Structural Equation Modeling (SEM) using R

Topic overview

1: CFA-SEM overview

2: CFA-SEM with Lavaan

3: Defining constructs

4: Developing the overall measurement model

5: Assessing measurement model validity

6: Specifying the structural model

7: Assessing structural model validity

CFA-SEM overview

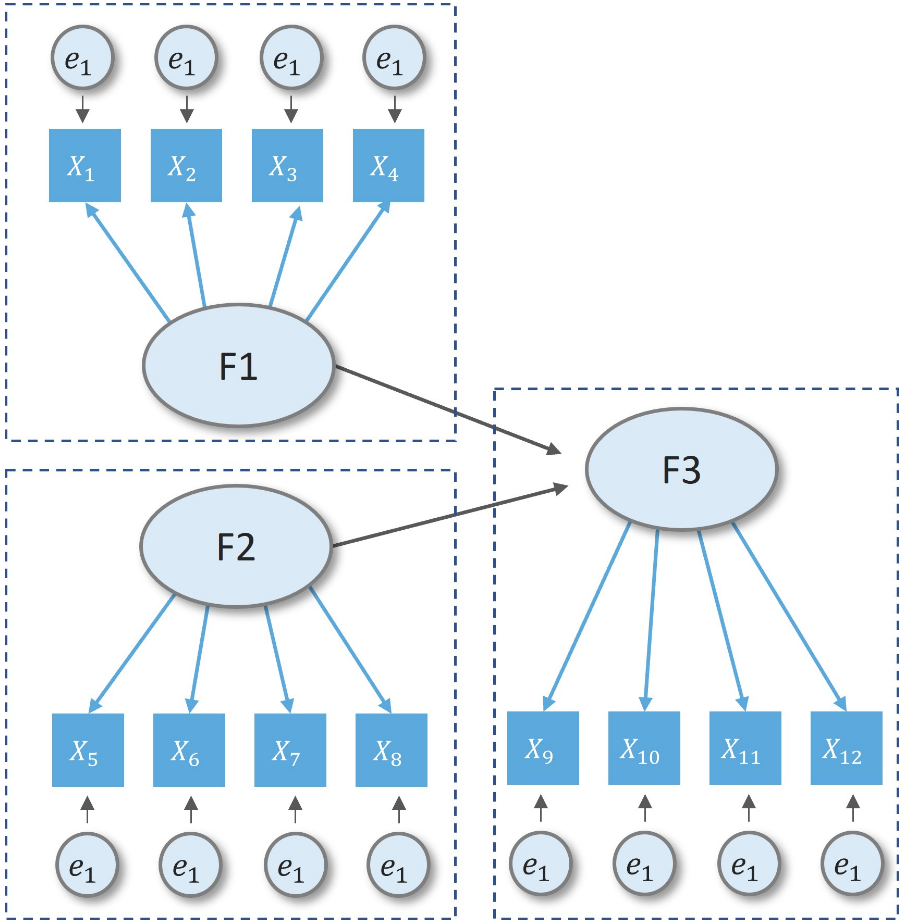
What is SEM?

- Not a one statistical "technique"
- Integrates a number of different multivariate technique
    - Factor analysis
    - Regression
    - Simultaneous equation
- Distinction between:
    - measurement model
    - structural model

What is SEM?

Measurement model

- measurement part of a a full SEM model
- confirmatory factor analysis



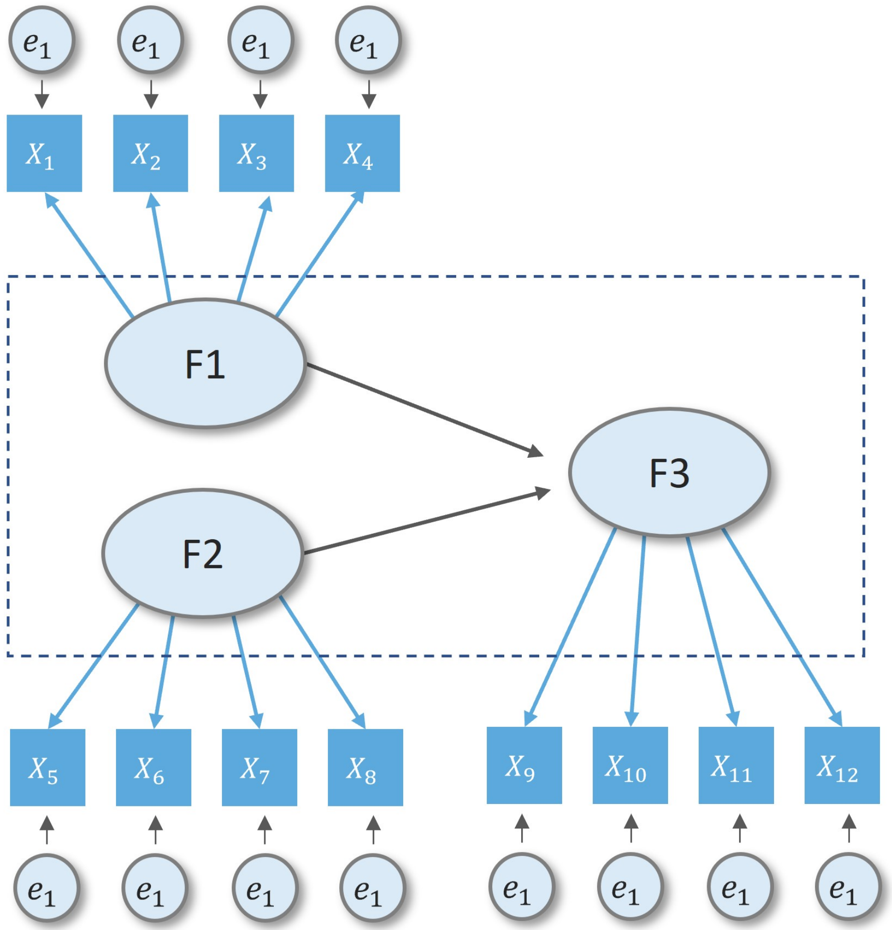
What is SEM?

