Theorem (1.6.9)**.** *The sum of an irrational number and a rational number is irrational.*

Proof. By contradiction. Suppose that m and n are rational numbers. By definition, there exist integers a, b, c, and d such that $m = \frac{a}{b}$ and $n = \frac{c}{d}$. Let x be an irrational number such that the sum of a rational number and an irrational number can be expressed as $m + x = n$ and $n + (-m) = x$. In terms of a, b, c, and d we have $\frac{-a}{b} + \frac{c}{d} = \frac{-ad}{bd} + \frac{cb}{bd} = \frac{-ad+cb}{bd} = x$. Note that the sum of products of integers is an integer. But this is impossible because x is irrational; thus a contradiction. ∎