Training Workshop on Structural Equation Modelling (SEM) using R

Session 3: CFA and SEM

Topic overview

1: CFA-SEM overview

2: CFA-SEM with Lavaan

3: Defining constructs

4: Developing the overall measurement model

5: Assessing measurement model validity

6: Specifying the structural model

7: Assessing structural model validity

CFA-SEM overview

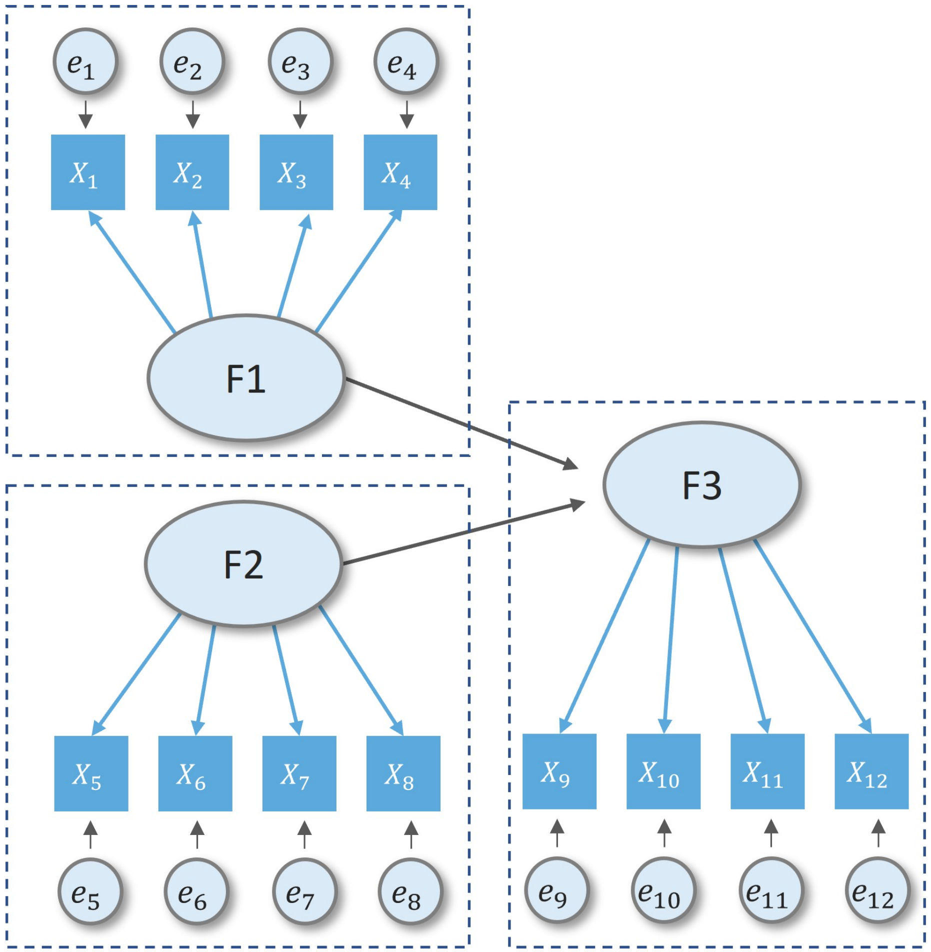
What is SEM?

- A multivariate technique combining aspects of:
    - factor analysis
    - multiple regression.
- Enables the researcher to simultaneously examine interrelated relationships.
    - among measured variables and latent constructs
    - between several latent constructs
- Distinction between:
    - measurement model
    - structural model

What is SEM?

Measurement model

- measurement part of a full SEM model
- confirmatory factor analysis



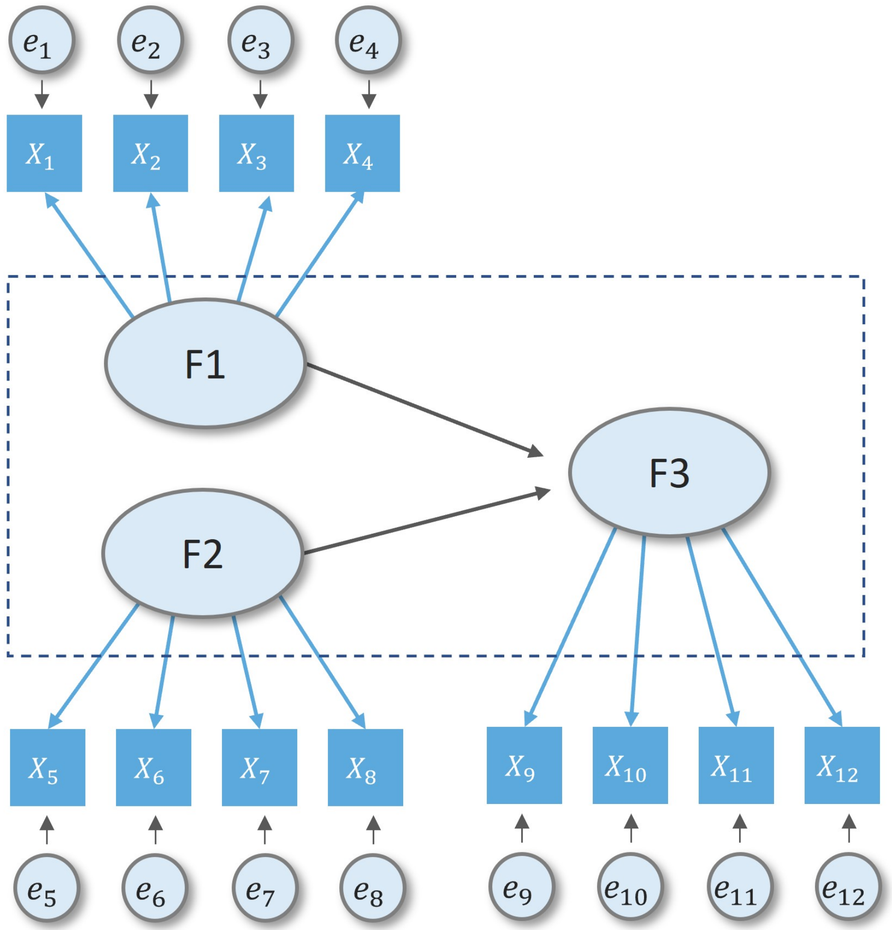
What is SEM?

