

**Theorem (2.3.25).** *Let  $f$  be a function  $f : \mathbb{R} \Rightarrow \mathbb{R}$  defined by  $f(x) = |x|$ .  $f(x)$  is not invertible.*

*Proof.* Let  $x$  be a positive real number.  $f(-x) = f(x) = x$ . If  $f$  had an inverse then  $f^{-1}(x) = x = -x$ , but this is not a function by definition. Also,  $f^{-1}$  is not a function by contradiction since  $\neg \exists x((x \in \mathbb{R}) \wedge (x = -x))$ . Thus,  $f$  is not a bijection, and  $f$  is not invertible. ■