

Building and Checking Suffix and LCP Arrays Using Induced Sorting Method

Yi Wu, Ge Nong, Wai Hong Chan, and Bin Lao

Abstract—Suffix and longest common prefix (LCP) arrays can be built by the induced sorting (IS) method on both internal and external memory models. We propose two methods that enable any IS builder to build and check suffix and LCP arrays simultaneously. The first method is for checking both suffix and LCP arrays, while the second method is for checking SA only. By combining the Karp-Rabin fingerprinting technique into our methods, we design two algorithms that perform checking correctly with a negligible error probability. Theoretically, the algorithm designed by the first method has a sorting complexity, while the algorithm designed by the second method only takes linear time to run. We integrate the algorithm designed by the second method into the existing disk-based suffix sorting algorithm DSA-IS and implement their combination for performance evaluation. From our experiments, the checking overhead is considerably smaller than the building consumption.

Index Terms—Suffix and LCP arrays, construction and verification, internal and external memory.



1 INTRODUCTION

A suffix array (SA) can be built within linear time and space by the internal-memory algorithm SA-IS [1]. According to the IS principle, the order of two suffixes is determined by comparing their heading characters and successors in sequence, where the order of their successors is assumed to be determined in advance. Recently, the IS method has been also applied to designing three disk-based suffix sorting algorithms eSAIS [2], DSA-IS [3] and SAIS-PQ [4], where eSAIS can build both suffix and LCP arrays at the same time. These works have better time complexities than the other alternatives (e.g., DC3 [5], bwt-disk [6], SAscan [7] and pSAscan [8]), but they all suffer from a bottleneck due to the large disk space for obtaining the heading characters of unsorted suffixes and the ranks of their sorted successors in a disk-friendly way. It is reported that the average peak disk use to construct an SA encoded by 40-bit integers for pSAscan is only $7.5n$, while that for eSAIS, DSA-IS and SAIS-PQ are $24n$, $18n$ and $15n$, respectively. The poor space performance of the IS algorithms is mainly because that their current programs fail to free the disk space for temporary data even when the data is no longer to use. A dramatic improvement can be achieved by storing temporary data in multiple files and deleting each file when the data residing on it is not needed any more. This technique has been used to implement a novel IS suffix sorting algorithm fSAIS [9], where the engineering of fSAIS takes no more than $8n$ disk space. This indicates a great potential for optimizing the performances of DSA-IS and SAIS-PQ.

A constructed SA should be checked to detect potential errors caused by implementation bugs and other malfunctions. The software packages for DC3 and eSAIS provide users a checker based on the idea presented in [5]. When running on external-memory model, the cost taken by this checker is rather high, because it performs two passes of integer sorts for arranging $\mathcal{O}(n)$ fixed-size tuples. In this paper, we describe two methods that enable any IS builder to build and check suffix and LCP arrays simultaneously. The first method is for checking both suffix and LCP arrays, while the second method is for checking SA only. We employ the Karp-Rabin fingerprinting technique [10] to design two probabilistic algorithms, in terms of that their checking results are wrong with a negligible probability. For analysis, we first use new substring sorting and naming methods to improve the design of DSA-IS and then combine the second checking method into the adapted DSA-IS for evaluating the checking overhead. Our experimental results indicate that the time, space and I/O volume for checking SA is considerable smaller than that for building SA. This implies that the IS method can be used to design efficient solutions for SA verification as well.

The rest of this paper is organized as follows. Section 2 introduces some notations and symbols for presentation convenience. Section 3 gives an overview of the existing IS suffix sorting algorithms and show the details of our new substring sorting and naming methods specific for DSA-IS. Section 4 presents the proposed checking methods and the probabilistic algorithms designed by them. Sections 5 and 6 show the experimental results and the concluding remarks, respectively.

2 PRELIMINARIES

Given a string $x[0, n)$ drawn from a full-ordered alphabet Σ , we assume the ending character $x[n - 1]$ to be unique and lexicographically smaller than any other characters in x .

- Y. Wu, G. Nong (corresponding author) and B. Lao are with the Department of Computer Science, Sun Yat-sen University, Guangzhou 510275, China. E-mails: wu.yi.christian@gmail.com, issng@mail.sysu.edu.cn, Laobin@mail3.sysu.edu.cn.
- Wai Hong Chan (corresponding author) is with the Department of Mathematics and Information Technology, The Education University of Hong Kong, Hong Kong. E-mail: waihchan@ied.edu.hk.

For convenience, we denote by $\text{suf}(i)$ and $\text{sub}(i, j)$ the suffix running from $x[i]$ to $x[n-1]$ and the substring running from $x[i]$ to $x[j]$, respectively. The following notations are also used in our presentation.

Characters in x are classified into two categories: L-type and S-type. We say $x[i]$ is S-type if (1) $i = n - 1$ or (2) $x[i] = x[i+1]$ and $x[i+1]$ is S-type; otherwise $x[i]$ is L-type. Further, if $x[i]$ and $x[i-1]$ are respectively S-type/L-type and L-type/S-type, then $x[i]$ is also called S*-type/L*-type. We use an array t to record the type of all the characters in x , where $t[i] = 1$ or 0 if $x[i]$ is S-type or L-type, respectively. The type of a substring or suffix is determined by that of its heading character.

Given two characters $x[i]$ and $x[i+1]$, we say $x[i]$ is the predecessor of $x[i+1]$ and $x[i+1]$ is the successor of $x[i]$. We define the predecessor-successor relationship between $\text{suf}(i)/\text{sub}(i, j)$ and $\text{suf}(i+1)/\text{sub}(i+1, j)$.

