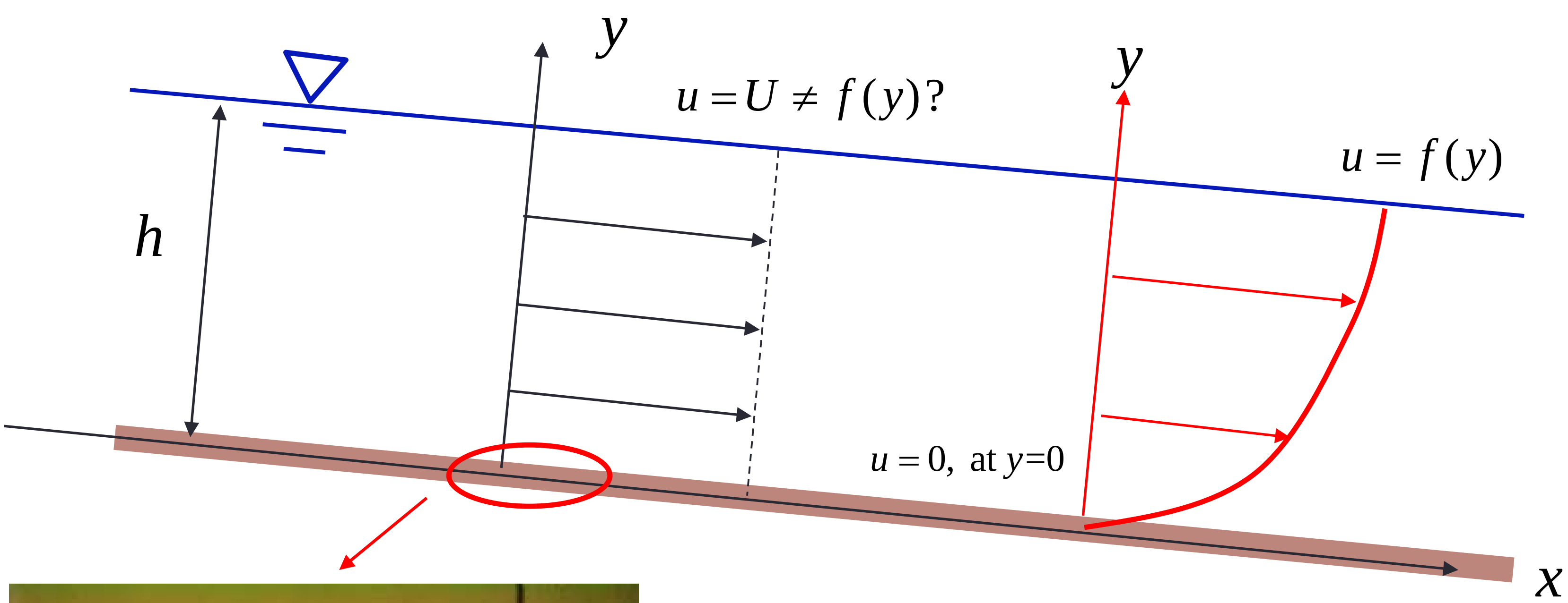
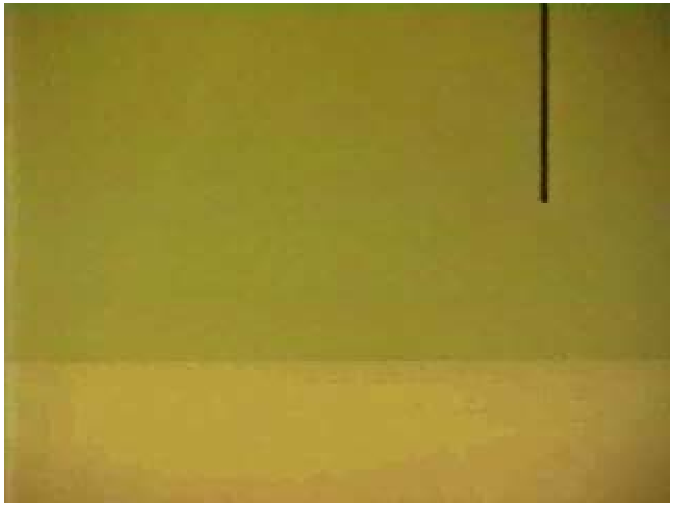
BOUNDARY LAYER FLOW IN OPEN CHANNELS

~~RIVER MECHANICS~~ (OPEN-CHANNEL HYDRAULICS) (CE5312)

Dr. Yuan Jing,
Department of Civil and Environmental Engineering
Office: E2-05-20
Phone: 65162160
Email: ceeyuan@nus.edu.sg

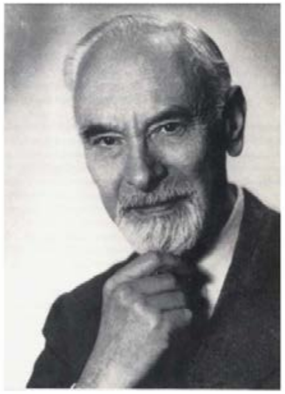
Vertical velocity distribution in open channels



