Theorem (2.2.19)**.** *Let A, and B be sets.* $A - B = A \cap \overline{B}$.

Proof. Let x be an element in $A - B$. By the definition for set difference, $(x \in A) \wedge (x \notin B)$. By the definition for set complementation this is the same as $(x \in A) \wedge (x \in \overline{B})$. Which is exactly the definition for $x \in (A \cap \overline{B})$.

Proving the converse trivially follows by reversing our steps in the direct form. Suppose there exists an element x such that $x \in (A \cap \overline{B})$. By the definition for set intersection we have $(x \in A) \wedge (x \in \overline{B})$. By the definition for set complementation we arrive at the definition for set difference
$(x \in A) \wedge (x \notin B)$. Therefore $x \in (A - B)$.

Since $(A - B) \subseteq (A \cap \overline{B})$ and $(A \cap \overline{B}) \subseteq (A - B)$, by the definition for set equality we have $(A - B) = (A \cap \overline{B})$. ∎