Structural Equation Modeling (SEM) using R

Session 1: Exploratory Factor Analysis (EFA)

23 August 2021

Factor analysis process

Stage 1: Objectives of factor analysis

Stage 2: Designing an Exploratory factor analysis

Stage 3: Assumptions in Exploratory factor analysis

Stage 4: Deriving factors and assessing overall fit

Stage 5: Interpreting the factors

Stage 1 : Objectives of factor analysis

Types of factor analysis

Exploratory factor analysis

- Use when you do not have a well-developed theory
- Estimate all possible variable/ factor relationships
- Looking for patterns in the data

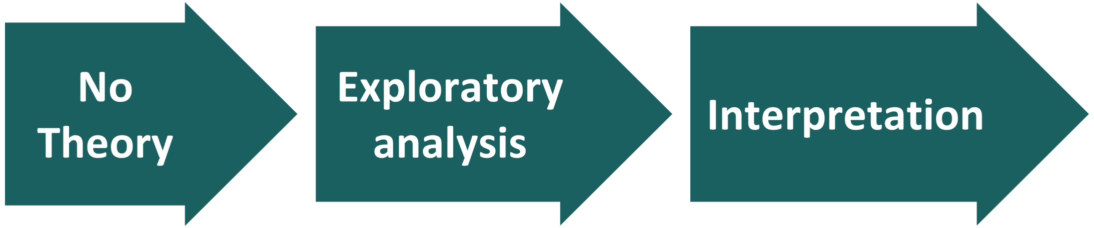
Confirmatory factor analysis

- Testing a theory that you know in advance
- Only specified variables/factor relationships

Types of factor analysis

Exploratory factor analysis

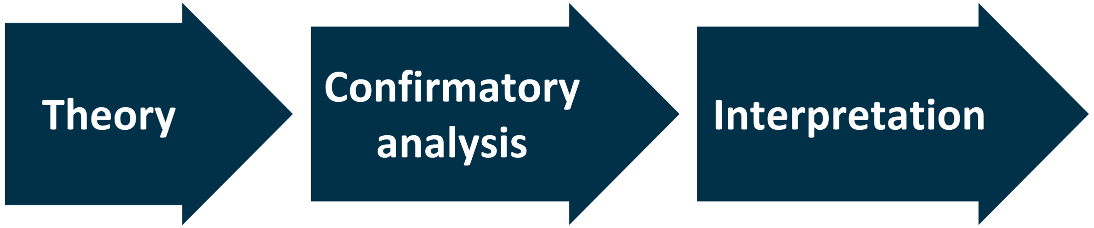
- Difficult to interpret without a theory.
- factor loadings: meanings can sometimes be inferred from patterns.



Types of factor analysis

Confirmatory factor analysis

- Model fit: how well the hypothesized model fits the data.
- Factor loadings: how well items measure their corresponding constructs.



Stage 2: Designing an EFA

Variable selection and measurement issues

What types of variables can be used in factor analysis?

- *Primary requirement: a correlation value can be calculated among all variables.*
- *e.g., metric variables, scale items, dummy variables to represent nonmetric variables.*

How many variables should be included?

- *Five or more per factor for scale development.*
- *Three or more per factor for factor measurement (based on how degrees of freedom is computed).*

Sample size

Some recommended guidelines:

Absolute size of the dataset

- *should not fewer than 50 observation*
- *preferably 100 and larger*
- *200 and larger as the number of variables and expected factors incerases*

Ratio of cases to variables

- *observation is 5x as the number of variables*
- *sample size is 10:1 ratio*
- *some proposes 20 cases per variables*

Stage 3: Assumptions in EFA

Sample Dataset

- HBAT Industries, manufacturer of paper products.
- Perceptions on a set of business functions.
- Rating scale:
  - `0 "poor"` to `10 "excellent"`

X_6	Product quality	Perceived level of quality of HBAT's paper products
X_7	E-commerce	Overall image of HBAT's website; user-friendliness
X_8	Technical support	Extent to which technical support is offered
X_9	Complaint resolution	Extent to which any complaints are resolved in timely and complete manner
X_{10}	Advertising	Perceptions of HBAT's product line to meet customer needs
X_{11}	Product line	Depth and breadth of HBAT's product line to meet customer needs
X_{12}	Salesforce image	Overall image of HBAT's salesforce
X_{13}	Competitive pricing	Extent to which HBAT offers competitive prices
X_{14}	Warranty and claims	Extent to which HBAT stands behind its product/ service warranties and claims
X_{15}	New products	Extent to which HBAT develops and sells new products
X_{16}	Ordering and billing	Perceptions that ordering and billing is handled efficiently and correctly
X_{17}	Price flexibility	Perceived willingness of HBAT sales reps to negotiate price on purchase of paper products
X_{18}	Delivery speed	Amount of time it takes to deliver the paper product once an order has been confirmed

