

# panel simulation notes

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## Designs

The simulation is set up to generate data from the following process. A set of  $K$  outcomes is observed at each of  $n$  time points, for each of  $m$  units. These units and/or time points are observed under  $H$  different treatment conditions, where the units may be completely nested within condition (i.e., a cluster-randomized design), completely crossed with condition (i.e., a randomized block design), or crossed with condition for some units but not for others (i.e., a difference-in-differences design). Suppose that there are  $G$  groups of units that share an identical pattern of treatment assignments, each of size  $m_g$ . Let  $n_{ghi}$  denote the number of time points at which unit  $i$  in group  $g$  is observed under condition  $h$ . All models used  $H = 3$  treatment conditions. Eight different designs were simulated:

1. Balanced randomized block design with an equal allocation, where all treatment conditions were observed for every unit ( $G = 1, m_1 = m$ ), with  $n_{1hi} = n/3$ .
2. Balanced randomized block design with an unequal allocation, with  $G = 1, m_1 = m$ ,  $n_{11i} = n/2, n_{12i} = n/3, n_{13i} = n/6$ .
3. Unbalanced randomized block design with an equal allocation, where  $G = 2, m_1 = m_2 = m/2$ ,  $n_{11i} = n/2, n_{12i} = n/3, n_{13i} = n/6$ , and  $n_{21i} = n/6, n_{22i} = n/3, n_{23i} = n/2$ .
4. Unbalanced randomized block design with an unequal allocation, where  $G = 2, m_1 = m_2 = m/2$ ,  $n_{11i} = n/2, n_{12i} = n/3, n_{13i} = n/6$ , and  $n_{21i} = n/3, n_{22i} = 5n/9, n_{23i} = n/9$ .
5. Balanced cluster-randomized design, where units were nested within treatment conditions, so that  $G = 3; m_g = m/3$ ; and  $n_{ghi} = n$  for  $g = h$  and zero otherwise.
6. Unbalanced cluster-randomized design, where units were nested within treatment conditions, so that  $G = 3; m_1 = 0.5m, m_2 = 0.3m, m_3 = 0.2m$ ; and  $n_{ghi} = n$  for  $g = h$  and zero otherwise.
7. Difference-in-differences design with  $G = 2$ ; where half of the observations remain in baseline throughout ( $m_1 = m/2$  and  $n_{11i} = n$ ) and the remaining half are observed for an **equal** number of time points under each treatment condition ( $m_2 = m/2$  and  $n_{2hi} = n/3$ ).
8. Difference-in-differences design with  $G = 2$ ; where  $2/3$  of the observations remain in baseline throughout ( $m_1 = 2m/3$  and  $n_{11i} = n$ ) and the remaining  $1/3$  are observed for an **equal** number of time points under each treatment condition ( $m_2 = m/3$  and  $n_{2hi} = n/3$ ).
9. Difference-in-differences design with  $G = 2$ ; where half of the observations remain in baseline throughout ( $m_1 = m/2$  and  $n_{11i} = n$ ) and the remaining half are observed for an **unequal** number of time points under each treatment condition ( $m_2 = m/2$  and  $n_{21i} = n/2, n_{22i} = n/3, n_{23i} = n/6$ ).
10. Difference-in-differences design with  $G = 2$ ; where  $2/3$  of the observations remain in baseline throughout ( $m_1 = 2m/3$  and  $n_{11i} = n$ ) and the remaining  $1/3$  are observed for an **unequal** number of time points under each treatment condition ( $m_2 = m/3$  and  $n_{21i} = n/2, n_{22i} = n/3, n_{23i} = n/6$ ).

