

Theorem (2.2.9b). *Let A be a set. $A \cap \overline{A} = \emptyset$.*

Proof. Let x be an element in $A \cap \overline{A}$. By definition, $(x \in A) \wedge (x \in \overline{A})$. The right-hand side of this conjunction, according to the definitions of set complementation and logical negation, can be restated as $(x \notin A) \equiv \neg(x \in A)$. Again, by logical negation, we have $(x \in A) \wedge \neg(x \in A) \equiv \perp$. This is the very meaning of $A \cap \overline{A} = \emptyset$. Thus proves the set complementation law for the intersection of sets. ■