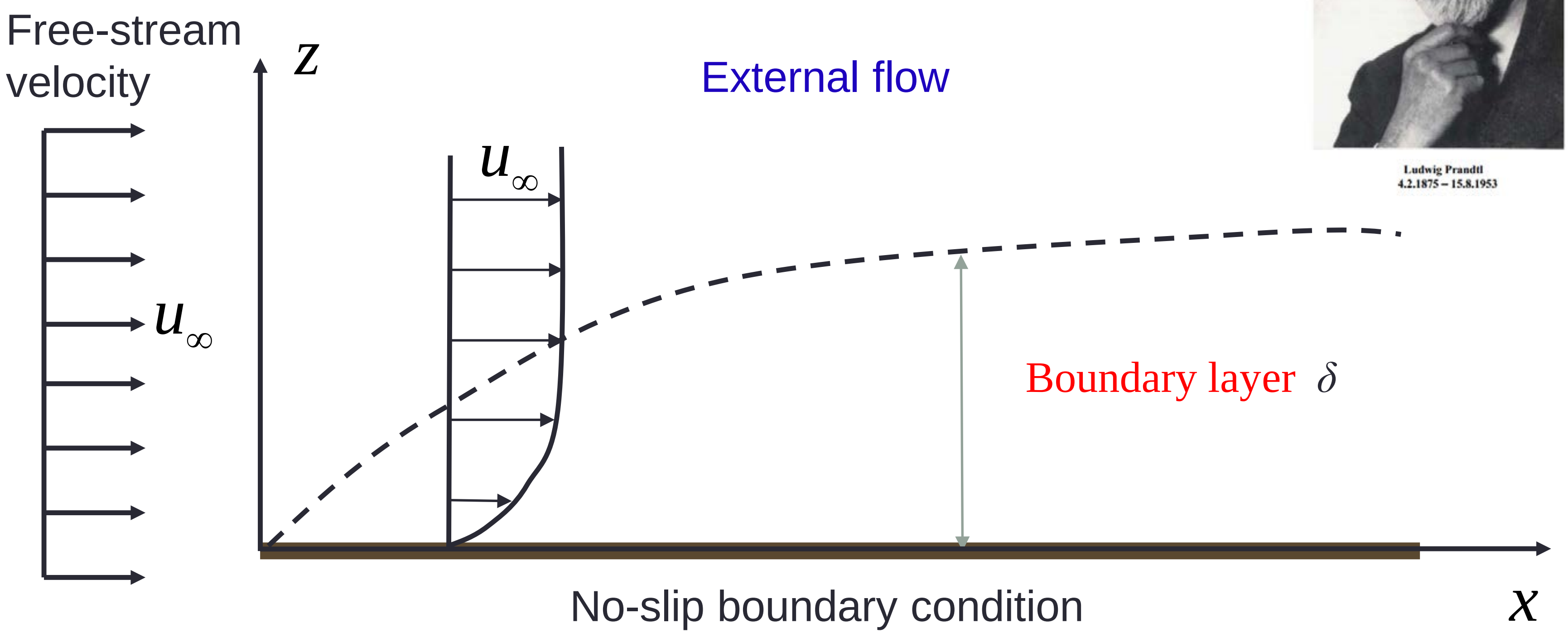


- Previously, we always assume the flow is depth-uniform.
- But this clearly violates the no-slip boundary condition.
- A realistic vertical velocity profile should decrease towards the bottom.

boundary-layer type flow



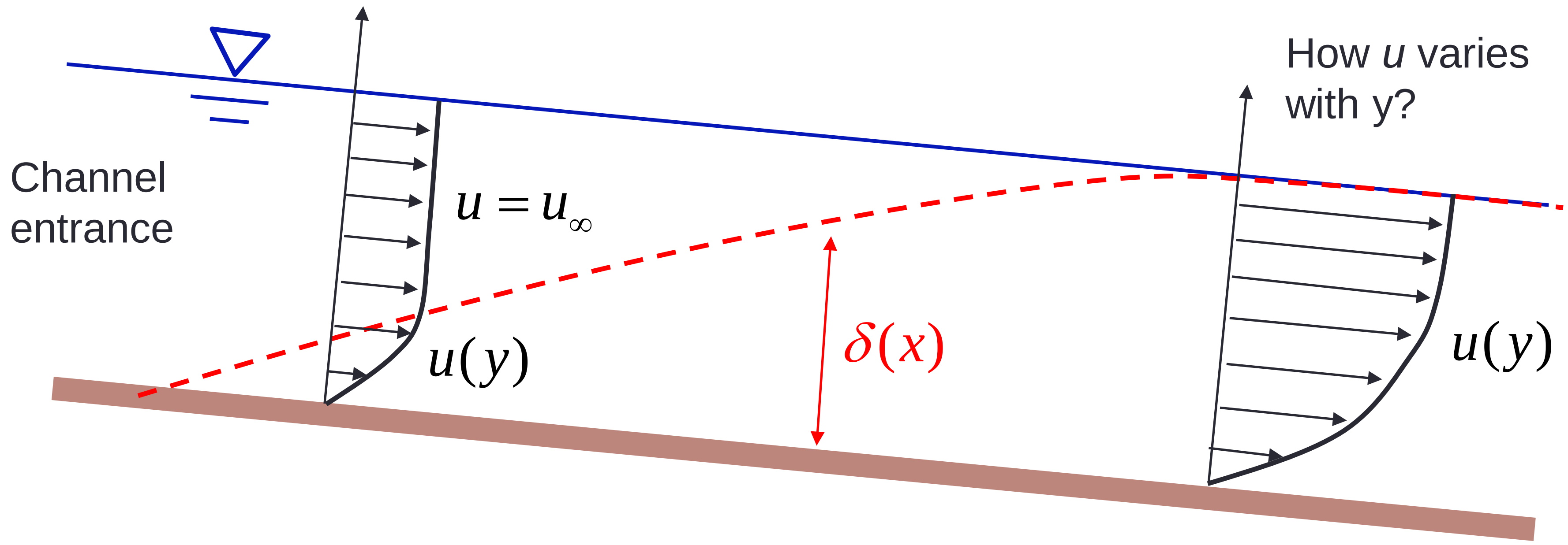
Ludwig Prandtl
4.2.1875 – 15.8.1953



Within a thin layer near the bottom, velocity varies significantly in the wall-normal direction, so the no-slip boundary condition is satisfied.

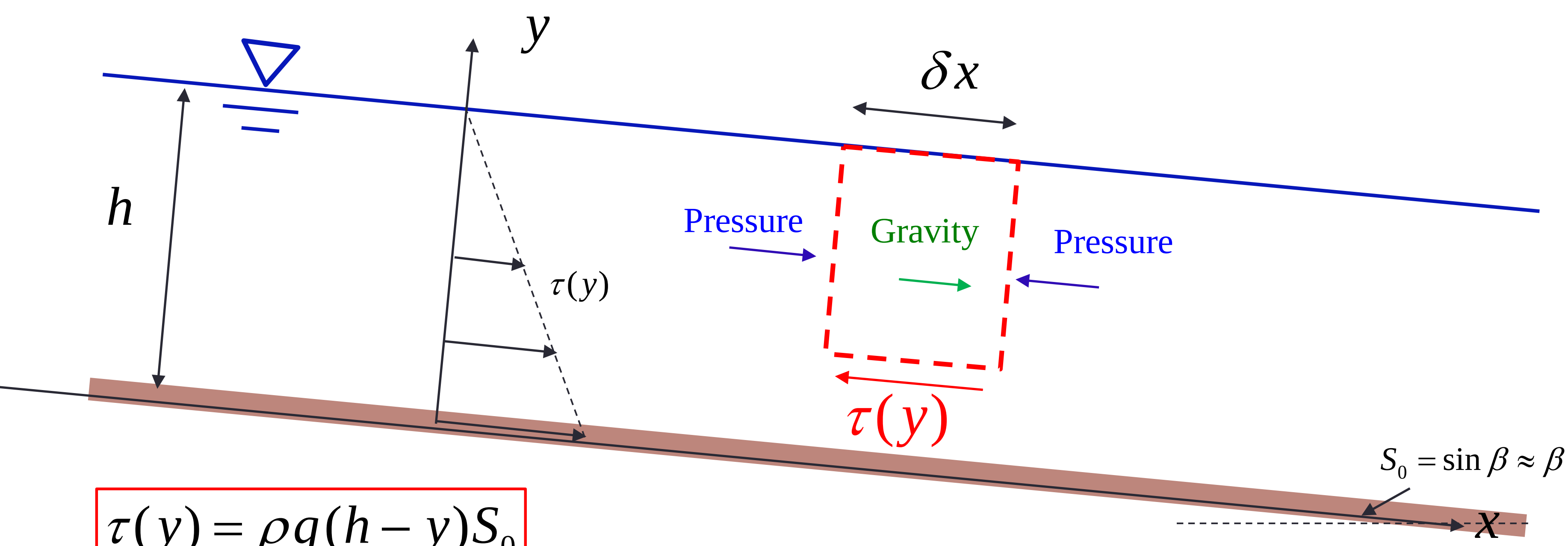
The boundary layer thickness increases with *x*.

