Prediction Market Performance and Market Liquidity: A Comparison of Automated Market Makers

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Prediction Market Performance and Market Liquidity: A Comparison of Automated Market Makers

Christian Slamka, Bernd Skiera, and Martin Spann

***Abstract*—The use of prediction markets (PMs) for forecasting is emerging in many fields because of its excellent forecasting accuracy. However, PM accuracy depends on its market design, including the choice of market mechanism. Standard financial market mechanisms are not well suited for small, usually illiquid PMs. To avoid liquidity problems, automated market makers (AMMs) always offer buy and sell prices. However, there is limited research that measures the relative performance of AMMs. This paper examines the properties of four documented and applied AMMs and compares their performance in a large-scale simulation study. The results show that logarithmic scoring rules and the dynamic pari-mutuel market attain the highest forecasting accuracy, good robustness against parameter misspecification, the ability to incorporate new information into prices, and the lowest losses for market operators. However, they are less robust in case of noisy trading, which makes them less appropriate in environments with high uncertainty about true prices for shares.**

***Index Terms*—Engineering management: decision making, engineering management: electronic commerce, mathematics: prediction algorithms.**

I. Introduction

THE use of prediction markets (PMs) as a forecasting tool has grown in the past decade. At PMs, participants can trade shares of securities. The final value of these securities is related to the realization of a future situation (akin to a state contingent claim), which is to be predicted by the PM [1]. Such markets began as niche applications in the field of political forecasting [2], but have since found further applications in areas such as sports [3] and business-related fields, including sales forecasting [4], new product concept evaluations [5]–[7], and even the generation and evaluation of ideas within companies [8]. Several studies show that in terms of accuracy, PMs are mostly superior to other comparable forecasting techniques such as polls [9], official company forecasts [4], or experts [1]—so that notable companies such as Google [10], Microsoft, Eli Lilly, and Hewlett-Packard [11] have started using PMs.

These examples point to an increasing acceptance of PMs, though they do not indicate which market design is best. This gap in existing research is surprising, because a well-designed market is essential for a PM to produce accurate forecasts [1] [12]. Market operators must make choices about several design characteristics, including the security design, incentive mechanisms, or the application of real- or play-money investments [3], [13]; perhaps the most crucial parameter is the market mechanism. Its choice becomes especially critical in a market with low liquidity, which is more likely when the number of different types of securities is large and/or the number of participating traders is low. Early PMs contained few types of securities and a rather high number of traders, but more recent applications in corporate forecasting and new product development feature many different types of securities and relatively fewer traders [5]. In these situations, market liquidity represents a central problem for applying PMs successfully, which may inhibit the application of a continuous double auction (CDA) trading mechanism. A possible solution is an automated market maker (AMM)

AMMs offer to buy and sell shares at any given point in time, creating unlimited liquidity in the market. However, unlike market makers (MMs) in real financial markets, AMMs are not human but rather rely on software. PMs applied four rather different AMM: logarithmic market scoring rules (LMSR) [14], [15], dynamic pari-mutuel market (DPM) [16], dynamic price adjustments (DPA) [8], [17], and a prominent AMM utilized by the Hollywood stock exchange (HSX, www.hsx.com [18]–[20]. Despite the apparent importance and various designs of AMMs, no existing research provides a thorough comparison of their suitability, so the differences among AMMs, as well as which one performs best, remain unclear. Such knowledge is critical to the providers of PMs, who could rather easily implement different AMMs if one AMM or another was found to be superior.

Therefore, the goal of this study is to present, describe, and compare AMMs that are currently documented and applied and evaluate their performance in a large-scale simulation study. To measure performance, we focus on four criteria. First, we analyze the overall forecasting accuracy of share prices, or *price efficiency*, which reflects the best predictions that can be achieved. Second, the *speed* at which new information appears, or when an event occurs, can be captured in market prices, which is important for measuring the "value" of an event [13], [21]. Third, we determine the amount of *losses* that might occur for AMM, which are of great importance when implementing AMMs for real-money markets, but less important for play-money markets. Fourth, an AMM should be *robust* against deviations that an operator can control, as well as against deviations not controlled by an operator that are caused by traders. An example for the first kind of deviation is a parameter misspecification, which means the parameters deviate from their optimal selection. An

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example for the second kind is a deviation resulting from noisy trading by less-than-perfectly informed traders.

We structure the remainder of this paper as follows. In Section II, we describe and conceptually compare the four mechanisms and illustrate their functions. In Section III, we explain the market model we use for the simulations. In Section IV, we outline the design of the simulation study, followed by the results we obtained. Finally, in Section V, we draw some conclusions.

II. Market Mechanisms

Market mechanisms in PMs could take inspiration from those known by financial exchanges, such as the Xetra Exchange, and that often employ CDAs throughout their trading phases. In CDAs, traders submit orders with a chosen maximum quantity of shares, usually with a price limit, into the order book. If a matching order appears, the order is executed immediately. Thus, trades can be executed among traders themselves on a continuous basis, given sufficient liquidity (i.e., orders) [22].

However, even in financial markets, liquidity may fall too low for sufficient trading activity, in which case MMs enable trading. These MMs are professional human traders who are willing and obliged to buy and sell shares. For example, the NASDAQ uses such a trading system.

The concepts of CDAs and MMs appear in PMs as well, e.g., the Iowa Electronic Markets [23], because liquidity problems tend to be much more severe in PMs than in financial markets. In small, often company-internal PMs, the number of traders is very low, and the number of different securities per trader is very high, which leads to a "chicken-and-egg" problem: Traders are attracted to liquid markets with high trading frequency, but liquid markets require many traders [16]. Fewer traders and trades lead to unreliable and inefficient market predictions, because fewer assessments by fewer traders regarding a forecasted event's outcome are reflected in the predictions.

Although in theory, (human) MMs might be employed in PMs, this approach generally is not feasible because of the costs for what is essentially a market research instrument. Most PMs, therefore, use AMMs, which set share prices according to an algorithm included within the software [14], [16]. Both human MMs in financial markets and AMMs in PMs share the primary goal of adding liquidity to markets, but human MMs must juggle more constraints in their trading actions than AMMs. For example, AMMs are not required to make profits or omit losses, as human MMs commonly must [24]. In the long run, the inventory of human MMs should be approximately level [25], but such considerations do not play any role for AMMs.

A. Overview

To date, four different types of AMMs have been documented for PMs. One of the most widely used (e.g., by Inkling, www.inkling.com; XPree, www.xpree.com; Washington Stock Exchange, www.thewsx.com) involves LMSRs introduced by Hanson [14], [15] and later also considered by Chen and Pennock [26]. The subsequent DPM by Pennock [16] appeared in the (now closed) Yahoo! Tech Buzz game [27] and AskMarkets (www.askmarkets.com).

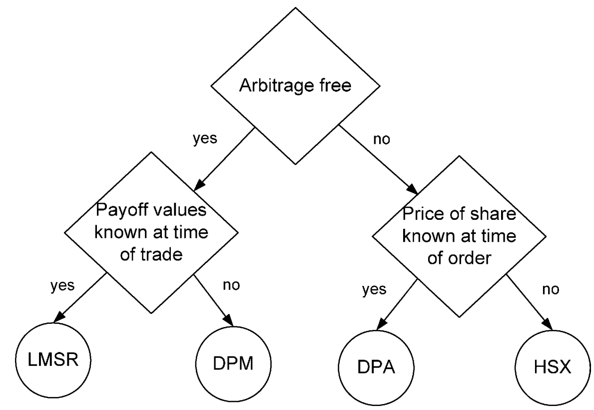


Fig. 1. Main characteristics of analyzed automated market makers (AMMs).

Both LMSR and DPM maintain continuous price functions to determine share prices, which mean they are arbitrage free (see Fig. 1). The price functions calculate the share prices according to the number of shares present in the market. Because the price function is continuous, every purchased (sold) share is more (less) expensive than the previous one. In other words, the prices of shares are not fixed for single trades, as they are in two other AMM cases. Therefore, the purchase costs per share for ten shares will be higher than the corresponding costs for five shares. Each trade of several shares can be thought of as several trades of one share. Whereas the possible payoff values are known with LMSR, in DPM, the final payoff values are not known, because they depend on further market trading actions.

Several studies [7], [8], [17] have introduced and employed an AMM that offers fixed purchase and sale prices of shares, which we call DPA. The last AMM is a prominent one used by the HSX) (www.hsx.com), the largest existing PM with several hundred thousand traders. Its underlying AMM has been described in three patents [18]–[20]. Neither DPA nor HSX implements a continuous price function; however, only the DPA mechanism transparently informs traders about the final price of a share at the time of the order.

B. Functionality of Mechanisms

In this section, we describe each AMM and complement our descriptions with three simple trading actions for each AMM, which we detail in the Appendix, including a description of the algorithms underlying each AMM.

1) (Logarithmic) Market Scoring Rules (LMSR): Market scoring rules (MSR), as the general concept behind LMSR, build on the long-standing concept of scoring rules, which can evaluate a forecaster's performance [28]–[30]. With simple scoring rules, forecasters give isolated, one-time predictions, whereas Hanson's MSR requires them to offer successive predictions about a particular forecasting goal by adjusting the most recent prediction [14], [15]. With an underlying continuous price function that depends on the particular scoring rule used, this AMM determines the price for each share sold or bought.

MSRs work with a set of *N* mutually exclusive and exhaustive outcomes, and the probabilities of all these outcomes sum to 1. The cost function $C(\vec{q_t})$, as the integral of the price function, reflects the amount of money that has already been spent on buying or selling $|\vec{q_t}| = \sum_{j=1}^{N} |q_{j,t}|$ shares, where $\vec{q_t} = (q_{1,t}, q_{2,t}, ..., q_{N,t})$ is the vector of the number of shares on the market after the *t*th trade in the market [31], including *N* entries for each of the securities. The amount of money that a trader must spend/receive for each trade is

$$\text{Costs}_{i,t}^{\text{LMSR}}\left(q_{i,t}^{\text{current}}\right) = C\left(\vec{q_t}\right) - C\left(\vec{q_{t-1}}\right)$$
$$\text{and } q_{i,t} - q_{i,t-1} = q_{i,t}^{\text{current}} \quad (1)$$

where $C(\vec{q_t})$ is the same vector as $C(\vec{q_{t-1}})$, except for one position *i*, which is the number of traded shares in transaction *i*, and a difference pertaining to the number of ordered shares of $q_{i,t}^{\text{current}}$. A trader who makes a purchase pays $C(\vec{q_t}) - C(\vec{q_{t-1}})$ if $q_{i,t}^{\text{current}} = q_{i,t} - q_{i,t-1} > 0$; one who makes a sale receives $C(\vec{q_{t-1}}) - C(\vec{q_t})$ if $q_{i,t}^{\text{current}} = q_{i,t} - q_{i,t-1} < 0$.

