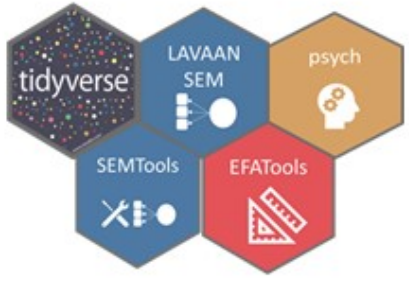
# Training Workshop on Structural Equation Modelling (SEM) using R

## Session 2: Exploratory Factor Analysis (EFA)



# Factor analysis process

**Stage 1**: Objectives of factor analysis

**Stage 2**: Designing an Exploratory factor analysis

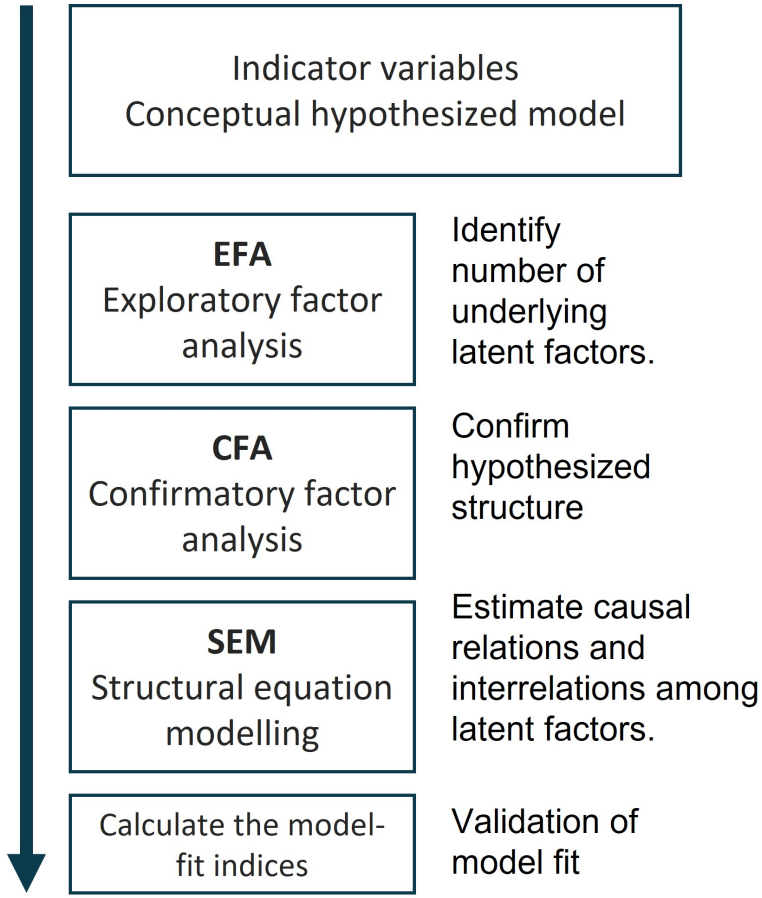
**Stage 3**: Assumptions in Exploratory factor analysis

**Stage 4**: Deriving factors and assessing overall fit

**Stage 5**: Interpreting the factors

# Stage 1 : Objectives of factor analysis

# Overview




*Source: Dragan and Darja (2014): Introduction to SEM: review, methodology and practical applications.*

# Overview

**Exploratory factor analysis**

- Exploratory or descriptive technique to determine the appropriate number of common factor.
- No specifications are made in regards to the number of factors (initially) or the pattern of relationships between factor and indicators
- Looking for patterns in the data

**Confirmatory factor analysis**

- Should be conducted prior to the specifications of a structural model.
- Researcher specifies the number of factors and the pattern of indicator-factor in advance.
- Testing a theory that you know in advance

# Stage 2: Designing an EFA

# Variable selection and measurement issues

What types of variables can be used in factor analysis?

- *Primary requirement: a correlation value can be calculated among all variables.*
- *e.g., metric variables, scale items, dummy variables to represent nonmetric variables.*

How many variables or items should be used per factor?

- *Five or more per factor for scale development.*
- *Three or more per factor for factor measurement (based on how degrees of freedom is computed).*

# Sample size

Some recommendations in literature:

- Five cases minimum per estimated parameter (Bentler and Chou, 1987)
- Monte carlo studies recommend 100 cases minimum and 200 is better for modest models (Loehlin, 2017)
- Larger or complicated models, models with more latent variables or parameter estimates, require more cases.

# Stage 3: Assumptions in EFA

# Sample Dataset

- HBAT Industries, manufacturer of paper products.
- Perceptions on a set of business functions.
- Rating scale:
  - `0 "poor"` to `10 "excellent"`

| | | |
|---|---|---|
| $X_6$ | Product quality | Perceived level of quality of HBAT's paper products |
| $X_7$ | E-commerce | Overall image of HBAT's website; user-friendliness |
| $X_8$ | Technical support | Extent to which technical support is offered |
| $X_9$ | Complaint resolution | Extent to which any complaints are resolved in timely and complete manner |
| $X_{10}$ | Advertising | Perceptions of HBAT's product line to meet customer needs |
| $X_{11}$ | Product line | Depth and breadth of HBAT's product line to meet customer needs |
| $X_{12}$ | Salesforce image | Overall image of HBAT's salesforce |
| $X_{13}$ | Competitive pricing | Extent to which HBAT offers competitive prices |
| $X_{14}$ | Warranty and claims | Extent to which HBAT stands behind its product/ service warranties and claims |
| $X_{15}$ | New products | Extent to which HBAT develops and sells new products |
| $X_{16}$ | Ordering and billing | Perceptions that ordering and billing is handled efficiently and correctly |
| $X_{17}$ | Price flexibility | Perceived willingness of HBAT sales reps to negotiate price on purchase of paper products |
| $X_{18}$ | Delivery speed | Amount of time it takes to deliver the paper product once an order has been confirmed |