Partition x into multiple S*-type substrings. Each substring only contains two S*-type characters and any two neighboring substrings overlap an S*-type character. Produce a reduced string x_1 by replacing each S*-type substring with its name, which represents the rank of the corresponding S*-type substring among all. In the following paragraphs, we use "rank" and "name" interchangeably to indicate the lexical order of the corresponding substring.

The suffix array sa arranges all the suffixes of x in their lexical order, where $sa[i]$ records the starting position of the $(i+1)$ -th smallest suffix. We also define the suffix array sa_1 for the reduced string x_1 in the same way.

The LCP array lcp records the LCP-value of each pair of neighboring suffixes in sa . We assume $lcp[0] = 0$ and let $lcp[i] = \ell$ for $i \in [1, n)$, where ℓ is the LCP-value of $\text{suf}(sa[i])$ and $\text{suf}(sa[i-1])$.

All the suffixes in sa are naturally grouped into multiple buckets. Each bucket occupies a contiguous interval in sa and contains all the suffixes with a same heading character. Without loss of generality, we denote by $\text{sa_bkt}(c_0)$ the bucket for suffixes starting with c_0 . It should be noticed that $\text{sa_bkt}(c_0)$ can be divided into two parts where the left part $\text{sa_bkt}_L(c_0)$ and the right part $\text{sa_bkt}_S(c_0)$ contain the L-type and S-type suffixes, respectively. We also define $\text{lcp_bkt}(c_0)$, $\text{lcp_bkt}_L(c_0)$ and $\text{lcp_bkt}_S(c_0)$ on lcp for each character in Σ .

Let $n_1 = \|x_1\|$, we use $sa^*[0, n_1)$ and $lcp^*[0, n_1)$ to denote the suffix and LCP arrays for S*-type suffixes in x , respectively. Specifically, $sa^*[i]$ records the starting position of the $(i+1)$ -th smallest S*-type substring, while $lcp^*[i]$ records the LCP-value of $\text{suf}(sa^*[i])$ and $\text{suf}(sa^*[i-1])$.

3 BUILDER

3.1 Introduction to IS Suffix Sorting Algorithms

Algorithm 1 shows the framework of any IS suffix sorting algorithm. In lines 2-3, a reduction phase for sorting and naming S*-type substrings is called to produce the reduced string x_1 . If there exist duplicate characters in x_1 , the reduction phase is called with x_1 as input at the higher recursion level in line 7; otherwise, all the S*-type suffixes in x are already sorted and sa_1 is directly computed from x_1

Algorithm 1: The Framework of an IS suffix sorting algorithm.

Input: x

Output: sa

```

1 /* Reduction Phase */
2 Sort S*-type substrings by the IS method.
3 Name the sorted S*-type substrings to produce  $x_1$ .
4
5 /* Check Recursion Condition */
6 if exist two equal characters in  $x_1$  then
7   Recursively call the reduction phase on  $x_1$ .
8 end
9 else
10   Compute  $sa_1$  from  $x_1$ .
11 end
12
13 /* Induction Phase */
14 Sort suffixes by the IS method.
```

in line 10. Afterward, an induction phase for sorting suffixes is called to produce sa for x at the current recursion level in line 14. The induction phase is recursively called with sa as input at the lower recursion level (if any).

During the execution of a reduction/induction phase, all the substrings/suffixes are sorted by comparing their heading characters and the ranks of their successors according to the IS principle. This involves a great number of random accesses to x and sa , which can be done very fast if both x and sa can wholly reside on RAM. However, if the size of input and output exceeds the capacity of internal memory, each access will take an individual I/O operation, leading to a performance degradation. The DSA-IS algorithm solves the problem by performing the following two steps during a reduction/induction phase:

- S1 Split x into blocks and sort substrings/suffixes of each block by calling SA-IS. The heading characters in need are copied to external memory in their access order.
- S2 Sort the substrings/suffixes by their heading characters and the ranks of their successors in an external-memory heap. Scan the sorted substrings/suffixes in the heap and induce their predecessors into the heap, where the heading characters of the induced substrings/suffixes have been retrieved from external memory in advance by sequential I/O operations.

As shown in Section 5, our program for DSA-IS requires less disk space than that for eSAIS, but the former runs slower than the latter due to the large I/O volume for sorting and naming S*-type substrings during the reduction phase. To improve the performance, we propose new substring sorting and naming methods in the following.

3.2 Improvements on DSA-IS

All the S*-type substrings are classified into long and short categories with respect to whether or not containing more than D characters. The new substring sorting method mainly consists of the three steps below:

- S1' Sort the long in each block by S1. During the process, copy the short to external memory in their sorted order.

- S2' Sort the long in x by S2. During the process, copy the leftmost D characters of the long to external memory in their sorted order.
- S3' Merge the short and long by a multi-way sorter.

After S1'-S2', the short S*-type substrings in each block and the long S*-type substrings in x are sorted and separately organized as a sequence in external memory. Assume x is split into k blocks, the multi-way sorter in S3' maintains an internal-memory heap to cache the current smallest of each sequence and continually retrieve the top item from the heap to determine the lexical order of substrings from different sequences. The heap contains at most $k + 1$ substrings at any point of time and it compares any two substrings in $\mathcal{O}(D)$ time by conducting a literal string comparison¹. The above sorting method can achieve a good performance if the majority of S*-type substrings in x are short, provided D is small. This is commonly satisfied in real-world datasets.

