Theorem (3.2.1f)**.** *Let f be the function defined by $f(x) = \lceil \frac{x}{2} \rceil$. $f(x)$ is $\mathcal{O}(x)$.*

Proof. Let g be the function defined by $g(x) = x$. By the properties for ceiling functions there exists an integer $\lceil \frac{x}{2} \rceil = n$ such that $(2n-2) < x \leq 2n$. Clearly, $n \leq (2n-2)$ for all $n \geq 2$. Also, $n = \lceil \frac{x}{2} \rceil \geq 2$ whenever $x > 2$. So we have $\lceil \frac{x}{2} \rceil \leq (2n-2) < x$, for all $x > 2$. From that, obviously $\lceil \frac{x}{2} \rceil \leq x$, for all $x > 2$, and since $x > 2$ we know that $|\lceil \frac{x}{2} \rceil| \leq 1|x|$ holds. By the definitions for f and g that is, $|f(x)| \leq 1|g(x)|$, for all $x > 2$. Therefore $f(x)$ is $\mathcal{O}(x)$ with constant witnesses $C = 1$, and $k = 2$. ∎