# Sample Dataset

- $X_6$ product quality
- $X_7$ e-commerce
- $X_8$ technical support
- $X_9$ complaint resolution
- $X_{10}$ advertising
- $X_{11}$ product line
- $X_{12}$ salesforce image
- $X_{13}$ competitive pricing
- $X_{14}$ warranty claims
- $X_{15}$ packaging
- $X_{16}$ order & billing
- $X_{17}$ price flexibility
- $X_{18}$ delivery speed

| x6 | x7 | x8 | x9 | x10 | x11 | x12 |
|---|---|---|---|---|---|---|
| <dbl+lbl> | <dbl+lbl> | <dbl+lbl> | <dbl+lbl> | <dbl+lbl> | <dbl+lbl> | <dbl+lbl> |
| 8.5 | 3.9 | 2.5 | 5.9 | 4.8 | 4.9 | 6.0 |
| 8.2 | 2.7 | 5.1 | 7.2 | 3.4 | 7.9 | 3.1 |
| 9.2 | 3.4 | 5.6 | 5.6 | 5.4 | 7.4 | 5.8 |
| 6.4 | 3.3 | 7.0 | 3.7 | 4.7 | 4.7 | 4.5 |
| 9.0 | 3.4 | 5.2 | 4.6 | 2.2 | 6.0 | 4.5 |
| 6.5 | 2.8 | 3.1 | 4.1 | 4.0 | 4.3 | 3.7 |
| 6.9 | 3.7 | 5.0 | 2.6 | 2.1 | 2.3 | 5.4 |
| 6.2 | 3.3 | 3.9 | 4.8 | 4.6 | 3.6 | 5.1 |
| 5.8 | 3.6 | 5.1 | 6.7 | 3.7 | 5.9 | 5.8 |
| 6.4 | 4.5 | 5.1 | 6.1 | 4.7 | 5.7 | 5.7 |

1-10 of 100 rows | 1-7 of 11 col... Previous **1** 2 3 4 5 6 … 10 Next

*Source: J.F. Hair (2019): Multivariate data analysis.*

# Conceptual assumptions

- Some uderlying structure does exist in the set of selected variables.
- correlated variables and subsequent definition of factors do not guarantee relevance
    - *even if they meet the statistical requirement!*
- It is the responsibility of the researcher to ensure that observed patterns are conceptually valid and appropriate.

# Determining appropriateness of EFA

1. Bartlett Test

2. Measure of Sampling Adequacy

# Determining the appropriateness of EFA

**1. Bartlett Test**

- Examines the entire correlation matrix
- Test the hypothesis that correlation matrix is an identity matrix.
- A significant result signifies data are appropriate for factor analysis.

```
library(EFAtools)
BARTLETT(data, N= nrow(data))
```

```
v The Bartlett's test of sphericity was significa
  These data are probably suitable for factor ana

  <U+0001D712>²(55) = 619.27, p < .001
```

# Determining the appropriateness of EFA

**2. Kaiser-Meyen-Olkin (KMO Test)**

- Measure of sampling adequacy
- Indicate the proportion of variance explained by the underlying factor.
- Guidelines:
  - $\geq 0.90$ - marvelous
  - $\geq 0.80$ - meritorious
  - $\geq 0.70$ - middling
  - $\geq 0.60$ - mediocre
  - $\geq 0.50$ - miserable
  - $< 0.50$ - unacceptable

# Determining the appropriateness of EFA

**2. Kaiser-Meyen-Olkin (KMO Test)**

```
-- Kaiser-Meyer-Olkin criterion (KMO) ------------------------------

! The overall KMO value for your data is mediocre.
  These data are probably suitable for factor analysis.

  Overall: 0.653

  For each variable:
   x6    x7    x8    x9   x10   x11   x12   x13   x14   x16   x18
0.509 0.626 0.519 0.787 0.779 0.622 0.622 0.753 0.511 0.760 0.666
```

# Determining the appropriateness of EFA

**2. Kaiser-Meyen-Olkin (KMO Test)**

- When overall MSA is less than 0.50
  - Identify variables with lowest MSA subject for deletion.
  - Recalculate MSA
  - Repeat unitl overall MSA is 0.50 and above

- Deletion of variables with MSA under 0.50 means variable's correlation with other variables are poorly representing the extracted factor.