Next, we describe a method for naming the S*-type substrings when they are being sorted. The key point here is to check equality of two substrings successively popped from the heap in S3' immediately. If either of the two substrings is short, then we literally compare them in $\mathcal{O}(D)$ time to check their equality. Otherwise, the two substrings are both long, we first determine their names when inducing them in S2' and compare these names in $\mathcal{O}(1)$ time to check their equality. This naming technique was originally proposed for SAIS-PQ for merging the sorting and naming processes into a whole. The experimental results in Section 5 indicate that, by using the new substring sorting and naming methods, the adapted DSA-IS, called DSA-IS+, only takes half as much disk space as eSAIS.

4 CHECKERS

4.1 Prior Art

We describe below the main idea of the existing checker presented in [5].

Lemma 4.1. $sa[0, n)$ is the SA for $x[0, n)$ if and only if the following conditions are satisfied:

- (1) sa is a permutation of $[0, n)$.
- (2) $r_i < r_j \Leftrightarrow (x[i], r_{i+1}) < (x[j], r_{j+1})$ for $i, j \in [0, n)$ and $i \neq j$, where r_i and r_j represent the ranks of $\text{suf}(i)$ and $\text{suf}(j)$ among all the suffixes, respectively.

Proof: The first condition indicates that sa is a permutation of all the suffixes in x . The second condition indicates that the order of suffixes in sa corresponds to that of their heading characters and successors. Because any two suffixes can be sorted by comparing their heading characters and successors, the above conditions are sufficient and necessary for SA verification. \square

The disk-based implementation of this checker conducts two passes of integer sorts and each sort arranges the order

1. If the leftmost D characters of a long S*-type substring is equal to a short S*-type substring, then the short is lexicographically greater than the long.

of $\mathcal{O}(n)$ fixed-size tuples using external memory. As can be observed from Section 5, the peak disk use and the I/O volume for an SA encoded by 40-bit integers are around $26n$ and $53n$, respectively.

4.2 Proposals

Recall that, an IS suffix sorting algorithm recursively perform the reduction phase to compute the reduced string x_1 until x_1 contains no duplicate characters. Afterward, it produces sa_1 from x_1 and recursively perform the induction phase to compute sa until reaching the top recursion level, where the induction phase consists of the following steps:

- S1'' sort the starting positions of all the S*-type suffixes with their ranks indicated by sa_1 to produce sa^* .
- S2'' Clear sa . Scan sa^* leftward with i decreasing from $n_1 - 1$ to 0. For each scanned item $sa^*[i]$, insert it into the rightmost empty position of $sa_bkt_S(x[sa^*[i]])$.
- S3'' Scan sa rightward with i increasing from 0 to $n - 1$. For each scanned non-empty item $sa[i]$, insert $sa[i] - 1$ into the leftmost empty position of $sa_bkt_L(x[sa[i] - 1])$ if $t[sa[i] - 1] = 0$.
- S4'' Clear $sa_bkt_S(c)$ for $c \in \Sigma$. Scan sa leftward with i decreasing from $n - 1$ to 0. For each scanned non-empty item $sa[i]$, insert $sa[i] - 1$ into the rightmost empty position of $sa_bkt_S(x[sa[i] - 1])$ if $t[sa[i] - 1] = 1$.

A running example of S2''-S4'' is shown in Fig. 1. Given sa^* is already known, line 6 inserts each S*-type suffix into the right part of the corresponding bucket and arranges them according to their sorted order indicated by sa^* . For example, $\text{suf}(8)$, $\text{suf}(5)$, $\text{suf}(2)$ are placed into the right part of $sa_bkt(i)$ and arranged in their sorted order indicated by sa^* . Next, we find the leftmost position of each bucket (marked by the symbol \wedge) and scan sa rightward for inducing the order of L-type suffixes. For this, we first check $sa[0] = 14$ (marked by the symbol $@$) and induce the predecessor of $\text{suf}(14)$ in lines 8-9. Because $x[13] = i$ is L-type, we put $\text{suf}(13)$ into the current leftmost empty position in $sa_bkt_L(i)$. To step through sa in this way, we get all the L-type suffixes sorted in line 17. Afterward, we find the rightmost position of each bucket and scan sa leftward for inducing the order of S-type suffixes. When scanning $sa[14] = 4$ in lines 19-20, we see $x[3] = i$ is S-type and thus put $\text{suf}(3)$ into the current rightmost empty position in $sa_bkt_S(i)$. Following the same idea, we get all the S-type suffixes sorted in line 28. Based on the discussion, we show in Lemma 4.2 a set of sufficient and necessary conditions for SA verification.

Lemma 4.2. For any IS suffix sorting algorithm, its output $sa[0, n)$ is the SA for $x[0, n)$ if and only if the following conditions are satisfied at the top recursion level:

- (1) sa^* is correctly computed.
- (2) S1''-S4'' are correctly implemented.

Proof: Given two suffixes placed at $sa[i]$ and $sa[j]$ and their successors placed at $sa[p]$ and $sa[q]$ at the top recursion level, we prove the statement as follows.

Because S1''-S4'' are correctly implemented, $i < j \Leftrightarrow (x[sa[i]], p) < (x[sa[j]], q)$. This satisfies the second condition of Lemma 4.1.

