Counting Problem of *PureCircuit*

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***Abstract*—Test.Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.**

***Index Terms*—Complexity Theory, Counting Complexity, Hazard Free Circuits, TFNP, PPAD, Search Problems, Kleene Logic**

I. Introduction

Over the last couple of years, there has been a revolutionary initiative in the field of combinatronics. Combinatronics has been a field of study in mathematics that primarly focused on the notion of counting objects with certain properties. Over time, this notion has shifted, especially in the subfield of algebraic combinatronics, where there is no clear notion of the object that we are counting and the numbers express something more abstract [1]. This gave a need to be able to assign a combinatorial interpretation to such numbers, or more simply, do these numbers correpsond to some counting over a set of objects. Being able to find such definitions or interpretations can be very important, it allows us to utilise tools from combinatronics as well as allow us to understand and reveal hidden structures and properties for such numbers [1]. Moreover there are several problems or numbers such as *Kronecker coefficients*, whose combinatorial interpretation, would give a step towards the resolution of the $P \neq NP$ conjecture [2].

To reiterate the previous statement, we can understand combinatorial interpretation as the process of: given a sequence of numbers $\{a_x\}$, find a set of combinatorial objects A_x such that $|A_x| = a_x$ To formalise the current idea, Igor Pak et al. has concluded that $f \in$ *#P*, implies that f has a combinatorial interpretation [1], [2]. We will explore this idea in much greater detail in the upcoming section, but the main benefit is the ability to use a very expressive but formal language that encapsulates this notion of a combinatorial interpretation.

In our current work, we focus on extending the work done by Ikenmeyer et al., where they focused on the creation of frameworks that determine whether $f \in^?$ **#P**, by looking at the complexity class of **#***TFNP -1*. This is a class of problems that are guaranteed to have a solution and their solutions are verifiable in polynomial time. In their paper, they were able to show that for a subclass of problems, also known as *PPAD* $\subseteq$ *TFNP*, different *PPAD-complete* problems, may or may not have a combinatorial interpretation. Our contribution, comes to the analysis of a specific problem, known as PureCircuit, which utilises Kleene logic, to find satisfying assingments in sequential circuits. We hope to demonstrate that such problem could help us bound, the counting complexity limits of #PureCircuit $- 1$.

A. Project objectives

Below we will present our table of objectives. We will denote updated objectives with *, new objectives with (!), completed objectives with (✓), deleted objectives with (-).

R.1) (✓) Find a parsimonious reduction from the *EndOfLine* to *EndOfLine*.
R.2) (!) Improve the combinatorial bound between the reduction from EndOfLine to PureCircuit
R.3) (!) Demonstrate that #PPAD$(PureCircuit) - 1 \nsubseteq$ **#P**
R.4) (*) Prove or disprove the following $\forall n \in \mathbb{N}_{\geq 2}$:

$$\exists c \in \mathbb{N} : \#\textsc{SourceOrExcess}(n, 1) \subseteq^c \#\textsc{PureCircuit}$$

R.5) (*) Prove or disprove the following claim: $\forall L \in$ **PPAD**

$$\exists c \in \mathbb{N} : \#L \subseteq^c \#\textsc{PureCircuit}$$

Below we will be representing the development portion of the project.

S.1)
S.2) Test 2
S.3) Test 3

II. Aims and Objectives

III. Preliminaries and Background review

IV. Research Methodology

V. Project Plan

VI. Conclusion

References

[1] I. Pak, “What is a combinatorial interpretation?” Sep. 2022.
[2] C. Ikenmeyer and I. Pak, “What is in #P and what is not?” in *2022 IEEE 63rd Annual Symposium on Foundations of Computer Science (FOCS)*, Oct. 2022, pp. 860–871.