**Theorem** (2.3.29a)**.** *Let f be a function* $f : B \implies C$*, and let g be a function* $g : A \implies B$*. If both* $f$ *and* $g$ *are injective, then* $f \circ g$ *is injective.*

*Proof.* By the contrapositive. Let the domain of discourse be $A$.
Suppose it were not the case that $(f \circ g)$ were injective. Then by the definition for injective functions, the following universally quantified statement is true, $\neg\forall a\forall b((f \circ g)(a) = (f \circ g)(b) \implies (a = b))$. Note that the composition of functions $(f \circ g)(x)$ is defined by $f(g(x))$. Thus, we have the equivalent universal quantification $\neg\forall a\forall b(f(g(a)) = f(g(b)) \implies (a = b))$. In other words, it is not the case that $f$ is injective, by the definition for injective functions. Also, because $f = f$, $\neg\forall a\forall b(g(a) = g(b) \implies (a = b))$ is a logically equivalent universal quantification. That is, it is not the case that $g$ is injective, by the definition for injective functions. Since the contrapositive follows directly from the negation of the conclusion, it is necessarily the case that if both f and g are injective, then $f \circ g$ is injective. ∎