Although several scoring rules can represent MSRs, the logarithmic scoring rule $s_i = a_i + b\log(r_i)$ has widely been applied. The cost function, which can be derived from the scoring rule (for details, see [26]) for the LMSR, is

$$C\left(\vec{q_t}\right) = b \cdot \ln\left(\sum_{j \in N} \exp\left(q_{j,t}/b\right)\right). \quad (2)$$

Furthermore, MSRs require a subsidy *b* that determines the maximum amount of money an AMM can lose. This subsidy also controls liquidity in the market, such that a higher subsidy adds more liquidity to the market and prices move more slowly. The current price for an infinitesimally small amount of shares is the derivative of the cost function (2), and the price function is [31]

$$p_i\left(\vec{q_t}\right) = \frac{\partial C\left(\vec{q_t}\right)}{\partial q_{i,t}} = \frac{\exp(q_{i,t}/b)}{\sum_{j \in N} \exp(q_{j,t}/b)} = \pi_{i,t} \quad (3)$$

where $\pi_{i,t}$ is the probability that the *i*th-security's underlying event will occur after the *t*th trade in the case of a winner-takes-all market. Because this price quote is only valid for an infinitesimally small number of shares, a trader that hopes to buy shares must pay more money per share than the current quote and receives less when selling shares. Because of the continuity of the price function, buying an arbitrary amount of shares and selling it again (or *vice versa*) will not lead to a change in a trader's portfolio value. Thus, this mechanism is arbitrage free.

2) Dynamic Pari-Mutuel Market (DPM): Standard pari-mutuel markets, known from horse races, can aggregate information efficiently at a single point in time [32]. At any time before the close of the market, money can be spent on each of the *N* mutually exclusive and exhaustive outcomes, such as the victory of a particular horse. After the close of the market, the total money spent on all possible outcomes is $M^{\text{final}} = \sum_{j \in N} m_j$, where m_j is the amount bet on outcome *j*. When the final outcome is known, each dollar invested in the "correct" event *i* is worth $M^{\text{final}}/m_i \geq 1$. Despite their ability to aggregate information efficiently at the latest point in time, pari-mutuel markets cannot update predictions after the arrival of fresh information, such as news [16].

The DPM overcomes this problem by introducing dynamic prices for shares that represent the final amount of money, rather than maintaining a fixed price, as with the standard pari-mutual mechanism [16]. Similar to MSRs, the price of a share depends on the number of shares in the market and the price function [31]. As in a standard pari-mutuel market, all money gets redistributed to all winning shares, and the price of a share does not correspond directly to actual probabilities in winner-takes-all markets but must be transformed into probabilities π [27]:

$$\pi_i\left(\vec{q_t}\right) = \frac{q_{i,t}^2}{\sum_{j \in N} q_{j,t}^2} = \pi_{i,t} \quad (4)$$

where $q_{i,t}$ is the number of shares of the *i*th security on the market after the *t*th trade. The cost function, then, is defined as

$$C\left(\vec{q_t}\right) = \sqrt{\sum_{j \in N} q_{j,t}^2} \quad (5)$$

which denotes a one-to-one mapping between the vector of the number of shares and the money in the market. When starting the market, an initial amount $C\left(\vec{q_0}\right) = M^{\text{ini}}$ and a vector $\vec{q_0}$ of shares that contain positive quantities of shares of each security must be assigned to allow for trading.[1]

Similar to the subsidy *b* with MSRs, the initial assignment of money/shares controls the liquidity of the market. The more seed money initially enters the market, the slower the prices and implied probabilities move. Identical to MSRs, the cost of a trade equals the difference in the costs of the *(t –1)*th trade and the *t*th trade in the market, again using (1).

At the close of the market at time *T*, there are $|\vec{q_T}|$ shares on the market, and the corresponding money $C(\vec{q_T}) = M^{\text{final}}$ is totally invested. If the event that occurs out of the *N* events is *i*, all the money invested in all shares is equally split among the holders of that share. That is, the value of the share of event *i* is $M^{\text{final}}/q_{i,T}$, where M^{final} denotes the total amount of money invested in all shares, including seed money, and $q_{i,T}$ is the number of shares of *i* at the close of the market. The price of an infinitesimal amount of shares in security *i* is the derivative of the cost function [see ([5])]:

$$p_i\left(\vec{q_t}\right) = \frac{q_{i,t}}{\sqrt{\sum_{j \in N} q_{j,t}^2}}. \quad (6)$$

In addition, the probabilities of events can be derived from not only the shares on the market (4) but also the share prices [27]:

$$\pi_i\left(\vec{q_t}\right) = \frac{p_{i,t}^2}{\sum_{j \in N} p_{j,t}^2} = \pi_{i,t}. \quad (7)$$

Because it uses a continuous price function, the DPM, like the LMSR, is arbitrage free.

[1] As with the MSR, we assume all *N* events to be equally likely, and thus, the elements of the initial vector of number of shares are equal.

3) Dynamic Price Adjustment (DPA): In both LMSR and DPM, a continuous price function exists that determines the price of a share, depending on the order quantity. Another mechanism used in previous research [8], [17], the DPA, does not implement a continuous price function but rather offers an equal buy and sell price for each share of a security up to a certain maximum quantity. Thus, for $q_{i,t}^{\text{current}}$ shares of the *i*th security at the *t*th trade of this particular security, a trader pays or receives:

$$\text{Costs}_{i,t}^{\text{DPA}}\left(q_{i,t}^{\text{current}}\right) = q_{i,t}^{\text{current}} \cdot p_{i,t-1} \tag{8}$$

where $p_{i,t-1}$ is the price of a share of the *i*th security after the *(t – 1)*st trade of this security.

After the trade, a new price is calculated according to the last executed trades within a moving window. Therefore, the price rises after a purchase, and even more so if the previous transactions were also purchases, because this trend indicates an increase in the underlying true value. The function for the price update of a share of security *i* is given by [17]

$$p_{i,t}\left(p_{i,t-1}, \overrightarrow{q_{i,t}^{\text{lag}}}\right) = p_{i,t-1} + \text{sign}\left(q_{i,t,0}^{\text{lag}}\right) \cdot \max \left\{ \left|q_{i,t,0}^{\text{lag}}\right| \cdot \frac{p_{i,\max}^2}{\gamma^2} \cdot \frac{\left(\sum_{k=0}^{I_t} \text{sig}\left(q_{i,t,0}^{\text{lag}}\right) = \text{sig}\left(q_{i,t,k}^{\text{lag}}\right) \left|q_{i,t,k}^{\text{lag}}\right|\right)}{I_t + 1}, \tau \right\} = \pi_{i,t} \tag{9}$$

with $\overrightarrow{q_{i,t}^{\text{lag}}} = (q_{i,t,0}^{\text{lag}}, q_{i,t,1}^{\text{lag}}, \ldots, q_{i,t,I_t}^{\text{lag}}) = (q_{i,t}, q_{i,t-1}, \ldots, q_{i,t-I_t})$ $I_t = \begin{cases} t, & \text{for } t < I \\ I, & \text{for } t \geq I \end{cases}$, $|q_{i,n}| \leq q_{\max}$, where

$\overrightarrow{q_{i,t}^{\text{lag}}}$	vector of quantities of orders of shares of ith security at the *t*th trade of the *i*th security;
$q_{i,t}$	quantity of order of shares of *i*th security at the *t*th trade of *i*th security;
$p_{i,\max}$	maximum price for shares of *i*th security;
	length of moving average window;
γ	liquidity parameter ("γ");
τ	minimum tick size;
$q_{\max}$	maximum number of tradable shares.

Thus, the higher the liquidity parameter γ, the slower the share prices move. In contrast with LMSR and DPM, this market mechanism is not arbitrage free, so traders can buy shares of a security at a fixed price p^*, initiate a price update, and then sell it again for a price $p^{**} > p^*$, which nets them a profit of $p^{**} - p^*$. To avoid this situation in real-world applications, DPA requires a restriction that prevents a trader from quickly executing opposing transactions with the same security.

4) (Basic) HSX Mechanism (bHSX): The HSX (www.hsx.com) has been one of the biggest PMs online, but its AMM has not been described publicly in detail; however, the basic idea has been published in three patents [18]–[20]. During a certain time frame, or "sweep phase," for a specific security, this mechanism collects buy and sell orders, comparable to call auctions in financial markets, but does not execute them immediately. At the end of the sweep phase, the AMM determines a net movement balance, which equals the difference in the number of shares of buy and sell orders. If this number is positive, demand for the shares of a security is higher than supply, which suggests a higher "true value" of the underlying shares.

The net movement balance is then multiplied by a scaling parameter, "alpha," to produce the projected price movement. The price movement may be attenuated by a "Virtual Specialist" function if it appears too strong.[2] The newly calculated price is the old price plus the price movement. At this point, the final buy/sell price for the ordered shares in the elapsed time frame can be calculated, and the user can be informed about the final buy or sell price.