Measurement model

- measurement part of a a full SEM model
- confirmatory factor analysis

Structural model

- relationship between constucts
- full sem model is combination of measurement and structural component



Basic SEM conventions

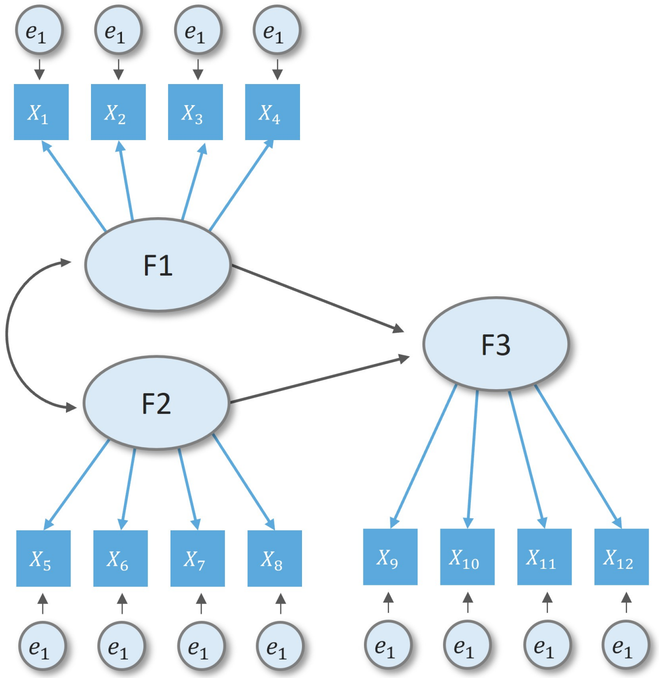
F1 measured latent variable

X_1 observed / manifest variable

e_1 error variance / disturbance term

Covariance / non-directional path

Regression / directional path



2. CFA-SEM with Lavaan R package

What is Lavaan?

- *"developed to provide useRs, researchers, and teachers a free open-source, but commercial quality"*, Yves Rosseel (2012)
- Check-out their lavaan tutorial

```
install.packages("lavaan")
library(lavaan)
example(cfa)
```

```
cfa> ## The famous Holzinger and Swineford (1939) example
cfa> HS.model <- ' visual  =~ x1 + x2 + x3
cfa+              textual =~ x4 + x5 + x6
cfa+              speed   =~ x7 + x8 + x9 '

cfa> fit <- cfa(HS.model, data = HolzingerSwineford1939)

cfa> summary(fit, fit.measures = TRUE)
lavaan 0.6-9 ended normally after 35 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        21
```

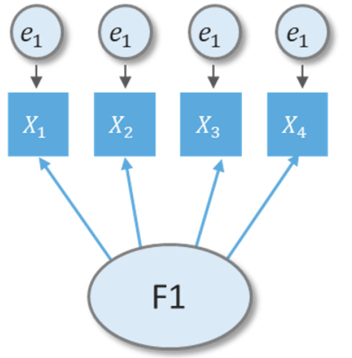
Major operators of lavaan syntax

Command	Operator	Illustration	Significance
Estimate covariance	~~	X ~~ Y	X is correlated with Y
Estimate regression	~	Y ~ X	Y is regressed on X
Define a reflective latent variable	= ~	F =~ item_1 + item_2 + item_3	The F factor is measured by indicators item 1, item 2, and item 3 over which it has effects
Label a parameter	*	F =~ b1*item_1 + b2*item2 + b3*item3	Item 1-3 is named “b1”, “b2”, and “b3”, respectively.
Create a new parameter	:=	B1b2 := b1*b2	Define a parameter that is not in the model. For example: b1b2 = indirect effect of b1 and b2
Insert a comment in the syntax	#	#indirect effects B1b2 := b1*b2	Explain to the reader the meaning of a command.

Major operators of lavaan syntax

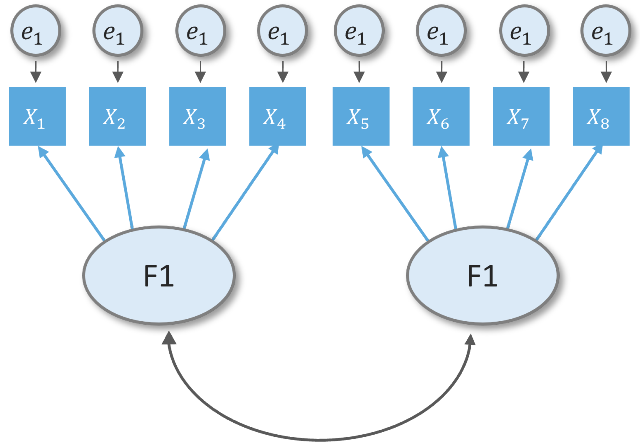
Defining a reflective latent variable

```
model <- "F1 =~ x1 + x2 + x3 + x4"
```



Estimate factor covariance

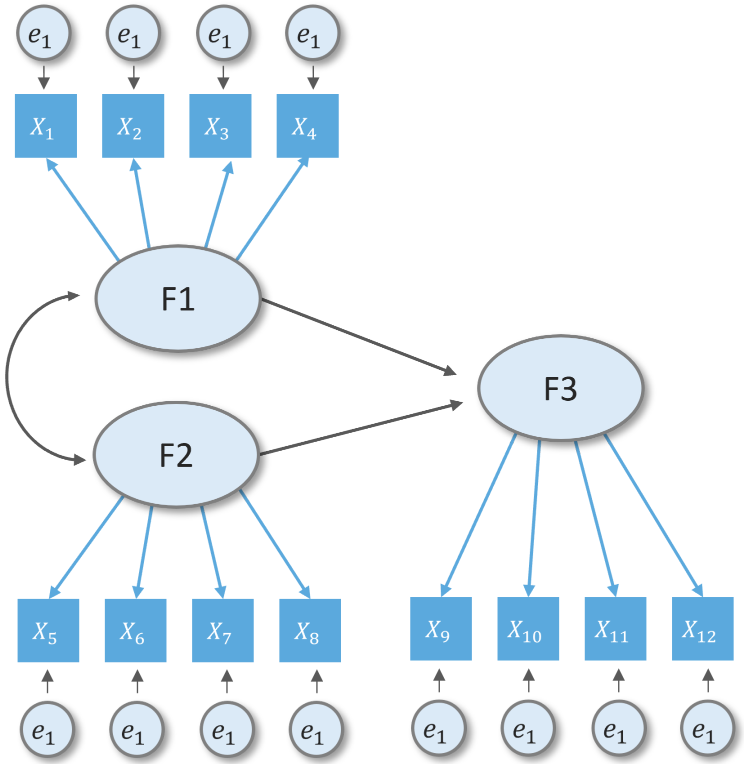
```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x6 + x8
          F1 ~~ F2"
```



