

Theorem (3.2.3). *Let f be the function defined by $f(x) = x^4 + 9x^3 + 4x + 7$. $f(x)$ is $\mathcal{O}(x^4)$.*

Proof. Let g be the function defined by $g(x) = x^4$. If $x \geq 2$, then $x^4 + 9x^3 + 4x + 7 \leq x^4 + 9x^4 + 4x^4 + x^4 = 15x^4$. Thus, $|f(x)| \leq 15|g(x)|$, for all $x > 2$. It immediately follows from the definition that $f(x)$ is $\mathcal{O}(x^4)$ with constant witnesses $C = 15$, and $k = 2$. ■