Theorem (3.2.23b)**.** *Let f be the function defined by $f(x) = x^2 + 1000$. $f(x)$ is $\Theta(x^2)$.*

Proof. Obviously $f(x) = x^2 + 1000 \geq x^2$, for all $x \in \mathbb{R}$. So $f(x)$ is $\Omega(x^2)$ with constant witnesses $C = 1$ and any $k \in \mathbb{R}$. Now, $x^2 + x^2$ cannot exceed $x^2 + 1000$ unless $x \geq \lceil\sqrt{1000}\rceil = 32$. Thus, $|f(x)| \leq 2|x^2|$ for all $x > 32$. So $f(x)$ is $\mathcal{O}(x^2)$ with constant witnesses $C = 2$ and $k = 32$. Finally, we have $|x^2| \leq |f(x)| \leq 2|x^2|$, for all $x > 32$. Therefore, $f(x)$ is $\Theta(x^2)$ with constant witnesses $C_1 = 1$, $C_2 = 2$, and $k = 32$. ■