Theorem (2.3.42)**.** *Let x be a real number. $\lfloor x + \frac{1}{2} \rfloor$ is the closest integer to x, except when x is midway between two integers, when it is the larger of these two integers.*

Proof. By cases. Let n be the integer such that $n \leq x < n + 1$ and $\lfloor x + \frac{1}{2} \rfloor = \lfloor n + \epsilon + \frac{1}{2} \rfloor$. ϵ is the decimal part of x.

(i) If $\epsilon \geq \frac{1}{2}$, then $\epsilon + \frac{1}{2} \geq \frac{1}{2} + \frac{1}{2}$. That is, if $\epsilon + \frac{1}{2} \geq 1$, then $\lfloor x + \frac{1}{2} \rfloor \geq \lfloor (x - \epsilon) + 1 \rfloor = n + 1$.

(ii) If $\epsilon < \frac{1}{2}$, then $\epsilon + \frac{1}{2} < 1$. That is, if $\epsilon + \frac{1}{2} < 1$, then $\lfloor x + \frac{1}{2} \rfloor = \lfloor n + (\epsilon + \frac{1}{2}) \rfloor = n$. ∎