00	p :	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
01	x :	m	m	i	i	s	i	i	s	i	i	p	p	i	i	#
02	t :	0	0	1	1	0	1	1	0	1	1	0	0	0	0	1
03	sa^* :	14	8	5	2											
04	insert the sorted S*-type suffixes:															
05	bkt:	#			i					m		p		s		
06	sa :	14	-1	-1	-1	-1	8	5	2	-1	-1	-1	-1	-1	-1	-1
07	induce L-type suffixes:															
08	sa :	14	-1	-1	-1	-1	8	5	2	-1	-1	-1	-1	-1	-1	-1
09		@^	^							^		^		^		
10		14	13	-1	-1	-1	8	5	2	-1	-1	-1	-1	-1	-1	-1
11		^	@	^						^		^		^		
12		14	13	12	-1	-1	8	5	2	-1	-1	-1	-1	-1	-1	-1
13		^		@	^					^		^		^		
14		14	13	12	-1	-1	8	5	2	-1	-1	11	-1	-1	-1	-1
15		^			^		@			^			^	^		
16							...									
17		14	13	12	-1	-1	8	5	2	1	0	11	10	7	4	
18	induce S-type suffixes:															
19	sa :	-1	13	12	-1	-1	-1	-1	-1	1	0	11	10	7	4	
20		^								^		^		^	@^	
21		-1	13	12	-1	-1	-1	-1	-1	3	1	0	11	10	7	4
22		^								^		^		^	@	^
23		-1	13	12	-1	-1	-1	-1	6	3	1	0	11	10	7	4
24		^							^			^		@^	^	
25		-1	13	12	-1	-1	-1	9	6	3	1	0	11	10	7	4
26		^							^			^	@	^	^	
27							...									
28		14	13	12	8	5	2	9	6	3	1	0	11	10	7	4
29	$\overline{sa^*}$:	14			8	5	2									

Fig. 1. An example for inducing sa from sa^* .

Further, suppose $sa[i] = sa[j]$ and $i \neq j$, then $sa[p] = sa[q]$ and $p \neq q$. Repeating this reasoning process by replacing (i, j) with (p, q) until $\text{suf}(sa[i])$ is S*-type, then sa^* must contain duplicate elements. However, each element in sa^* is unique as sa^* is corrected computed. This leads to a contradiction. \square

It should be noticed that, we can check the correctness of sa^* instead of sa at the top recursion level to perform SA verification if the first condition of Lemma 4.2 is always true. In fact, the code snippet for the induction phase at the top recursion level only consists of tens of C++ code lines. This is considerable smaller than the whole program for a disk-based suffix sorting algorithm. Following the idea, we assume S1"-S4" are correctly implemented and propose two methods for checking computation errors caused by implementation bugs and other malfunctions. The algorithms designed by these methods can be seamlessly integrated into any IS suffix sorting algorithm for building and checking an SA at the same time.

4.2.1 Method A

Method A is based on Lemma 4.3, which is extended from Lemma 4.2.

Lemma 4.3. Assume the induction phase is correctly implemented at the top recursion level, the output $sa[0, n)$ of an IS suffix sorting algorithm is the SA for the input string $x[0, n)$ if and only if the following

conditions are satisfied:

- (1) sa^* is correct.
- (2) $sa[i]$ is equal to the value calculated by S1"-S4" for $i \in [0, n)$.

The first condition of Lemma 4.3 can be checked by a sparse SA checker like [11]. The second condition intends for detecting malfunctions other than implementation bugs (e.g., I/O errors). This is checked by determining whether the induced and scanned values for each suffix are equal². The problem to be solved here is that when a suffix is induced into a bucket, its corresponding value in sa may not be scanned at once. Our solution is to integrate the checking process into the building process, by ensuring the sequence of values placed at a bucket is identical to that scanned later. For the purpose, we use a fingerprinting function to increasingly compute the fingerprints of both sequences and check their equality in constant time at the end of the induction phase. If the two fingerprints for each bucket are equal, then the second condition of Lemma 4.3 will be seen with a high probability. As a result, sa can be built and probabilistically checked at the same time. It should be noticed that this method can be also applied to checking the LCP array when it is being built from lcp^* following the IS principle [12]. As a result, we design Algorithm 2 to probabilistically check sa and lcp at the top recursion level. In the algorithm, $sa_{11}(c)$ and $sa_{12}(c)$ are two sequences respectively induced into $sa_bkt_L(c)$ and $sa_bkt_S(c)$, while $sa_{S1}(c)$ and $sa_{S2}(c)$ are two sequences respectively scanned from $sa_bkt_L(c)$ and $sa_bkt_S(c)$.

4.2.2 Method B

According to Lemma 4.4, Method B uses a different way to check the correctness of sa^* . Before our presentation, we first introduce a notation called $\overline{sa^*}$. The same as sa^* , $\overline{sa^*}$ also represents the suffix array for all the S*-type suffixes. Both sa^* and $\overline{sa^*}$ are computed during the induction phase at the top recursion level, where the former is calculated from sa_1 in S1" and the latter is calculated when inducing the S*-type suffixes into sa in S4".

Lemma 4.4. Assume the induction phase is correctly implemented at the top recursion level, the output $sa[0, n)$ of an IS suffix sorting algorithm is the SA for the input string $x[0, n)$ if and only if the following conditions are satisfied:

- (1) $sa^*[i]$ is a permutation of all the S*-type suffixes.
- (2) $sa^*[i] = \overline{sa^*}[i]$ for $i \in [0, n_1)$.
- (3) $sa[i]$ is equal to the value calculated by S1"-S4" for $i \in [0, n)$.

Proof:

We only prove the sufficiency as the necessity is clear.

Case 1: Suppose all the S*-type suffixes are correctly sorted in sa^* , then sa is right according to Lemma 4.3.

Case 2: Suppose any two S*-type suffixes are not correctly sorted, i.e., $\text{suf}(sa^*[i_0]) > \text{suf}(sa^*[j_0])$ and $i_0 < j_0$. By condition (2), we have $\text{suf}(\overline{sa^*}[i_0]) > \text{suf}(\overline{sa^*}[j_0])$. Given

2. Notice that each suffix induced into sa will be latter scanned for inducing the order of its predecessor during S3" and S4".