Sample Dataset

- X_6 product quality
- X_7 e-commerce
- X_8 technical support
- X_9 complaint resolution
- X_{10} advertising
- X_{11} product line
- X_{12} salesforce image
- X_{13} competitive pricing
- X_{14} warranty claims
- X_{15} packaging
- X_{16} order & billing
- X_{17} price flexibility
- X_{18} delivery speed

x6	x7	x8	x9	x10	x11	x12
<dbl+lbl>	<dbl+lbl>	<dbl+lbl>	<dbl+lbl>	<dbl+lbl>	<dbl+lbl>	<dbl+lbl>
8.5	3.9	2.5	5.9	4.8	4.9	6.0
8.2	2.7	5.1	7.2	3.4	7.9	3.1
9.2	3.4	5.6	5.6	5.4	7.4	5.8
6.4	3.3	7.0	3.7	4.7	4.7	4.5
9.0	3.4	5.2	4.6	2.2	6.0	4.5
6.5	2.8	3.1	4.1	4.0	4.3	3.7
6.9	3.7	5.0	2.6	2.1	2.3	5.4
6.2	3.3	3.9	4.8	4.6	3.6	5.1
5.8	3.6	5.1	6.7	3.7	5.9	5.8
6.4	4.5	5.1	6.1	4.7	5.7	5.7

1-10 of 100 rows | 1-7 of 11 col... Previous **1** 2 3 4 5 6 ... 10 Next

Source: J.F. Hair (2019): Multivariate data analysis.

Conceptual assumptions

- Some uderlying structure does exist in the set of selected variables.
- correlated variables and subsequent definition of factors do not guarantee relevance
  - *even if they meet the statistical requirement!*
- It is the responsibility of the researcher to ensure that observed patterns are conceptually valid and appropriate.

Determining appropriateness of EFA

1. Bartlett Test

2. Measure of Sampling Adequacy

Determining the appropriateness of EFA

1. Bartlett Test

- Examines the entire correlation matrix
- Test the hypothesis that correlation matrix is an identity matrix.
- A significant result signifies data are appropriate for factor analysis.

```r
library(EFAtools)
BARTLETT(data, N = 100)
```

```
v The Bartlett's test of sphericity was significa
  These data are probably suitable for factor ana

  <U+0001D712>²(55) = 619.27, p < .001
```

Determining the appropriateness of EFA

2. Kaiser-Meyen-Olkin (KMO Test)

- Measure of sampling adequacy
- Indicate the proportion of variance explained by the underlying factor.
- Guidelines:
    - ≥ 0.90 - marvelous
    - ≥ 0.80 - meritorious
    - ≥ 0.70 - middling
    - ≥ 0.60 - mediocre
    - ≥ 0.50 - miserable
    - < 0.50 - unacceptable

Determining the appropriateness of EFA

2. Kaiser-Meyen-Olkin (KMO Test)

```
library(psych)
KMO(data)
```

```

-- Kaiser-Meyer-Olkin criterion (KMO) ----------------------------------------------------

! The overall KMO value for your data is mediocre.
  These data are probably suitable for factor analysis.

  Overall: 0.653

  For each variable:
   x6    x7    x8    x9   x10   x11   x12   x13   x14   x16   x18
0.509 0.626 0.519 0.787 0.779 0.622 0.622 0.753 0.511 0.760 0.666
```

Determining the appropriateness of EFA

2. Kaiser-Meyen-Olkin (KMO Test)

- When overall MSA is less than 0.50
  - Identify variables with lowest MSA subject for deletion.
  - Recalculate MSA
  - Repeat unitl overall MSA is 0.50 and above

- Deletion of variables with MSA under 0.50 means variable's correlation with other variables are poorly representing the extracted factor.

Let's practice!

