Theorem (3.2.8c)**.** *Let f be the function defined by $f(x) = \frac{x^4+x^2+1}{x^4+1}$. $f(x)$ is $\mathcal{O}(1)$.*

Proof. Let g be the function defined by $g(x) = 1$. If $x \geq 1$, then

$$f(x) = \left(\frac{x^4+x^2+1}{x^4+1}\right) = \left(\frac{x^2}{x^4+1} + 1\right) \leq \left(\frac{x^2}{x^4} + 1\right) = \left(\frac{1}{x^2} + 1\right).$$

$\frac{1}{x^2} + 1$ is a decreasing function of x with respect to $2 \cdot g(x)$, for all $x \geq 1$. Therefore, $|f(x)| \leq 2|g(x)|$ for all $x > 1$. It follows that $f(x)$ is $\mathcal{O}(1)$ with constant witnesses $C = 2$, and $k = 1$. ■