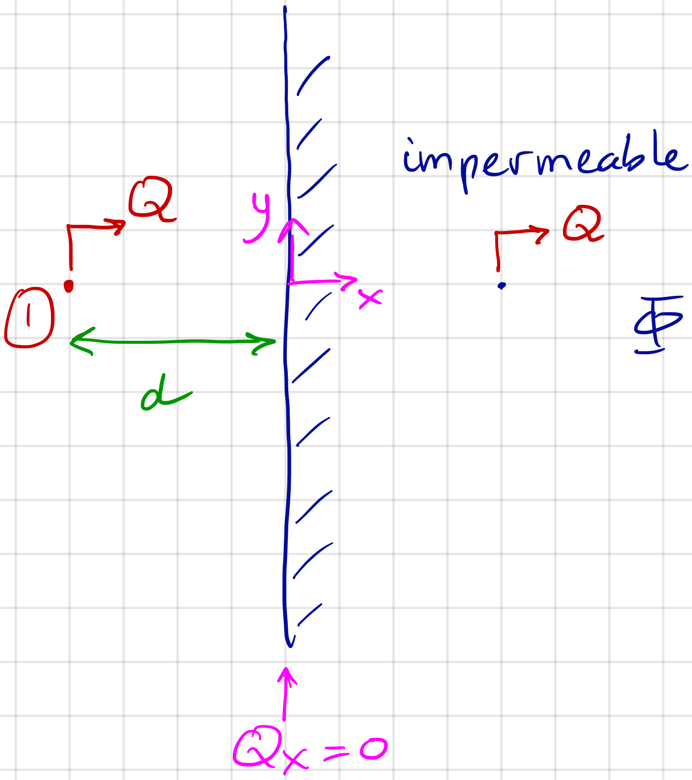
$$\frac{x_s - x_w}{r^2} = \frac{2\pi Q_{x0}}{Q} \qquad r = x_s - x_w$$

$$\cancel{\frac{x_s - x_w}{(x_s - x_w)^{\cancel{2}}}} = \frac{2\pi Q_{x0}}{Q}$$

$$\boxed{x_s - x_w = \frac{Q}{2\pi Q_{x0}}}$$

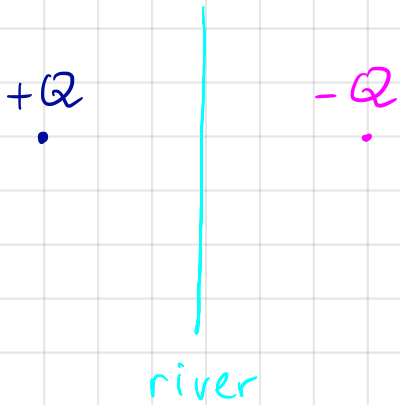
$$\Phi = \frac{Q}{2\pi} \ln\left(\frac{r_1}{r_2}\right) + \Phi_0$$

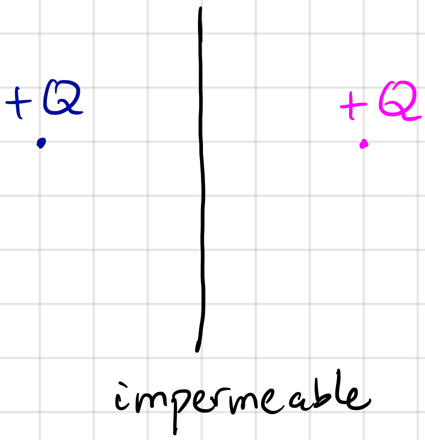


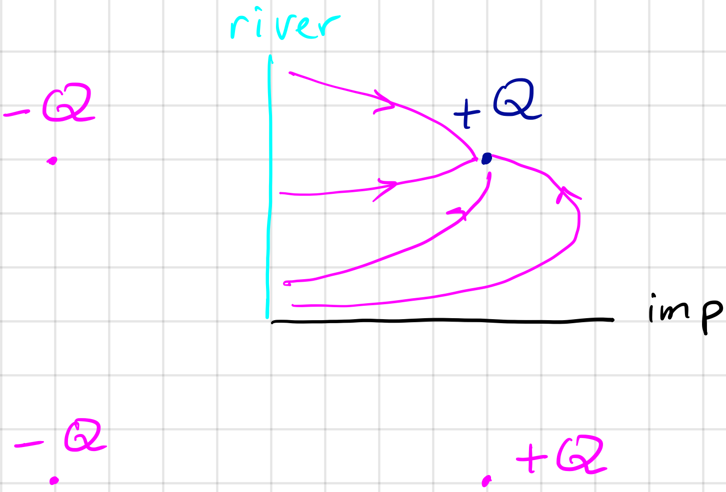


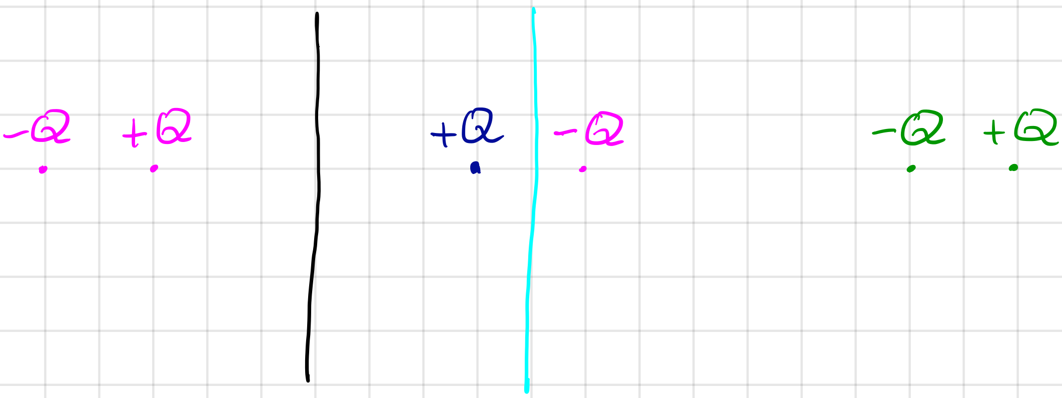
$$\Phi = \frac{Q}{2\pi} \ln r_1 + \frac{Q}{2\pi} \ln r_2 + C$$

$$= \frac{Q}{2\pi} \ln\left(r_1 r_2\right) + C$$


+Q
-Q
river


+Q
+Q
impermeable


river
-Q
+Q
imp
-Q
+Q


-Q
+Q
+Q
-Q
-Q
+Q