Theorem (2.3.67a)**.** *Let A, and B be sets with universal set U. Let $f_{A \cap B}$ be the characteristic function $f_{A \cap B} : U \implies \{0, 1\}$. Let f_A be the characteristic function $f_A : U \implies \{0, 1\}$. Let f_B be the characteristic function $f_B : U \implies \{0, 1\}$. $f_{A \cap B}(x) = f_A(x) \times f_B(x)$.*

Proof. Let x be an element in $A \cap B$. By the definition for characteristic functions, $f_{A \cap B}(x) = 1$. Since the definition for set intersection says that $(x \in A) \wedge (x \in B)$, we know by the definition for characteristic functions that $f_A(x) = f_B(x) = 1$. Thus, it follows immediately by the multiplicative identity law from the field axioms that $f_{A \cap B}(x) = f_A(x) \times f_B(x)$.

Suppose it were not the case that x were an element in $A \cap B$. That is, $x \notin (A \cap B) \equiv [(x \notin A) \vee (x \notin B)]$, by DeMorgans law. By the definition for characteristic functions, $f_{A \cap B}(x) = 0$. Also, again by the definition for characteristic functions we know that $(f_A(x) = 0) \vee (f_B(x) = 0)$. Without loss of generality we can suppose $f_A(x) = 0$. It follows immediately from the multiplicative property of zero that $f_{A \cap B}(x) = 0 \times f_B(x) = 0$. Thus, $f_{A \cap B}(x) = f_A(x) \times f_B(x)$. ■