

Theorem (1.6.6). *The product of two odd numbers is odd.*

Proof. Suppose that x and y are odd numbers. By definition, there exist integers m and n such that $x = 2m+1$ and $y = 2n+1$. $xy = (2m+1)(2n+1) = 2m2n + 2m + 2n + 1 = 2(mn + m + n) + 1$. $mn + m + n$ is an integer because the sum of integers is an integer. Thus, xy is odd by definition. ■