Boundary layer in open channel



- The boundary layer (BL) have enough space to develop until the BL thickness is equal to water depth.
- From that point downstream the boundary layer is fully developed.
- We are primarily interested with the fully developed BL.

Shear stress in open-channel flow



$$\boxed{\begin{aligned}\tau(y) &= \rho g(h-y)S_0 \\ &= \tau_b\left(\frac{h-y}{h}\right)\end{aligned}}$$

$$\tau(y) = f(u(y))$$

$$\Rightarrow f(u(y)) = \rho g(h-y)S_0$$

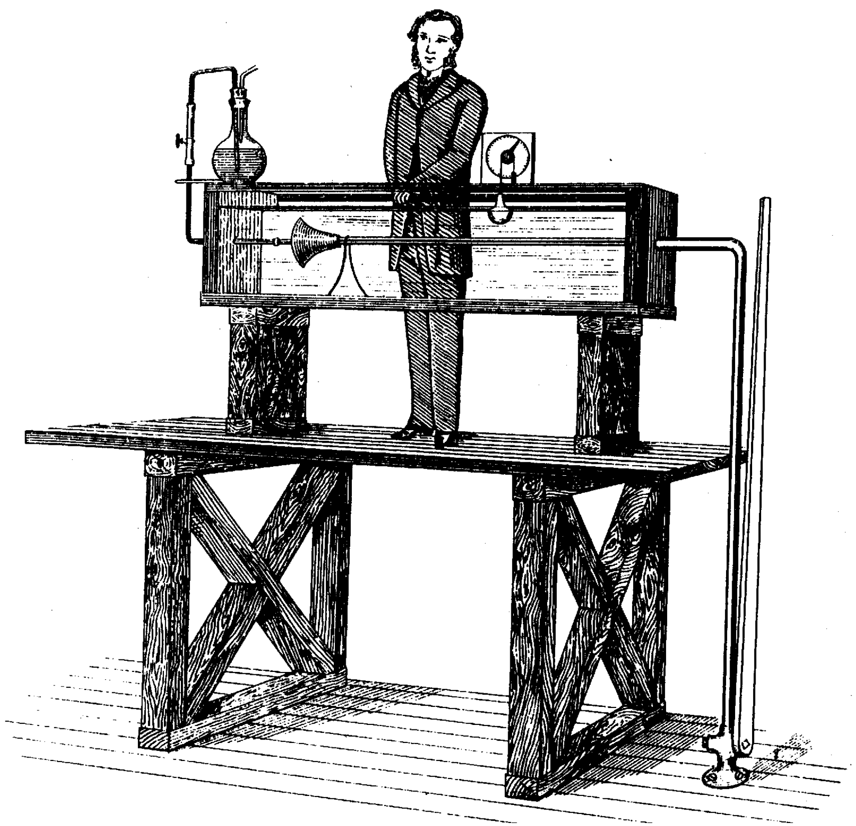
Shear stress at level y balances the channel-parallel component of gravity above y.

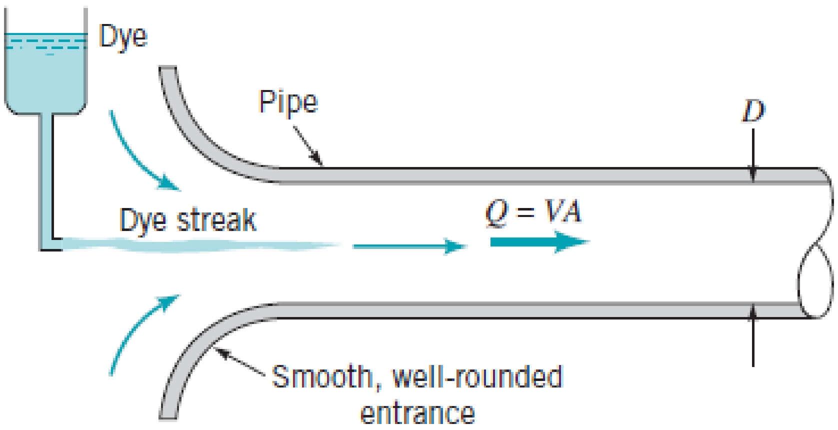
If we understand how shear stress varies with flow velocity, we will have an equation for u(y)!

Laminar and turbulent flows:
----Reynolds dye experiment (1883)

How does pipe flow's behavior change with Re?

