Theorem (2.2.16e)**.** *Let A and B be sets.* $A \cup (B - A) = A \cup B$.

Proof. Let x be an element in $A \cup (B - A)$. Note that $(B - A) \equiv (B \cap \overline{A})$. So by definition we have $(x \in A) \vee [(x \in B) \wedge (x \notin A)]$. Distributing logical disjunction over logical conjunction yields
$[(x \in A) \vee (x \in B)] \wedge [(x \in A) \vee (x \notin A)]$. Which by logical negation and by logical identity reduces to $(x \in A) \vee (x \in B)$, that is the very definition for $A \cup B$.

Suppose the converse case in which x is an element of $A \cup B$. That is, of course as already stated, defined as $(x \in A) \vee (x \in B)$. Note the fact that the conjunction of this proposition with another true proposition is true. Let p be that proposition, $(x \in A)$. Then $p \vee \neg p \equiv (x \in A) \vee (x \notin A) \equiv T$, and thus we can make the following statement $[(x \in A) \vee (x \in B)] \wedge [(x \in A) \vee (x \notin A)]$, which holds. Factoring the term $(x \in A)$ out on the logical operators gives the form $(x \in A) \vee [(x \in B) \wedge (x \notin A)]$. This statement is the definition for $A \cup (B - A)$.

Since $A \cup (B - A) \subseteq A \cup B$ and $A \cup B \subseteq A \cup (B - A)$,
$A \cup (B - A) = A \cup B$ by definition. ■