Stage 4: Deriving factors and assessing overall fit

Partitioning the variance of a variable

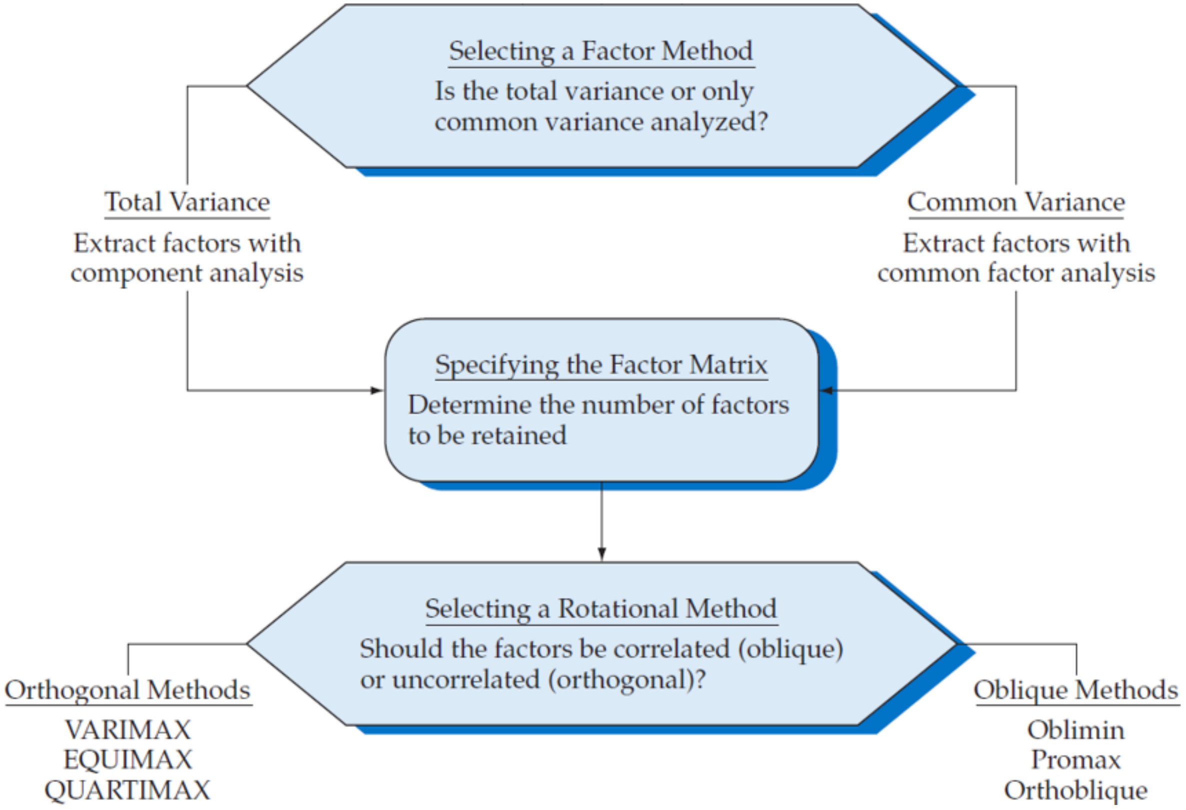
Unique variance

- Variance associated with only a specific variable.
- Not represented in the correlations among variables.
- *Specific variance*
  - associated uniquely with a single variable.
- *Error variance*
  - May be due to unreliability of data gathering process, measurement error, or a random component in the measured phenomenom.

Common variance

- Shared variance with all other variables.
- High common variance are more amenable for factor analysis.
- Derived factors represents the shared or common variance among the variables.

Partitioning the variance of a variable



Source: JF Hair et al. (2019) Multivariate data analysis.

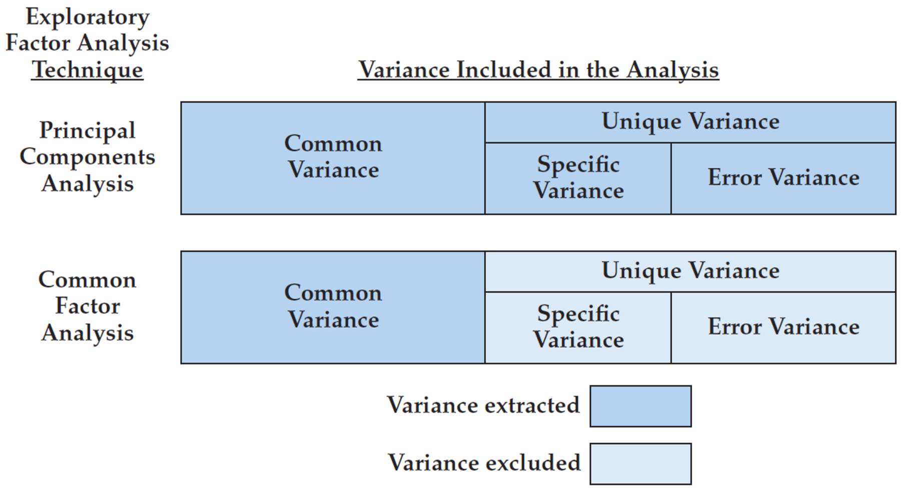
PCA vs Common factor analysis

Principal component analysis (PCA)

- Considers the total variance
- data reduction is a primary concern

Common factor analysis

- Considers only the common variance or shared variance
- Primary objective si to identify the laten dimensions or constructs



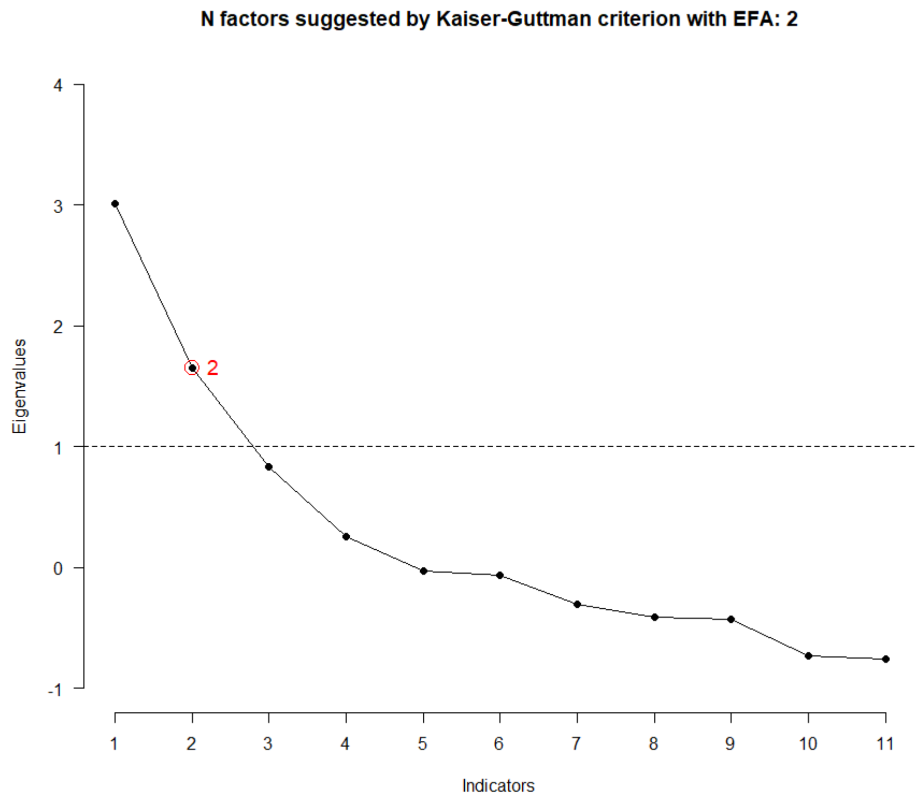
Source: JF Hair et al. (2019) Multivariate data analysis.