Algorithm 2: The Algorithm Based on Lemma 4.3.

```

1 Function CheckByMethodA( $x, sa^*, lcp^*$ )
2   Verify  $sa^*$  and  $lcp^*$  by the checker presented in [11].
3   Compute the fingerprints of  $sa_{l1}(c)$ ,  $sa_{s1}(c)$ ,  $lcp_{l1}(c)$  and  $lcp_{s1}(c)$  when inducing the order and the LCP-values
   of L-type suffixes.
4   Check if  $sa_{l1}(c) = sa_{s1}(c)$  and  $lcp_{l1}(c) = lcp_{s1}(c)$ .
5   Compute the fingerprints of  $sa_{l2}(c)$ ,  $sa_{s2}(c)$ ,  $lcp_{l2}(c)$  and  $lcp_{s2}(c)$  when inducing the order and the LCP-values
   of S-type suffixes.
6   Check if  $sa_{l2}(c) = sa_{s2}(c)$ ,  $lcp_{l2}(c) = lcp_{s2}(c)$ .

```

that the order of $\text{suf}(\overline{sa^*}[i_0])$ and $\text{suf}(\overline{sa^*}[j_0])$ are induced from $\text{suf}(sa^*[i_1])$ and $\text{suf}(sa^*[j_1])$, then $\text{sub}(\overline{sa^*}[i_0], sa^*[i_1])$ and $\text{sub}(\overline{sa^*}[j_0], sa^*[j_1])$ are two S*-type substrings and there must be $\text{suf}(sa^*[i_1]) > \text{suf}(sa^*[j_1])$ and $i_1 < j_1$. Repeating this reasoning process, because condition (1), we must see $\text{suf}(sa^*[i_k]) > \text{suf}(sa^*[j_k])$ and $\text{suf}(sa^*[i_{k+1}]) < \text{suf}(sa^*[j_{k+1}])$, where $i_k < j_k$ and $i_{k+1} < j_{k+1}$. However, given $\text{suf}(sa^*[i_{k+1}]) < \text{suf}(sa^*[j_{k+1}])$, the inducing process will produce $\text{suf}(\overline{sa^*}[i_k]) < \text{suf}(\overline{sa^*}[j_k])$, which implies $\text{suf}(sa^*[i_k]) < \text{suf}(sa^*[j_k])$ because condition (2), leading to a contradiction. \square

The first condition of Lemma 4.4 is naturally satisfied when computing sa^* from sa_1 in $S1''$. The second condition is probabilistically checked by computing and comparing the fingerprints of sa^* and $\overline{sa^*}$. The fingerprint of sa^* is computed when placing the elements of sa^* into sa in $S2''$, while the fingerprint of $\overline{sa^*}$ is computed when inducing the S*-type suffixes into sa .

4.3 Fingerprinting Function

Formula 4.5. $\text{fp}(A[0, -1]) = 0$.

Formula 4.6. $\text{fp}(A[0, i]) = \text{fp}(A[0, i-1]) \cdot \delta + A[i] \mod L$ for $i \geq 0$.

The proposed two methods check equality of two arrays (or sub-arrays) by comparing their fingerprints, which can be calculated using a hash function. Notice that two equal arrays must share an identical hash value, but the inverse is not always true. To lower the error probability of a false match, we prefer using the Karp-Rabin fingerprinting function to compute the required values following Formulas 4.5-4.6, where L is a prime and δ is an integer randomly chosen from $[1, L)$. By setting L to a large value, the error probability can be reduced to a negligible level. In Fig. 2, we depict an example for computing the integer array A , where A is identical to sa^* and $\overline{sa^*}$ in Fig. 1. Given $L = 23$ and $\delta = 5$, we first compute the fingerprint of $A[0, 0]$ in line 4. Because $\text{fp}(A[0, -1]) = 0$, $\text{fp}(A[0, 0]) = (0 \cdot 5 + 14) \mod 23 = 14$. By iteratively computing the fingerprints of the prefixes of A , we finally obtain the fingerprint of A in line 7.

4.4 Complexity Analysis

Assuming the induction phase at the top recursion level is correctly implemented, the bottleneck of Algorithms 2 and 3 occur when checking sa^* . For Algorithm 2, we apply the method proposed in [11] to ensure the correctness of sa^* within the sorting complexity. Specifically, given two

```

01  p:      0          1          2          3
02  A:      14         8          5          2
03  compute fp(A[0, p]) with L = 23, δ = 5:
04  fp(A[0, 0]) = fp(A[0, -1]) * 5 + A[0] mod 23 = 14
05  fp(A[0, 1]) = fp(A[0, 0]) * 5 + A[1] mod 23 = 9
06  fp(A[0, 2]) = fp(A[0, 1]) * 5 + A[2] mod 23 = 4
07  fp(A[0, 3]) = fp(A[0, 2]) * 5 + A[3] mod 23 = 22

```

Fig. 2. An example for calculating fingerprints by Karp-Rabin fingerprinting function.

suffixes, i.e. $\text{suf}(i)$ and $\text{suf}(j)$, and their LCP-value ℓ , we compute and compare the fingerprints of $\text{sub}(i, i + \ell - 1)$ and $\text{sub}(j, j + \ell - 1)$. If the fingerprints are equal and the order of $x[i + \ell]$ and $x[j + \ell]$ corresponds to that of $\text{suf}(i)$ and $\text{suf}(j)$, then the suffixes are correctly sorted and their LCP-value is right with a high probability. Because at most one out of every two suffixes is S*-type (commonly one-third in real-world data sets) and the checking process is only executed during the induction phase at the top recursion level, the checking overhead is much less than the building consumption. This conclusion is also true for Algorithm 3, where the checking overhead is mainly taken by computing fingerprints in need.