Measurement model

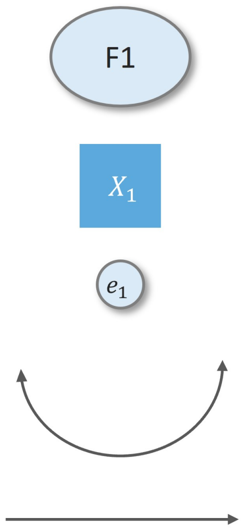
- measurement part of a full SEM model
- confirmatory factor analysis

Structural model

- relationship between constucts
- full SEM model is a combination of measurement and structural component



Basic SEM conventions



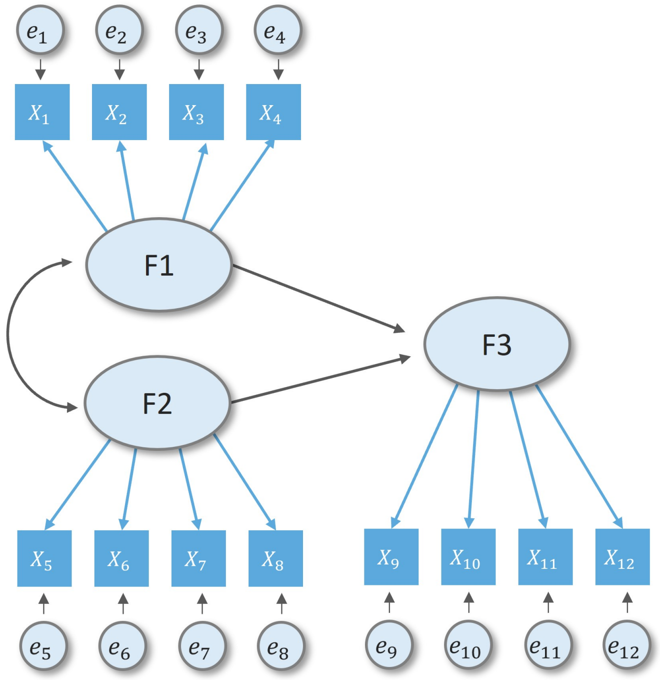
measured latent variable

observed / manifest variable

error variance / disturbance term

Covariance / non-directional path

Regression / directional path



2. CFA-SEM with Lavaan R package

What is Lavaan?

- *"developed to provide useRs, researchers, and teachers a free open-source, but commercial quality"*, Yves Rosseel (2012)
- Check-out the lavaan tutorial

```
install.packages("lavaan")
library(lavaan)
example(cfa)
```

```
cfa> ## The famous Holzinger and Swineford (1939) example
cfa> HS.model <- ' visual  =~ x1 + x2 + x3
cfa+              textual =~ x4 + x5 + x6
cfa+              speed   =~ x7 + x8 + x9 '

cfa> fit <- cfa(HS.model, data = HolzingerSwineford1939)

cfa> summary(fit, fit.measures = TRUE)
lavaan 0.6-9 ended normally after 35 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        21
```

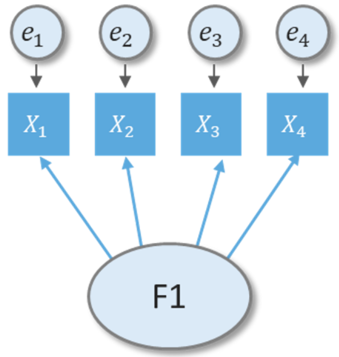
Major operators of lavaan syntax

Command	Operator	Illustration	Significance
Estimate covariance	~~	X ~~ Y	X is correlated with Y
Estimate regression	~	Y ~ X	Y is regressed on X
Define a reflective latent variable	= ~	F =~ item_1 + item_2 + item_3	The F factor is measured by indicators item 1, item 2, and item 3 over which it has effects
Label a parameter	*	F =~ b1*item_1 + b2*item2 + b3*item3	Item 1-3 is named “b1”, “b2”, and “b3”, respectively.
Create a new parameter	:=	B1b2 := b1*b2	Define a parameter that is not in the model. For example: b1b2 = indirect effect of b1 and b2
Insert a comment in the syntax	#	#indirect effects B1b2 := b1*b2	Explain to the reader the meaning of a command.

Major operators of lavaan syntax

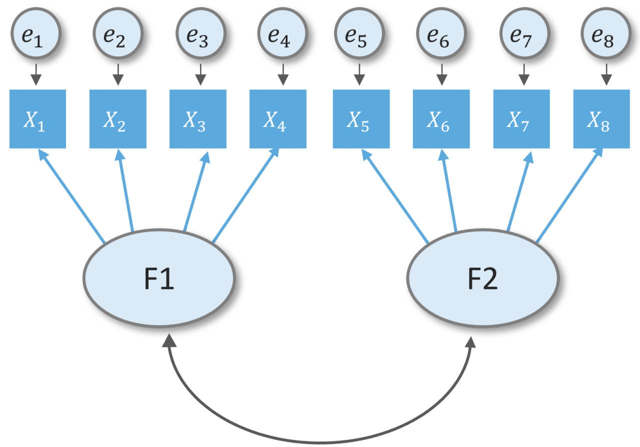
Defining a reflective latent variable

```
model <- "F1 =~ x1 + x2 + x3 + x4"
```



Estimate factor covariance

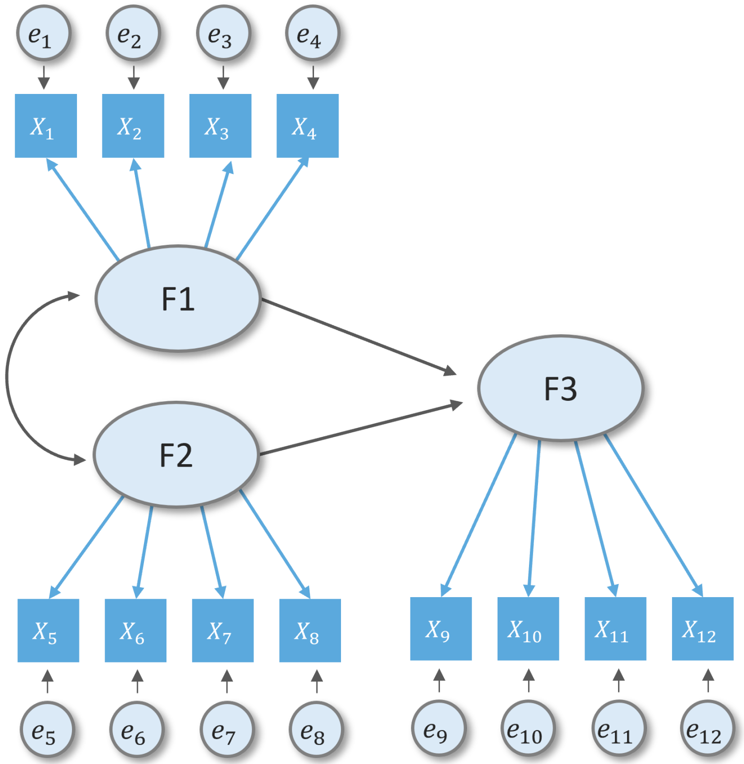
```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x6 + x8
          F1 ~~ F2"
```



Major operators of lavaan syntax

Estimate regression

```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12
          F1 ~~ F2
          F3 ~ F1 + F2"
```



