Theorem (2.3.20)**.** *Let f be the function f: $\mathbb{R} \implies \mathbb{R}$, such that $\forall x((x \in \mathbb{R}) \implies (f(x) > 0))$. Let g be the function $g : \mathbb{R} \implies \mathbb{R}$ defined by g(x) = 1/f(x). f(x) is strictly increasing if and only if g(x) is strictly decreasing.*

Proof. Suppose there exist real numbers x and y such that $x < y$, and suppose that $f(x) < f(y)$. f is a strictly increasing real-valued function by definition. It follows that $g(x) = 1/f(x) > g(y) = 1/f(y)$, which is the definition for strictly decreasing real-valued functions.

Conversely, suppose there exist real numbers x and y such that $x < y$, and suppose that $g(x) > g(y)$. g is a strictly decreasing real-valued function by definition. It follows that $f(x) = 1/g(x) < f(y) = 1/f(y)$, which is the definition for strictly increasing real-valued functions.

Thus, $f(x)$ is strictly increasing if and only if $g(x)$ is strictly decreasing. ■