**Theorem (2.4.20).**

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$$

*Proof.* The identity of $\left(\frac{1}{k(k+1)}\right)$ is $\left(\frac{1}{k} - \frac{1}{(k+1)}\right)$. This can be demonstrated by the equation

$$k\left(\frac{1}{k} - \frac{1}{(k+1)}\right) = \left(\frac{k+1}{k+1} - \frac{k}{k+1}\right) = \left(\frac{k+1-k}{k+1}\right) = \left(\frac{1}{k+1}\right)$$

Dividing both sides of this equation by $k$ gives the desired identity such that

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right)$$

The sequence for which is the telescopic summation

$$\left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n-2} - \frac{1}{n-1}\right) + \cdots + \left(\frac{1}{1} - \frac{1}{2}\right)$$

Thus, by Theorem 2.4.19

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \left(-\frac{1}{n+1} + \frac{1}{1}\right) = \left(\frac{(-1) + (n+1)}{n+1}\right) = \frac{n}{n+1}$$

$\blacksquare$