Major operators of lavaan syntax

Estimate regression

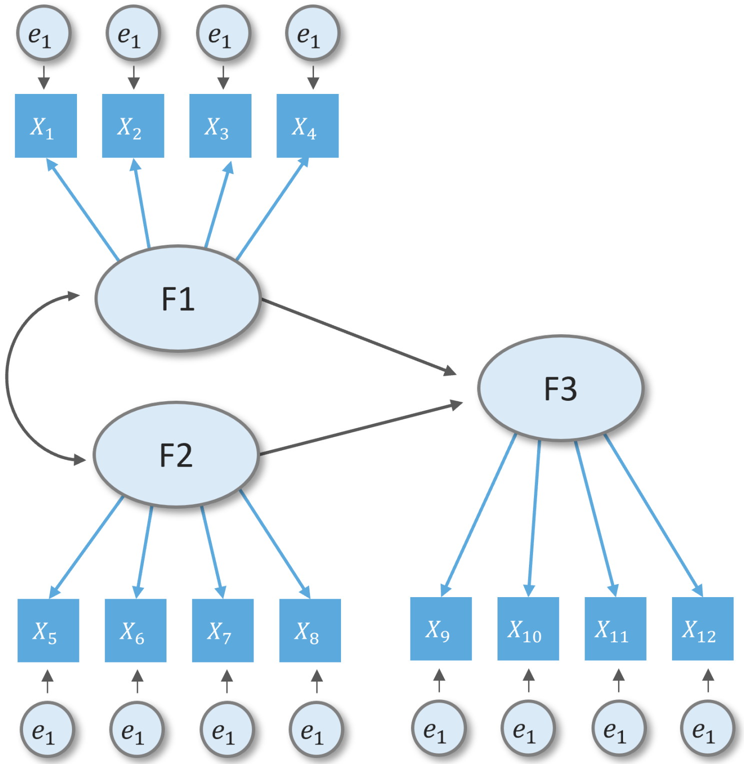
```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12
          F1 ~~ F2
          F3 ~ F1 + F2"
```



Major operators of lavaan syntax

Insert a comment in the syntax

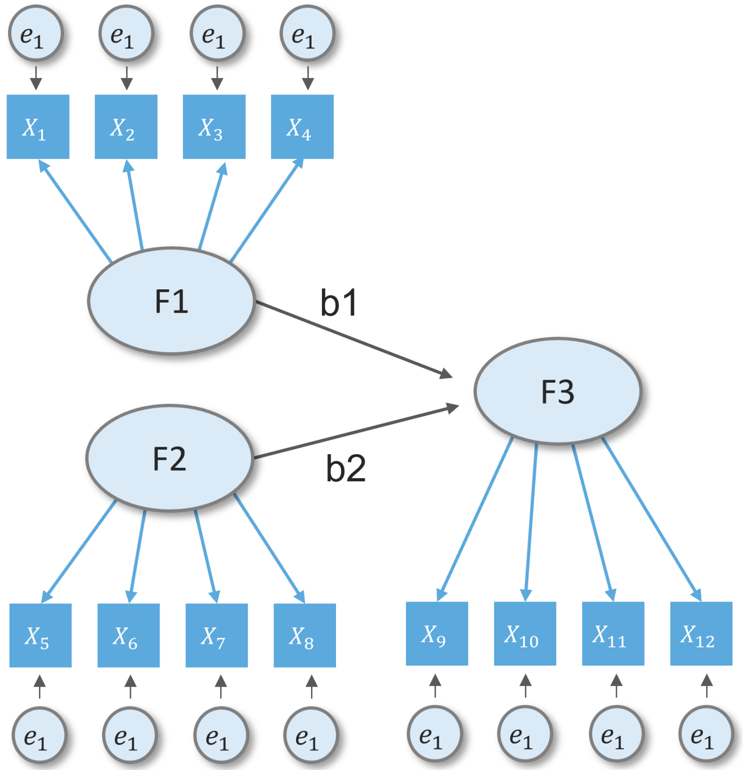
```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12

          # covariance
          F1 ~~ F2

          # F3 is regressed on F1 and F2
          F3 ~ F1 + F2"
```



Major operators of lavaan syntax

Label a parameter

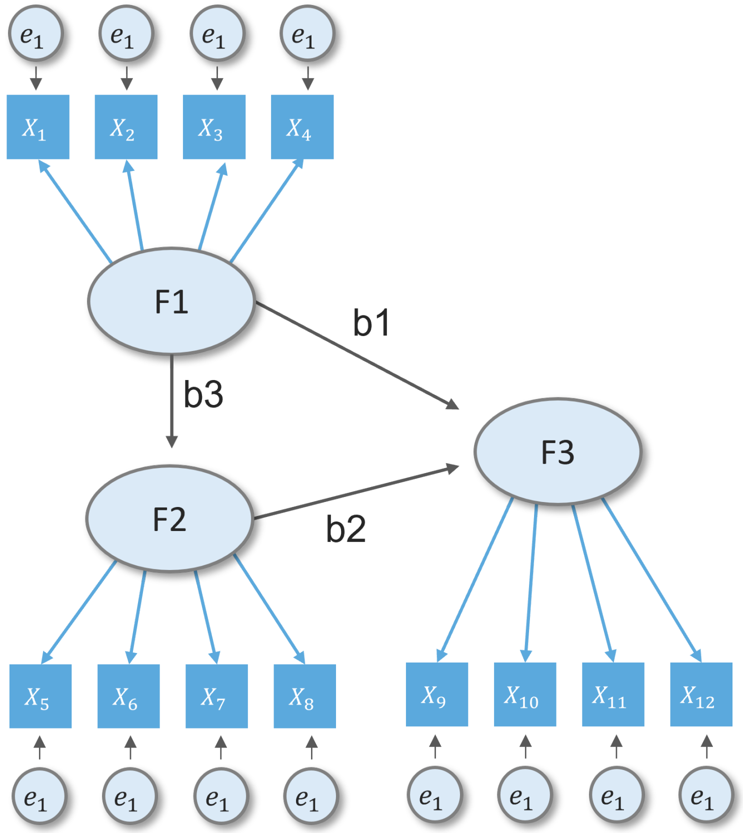
```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12

          # covariance
          F1 ~~ F2

          # F3 is regressed on F1 and F2
          F3 ~ b1*F1 + b2*F2"
```



Major operators of lavaan syntax

Create a new parameter

```
model <- "F1 =~ x1 + x2 + x3 + x4
          F2 =~ x5 + X6 + x7 + x8
          F3 =~ x9 + X10 + x11 + x12

          # regression
          F3 ~ b1*F1 + b2*F2
          F2 ~ b3*F1

          # F1 indirect effect
          ie := b3*b2

          # F1 total effect
          te := b3*b2 + b1"
```



Main steps in SEM

Main steps in SEM

1. Defining constructs
2. Developing the overall measurement model
3. Assessing measurement model validity
4. Specifying the structural model
5. Assessing structural model validity

1. Defining Constructs

Dataset

- HBAT company
- HBAT is interested in understanding what affects employee's attitudes and behaviors that contributes to employee's retension.

