Technical Report for Sample Code

Technical Report STARS

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Contents

1 Problem Specification

The problem is to accurately estimate the state of a mass being carried by a balloon using an onboard accelerometer and GPS. The problem configuration and coordinate frame can be seen in 1. The sample code consists of the following four ROS nodes:

1. **System simulation node:** simulates the system dynamics.

2. **Accelerometer simulation node:** simulates noisy accelerometer measurements based on the system's current position.

3. **GPS simulation node:** simulates noisy GPS measurements based on the system's current position.

4. **State estimation node:** receives noisy accelerometer and GPS measurements at different different rates and provides a state estimate at a user specified update rate.

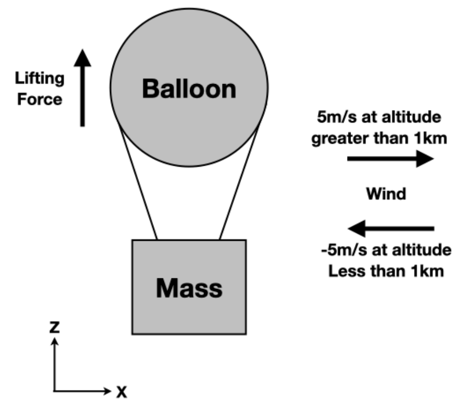

Figure 1: Problem diagram. A mass is lifted using a balloon. The problem solved by this sample code is estimating the state of the mass as it is moved by the lifting force of the balloon and drag force of wind. Noisy accelerometer and GPS measurements are available.

The available details are the following:

- The mass being carried is $100kg$ and the balloon skin is $20kg$.
- A lifting force of $1500N$ exists from the gas inside the balloon.
- The balloon Cd is 0.35
- A drag force from wind exists on the balloon. The wind speed is $5m/s$ in the negative x direction at altitudes lower than $1000m$ and $5m/s$ in the positive x direction at altitudes greater than $1000m$.
- The system process noise is zero mean with a $5m$ standard deviation.
- The GPS provides x and z position measurements that are zero mean with a $60m$ standard deviation and a $1Hz$ update rate.
- The accelerometer provides x and z acceleration measurements that are zero mean with a $0.1g$ standard deviation and a $40Hz$ update rate.

2 Preliminaries

The balloon is exerting a lifting force of $1500N$. If we assume an air temperature of 10°C (air density at that temperature is $P_{10}^{air} = 1.25$ kg/m^3). Therefore the balloon volume V_b [m^3] can be determined (from $F_b = PVg$) as,

$$V_b = \frac{F_b}{P_{10}^{air} g} = \frac{1500}{1.25 * 9.8} = 122.4 \tag{1}$$

Since the design specification states that the ballon should be assumed spherical, the radius of the balloon r_b [m] is,

$$r_b = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}} = 3.08 \tag{2}$$

and the largest cross-sectional area A_c [m^2] is,

$$A_c = \pi r_b^2 = 29.8 \tag{3}$$

3 System Model and State Estimation

3.1 Equations of Motion

The equations of motion in the x and z directions respectively are[1],

$$m\ddot{x} = F_{drag,x} = C_d \frac{1}{2}\rho(v_x - v_{wind})^2 \tag{4}$$

$$m\ddot{z} = F_{lift} - F_{drag,z} - F_{grav} = F_{lift} - C_d \frac{1}{2}\rho(v_z)^2 A_c - mg \tag{5}$$

3.2 Simulated Noise-Free Dynamics

Using Equation 4 and Equation 5 the noise free "true" dynamics are simulated to produce Figure 2.

