

Theorem (3.2.20a). *Let f be the function defined by*
 $f(n) = (n^3 + n^2 \log n)(\log n + 1) + (17 \log n + 19)(n^3 + 2)$.
 $f(n)$ is $\mathcal{O}(n^3 \log n)$.

Proof. f is the sum of functions $(f_1 + f_2)$ where
 $f_1(n) = (n^3 + n^2 \log n)(\log n + 1)$ and $f_2(n) = (17 \log n + 19)(n^3 + 2)$. Each of
 f_1 , and f_2 are polynomials. $f_1(n) = n^3 \log n + n^3 + n^2(\log n)^2 + n^2 \log n$, and
 $f_2(n) = 17n^3 \log n + 34 \log n + 19n^3 + 38$. Since a k^{th} degree polynomial is
 $\mathcal{O}(k)$, $f_1(n)$ is $\mathcal{O}(n^3 \log n)$ and $f_2(n)$ is $\mathcal{O}(n^3 \log n)$. It follows from the fact
that the maximum bounding function in a sum of functions is the bounding
function for the sum, that $f(n)$ is $\mathcal{O}(n^3 \log n)$. ■