Because we lack complete documentation, we refer to this AMM as the basic Hollywood stock exchange mechanism (bHSX) and rely on publicly available details. In this mechanism, the final buy and sell price for each share in the *s*th sweep phase is

$$p_{i,s}\left(p_{i,s-1}, \overrightarrow{q_{i,s}^{\text{sweep}}}\right) = p_{i,s-1} + \alpha \cdot \sum_{z=1}^{\left|\overrightarrow{q_{i,s}^{\text{sweep}}}\right|} q_{i,s,z}^{\text{sweep}} = \pi_{i,s} \tag{10}$$

with $q_{i,t} \in \overrightarrow{q_{i,s}^{\text{sweep}}}$ if tth trade in sth sweep phase, $|\sum_{z=1}^{|\overrightarrow{q_{i,s}^{\text{sweep}}}|} q_{i,s,z}^{\text{sweep}}| \leq q_{\max}$, where

$p_{i,s}$	final bHSX price for shares of the *i*th security in the *s*th sweep phase for all trades in this phase;
$\overrightarrow{q_{i,s}^{\text{sweep}}}$	vector of quantities of orders of shares of *i*th security in the *s*th sweep phase;
$q_{i,t}$	quantity of order of shares of *i*th security of *t*th trade;
I	length of sweep phase;
α	scaling parameter ("alpha");
$q_{\max}$	maximum number of tradable shares.

For $q_{i,t}^{\text{current}}$ shares of the *i*th security at the *t*th trade in the *s*th sweep phase, a trader pays or receives:

$$\text{Costs}_{i,t}^{\text{bHSX}}\left(q_{i,t}^{\text{current}}\right) = q_{i,t}^{\text{current}} \cdot p_{i,s}. \tag{11}$$

The higher the liquidity parameter alpha, the faster the share prices move. Similar to the DPA, the bHSX is not arbitrage free, so losses potentially grow to infinity for the operators.

C. Comparison

We compared the mechanisms with respect to their general properties and usability for traders, implementation effort demanded of operators, and possible security designs (see Table I). All AMMs provide unlimited buy and sell liquidity. LMSR, DPM, and DPA execute all orders immediately, whereas the HSX accumulates the orders during the sweep phase and executes them only at the end. In contrast with the other AMMs, the price of shares in the DPM depends on the money invested in the markets. Without continuous price functions, DPA and

[2] Due to lack of documentation, we do not model the Virtual Specialist function.

TABLE I
CONCEPTUAL COMPARISON OF AMMs

	LMSR	DPM	DPA	(b)HSX
General properties				
Unlimited buy/sell liquidity	Yes	Yes	Yes	Yes
Immediate order execution	Yes	Yes	Yes	No
Price of shares dependent on money invested in markets/length of market	No	Yes	No	No
Arbitrage free	Yes	Yes	No	No
Usability				
Potential payoff values per share known at time of trade	Yes	No	Yes	Yes
Price of shares reflecting probabilities in case of winner-takes-all markets	Yes	No, to be transformed to probabilities	Yes	Yes
Final price of shares known to user before trade	Yes	Yes	Yes	No
Trading price of shares dependent on size of order	Yes	Yes	No	Yes, but not directly observable
Implementation effort				
Number of parameters to set	1	1	3	≥ 3
Monetary losses	Bounded	Bounded	Not bounded	Not bounded
Possible security designs				
Winner-takes-all	Yes	Yes	Yes	Yes
Linear with prices/impl. probabilities of shares in market summing up to 1	Yes	Yes, with transformed probabilities	Yes	Yes
Linear in general	Yes	No	Yes	Yes

LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.

HSX are not arbitrage free and require restraints to suppress such behavior, such as limiting the combination of subsequent purchases and sales. However, such restraints may not always work and limit their feasibility.

Their main characteristics also affect the usability of the mechanisms. For example, in the DPM, the potential payoff values of each share are unknown at the time of the trade, because they depend on the final amount of money and number of shares invested in the market. Unless a trade is the last one, a trader can only speculate about the final payoff value, which adds uncertainty to the investment decision. For the remaining AMMs, payoff values for each share are well defined and depend on the outcome of the event to be forecasted. Another drawback for the DPM's usability is that its share prices do not directly correspond to the probabilities in winner-takes-all markets. Therefore, a trader must explicitly or implicitly convert share prices to probabilities [see ([4])] to make investment decisions. In contrast, share prices for LMSR, DPA, and HSX correspond to the probabilities.

In the case of the HSX, a lack of knowledge of the final purchase or sale price of shares might be disadvantageous for traders, insofar as it adds uncertainty to the investment decision. In contrast, the DPA makes the process very easy for traders: they need only base their decisions on the single purchase and sale price, rather than have to incorporate a rising or falling price per share that depends on the number of bought or sold shares.

From a PM operator's point of view, a key criterion is the effort required to implement markets that run a specific AMM. As the number of parameters that must be set prior to running a market grows, so does the complexity and effort faced by the operator. Whereas LMSR and DPM require a single liquidity parameter, DPA and HSX need both liquidity parameters and lengths of the trading windows during which trades may be considered. The maximum number of tradable shares should be set in both AMMs. Moreover, whereas LMSR and DPM feature strict bounds on losses, DPA and bHSX lack any such bounds, which are important in real-world markets. Losses for operators could grow to infinity in these latter cases, though this restraint assumes only a minor importance in most play-money markets.

With regard to general applicability, the different possible security designs for an AMM are critical [1], [33]. All AMMs support so-called winner-takes-all securities, in which a single winning share of a security cashes out at \$1 in the case of LMSR, DPA and HSX, and $M^{\text{final}}/q_{i,T}$ in the case of DPM. The remaining shares cash out at \$0. In the case of linear securities, shares of a security cash out between \$0 and \$1, and all share

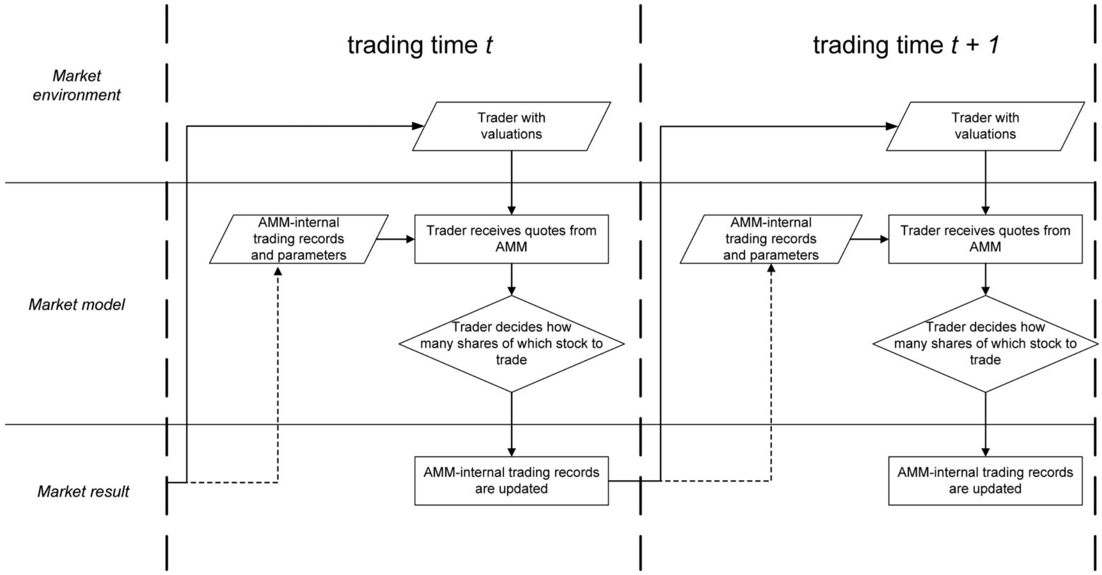


Fig. 2. Overview of market microstructure.

prices sum to \$1. Thus, outcomes of elections might be forecast such that each party receives a share of p_i votes, and shares of security *i* cashing out receives \$$p_i$ per share. In the case of the DPM, each share's cash out is $(M^{\text{final}}/q_{i,T}) \cdot p_i$, and all cash outs sum to M^{final}. In general, linear securities, whose shares of a security can cash out at any value, are not supported by DPM, though LMSR can be configured to support this design [14]; DPA and HSX both support it.

One might argue that because of the different properties of each AMM, each might be used in different, specific settings. The LMSR is used in a variety of settings, as discussed in Section II-A, such as forecasting in different topics with different market sizes [www.inkling.com, www.thewsx.com, and even innovation assessment (www.xpree.com)]. The DPM has only been used in the Tech Buzz game [27], [34], though the variety of topics was large and the number of traders varied across the markets. The DPA, as well as the LMSR, has also been employed in relatively small-sized forecasting tasks for a variety of topics [17], as well as in larger innovation assessment applications [8]. The HSX, though only practiced by one market provider (www.hsx.com), has been used for markets of different sizes [1], and even been utilized for event studies [21].

III. Simulation Study Design

A. Market Microstructure

The general idea behind the evaluations in this section is to create a market environment that contains traders with certain properties, such as valuations, that are independent from any AMM (see Fig. 2). Then, by specifying the market model, we can determine how the shares are exchanged using a specific AMM, including the specific parameters. By keeping the market environment identical for all AMMs, we can measure the quality of the market outcome for each AMM and its parameter selection, which provides our market result. Deviations in the market results thus reflect upon the chosen AMMs and their selected parameters. This complete framework constitutes the market microstructure.

At a certain trading time *t*, a trader possesses valuations, which we describe in greater detail subsequently. The trader then receives quotes from the AMM (e.g., share prices or the number of shares in the market. Using this information, the trader can decide which and how many shares to trade. When the trader submits an order, the AMM internal trading records will update, thus producing new prices and predictions.

B. Model

As a basis for our simulations, we submit a model that represents an extension of the model introduced by Das [35], which itself is an extended version of Glosten and Milgrom's [36] model. Our extension is necessary to reflect the PM- and AMM-specific properties of the markets, such as the existence of two securities in a market and varying order quantities rather than one security or a fixed order quantity.

1) Base Model (Assumptions): In Das's [35] model, a security *i* in the market exists whose shares can be traded only through the intermediation of an MM; thus, one transaction side is always the MM, and the opposite side is always a trader. The share of security *i* has an underlying true value of $V_{i,t}$, which is exogenously given (i.e., the event outcome in a PM). There are two types of risk-neutral traders: perfectly informed traders who are aware of the security's true value $V_{i,t}$ and noisy informed traders who receive a noisy signal $\omega(0, \sigma_{\text{noisy}})$ with a mean of 0 and a standard deviation of σ_{noisy}. Thus, a single trader at time *t* who trades the *i*th security receives the following signal of the share of this security's valuation:

$$W_{i,t} = \begin{cases} V_{i,t}, & \text{if trader at time } t \text{ trading } i\text{th stock is perfectly informed} \\ V_{i,t} + \omega(0, \sigma_{\text{noisy}}), & \text{if trader at time } t \text{ trading } i\text{th stock is noisy informed.} \end{cases} \quad (12)$$

The sequence of trading is the following.

