Theorem (3.2.17)**.** *Let f, g, and h be functions such that f(x) is $\mathcal{O}(g(x))$, and g(x) is $\mathcal{O}(h(x))$. f(x) is $\mathcal{O}(h(x))$.*

Proof. $f(x)$ is $\mathcal{O}(h(x))$ trivially follows from the definition of big-O. If $f(x)$ is $\mathcal{O}(g(x))$, then there exist constant witnesses C and k such that $|f(x)| \leq C|g(x)|$, for all $x > k$. Likewise, if $g(x)$ is $\mathcal{O}(h(x))$, then there exist constant witnesses C' and k such that $|g(x)| \leq C'|h(x)|$, for all $x > k$. Thus, $C|g(x)| \leq C \cdot C'|h(x)|$, for all $x > k$. It follows that $|f(x)| \leq C|g(x)| \leq C \cdot C'|h(x)|$, for all $x > k$. Let $C'' = C \cdot C'$. Then $|f(x)| \leq C''|h(x)|$, for all $x > k$. Hence $f(x)$ is $\mathcal{O}(h(x))$ with constant witnesses C'' and k. ∎