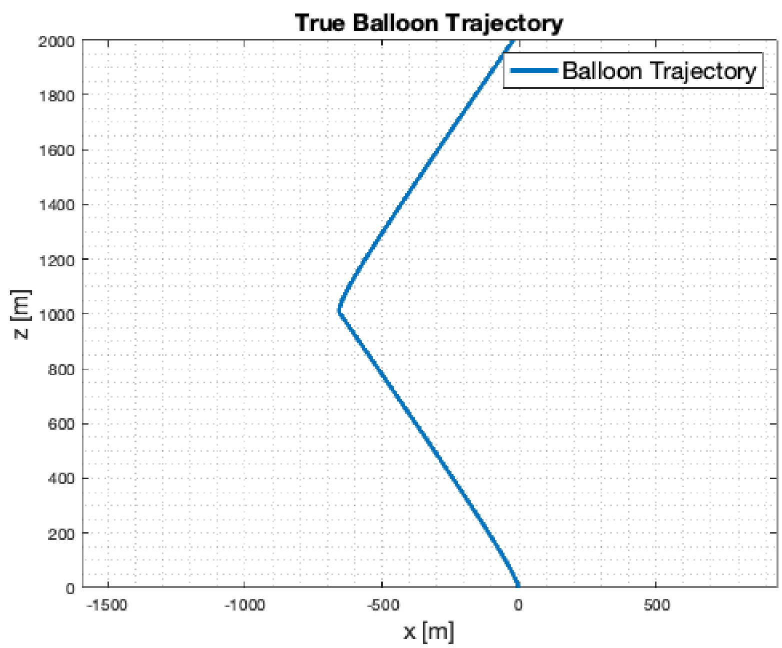

Figure 2: Plot of the true balloon dynamics without noise. The trajectory is simulated to an altitude of 2 km.

3.3 Simulate Noisy Measurements

Zero-mean GPS and accelerometer readings are simulated on the system with zero mean process noise (standard deviation of $5m$). The GPS and accelerometer measurements have a standard deviation of 60 m and $0.1g$ m/s^2 respectively. The noisy measurements over time are shown in Figure 3.

3.4 Estimation

3.4.1 Models

The system state is first defined as 2D position and velocity,

$$\mathbf{x} = \begin{bmatrix} x & z & \dot{x} & \dot{z} \end{bmatrix}^T \tag{6}$$

[1]The wind direction is assumed to be in the x direction as the figure does not appear to specify a wind direction

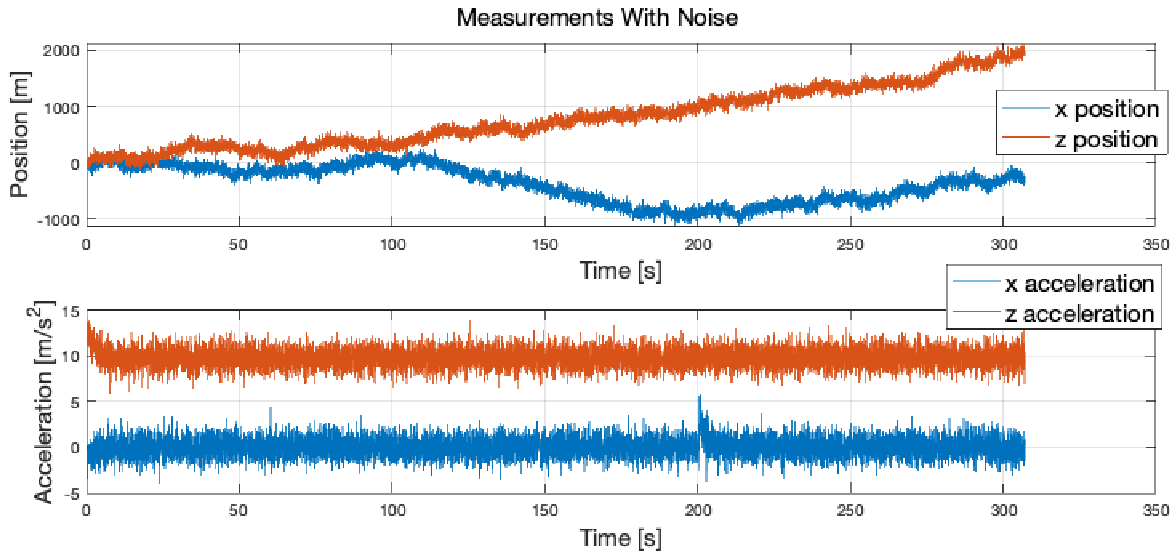

Figure 3: Plot of the noisy position and acceleration measurements from the GPS and accelerometers respectively.

