Theorem (3.2.21b)**.** *Let f be the function defined by*
$f(n) = (n \log n + 1)^2 + (\log n + 1)(n^2 + 1)$. $f(n)$ *is* $\mathcal{O}(n^2(\log n)^2)$

Proof. f is the sum of functions $(f_1 + f_2)$ where $f_1(n) = (n \log n + 1)^2$, and $f_2(n) = (\log n + 1)(n^2 + 1)$.

Consider f_1. f_1 is the product of functions $(f_1' f_1')$ where $f_1'(n) = n \log n + 1$. By the fact that the bounding function for the sum of functions is the maximum bounding function in the addends, $f_1'(n)$ is $\mathcal{O}(n \log n)$. Since the bounding function for the product of functions is the product of the bounding functions for those functions, $f_1(n) = (f_1' f_1')(n)$ is $\mathcal{O}((n \log n)^2) = \mathcal{O}(n^2(\log n)^2)$.

f_2 is the product of functions $(f_2' f_2'')$ where $f_2'(n) = \log n + 1$ and $f_2''(n) = n^2 + 1$. Both of these functions are binomials, and since a k^{th} degree polynomial is $\mathcal{O}(x^k)$, $f_2'(n)$ is $\mathcal{O}(\log n)$, and $f_2''(n)$ is $\mathcal{O}(n^2)$. Thus, $f_2(n)$ is $\mathcal{O}(n^2 \log n)$.

The tightest bounding function for f is the maximum of the bounding functions for f_1 and f_2. So $f(n)$ is $\mathcal{O}(n^2(\log n)^2)$. ■