# Let's practice!

# Stage 4: Deriving factors and assessing overall fit

# Partitioning the variance of a variable

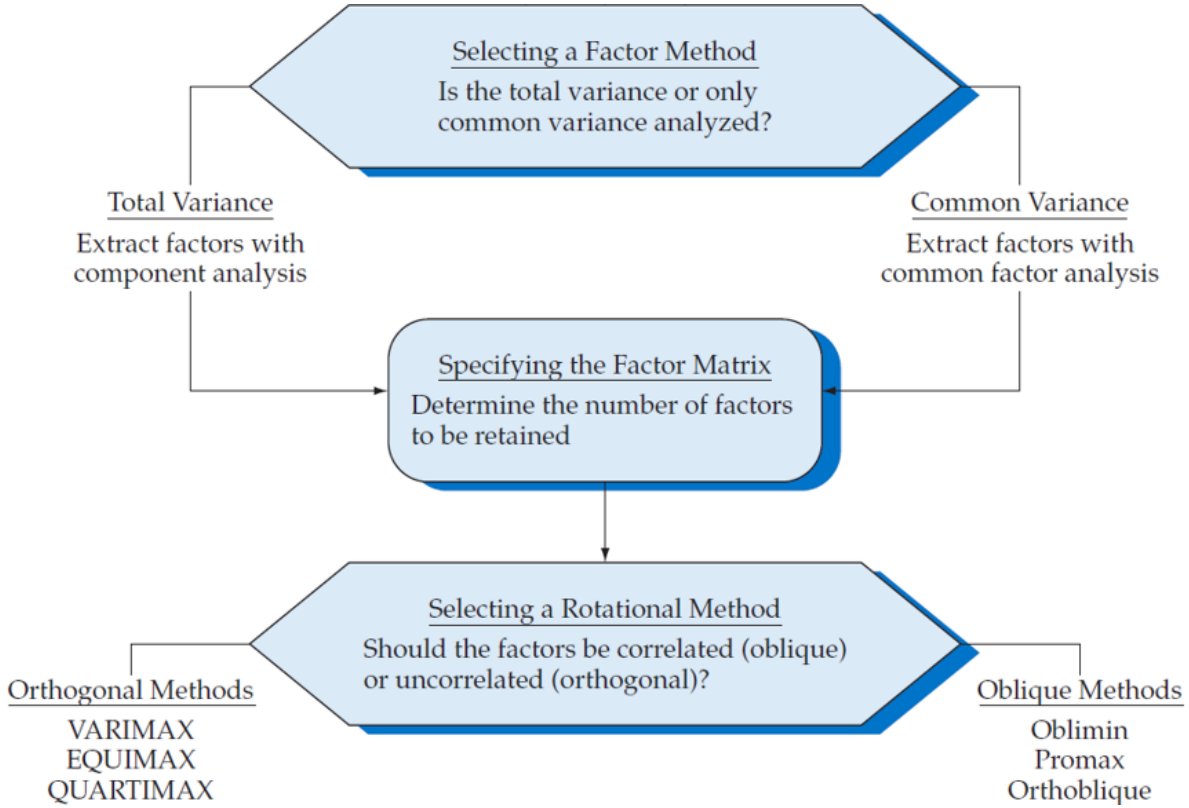
**Unique variance**

- Variance associated with only a specific variable.
- Not represented in the correlations among variables.
- *Specific variance*
  - associated uniquely with a single variable.
- *Error variance*
  - May be due to unreliability of data gathering process, measurement error, or a random component in the measured phenomenom.

**Common variance**

- Shared variance with all other variables.
- High common variance are more amenable for factor analysis.
- Derived factors represents the shared or common variance among the variables.

# Partitioning the variance of a variable




*Source: JF Hair et al. (2019) Multivariate data analysis.*

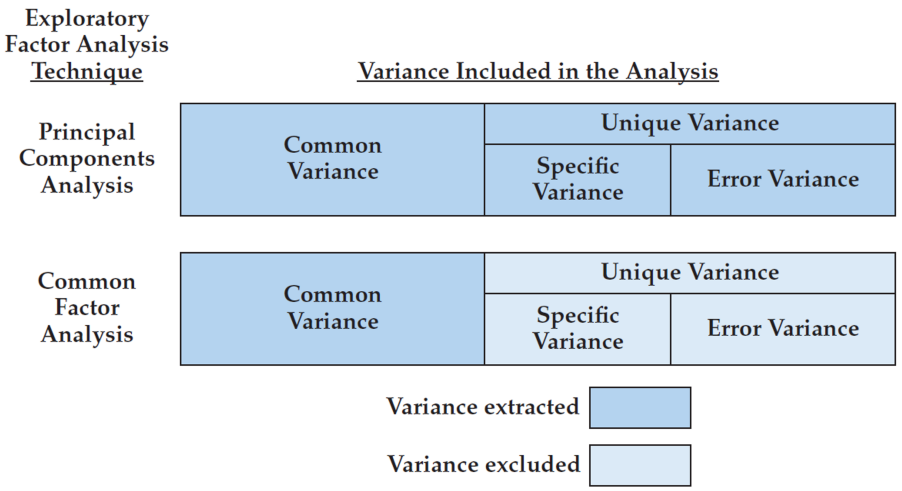
# PCA vs Common factor analysis

**Principal component analysis (PCA)**

- Considers the total variance
- data reduction is a primary concern

**Common factor analysis**

- Considers only the common variance or shared variance
- Primary objective is to identify the latent dimensions or constructs



