Theorem (2.3.48b)**.** *Let x be a real number, and let n be an integer.* $n \leq x \iff n \leq \lfloor x \rfloor$.

Proof. By the direct form contrapositive. Suppose $n > \lfloor x \rfloor$. Since n and $\lfloor x \rfloor$ are integers, $n \geq \lfloor x \rfloor + 1$. Now, by the properties of the floor function, we have the following tautology, $\lfloor x \rfloor = \lfloor x \rfloor \iff \lfloor x \rfloor + 1 > x \geq \lfloor x \rfloor$. Combining these two inequalities yields $n \geq \lfloor x \rfloor + 1 > x \geq \lfloor x \rfloor$ This statement says that $n > x$.

Proving the converse form by the contrapositive. Note that $x \geq \lfloor x \rfloor$, by the properties of the floor function. So if $n > x$, then $n > x \geq \lfloor x \rfloor$, and of course $n > \lfloor x \rfloor$.

$\therefore n \leq x \iff n \leq \lfloor x \rfloor$ ■