Small sample correction methods for cluster-robust variance estimators and hypothesis tests

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Abstract

Keywords:

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Cluster-robust variance estimators (CRVE) and hypothesis tests based upon such estimators are ubiquitous in applied econometric work. Nearly every respectable paper in the past 15 years uses cluster-robust variance estimators because to do otherwise would be to risk being seen as insufficiently rigorous (or worse, anti-conservative....ughh....how gauche!).

There's been a lot of fretting recently that even CRVE may actually not be rigorous enough. Cite the following people so as not to get their ire up:

- Brewer, Crossley, and Joyce (2013)
- Cameron, Gelbach, and Miller (2008)
- Cameron and Miller (2015)
- Carter, Schnepel, and Steigerwald (2013)
- Ibragimov and Müller (2010)
- Imbens and Kolesar (2012)
- Kezdi (2004)
- McCaffrey, Bell, and Botts (2001)
- McCaffrey and Bell (2006)
- Webb and MacKinnon (2013)
- Kline and Santos (2012)

Econometric framework

We will consider linear regression models in which the errors within a cluster have an unknown variance structure. The model is

$$\mathbf{Y}_j = \mathbf{X}_j\boldsymbol{\beta} + \mathbf{e}_j, \tag{1}$$

for $j = 1, ..., m$, where $\mathbf{Y}_j$ is $n_j \times 1$, $\mathbf{X}_j$ is an $n_j \times p$ matrix of regressors for cluster j, $\boldsymbol{\beta}$ is a $p \times 1$ vector, and $\mathbf{e}_j$ is an $n_j \times 1$ vector of errors. Assume that $\mathrm{E}\left(\mathbf{e}_j \left|\mathbf{X}_j\right.\right) = \mathbf{0}$ and $\mathrm{Var}\left(\mathbf{e}_j \left|\mathbf{X}_j\right.\right) = \boldsymbol{\Sigma}_j$, for $j = 1, ..., m$, where $\boldsymbol{\Sigma}_1, ..., \boldsymbol{\Sigma}_m$ may be unknown. Let $\boldsymbol{\Sigma} = \bigoplus_{j=1}^m \boldsymbol{\Sigma}_j$.

The vector of regression coefficients is estimated by weighted least squares (WLS). Given a set of m weighting matrices $\mathbf{W}_1, ..., \mathbf{W}_m$, the WLS estimator is

$$\hat{\boldsymbol{\beta}} = \mathbf{M} \sum_{j=1}^m \mathbf{X}_j' \mathbf{W}_j \mathbf{Y}_j, \tag{2}$$

where $\mathbf{M} = \left(\sum_{j=1}^m \mathbf{X}_j' \mathbf{W}_j \boldsymbol{X}_j\right)^{-1}$.

Common choices for weighting include the unweighted case, in which $\mathbf{W}_j$ is an identity matrix of dimension $n_j \times n_j$, and inverse-variance weighting under a working model. In the latter case, the errors are assumed to follow some known structure, $\mathrm{Var}\left(\mathbf{e}_j \left|\mathbf{X}_j\right.\right) = \boldsymbol{\Phi}_j$, where $\boldsymbol{\Phi}_j$ is either known or a function of a low-dimensional parameter. The weighting matrices are then taken to be $\mathbf{W}_j = \boldsymbol{\Phi}_j^{-1}$, possibly based on estimates of the variance parameters.

The WLS estimator also encompasses the estimator proposed by Ibragimov and Müller (2010) for clustered data. Assuming that $\mathbf{X}_j$ has rank p, their proposed approach involves estimating $\boldsymbol{\beta}$ separately within each cluster and taking the simple average of these estimates. The resulting average is equivalent to the WLS estimator with weights $\mathbf{W}_j = \mathbf{X}_j \left(\mathbf{X}_j'\mathbf{X}_j\right)^{-2} \mathbf{X}_j$.

Cluster-robust variance estimators

Considerations with panel models

Hypothesis testing

Single-constraint tests

Multiple-constraint tests

Examples

Simulation evidence

Discussion

References

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