Theorem (1602). *Let* $\boldsymbol{\chi}$ *and* $\boldsymbol{\zeta}$ *be integers. If* $\boldsymbol{\chi}$ *and* $\boldsymbol{\zeta}$ *are even, then* $\boldsymbol{\chi} + \boldsymbol{\zeta}$ *is even.*

Proof. By the definition for even numbers, there exist integers $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ such that $\mathbf{2}\boldsymbol{\mu} = \boldsymbol{\chi}$ and $\mathbf{2}\boldsymbol{\nu} = \boldsymbol{\zeta}$. Hence,

$$\chi + \zeta = \Big[\langle 2\mu\rangle + \langle 2\nu\rangle\Big] = \Big[2\langle \mu + \nu\rangle\Big]$$

Integers are closed under addition. Thus, the factor $\langle \boldsymbol{\mu} + \boldsymbol{\nu} \rangle$ is an integer. It follows that $\boldsymbol{\chi} + \boldsymbol{\zeta}$ is even, by the definition for even numbers. ∎