JS1	OC1	OC2	EP1	OC3	OC4	EP2	EP3	AC1	EP4
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
5	3	5	10	10	10	10	5	1	2
3	0	5	10	3	7	10	10	2	7
4	6	10	10	10	10	10	10	1	7
4	7	7	10	10	7	10	9	2	7
5	2	10	10	9	9	9	10	1	6
6	5	8	8	7	7	10	7	1	7
2	6	10	9	10	9	9	9	2	6
2	4	9	10	9	7	10	10	1	7
4	9	10	8	10	10	6	8	3	3
5	5	9	10	9	10	10	8	2	7

1-10 of 400 rows | 1-1... Previous **1** 2 3 4 5 6 ... 40 Next

Defining individual constructs

- Based on literature and preliminary interviews, a study was designed focusing on five key constructs.
    - *Job satisfaction (JS)* : reactions resulting from an appraisal of one's job situation.
    - *Organizational commitment (OC)*: extent to which an employees indentifies and feels part of HBAT.
    - *Staying intention (SI)*: extent to which an employee intends to continue working for HBAT.
    - *Environmental perceptions (EP)*: beliefs an employee has about day-to-day, physical working conditions.
    - *Attitudes towards cowrokers (AC)*: attitudes an employee has toward the coworkers he/she interacts with on a regular basis.

Defining individual constructs

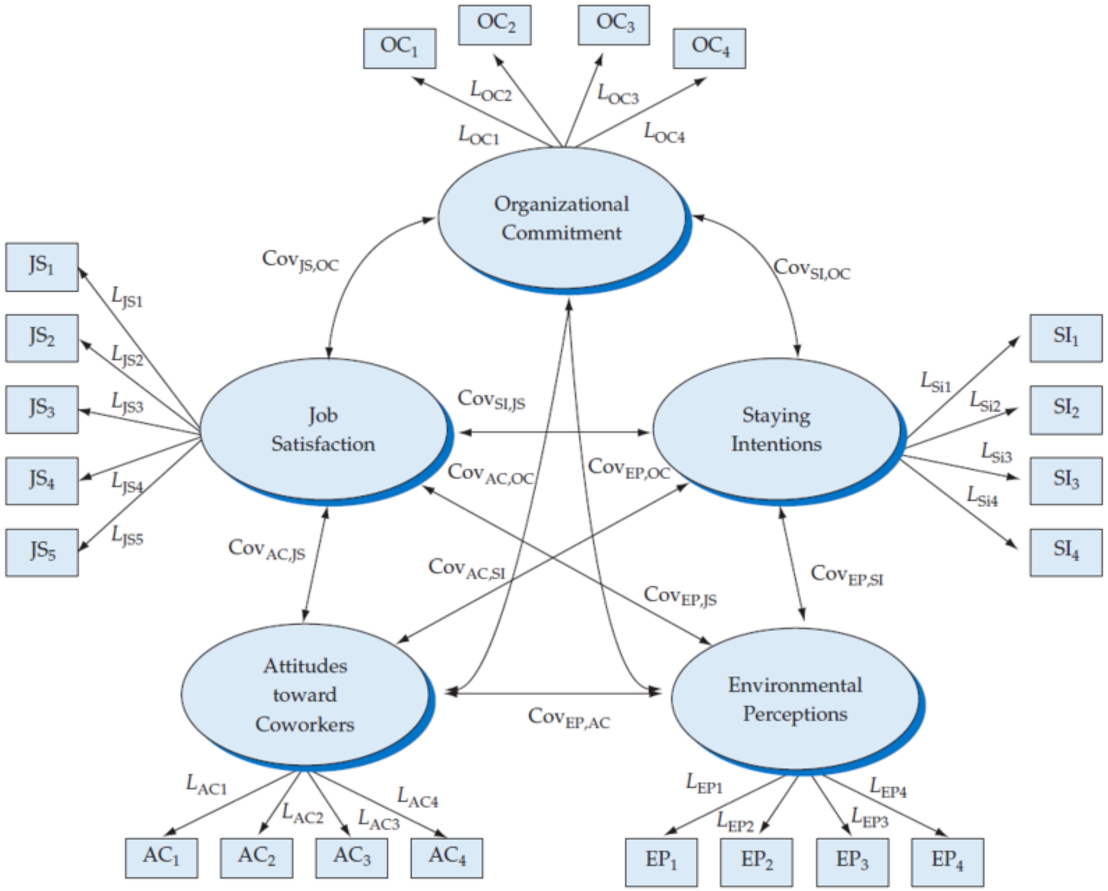
Item	Scale Type	Description	Construct
JS_1	0–10 Likert Disagree–Agree	All things considered, I feel very satisfied when I think about my job.	JS
OC_1	0–10 Likert Disagree–Agree	My work at HBAT gives me a sense of accomplishment.	OC
OC_2	0–10 Likert Disagree–Agree	I am willing to put in a great deal of effort beyond that normally expected to help HBAT be successful.	OC
EP_1	0–10 Likert Disagree–Agree	I am comfortable with my physical work environment at HBAT.	EP
OC_3	0–10 Likert Disagree–Agree	I have a sense of loyalty to HBAT.	OC
OC_4	0–10 Likert Disagree–Agree	I am proud to tell others that I work for HBAT.	OC
EP_2	0–10 Likert Disagree–Agree	The place I work in is designed to help me do my job better.	EP
EP_3	0–10 Likert Disagree–Agree	There are few obstacles to make me less productive in my workplace.	EP
AC_1	5-point Likert	How happy are you with the work of your coworkers? ___ Not happy ___ Somewhat happy ___ Happy ___ Very happy ___ Extremely happy	AC
EP_4	7-point Semantic Differential	What term best describes your work environment at HBAT? Too hectic ________ Very soothing	EP
JS_2	7-point Semantic Differential	When you think of your job, how satisfied do you feel? Not at all satisfied ________ Very much satisfied	JS
JS_3	7-point Semantic Differential	How satisfied are you with your current job at HBAT? Very unsatisfied ________ Very satisfied	JS
AC_2	7-point Semantic Differential	How do you feel about your coworkers? Very unfavorable ________ Very favorable	AC
SI_1	5-point Likert Disagree–Agree	I am not actively searching for another job. Strongly disagree ________ Strongly agree	SI
JS_4	5-point Likert	How satisfied are you with HBAT as an employer? ___ Not at all ___ Little ___ Average ___ A lot ___ Very much	JS
SI_2	5-point Likert Disagree–Agree	I seldom look at the job listings on monster.com. Strongly disagree ________ Strongly agree	SI
JS_5	Percent Satisfaction	Indicate your satisfaction with your current job at HBAT by placing a percentage in the blank, with 0% = Not satisfied at all, and 100% = Highly satisfied. ________	JS
AC_3	5-point Likert	How often do you do things with your coworkers on your days off?___ Never ___ Rarely ___ Occasionally ___ Often ___ Very often	AC
SI_3	5-point Likert Disagree–Agree	I have no interest in searching for a job in the next year. Strongly disagree ________ Strongly agree	SI
AC_4	6-point Semantic Differential	Generally, how similar are your coworkers to you? Very different ________ Very similar	AC
SI_4	5-point Likert	How likely is it that you will be working at HBAT one year from today? ___Very unlikely ___ Unlikely ___ Somewhat likely ___ Likely ___ Very likely	SI

Source: JF Hair et al. (2019) : Multivariate data analysis

Step 2. Developing overall measurement model

Developing overall measurement model

- Measurement theory model (CFA) for HBAT employees
- Direction of the relationship between factors is not yet defined.
- Focus on confirming the specified model with empirical model (using empirical data), hence confirmatory.



