Theorem (1.6.5)**.** *Let m, n, and p be integers. If $m + n$ and $n + p$ are even integers, then $m + p$ is even.*

Proof. For the purpose of contraposition suppose that $m + p$ is odd. By definition there exists an integer x such that $2x + 1 = m + p$. Let k, j, and n be integers such that $k + j - n = x$. We have $m + p = 2(k + j - n) + 1 = 2k + 2j - 2n - 1$. Adding $2n$ to both sides we get $m + p + 2n = 2(k + j) - 1$. This sum is odd, so at least one of the terms in $(m + n) + (p + n)$ has to be odd; that is, the negation of the hypothesis by DeMorgans law. ∎