Theorem (2.3.47a)**.** *Let x be a real number, and let n be an integer.* $x < n \iff \lfloor x \rfloor < n$.

Proof. $\lfloor x \rfloor \leq x$, by the properties of the floor function. So if $x < n$, then $\lfloor x \rfloor \leq x < n$, and of course $\lfloor x \rfloor < n$.

Proving the converse, suppose $\lfloor x \rfloor < n$. Since $\lfloor x \rfloor$ and n are integers, $\lfloor x \rfloor + 1 \leq n$. Now, by the properties of the floor function, we have the following tautology, $\lfloor x \rfloor = \lfloor x \rfloor \iff \lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$. Since we know that $\lfloor x \rfloor + 1 \leq n$, it must be that $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1 \leq n$. This statement says that $x < n$. ∎