Theorem (2.2.22). *Let A, B, and C be sets. $A \cap (B \cap C) = (A \cap B) \cap C$, such that set intersection is associative.*

Proof. Let x be an element in $A \cap (B \cap C)$. The logical definition being $(x \in A) \wedge [(x \in B) \wedge (x \in C)]$. It trivially follows from the logical law of association for conjunction that $[(x \in A) \wedge (x \in B)] \wedge (x \in C)$. Therefore by definition x is an element of $(A \cap B) \cap C$.

Proving the converse, let x be an element in $(A \cap B) \cap C$. The logical definition being $[(x \in A) \wedge (x \in B)] \wedge (x \in C)$. It trivially follows from the logical law of association for conjunction that $(x \in A) \wedge [(x \in B) \wedge (x \in C)]$. Therefore by definition x is an element of $A \cap (B \cap C)$.

Since $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ and $(A \cap B) \cap C \subseteq A \cap (B \cap C)$ it immediately follows from the definition for set equality that $A \cap (B \cap C) = (A \cap B) \cap C$. Thus, the intersection of three sets is associative. ∎