Source: JF Hair et al. (2019): Multivariate data analysis

Let's practice!

Step 3. Assessing measurement model validity

Basic principles

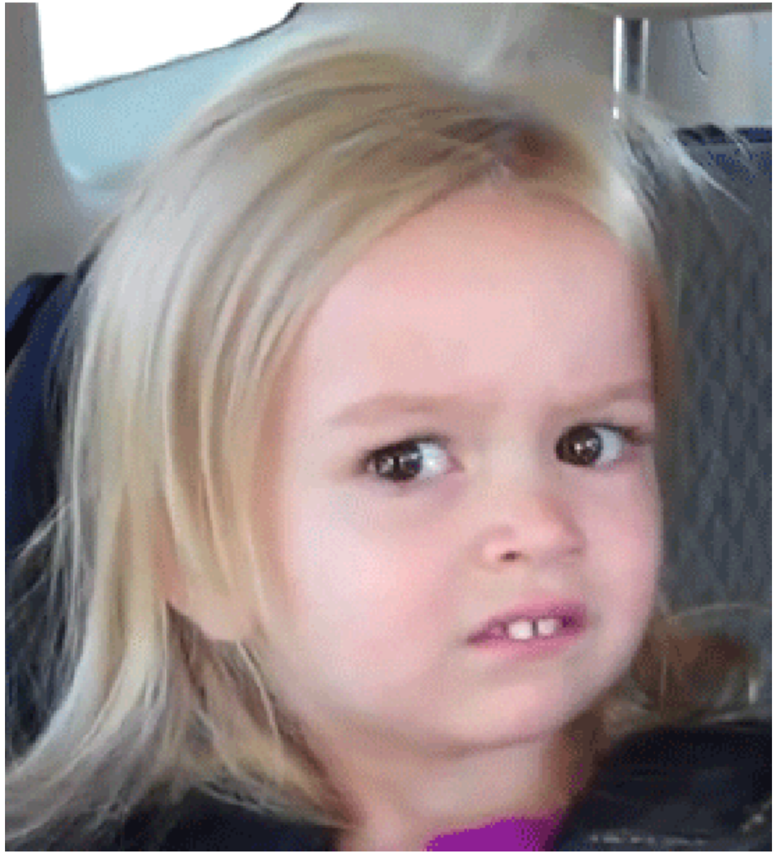
- Compare covariance matrix of the research data S and reproduced covariance Σ
- Hypothesis:
  - Null: $\mathrm{S} = \Sigma$
  - Alternative: $\mathrm{S} \neq \Sigma$
- Idea is to arrived with a parameter that minimizes the difference of S and Σ

```
cfa_fit <- cfa(cfa_model, data = hbat_data)
cfa_fit %>% summary()
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
  P-value (Chi-square)                           0.001

Parameter Estimates:
```

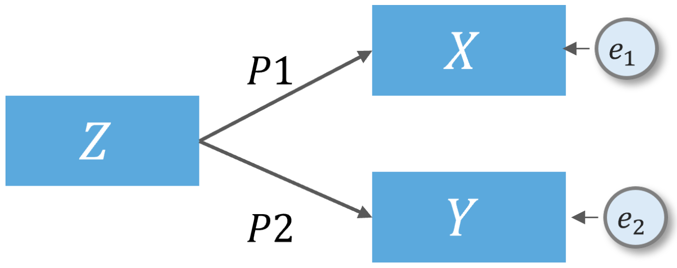
Basic principles

- Compare covariance matrix of the research data S and reproduced covariance Σ
- Hypothesis:
  - Null: $\mathrm{S} = \Sigma$
  - Alternative: $\mathrm{S} \neq \Sigma$
- Idea is to arrived with a parameter that minimizes the difference of S and Σ



Basic principles

- To understand the SEM process, consider the Table on the right.
- e.g., iterative procedure using least square method.



	Parameters		$r_{XZ} = 0.79$	$r_{YZ} = 0.59$	$r_{XY} = 0.49$	$\sum d^2$
Iteration cycles	**P1**	**P2**	**Reproduced correlations**			**Least squares**
1	0.50	0.50	0.50	0.50	0.250	0.149
1a	0.49	0.49	0.49	0.49	0.240	0.162
1b	0.49	0.50	0.49	0.50	0.245	0.158
1c	0.50	0.49	0.50	0.49	0.245	0.158
2	0.55	0.55	0.55	0.55	0.300	0.094
2a	0.60	0.60	0.60	0.60	0.360	0.024
3	0.65	0.61	0.65	0.61	0.400	0.027
3a	0.65	0.62	0.65	0.62	0.403	0.028
4	0.67	0.61	0.67	0.61	0.408	0.021
4a	0.70	0.61	0.70	0.61	0.427	0.012
4b	0.75	0.61	0.75	0.61	0.457	0.003
4c	0.80	0.61	0.80	0.61	0.480	0.0006
5	0.81	0.61	0.81	0.61	0.494	0.0008

Summary output

- Overall results
- Loadings
- Variances

```
cfa_fit <- cfa(cfa_model, data = hbat_data)
summary(cfa_fit)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
  P-value (Chi-square)                           0.001

Parameter Estimates:
```

Summary output

Overall results

- Degrees of freedom (df)
  - $df = \frac{1}{2}p(p+1) - k$
  - p = total observed variables
  - k = total estimated parameters
- Identification
  - Include at least three manifest variables
  - Create models with $df > 0$

```
cfa_fit <- cfa(cfa_model, data = hbat_data)
summary(cfa_fit)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
  P-value (Chi-square)                           0.001

Parameter Estimates:
```

Summary output

Loadings

- Measures the strength of the relationship between items and factor.

```r
cfa_fit <- cfa(cfa_model, data = hbat_data)
summary(cfa_fit, standardized = TRUE)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
  P-value (Chi-square)                           0.001

Parameter Estimates:
```

Summary output

Variances

Refer to unique variance that the factor unable to account for. Similar to error term in OLS, hence it is also term as error variance.

```
cfa_fit <- cfa(cfa_model, data = hbat_data)
summary(cfa_fit, standardized = TRUE)
```

```
lavaan 0.6-9 ended normally after 54 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        52

  Number of observations                           400

Model Test User Model:

  Test statistic                               240.738
  Degrees of freedom                               179
  P-value (Chi-square)                           0.001

Parameter Estimates:
```

Fit indices

Goodness of fit indices

- Goodness-of-fit index (GFI)
- Adjusted goodness-fit-index (AGFI)
- Comparative fit index (CFI)
- Normed fit index (NFI)
- Non-normed fit index (NNF)

Badness of fit indices

- Standard root mean square of the residuals (SRMR)
- Root mean square error of approximation (RMSEA)

Table 3. Goodness of fit of the measurement model.