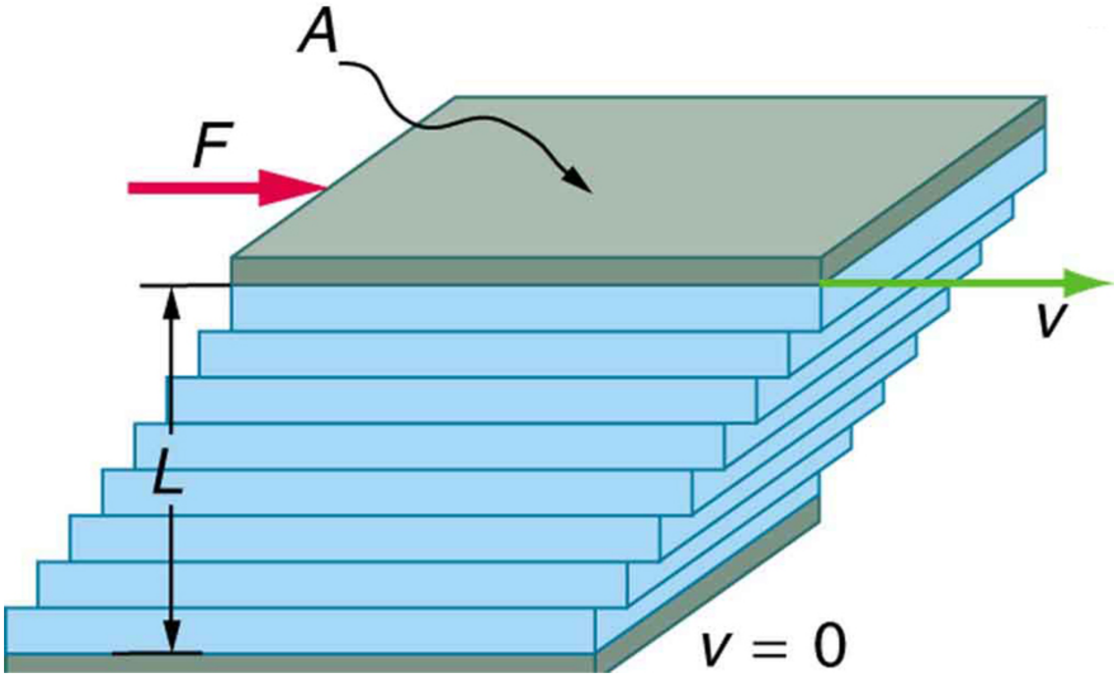
$$\mathrm{Re} = \frac{DV}{\nu}$$





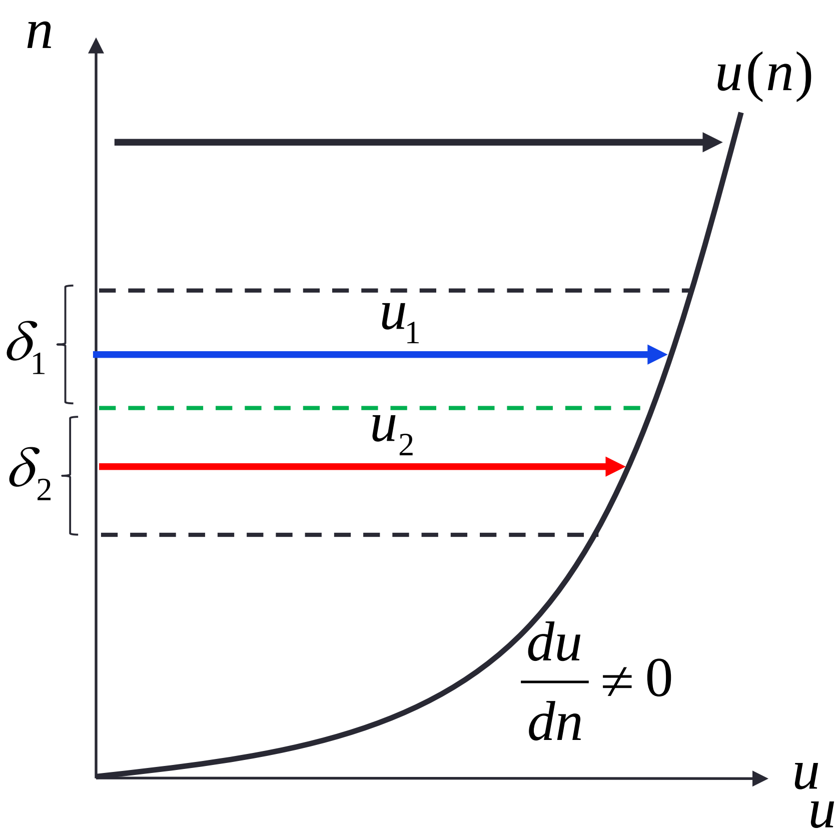
- Depends on flow and wall conditions, the boundary layer flow can be either laminar or turbulent.
- Reynolds number $$\mathrm{Re} = \frac{U \cdot L}{\nu} \begin{cases} \text{laminar if Re is small} \\ \text{turbulent if Re is large} \end{cases}$$
- The physics for shear stress is not the same for laminar and turbulent flows, so we need to look at them separately.

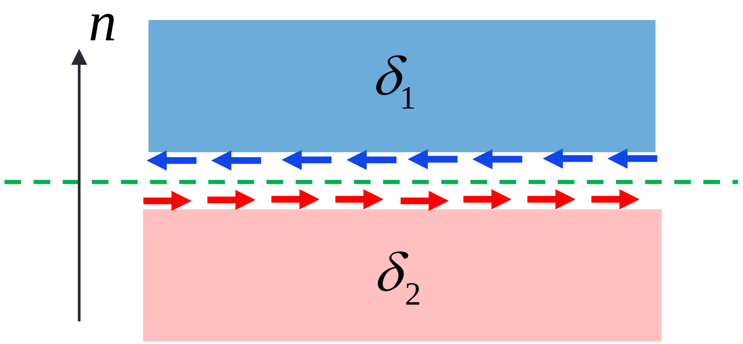
Laminar flow



laminar flow occurs when a fluid flows in parallel layers
Very low velocity or very viscous (“thick”) fluid, e.g. blood flow

The nature of shear stress for laminar flow





Viscous stress is the shear stress

$$\tau = \mu \frac{du}{dy} = \rho \nu \frac{du}{dy}$$

μ : dynamic viscosity [kg/(sm)]

ν : kinematic viscosity=μ / ρ [m^2/s]

Laminar boundary layer in open channels

$G.E.:$

$$\tau(y) = \tau_b\left(1 - \frac{y}{h}\right) = \mu\frac{\partial u}{\partial y}$$

