Theorem (2.2.38b)**.** *Let A, and B be sets. The symmetric difference of sets is subject to absorption such that $(A \oplus B) \oplus B = A$.*

Proof. By Theorem 2.2.35, $(A \oplus B) \oplus B = [(A \oplus B) \cup B] - [(A \oplus B) \cap B]$. By Theorem 2.2.19 that is $[(A \oplus B) \cup B] \cap \overline{[(A \oplus B) \cap B]}$, and by DeMorgans law $[(A \oplus B) \cup B] \cap [\overline{(A \oplus B)} \cup \overline{B}]$. Again, by Theorem 2.2.35, that is $\{[(A \cup B) - (A \cap B)] \cup B\} \cap \{\overline{[(A \cup B) - (A \cap B)]} \cup \overline{B}\}$. By Theorem 2.2.19, $\{[(A \cup B) \cap \overline{(A \cap B)}] \cup B\} \cap \{[(A \cup B) \cap \overline{(A \cap B)}] \cup \overline{B}\}$. Applying DeMorgans law, three times in succession we get $\{[(A \cup B) \cap (\overline{A} \cup \overline{B})] \cup B\} \cap \{[(\overline{A} \cap \overline{B}) \cup (A \cap B)] \cup \overline{B}\}$. By distribution and association we make the following equivalent statement, $[(A \cup B \cup B) \cap (\overline{A} \cup \overline{B} \cup B)] \cap \{[(\overline{A} \cap \overline{B}) \cup \overline{B}] \cup [(A \cap B) \cup \overline{B}]\}$. Now this expression shall be reduced. By the idempotent law, the complementation law, set absorption, and distribution we have $[(A \cup B) \cap (\overline{A} \cup U)] \cap \{\overline{B} \cup [(A \cup \overline{B}) \cap (B \cup \overline{B})]\}$. By complementation and domination we make the following equivalent statement, $[(A \cup B) \cap U] \cap \{\overline{B} \cup [(A \cup \overline{B}) \cap U]\}$. By set identity, and the idempotent law that is $(A \cup B) \cap (A \cup \overline{B})$. Factoring out A by the law of distribution, and by the set complementation laws, and by set identity we find that
$A \cup (B \cap \overline{B}) \equiv A \cup \emptyset \equiv A$

$\therefore (A \oplus B) \oplus B = A.$ ∎