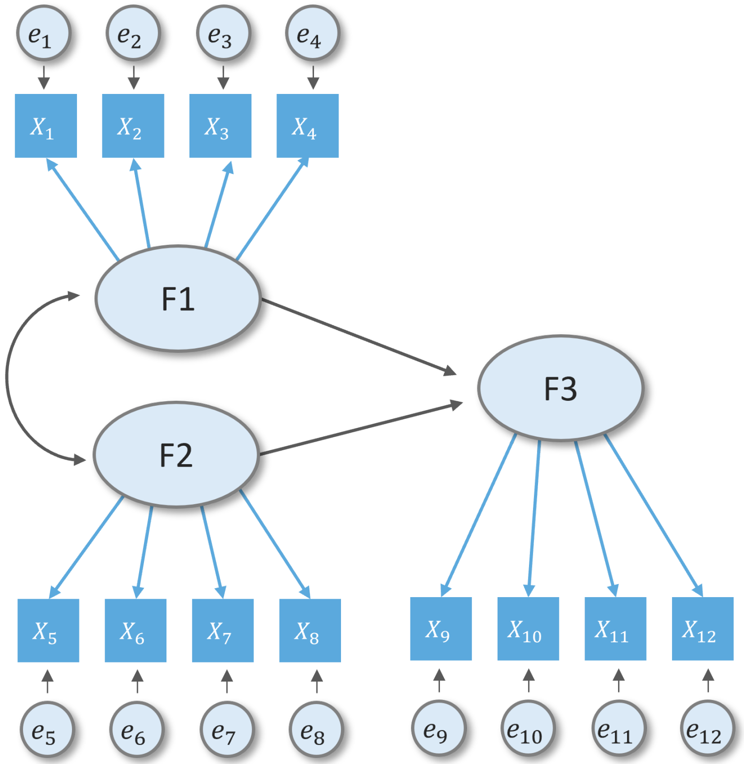
Major operators of lavaan syntax

Insert a comment in the syntax

```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12

          # covariance
          F1 ~~ F2

          # F3 is regressed on F1 and F2
          F3 ~ F1 + F2"
```



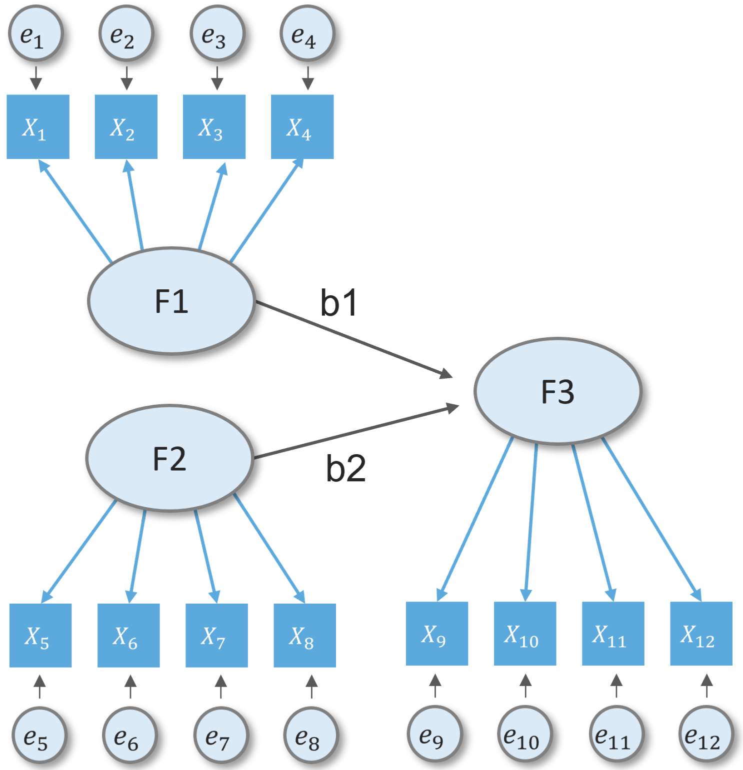
Major operators of lavaan syntax

Label a parameter

```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12

          # covariance
          F1 ~~ F2

          # F3 is regressed on F1 and F2
          F3 ~ b1*F1 + b2*F2"
```



Major operators of lavaan syntax

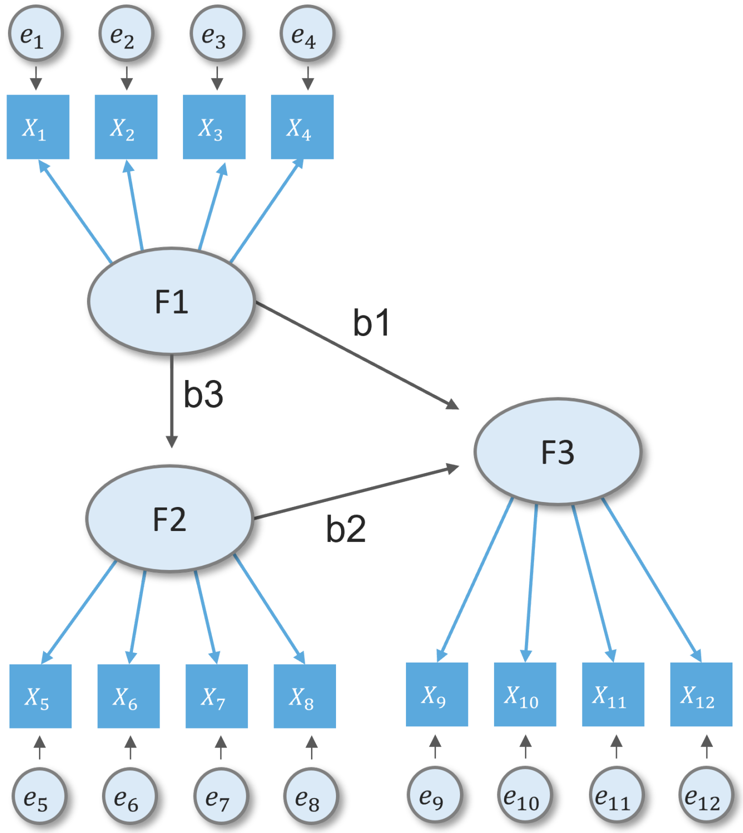
Create a new parameter

```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12

          # regression
          F3 ~ b1*F1 + b2*F2
          F2 ~ b3*F1

          # F1 indirect effect
          ie := b3*b2

          # F1 total effect
          te := b3*b2 + b1"
```



Main steps in SEM

Main steps in SEM

1. Defining constructs
2. Developing the overall measurement model
3. Assessing measurement model validity
4. Specifying the structural model
5. Assessing structural model validity

1. Defining Constructs

Dataset

- HBAT company
- HBAT is interested in understanding what affects employee's attitudes and behaviors that contributes to employee's retension.

