

**Theorem (2.3.67c).** *Let  $A$  be a set with universal set  $U$ . Let  $f_A$  be the characteristic function  $f_A: U \Rightarrow \{0, 1\}$ .  $f_{\bar{A}}(x) = 1 - f_A(x)$ .*

*Proof.* Let  $x$  be an element in  $A$ . Then clearly  $x \notin \bar{A}$ . By the definition for characteristic functions  $f_{\bar{A}}(x) = 0$ , and  $f_A(x) = 1$ . It follows immediately that  $f_{\bar{A}}(x) = 1 - f_A(x)$ .

Now suppose  $(x \notin A) \wedge (x \in \bar{A})$ . By the definition for characteristic functions that is  $f_{\bar{A}}(x) = 1$ , and  $f_A(x) = 0$ . It follows immediately that  $f_{\bar{A}}(x) = 1 - f_A(x)$ . ■