5 EXPERIMENTS

For performance comparison, we engineer DSA-IS and DSA-IS+ by the STXXL's containers (vector, sorter, priority queue and stream). The experimental platform is a desktop computer equipped with an Intel Xeon E3-1220 V2 CPU, 4GiB RAM and 500GiB HD. All the programs are compiled by gcc/g++ 4.8.4 with -O3 options under Ubuntu 14.04 64-bit operating system. In our experiments, three performance metrics are investigated for the programs running on the corpora listed in Table 1, where each metric is measured as a mean of two runs.

- construction time (CT): the running time, in units of microseconds per character.
- peak disk use (PDU): the maximum disk space requirement, in units of bytes per character.
- I/O volume (IOV): as the term suggests, in units of bytes per character.

Algorithm 3: The Algorithm Based on Lemma 4.4.

- 1 **Function** CheckByMethodB(x, sa^*)
 - 2 Compute the fingerprints of sa^* , $sa_{l1}(c)$ and $sa_{s1}(c)$ when inducing the order of L-type suffixes.
 - 3 Check if $sa_{l1}(c) = sa_{s1}(c)$ by comparing their fingerprints.
 - 4 Compute the fingerprints of $\overline{sa^*}$, $sa_{l2}(c)$ and $sa_{s2}(c)$ when inducing the order of S-type suffixes.
 - 5 Check if $sa_{l2}(c) = sa_{s2}(c)$ by comparing their fingerprints.
 - 6 Check if $sa^* = \overline{sa^*}$ by comparing their fingerprints.
-

TABLE 1
Corpus, n in Gi, 1 byte per character

Corpora	n	$\ \Sigma\ $	Description
guten	22.5	256	Gutenberg, at http://algo2.iti.kit.edu/bingmann/esais-corpus .
enwiki	74.7	256	Enwiki, at https://dumps.wikimedia.org/enwiki , dated as 16/05/01.
proteins	1.1	27	Swissprot database, at http://pizzachili.dcc.uchile.cl/texts/protein , dated as 06/12/15.
uniprot	2.5	96	UniProt Knowledgebase release 4.0, at ftp://ftp.expasy.org/databases/... /complete, dated as 16/05/11.

5.1 Building Performance

Because fSAIS is not available online, we use eSAIS as a baseline for analyzing the performance of DSA-IS and DSA-IS+, where the program for eSAIS is also implemented by the STXXL library. Fig. 3 shows a comparison between the programs for these three algorithms in terms of the investigated metrics. As depicted, the program for DSA-IS requires less disk space than that for eSAIS when running on "enwiki" and "guten". In details, the peak disk use of DSA-IS and eSAIS are around $18n$ and $24n$, respectively. However, eSAIS runs much faster than DSA-IS due to the different I/O volumes. In order for a deep insight, we collect in Table 2 the statistics of their I/O volumes in the reduction and induction phases. As can be seen, although DSA-IS and eSAIS have similar performances when sorting suffixes in the induction phase, the latter consumes much less I/O volume than the former when sorting substrings in the reduction phase. More specifically, the mean ratio of induction I/O volume to reduction I/O volume are 0.23 and 0.71 for them, respectively. We can also see from the same figure that DSA-IS+ achieves a substantial improvement against DSA-IS, it runs as fast as eSAIS and takes half as much disk space as the latter. This is because the reduction I/O volume for DSA-IS+ is only half as much as that for DSA-IS (Table 2). Notice that the new substring sorting and naming methods adopted by DSA-IS+ take effect when most of the S*-type substrings are short. From our experiments, given $D = 8$, the ratio of long S*-type substrings in the investigated corpus nearly approaches one hundred percent, indicating that these methods are practical for real-world datasets.

5.2 Checking Performance

For evaluation, we integrate Method B into DSA-IS+ to constitute "Solution A" and compare it with "Solution B" composed of eSAIS and the existing checking method in [5].

Fig. 4 gives a glimpse of the performance of two solutions on various corpora. It can be observed that, the time, space and I/O volume for verification by Method B is negligible in comparison with that for construction by DSA-IS+, while the overhead for checking SA in Solution B is relatively large. Table 3 shows the performance breakdown of Solution B, where the checking time is one-fifth as the running time of the plain eSAIS and the peak disk use for verification is also a bit larger than that for construction. As a result, the combination of DSA-IS+ and Method B can build and check an SA in better total time and space.

5.3 Discussion

Rather than designing an I/O layer for efficient I/O operations, we currently use the containers provided by the STXXL library to perform reading, writing and sorting in external memory, these containers do not free the disk space for storing temporary data even if it is not needed any more, leading to a space consumption higher than our expectation. This is an implementation issue that can be solved by storing the data into multiple files and deleting each file when it is obsolete. In this way, our program can be further improved to achieve a space performance comparable to fSAIS. Our next paper will describe a novel disk-based IS suffix sorter that only takes $1n$ work space excluding the disk space for storing input and output.

6 CONCLUSION

By assuming the induction phase at the top recursion level is correctly implemented, we proposed two methods that enable any IS suffix sorter to build and check an SA simultaneously. The probabilistic algorithm designed by the second method is rather lightweight, it takes negligible time and space compared with the existing IS suffix sorting and checking algorithms. We also made our first attempt to improve the space performance of DSA-IS using new substring and naming methods. Our experimental results show that the program for the adapted algorithm DSA-IS+ runs as fast as that for eSAIS and consumes only half as much disk space as the latter on various real-world datasets. We are now designing and implementing a novel IS suffix sorter that takes no more than $1n$ work space on external memory model. Theoretically, this suffix sorter has a better space performance than fSAIS under the same circumstances.