## Data-generating model

Let  $y_{hijk}$  denote a measurement of outcome  $k$  at time point  $j$  for unit  $i$  under condition  $h$ , for  $h = 1, \dots, H$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, n$ , and  $k = 1, \dots, K$ . The outcomes follow the model

$$y_{hijk} = \mu_h + \nu_{hi} + \epsilon_{ijk},$$

where  $\mu_h$  is the mean outcome under condition  $h$ ,  $\nu_{hi}$  is a random effect for unit  $i$  under condition  $h$ , and  $\epsilon_{ijk}$  is the idiosyncratic error for unit  $i$  at time point  $j$  on outcome  $k$ . The errors at a given time point are assumed to be correlated, with

$$\text{Var}(\epsilon_{ijk}) = 1, \quad \text{corr}(\epsilon_{ijk}, \epsilon_{ijl}) = \rho$$

for  $k \neq l, k, l = 1, \dots, K$ . The random effects for unit  $i$  have variance

$$\text{Var}(\nu_{hi}) = \tau^2 = ICC/(1 - ICC)$$

for some specified intra-class correlation. The random effects for a given individual are also assumed to be equi-correlated in order to induce a degree of mis-specification into the analytic models described below. Specifically,

$$\text{corr}(\nu_{gi}, \nu_{hi}) = 1 - \frac{\sigma_\delta^2(1 + \tau^2)}{2\tau^2},$$

where  $\sigma_\delta^2 = \text{Var}(\nu_{gi} - \nu_{hi}) / \text{Var}(y_{hijk})$  is the variance of the differences between treatment conditions for each unit (i.e., the variance of the treatment effects), scaled in terms of the variance of the outcome at a given point in time.

The simulation examined the following combinations of sample size and parameters of the data-generating process:

Parameter	Meaning	Levels
$m$	number of units	15, 30, 50
$n$	number of time-points	18, 30
$k$	number of outcomes	3
$\rho$	correlation between outcome measures	0.2, 0.8
$ICC$	intra-class correlation	0.05, 0.15, 0.25
$\sigma_\delta^2$	treatment effect variability	0.0, 0.01, 0.04

The mean outcomes were set to  $\mu_h = 0$  across all  $H$  conditions, so that the null hypotheses to be tested are true. Each combination of parameters was tested for all eight designs.

## Analytic models

Given a set of simulated data, treatment effects on each outcome are estimated using the SUR framework. The general analytic model for the difference-in-differences design is

$$y_{hijk} = \mu_{hk} + \alpha_i + \gamma_j + \epsilon_{ijk},$$

where  $\mu_{hk}$  is the mean of outcome  $k$  under condition  $h$ ,  $\alpha_i$  is a fixed effect for each unit (cluster),  $\gamma_j$  is a fixed effect for each time-point, and  $\epsilon_{ijk}$  is residual error. The model is fit by OLS after absorbing the fixed effects for units and time-points, and so the “working” model amounts to assuming that the residuals are all independent and identically distributed (which isn’t true if  $\rho > 0$  or both  $ICC > 0$  and  $\sigma_\delta^2 > 0$ ). For cluster-randomized designs, the fixed effects for units are omitted (because units are nested within treatment conditions). For randomized block designs, the fixed effects for time-points are omitted for simplicity.

# Hypotheses

For each fitted model, six different hypotheses are tested, ranging in dimension from  $q = 1$  to  $q = 6$ :

Label	Dimension	Hypothesis
$t_B$	1	$\mu_{11} = \mu_{12}$
$t_C$	1	$\mu_{11} = \mu_{13}$
$F_1$	2	$\mu_{11} = \mu_{12} = \mu_{13}$
$F_B$	3	$\mu_{11} = \mu_{12}, \mu_{21} = \mu_{22}, \mu_{31} = \mu_{32}$
$F_C$	3	$\mu_{11} = \mu_{13}, \mu_{21} = \mu_{23}, \mu_{31} = \mu_{33}$
$F_{all}$	6	$\mu_{11} = \mu_{12} = \mu_{13}, \mu_{21} = \mu_{22} = \mu_{23}, \mu_{31} = \mu_{32} = \mu_{33}$

In words:

- $t_B$  is the hypothesis that there is no difference between treatment conditions 1 and 2 on the first outcome;
- $t_C$  is the hypothesis that there is no difference between treatment conditions 1 and 3 on the first outcome;
- $F_1$  is the hypothesis that there is no difference among the treatment conditions on the first outcome;
- $F_B$  is the hypothesis that there is no difference between treatment conditions 1 and 2 on any of the outcomes;
- $F_C$  is the hypothesis that there is no difference between treatment conditions 1 and 3 on any of the outcomes;
- $F_{all}$  is the hypothesis that there is no difference among the treatment conditions on any of the outcomes.