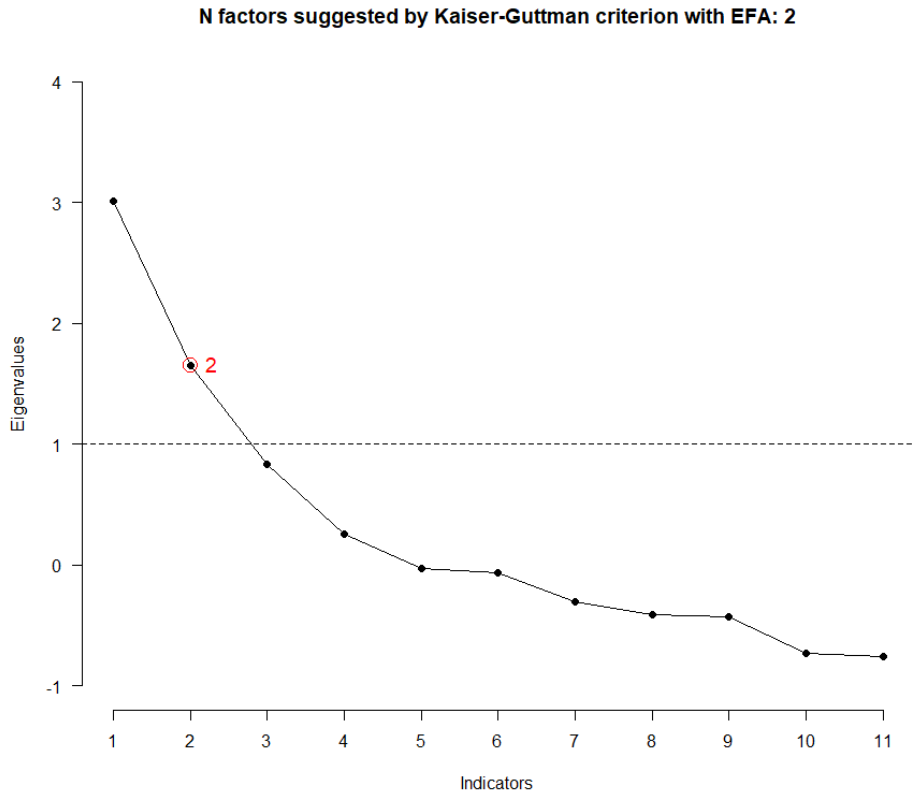

*Source: JF Hair et al. (2019) Multivariate data analysis.*

# Exploring possible factors

**1. Kaiser-Guttman Criterion**

- Only consider factors whose eigenvalues is greater than 1.
- Rationale is that factor should account for the variance of at least a single variable if it is to be retained for interpretation.

```
library(EFAtools)
KGC(Data, eigen_type = "EFA")
```

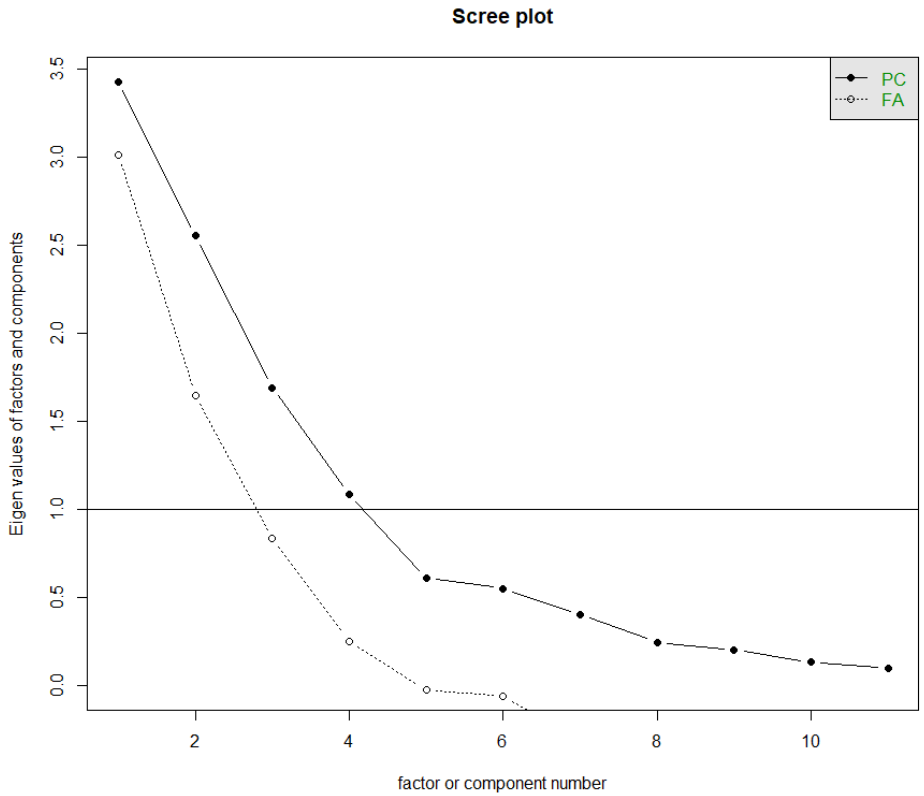


# Exploring possible factors

**2. Scree test**

- Identify the optimum number of factors that can be extracted before the amount of unique variance begins to dominate the common variance.
- Inflection point or the "elbow"

```
library(psych)
scree(data)
```

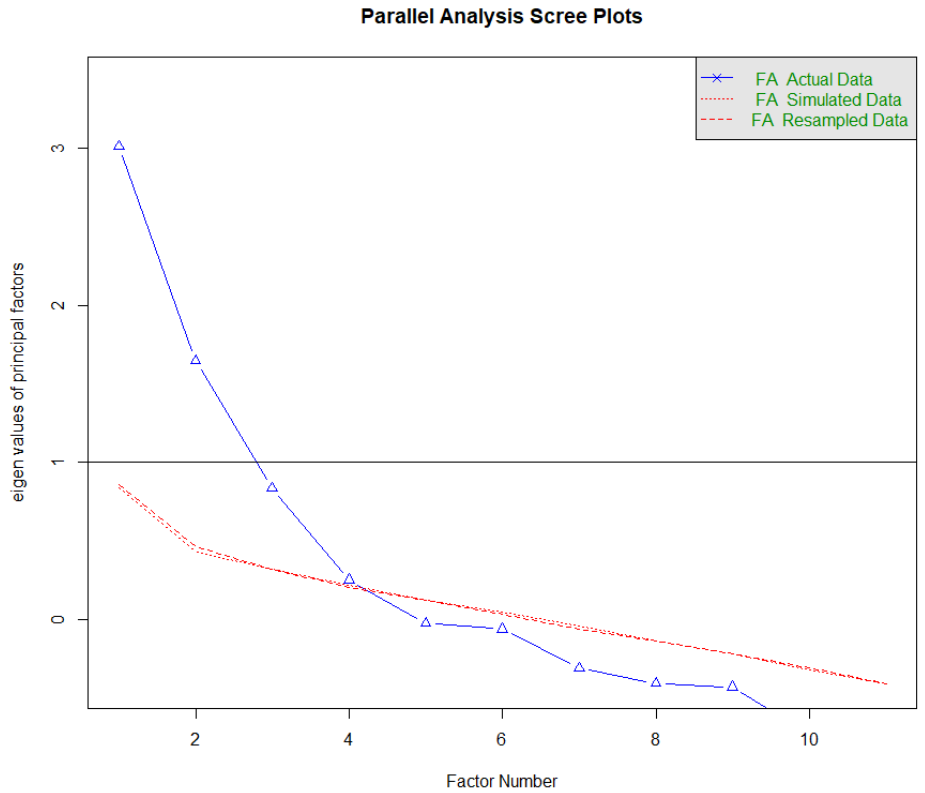


# Exploring possible factors

### 3. Parallel Test

- Generates a large number of simulated dataset.
- Each simulated dataset is factor analyzed.
  - Results is the average eigenvalues across simulation.
  - Values are then compared to the eigenvalues extracted from the original dataset.
  - All factors with eigenvalues above those average eigenvalues are retained.

```
library(psych)
fa.parallel(data, fa = "fa")
```



