Theorem (2.2.18a)**.** *Let A, B, and C be sets.* $(A \cup B) \subseteq (A \cup B \cup C)$.

Proof. Let x be an element in $(A \cup B)$. We have $(x \in A) \vee (x \in B)$, by definition. Let this definition statement be represented by p. Trivially, the fact of p would be unaffected were p disjunct any proposition q. Supposing such a q existed, we would have $p \vee q \equiv T$ by logical domination (because the hypothetical supposition assumes $p \equiv T$.) Let q be the proposition $(x \in C)$. Then $p \vee q \equiv (x \in A) \vee (x \in B) \vee (x \in C)$ is the well formed statement defining the superset under interrogation. Since we know that x is in the union of A and B by the hypothesis, and because $p \vee q \equiv T$ means that x is in the union of A, B, and C, it immediately follows that
$(A \cup B) \subseteq (A \cup B \cup C)$. ∎