JS1	OC1	OC2	EP1	OC3	OC4	EP2	EP3	AC1	EP4
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
5	3	5	10	10	10	10	5	1	2
3	0	5	10	3	7	10	10	2	7
4	6	10	10	10	10	10	10	1	7
4	7	7	10	10	7	10	9	2	7
5	2	10	10	9	9	9	10	1	6
6	5	8	8	7	7	10	7	1	7
2	6	10	9	10	9	9	9	2	6
2	4	9	10	9	7	10	10	1	7
4	9	10	8	10	10	6	8	3	3
5	5	9	10	9	10	10	8	2	7

1-10 of 400 rows | 1-1... Previous **1** 2 3 4 5 6 ... 40 Next

Defining individual constructs

- Based on literature and preliminary interviews, a study was designed focusing on five key constructs.
  - **Job satisfaction(JS)** : reactions resulting from an appraisal of one's job situation.
  - **Organizational commitment(OC)**: extent to which an employees identifies and feels part of HBAT.
  - **Staying intention(SI)**: extent to which an employee intends to continue working for HBAT.
  - **Environmental perceptions(EP)**: beliefs an employee has about day-to-day, physical working conditions.
  - **Attitudes towards coworkers(AC)**: attitudes an employee has toward the coworkers he/she interacts with on a regular basis.

Defining individual constructs

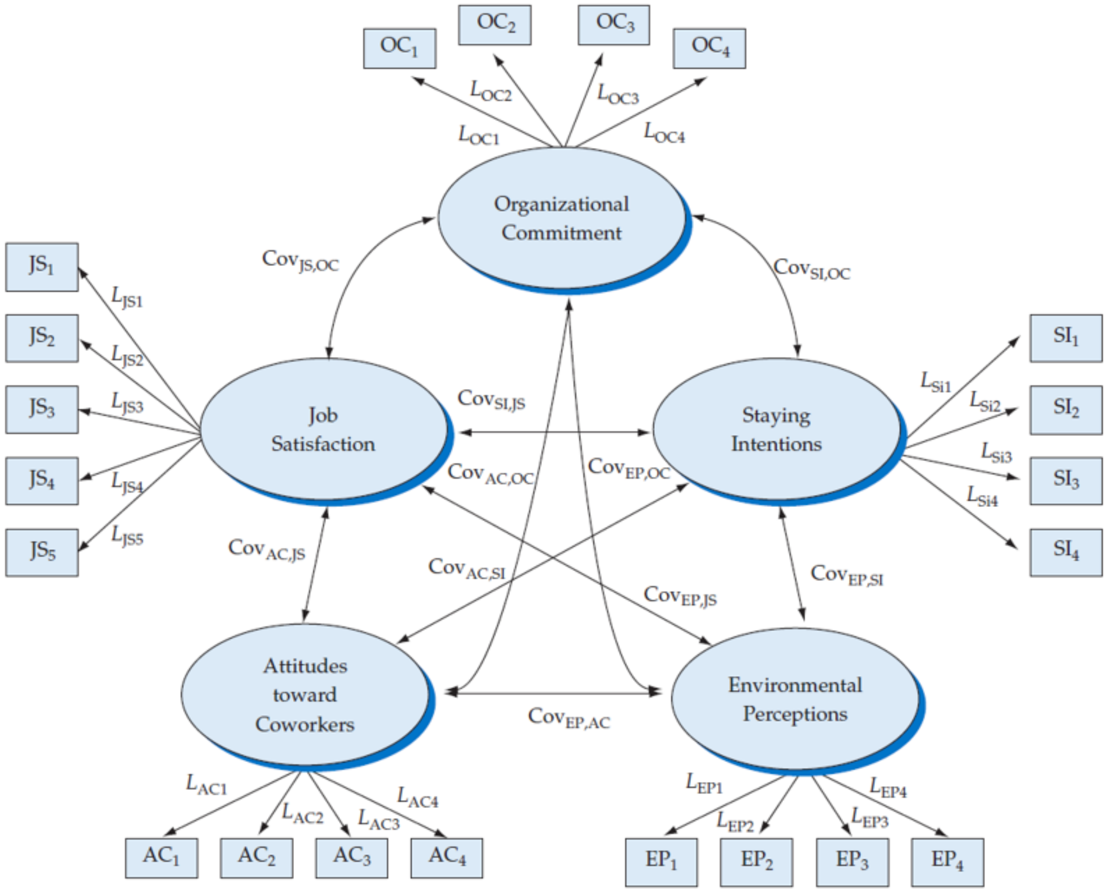
Item	Scale Type	Description	Construct
JS_1	0–10 Likert Disagree–Agree	All things considered, I feel very satisfied when I think about my job.	JS
OC_1	0–10 Likert Disagree–Agree	My work at HBAT gives me a sense of accomplishment.	OC
OC_2	0–10 Likert Disagree–Agree	I am willing to put in a great deal of effort beyond that normally expected to help HBAT be successful.	OC
EP_1	0–10 Likert Disagree–Agree	I am comfortable with my physical work environment at HBAT.	EP
OC_3	0–10 Likert Disagree–Agree	I have a sense of loyalty to HBAT.	OC
OC_4	0–10 Likert Disagree–Agree	I am proud to tell others that I work for HBAT.	OC
EP_2	0–10 Likert Disagree–Agree	The place I work in is designed to help me do my job better.	EP
EP_3	0–10 Likert Disagree–Agree	There are few obstacles to make me less productive in my workplace.	EP
AC_1	5-point Likert	How happy are you with the work of your coworkers? ___ Not happy ___Somewhat happy ___ Happy ___ Very happy ___ Extremely happy	AC
EP_4	7-point Semantic Differential	What term best describes your work environment at HBAT? Too hectic ________ Very soothing	EP
JS_2	7-point Semantic Differential	When you think of your job, how satisfied do you feel? Not at all satisfied __________________ Very much satisfied	JS
JS_3	7-point Semantic Differential	How satisfied are you with your current job at HBAT? Very unsatisfied __________________ Very satisfied	JS
AC_2	7-point Semantic Differential	How do you feel about your coworkers? Very unfavorable ___________ Very favorable	AC
SI_1	5-point Likert Disagree–Agree	I am not actively searching for another job. Strongly disagree ___________ Strongly agree	SI
JS_4	5-point Likert	How satisfied are you with HBAT as an employer? ___ Not at all ___ Little ___ Average ___ A lot ___ Very much	JS
SI_2	5-point Likert Disagree–Agree	I seldom look at the job listings on monster.com. Strongly disagree ___________ Strongly agree	SI
JS_5	Percent Satisfaction	Indicate your satisfaction with your current job at HBAT by placing a percentage in the blank, with 0% = Not satisfied at all, and 100% = Highly satisfied. ________	JS
AC_3	5-point Likert	How often do you do things with your coworkers on your days off?___ Never ___ Rarely ___ Occasionally ___ Often ___ Very often	AC
SI_3	5-point Likert Disagree–Agree	I have no interest in searching for a job in the next year. Strongly disagree ____________ Strongly agree	SI
AC_4	6-point Semantic Differential	Generally, how similar are your coworkers to you? Very different ___________ Very similar	AC
SI_4	5-point Likert	How likely is it that you will be working at HBAT one year from today? ___Very unlikely ___ Unlikely ___ Somewhat likely ___ Likely ___ Very likely	SI

Source: JF Hair et al. (2019) : Multivariate data analysis

Step 2. Developing overall measurement model

Developing overall measurement model

- Measurement theory model (CFA) for HBAT employees
- Direction of the relationship between factors is not yet defined.
- Focus on confirming the specified model with empirical model (using empirical data), hence confirmatory.



