Checking Big Suffix and LCP Arrays by Probabilistic Methods

Yi Wu, Ge Nong, Wai Hong Chan, Ling Bo Han

Abstract—For full-text indexing of massive data, the suffix and LCP (longest common prefix) arrays have been recognized as the fundamental data structures, and there are at least two needs in practice for checking their correctness, i.e. program debugging and verifying the arrays constructed by probabilistic algorithms. In this paper, we propose two methods to check the suffix and LCP arrays in external memory by using a Karp-Rabin fingerprinting technique, where the checking result is wrong only with a negligible error probability. The first method checks the lexicographical order and the LCP-value of two neighboring suffixes in the given suffix array by computing and comparing the fingerprints of their LCPs. This idea is also employed in the second method to verify a subset of the given suffix and LCP arrays, from which then a copy of the final suffix and LCP arrays is produced following the induced sorting principle and compared with the given arrays for verification.

Index Terms—Suffix and LCP arrays verification, Karp-Rabin fingerprinting technique, external memory.

1 Introduction

1.1 Background

Suffix and longest common prefix (LCP) arrays play an important role in various string processing tasks, such as data compression, pattern matching and genome assembly. In many applications, these two data structures make up the core part of a powerful full-text index, called enhanced suffix array [1], which is more space efficient than a suffix tree and applicable to emulating any searching functionalities provided by the latter in the same time complexity. The first algorithm for building suffix array in internal memory was presented in [2]. From then on, much more effort has been put on designing efficient constructors for suffix array (SA) on different memory models [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. In respect of the research on LCP-array construction algorithms, the existing works can be classified into two categories with regard to the input requirements, where the algorithms from the first category compute both suffix and LCP arrays at the same time with the original text only [10], [14], [15] and those from the second category carry out the computation by taking SA and/or Burrows-Wheeler transform (BWT) as additional input [14], [16], [17], [18], [18], [19]. Among all, the algorithms designed by the induced sorting (IS) principle take linear time and space to run and outperform previous arts on both internal and external memory models [6], [11]. Recently, there appear some novel works that are competitive with the IS-based ones and even achieve better performance when adequate computation resources are available. These algorithms can maximize their throughput by making use of the multi-core CPUs and/or GPUs in computers [19], [20], [21], [22], [23].

While the research on efficient construction of suffix and LCP arrays is evolving, the algorithms proposed recently are becoming more complicated than before. This reveals a need for program debugging because a program gives no guarantee that it has correctly implemented the underlying algorithm. As a common practice, a suffix or LCP checker is provided to check the correctness of a constructed array. For example, such a checker can be found in some software packages for SA-IS [6], eSAIS [10], DC3 [24] and so forth. In addition to help avoid implementation bugs, a checker is also demanded for an array constructed by a probabilistic algorithm [25]. In this case, the array is correctly constructed with a probability and hence must be verified by a checker to ensure its correctness. As far as we know, the work in [26] describes the only SA checking method that can be found in the existing literature, and no efficient checking method for LCP array has been reported yet. In particular, there is currently no reported solution that can check both the suffix and the LCP arrays in external memory. This motivates our work here to design high-performance SA and LCP checkers for massive data.

1.2 Contribution

Our contribution mainly includes two methods to probablistically verify the given suffix and LCP arrays. Method A checks the lexical order and LCP-value of two neighboring suffixes in SA by literally comparing the characters of their LCPs in pairs. For reducing the time complexity of a comparison between two sequences of characters, we use a Karp-Rabin fingerprinting technique to convert each sequence into a single integer, called fingerprint, and compare the fingerprints instead to check equality of these sequences. The algorithm for this method involves multiple scans and sorts on sets of $\mathcal{O}(n)$ fixed-size items. When implemented in external memory, it suffers from a space bottleneck owing

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to the large disk volume taken by each sort. To overcome the drawback, Method B first employs the fingerprinting technique to check a subset selected from the given suffix and LCP arrays, then it reuses the inducing process of an IS-based algorithm to produce the final suffix and LCP arrays from the verified subset and literally compares them with the input arrays to ensure the correctness of the latter. Our experiments indicate that the peak disk use of the program for the algorithm designed by Method B is only half as that of the program for the algorithm designed by Method A.

The remainder of this paper is organized as follows. We first describe the checking methods and their algorithmic designs in Sections 2 and 3, and then evaluate the performance of our programs for the algorithms of these methods in Section 4. Finally, we present the concluding remarks in Section 5.