Next the kinematic motion and measurement model for the system are defined. The motion model is,

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}(\mathbf{a}_{m,k} - \mathbf{g}) + \mathbf{m}_k \tag{7}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$\mathbf{B} = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 \\ 0 & \frac{\Delta t^2}{2} \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \tag{9}$$

where $\mathbf{m}_k \sim \mathcal{N}(0, \mathbf{Q}_k)$ and $\mathbf{Q}_k$ is the process covariance and the interceptive (accelerometer) measurement is,

$$\mathbf{a}_{m,k} = \mathbf{a}_k + \mathbf{g} + \mathbf{v}_k \tag{10}$$

where $\mathbf{g}$ is gravity, $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{V}_k)$ and $\mathbf{V}_k$ is interoceptive measurement covariance. The exteroceptive (GPS) measurement model is,

$$\mathbf{h}_k = \begin{bmatrix} h_{1,k} \\ h_{2,k} \end{bmatrix} = \mathbf{C}\mathbf{x}_k + \mathbf{n}_k \tag{11}$$

$$\mathbf{C} = \mathbf{I}_{2\times 2} \tag{12}$$

where $\mathbf{n}_k \sim \mathcal{N}(0, \mathbf{R}_k)$ and $\mathbf{R}_k$ is the exteroceptive measurement covariance.

3.4.2 Estimator

The state and state covariance prediction equations are,

$$\check{\mathbf{P}}_k = \mathbf{A}\hat{\mathbf{P}}_{k-1}\mathbf{A}^T + \tilde{\mathbf{Q}}_{\mathbf{k}} \tag{13}$$

$$\check{\mathbf{x}} = \mathbf{A}\check{\mathbf{x}}_{k-1} + \mathbf{B}(\mathbf{a}_{m,k} - \mathbf{g}) \tag{14}$$

The Kalman gain is,

$$\mathbf{K}_k = \check{\mathbf{P}}_k \mathbf{C}^T (\mathbf{C}\check{\mathbf{P}}_k \mathbf{C}^T + \mathbf{R}_k)^{-1} \tag{15}$$

The state and state covariance correction is,

$$\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{C})\check{\mathbf{P}}_k \tag{16}$$

$$\hat{\mathbf{x}}_k = \check{\mathbf{x}}_k + \mathbf{K}_k(\mathbf{h}_k - \mathbf{C}\check{\mathbf{x}}_k) \tag{17}$$

Notably missing a definition is the process covariance $\tilde{\mathbf{Q}}$. This covariance can be tailored to the characteristics of the problem. In the kinematic model defined in Equation 7 both both process noise and interoceptive measurement noise contribute to $\tilde{\mathbf{Q}}$. The following will unify these noise quantities into a single process noise. First by substituting Equation 10 into Equation 7,

$$\tilde{\mathbf{m}} = \mathbf{B}\mathbf{v}_k + \mathbf{m}_k \tag{18}$$

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{a}_k + \tilde{\mathbf{m}}_k \tag{19}$$

Now obtaining the covariance $\tilde{\mathbf{Q}}_k$,

$$\tilde{\mathbf{Q}}_k = \mathbb{E}[\tilde{\mathbf{m}}\tilde{\mathbf{m}}^T] = \mathbf{B}\mathbb{E}[\mathbf{v}_k\mathbf{v}_k^T]\mathbf{B}^T + \mathbb{E}[\mathbf{m}_k\mathbf{m}_k^T] = \mathbf{B}\mathbf{W}_k\mathbf{B}^T + \mathbf{Q}_k \tag{20}$$

where the interoceptive covariance is,

$$\mathbf{W}_k = \begin{bmatrix} \sigma_{ax}^2 & 0 \\ 0 & \sigma_{az}^2 \end{bmatrix} \tag{21}$$

and I use a constant velocity process for $\mathbf{Q}_k$.

