

Theorem (3.2.20b). *Let f be the function defined by $f(n) = (2^n + n^2)(n^3 + 3^n)$. $f(x)$ is $\mathcal{O}(6^n)$.*

Proof. Let g be the function defined by $g(n) = 6^n$. If $n \geq 4$, then

$$f(n) = (2^n n^3 + 2^n 3^n + n^2 n^3 + 3^n n^2) \leq (2^n 3^n + 2^n 3^n + 2^n 3^n + 2^n 3^n).$$

$4(2^n 3^n) = 4(6^n)$, so $f(n)$ is $\mathcal{O}(6^n)$ with constant witnesses $C = 4$ and $k = 4$. ■