1) Traders in the market arrive one by one in discrete points in time $t \in \{1, ..., T\}$ and may execute either buy or sell orders or possibly no orders at all.
2) The size of a trade is fixed to one unit for each trade, and in every period the MM issues bid and ask prices for one unit, $P_{i,t,b}$ and $P_{i,t,a}$, respectively.
3) A trader issues a sell order if $W_{i,t} < P_{i,t,b}$, such that a share of security i at time t appears overvalued, or issues a buy order if $W_{i,t} > P_{i,t,a}$ for a (subjectively) undervalued share.
4) If the signal lies within the spread, such that $P_{i,t,b} \leq W_{i,t} \leq P_{i,t,a}$, the trader issues no order.

2) Model Extension (Assumptions): Not all AMMs support all market designs, but each one supports the popular winner-takes-all and linear outcomes, with shares in the market summing to 1. Therefore, we assume that two securities are present, and that their shares' true values always add to 1, such that $V_{2,t} = 1 - V_{1,t}$ holds at all times *t*. The true values of each share of a security, depending on the interpretation, reveal either the probability that a security's underlying event will happen (winner-takes-all) or the expected value (linear) of the outcome [1]. All traders are aware of this property; we can thus deduce that the signal for the trader's share of the second security at time *t*, $W_{2,t}$, can be inferred by calculating $W_{2,t} = 1 - W_{1,t}$. Moreover, the choice to execute a buy or sell order is exogenous for the trader: If a buy order is chosen, the undervalued share (relative to $W_{i,t}$) is traded, whereas if a sell order is chosen, the overvalued share is traded. Because $V_{2,t} = 1 - V_{1,t}$ and $W_{2,t} = 1 - W_{1,t}$, it is evident as to which share of a security is subjectively under-/overvalued or whose share prices (transformed in DPM) exactly correspond to the valuations, in which case no transaction occurs.

Trading. The second extension of the model relaxes the restriction of trading only one share per period, because different order quantities are essential for the price movements of the AMMs. Therefore, the trader's decision problem is tied to not only the question of buying or selling a certain share of a security, but also to the quantity. To determine the number of shares of a security to trade, we assume that the risk-neutral trader maximizes his or her utility for a security $i \in \{1, 2\}$ in a particular trade *t*:

$$\max_{q_{i,t}^{\text{current}}} U(E(V_{i,t}), \text{Costs}_{i,t}^{\text{AMM}}(q_{i,t}^{\text{current}})) \\ = U(W_{i,t}, \text{Costs}_{i,t}^{\text{AMM}}(q_{i,t}^{\text{current}})). \tag{13}$$

The utility depends on two factors: the expected value of a share of security *i* at time *t*, $E(V_{i,t})$, which a myopic trader subjectively believes to be $W_{i,t}$ [see (12)], and the total costs of the shares, $\text{Costs}_{i,t}$, for a trade of $q_{i,t}^{\text{current}}$ shares. This number, $q_{i,t}^{\text{current}}$, must be optimized to maximize utility. See the Appendix for details on the utility maximization.

To obtain a more realistic setting, we set a maximum amount c_t as a budget constraint in order to control for what a trader can spend (buy order) or redeem (sell order). Moreover, for DPA and bHSX, the budget constraint imposes a necessary bound on the number of tradable shares. Without this bound, traders would always trade the maximum allowable number of shares. Consequently, for a buy order, the total cost must fall below c_t, and the total redemption value of a sell order must be below c_t:

$$\left|\text{Costs}_{i,t}^{\text{AMM}}(q^{\text{current}})\right| \leq c_t. \tag{14}$$

C. Determination of Optimal Number of Traded Shares

For LMSR, DPA, and bHSX, the share prices directly correspond to probabilities and expected values. In the case of LMSR, we can easily observe that utility becomes maximized when the shares are ordered up to the point that the price of the $q_{i,t}^{\text{current}}$th share ordered is less than the valuation $W_{i,t}$ but the price of the $(q_{i,t}^{\text{current}} + 1)$st share is greater than that valuation. Thus, the optimal number of traded shares $q_{i,t}^{\text{current}}$ for a buy order and the case of LMSR is

$$\begin{aligned} &U(E(V_{i,t}), \text{Costs}_{i,t}^{\{\text{LMSR}\}}(q_{i,t}^{\text{current}})) \\ &= U(W_{i,t}, \text{Costs}_{i,t}^{\{\text{LMSR}\}}(q_{i,t}^{\text{current}})) \\ &= \sum_{s=1}^{q_{i,t}^{\text{current}}} \left(W_{i,t} - p_{i,t}^{\{\text{LMSR}\}}(s)\right) \\ &\rightarrow \text{to be maximized by optimizing } \; q_{i,t}^{\text{current}} \\ &\Leftrightarrow \\ &\left[p_{i,t}^{\text{LMSR}}(q_{i,t}^{\text{current}}) \leq W_{i,t} \wedge p_{i,t}^{\text{LMSR}}(q_{i,t}^{\text{current}} + 1) > W_{i,t}\right]. \end{aligned} \tag{15}$$

The function $p_{i,t}^{\text{LMSR}}(\cdot)$ is increasing because a continuous price function for the pricing of the shares exists. We optimize $q_{i,t}^{\text{current}}$by completely enumerating $W_{i,t}$.

For the DPA, the function $p_{i,t}^{\text{DPA}}(\cdot)$, and thus the price of each share, is constant. For the bHSX, the rationale is similar, because traders cannot infer the final (constant) price and must assume that the last price is the price they will have to pay. The optimal number of traded shares is

$$\begin{aligned} &U(E(V_{i,t}), \text{Costs}_{i,t}^{\{\text{DPA,HSX}\}}(q_{i,t}^{\text{current}})) \\ &= U(W_{i,t}, \text{Costs}_{i,t}^{\{\text{DPA,HSX}\}}(q_{i,t}^{\text{current}})) \\ &= \sum_{s=1}^{q_{i,t}^{\text{current}}} \left(W_{i,t} - p_{i,t}^{\{\text{DPA,HSX}\}}(s)\right) \\ &\rightarrow \text{maximize } q_{i,t}^{\text{current}} \text{ within budget constraint } c_t. \end{aligned} \tag{16}$$

In the DPM, prices do not correspond directly to probabilities, and expected payoff values must be considered, as analyzed by Nikolova and Sami [37], lemma [5]. They show that a risk-neutral trader maximizes utility when trading with the DPM (for a purchase and the first stock), inferring the optimal number

TABLE II
AMM PARAMETER SELECTION IN SIMULATION STUDY

	LMSR	**DPM**	**DPA**	**bHSX**
1st (liquidity) parameter	Subsidy	Seed money	Gamma	Alpha
Values	To be optimized	To be optimized	To be optimized	To be optimized
2nd (memory) parameter	-	-	Length window	Length window
Values			5, 10, 20	5, 10, 20
3rd (restriction) parameter	-	-	Max. number of shares per trade	Max. number of shares in window
Values			Fixed at 50	Fixed at 50

LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.

of shares as

$$U(E(V_{i,t}), \mathrm{Costs}_{i,t}^{\mathrm{DPM}}(q_{i,t}^{\mathrm{current}}))$$

to be maximized by optimizing $q_{i,t}^{\mathrm{current}}$

$$\Leftrightarrow$$

$$\frac{q_{1,t}+q_{i,t}^{\mathrm{current}}}{q_{2,t}} \leq \sqrt{\frac{\pi_1\left(\vec{q_t}+\left(q_{i,t}^{\mathrm{current}},0\right)\right)}{\pi_2\left(\vec{q_t}\right)}}$$

$$\wedge \quad \frac{q_{1,t}+q_{i,t}^{\mathrm{current}}+1}{q_{2,t}} > \sqrt{\frac{\pi_1\left(\vec{q_t}+\left(q_{i,t}^{\mathrm{current}}+1,0\right)\right)}{\pi_2\left(\vec{q_t}\right)}}. \tag{17}$$

D. Overview and Optimization Criterion

The goal of the simulation study is to analyze the AMMs according to the four criteria that we discussed in the introduction, namely, the highest forecasting accuracy with optimal parameters; robustness against parameter misspecifications and noisy trading; the speed of information incorporation, and possible losses.

As the basis for our evaluations and the parameter optimization, we consider the (implied for DPM) price efficiency, or the degree to which market prices reflect the shares of a security's true underlying values. We measure price efficiency as the mean absolute error (MAE) over all trading periods, which equals the absolute difference between the market prediction $\mathrm{pred}_{1,T,r}$ and the true value $V_{1,t,r}$:

$$\mathrm{MAE}_r = \frac{1}{T}\sum_{t\in\{1,..,T\}}\left|\mathrm{pred}_{1,t,r}-V_{1,t,r}\right| \tag{18}$$

where $\mathrm{pred}_{i,t,r}$ is the prediction about the true value of shares of the *i*th security at time *t* and in the *r*th replication, and $V_{1,t,r}$ is the true value of the shares of the *i*th security at time *t* and for the *r*th replication. By compiling all periods, we capture the complete AMM behavior and quality of the predictions, rather than focusing on a single point in time. Using the predictions and the true values of the shares of the first security to measure the error is essentially the same as using the predictions and true values of the shares of the second security, because $\left(\mathrm{pred}_{1,t,}-V_{1,t,r}\right)^2 = \left((1-\mathrm{pred}_{2,t,r})-(1-V_{2,t,r})\right)^2 = \left(\mathrm{pred}_{2,t,r}-V_{2,t,r}\right)^2$.

However, because the final prediction may play a role in cases in which the true value does not change, we also report the error obtained after the last trade, or the final absolute error (FAE), for the first criterion under which the true value does not change

$$\mathrm{FAE}_r = \left|\mathrm{pred}_{1,T,r}-V_{1,T,r}\right|. \tag{19}$$

E. Selection of AMM Parameters

Table II displays the variations of the liquidity parameters, which indicate how fast prices react to trading actions. For LMSR (subsidy), DPM (seed money), and DPA (gamma), a higher liquidity parameter means a slower movement of prices; for bHSX, setting the alpha higher signifies a faster movement.

