Theorem (2.3.45)**.** *Let x be a real number.*
$(x-1) < \lfloor x \rfloor \leq x \leq \lceil x \rceil < (x+1)$.

Proof. Notice that $\epsilon = x - \lfloor x \rfloor$, so $(0 \leq \epsilon < 1)$. It is important to note too, that multiplying this inequality by -1 on every side yields $(0 \geq -\epsilon > -1) = (-1 < -\epsilon \leq 0)$. Finally, note that $\sigma = \lceil x \rceil - x$, so $(0 \leq \sigma < 1)$. But these inequalities together state that $-1 < -\epsilon \leq 0 \leq \sigma < 1$. Since this inequality is true, by adding x to every side we find that the following statement is also true: $(x-1) < \lfloor x \rfloor \leq x \leq \lceil x \rceil < (x+1)$.

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