2 METHOD A

2.1 Preliminaries

Given a string $x[0, n)$ drawn from an alphabet Σ, the suffix array of x, denoted by sa, is a permutation of $\{0, 1, ..., n-1\}$ such that $\mathsf{suf}(sa[i]) < \mathsf{suf}(sa[j])$ for $i, j \in [0, n)$ and $i < j$, where $\mathsf{suf}(sa[i])$ and $\mathsf{suf}(sa[j])$ are two suffixes starting with $x[sa[i]]$ and $x[sa[j]]$, respectively. Particularly, we say $\mathsf{suf}(sa[j])$ is a lexical neighbor of $\mathsf{suf}(sa[i])$ if $|i-j| = 1$. The LCP array of x, denoted by lcp, consists of n integers, where $lcp[0] = 0$ and $lcp[i]$ records the LCP-value of $\mathsf{suf}(sa[i])$ and $\mathsf{suf}(sa[i-1])$ for $i \in [1, n)$.

2.2 Idea

According to the definitions of sa and lcp, we show in Lemma 1 the sufficient and necessary conditions for checking the given suffix and LCP arrays. Notice that the lexical order and the LCP-value of two suffixes in x can be computed by literally comparing their characters from left to right. Because all the suffixes differ in length and end with a common character, there must exist $k \in [0, n)$ such that $x[i, i+k) = x[j, j+k)$ and $x[i+k] \neq x[j+k]$ for any $i, j \in [0, n)$ and $i \neq j$. This approach can be also applied to verifying the last two conditions in Lemma 1, but it takes at worst $\mathcal{O}(n)$ character-wise comparisons for each pair of neighboring suffixes in sa to compare the two substrings, that is the LCPs, indicated by their LCP-value.

Lemma 1. Both $sa[0, n)$ and $lcp[0, n)$ are correct if and only if the following conditions are satisfied, for all $i \in [1, n)$:

(1) sa is a permutation of $\{0, 1, \ldots, n-1\}$.
(2) $x[sa[i], sa[i]+lcp[i]-1] = x[sa[i-1], sa[i-1]+lcp[i]-1]$.
(3) $x[sa[i] + lcp[i]] > x[sa[i-1] + lcp[i]]$.

Proof: Both the sufficiency and necessity are immediately seen from the definition of suffix and LCP arrays. Specifically, condition (1) demonstrates that all the suffixes in x are sorted in sa, while conditions (2)-(3) indicate that the lexical order and the LCP-value of any two neighboring suffixes in sa are both correct. □

An alternative is to exploit a perfect hash function (PHF) to convert each substring into a single integer such that any two substrings have a common hash value if and only if they are literally equal to each other. This implies that we can compare the hash values instead to check equality of their corresponding substrings. The key point here is how to quickly calculate the hash values of $x[sa[i], sa[i]+lcp[i]-1]$ and $x[sa[i-1], sa[i-1]+lcp[i]-1]$ for all $i \in [1, n)$. Taking into consideration the high difficulty of finding a PHF to meet this requirement, we prefer using a Karp-Rabin fingerprinting function [27] to transform a substring into its integer form, called fingerprint. Specifically, suppose L is a prime and δ is randomly chosen from $[1, L)$, the fingerprint $\mathsf{fp}(i, j)$ of a substring $x[i, j]$ can be calculated according to the formulas below as following: scan x rightward to iteratively compute $\mathsf{fp}(0, k)$ for all $k \in [0, n)$ using Formulas 1-2, record $\mathsf{fp}(0, i-1)$ and $\mathsf{fp}(0, j)$ during the calculation and subtract the former from the latter to obtain $\mathsf{fp}(i, j)$ using Formula 3.

Formula 1. $\mathsf{fp}(0, -1) = 0$.

Formula 2. $\mathsf{fp}(0, i) = \mathsf{fp}(0, i-1) \cdot \delta + x[i] \mod L$ for $i \geq 0$.

Formula 3. $\mathsf{fp}(i, j) = \mathsf{fp}(0, j) - \mathsf{fp}(0, i-1) \cdot \delta^{j-i+1} \mod L$.

We point out that two equal substrings always share a common fingerprint, but the inverse is not true. Fortunately, it has been proved in [27] that the probability of a false match can be reduced to a negligible level by setting L to a large value[1]. This leads us to the conclusion in Corollary 1.

Corollary 1. Both $sa[0, n)$ and $lcp[0, n)$ are correct with a high probability given the following conditions, for all $i \in [1, n)$:

(1) sa is a permutation of $\{0, 1, \ldots, n-1\}$.
(2) $\mathsf{fp}(sa[i], sa[i]+lcp[i]-1) = \mathsf{fp}(sa[i-1], sa[i-1]+lcp[i]-1)$.
(3) $x[sa[i] + lcp[i]] > x[sa[i-1] + lcp[i]]$.

