Theorem (2.3.66)**.** *Let f be the invertible function $f : Y \implies Z$, and let g be the invertible function $g : X \implies Y$. The inverse of the composition $f \circ g$ is given by $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.*

Proof. By Theorem 2.3.29a and Theorem 2.3.29b, and by the definition for bijective functions, $f \circ g$ is invertible. Thus, $(f \circ g)^{-1} \circ (f \circ g) = \iota_X$.

What remains to be determined is whether $(g^{-1} \circ f^{-1}) \circ (f \circ g) = \iota_X$. Let x be an element in the domain of g such that $((g^{-1} \circ f^{-1}) \circ (f \circ g))(x) = x$. By the definition for the composition of functions, that is $g^{-1}(f^{-1}(f(g(x)))) = x$. Clearly, $(g^{-1} \circ f^{-1}) \circ (f \circ g) = \iota_X$.

Thus, the inverse of the composition $f \circ g$ is indeed given by
$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

■