# Let's practice!

# Stage 5: Interpreting the factors

# Three process of factor intepretation

**1. Factor extraction**

**2. Factor rotation**

**3. Factor interpretation and re-specification**

# Factor extraction

**Loadings**

- Correlation of each variable and the factor.
- Indicate the degree of correspondence between variable and factor.
- Higher loadings making the variable representative of the factor.

```
fa_unrotated <- fa(r = data, nfactors = 4,rotate
print(fa_unrotated$loadings)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x6   0.201 -0.408         0.463
x7   0.290  0.656  0.267  0.210
x8   0.278 -0.382  0.744 -0.169
x9   0.862        -0.255 -0.184
x10  0.287  0.456         0.127
x11  0.689 -0.454 -0.141  0.316
x12  0.398  0.807  0.348  0.255
x13 -0.231  0.553        -0.287
x14  0.378 -0.322  0.730 -0.151
x16  0.747        -0.176 -0.181
x18  0.895        -0.304 -0.198

                 MR1   MR2   MR3   MR4
SS loadings    3.215 2.226 1.500 0.679
Proportion Var 0.292 0.202 0.136 0.062
Cumulative Var 0.292 0.495 0.631 0.693
```

# Factor extraction

**Loadings**

- $\leq \pm 0.10 \approx$ zero
- $\pm 0.10$ to $\pm 0.40$ meet the minimal level
- $\geq \pm 0.50$ practically significant
- $\geq \pm 0.70 \approx$ well-defined structure

**SS loadings**

- Eigenvalues - column sum of squared factor loadings.
- Relative importance of each factor in accounting for the variance associated with the set of variables.

```
fa_unrotated <- fa(r = data, nfactors = 4,r
print(fa_unrotated$loadings)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x6   0.201 -0.408         0.463
x7   0.290  0.656  0.267  0.210
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SS loadings    3.215 2.226 1.500 0.679
Proportion Var 0.292 0.202 0.136 0.062
Cumulative Var 0.292 0.495 0.631 0.693
```
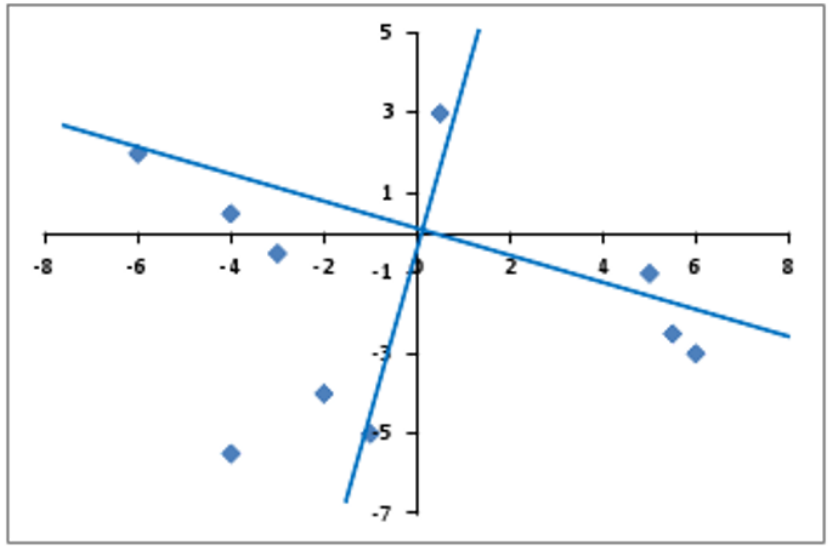

# Factor rotation

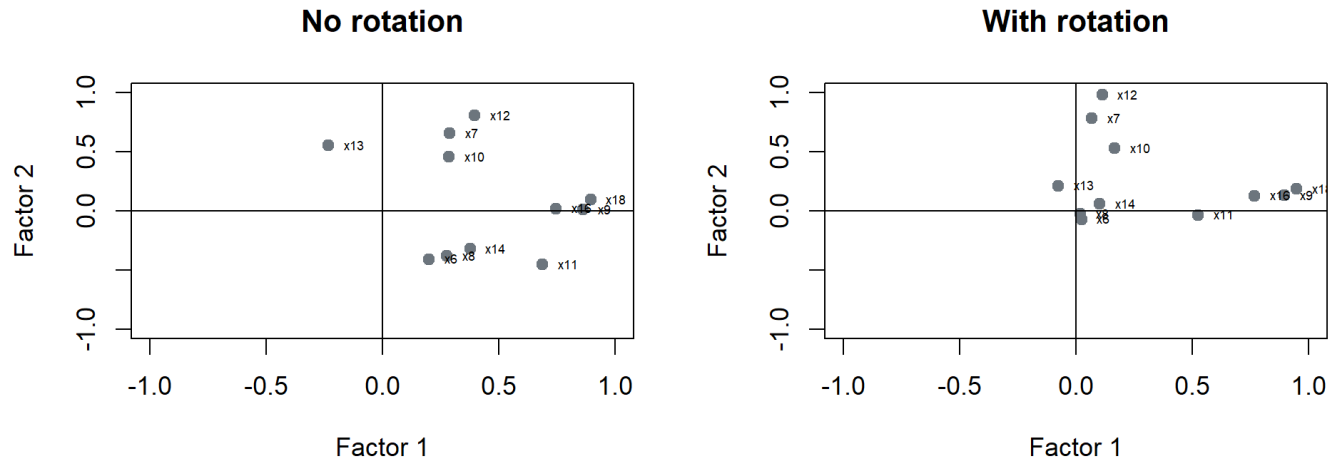
**Why do factor rotation?**

- To simplify the complexity of factor loadings.
- Distribute the loadings more clearly into the factors.
- Facilitate interpretation.



