Theorem (2.3.29b)**.** *Let f be a function $f : B \implies C$, and let g be a function $g : A \implies B$. If both f and g are surjective, then $f \circ g$ is surjective.*

Proof. Let C be the domain of discourse. By the hypothesis, and by the definition for surjective functions, the following universally quantified statement must be true, $\forall c \exists b(f(b) = c)$. Note that $g(x)$ is in the domain of f, for every x in the domain of g, by the definition of g. It immediately follows from the general definition for functions that $\forall c \exists a(f(g(a)) = c)$ must be a logically equivalent universal quantification. Since the composition of functions $(f \circ g)(x)$ is defined by $f(g(x))$, it follows directly from the hypothesis that $f \circ g$ is surjective, by the definition for surjective functions. ■