Source: JF Hair et al. (2019): Multivariate data analysis

Let's practice!

Step 3. Assessing measurement model validity

Model fit statistics

Running a CFA model in lavaan

```r
hbat_cfa_model <- "si =~ si1 + si2 + s
                js =~ js1 + js2 + js3 +
                ac =~ ac1 + ac2 + ac3 +
                ep =~ ep1 + ep2 + ep3 +
                oc =~ oc1 + oc2 + oc3 +

hbat_cfa_fit <- cfa(model = hbat_cfa_m
```

```r
summary(hbat_cfa_fit, fit.measures = TRUE)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
```

Model fit statistics

The hypothesis:

$H_0 : \Sigma(\theta) = \Sigma$

$H_1 : \Sigma(\theta) \neq \Sigma$

But we do not have the population covariance, so we will use the sample extimates to evaluate model fit.

$H_0 : \Sigma(\hat{\theta}) = S$

$H_1 : \Sigma(\hat{\theta}) = S$

Let:

Σ = population covariance matrix

S = sample covariance matrix

$\Sigma(\theta)$ = model-implied covariance

$\Sigma(\hat{\theta})$ = sample model-implied covariance

Model Fit Stastistics

$$H_0 : \Sigma(\hat{\theta}) = S$$

$$H_1 : \Sigma(\hat{\theta}) = S$$

Sample covariance S

```
data_subset <- hbat_data %>%
 select(starts_with("SI"))
cov(data_subset) %>% round(3)
```

```
      SI1   SI2   SI3   SI4
SI1 0.758 0.560 0.513 0.564
SI2 0.560 0.770 0.557 0.616
SI3 0.513 0.557 1.032 0.657
SI4 0.564 0.616 0.657 0.937
```

Model implied covariance $\Sigma(\hat{\theta})$

```
model <- "SI =~ SI1 + SI2 + SI3 + SI4"
model_cfa <- cfa(model, data = data_subset)
inspect(model_cfa, "cov.ov")
```

```
    SI1   SI2   SI3   SI4
SI1 0.756
SI2 0.540 0.768
SI3 0.532 0.573 1.029
SI4 0.577 0.622 0.613 0.935
```

Model fit statistics

Known values, parameters, and degrees of freedom

- Known values are the number of unique variance-covariance value
- Formula: $[p(p+1)]/2$ where p is the number of observed items
- Example: $[4(4+1)]/2 = 10$

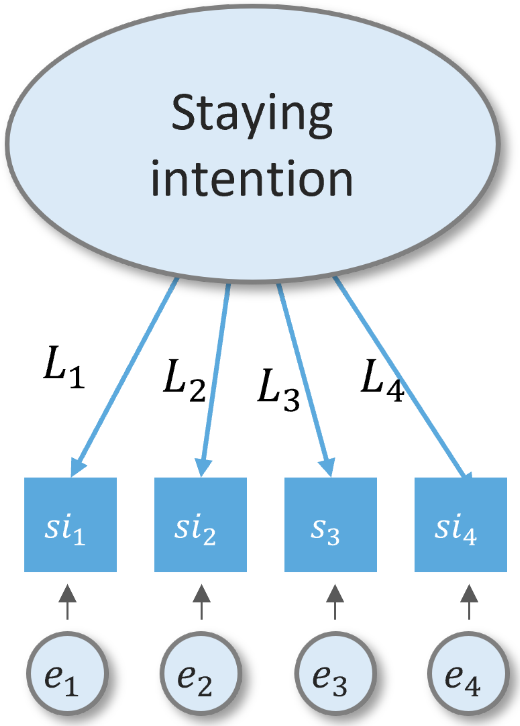
```
cov(data_subset) %>% round(3)
```

```
      SI1   SI2   SI3   SI4
SI1 0.758 0.560 0.513 0.564
SI2 0.560 0.770 0.557 0.616
SI3 0.513 0.557 1.032 0.657
SI4 0.564 0.616 0.657 0.937
```

Model fit statistics

Known values, parameters, and degrees of freedom

- The known values are the primary restrictions of how many parameters(k) we can estimate.
- e.g., 4 loading and 4 error variance



Model fit statistics

Known values, parameters, and degrees of freedom

degrees of freedom (df)

df = known values - estimated parameters

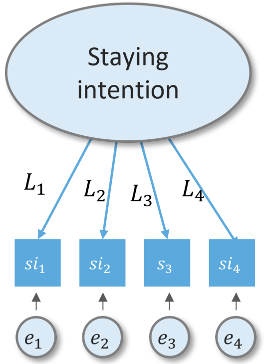
$df = [p(p+1)]/2 - k$

e.g., $df = 10 - 8 = 2$

- 10 unique variance-covariance values

```
      SI1   SI2   SI3   SI4
SI1 0.758 0.560 0.513 0.564
SI2 0.560 0.770 0.557 0.616
SI3 0.513 0.557 1.032 0.657
SI4 0.564 0.616 0.657 0.937
```

- 8 parameters to be estimated



Model fit statistics

Known values, parameters, and degrees of freedom

degrees of freedom (df)

e.g., $df = 10 - 8 = 2$

In a factor with 3 observed indicator, what is the degrees of freedom?

```
model <- "SI =~ SI1 + SI2 + SI3 + SI4"
model_cfa <- cfa(model, data = data_subset)
summary(model_cfa)
```

```
lavaan 0.6-9 ended normally after 22 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         8

  Number of observations                           400

Model Test User Model:

  Test statistic                                12.582
  Degrees of freedom                                 2
```

Summary output

Loadings

1. Marker method

- fixes the first loading of each factor to 1.

2. factor variance standardization

- fixed variance of each factor to 1 but freely estimates all loadings.

```
cfa_fit <- cfa(hbat_cfa_model, data = hbat_data_lower)
summary(cfa_fit, standardized = TRUE)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
```

Summary output

Covariances

```
cfa_fit <- cfa(hbat_cfa_model, data = hbat_data_lower)
summary(cfa_fit, standardized = TRUE)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
```

Summary output

Error variances

Refer to unique variance that the factor unable to account for.