$$\mathbf{Q}_k = q \begin{bmatrix} \frac{\Delta t^3}{3} & 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & \frac{\Delta t^3}{3} & 0 & \frac{\Delta t^2}{2} \\ \frac{\Delta t^2}{2} & 0 & \Delta t & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 & \Delta t \end{bmatrix} \tag{22}$$

where q is a scaling factor. Other methods for process covariance exist and their performance depends on the trajectory. In the balloon test case, velocity is near constant for significant portions of the flight. For this reason the estimator performed better with $\tilde{\mathbf{Q}}_k$ simply equal to the constant velocity process $\mathbf{Q}_k$.

3.4.3 Results

The estimation and ground truth balloon positions are shown in Figure 4.

3.5 Methods of Improvement

- Include change in air pressure with altitude which will change the size of balloon and thus the drag and lifting forces as well.
- Account for change in gravity with altitude (minimal change for 2000 m max considered in this work).
- Incorporate prior wind knowledge and/or wind estimation into the state estimator. This would allow for a drag estimation leading to improved state propagation.

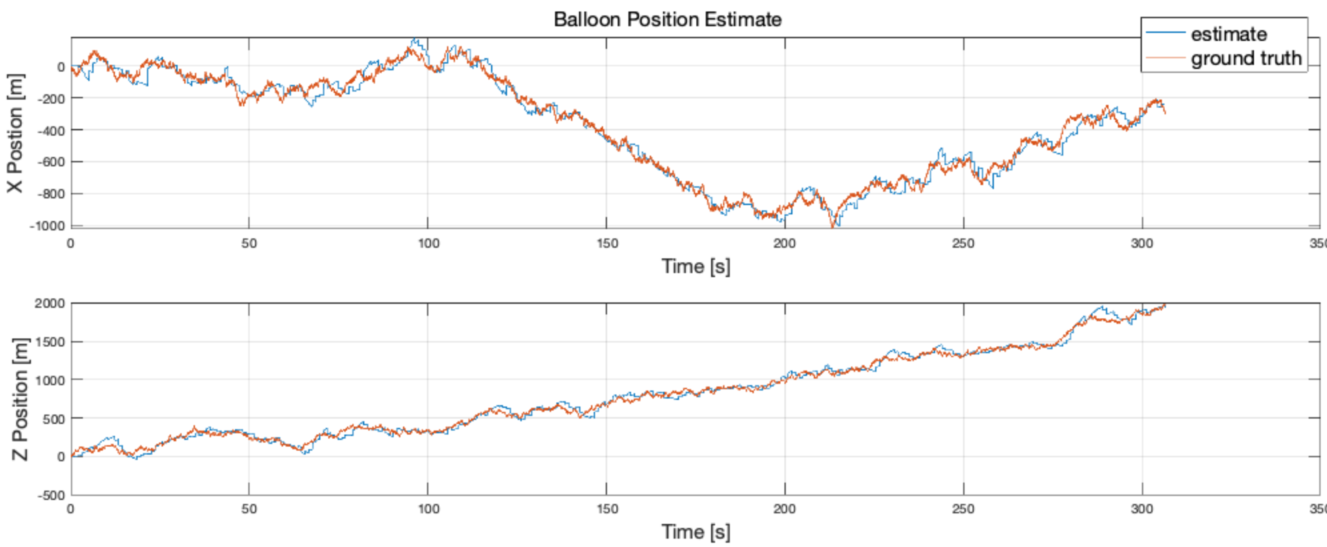

Figure 4: Plot of the estimated and ground truth ballon position.