REFERENCES

- [1] G. Nong, S. Zhang, and W. H. Chan, "Two Efficient Algorithms for Linear Time Suffix Array Construction," *IEEE Transactions on Computers*, vol. 60, no. 10, pp. 1471–1484, October 2011.

TABLE 2
A Comparison of Reduction and Induction I/O Volumes Amongst DSA-IS, DSA-IS+ and eSAIS on enwiki

	eSAIS				DSA-IS				DSA-IS+ ($D = 4$)			
Size	Red.	Ind.	Total	Ratio	Red.	Ind.	Total	Ratio	Red.	Ind.	Total	Ratio
1G	36.6	132.8	169.4	0.27	81.3	109.6	190.9	0.74	45.4	91.7	137.1	0.33
2G	36.0	141.9	177.9	0.25	83.5	111.6	195.1	0.75	47.2	93.4	140.6	0.34
4G	35.6	152.1	187.7	0.23	94.3	144.1	238.4	0.65	54.1	111.5	165.6	0.33
8G	35.2	165.7	200.9	0.21	107.8	159.6	267.4	0.68	60.1	122.1	182.2	0.33
16G	35.0	172.1	207.1	0.20	121.9	166.1	288.0	0.73	62.7	128.7	191.4	0.33

TABLE 3
Performance Breakdown of Solution B on various Corpora

Corpus	checking			building		
	PDU	IOV	CT	PDU	IOV	CT
enwiki_16G	26.0	53.0	0.71	23.5	205.6	3.49
guten_16G	26.0	53.0	0.79	23.4	195.2	3.20
uniprot	25.9	53.0	0.74	22.7	162.0	2.50
proteins	25.9	53.0	0.58	24.1	172.3	2.33

- [2] T. Bingmann, J. Fischer, and V. Osipov, "Inducing Suffix and LCP Arrays in External Memory," in *Proceedings of the 15th Workshop on Algorithm Engineering and Experiments*, 2012, pp. 88–102.
- [3] G. Nong, W. H. Chan, S. Q. Hu, and Y. Wu, "Induced Sorting Suffixes in External Memory," *ACM Transactions on Information Systems*, vol. 33, no. 3, pp. 12:1–12:15, March 2015.
- [4] W. J. Liu, G. Nong, W. H. Chan, and Y. Wu, "Induced Sorting Suffixes in External Memory with Better Design and Less Space," in *Proceedings of the 22nd International Symposium on String Processing and Information Retrieval*, London, UK, September 2015, pp. 83–94.
- [5] R. Dementiev, J. Kärkkäinen, J. Mehnert, and P. Sanders, "Better External Memory Suffix Array Construction," *ACM Journal of Experimental Algorithmics*, vol. 12, no. 3, pp. 4:1–4:24, August 2008.
- [6] P. Ferragina, T. Gagie, and G. Manzini, "Lightweight Data Indexing and Compression in External Memory," *Algorithmica*, vol. 63, no. 3, pp. 707–730, 2012.
- [7] J. Kärkkäinen and D. Kempa, "Engineering a Lightweight External Memory Suffix Array Construction Algorithm," in *Proceedings of the 2nd International Conference on Algorithms for Big Data*, Palermo, Italy, April 2014, pp. 53–60.
- [8] J. Kärkkäinen, D. Kempa, and S. J. Puglisi, "Parallel External Memory Suffix Sorting," in *In proceedings of the 26th Annual Symposium on Combinatorial Pattern Matching*, Ischia Island, Italy, July 2015, pp. 329–342.
- [9] J. Kärkkäinen, D. Kempa, S. J. Puglisi, and et al., "Engineering External Memory Induced Suffix Sorting," in *In Proceedings of the 19th Workshop on Algorithm Engineering and Experiments*, 2017, pp. 98–108.
- [10] R. Karp and M. Rabin, "Efficient Randomized Pattern Matching Algorithms," *IBM Journal of Research and Development*, vol. 31, no. 2, pp. 249–260, March 1987.
- [11] Y. Wu, G. Nong, W. H. Chan, and L. B. Han, "Checking Big Suffix and LCP Arrays by Probabilistic Methods," *IEEE Transactions on Computers*, 2017.
- [12] J. Fischer, "Inducing the LCP-array," in *In Workshop on Algorithms and Data Structures*, 2011, pp. 374–385.