Similar to the error term in OLS, hence error variance.

```r
cfa_fit <- cfa(hbat_cfa_model, data = hbat_data_lower)
summary(cfa_fit, standardized = TRUE)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
```

Fit indices

Goodness of fit indices

- Goodness-of-fit index (GFI)
- Adjusted goodness-fit-index (AGFI)
- Comparative fit index (CFI)
- Normed fit index (NFI)
- Non-normed fit index (NNF)

Badness of fit indices

- Standard root mean square of the residuals (SRMR)
- Root mean square error of approximation (RMSEA)

Table 3. Goodness of fit of the measurement model.

Fit indices	Recommended value	Sources	Research model
χ^2	–	–	369.4
df	–	–	120
χ^2/df	<5	Bollen (1989)	3.08
GFI	>0.9	Scott (1995)	0.91
AGFI	>0.8	Scott (1995)	0.87
SRMR	<0.1	Hu and Bentler (1999)	0.034
CFI	>0.9	Bagozzi and Yi (1988)	0.96
RMSEA	<0.08	MacCallum *et al.* (1996)	0.071
NFI	>0.9	Bentler and Bonett (1980)	0.95
NNFI	>0.9	Bentler and Bonett (1980)	0.95

Sample GOF results from W. Shiau & M. Luo (2013). Continuance intention of blog users: The impact of perceived enjoyment, habit, user involvement and blogging time.

Fit indices

Goodness of fit indices

- Goodness-of-fit index (GFI)
- Adjusted goodness-fit-index (AGFI)
- Comparative fit index (CFI)
- Normed fit index (NFI)
- Non-normed fit index (NNF)

```
fitMeasures(cfa_fit)
```

```
            npar               fmin              chis
          52.000              0.301            240.73
          pvalue     baseline.chisq        baseline.d
           0.001           4452.567            210.00
             cfi                tli               nnf
           0.985              0.983              0.98
             nfi               pnfi                if
           0.946              0.806              0.98
            logl  unrestricted.logl                ai
      -13916.782         -13796.413          27937.56
          ntotal               bic2              rmse
         400.000          27980.120              0.02
 rmsea.ci.upper        rmsea.pvalue                rm
           0.039              1.000              0.41
            srmr       srmr_bentler srmr_bentler_nomea
           0.036              0.036              0.03
     crmr_nomean        srmr_mplus   srmr_mplus_nomea
           0.037              0.036              0.03
           cn_01                gfi               agf
         376.401              0.947              0.93
```

Fit indices

Goodness of fit indices

- Goodness-of-fit index (GFI)
- Adjusted goodness-fit-index (AGFI)
- Comparative fit index (CFI)
- Normed fit index (NFI)
- Non-normed fit index (NNF)

```
fitMeasures(cfa_fit,  fit.measures = c("gfi", "agfi", "cfi
```

```
  gfi  agfi   cfi   nfi  nnfi
0.947 0.932 0.985 0.946 0.983
```

Fit indices

Badness of fit indices

- Standard root mean squrare residual (SRMR)
- Root mean square error of approximation (RMSEA)

```
fitMeasures(cfa_fit,  fit.measures = c("srmr", "rmsea"))
```

```
 srmr rmsea 
0.036 0.029 
```

Reliability and validity test

Reliability test

- Composite reliability

Validity test

- Convergent validity
- Discriminant validity

	α	CR	AVE	CU	TD	FI	HE	IN	INTE	SA
CU	0.94	0.96	0.89	0.95[a]						
TD	0.88	0.93	0.81	0.58	0.90					
FI	0.92	0.94	0.76	0.74	0.78	0.87				
HE	0.94	0.96	0.89	0.79	0.59	0.74	0.94			
IN	0.88	0.92	0.74	0.51	0.48	0.51	0.59	0.86		
INTE	0.88	0.92	0.80	0.61	0.54	0.60	0.66	0.70	0.89	
SA	0.88	0.93	0.80	0.41	0.38	0.44	0.56	0.59	0.59	0.90

Notes: α, Cronbach's α; CR, composite reliability. CU, CUriosity; HE, Heightened Enjoyment; TD: Temporal Dissociation; FI: Focused Immersion; IN: INteractivity; INTE: INTEreat; SA: SAtisfaction.
[a]The square root of AVE

Source: A. Hou, W. Shiau, & R. Shang (2019). The involvement paradox. The role of cognitive absorption in mobile instant messaging user satisfaction.

Reliability and validity test

- Composite reliability: `alpha` > `0.70`
- Convergent validity: AVE (`avevar`) > `0.50`
- Discriminant validity: `omega` > `0.7`

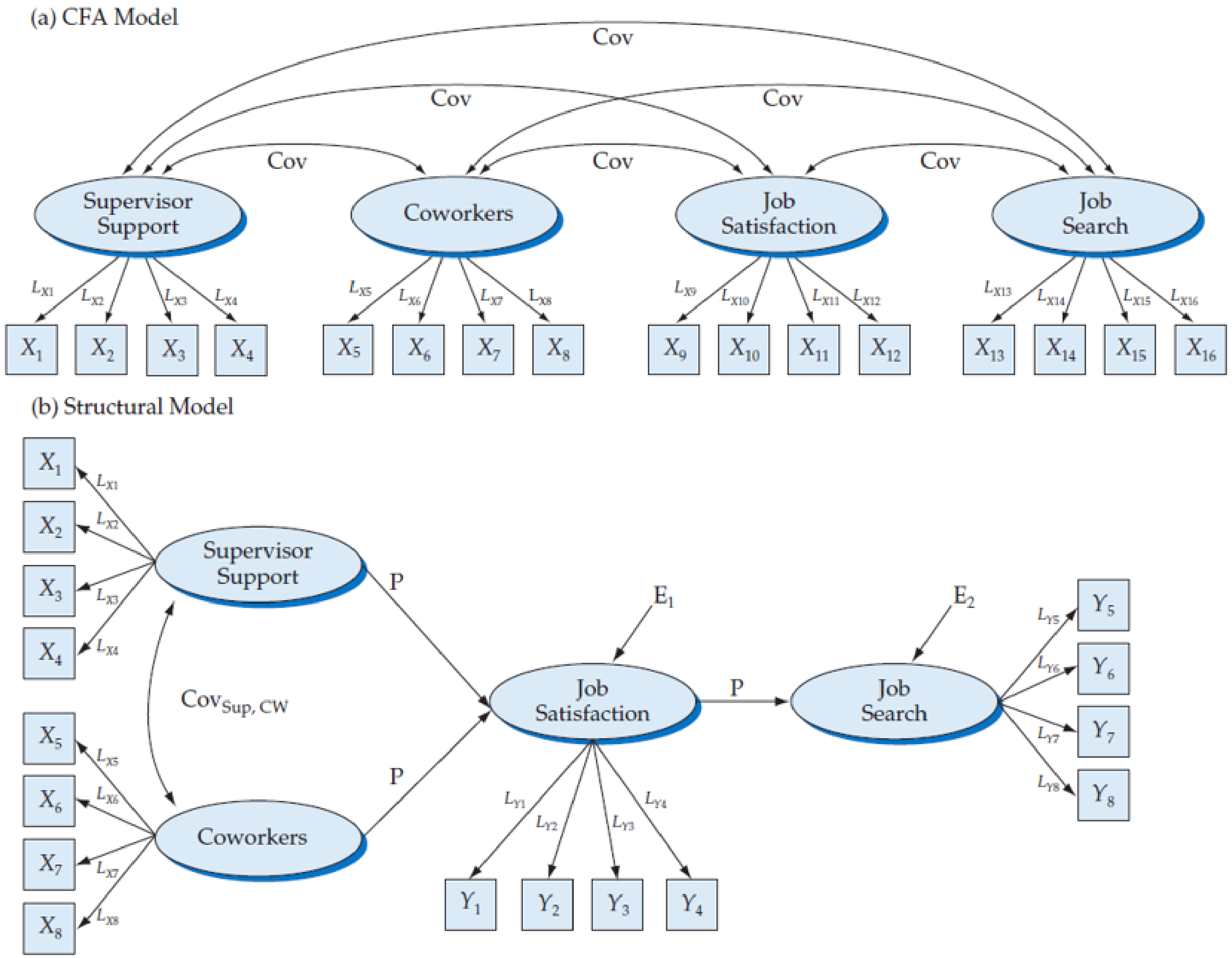
```
library(semTools)
reliability(cfa_fit) %>% round(3)
```

```
          si    js    ac    ep    oc
alpha  0.886 0.281 0.891 0.847 0.823
omega  0.887 0.640 0.893 0.850 0.827
omega2 0.887 0.640 0.893 0.850 0.827
omega3 0.887 0.641 0.893 0.850 0.818
avevar 0.664 0.535 0.677 0.587 0.552
```