We show in Fig. 2.2 an example for better understanding. Assume $L = 197$ and $\delta = 101$, lines 4-8 compute $\mathsf{fp}(0, p)$ iteratively according to Formulas 1-2. These values are used to compute the fingerprints for the target substrings. Consider the leftmost pair of neighboring suffixes in sa, that is $\mathsf{suf}(sa[0])$ and $\mathsf{suf}(sa[1])$, the substrings indicated by their LCP-value are $x[sa[0], sa[0] + lcp[1] - 1]$ and $x[sa[1], sa[1] + lcp[1] - 1]$, respectively. According to Formula 3, $\mathsf{fp}(sa[0], sa[0] + lcp[1] - 1)$ is equal to the difference between $\mathsf{fp}(0, sa[0])$ and $\mathsf{fp}(0, sa[0] + lcp[1] + 1)$, which have been calculated beforehand. In lines 10-12, we obtain the fingerprints for these two substrings and find that they are equal to each other.

2.3 Algorithm

We first describe an algorithm for checking conditions in Corollary 1 on random access models, of which the core part is to check the lexical order and LCP-value for each pair of neighboring suffixes in sa on-the-fly during the scan of sa and lcp. This is done by using Formulas 1-3 following our discussion in the previous subsection. As can be seen, two zero-initialized array, namely fp and mk, are introduced to facilitate the checking process, where fp is for storing the fingerprints of prefixes and mk is for determining whether or not each number of $\{0, 1, ..., n-1\}$ is present in sa.

1. This property is utilized in [25] to design a probabilistic algorithm for computing a sparse suffix array.

00	p:	0	1	2	3	4	5	6	7	8	9	10	11	12	13
01	$x[p]$:	2	1	3	1	3	1	2	1	3	1	3	1	2	1
02	$sa[p]$:	13	11	5	9	3	7	1	12	6	0	10	4	8	2
03	$lcp[p]$:	0	1	3	1	5	3	7	0	2	8	0	4	2	6

04 Compute fp(0, p) for p ∈ [0, n):

05 fp(0,0) = fp(0, −1) · 101 + x[0] mod 197 = 2,

06 fp(0,1) = fp(0,0) · 101 + x[1] mod 197 = 6,

07 fp(0,2) = fp(0,1) · 101 + x[2] mod 197 = 18,

08 .....

09	fp(0, p):	2	6	18	46	118	99	151	83	112	84	16	41	6	16

10 For suf(sa[0]) and suf(sa[1]):

11 $\text{fp}(sa[1], sa[1] + lcp[1] - 1) = fp(11) - fp(10) \cdot 101^1 \bmod 197 = 1$

12 $\text{fp}(sa[0], sa[0] + lcp[1] - 1) = fp(13) - fp(12) \cdot 101^1 \bmod 197 = 1$

13 For suf(sa[1]) and suf(sa[2]):

14 $\text{fp}(sa[2], sa[2] + lcp[2] - 1) = fp(7) - fp(4) \cdot 101^3 \bmod 197 = 160$

15 $\text{fp}(sa[1], sa[1] + lcp[2] - 1) = fp(13) - fp(10) \cdot 101^3 \bmod 197 = 160$

16 .....

Fig. 1. An Example for Computing the Fingerprints for the LCPs of the Neighboring Suffixes in SA.

S1 Scan x rightward with i increasing from 0 to $n-1$. For each scanned $x[i]$, compute $\text{fp}(0, i)$ and assign the value to $fp[i]$.

S2 Scan sa and lcp rightward with i increasing from 1 to $n-1$. For each scanned $sa[i]$ and $lcp[i]$, let $u = sa[i], v = lcp[i], w = sa[i-1]$ and perform substeps (a)-(c) in sequence:
   (a) Retrieve $fp[u-1]$ and $fp[u+v-1]$ from fp to compute $\text{fp}(u, u+v-1)$. Set $mk[u]$ to 1.
   (b) Retrieve $fp[w-1]$ and $fp[w+v-1]$ from fp to compute $\text{fp}(w, w+v-1)$.
   (c) Check if $\text{fp}(u, u+v-1) = \text{fp}(w, w+v-1)$ and $x[u+v] > x[w+v]$.
   (d) Set $mk[sa[0]] = 1$.

S3 Check if $mk[i] = 1$ for all $i \in [0, n)$.

