Theorem (2.4.43). *The set of all finite bit strings is countable.*

Proof. Let $\{a_{n-1}\}$ be the sequence of bits for any finite bit string a(base-2) of length n. The unique base-2 expansion for $\{a_{n-1}\}$ is the integer $a(\text{base-10}) = \sum_{i\in\mathbb{N}}^{n-1} a_i 2^i$. Also, this integer can be converted to the unique base-2 bit string for a(base-10) by $a(\text{base-2}) = \sum_{i\in\mathbb{N}}^{n-1} [a(\text{base-10})(\text{mod } 2^{i+1})]10^i$. Hence, there exists a one-to-one correspondence between $\mathbb{Z}$ and the set of all finite bit strings. So the cardinality for the set of all finite bit strings is $\aleph_0$. It follows that the set of all finite bit strings is countably infinite, by definition. ∎