Theorem (2.3.41)**.** *Let f be the function $f : A \implies B$. Let S be a subset of B. $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$.*

Proof. By the definition for the inverse image of $\overline{S}$ under the function f^{-1}, we have $f^{-1}(\overline{S}) \equiv \{a \in A | f(a) \in \overline{S}\}$. Factoring the complementation out from the right-hand side of the equivalence, $f^{-1}(\overline{S}) \equiv \overline{\{a \in A | f(a) \in S\}}$. But this statement is the negation of the formal definition for the inverse image of S under the function f^{-1}. In other words, $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$. ■