It is clear that the above algorithm consumes $\mathcal{O}(n)$ time and space when implemented in internal memory. However, if the two auxiliary arrays cannot be wholly accommodated into RAM during the execution of S2, it suffers from a performance degradation caused by frequent random accesses to disks. Assume that x, sa and lcp are stored in external memory, we design Algorithm 1 for conducting these I/O operations in a disk-friendly way. The main idea is to first sort data in the order that they are visited and then access them by sequential reads and writes. To the end, our algorithm first scans sa and lcp to produce ST_1, ST_2, ST_3 and sorts their tuples by 1st component in ascending order at the very beginning (lines 2-5). Afterward, it iteratively computes the fingerprints of all the prefixes according to Formulas 1-2 and assigns them to the sorted tuples as following (lines 6-21): when figuring out $\text{fp}(0, i-1)$, extract each tuple e with $e.1st = i$ from $ST_1/ST_2/ST_3$, update e with $\text{fp}(0, i-1)$, and then forward e to $ST_1'/ST_2'/ST_3'$. Because the 1st component of the tuples in ST_1 constitute a copy of sa, the algorithm can check condition (1) in lines 9-14 when scanning them in the sorted order. Finally, it sorts the updated tuples back to their original order (line 22) and visits them sequentially to check conditions (2)-(3) following the same way of S2.

The last point to be mentioned here is how to obtain the value of $\delta^{lcp[i]}$ quickly when computing $\hat{fp}_1$ and $\hat{fp}_2$ in lines 25 and 27. One method is to keep a lookup table in internal memory to store $\delta^{lcp[i]}$ for all $i \in [0, n)$. This method can answer the question in constant time, but it is space-consuming and impractical to use when the table is bigger than the memory bank capacity. We provide another method that returns the answer in $\mathcal{O}(\lceil \log 2^n \rceil)$ time using $\mathcal{O}(\lceil \log 2^n \rceil)$ internal memory space. Specifically, suppose e is an integer from $[0, n)$, it can be decomposed into the form of $\Sigma_{i=0}^{\lceil \log 2^n \rceil} k_i \cdot 2^i$, where $k_0, k_1, ..., k_{\lceil \log 2^n \rceil}$ are determined by performing at most $\lceil \log 2^n \rceil$ divisions. Hence, we have $\delta^e = \Pi_{i=0}^{\lceil \log 2^n \rceil} \delta^{k_i \cdot 2^i}$ by replacing e with its decomposition, where the expression on the right side of the equation can be easily computed with $\{\delta^1, \delta^2, \ldots, \delta^{2^{\lceil \log 2^n \rceil}}\}$ already known.

2.4 Analysis

Algorithm 1 is I/O-intensive, it performs multiple scans and sorts on the arrays of $\mathcal{O}(n)$ fixed-size tuples residing on disks. Given an external memory model with RAM size M,

Algorithm 1: The Algorithm Based on Corollary 1.

1 **Function** CheckByFP(x, sa, lcp, n)
2 $ST_1 := [(sa[i], i, null) | i \in [0, n)]$
3 $ST_2 := [(sa[i] + lcp[i+1], i, null, null) | i \in [0, n-1)]$
4 $ST_3 := [(sa[i] + lcp[i], i, null, null) | i \in [1, n)]$
5 sort tuples in ST_1, ST_2 and ST_3 by 1st component
6 $fp := 0$
7 **for** $i \in [0, n]$ **do**
8 **if** ST_1.notEmpty() and ST_1.top().$1st = i$ **then**
9 $e := ST_1$.top(), ST_1.pop(), $e.3rd := fp$, ST'_1.push(e)
10 **end**
11 **else**
12 **return** false // condition (1) is violated
13 **end**
14 **while** ST_2.notEmpty() and ST_2.top().$1st = i$ **do**
15 $e := ST_2$.top(), ST_2.pop(), $e.3rd := fp$, $e.4th := x[i]$, ST'_2.push(e)
16 **end**
17 **while** ST_3.notEmpty() and ST_3.top().$1st = i$ **do**
18 $e := ST_3$.top(), ST_3.pop(), $e.3rd := fp$, $e.4th := x[i]$, ST'_3.push(e)
19 **end**
20 $fp := fp \cdot \delta + x[i] \mod P$
21 **end**
22 sort tuples in ST_1, ST_2 and ST_3 by 2nd component.
23 **for** $i \in [1, n-1)$ **do**
24 $fp_1 := ST'_1$.top().$3rd$, ST'_1.pop(), $fp_2 := ST'_2$.top().$3rd$, $ch_1 := ST'_2$.top().$4th$, ST'_2.pop()
25 $\hat{fp_1} = fp2 - fp1 \cdot \delta^{lcp[i]} \mod P$
26 $fp_1 := ST'_1$.top().$3rd$, $fp_3 := ST'_2$.top().$3rd$, $ch_2 := ST'_3$.top().$4th$, ST'_3.pop()
27 $\hat{fp_2} = fp3 - fp1 \cdot \delta^{lcp[i]} \mod P$
28 **if** $\hat{fp_1} \neq \hat{fp_2}$ or $ch_1 \leq ch_2$ **then**
29 **return** false // condition (2) or (3) is violated
30 **end**
31 **end**
32 **return** true

