Theorem (3.2.19b)**.** *Let f be the function defined by $f(n) = (n \log n + n^2)(n^3 + 2)$. f(n) is $\mathcal{O}(n^5)$.*

Proof. $f(n)$ is the product of functions $(f_1 f_2)(n)$, where $f_1(n) = (n \log n + n^2)$ and $f_2(n) = (n^3 + 2)$.

Consider f_1, which is the sum of functions $(f_1' + f_1'')$, where $f_1'(n) = n \log n$, and $f_1'' = n^2$. Since $n \log n \leq n \cdot n$, for all $n \geq 1$, it follows that $f_1'(n)$ is $\mathcal{O}(n^2)$ with constant witnesses $C = 1$ and $k = 1$. Clearly, $f_1''(n)$ is $\mathcal{O}(n^2)$ with constant witnesses in $\mathbb{N}$. Now, the bounding function for the sum of functions is the maximum of the bounding functions in the addends for the sum of functions. So $(f_1' + f_1'')(n)$ is $\mathcal{O}(n^2)$. Thus, f_1 is $\mathcal{O}(n^2)$.

We now turn our attention to f_2. f_2 is a 3^{rd} degree binomial. Since a k^{th} degree polynomial is $\mathcal{O}(x^k)$, it follows that $f_2(n)$ is $\mathcal{O}(n^3)$.

The bounding function for the product of functions is the product of the bounding functions for each function in the product of functions. So $f(n)$ is $\mathcal{O}(n^2 \cdot n^3)$. This means that $f(n)$ is $\mathcal{O}(n^5)$. ∎