Let's practice

Step 4: Specifying the structural model

CFA model to structural model

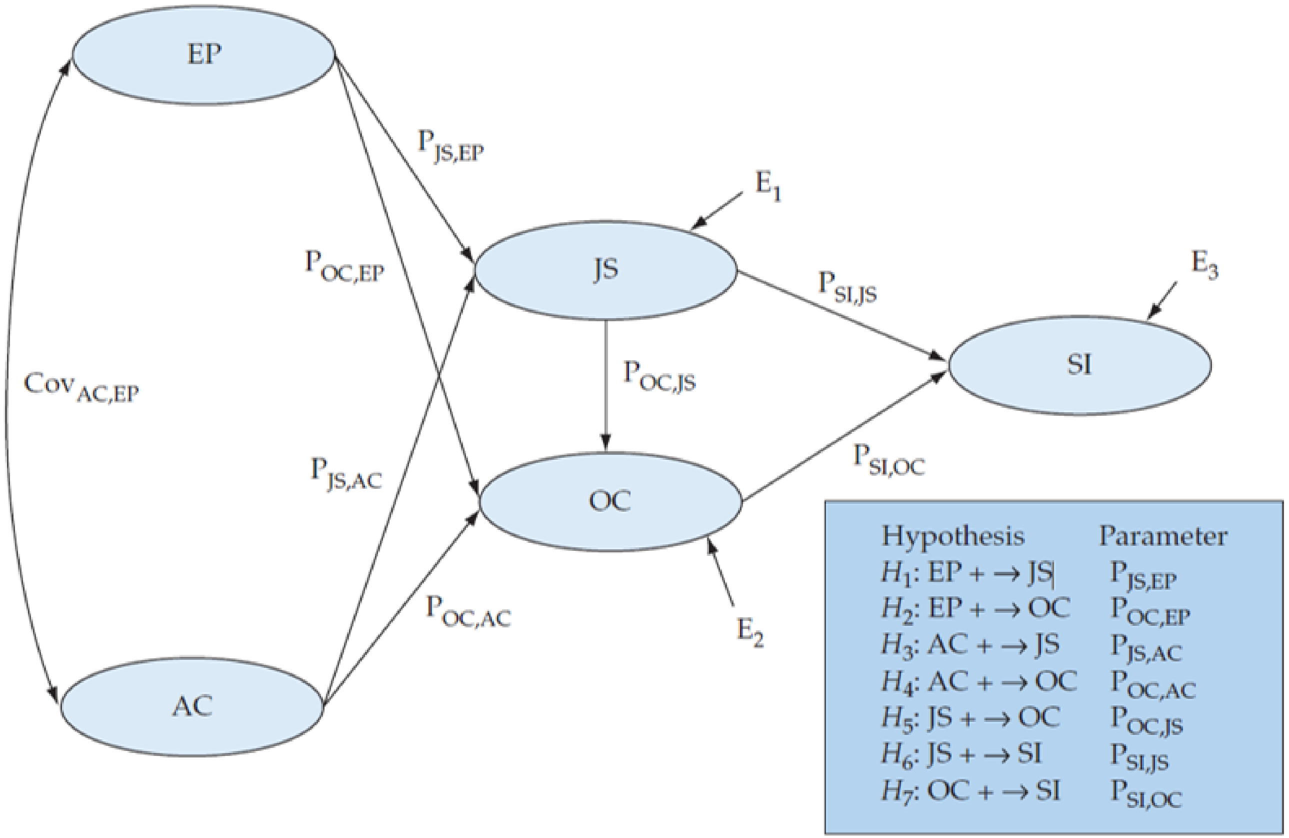


Defining structural model

Hypothesis:

- H1: Environmental perceptions are positively related to job satisfaction.
- H2: Environmental perceptions are positively related to organizational commitment.
- H3: Attitudes toward coworkers are positively related to job satisfaction.
- H4: Attitudes toward coworkers are positively related to organizational commitment.
- H5: Job satisfaction is related positively to organizational commitment.
- H6: Job satisfaction is related positively to staying intentions.
- H7: Organizational commitment is related positively to staying intention.

