Theorem (1.6.12). The product of a nonzero rational number and an irrational number is irrational.

Proof. For the purpose of contradiction, suppose that the product of a nonzero rational number and an irrational number is rational. This can be expressed as $\frac{a}{b} \cdot x = \frac{c}{d}$, where a, b, c, and d are integers and x is irrational. Since $a \neq 0$ we equivalently have $x = \frac{c}{d} \cdot \frac{b}{a} = \frac{cb}{da}$. A contradiction.