Fit indices	Recommended value	Sources	Research model
χ^2	–	–	369.4
df	–	–	120
χ^2/df	<5	Bollen (1989)	3.08
GFI	>0.9	Scott (1995)	0.91
AGFI	>0.8	Scott (1995)	0.87
SRMR	<0.1	Hu and Bentler (1999)	0.034
CFI	>0.9	Bagozzi and Yi (1988)	0.96
RMSEA	<0.08	MacCallum *et al.* (1996)	0.071
NFI	>0.9	Bentler and Bonett (1980)	0.95
NNFI	>0.9	Bentler and Bonett (1980)	0.95

Sample GOF results from W. Shiau & M. Luo (2013). Continuance intention of blog users: The impact of perceived enjoyment, habit, user involvement and blogging time.

Fit indices

Goodness of fit indices

- Goodness-of-fit index (GFI)
- Adjusted goodness-fit-index (AGFI)
- Comparative fit index (CFI)
- Normed fit index (NFI)
- Non-normed fit index (NNF)

```
fitMeasures(cfa_fit)
```

```
               npar                fmin                chisq
             52.000               0.301              240.738
             pvalue      baseline.chisq          baseline.df
              0.001            4452.567              210.000
                cfi                 tli                  nnfi
              0.985               0.983                0.983
                nfi                pnfi                   ifi
              0.946               0.806                0.986
               logl   unrestricted.logl                  aic
         -13916.782          -13796.413            27937.564
             ntotal                bic2                rmsea
            400.000           27980.120                0.029
     rmsea.ci.upper        rmsea.pvalue                  rmr
              0.039               1.000                0.414
               srmr        srmr_bentler  srmr_bentler_nomean
              0.036               0.036                0.036
        crmr_nomean         srmr_mplus    srmr_mplus_nomean
              0.037               0.036                0.036
              cn_01                 gfi                 agfi
            376.401               0.947                0.932
```

Fit indices

Goodness of fit indices

- Goodness-of-fit index (GFI)
- Adjusted goodness-fit-index (AGFI)
- Comparative fit index (CFI)
- Normed fit index (NFI)
- Non-normed fit index (NNF)

```
fitMeasures(cfa_fit,  fit.measures = c("gfi", "agfi", "cfi
```

```
  gfi  agfi   cfi   nfi  nnfi
0.947 0.932 0.985 0.946 0.983
```

Fit indices

Badness of fit indices

- Standard root mean squrare residual (SRMR)
- Root mean square error of approximation (RMSEA)

```
fitMeasures(cfa_fit,  fit.measures = c("srmr", "rmsea"))
```

```
 srmr rmsea
0.036 0.029
```

Reliability and validity test

Reliability test

- Composite reliability

Validity test

- Convergent validity
- Discriminant validity

	α	CR	AVE	CU	TD	FI	HE	IN	INTE	SA
CU	0.94	0.96	0.89	0.95[a]						
TD	0.88	0.93	0.81	0.58	0.90					
FI	0.92	0.94	0.76	0.74	0.78	0.87				
HE	0.94	0.96	0.89	0.79	0.59	0.74	0.94			
IN	0.88	0.92	0.74	0.51	0.48	0.51	0.59	0.86		
INTE	0.88	0.92	0.80	0.61	0.54	0.60	0.66	0.70	0.89	
SA	0.88	0.93	0.80	0.41	0.38	0.44	0.56	0.59	0.59	0.90

Notes: α, Cronbach's α; CR, composite reliability. CU, CUriosity; HE, Heightened Enjoyment; TD: Temporal Dissociation; FI: Focused Immersion; IN: INteractivity; INTE: INTEreat; SA: SAtisfaction.
[a]The square root of AVE

Source: A. Hou, W. Shiau, & R. Shang (2019). The involvement paradox. The role of cognitive absorption in mobile instant messaging user satisfaction.

Reliability and validity test

- Composite reliability: `alpha` > `0.70`
- Convergent validity: AVE (`avevar`) > `0.50`
- Discriminant validity: `omega` > `0.7`

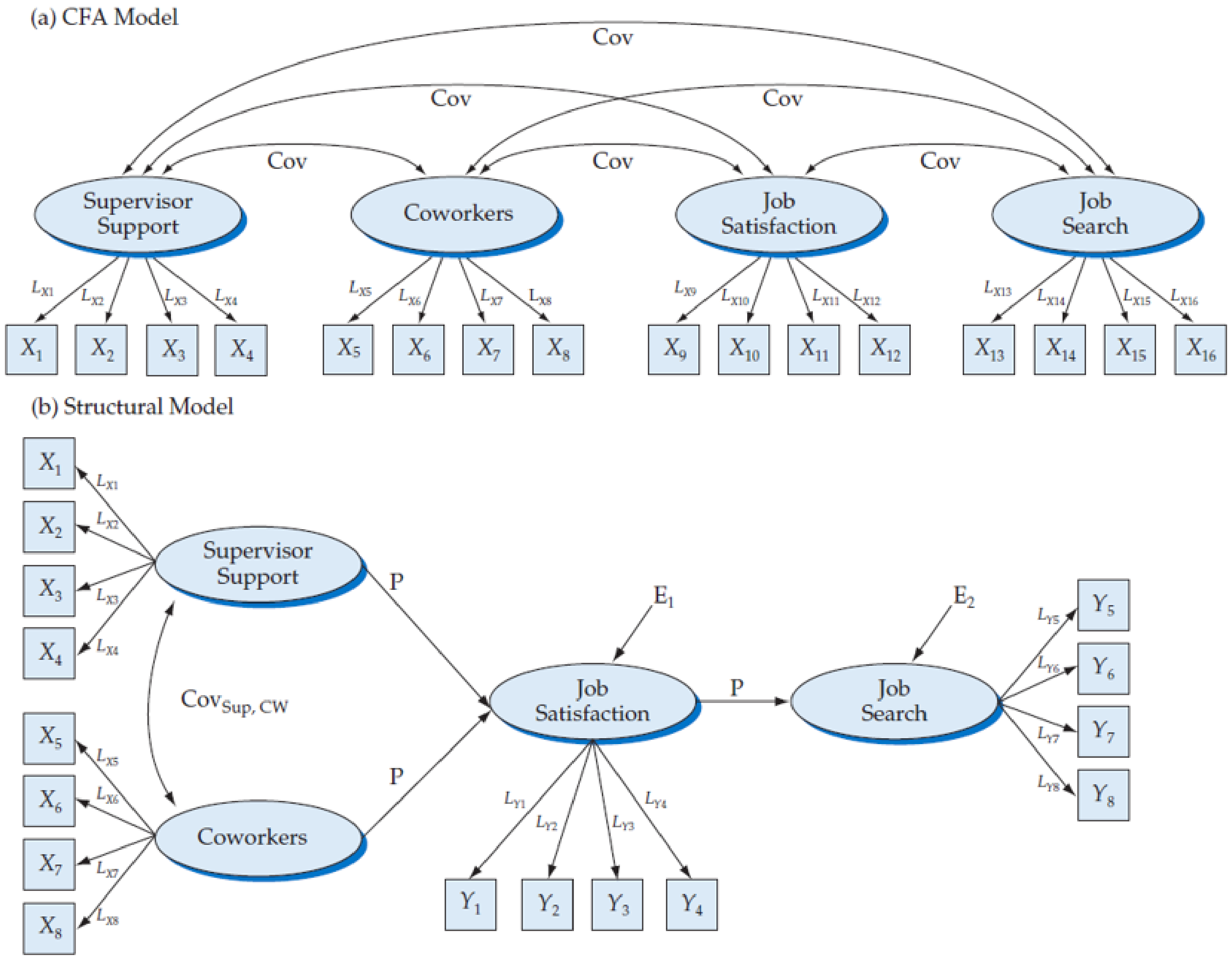
```r
library(semTools)
reliability(cfa_fit) %>% round(3)
```

```
          SI    JS    AC    EP    OC
alpha  0.886 0.281 0.891 0.847 0.823
omega  0.887 0.640 0.893 0.850 0.827
omega2 0.887 0.640 0.893 0.850 0.827
omega3 0.887 0.641 0.893 0.850 0.818
avevar 0.664 0.535 0.677 0.587 0.552
```