# Factor rotation

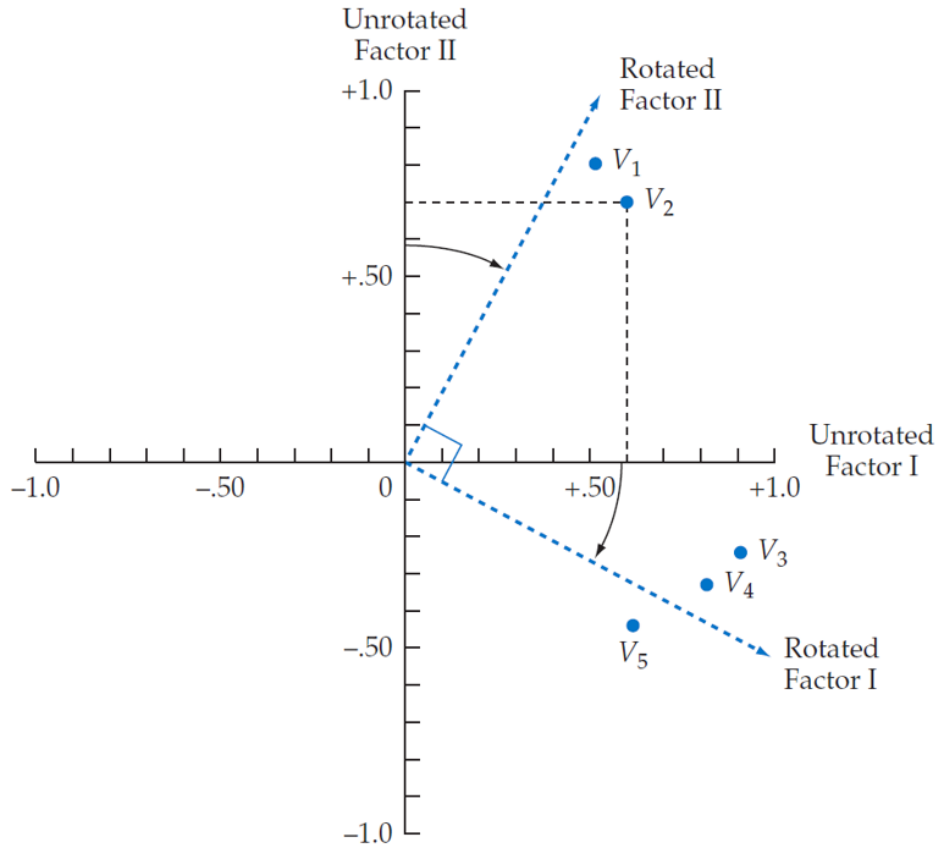
```
par(mfrow = c(1, 2))
plot(fa_unrotated$loadings[,1]
     xlab = "Factor 1", ylab =
     ylim = c(-1, 1), xlim = c
     main = "No rotation",
     pch = 19, col = "#6c757d"
     abline(h=0, v=0)
     text(fa_unrotated$loading
          labels = rownames(fa
          pos = 4, cex = 0.5)

plot(fa_rotated$loadings[,1],
     xlab = "Factor 1", ylab =
     ylim = c(-1, 1), xlim = c
     main = "With rotation",
     pch = 19, col = "#6c757d"
     abline(h=0, v=0)
     text(fa_rotated$loadings[
          labels = rownames(fa
          pos = 4, cex = 0.5)
```