LMSR and DPM need only one parameter. DPA and bHSX must feature at least two more parameters and we take a window length of 10 as our base value [17] for both AMMs, but also test half and twice the window length (i.e., 5, 20 periods).

A third parameter that applies to DPA and bHSX, the restriction parameter, involves the maximum number of tradable shares. Again, we follow Van Bruggen *et al.* [17] and restrict the maximum number to 50 shares per type of security. This restriction applies to each trade of the DPA and the total number of trades in the window of the bHSX. The rationale behind this selection states that only a certain number of shares should influence the movement of prices, accomplished in a single trade in the case of DPA, but over the whole window for bHSX.

We set all AMMs initially to have share prices of \$0.50 (LMSR, DPA, bHSX) or an implied probability of 50% (DPM).

F. Market Environment

For the simulation, we created a market environment in which the true values of the shares of a security, signals of traders, type of transaction, and maximum amount to spend or redeem are set for each period.

Major settings (full list in the Appendix).

1) The number of trading periods is set to 300 in which either of the two securities can be traded.

2) The true value volatility is either 'no', 'low', or 'high'. Usually, a PM operator must set the AMM parameters without knowledge of the true underlying value. However, an operator still should have some knowledge about the potential volatility at which the true value changes.
   a) *No true value volatility:* true value is uniformly distributed over the whole range between 0 and 1, determined once per replication.
   b) *Low true value volatility:* the true value maintains a low volatility, modeled by incorporating a jump in the true value from 0.5 to $0.5 + d$, where d is normally distributed with mean 0 and a standard deviation of 0.1. The jump occurs in period 101.
   c) *High true value volatility:* the true value maintains a low volatility, modeled by incorporating a jump in the true value from 0.5 to $0.5 + d$, where d is normally distributed with mean 0 and a standard deviation of 0.25. The large standard deviation of 0.25 implies new true values mostly near the extremes of 0, or 1, respectively, expressing large jumps on average, which is exactly what we try to achieve. The jump always occurs in period 101.

Parameter optimization. The crucial difference here compared to Brahma *et al.'s* [12] model and simulation design is that we optimize the parameters according to the smallest MAE. This way, we are simulating the choice that a market operator has to make when preparing the market and the AMMs.

1) The relation between liquidity parameter values and the obtained MAE is a convex function, with too high or too low liquidity parameter values leading to high MAEs. Thus, the optimal liquidity parameter value, which signifies neither too much nor too little liquidity, exists.
2) With the assumed market operator's estimation of whether the market will contain large, small or even no jumps, the optimal parameter values are determined for each of the three cases of volatility in the true value. In the simulation, we obtain the optimal liquidity parameters by conducting simulation runs with all other parameters except the liquidity parameters fixed, and then recursively conduct the simulations with improved liquidity parameter values, until we have reached the optimum, i.e., the lowest MAE.
3) We then ascertain the optimal liquidity parameter value for each of the remaining parameters of DPA and bHSX. We do not set any preconditions on the choice of liquidity parameters, as we assume that the market operator's main goal is to achieve the most efficient prices.

IV. Results

In this section, we describe our results for every AMM according to forecasting accuracy, speed of information incorporation, losses, and robustness.

A. Forecasting Accuracy

Table III provides the results from the parameter optimization for every AMM. Averaged across all 6000 cases, DPM performed best with an MAE of 1.10, and LMSR was slightly worse with an MAE of 1.25. The DPA resulted in more than twice the error with an MAE of 2.25. The worst performer, bHSX, attained an error of 3.23.

For the no-volatility case, the optimal subsidy parameter was 16, which returned an MAE of 1.62. The DPM in this case delivered slightly better results (MAE 1.62), obtained with seed money of 80. However, DPA and bHSX performed substantially worse. The best DPA result occurred with a window length of 5 and gamma of 93 (MAE 3.71). The results barely fluctuate with respect to the length of the window though. We can infer that the DPA with a window length of 10 or 20 has a higher error of only 0.02 and 0.05, respectively. Thus, with regard to window length, the DPA seems to be robust. Again, bHSX delivered the worst results, with a window length of 5 being the optimal solution, given a liquidity parameter value alpha of 0.00145 (MAE 4.53). In contrast with DPA, bHSX was very sensitive to the length of the window: With window lengths of 10 and 20, the MAE became 5.63 and 7.34, respectively.

In the case of low volatility, the differences among the AMMs were less pronounced. The LMSR achieved an MAE of 0.72, and the DPM again offered a lower MAE of 0.66. In contrast with the no-volatility case, the DPA performed only 0.16 points worse (0.82) than the DPM with window lengths of 5 and 10. Once again, the bHSX performed worst (MAE 1.27); however, compared with the no-volatility case, this level represents a clear improvement.

The high-volatility case returned similar results compared to the previous two cases. Because the last price is frequently used for predicting the event, we display these results in Table III as well. The mean FAE across all cases was lower for the DPM (0.64) than for the LMSR (0.95). The bHSX once again performed worst (mean FAE 1.82), though the span with the DPA did see a reduction (mean FAE 1.40).

B. Speed of Information Incorporation

Table IV presents the results that we obtained by measuring the number of periods needed to arrive at a new price level in the case of a jump in the true underlying value. We distinguish the low and high-volatility cases (with small and large jumps) that occur during the trading phase. In the low-volatility case, LMSR and DPM performed comparably well, with the mean number of periods needed for information incorporation being 12.60 and 12.44, respectively. In contrast, the mean number was approximately 9 periods higher for the DPM (21.34). The bHSX was by far the slowest AMM, with an average of 33.91 periods required to reach the new true value. If the jump is large, as in the case of high volatility, then the DPM again performs best with an average of 8.13 periods. The margin between DPM and LMSR widened some, too: the latter showed 9.79 periods on average. The DPA required 18.24 periods, followed by the bHSX with almost 30 periods.

C. Losses

We determined both the 1) maximum and 2) expected loss for each replication per AMM. We define losses as the amount in dollars as the sum of money with which a PM operator has to

TABLE III
OPTIMAL PARAMETER VALUES AND FORECASTING ACCURACY ACROSS VOLATILITY CASES

	LMSR	DPM	DPA			bHSX		
Liquidity par. optimized	*Subsidy*	*Seed money*		*Gamma (γ)*			*Alpha (α)*	
Length window	*n/a*	*n/a*	*5*	*10*	*20*	*5*	*10*	*20*
Max.# shares tradable	*n/a*	*n/a*	*50*	*50*	*50*	*50*	*50*	*50*
All cases								
Mean / Median AE	**1.25** / 1.08	**1.10** / 0.99	**2.25** / 1.20	2.28 / 1.20	2.31 / 1.22	**3.23** / 2.14	3.94 / 2.85	5.14 / 3.89
Mean / Median FAE	0.95 / 0.31	0.64 / 0.24	1.40 / 0.61	1.43 / 0.58	1.42 / 0.63	1.82 / 1.22	2.38 / 1.74	3.14 / 2.51
No volatility in true val.								
Opt. liquidity par. Val.	*16*	*80*	*93*	*93*	*104*	*0.00145*	*0.00160*	*0.00200*
Mean / Median AE	**1.62** / 1.44	**1.36** / 1.25	**3.71** / 2.96	3.73 / 2.88	3.76 / 2.95	**4.53** / 4.08	5.63 / 4.98	7.34 / 6.46
Mean / Median FAE	1.30 / 0.47	0.87 / 0.33	2.16 / 1.14	2.28 / 1.15	1.98 / 1.02	2.72 / 2.28	3.29 / 2.91	4.40 / 4.15
Low volatility in true val.								
Opt. liquidity par. value	*280*	*643*	*117*	*119*	*108*	*0.00063*	*0.00071*	*0.00087*
Mean / Median AE	**0.72** / 0.62	**0.66** / 0.57	0.82 / 0.64	**0.82** / 0.63	0.84 / 0.66	**1.27** / 0.96	1.67 / 1.28	2.40 / 1.88
Mean / Median FAE	0.46 / 0.17	0.34 / 0.13	0.45 / 0.31	0.42 / 0.29	0.49 / 0.35	0.72 / 0.63	0.96 / 0.83	1.47 / 1.24
High volatility in true val.								
Opt. liquidity par. Val.	*49*	*141*	*88*	*86*	*82*	*0.00125*	*0.00160*	*0.00172*
Mean / Median AE	**1.43** / 1.18	**1.28** / 1.15	**2.24** / 1.60	2.28 / 1.60	2.34 / 1.65	**3.24** / 2.55	4.31 / 3.42	5.86 / 4.51
Mean / Median FAE	1.09 / 0.19	0.71 / 0.24	1.58 / 0.78	1.60 / 0.79	1.80 / 0.86	2.04 / 1.61	2.88 / 2.53	3.56 / 3.23

AE: absolute error; FAE: final absolute error; Val.: value; LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.

TABLE IV
SPEED OF INFORMATION INCORPORATION

	LMSR	DPM	DPA	bHSX
Low volatility in true value				
Mean / Median # periods	**12.60** / 9	**12.44** / 11	**21.34** / 21	**33.91** / 30
Min / Max / SD # periods	0 / 92 / 14.56	0 / 71 / 10.81	0 / 98 / 13.17	0 / 140 / 22.49
High volatility in true value				
Mean / Median # periods	**9.79** / 4	**8.13** / 7	**18.24** / 18	**29.96** / 30
Min / Max / SD # periods	0 / 106 / 17.29	0 / 37 / 6.86	0 / 98 / 12.74	0 / 140 / 22.67

LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.

start the market and the amount he possibly has to pay off the traders for their winning shares. When determining the expected losses, we weighed the probability that each of the two losses of each stock occur with the probability that the event related to the respective security wins by taking the security's last true value.