Let's practice

Step 4: Specifying the structural model

CFA model to structural model

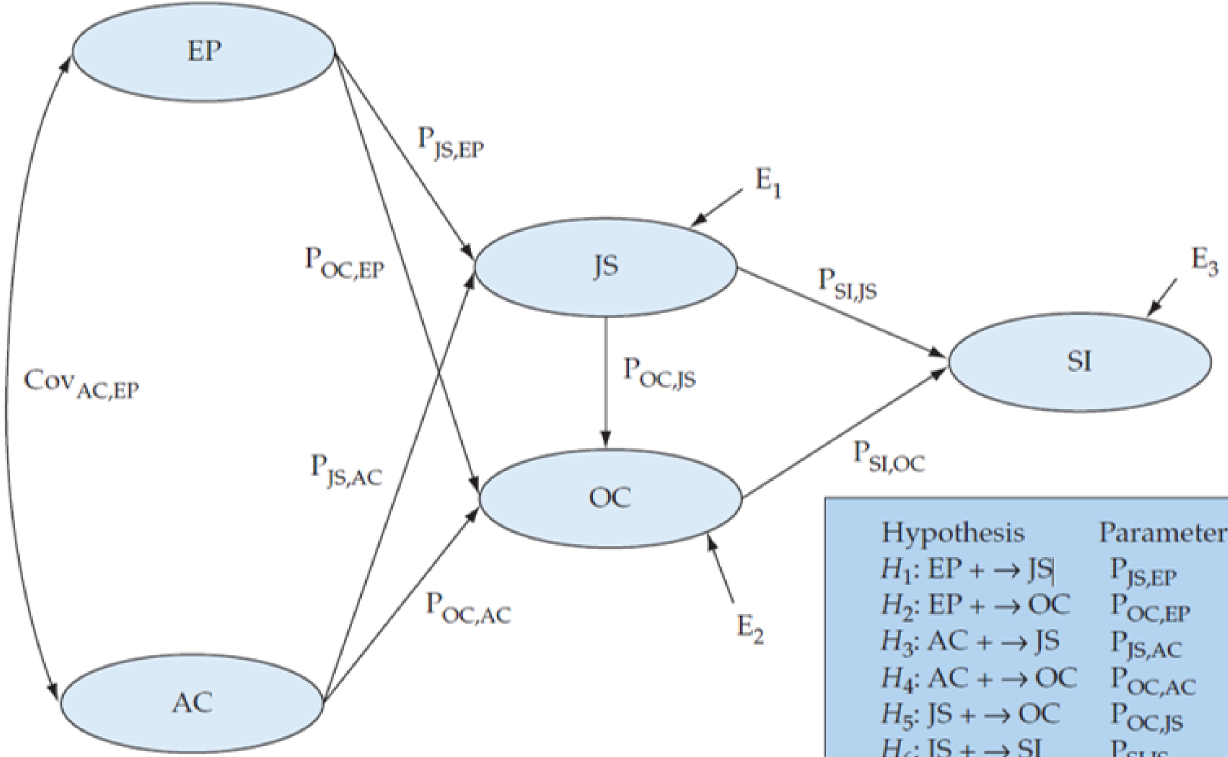


Defining structural model

Hypothesis:

- H1: Environmental perceptions are positively related to job satisfaction.
- H2: Environmental perceptions are positively related to organizational commitment.
- H3: Attitudes toward coworkers are positively related to job satisfaction.
- H4: Attitudes toward coworkers are positively related to organizational commitment.
- H5: Job satisfaction is related positively to organizational commitment.
- H6: Job satisfaction is related positively to staying intentions.
- H7: Organizational commitment is related positively to staying intention.