# Factor rotation

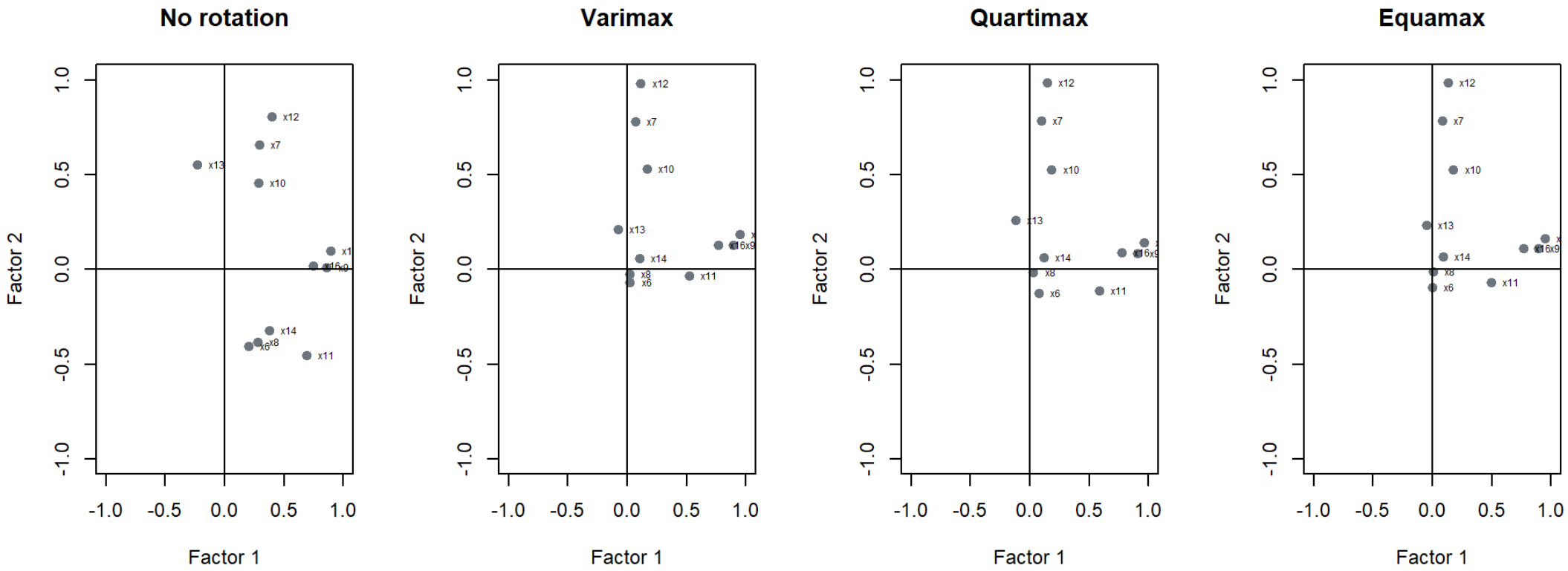
**Orthogonal rotation**

- axes are maintained at 90 degrees
- orthogonal rotation methods
  - Varimax - *most commonly used*
  - Quartimax
  - Equimax
- Check-out some of these references
  - IBM
  - Factor analysis