disk size D and block size B, all are in words, the time and I/O complexities for each scan are $\mathcal{O}(n)$ and $\mathcal{O}(n/B)$, respectively, while those for each sort are $\mathcal{O}(n \log_{M/B}(n/B))$ and $\mathcal{O}((n/B) \log_{M/B}(n/B))$, respectively [28]. This algorithm reaches its peak disk use when sorting tuples in lines 5 and 22. A trick for reducing the space requirements is to compute the fingerprints of prefixes specified in ST_1, ST_2 and ST_3, respectively. However, our experimental study in Section 4 shows that Algorithm 1 is still rather space hungry, its peak disk use is 40 bytes per input character. In the next section, we will describe an alternative checking method based on the induced-sorting principle. Compared with Algorithm 1, the program for our algorithm designed by this method only takes half space on real-world datasets.

3 Method B

3.1 Preliminaries

We introduce some symbols and notations used in the following presentation.

Character and suffix classification. All the characters in x are classified into three types, namely L-, S- and S*-type. In detail, $x[i]$ is L-type if (1) $i = n-1$ or (2) $x[i] > x[i+1]$ or (3) $x[i] = x[i+1]$ and $x[i+1]$ is L-type; otherwise, $x[i]$ is S-type. Further, if $x[i]$ and $x[i+1]$ are respectively S- and L-type, then $x[i]$ is also an S*-type character. Moreover, a suffix is L-, S- or S*-type if its heading character is L-, S-, or S*-type, respectively.

Suffix and LCP buckets. Suppose sa is correct, all the suffixes in sa are naturally partitioned into multiple buckets and those of a common heading character are grouped into a single bucket that occupies a contiguous interval in sa. Each bucket can be further divided into two sub-buckets, where the left and right parts only contain L- and S-type suffixes, respectively. For short, we use $\mathsf{sa\_bkt}(c)$ to denote the bucket storing suffixes starting with character c and $\mathsf{sa\_bkt_L}(c)$/$\mathsf{sa\_bkt_S}(c)$ to denote its left/right sub-bucket. Accordingly, lcp can be also split into multiple buckets, where $\mathsf{lcp\_bkt}(c)$/$\mathsf{lcp\_bkt_L}(c)$/$\mathsf{lcp\_bkt_S}(c)$ stores the LCP-values of suffixes in $\mathsf{sa\_bkt}(c)$/$\mathsf{sa\_bkt_L}(c)$/$\mathsf{sa\_bkt_S}(c)$.

Suffix and LCP arrays for S-type suffixes.* Given that the number of S*-type suffixes is n_1, $sa^*[0, n_1)$ stores all the S*-type suffixes and arrange them in lexical order, while the $(i+1)$-th item of $lcp^*[0, n_1)$ records the LCP-value of $\mathsf{suf}(sa^*[i])$ and $\mathsf{suf}(sa^*[i-1])$.

3.2 Idea

The induced sorting principle has been employed to invent efficient algorithms for constructing suffix and LCP

arrays on both internal and external memory models. These algorithms mainly consist of a reduction phase for computing sa^* and lcp^*, followed by an induction phase for inducing sa and lcp from sa^* and lcp^*. Given that sa^* and lcp^* are already known, we can directly construct the suffix and LCP arrays by calling the inducing process of any existing IS-based construction algorithm. This enlightens us to design a checker following Lemma 2.

Lemma 2. Both $sa[0,n)$ and $lcp[0,n)$ are correct if and only if the conditions below are satisfied:

(1) sa^* and lcp^* are both correct.
(2) $sa = sa'$ and $lcp = lcp'$, where sa' and lcp' are induced from sa^* and lcp^* by calling the inducing process of any existing IS-based construction algorithm.

Similar to Method A, we can employ the fingerprinting technique to probablistically check the correctness of sa^* and lcp^*. Based on this idea, we describe in the subsequent paragraphs an external-memory algorithm for checking the conditions in Corollary 2.

