Theorem (2.2.9a)**.** *Let A be a set with universal set U. $A \cup \overline{A} = U$.*

Proof. Let x be an element in $A \cup \overline{A}$. By the definition for set union and the complement of sets we have $(x \in A) \vee (x \notin A)$. The right-hand side of this disjunction is equivalent to $x \in (U \cap \overline{A})$ according to the definition for set complementation. That is, $(x \in U) \wedge (x \notin A)$. So the original disjunction is the same as $(x \in A) \vee [(x \in U) \wedge (x \notin A)]$. We must distribute the left-hand side of this disjunction over the conjunction occurring in the right-hand side. We get $[(x \in A) \vee (x \in U)] \wedge [(x \in A) \vee (x \notin A)]$. By the logical law of negation the identity for the right-hand side of this conjunction is true. The left-hand side of this conjunction is dominated by U, according to Theorem 2.2.7a. Therefore the statement $x \in A \cup \overline{A}$ can be equivalently stated as $(x \in U) \wedge T$; the logical identity of which is $x \in U$. Thus proving the set complementation law for the union of sets, $A \cup \overline{A} = U$. ∎