Theorem (3.2.7b)**.** *Let f be the function defined by f(x)* $= 3x^3 + (\log x)^4$*. f(x) is* $\mathcal{O}(x^3)$*.*

Proof. Let g be the function defined by $g(x) = x^3$. $f(x)$ is the sum of functions. The function $3x^3$ in the sum of functions $f(x)$ is less than or equal to $3 \cdot g(x)$, for all $x \in \mathbb{R}$ Therefore, $3x^3$ is $\mathcal{O}(g(x))$ with constant witnesses $C = 3$, and any $k \in \mathbb{R}$. If $x \geq 1$, then the function $(\log x)^4$ is less than $g(x)$. Therefore, $(\log x)^4$ is $\mathcal{O}(g(x))$ with constant witnesses $C = 1$, and $k = 1$. By the theorem stating that the bounding function for the sum of functions is the maximum bounding function of those functions, it follows that $f(x)$ is $\mathcal{O}(x^3)$ with constant witnesses $C = 3$ and $k = 1$. ∎