Theorem (2.3.44)**.** *Let x be a real number.* $\lceil x \rceil - \lfloor x \rfloor = 1$*, if* $x \notin \mathbb{Z}$*.*
$\lceil x \rceil - \lfloor x \rfloor = 0$*, if* $x \in \mathbb{Z}$*.*

Proof. By cases.

(i) Suppose $x \notin \mathbb{Z}$. $\lceil x \rceil = \lfloor x \rfloor + 1$. Thus, $\lceil x \rceil - \lfloor x \rfloor = (\lfloor x \rfloor + 1) - \lfloor x \rfloor = 1$.

(ii) Suppose $x \in \mathbb{Z}$. $\lceil x \rceil = \lfloor x \rfloor = x$. Thus, $\lceil x \rceil - \lfloor x \rfloor = x - x = 0$. ■