Theorem (1.6.17)**.** *Let n be an integer. If $n^3 + 5$ is odd, then n is even.*

Proof. By the contrapositive. Suppose that n is odd. By definition there exists and integer k such that $n = 2k + 1$. By the Binomial Theorem, $(2k+1)^3 + 5 = 5 + \sum_{i=0}^{3} \binom{3}{i} 2k^{(3-i)} = 2(4k^3 - 6k^2 + 3k + 3)$. That is an integer factor with a coefficient of 2, even by definition. ■