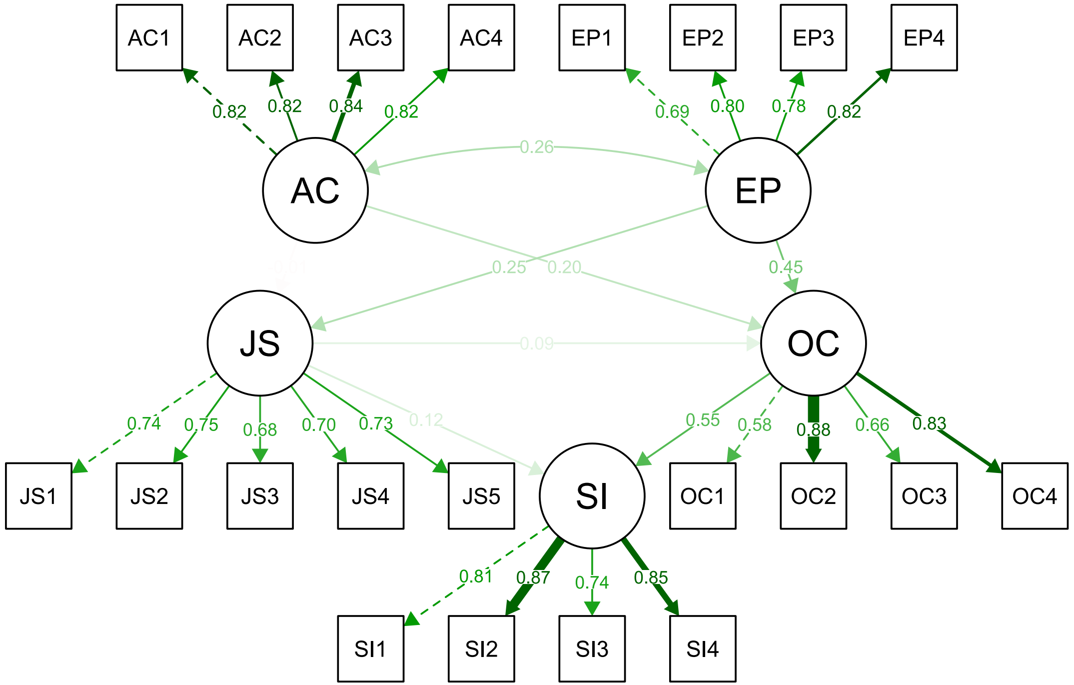
Defining structural model



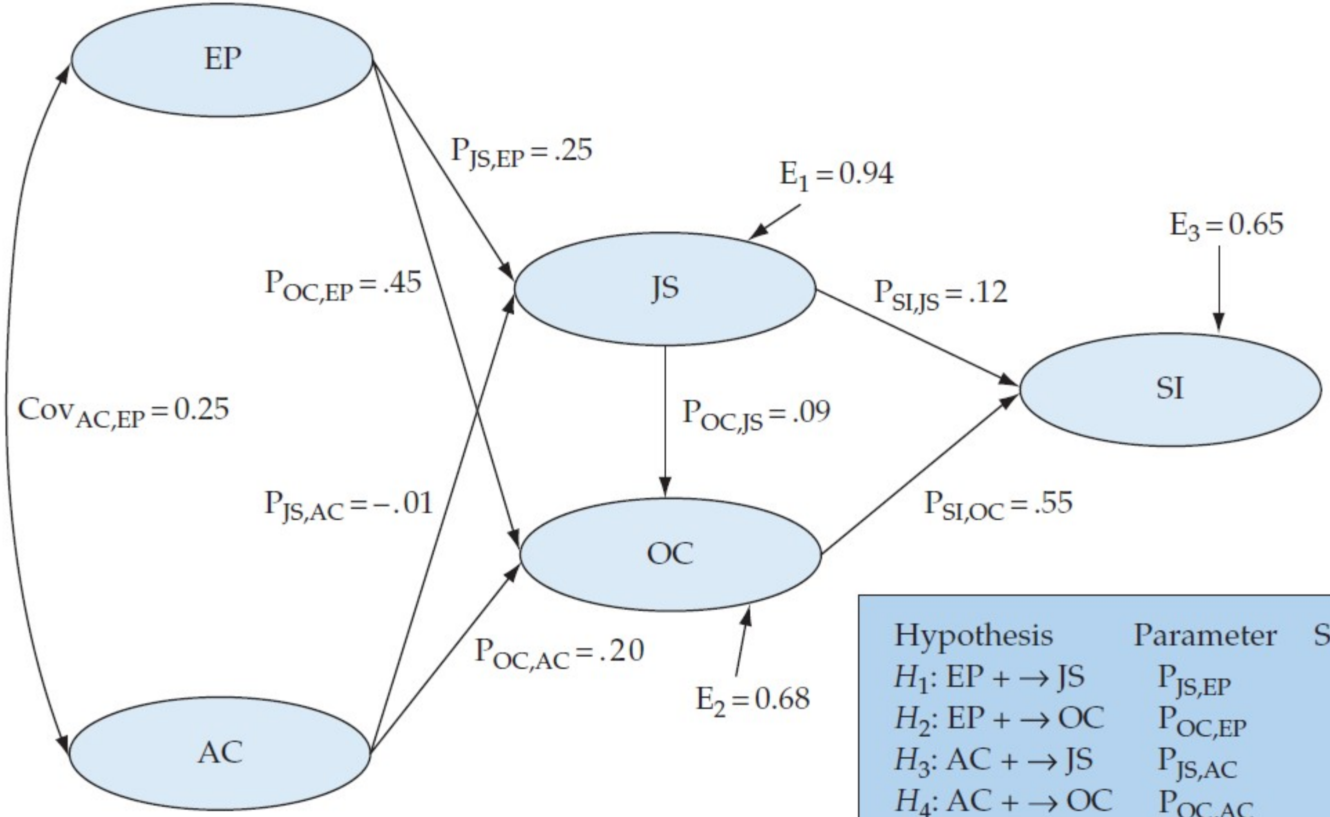
Let's practice

Defining structural model

```r
library(semPlot)
semPaths(object = sem_fit,
         what = "std",
         layout = "tree2",
         intercepts = FALSE,
         residuals = FALSE)
```



Defining structural model



Hypothesis	Parameter	Supported?
H_1: EP + → JS	$P_{JS,EP}$	Yes
H_2: EP + → OC	$P_{OC,EP}$	Yes
H_3: AC + → JS	$P_{JS,AC}$	No
H_4: AC + → OC	$P_{OC,AC}$	Yes
H_5: JS + → OC	$P_{OC,JS}$	No
H_6: JS + → SI	$P_{SI,JS}$	Yes
H_7: OC + → SI	$P_{SI,OC}$	Yes

GOF measures between structural and CFA model

```
gof_indices <- c('chisq', 'df','pvalue
                 'rmsea', 'rmr', 'srmr
                 'nnfi', 'cfi', 'agfi'
fitmeasures(sem_fit, fit.measures = go
fitmeasures(cfa_fit, fit.measures = go
```

```
  chisq       df  pvalue     gfi    rmse
287.179  181.000   0.000   0.938   0.03
   agfi
  0.921
```

```
  chisq       df  pvalue     gfi    rmse
240.738  179.000   0.001   0.947   0.02
   agfi
  0.932
```

GOF index	Employee retention model	CFA model
χ^2(chi-square)	287.179	240.738
Degrees of freedom	181	179
Probability	0.000	0.001
GFI	0.938	0.947
RMSEA	0.038	0.029
RMR	0.410	0.414
SRMR	0.060	0.036
NFI	0.936	0.946
NNFI	0.971	0.983
CFI	0.975	0.985
AGFI	0.921	0.932

What's next?

- Modification indeces
- Handling heywood cases
- Comparing competing models
- Formative scales in SEM
- Higher-order factor analysis
- Multigroup analysis

Thank you!

Slides created via the R packages:

xaringan by Yihui

xaringanthemer and xaringanExtra by Garrick