Theorem (2.3.47b)**.** *Let x be a real number, and let n be an integer.* $n < x \iff n < \lceil x \rceil$.

Proof. $x \leq \lceil x \rceil$, by the properties of the ceiling function. So if $n < x$, then $n < x \leq \lceil x \rceil$, and $n < \lceil x \rceil$.

Proving the converse, suppose $n < \lceil x \rceil$. Since n and $\lceil x \rceil$ are integers, $n \leq \lceil x \rceil - 1$. By the properties of ceiling functions we have the following tautology, $\lceil x \rceil = \lceil x \rceil \iff \lceil x \rceil - 1 < x \leq \lceil x \rceil$. Combining these two inequalities yields $n \leq \lceil x \rceil - 1 < x \leq \lceil x \rceil$. Thus, $n < x$. ■