$B.C.:$

$$u = 0, y = 0$$

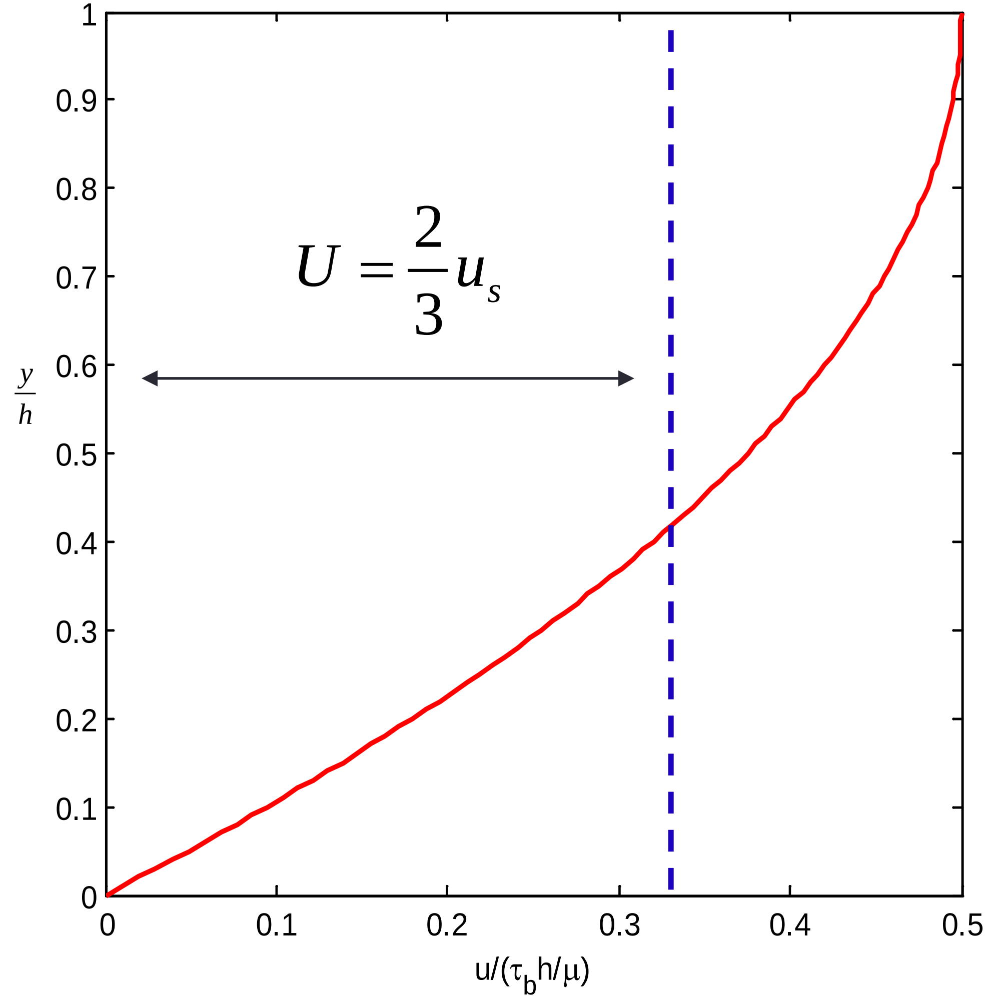
Solution:

$$u = \frac{\tau_b h}{\mu}\left(\frac{y}{h} - \frac{1}{2}\left(\frac{y}{h}\right)^2\right)$$

Depth-averaged :

$$U = \frac{1}{h}\int_0^h u\,dy = \frac{2}{3}u_s$$

$$y = h: \quad u_s = \frac{1}{2}\left(\frac{\tau_b h}{\mu}\right)$$



Friction factor for laminar BL open-channel flow

$$\tau_b = \frac{1}{8} f \rho U^2$$

f is Darcy-Weisbach's friction factor

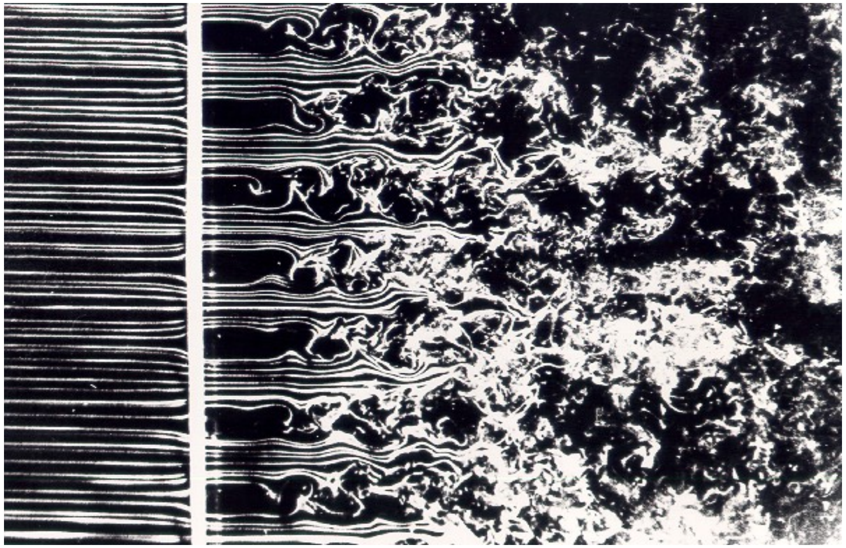
$$\left.\begin{aligned} U &= \frac{2}{3} u_s \\ u_s &= \frac{1}{2}\left(\frac{\tau_b h}{\mu}\right) \end{aligned}\right\} \Rightarrow U = \frac{1}{3}\left(\frac{\tau_b h}{\mu}\right) \Rightarrow \tau_b = \mu \frac{3U}{h}$$

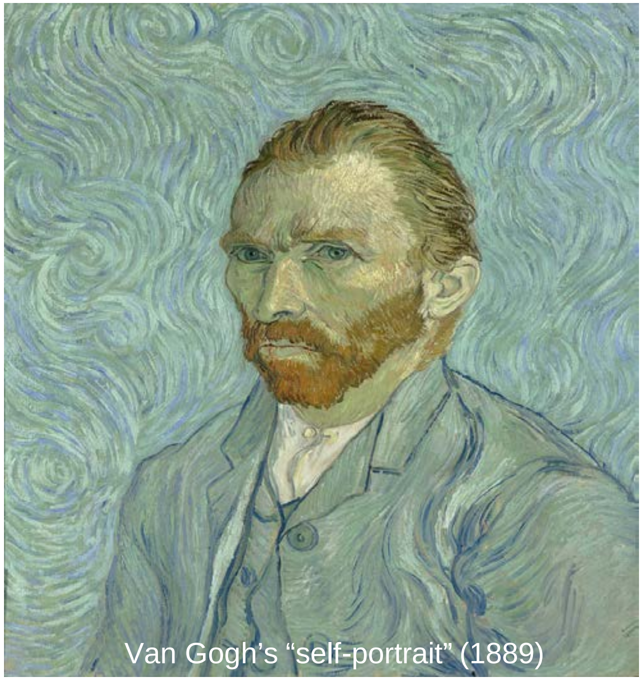
Some algebra: $\boxed{f = \frac{24}{\mathrm{Re}}, \quad \mathrm{Re} = \frac{Uh}{\nu}}$

Roughly speaking, the flow turns turbulent when
Re>~1000 (very easy to exceed this limit)

Turbulent flow

- Trigged by some irregularities
- Chaotic but with <u>coherent structure</u>
- An unsolved mystery!