The results of the simulations (shown in Fig. 3) also display the theoretical bound for the losses of the LMSR and DPM. While the LMSR saw a median maximum loss of \$10.77, the median expected loss was higher for the DPM at \$19.64. The maximum losses hardly differed between those two AMMs—\$137.32 and \$139.66, respectively—but were much higher for the AMMs with theoretically unbounded losses. The median maximum loss for the DPA reached \$122.36 (\$500.94 at maximum), while the bHSX sustained a lower median maximum loss of \$68.59 (\$393.55 at maximum).

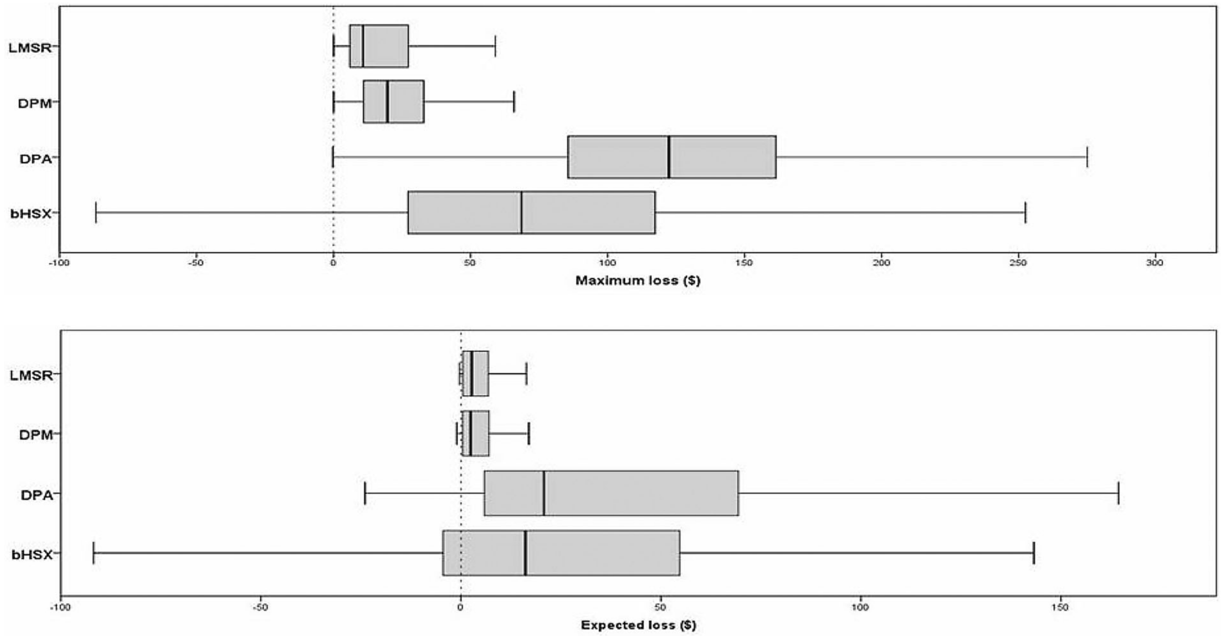
In contrast to the maximum losses, the median expected loss was \$2.69 for the LMSR, and slightly lower for the DPM at \$2.40. The maximum expected losses are, in reverse, lowest for the LMSR with \$59.66, and higher for the DPM at \$92.42. For the DPA, the median expected losses were \$20.77 (maximum \$419.25), and \$16.14 for the bHSX (maximum \$366.24).

D. Robustness

1) Robustness Against Parameter Misspecification: We allowed the parameters to deviate –75%, –50%, –25%, 25%, 50%, 75%, and 100% from their optimal values and computed the resulting MAEs. In Fig. 4, we depict the aggregated results across all volatility cases in relative terms. The results for each of the volatility cases are consistent with those in the aggregated form.

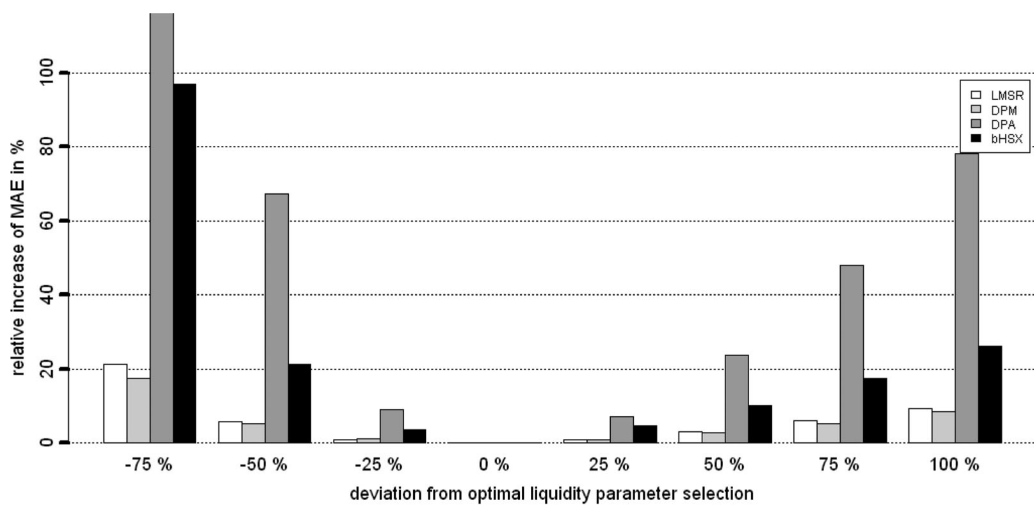
The LMSR and DPM results were the least sensitive to the parameter selection on average. We perceived only a weak rise of MAE for both AMMs. Even when we lowered the liquidity parameters by 75%, the error increase remained around 20%. A variation in the lower direction yielded a stronger negative effect on the accuracy than that in the upper direction. For example, when the parameter deviated +75% from the optimum, the increases in error were 6.10% and 5.27% for LMSR and DPM, respectively. When the deviation dropped to –75%, however, the errors jumped to 21.22% and 17.39% (again, respectively). Thus, if those who set market parameters happen to be in doubt over parameter selection, a higher liquidity parameter that lets prices move more slowly should be preferable.

For the bHSX and especially DPA, the effects of the parameter deviation were stronger. When the bHSX parameter value alpha



LMSR: Logarithmic Market Scoring Rules; DPM: Dynamic Pari-Mutuel Market;
DPA: Dynamic Price Adjustments; bHSX: Basic Hollywood Stock Exchange Mechanism

Fig. 3. Box plots for maximum and expected losses for market operator.



MAE: Mean Absolute Error; LMSR: Logarithmic Market Scoring Rules; DPM: Dynamic Pari-Mutuel Market;
DPA: Dynamic Price Adjustments; bHSX: Basic Hollywood Stock Exchange Mechanism

Fig. 4. Relative increase of MAEs when deviating from the optimal liquidity parameter value (all volatility cases).

decreased by 75% (which increased the liquidity, in contrast with the other AMMs), the MAE improved nearly 100%. However, raising the parameter by 75% to allow less liquidity generated an error increase of barely 20%. For bHSX, then, it seems beneficial to apply higher liquidity parameter values rather than lower ones in doubtful situations.

For DPA, gamma liquidity parameter values that were too low produced strong consequences for the MAE. The largest increase of error occurred when the deviation dropped to –75% from the optimum, which resulted in a relative increase of the MAE of more than 500%.

In relative terms, the DPA is by far the most sensitive AMM, even compared with the bHSX. A small liquidity parameter value in the DPA results in a larger error than a comparable higher parameter value. For example, when we allowed a deviation of ±50%, the resulting increase of MAE was nearly 70% for –50% and slightly over 20% for +50% from the optimum. Thus, more liquidity functions better than less liquidity, similar to the findings for LMSR and DPM.

2) Impact of Noisy Trading: We used an analysis of variance to identify the extent to which the factors for noisy trading—that is, the proportion of noisy traders and the degree of

TABLE V
INFLUENCE OF PROPORTION OF NOISY TRADERS AND DEGREE OF NOISE OF NOISY TRADERS ON MAE

		LMSR		DPM		DPA		bHSX	
All volatility cases	*Factor levels for MCA*	*Ex-plained variance*	*MCA value*	*Ex-plained variance*	*MCA value*	*Ex-plained variance*	*MCA value*	*Ex-plained variance*	*MCA value*
Average			1.254		1.101		2.253		3.233
Proportion of noisy traders	high	13.99% **	0.279	12.17% **	0.206	0.30% **	0.123	0.00% n.s.	0.011
	low		-0.279		-0.206		-0.123		-0.011
Degree of noise of noisy traders	high	16.73% **	0.305	11.19% **	0.105	0.22% *	0.105	0.13% n.s.	0.087
	low		-0.305		-0.105		-0.105		-0.087
% Factors		30.73%		23.36%		0.52%		0.13%	

(*: $p < 0.005$, **: $p < 0.001$; $N = 6000$ for each AMM), MCA: multiple classification analysis; LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.

TABLE VI
OVERVIEW OF STUDY RESULTS

	LMSR	DPM	DPA	bHSX
1. Accuracy	+	++	–	– –
2. Speed of information incorporation	+	++	–	– –
3. Losses	+	+	– –	–
4. Robustness				
a. parameter selection	+ +	++	– –	–
b. noisy trading	– –	– –	+ / –	++

LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.

noise—influence the MAE. With a multiple-classification analysis (MCA), we also determined how much a factor influences error. We depict the aggregated results over all volatility cases in Table V; these results are consistent with the single volatility cases.

For both LMSR and DPM, the proportion and degree of noisy traders had significant influences on the MAE ($p < 0.001$). The proportion of noisy traders explains 13.99% and 12.17% of the variance in LMSR and DPM, respectively. The influence of the degree of noise is comparably high at 16.73% and 11.19%, respectively. For the LMSR, the MCA showed that a high proportion of noisy traders increased the MAE by 0.279; the degree of noise increased the error by 0.305. For the DPM, results are comparable though not as strong: 0.206 for the proportion of noisy traders and 0.105 for the degree of noise.

The results for the DPA are less clear. The variance explained by the two factors is very low at 0.30% and 0.22%, respectively, though it remains significant at the 0.1% level for the proportion of noisy traders and at the 0.5% level for the degree of noise. The MCA revealed a value of 0.123 for the proportion and 0.105 for the degree of noisy traders. Thus, both factors influence the MAE of all the DPA, but less strongly than for the LMSR and DPA, especially when considering the proportion of noisy traders.

