Theorem (2.2.18e). *Let A, B, and C be sets.* $(B-A)\cup(C-A) = (B\cup C)-A$.

Proof. Let x be an element in $(B-A)\cup(C-A)$. This is the same as saying $x \in [(B\cap\overline{A})\cup(C\cap\overline{A})]$. By definition,
$[(x\in B)\wedge(x\in\overline{A})]\vee[(x\in C)\wedge(x\in\overline{A})]$. By the associative property for logical conjunction, and factoring out the term $(x\in\overline{A})$, we get
$[(x\in B)\vee(x\in C)]\wedge(x\in\overline{A})$. This statement is the definition for
$(B\cup C)-A$.

Proving the converse case, suppose that x is an element in $(B\cup C)-A$. Note that the expression is equivalent to $(B\cup C)\cap\overline{A}$. Thus we have the following definition, $[(x\in B)\vee(x\in C)]\wedge(x\in\overline{A})$. By logical distribution for conjunction over disjunction $[(x\in B)\wedge(x\in\overline{A})]\vee[(x\in C)\wedge(x\in\overline{A})]$. This statement defines the expression $x\in[(B\cap\overline{A})\cup(C\cap\overline{A})]$. Which, as argued in the first paragraph, is logically equivalent to $x\in[(B-A)\cup(C-A)]$.

Since $[(B-A)\cup(C-A)]\subseteq[(B\cup C)-A]$ and
$[(B\cup C)-A]\subseteq[(B-A)\cup(C-A)]$. It immediately follows from the definition of set equivalence that $(B-A)\cup(C-A)=(B\cup C)-A$. ■