Exploring possible factors

1. Kaiser-Guttman Criterion

- Only consider factors whose eigenvalues is greater than 1.
- Rationale is that factor should account for the variance of at least a single variable if it is to be retained for interpretation.

```
library(EFAtools)
KGC(Data, eigen_type = "EFA")
```

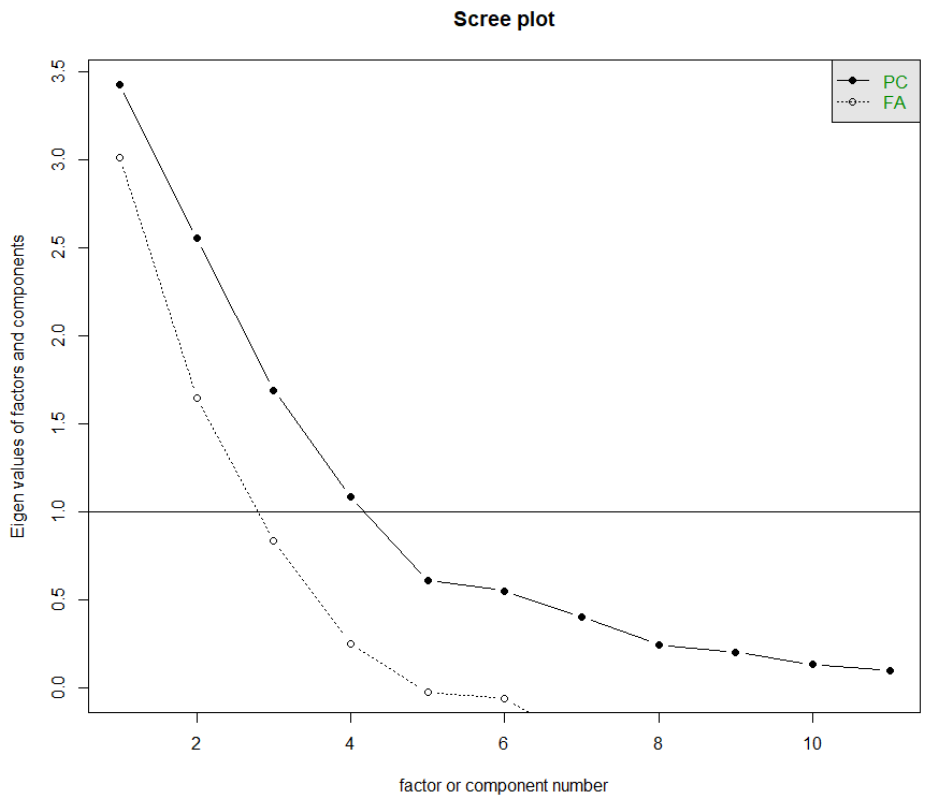


Exploring possible factors

2. Scree test

- Identify the optimum number of factors that can be extracted before the amount of unique variance begins to dominate the common variance.
- Inflection point or the "elbow"

```r
library(psych)
scree(data)
```