Defining structural model

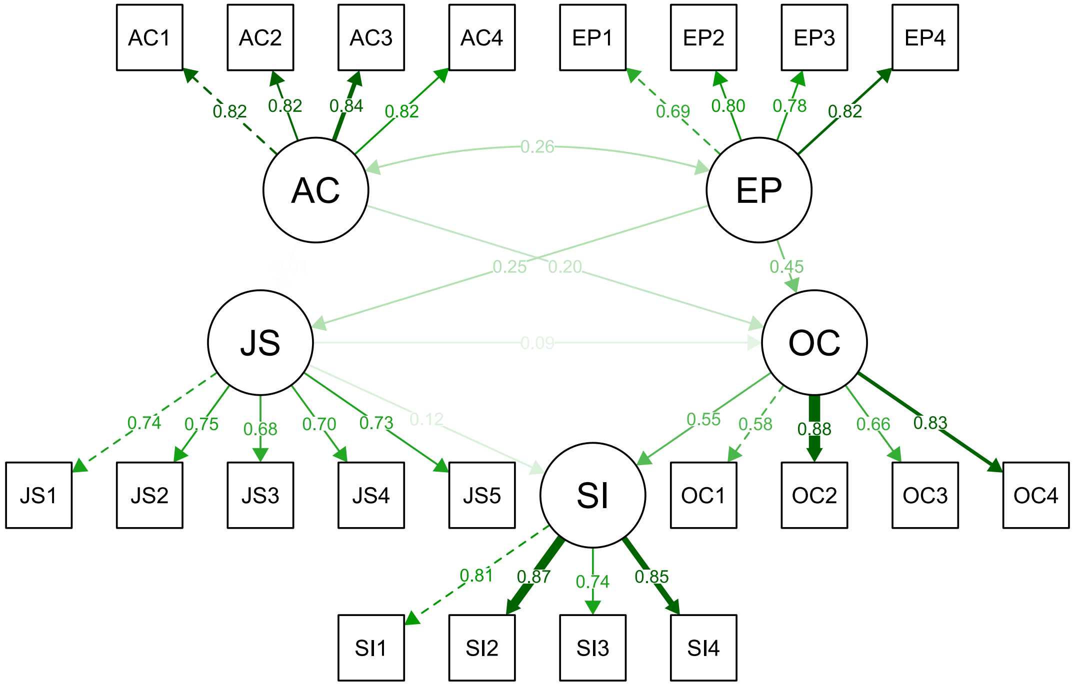


Hypothesis	Parameter
H_1: EP + → JS	$P_{JS,EP}$
H_2: EP + → OC	$P_{OC,EP}$
H_3: AC + → JS	$P_{JS,AC}$
H_4: AC + → OC	$P_{OC,AC}$
H_5: JS + → OC	$P_{OC,JS}$
H_6: JS + → SI	$P_{SI,JS}$
H_7: OC + → SI	$P_{SI,OC}$

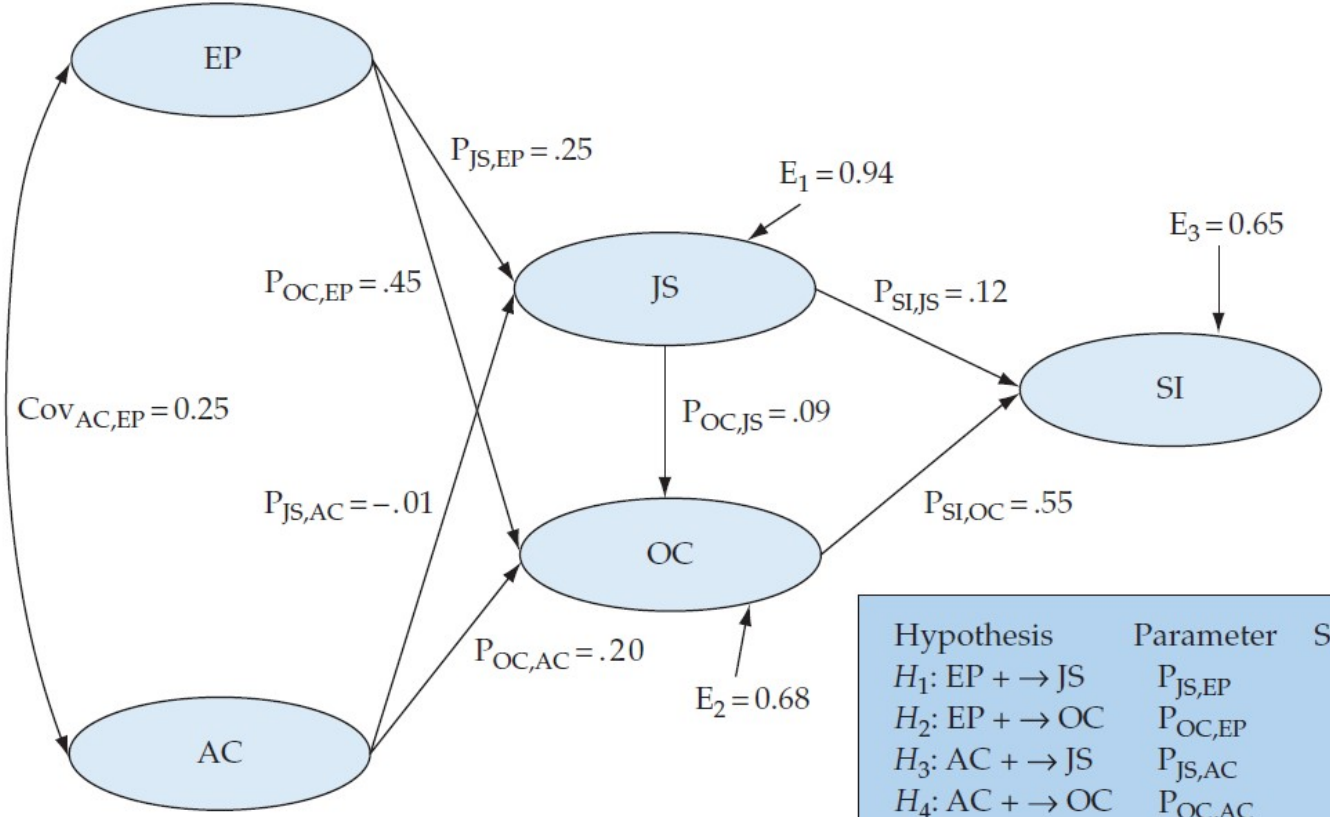
Let's practice

Defining structural model

```r
library(semPlot)
semPaths(object = sem_fit,
         what = "std",
         layout = "tree2",
         intercepts = FALSE,
         residuals = FALSE)
```



Defining structural model



Hypothesis	Parameter	Supported?
H_1: EP + → JS	$P_{JS,EP}$	Yes
H_2: EP + → OC	$P_{OC,EP}$	Yes
H_3: AC + → JS	$P_{JS,AC}$	No
H_4: AC + → OC	$P_{OC,AC}$	Yes
H_5: JS + → OC	$P_{OC,JS}$	No
H_6: JS + → SI	$P_{SI,JS}$	Yes
H_7: OC + → SI	$P_{SI,OC}$	Yes

GOF measures between structural and CFA model

```
gof_indices <- c('chisq', 'df','pvalue
                 'rmsea', 'rmr', 'srmr
                 'nnfi', 'cfi', 'agfi'
fitmeasures(sem_fit, fit.measures = go
fitmeasures(cfa_fit, fit.measures = go
```

```
  chisq       df  pvalue     gfi    rmse
287.179 181.000   0.000   0.938   0.03
   agfi
  0.921
```

```
  chisq       df  pvalue     gfi    rmse
240.738 179.000   0.001   0.947   0.02
   agfi
  0.932
```

GOF index	Employee retention model	CFA model
χ^2(chi-square)	287.179	240.738
Degrees of freedom	181	179
Probability	0.000	0.001
GFI	0.938	0.947
RMSEA	0.038	0.029
RMR	0.410	0.414
SRMR	0.060	0.036
NFI	0.936	0.946
NNFI	0.971	0.983
CFI	0.975	0.985
AGFI	0.921	0.932

What's next?

- Modification indeces
- Handling heywood cases
- Comparing competing models
- Formative scales in SEM
- Higher-order factor analysis
- Multigroup analysis

Thank you!

Slides created via the R packages:

xaringan by Yihui

xaringanthemer and xaringanExtra by Garrick