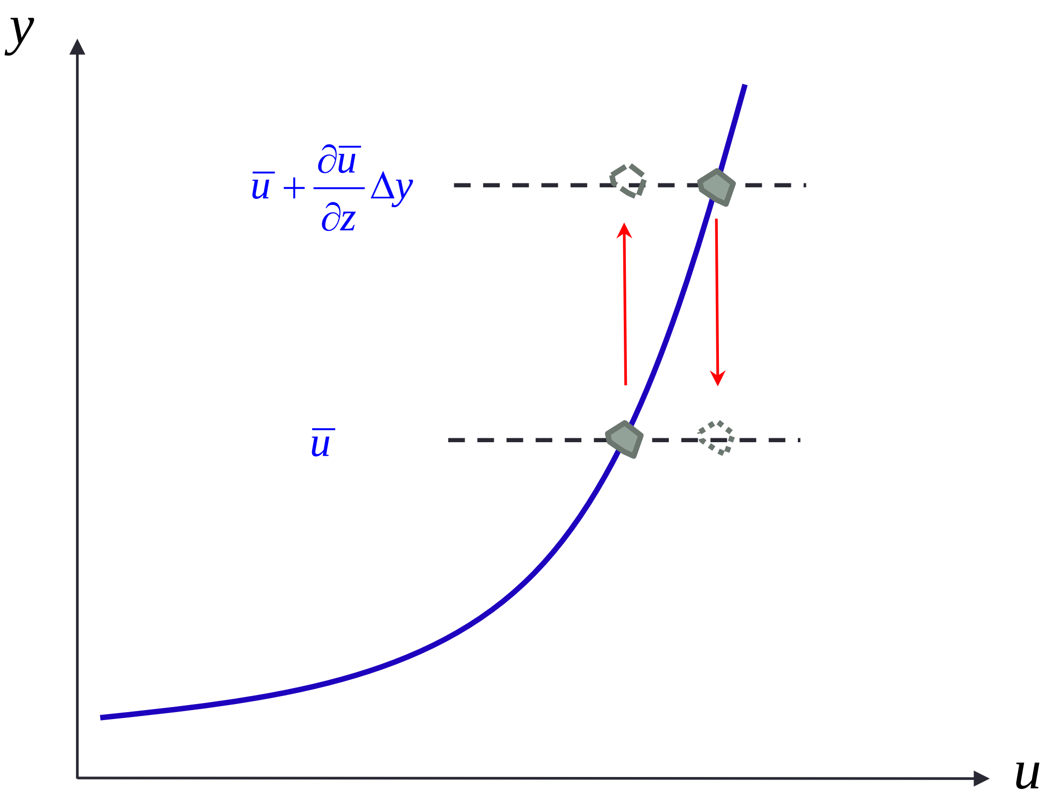


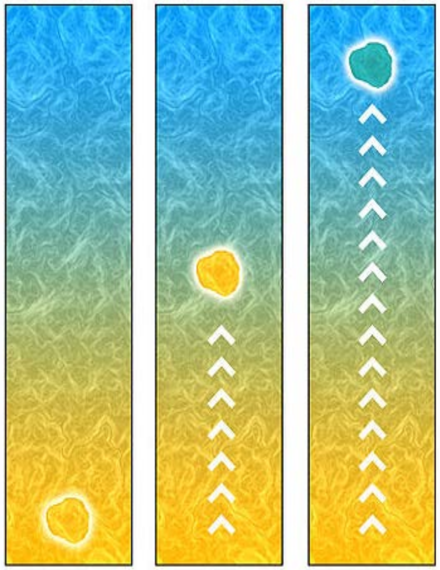


Van Gogh's "self-portrait" (1889)

Van Gogh's "The Starry Night" (1889)

Shear stress for turbulent flows





Turbulences lead to exchange of momentum in the y direction, which is equivalent to a shear stress, and we call it Reynolds stress

Reynolds stress

Reynolds stress: the exchange of momentum due to turbulence mixing

$$\boxed{\frac{\tau(y)}{\rho} = \nu_T \frac{\partial u}{\partial y}}$$

ν_T : turbulent eddy viscosity

$[\nu_T] = [\text{length} \cdot \text{velocity}]$

In the very near-bottom region: *y*<<*h*

[velocity]: $u_* = \sqrt{\tau_b / \rho}$ (shear velocity)

[length]: y

$$\boxed{\nu_T = \kappa u_* y} \qquad \boxed{\frac{\tau(y)}{\rho} = \kappa u_* y \frac{\partial u}{\partial y}}$$

$\kappa \approx 0.4$: von Karman constant

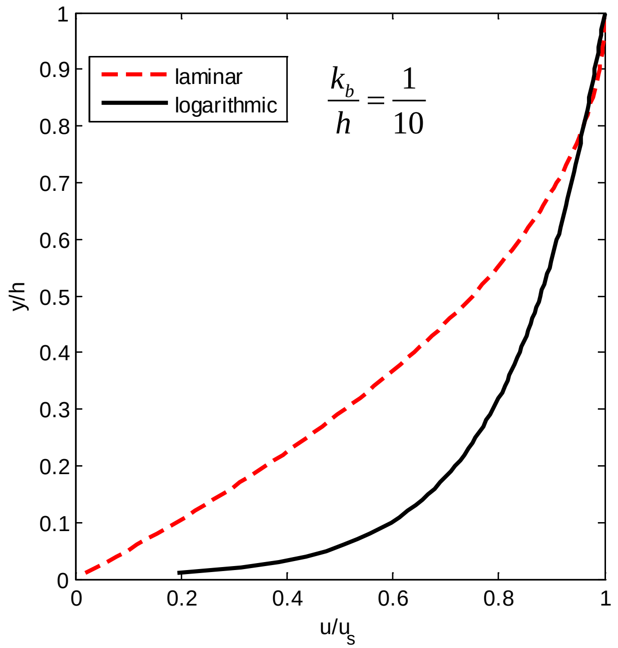
Logarithmic velocity profile (*y*<0.15*h*)

neglect shear stress variation for $\frac{y}{h} < 0.15$

$$\frac{\tau(y)}{\rho} \approx \frac{\tau_b}{\rho} = \kappa u_* y \frac{\partial u}{\partial y}$$

$$\boxed{u = \frac{u_*}{\kappa} \ln \frac{30y}{k_b}}$$

K_b: bottom roughness (a function of flow and bottom condition)