Finally, for bHSX, the results indicated that neither the proportion nor the degree of noisy trades has a significant influence ($p > 0.005$) on the MAE. Both factors explain only 0.13% of the variance and reveal MCA values of less than 0.1. Thus, the bHSX is robust against noisy trading on both dimensions.

E. Summary of Results

Table VI summarizes the results of the four AMM across the four main criteria.

Each AMM has strengths and weaknesses in different dimensions, though LMSR and DPM offer clear advantages in terms of forecast accuracy and speed of information incorporation (i.e., how quickly new information is reflected in forecasts). Furthermore, LMSR and DPM are the only viable alternatives with regard to possible losses, having produced substantially fewer losses than DPA or bHSX. Evaluating the robustness of parameter selection (i.e., deviation of parameters from the optimal selection), LMSR and DPM are again superior to DPA and bHSX. The liquidity parameter in the case of the DPA was particularly sensitive to deviations, as was window length in the case of bHSX. However, with respect to the robustness against noisy trading (i.e., in case of many traders with imperfect information), bHSX outperforms the other AMMs. The results pertaining to DPA indicated a slightly lower robustness to noisy trading. Possessing this property is especially important when many poorly informed traders participate in markets or a significant degree of uncertainty surrounds the true values. To their fault, then, both LMSR and DPM were highly exposed to noisy trading. Apparently, there is a tradeoff between robustness against noisy trading and speed of information incorporation. In summary, LMSR and DPM outperform DPA and bHSX on all but one dimension.

V. Conclusion

Choosing an appropriate market mechanism is crucial for forecast accuracy and the success of any PM. Many applications show that standard CDAs are not well suited for PMs because of their liquidity problems, which implies the need for AMMs. Yet, the prior literature has only proposed and applied different AMMs; it has not compared their performance, with exception of [12], which only compares the LMSR with a new, not yet used AMM. Hence, providers of PMs have received little guidance for selecting among different AMMs.

Such a choice is far from being easy considering how strongly the available AMMs differ. The LMSR and DPM use continuous price functions, making them arbitrage free, while these two AMMs differ in the trader's knowledge of payoff values at the time of trade. By contrast, the DPA and bHSX mainly differ on this point: the former grants knowledge of the trading price at the time of the trade, while the trading price of the latter is only known after a certain period of time. However, previous research provides no support for choosing appropriate parameter values, nor insights on whether the choice of parameters strongly influences the performance of the AMM.

We are the first to propose a market microstructure that creates space for comparing AMMs of PMs. Our extensive set of simulations and the summary provided in Table VI reveal that no single best mechanism exists. Rather, each AMM has strengths and weaknesses in different dimensions. LMSR belongs to the group of most versatile AMMs that support multiple security designs. Moreover, LSMR delivers good forecasting results in most cases, making it the most complete AMM of all those we have analyzed.

The DPM also offers strength in its forecasting results, speed of information incorporation, losses and robustness against parameter misspecification. However, its major drawbacks include complex usability for traders (which might prevent less experienced participants from trading) and limited support of different market designs.

With DPA, the only real strength we detected lies in its offer of a fixed buy and sell price, which can be useful for inexperienced traders; however, the simulations returned weak results in most dimensions.

The bHSX, meanwhile, returned poor forecasting accuracy in the simulations. Granted, we did not use the complete mechanism due to having only partial information about this AMM; the actual HSX mechanism might employ many more constraints and parameters and, therefore, may provide greater accuracy. However, additional parameters would induce higher implementation costs in order to determine optimal parameter values. A major strength of the bHSX, which should also hold true for the HSX, is the AMM's high resistance to noisy trading, which might render this mechanism useful in markets containing heterogeneous, poorly informed traders.

We have to acknowledge several limitations, which provide avenues for future research. First, we did not account for the potential strategic behavior of traders, which might play a role in real-world settings [37]. For instance, manipulations can possibly occur. Future research should place special emphasis on analyzing this problem, since traders with great technical knowledge about market mechanisms might compete against traders with less technical knowledge. Further, traders' behavior may change due to learning, which is also a fruitful area for future research. Second, we assumed two types of traders: informed ones with perfect knowledge about the true share values, and uninformed ones, with a distribution around the true value. The assumption of perfect knowledge about true values is a strong one, compared to the assumption of limited uncertainty (i.e., a known distribution) of true values. However, because we varied the proportion of noisy traders, we believe this effect to be negligible. Third, we fixed the budget constraint. Future research may want to analyze how different levels of budget constraint impact market results: For example, it may be interesting to explore the relationship between the available budget and the AMM's required subsidy. Fourth, we limited our analysis to the AMMs which are already implemented in applications of PMs. Analyzing AMMs that have been developed but remain unapplied might prove insightful for PM applications. Examples of such AMMs include Das' promising AMM based on the Glosten-Milgrom model [35]; Brahma *et al.'s* [12] Bayesian MM; or Othman *et al.*'s [38] MM.

Appendix

A. Examples for Trading Actions

As in the simulations implemented herein, we assume a winner-takes-all market with two stocks, such that one stock describes an event (e.g., a Democrat becomes President), and the second stock describes a complementary event (e.g., a Republican becomes President). Share prices then reflect (implied) probabilities of the events to happen. All AMMs, if needed, begin with a prediction of 50% for both stock 1 and 2. The trading actions are "Buy 1 share of stock 1," "Buy 3 shares of stock 1," or "Sell 2 shares of stock 2." Share prices of both stocks do not generally equal 1 in DPA and bHSX, so we normalized both share prices after every price change to ensure that they do. In the case of the DPA, the shares of stock that have not been traded are adjusted by setting their prices to 1, minus the traded shares of stock. For bHSX, because the shares of both stocks can be traded, we normalize their prices by using their relative prices after the so-called "sweep phase." As we only want to illustrate how the AMMs work, we did not optimize the parameter values of the AMMs for the numerical example outlined as follows.

1) LMSR: The LMSR's only parameter, subsidy, is initially set to 50. There are no shares of stock in the market. Thus, according to (3) in the paper, the initial prices of the (infinitesimal) shares of stock are $p_i(\vec{q_0}) = \frac{\exp(q_{i,0}/b)}{\sum_{j\in N}\exp(q_{j,0}/b)} = \frac{\exp(0/50)}{\exp(0/50)+\exp(0/50)} = \$0.50$ each. Because the prices of shares of stock correspond to probabilities, the current probability of each event is 50%. For the first trading action, the costs of buying one share of stock 1, according to (1) and (2) in the paper, are

$$\begin{aligned}\mathrm{Costs}_{i,t}^{\mathrm{LMSR}}\left(q_{i,t}^{\mathrm{current}}\right) &= C\left(\vec{q_0}\right) - C\left(\vec{q_{t-1}}\right) = C\left(\vec{q_1}\right) \\ &\quad - C\left(\vec{q_0}\right) = C\left((1,0)\right) - C\left((0,0)\right) \\ &= 50 \cdot \ln(\exp(1/50) + \exp(0/50)) \\ &\quad - 50 \cdot \ln(\exp(0/50) + \exp(0/50)) \\ &= \$0.502\end{aligned}$$

with $q_{1,1}^{\mathrm{current}} = 1$. After the trade, there is one share of stock 1 on the market, and no shares of stock 2. According to (3), in this paper, the new price for the share and thus the probability that the event underlying stock 1 will happen is $p_1(\vec{q_1}) = \frac{\exp(1/50)}{\exp(1/50)+\exp(0/50)} = 0.505\$ \triangleq 50.5\%$. Accordingly, the new price for stock 2 is $p_2(\vec{q_1}) = \frac{\exp(0/50)}{\exp(0/50)+\exp(1/50)} = \$0.495 = \$1 - p_1(\vec{q_1}) = \$1 - \$0.505$. The next trade, "Buy 3 shares of stock 1," can then be computed, with costs of $\mathrm{Costs}_{1,2}^{\mathrm{LMSR}}(3) = C((4,0)) - C((1,0)) = \$1.537$, which drive the prediction of stock 1's underlying event happening to $p_1(\vec{q_2}) = \frac{\exp(4/50)}{\exp(4/50)+\exp(0/50)} = \$0.520 \triangleq 52\%$. In the next step, two shares of stock 2 sell at costs of $\mathrm{Costs}_{2,3}^{\mathrm{LMSR}}(-2) = C((4,-2)) - C((4,0)) = -\$0.950$. Because it is negative, the trader receives \$0.95 instead of having to pay for the shares in the case of a purchase. The new and final price of stock 1 is $p_1(\vec{q_3}) = \frac{\exp(4/50)}{\exp(4/50)+\exp(-2/50)} = \$0.530$, and event 1 (event 2) happens with a probability of 53% (47%). If the underlying event occurs, the shares' payoffs are \$1; otherwise, they are \$0.

2) DPM: The market begins with 100 shares per stock, which, according to (5) in the paper, corresponds to $C(\vec{q_0}) = M^{\mathrm{ini}} = \sqrt{\sum_{i\in N} q_{i,0}^2} = \sqrt{100^2 + 100^2} = \$141.421$, the seed money invested in the market. With 100 shares each, according to (6) in this paper, the current price for a share of stock 1 or 2 is $p_i(\vec{q_0}) = \frac{q_{i,0}}{\sqrt{\sum_{j\in N} q_{j,0}^2}} = \frac{100}{\sqrt{100^2+100^2}} = \$0.707$. However, because share prices do not correspond to probabilities and must be transformed, the current probability that the event underlying stock 1 or 2 will happen, according to (4) in this paper, is $\pi_i(\vec{q_t}) = \frac{q_{i,t}^2}{\sum_{j\in N} q_{j,t}^2} = \frac{100^2}{100^2+100^2} = 0.5 = 50\%$. Buying and selling for DPM is similar to the LMSR. According to (1) and (5) in this paper, buying one share costs:

$$\begin{aligned}\mathrm{Costs}_{i,t}^{\mathrm{DPM}}\left(q_{i,t}^{\mathrm{current}}\right) &= C\left(\vec{q_t}\right) - C\left(\vec{q_{t-1}}\right) \\ &= C\left(\vec{q_1}\right) - C\left(\vec{q_0}\right) \\ &= C\left((101,100)\right) - C\left((100,100)\right) \\ &= \sqrt{101^2 + 100^2} - \sqrt{100^2 + 100^2}. \\ &= \$0.709\end{aligned}$$

The new probability of the event is $\pi_i(\vec{q_2}) = \frac{q_{i,2}^2}{\sum_{j\in N} q_{j,2}^2} = \frac{101^2}{101^2+100^2} = 50.5\%$, and the remaining calculations work accordingly. The payoff, in contrast to that of the other AMMs, is not \$1 or \$0, but instead depends on the amount of money invested in the market. The total money in the market is, according to (5) in the paper, $C(\vec{q_t}) = \sqrt{\sum_{j\in N} q_{j,t}^2} = \sqrt{104^2 + 98^2} = \$142.9$. If event 1 occurs, with 104 shares of the corresponding stock on the market, the money gets split among all shares of stock 1, and the payoff is $\frac{M^{\mathrm{final}}}{q_{i,T}} = \frac{\$142.9}{104} = \$1.37$, but \$0 for all shares of stock 2. If event 2 occurs, all shares of stock 2 are valued at $\$\frac{142.9}{98} = \$1.46$.