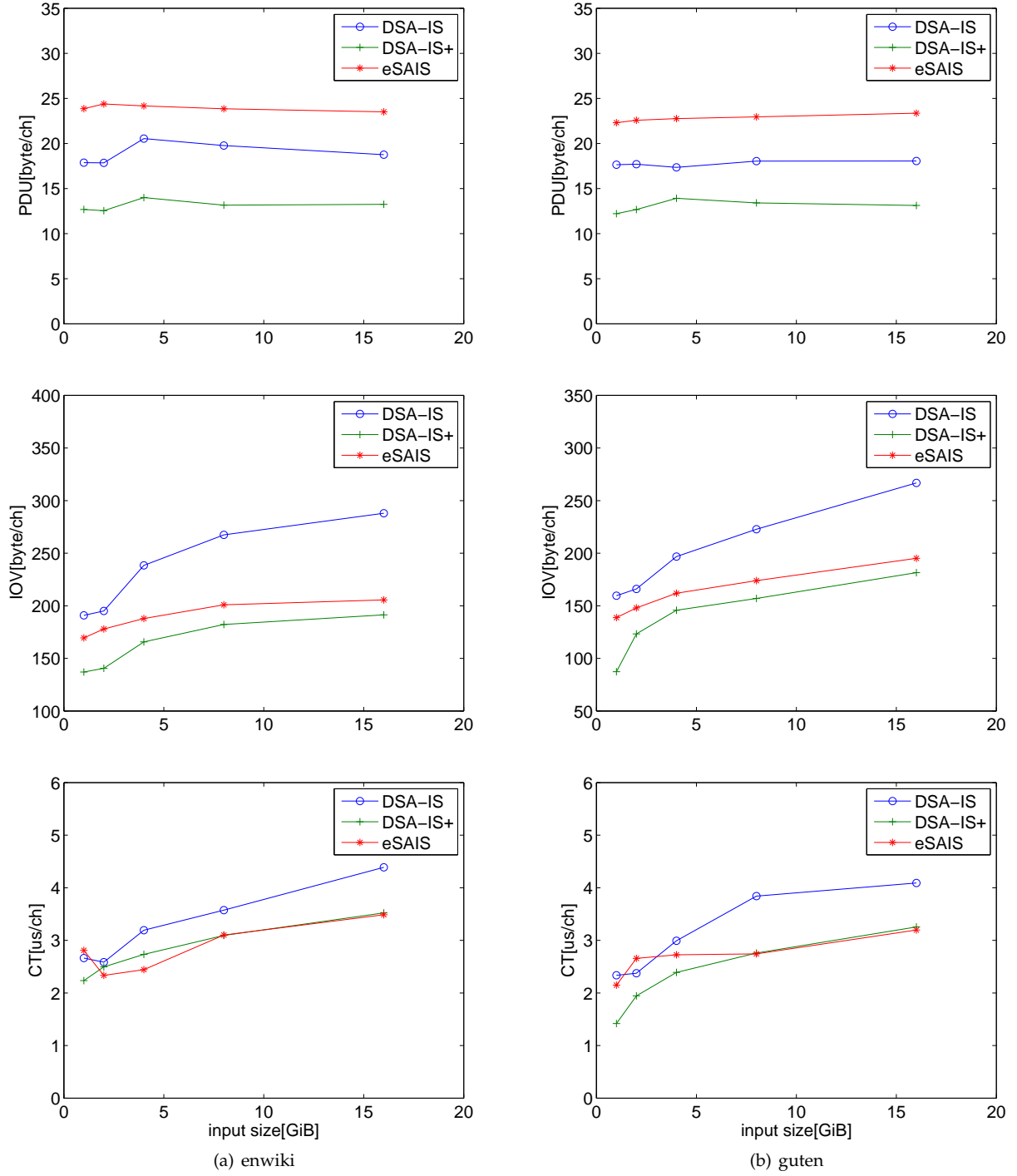


Fig. 3. A comparison of DSA-IS, DSA-IS+ and eSAIS on guten and enwiki in terms of peak disk usage, I/O volume and construction time, where $D = 4$ and the input size varies in $\{1, 2, 4, 8, 16\}$ GiB.

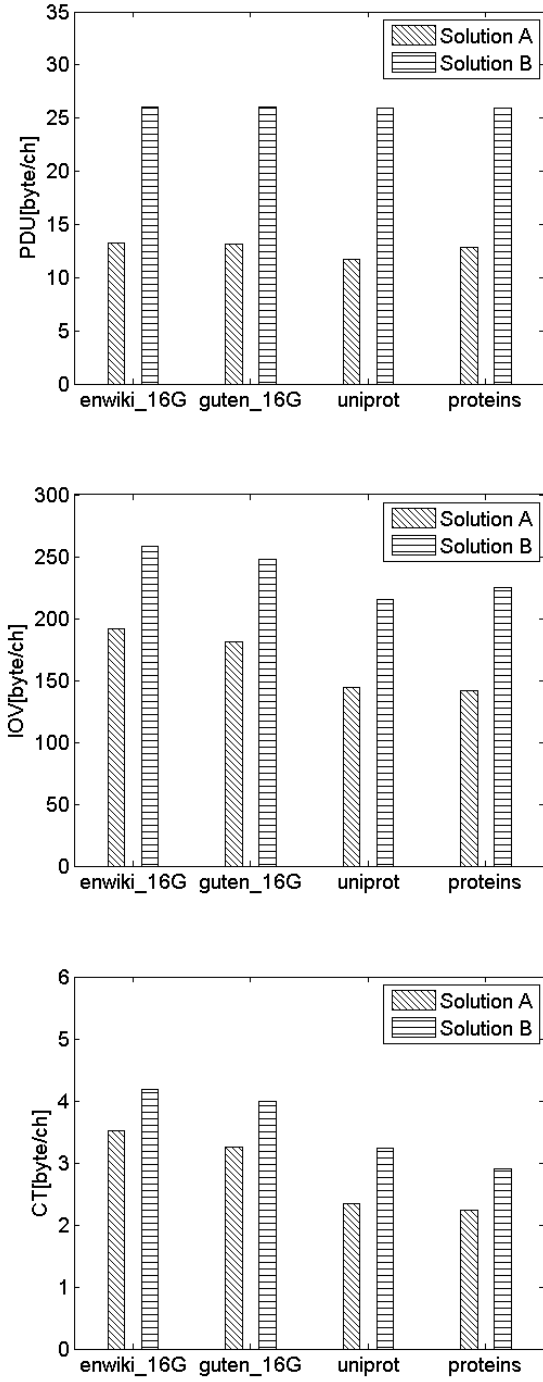


Fig. 4. A comparison of Solutions A and B on various corpora in terms of peak disk usage, I/O volume and construction time, where $D = 4$.