Corollary 2. Both $sa[0,n)$ and $lcp[0,n)$ are correct with a high probability given the following conditions, for $i \in [0,n)$, $\{j,k\} \in [1,n_1)$ and $j \neq k$:

(1) $sa^*[j] \neq sa^*[k]$.
(2) $\mathsf{fp}(sa^*[j], sa^*[j] + lcp^*[j] - 1) = \mathsf{fp}(sa^*[j-1], sa^*[j-1] + lcp^*[j] - 1)$.
(3) $x[sa^*[j] + lcp^*[j]] > x[sa^*[j-1] + lcp^*[j]]$.
(4) $sa[i] = sa'[i]$ and $lcp[i] = lcp'[i]$, where sa' and lcp' are induced from sa^* and lcp^* by calling the inducing process of an existing IS-based construction algorithm.

3.3 Algorithm

The first step of Algorithm 2 is to compute and verify sa^* and lcp^*. According to the definition, sa^* can be produced by sequentially retrieving the S*-type suffixes from sa while the LCP-value of two successive S*-type suffixes in sa, say $\mathsf{suf}(sa[i])$ and $\mathsf{suf}(sa[j])$, is equal to the minimal of $\{lcp[i+1], ..., lcp[j-1], lcp[j]\}$. Hence, the proposed algorithm first sorts all the suffixes in sa by their starting positions (lines 2-3) and then scans x only once to pick out the S*-type suffixes (lines 4-13). After that, it puts these S*-type suffixes back in their lexical order and outputs them one by one to generate sa^* (lines 14-27). Meanwhile, it calculates the LCP-value for each pair of neighboring suffixes in sa^* by tracing the minimal over an interval of lcp specified by the two suffixes. Notice that, we check condition (1) in Corollary 2 during the time when visiting the suffixes in position order (lines 7-12) and conditions (2)-(3) by calling Algorithm 1 with sa^* and lcp^* as input. Suppose sa^* and lcp^* are both correct, then Algorithm 1 invokes the inducing process of an existing construction algorithm to induce sa' and lcp', which are a copy of the final suffix and LCP arrays, from the two verified arrays (line 31) and literally compares them with sa and lcp to complete the whole checking process (lines 32-36).

Assume that the alphabet Σ is of a constant size, we can check sa and lcp without producing a copy of the final suffix and LCP arrays in Algorithm 2. The idea is to compare the induced suffix/LCP items with their corresponding items in sa/lcp during the inducing process. Specifically, when a suffix/LCP item v_1 is induced into a bucket, we check if it is equal to the corresponding item v_2 in sa/lcp. If $v_1 = v_2$, then v_2 is correct and we can further use it to induce the lexical order and LCP-value of its preceding suffix later. The key point here is how to retrieve items from sa/lcp quickly. This can be done by performing sequential I/O operations when we provide a read pointer together with a buffer for each suffix/LCP sub-bucket. We show more details of the adapted inducing process as below, where $c \in [0,\Sigma)$ and lp_1/lp_2 and sp_1/sp_2 are read pointer arrays for retrieving items from the L-type/S-type sub-buckets in sa and lcp.

S1 (a) Let $lp_1[c]$ and $lp_2[c]$ point to the leftmost items of $\mathsf{sa\_bkt_L}(c)$ and $\mathsf{lcp\_bkt_L}(c)$, respectively.
(b) Scan sa/lcp rightward to induce L-type suffixes and their LCP-values. For each induced suffix p (with a heading character c_0) and its LCP-value q: (1) check if $p = lp_1[c_0]$ and $q = lp_2[c_0]$; (2) let $lp_1[c_0]$ and $lp_2[c_0]$ point to the next items on the right side.

S2 (a) Let $sp_1[c]$ and $sp_2[c]$ point to the rightmost items of $\mathsf{sa\_bkt_S}(c)$ and $\mathsf{lcp\_bkt_S}(c)$, respectively.
(b) Scan sa/lcp leftward to induce S-type suffixes and their LCP-values. For each induced suffix p (with a heading character c_0) and its LCP-value q: (1) check if $p = sp_1[c_0]$ and $q = sp_2[c_0]$; (2) let $sp_1[c_0]$ and $sp_2[c_0]$ point to the next items on the left side.

3.4 Analysis

Algorithm 2 mainly consists of two parts, where the first part for checking sa^* and lcp^* can be implemented within sorting complexity and the second part for checking sa and lcp can be implemented in linear time under the condition that $|\Sigma| = \mathcal{O}(1)$. As demonstrated in Section 2, the peak disk use of our program for Algorithm 2 is around 20n and it has a similar performance compared to that for Algorithm 1 in terms of time and I/O volume.