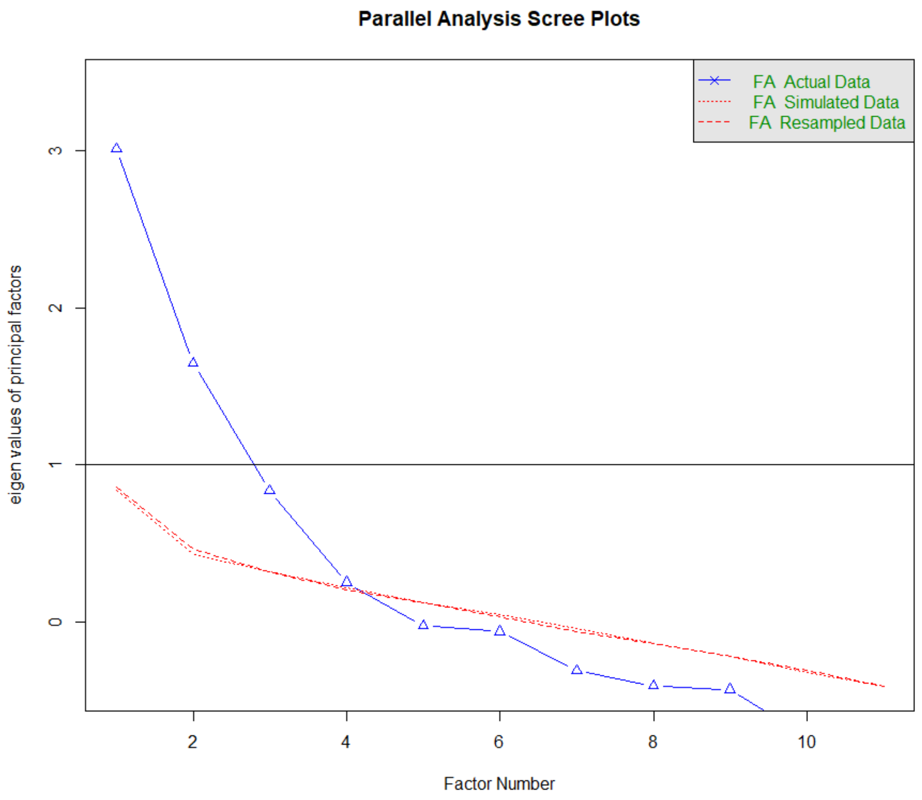


Exploring possible factors

3. Parallel Test

```r
library(psych)
fa.parallel(data, fa = "fa")
```

- Generates a large number of simulated dataset.
- Each simulated dataset is factor analyzed.
  - Results is the average eigenvalues across simulation.
  - Values are then compared to the eigenvalues extracted from the original dataset.
  - All factors with eigenvalues above those average eigenvalues are retained.



Let's practice!

Stage 5: Interpreting the factors

Three process of factor intepretation

1. **Factor extraction**

2. **Factor rotation**

3. **Factor interpretation and re-specification**

Factor extraction

Loadings

- Correlation of each variable and the factor.
- Indicate the degree of correspondence between variable and factor.
- Higher loadings making the variable representative of the factor.

```
fa_unrotated <- fa(r = data, nfactors = 4,rotate
print(fa_unrotated$loadings)
```

```
Loadings:
    MR1    MR2    MR3    MR4   
x6   0.201 -0.408         0.463
x7   0.290  0.656  0.267  0.210
x8   0.278 -0.382  0.744 -0.169
x9   0.862        -0.255 -0.184
x10  0.287  0.456         0.127
x11  0.689 -0.454 -0.141  0.316
x12  0.398  0.807  0.348  0.255
x13 -0.231  0.553        -0.287
x14  0.378 -0.322  0.730 -0.151
x16  0.747        -0.176 -0.181
x18  0.895        -0.304 -0.198

                 MR1   MR2   MR3   MR4
SS loadings    3.215 2.226 1.500 0.679
Proportion Var 0.292 0.202 0.136 0.062
Cumulative Var 0.292 0.495 0.631 0.693
```

Factor extraction

Loadings

- $\leq \pm 0.10 \approx$ zero
- ± 0.10 to ± 0.40 meet the minimal level
- $\geq \pm 0.50$ practically significant
- $\geq \pm 0.70 \approx$ well-defined structure

SS loadings

- Eigenvalues - column sum of squared factor loadings.
- Relative importance of each factor in accounting for the variance associated with the set of variables.

```
fa_unrotated <- fa(r = data, nfactors = 4,r
print(fa_unrotated$loadings)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x6   0.201 -0.408         0.463
x7   0.290  0.656  0.267  0.210
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Proportion Var 0.292 0.202 0.136 0.062
Cumulative Var 0.292 0.495 0.631 0.693
```
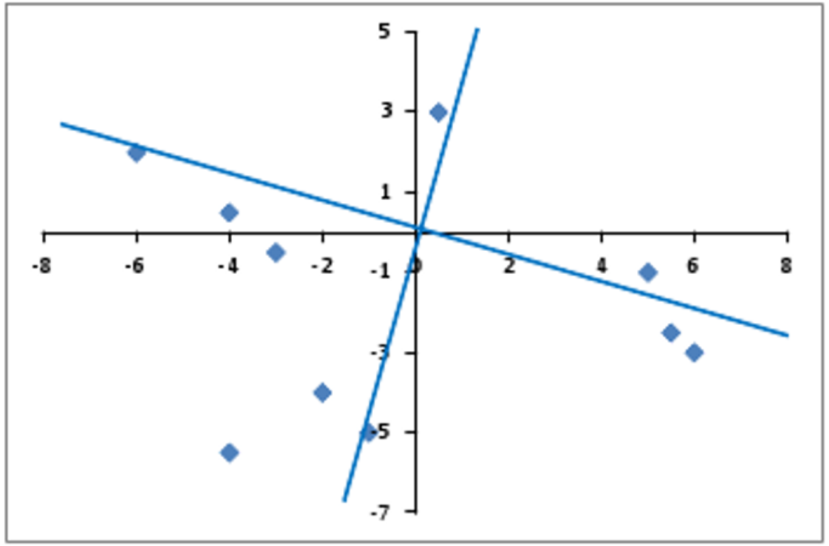