Viscous sublayer : (usually less than 1mm thick)

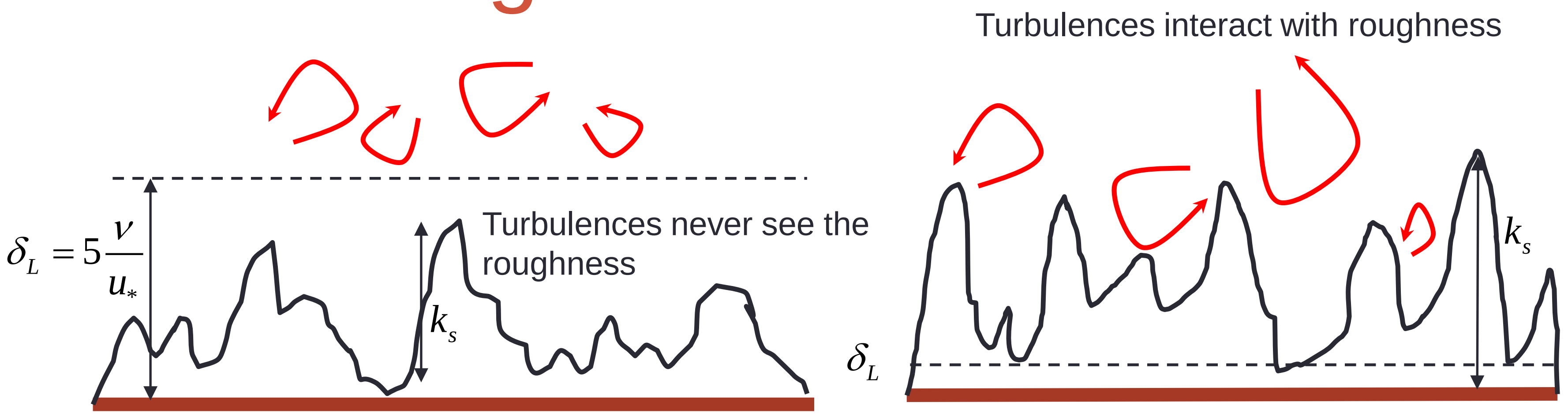
In the immediate vicinity of the solid wall, local flow is weak enough to be laminar

$$\frac{\tau}{\rho} \approx \frac{\tau_b}{\rho} = u_*^{\ 2} = \nu \frac{\partial u}{\partial y}$$

$$\Rightarrow \quad \boxed{u = \frac{u_*^{\ 2}}{\nu} y, \quad y < 5\frac{\nu}{u_*}}$$

Viscous length scale

Bottom roughness



(a) Hydrodynamically smooth **(b) Fully rough turbulent flow**

k_s: Nikuradse equivalent sand grain roughness

$$\mathrm{Re}_* = \frac{k_s}{\nu / u_*} = \frac{k_s u_*}{\nu} = \frac{\text{roughness}}{\text{viscous scale}}$$

$$u = \frac{u_*}{\kappa} \ln \frac{30y}{k_b}$$

$\mathrm{Re}_* < 5;$	$k_b = 3.3\nu / u_*$	Smooth turbulent Flow
$5 < \mathrm{Re}_* < 100;$	$k_b = f(k_s, \nu / u_*)$	Transitional rough turbulent Flow
$\mathrm{Re}_* > 100;$	$k_b = k_s$	Fully rough turbulent flow

Generalized Logarithmic velocity profile for turbulent open-channel flow

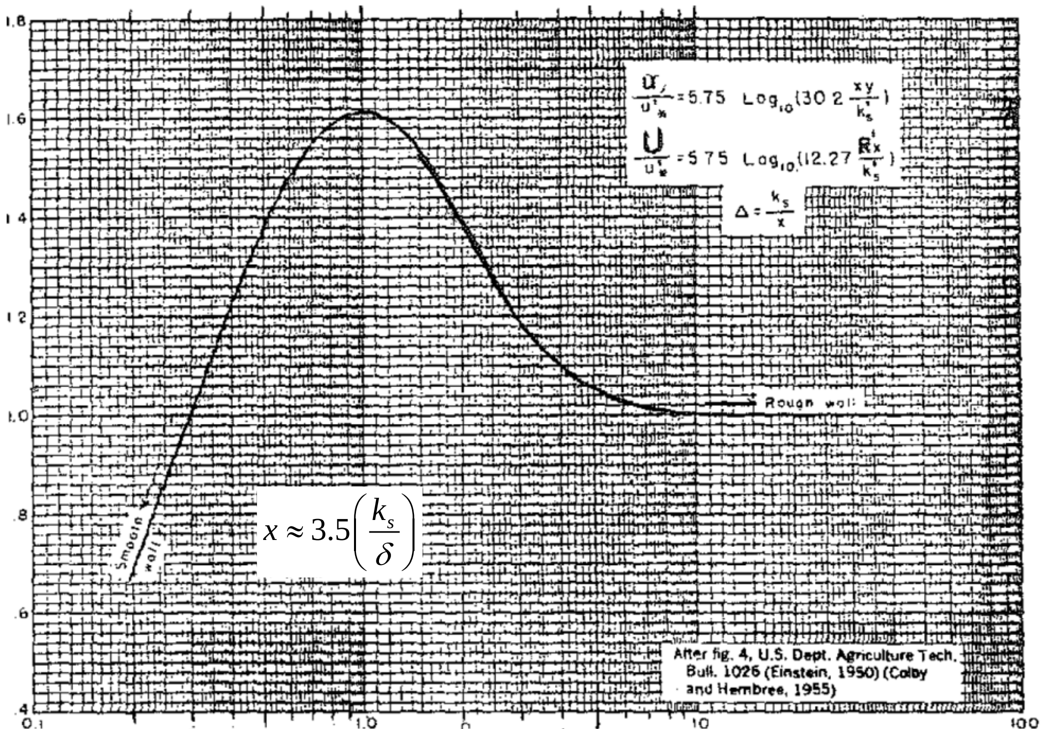
Logarithmic layer can reasonably approximate laminar sublayer (if any) and outer layer, so we simply adopt it as the entire velocity profile:

$$\boxed{\frac{u}{u_*} = \frac{1}{\kappa}\ln\frac{30xy}{k_s} \qquad \left(\frac{k_s}{30x} \le y \le h\right)}$$

x makes this distribution valid for smooth; transition and rough turbulent flow.
x=1 for fully rough turbulent flow