4 Experiments

4.1 Setup

The experimental platform is a work station equipped with an Intel Xeon E3-1220 V2 CPU, 4GiB RAM and 500GiB HD. To achieve high I/O efficiency, the algorithms proposed in the previous sections are implemented using the external-memory containers provided by the STXXL library [29]. Our programs are compiled by gcc/g++ 4.8.4 with -O3 options under ubuntu 14.04 64-bit operating system. For analysis, we evaluate the performance on real-world datasets listed in Table 1 by investigating the following three measurements normalized by the size of input string:

- RT: running time, in nanoseconds. Measured using the Linux 'time' command.
- PDU: peak disk use of external memory, in bytes.
- IOV: amount of data read from and write to external memory, in bytes.

4.2 Result

Fig. **??** demonstrates the performance comparison between the programs for Algorithms 1 and 2. As depicted,

Algorithm 2: The Algorithm Based on Corollary 2.

```
Function CheckByIS(x, sa, lcp, n)
    ST_1 := [(sa[i], i, null)|i ∈ [0, n)]
    sort tuples in ST_1 by 1st component
    pos := −1
    for i ∈ (n, 0] do
        e := ST_1.top(), ST_1.pop()
        if x[i] is S*-type then
            if pos ≥ e.1st then
                return false            // condition (1) is violated
            end
            ST_2.push(e), pos := e.1st
        end
    end
    sort tuples in ST_2 by 2nd component
    i := 0, j := 0, lcp_min := max_val
    while ST_2.NotEmpty() do
        e := ST_2.top(), ST_2.pop()
        while true do
            lcp_min := min(lcp_min, lcp[i])
            if e.2nd = i then
                sa*[j] := e.1st, lcp*[j] := lcp_min, j := j + 1, i := i + 1
                break
            end
            i := i + 1
        end
        lcp_min := max_val
    end
    if CheckByFP(x, sa*, lcp*, lcp*.size()) = false then
        return false;           // conditions (2) or (3) is violated
    end
    (sa', lcp') := InducingProcess(x, sa*, lcp*)
    for i ∈ [0, n) do
        if sa[i] ≠ sa'[i] ∥ lcp[i] ≠ lcp'[i] then
            return false // condition (4) is violated
        end
    end
    return true
```

TABLE 1
Corpus, n in Gi, 1 byte per character

Corpora	n	$\|\Sigma\|$	Description
enwiki	8	256	The 8-GiB prefix of an XML dump of English Wikipedia, available at https://dumps.wikimedia.org/enwiki, dated as 16/05/01.
uniprot	2.5	96	UniProt Knowledgebase, available at ftp://ftp.expasy.org/databases/.../complete, dated as 16/05/11.
proteins	1.1	27	Swissprot database, available at http://pizzachili.dcc.uchile.cl/texts/protein, dated as 06/12/15.

The speed gap between them is mainly due to the difference in their I/O efficiencies. Specifically, the I/O volume of ProgB is $190n$ in average, while that of ProgA is kept at $155n$ for different corpora. Notice that, although Algorithm **??** reuses Algorithm 1 to check sa_{LMS} and lcp_{LMS}, the consumption for the verification of sa_{LMS} and lcp_{LMS} in ProgB is at most half of ProgA because the number of LMS suffixes is no more than $\frac{1}{2}n$. It can be also observed that both programs are insensitive to the input corpus in terms of the space requirement. In details, the peak disk uses of ProgA and ProgB are respectively $26n$ and $40n$ on the three corpora.

We also investigate the performance trend of the two programs on the prefix of "enwiki" with the length varying on $\{1, 2, 4, 8\}$ GiB. Figure 3 illustrates that, as the prefix length increases, a performance degradation occurs to ProgB in both time and I/O efficiencies, but the fluctuation of ProgA can be ignored.

