Theorem (2.4.20)**.** $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$.

Proof. $\sum_{k=1}^{n} \frac{1}{k(k+1)} = \sum_{k=1}^{n}(\frac{1}{k} - \frac{1}{k+1}) =$
$(\frac{1}{n} - \frac{1}{n+1}) + (\frac{1}{n-1} - \frac{1}{n}) + (\frac{1}{n-2} - \frac{1}{n-1}) + \cdots + (\frac{1}{1} - \frac{1}{2})$ By Theorem 2.4.19, that is, $-\frac{1}{n+1} + \frac{1}{1} = \frac{(-1)+(n+1)}{n+1} = \frac{n}{n+1}$. ∎