Mean velocity $$\boxed{\frac{U}{u_*} = \frac{1}{\kappa}\ln\frac{11xh}{k_s}}$$

x



k_s/δ

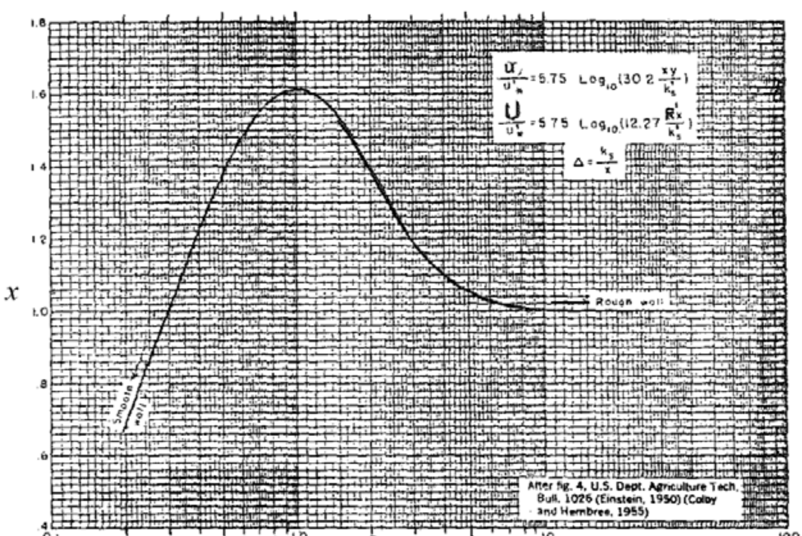
$$\delta = 11.6\frac{\nu}{u_*}$$

Summary

$$\tau_b = \rho g R_h S_0 \approx \rho g h S_0$$

Laminar flow (Re=Uh/v<1000): $$u(y) = \frac{u_*^2 h}{\nu}\left(\frac{y}{h} - \frac{1}{2}\left(\frac{y}{h}\right)^2\right)$$

Turbulent flow (Re=Uh/v>1000): $$u(y) = \frac{u_*}{\kappa}\ln\frac{30xy}{k_s} \quad \left(\frac{k_s}{30x} \le y \le h\right)$$



$k_s / \delta \quad \delta = 11.6\frac{\nu}{u_*}$

k_s: Nikuradse equivalent sand grain roughness

$u_* = \sqrt{\tau_b / \rho}$ (shear velocity)

$\kappa \approx 0.4$: von Karman constant

x makes this distribution valid for smooth; transition and rough turbulent flow.

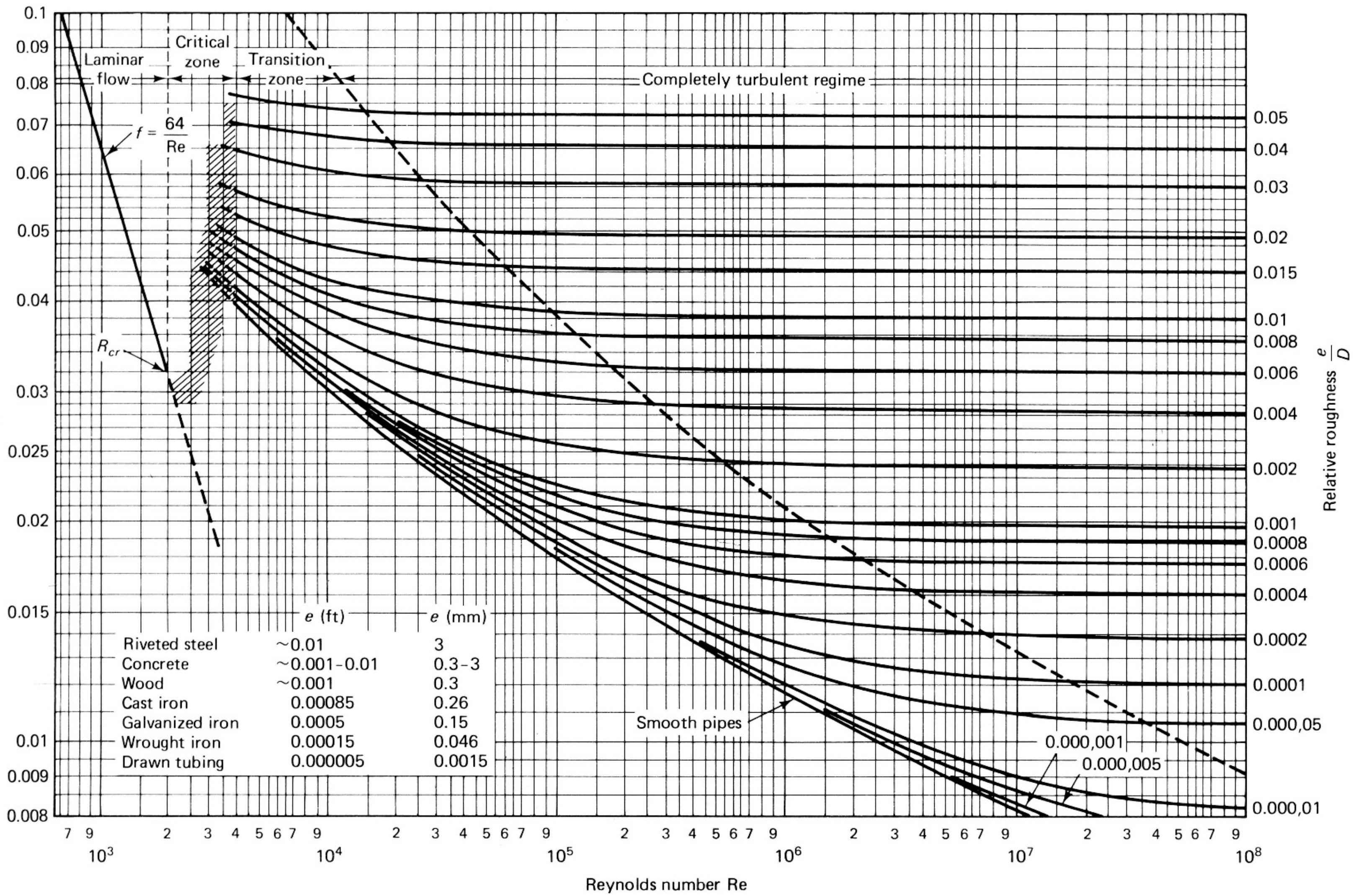
How to use Moody for open-channel flow?

Given: V, h, ν, k_s

Find: f, τ_s

$h --> R_h(h)$

$D --> 4R_h(h)$



$$\tau_s = \frac{1}{8} f \rho V^2$$

Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)