Theorem (3.2.8d). *Let f be the function defined by $f(x) = \frac{x^3+5\log x}{x^4+1}$. $f(x)$ is $\mathcal{O}(1)$.*

Proof. Let g be the function defined by $g(x) = 1$. If $x \geq 1$, then

$$f(x) = \left(\frac{x^3+5\log x}{x^4+1}\right) \leq \left(\frac{x^3+5\log x}{x^4}\right) = \left(\frac{x^3}{x^4} + \frac{5\log x}{x^4}\right) \leq \left(\frac{x^3}{x^4} + \frac{5x}{x^4}\right) = \left(\frac{1}{x} + \frac{5}{x^3}\right).$$

If $x \geq 1$, then $\frac{1}{x} + \frac{5}{x^3}$ is a decreasing function of x with respect to $6 \cdot g(x)$. Hence, $|f(x)| \leq 6|g(x)|$, for all $x > 1$. It follows that $f(x)$ is $\mathcal{O}(1)$ with constant witnesses $C = 6$, and $k = 1$. ∎