Factor rotation

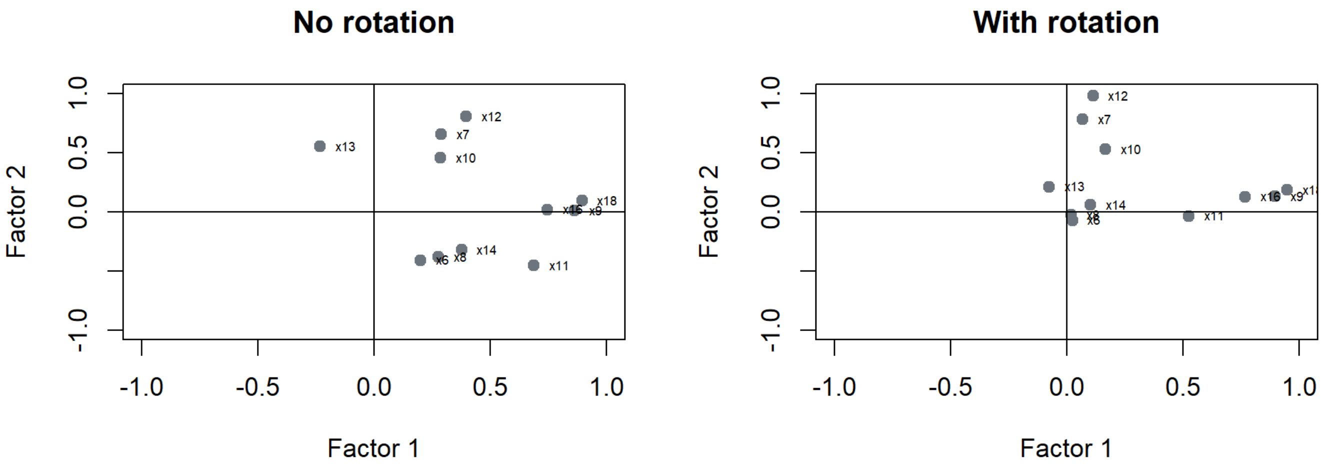
Why do factor rotation?

- To simplify the complexity of factor loadings.
- Distribute the loadings more clearly into the factors.
- Facilitate interpretation.



Factor rotation

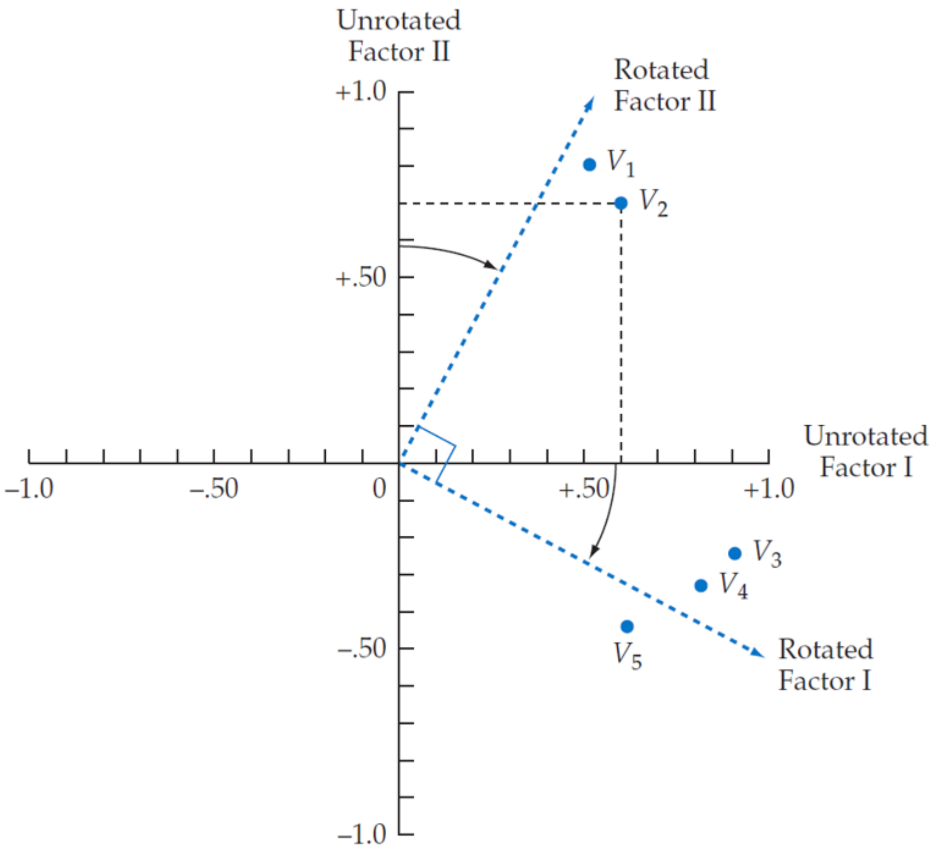
```
par(mfrow = c(1, 2))
plot(fa_unrotated$loadings[,1]
     xlab = "Factor 1", ylab =
     ylim = c(-1, 1), xlim = c
     main = "No rotation",
     pch = 19, col = "#6c757d"
     abline(h=0, v=0)
     text(fa_unrotated$loading
          labels = rownames(fa
          pos = 4, cex = 0.5)

plot(fa_rotated$loadings[,1],
     xlab = "Factor 1", ylab =
     ylim = c(-1, 1), xlim = c
     main = "With rotation",
     pch = 19, col = "#6c757d"
     abline(h=0, v=0)
     text(fa_rotated$loadings[
          labels = rownames(fa
          pos = 4, cex = 0.5)
```



