

Theorem (3.2.22e). *Let f be the function defined by $f(x) = \lfloor x \rfloor$. $f(x)$ is $\Theta(x)$.*

Proof. By the properties for floor functions $\lfloor x \rfloor \leq x$, for all $x \geq 1$. Thus, $f(x)$ is $\mathcal{O}(x)$ with constant witnesses $C = 1$ and $k = 1$. Also, $\lfloor x \rfloor \geq \frac{1}{2}x$, for all $x \geq 2$. So $f(x)$ is $\Omega(x)$ with constant witnesses $C = \frac{1}{2}$ and $k = 2$. It follows immediately from the definition that $f(x)$ is $\Theta(x)$. ■