Theorem (3.2.6)**.** *Let f be the function defined by $f(x) = \frac{x^3+2x}{2x+1}$. $f(x)$ is $\mathcal{O}(x^2)$.*

Proof. Let g be the function defined by $g(x) = x^2$. If $x \geq 1$, then

$$f(x) = \left(\frac{x^3+2x}{2x+1}\right) \leq \left(\frac{x^3+2x}{2x} = \frac{x^3}{2x} + 1\right) \leq \left(\frac{x^3}{x} + 1\right) \leq (x^2 + 1).$$

Clearly, if $x > 1$, then $x^2 + 1 \leq 2x^2$. So $|f(x)| \leq 2|g(x)|$, for all $x > 1$. It follows from the definition that $f(x)$ is $\mathcal{O}(x^2)$ with constant witnesses $C = 2$, and $k = 1$. ■