Theorem (3.2.20c)**.** *Let f be the function defined by $f(n) = (n^n + n2^n + 5^n)(n! + 5^n)$. $f(n)$ is $\mathcal{O}(n!n^n)$.*

Proof. f is the product of functions $(f_1 f_2)$ where $f_1(n) = n^n + n2^n + 5^n$ and $f_2(n) = n! + 5^n$. If $n \geq 5$, then

$$f_1(n) = n^n + n2^n + 5^n \leq n^n + n^n + n^n = 3n^n.$$

Thus, $f_1(n)$ is $\mathcal{O}(n^n)$ with constant witnesses $C = 3$ and $k = 5$. Now, if $n \geq 12$, then

$$f_2(n) = n! + 5^n \leq n! + n! = 2n!.$$

So, $f_2(n)$ is $\mathcal{O}(n!)$ with constant witnesses $C = 2$, and $k = 12$. Since the bounding function for the product of functions is the product of those functions bounding functions, $f(n)$ is $\mathcal{O}(n!n^n)$. ∎