# Factor rotation

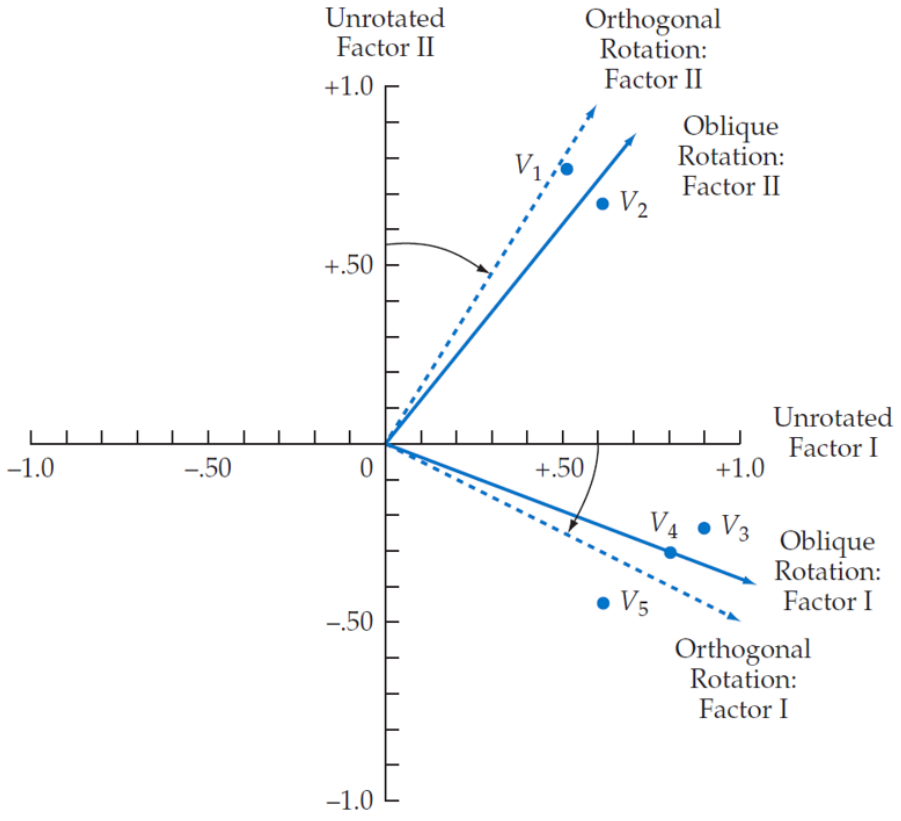
## Orthogonal rotation



# Factor rotation

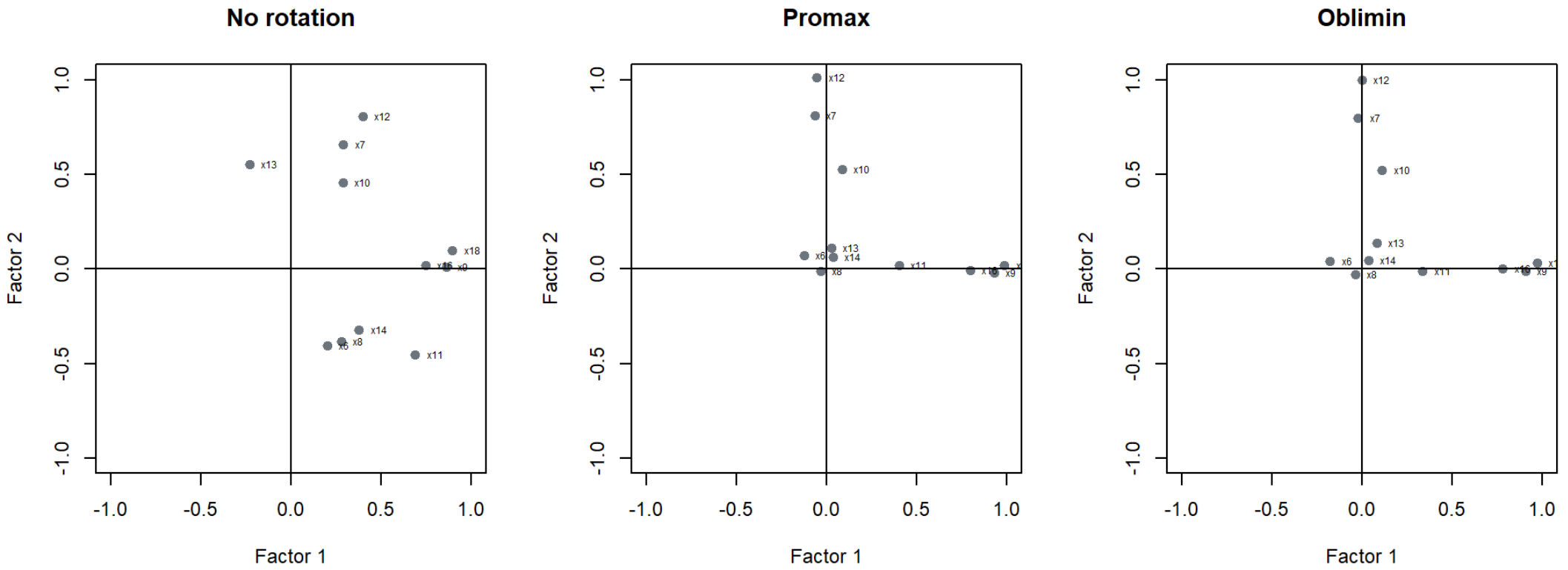
**Oblique rotation rotation**

- allow correlated factors
- suited to the goal of theoretically meaningful constructs
- oblique rotation methods
  - Promax
  - Oblimin



# Factor rotation

## Oblique rotation



# Factor interpretation and respecification

- each variable has a high loadings on one factor only
- each factor has a high loadings for only a subset of the items.

```
fa_varimax <- fa(r = data, nfactors = 4, rotate = "varimax
print(fa_varimax$loadings, sort = TRUE)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x9   0.897  0.130          0.132
x16  0.768  0.127
x18  0.949  0.185
x7           0.781        -0.115
x10  0.166  0.529
x12  0.114  0.980        -0.133
x8                  0.890  0.115
x14  0.103          0.879  0.129
x6                         0.647
x11  0.525          0.127  0.712
x13          0.213 -0.209 -0.590

                 MR1   MR2   MR3   MR4
SS loadings    2.635 1.973 1.641 1.371
Proportion Var 0.240 0.179 0.149 0.125
Cumulative Var 0.240 0.419 0.568 0.693
```

# Factor interpretation and respecification

- each variable has a high loadings on one factor only
- each factor has a high loadings for only a subset of the items.

```
fa_varimax <- fa(r = data, nfactors = 4, rotate = "varimax
print(fa_varimax$loadings, sort = TRUE, cutoff = 0.4)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x9   0.897
x16  0.768
x18  0.949
x7          0.781
x10         0.529
x12         0.980
x8                 0.890
x14                0.879
x6                        0.647
x11  0.525                0.712
x13                      -0.590

                 MR1   MR2   MR3   MR4
SS loadings    2.635 1.973 1.641 1.371
Proportion Var 0.240 0.179 0.149 0.125
Cumulative Var 0.240 0.419 0.568 0.693
```

# Factor interpretation and respecification

What to do with cross-loadings?

Ratio of variance (*JF Hair et al. 2019*)

- 1 to 1.5 - problematic
- 1.5 to 2.0 - potential cross-loading
- 2.0 and higher - ignorable

Example:

- $X_{11}$
- MR1: 0.525
- MR2: 0.712
- $0.712^2 \div 0.525^2 = 1.8$

```
fa_varimax <- fa(r = data, nfactors = 4, rotate = "varimax
print(fa_varimax$loadings, sort = TRUE, cutoff = 0.4)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x9   0.897
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x18  0.949
x7          0.781
x10         0.529
x12         0.980
x8                 0.890
x14                0.879
x6                        0.647
x11  0.525                0.712
x13                      -0.590

                 MR1   MR2   MR3   MR4
SS loadings    2.635 1.973 1.641 1.371
Proportion Var 0.240 0.179 0.149 0.125
Cumulative Var 0.240 0.419 0.568 0.693
```

# Factor interpretation and respecification

## Naming of factors

- **MR1: Postsale customer service**
  - x9 Complaint resolutions
  - x16 Order & Billing
  - x18 Delivery speed
- **MR2: Marketing**
  - x7 E-Commerce
  - x10 Advertising
  - x12 Salesforce image
- **MR3: Technical support**
  - x8 Technical support
  - x14 Warranty and claims
- **MR4: Product value**
  - x6 Product quality
  - x11 Product line
  - x13 Competitive pricing

# Factor interpretation and respecification

## Extracting factor scores

- **MR1: Postsale customer service**
- **MR2: Marketing**
- **MR3: Technical support**
- **MR4: Product value**

```
fa_varimax$scores %>% round(4)
```

```
           MR1     MR2     MR3     MR4
 [1,] -0.1390  0.9401 -1.7237  0.0968
 [2,]  1.6357 -2.0398 -0.5913  0.6504
 [3,]  0.3557  0.8757  0.0160  1.3821
 [4,] -1.2209 -0.5569  1.2497 -0.6456
 [5,] -0.4861 -0.4206 -0.0310  0.4755
 [6,] -0.5923 -1.3136 -1.1969 -0.9591
 [7,] -2.5355  0.4167 -0.5692 -1.2967
 [8,] -0.1153 -0.1178 -0.7072 -1.3628
 [9,]  0.9518  0.3721 -0.1393 -0.9290
[10,]  0.5865  0.4077 -0.4618 -0.6741
[11,] -0.0498 -0.3312 -0.4837  0.6250
[12,] -1.2272  1.2108  0.3349 -1.0737
[13,]  0.7127  1.3501 -0.1129  0.6064
```

# Let's practice!

# Thank you!

**Slides created via the R packages:**


xaringan by Yihui


xaringanthemer and xaringanExtra by Garrick