Theorem (2.4.21b). *The summation of natural numbers from 1 to n is*

$$\frac{n(n+1)}{2}$$

Proof. From Theorem 2.4.21a we know that

$$\sum_{k=1}^{n} 2k - 1 = n^2$$

This is the same as saying

$$n^2 = \left(-n + \sum_{k=1}^{n} 2k\right) \equiv \left(\sum_{k=1}^{n} 2k\right) = (n^2 + n) = n(n+1)$$

We can factor the coefficient 2 out of the term of summation,

$$2\sum_{k=1}^{n} k = n(n+1)$$

And of course dividing both sides by 2 gives

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Thus, indeed, the summation of natural numbers from 1 to n is $\frac{n(n+1)}{2}$. ∎