Counting Problem of *PureCircuit*

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Abstract

Index Terms

Complexity Theory, Counting Complexity, Hazard-Free Circuits, PPAD, Search Problems

I. Introduction

Over the last couple of years, there has been a revolutionary initiative in the field of combinatronics. Combinatronics has been a field of study in mathematics that primarly focused on the notion of counting objects with certain properties. Over time, this notion has shifted, especially in the subfield of algebraic combinatronics, where there is no clear notion of the object that we are counting and the numbers express something more abstract [1]. This gave a need to be able to assign a combinatorial interpretation to such numbers, or more simply, do these numbers correpsond to some counting over a set of objects. Being able to find such definitions or interpretations can be very important, it allows us to utilise tools from combinatronics as well as allow us to understand and reveal hidden structures and properties for such numbers [1]. Moreover there are several problems or numbers such as *Kronecker coefficients*, whose combinatorial interpretation, would give a step towards the resolution of the *P* $\neq$ *NP* conjecture [2].

To reiterate the previous statement, we can understand combinatorial interpretation as the process of: given a sequence of numbers $\{a_x\}$, find a set of combinatorial objects A_x such that $|A_x| = a_x$ To formalise the current idea, Igor Pak et al. has concluded that $f \in$ *#P*, implies that f has a combinatorial interpretation [1], [2]. We will explore this idea in much greater detail in the upcoming section, but the main benefit is the ability to use a very expressive but formal language that encapsulates this notion of a combinatorial interpretation.

In our current work, we focus on extending the work done by Ikenmeyer et al., where they focused on the creation of frameworks that determine whether $f \in^{?}$ **#P**, by looking at the complexity class of ***#****TFNP -1*. This is a class of problems that are guaranteed to have a solution and their solutions are verifiable in polynomial time. In their paper, they were able to show that for a subclass of problems, also known as *PPAD* $\subseteq$ *TFNP*, different *PPAD-complete* problems, may or may not have a combinatorial interpretation. Our contribution, comes to the analysis of a specific problem, known as PureCircuit, which utilises Kleene logic, to find satisfying assingments in sequential circuits. We hope to demonstrate that such problem could help us bound, the counting complexity limits of #PureCircuit $-$ 1.

We define the following updated list of software and research objectives:

R1 Test 1
R2 Test 2
R3 Test 3

II. Aims and Objectives

III. Preliminaries and Background review

IV. Research Methodology

V. Project Plan

VI. Conclusion

References

[1] I. Pak, “What is a combinatorial interpretation?” Sep. 2022.
[2] C. Ikenmeyer and I. Pak, “What is in #P and what is not?” in *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, Oct. 2022, pp. 860–871.