

**Theorem (2.3.69a).** *Let  $x$  be a real number.  $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$ .*

*Proof.* Let  $n$  be the integer such that  $n \leq x < n + 1$ . By the properties for floor functions,  $\lfloor x \rfloor = n$ . So  $\lceil \lfloor x \rfloor \rceil = \lceil n \rceil$ . Since  $n - 1 < n \leq n$  is a tautology, by the properties for ceiling functions it must be the case that  $\lceil n \rceil = n$ . But  $n = \lfloor x \rfloor$ , so  $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$ . ■