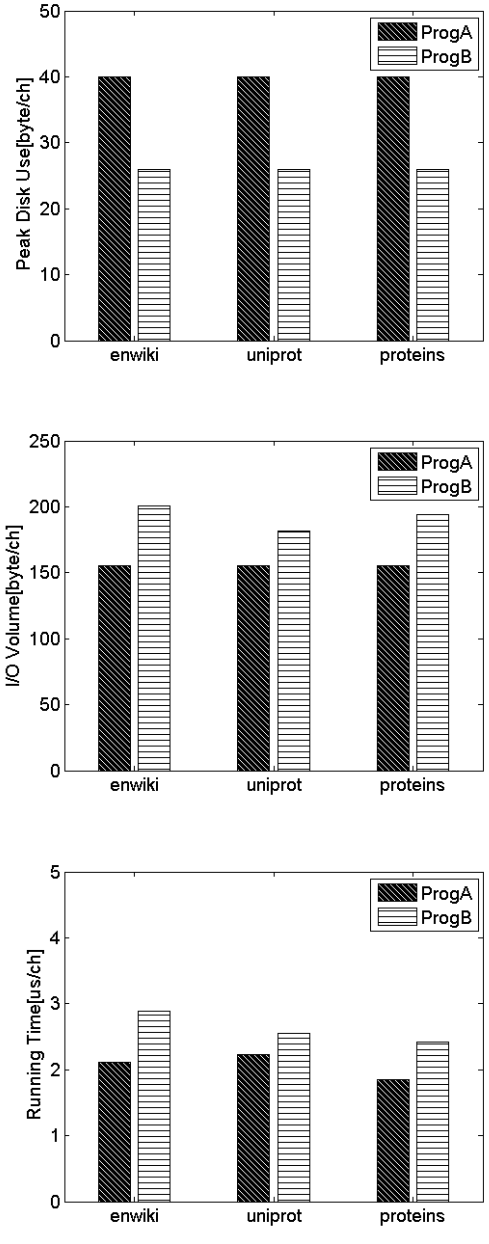

Fig. 2. Experimental results for various corpora.

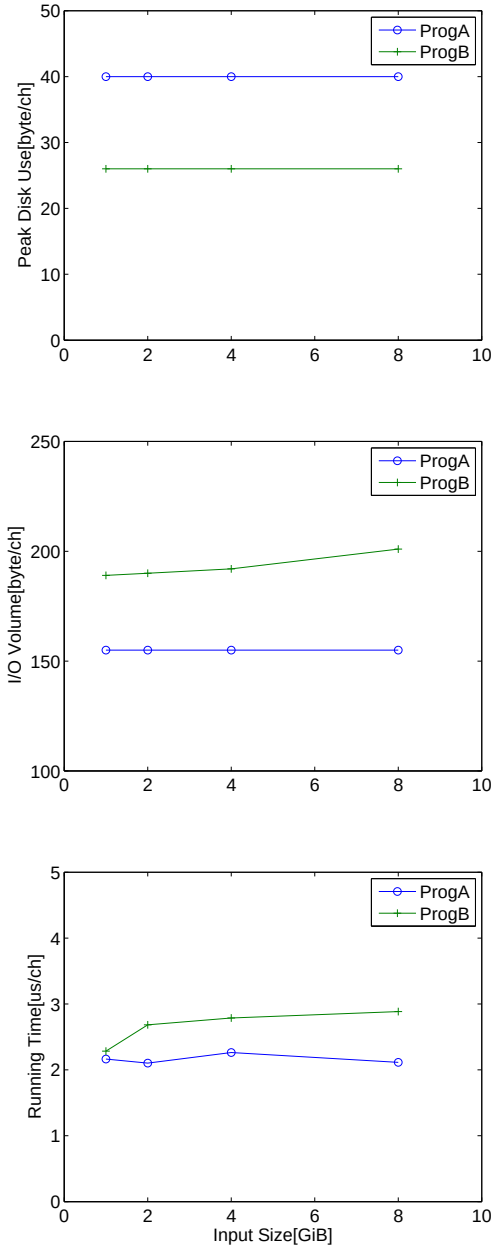

Fig. 3. Experimental results for prefixes of "enwiki".

4.3 Discussion

It is identified that both programs heavily rely on the performance of the external memory sorter in use. A potential candidate for improving their speed is to adapt a GPU-based multi-way sorter (e.g., [30], [31]) for sorting massive data using external memory. By the aid of these fast sorting algorithms, the throughputs of the programs are expected to nearly approach the I/O bandwidth. Besides, the first two steps of Algorithm 1 are independent of each other and thus can be executed in parallel for acceleration. This technique can be also applied to check the suffix and LCP arrays of the LMS suffixes in Algorithm **??**.

Currently, for Algorithm **??**, step 2 constitutes the space bottleneck. It is worthy of mentioning that this step produces a copy of the suffix and LCP array during the inducing and checking processes. Actually, given that Σ is of a constant size and sa/lcp are known already, we can simply scan the input sa/lcp to perform the inducing process and compare each induced suffix/LCP value with that in the given sa/lcp to perform the checking process, resulting in less space consumption. To the end, we must maintain a read pointer for each suffix/LCP bucket in sa/lcp to scan elements in sequence.

5 Conclusions

In this article, we propose two methods for probablistically checking the give suffix and LCP arrays using external memory. According to the experimental results, our program for Method A has a better performance than that for Method B with respect to the running time and I/O

volume by about 20 percent, while the peak disk use of the latter is about 26/40=0.65 as that of the former and can be further reduced to around 21n without a sacrifice in the time and I/O effciency. We think these two methods could potentially be a xxxx .

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