Theorem (2.3.43)**.** *Let x be a real number. $\lceil x - \frac{1}{2} \rceil$ is the closest integer to x, except when x is midway between two integers, when it is the smaller of these two integers.*

Proof. By cases. Let n be the integer such that $n \leq x < n + 1$ and
$\lceil x - \frac{1}{2} \rceil = \lceil (n + \epsilon) - \frac{1}{2}) \rceil$. ϵ is the decimal part of x.

(*i*) If $\epsilon > \frac{1}{2}$, then $\epsilon - \frac{1}{2} > \frac{1}{2} - \frac{1}{2} = 0$. So $\lceil (n + \epsilon) - \frac{1}{2} \rceil = n + 1$.

(*ii*) If $\epsilon \leq \frac{1}{2}$, then $\epsilon - \frac{1}{2} \leq 0$. So $(n-1) \leq [(n+\epsilon) - \frac{1}{2}] < n$, and $\lceil x - \frac{1}{2} \rceil = n$. ■