Theorem (2.2.24)**.** *Let A, B, and C be sets.* $(A-B)-C=(A-C)-(B-C)$.

Proof. Let x be an element in $(A-B)-C$. By the definition for set difference we have $[(x\in A)\wedge(x\notin B)]\wedge(x\notin C)$. Now, the logical identity for the proposition $(x\notin B)$ is $(x\notin B)\vee\bot\equiv(x\notin B)$, and since $(x\in C)\equiv\bot$ by reason of the hypothesis, it necessarily follows from logical identity that $(x\notin B)\equiv[(x\notin B)\vee(x\in C)]$. Thus, by these facts, the law of logical commutativity, and the law of logical association we make the logically equivalent statement with respect to the definition given by the first expression, $[(x\in A)\wedge(x\notin C)]\wedge[(x\notin B)\vee(x\in C)]$. Since the right-hand side of the conjunction is true and in its proper logical identity, if it were double negated, it would remain intact by the law of double negation. Thus applying first a negation by DeMorgans law, and second a negation directly on the propositional statement, we have $[(x\in A)\wedge(x\notin C)]\wedge\neg[(x\in B)\wedge(x\notin C)]$. By the definition for set difference and set complementation x is an element in $(A-C)\cap\overline{(B-C)}$. Which is equivalent to the expression $(A-C)-(B-C)$, by Theorem 2.2.19.

To prove the converse case, let x be an element in $(A-C)-(B-C)$. By Theorem 2.2.19 $(A-C)\cap\overline{(B-C)}$ is an equivalent expression. This expression containing the element x is defined by $[(x\in A)\wedge(x\notin C)]\wedge\neg[(x\in B)\wedge(x\notin C)]$. Applying DeMorgans law to the statement on the right-hand side of the conjunction we get $[(x\in A)\wedge(x\notin C)]\wedge[(x\notin B)\vee(x\in C)]$. Note that as demonstrated in the first paragraph we have the following identity $[(x\notin B)\vee(x\in C)]\equiv(x\notin B)$. Thus, by that fact, the law of logical commutativity, and by the law of logical association it follows that the identity of our statement is $[(x\in A)\wedge(x\notin B)]\wedge(x\notin C)$. This is the definition for $x\in[(A-B)-C]$.

Since $(A-B)-C\subseteq(A-C)-(B-C)$ and $(A-C)-(B-C)\subseteq(A-B)-C$, it follows immediately from the definition for set equality that $(A-B)-C=(A-C)-(B-C)$ ∎