Theorem (1.6.25)**.** *There does not exist a rational number r such that $r^3 + r + 1 = 0$.*

Proof. By contradiction. Assume that there exists a rational number r satisfying the equation $r^3+r+1 = 0$. By definition there exist integers a and b (b is nonzero,) such that $\frac{a^3}{b^3}+\frac{a}{b}+1 = a^3+ab^2+b^3 = 0$. Clearly $a^3 = -(ab^2+b^3)$ and $b^3 = -(a^3+ab^2)$. So we have $-(ab^2+b^3)+ab^2-(a^3+ab^2) = 0$. Simplifying we find that $-a^3 - ab^2 - b^3 = a^3 + ab^2 + b^3$. This can only happen when $b = 0$, but $b = 0$ is a contradiction because b is a divisor in r. ■