Theorem (3.2.13). *Let f be the function defined by $f(n) = 2^n$, and let g be the function defined by $g(n) = 3^n$. $f(n)$ is $\mathcal{O}(g(n))$, but $g(n)$ is not $\mathcal{O}(f(n))$.*

Proof. $f(n)$ is $\mathcal{O}(g(n))$ trivially follows from the fact that $|2^n| \leq |3^n|$, for all $n > 1$.

If it were that $g(x) \in \mathcal{O}(f(x))$, then there would exists constant witnesses C and k such that $3^n \leq C \cdot 2^n$, for all $n > k$. This inequality also means that $\left(\frac{3}{2}\right)^n \leq C$, for all $n > k$. Clearly, no constant C exists such that $\left(\frac{3}{2}\right)^n \leq C$, for all $n > k$. Thus, $g(n) \notin \mathcal{O}(f(n))$. ∎