Factor rotation

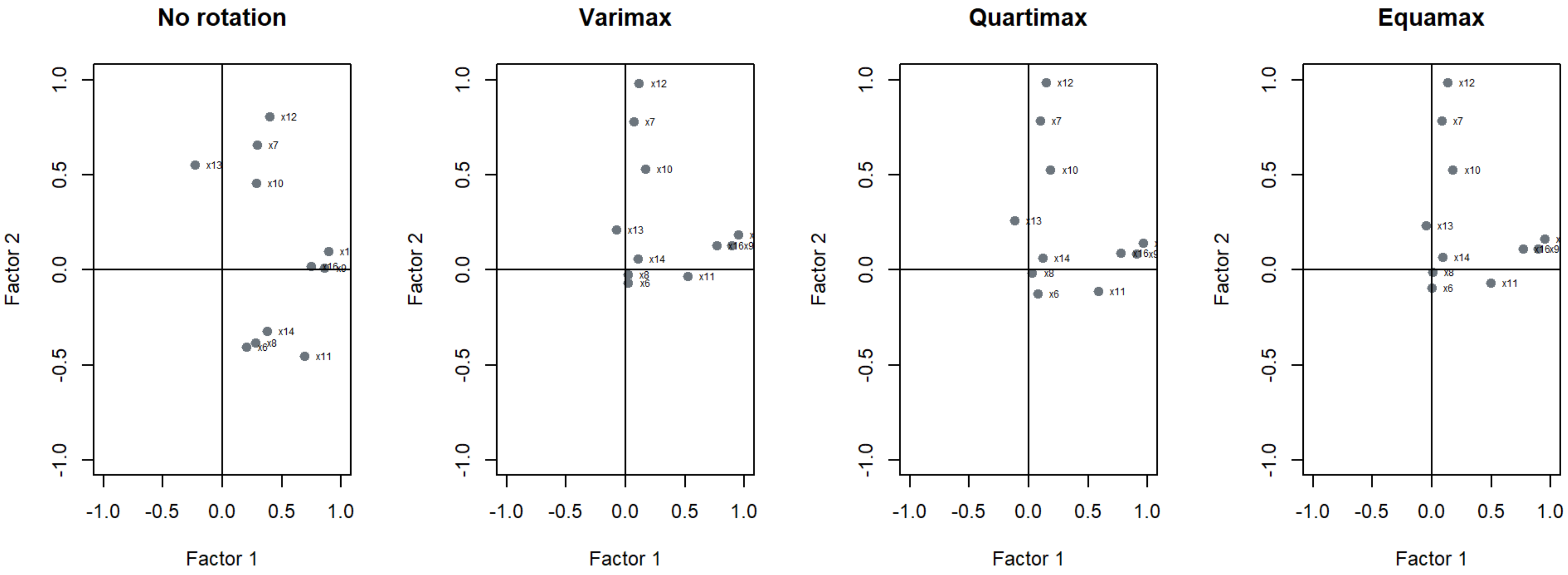
Orthogonal rotation

- axes are maintained at 90 degrees
- orthogonal rotation methods
  - Varimax - *most commonly used*
  - Quartimax
  - Equimax
- Check-out some of these references
  - IBM
  - Factor analysis