3) DPA: In contrast to the LMSR, DPA requires that we initialize each stock explicitly with a price for a share of stock, or \$0.5. The liquidity parameter gamma is set to 15, and the length of the moving average window is set to 3. Because the prices are fixed, each share can be bought or sold at a predetermined price: \$0.5 for one share of stocks 1 or 2, and $\mathrm{Costs}_{i,t}^{\mathrm{DPA}}(q_{i,t}^{\mathrm{current}}) = q_{i,t}^{\mathrm{current}} \cdot p_{i,t-1} = 1 \cdot \$0.5 = \$0.5$. After the trade, the prices are adjusted according to (9) in this paper:

$$\begin{aligned}p_{i,t}\left(p_{i,t-1}, \vec{q_{i,t}^{\mathrm{lag}}}\right) &= p_{i,t-1} + \mathrm{sig}\left(q_{i,t,0}^{\mathrm{lag}}\right) \cdot \left|q_{i,t,0}^{\mathrm{lag}}\right| \cdot \frac{p_{i,\max}^2}{\gamma^2} \\ &\cdot \frac{\left(\sum_{\substack{k=0 \\ \mathrm{sig}\left(q_{i,t,0}^{lag}\right) = \mathrm{sig}\left(q_{i,t,k}^{\mathrm{lag}}\right)}}^{I_t} \left|q_{i,t,k}^{\mathrm{lag}}\right|\right)}{I_t + 1} = \$0.502 = p_{1,1}.\end{aligned}$$

Now the prices for the shares of the second stock can be adjusted, $p_{2,1} = \$1 - p_{1,1} = \$1 - \$0.502 = \$0.498$, which corresponds to probabilities of 50.2% and 49.8% for event 1 and event 2. In the second trading action, a trader can buy three shares of stock 1 for the new fixed price of $\mathrm{Costs}_{1,2}^{\mathrm{DPA}}(3) = 3 \cdot \$0.502 = \$1.506$. Thereafter, the price of shares of stock 1 adjusts again to $p_{1,2} = \$0.502 + \$1 \cdot |3| \cdot \frac{1}{15^2} \cdot \frac{|1|+|3|}{3} = \$0.52$, and share prices of stock 2 also adjust. For the last trade, two shares of stock 2 sell for \$0.48 each: $\mathrm{Costs}_{2,3}^{\mathrm{DPA}}(-2) = (-2) \cdot \$0.48 = -\$0.96$. The price is then adjusted, such that $p_{2,3} = \$0.48 - \$1 \cdot |-2| \cdot \frac{1}{15^2} \cdot \frac{|2|}{4} = \$0.476 = \$1 - \$0.524 = \$1 - p_{1,3}$, which drives the last predictions to 52.4% and 47.6%, respectively. The shares' payoffs are either \$1 or \$0, depending on which event occurs.

4) bHSX: Similar to the DPA, share prices are initialized to \$0.5, and the liquidity parameter alpha is set to 0.01. During the sweep phase, orders are accumulated but not executed. Therefore, the bHSX receives two buy orders for $1 + 3 = 4$ shares of stock 1 and one sell order of two shares of stock 2. During this phase, the final price at which the shares are traded is not known to the traders. At the end of the sweep phase, which has a length of 3 periods, the final trading price can be determined (see (10) in this paper):

$$p_{i,s}\left(p_{j,s-1}, \vec{q_{i,s-1}^{\mathrm{sweep}}}\right) = p_{i,s-1} + \alpha \cdot \sum_{z=1}^{\left|\vec{q_{i,s-1}^{\mathrm{sweep}}}\right|} q_{i,s-1,z}^{\mathrm{sweep}} = \$0.54 = p_{1,1}$$

TABLE VII
DESIGN OF SIMULATION STUDY

Factor	# Levels	Levels	Specifications
Number of trading periods	1	Fixed	300
Expected volatility in true value	3	None	True value constant and ~ U(0,1)
		Low	True value jumps in period 101 to 0.5 + d ~ N(0,0.1)
		High	True value jumps in period 101 from to 0.5 + d ~ N(0,0.25)
Proportion of noisy traders	2	Low	Noisy to informed traders: 1:2
		High	Noisy to informed traders: 2:1
Degree of noise of noisy traders	2	Low	Deviation of noisy traders from true value ~ N(0,0.02)
		High	Deviation of noisy traders from true value ~ N(0,0.05)
Max. amount to spend/redeem	1	Fixed	~ N($5, $1)
Type of transaction	1	Fixed	Type ~ Ber(0.6)
Criterion	36		1. Optimal parameters for minimal error - 1 for LMSR - 1 for DPM - 3 for DPA (lengths window) - 3 for bHSX (lengths window) 2.1. Deviation from optimal parameters - 28 (7 deviations per AMM) 2.2. Noisy trading - 0 (data from criterion 1) 3. Speed of information incorporation - 0 (data from criterion 1)
Number of cases	3 × 2 × 2 × 36 = 432		
Replications per case	500		
Total number of observations	432 × 500 = **216,000**		

AMM: automated market maker; LMSR: logarithmic market scoring rules; DPM: dynamic pari-mutuel market; DPA: dynamic price adjustments; bHSX: basic Hollywood stock exchange mechanism.
N: normal distribution; Ber: Bernoulli distribution; U: uniform distribution.

for the shares of the first stock. For the shares of the second stock, the calculation is $p_{2,1} = \$0.5 + \$0.01 \cdot (-2) = \$0.48$. Because the prices of the two shares must sum to 1, we normalize their share prices, such that $p_{1,1}^{\text{final}} = \frac{p_{1,1}}{p_{1,1}+p_{2,1}} = \frac{\$0.54}{\$0.54+\$0.48} = \$0.529 = \$1 - p_{2,1}^{\text{final}}$. Thus, the final execution price for buying and selling shares in this sweep phase is \$0.529 for a share of stock 1 and \$0.471 for a share of stock 2, which implies final predictions of 52.9% and 47.1%, respectively. The costs for the first trade are $\text{Costs}_{i,t}^{\text{bHSX}}\left(q_{i,t}^{\text{current}}\right) = q_{i,t}^{\text{current}} \cdot p_{i,s} = \$0.529$; those for the second trade are $\text{Costs}_{1,2}^{\text{bHSX}}(3) = 3 \cdot \$0.529 = 1.587$; and those for the third trade are $\text{Costs}_{2,3}^{\text{bHSX}}(-2) = -\$0.942$. The payoffs are the same as those for LMSR and DPA.

B. Detailed Setup of Market Environment

To test the mechanisms' robustness against noisy trading, we analyzed the effects of two dimensions: the proportion of noisy traders and the degree of noise when a trader receives a noisy signal. We assumed low and high levels of the proportion of noisy traders in the trading crowd. At the low level, there were twice as many informed traders as there were noisy traders; at the high level, the situation was reversed. For the degree of noise, we used a distribution of deviation from the true value of $\sim N(0,0.05)$, as in [35]. We left this setting as our high level and specified the low level as $\sim N(0,0.02)$.

The budget constraint c_t (see (14) in this paper) is drawn from a normal distribution. We set the mean of the distribution to \$5, which was sufficiently high for avoiding an effect at different levels of share prices on the trading outcome. For a small deviation from \$5, we left the standard deviation rather low at \$1. The type of transaction, buy or sell, must be specified for all periods, so we also randomly determined this type. Because the DPM is sensitive to the amount of money in the market, we simulated increasing numbers of traders by setting the probability of a purchase to $\Pr(\text{Type} = 1 = \text{purchase}) = 0.6$ versus $\Pr(\text{Type} = 0 = \text{sale}) = 0.4$. Thus, this type is Bernoulli distributed, such that $\text{Type} \sim \text{Ber}(0.6)$.

For the first criterion, we optimized one (liquidity) parameter for LMSR and DPM. However, we had three window lengths in the case of the DPA and bHSX (5, 10, and 20 periods), so we had to also optimize the liquidity parameters individually for each window length. In Table VII, we display only the optimal liquidity parameters. The (much larger) number of parameter combinations required to arrive at this optimal point is omitted for simplicity. We also used the optimal liquidity parameters (for all AMMs) and optimal liquidity parameter/length of window combination to deviate from these values, which revealed seven cases for each AMM and 28 total cases. The data come from the first criterion. To reduce variance and decrease the number of replications, we employed common random numbers for each factor. We set the number of replications to 500, which allowed for a wide range of possible trading situations. Thus, we obtained a total of $3 \times 2 \times 2 \times 36 \times 6000 = 216\ 000$ observations.

With regard to the first criterion, we acquired three optimal parameters (parameter combinations for DPA/bHSX) per AMM for each volatility case, so that we could optimize the parameters for three different volatility cases. The global optimum is a function of the liquidity parameter, and the resulting MAE is convex. Too much liquidity moves prices too slowly and increases the error, whereas insufficiently low liquidity moves prices too fast, which also increases the error. We conducted the parameter optimization by choosing a very low minimum and very high maximum liquidity value, and then used nested intervals to obtain the optimal solution.

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