Theorem (2.2.41)**.** *Let A, B, and C be sets. If $A \oplus C = B \oplus C$, then $A = B$.*

Proof. By contraposition. Note that the statement $A \oplus C = B \oplus C$ is by definition $(A \cap \overline{C}) \cup (\overline{A} \cap C) = (B \cap \overline{C}) \cup (\overline{B} \cap C)$. Assume there exists an element x such that $x \in A$ and $x \notin B$. Thus, $A \not\subseteq B$. By the hypothesis, x has to be in $(A \cap \overline{C})$ and cannot be in $(\overline{A} \cap C)$. This means that x is not in C. Neither can x be in $(B \cap \overline{C})$. And since $x \notin C$, x cannot be in $(\overline{B} \cap C)$. So x is in $A \oplus C$ but not $B \oplus C$. Therefore, $A \oplus C \not\subseteq B \oplus C$. The implication, if $B \not\subseteq A$, then $B \oplus C \not\subseteq A \oplus C$, trivially follows without loss of generality. Conclusively, $A \neq B$ implies $A \oplus C \neq B \oplus C$. ■