Factor rotation

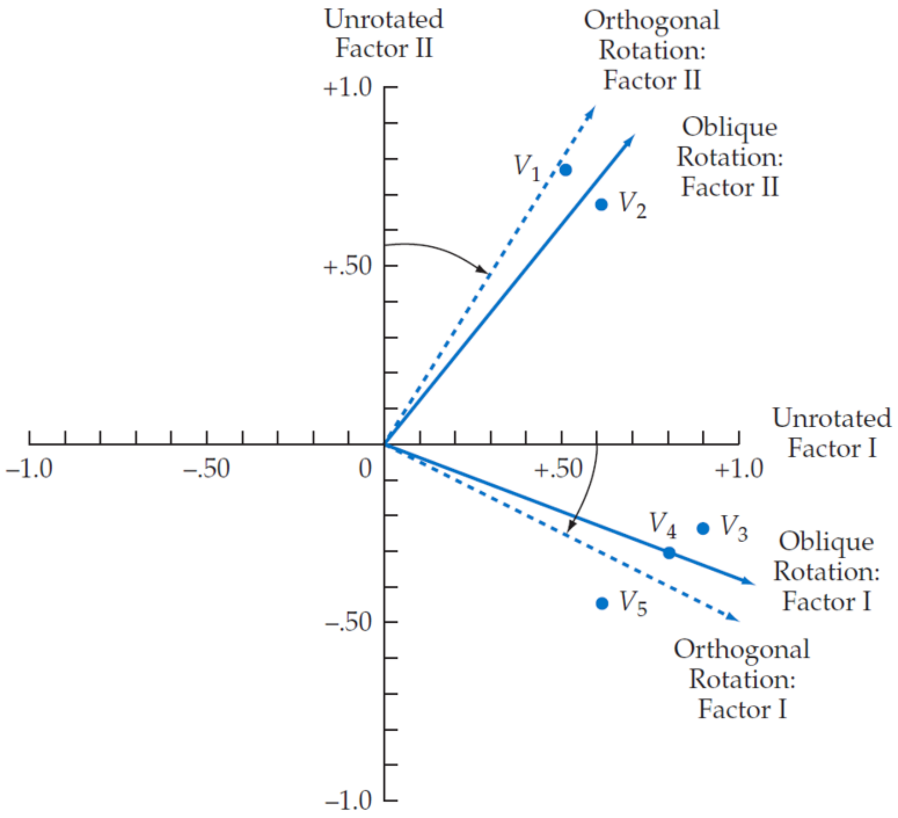
Orthogonal rotation



Factor rotation

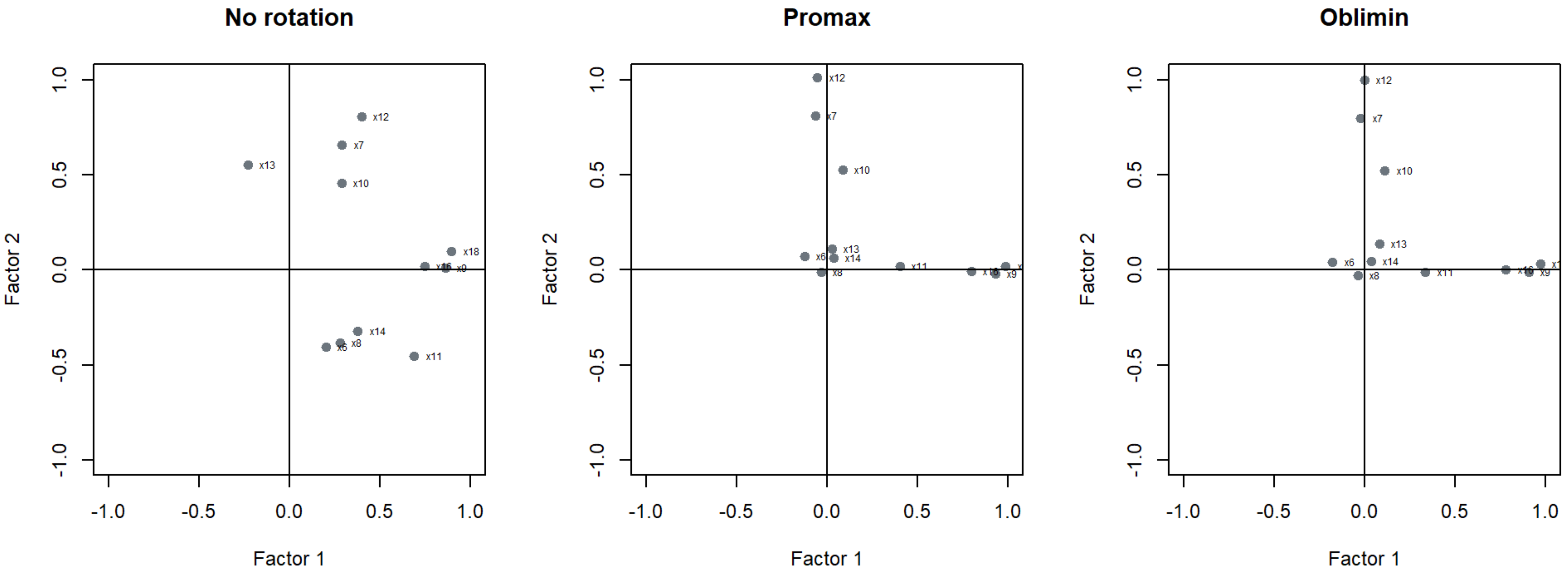
Oblique rotation rotation

- allow correlated factors
- suited to the goal of theoretically meaningful constracts
- oblique rotation methods
    - Promax
    - Oblimin
- Note: no specific rules in selecting a between rotation method.



Factor rotation

Oblique rotation



Let's practice!

Factor interpretation and respecification

- each variable has a high loadings on one factor only
- each factor has a high loadings for only a subset of the items.

```
fa_varimax <- fa(r = data, nfactors = 4, rotate = "varimax
print(fa_varimax$loadings, sort = TRUE)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x9   0.897  0.130         0.132
x16  0.768  0.127
x18  0.949  0.185
x7          0.781        -0.115
x10  0.166  0.529
x12  0.114  0.980        -0.133
x8                 0.890  0.115
x14  0.103         0.879  0.129
x6                        0.647
x11  0.525         0.127  0.712
x13         0.213 -0.209 -0.590

                 MR1   MR2   MR3   MR4
SS loadings    2.635 1.973 1.641 1.371
Proportion Var 0.240 0.179 0.149 0.125
Cumulative Var 0.240 0.419 0.568 0.693
```

Factor interpretation and respecification

- each variable has a high loadings on one factor only
- each factor has a high loadings for only a subset of the items.

```
fa_varimax <- fa(r = data, nfactors = 4, rotate = "varimax
print(fa_varimax$loadings, sort = TRUE, cutoff = 0.4)
```

```
Loadings:
    MR1    MR2    MR3    MR4
x9   0.897
x16  0.768
x18  0.949
x7          0.781
x10         0.529
x12         0.980
x8                 0.890
x14                0.879
x6                        0.647
x11  0.525                0.712
x13                      -0.590

                 MR1   MR2   MR3   MR4
SS loadings    2.635 1.973 1.641 1.371
Proportion Var 0.240 0.179 0.149 0.125
Cumulative Var 0.240 0.419 0.568 0.693
```

Factor interpretation and respecification

What to do with cross-loadings?

Ratio of variance (*JF Hair et al. 2019*)

- 1 to 1.5 - problematic
- 1.5 to 2.0 - potential cross-loading
- 2.0 and higher - ignorable

Example:

- X_{11}
- `MR1`: 0.525
- `MR2`: 0.712
- $0.712^2 \div 0.525^2 = 1.8$

```
fa_varimax <- fa(r = data, nfactors = 4, rotate = "varimax
print(fa_varimax$loadings, sort = TRUE, cutoff = 0.4)
```

```

Loadings:
    MR1    MR2    MR3    MR4
x9   0.897
x16  0.768
x18  0.949
x7          0.781
x10         0.529
x12         0.980
x8                 0.890
x14                0.879
x6                        0.647
x11  0.525                0.712
x13                      -0.590

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```

Let's practice!