Theorem (2.4.19)**.** *Let* $\{a_n\}$ *be a sequence of real numbers.* $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$.

Proof. $\sum_{j=1}^{n}(a_j - a_{j-1}) =$ $(a_n - a_{n-1}) + (a_{n-1} - a_{n-2}) + (a_{n-2} - a_{n-3}) + \cdots + (a_1 - a_0)$. By the associativity for addition from the field axioms for real numbers, that is $a_n + (-a_{n-1} + a_{n-1}) + (-a_{n-2} + a_{n-2}) + (-a_{n-3} + a_{n-3}) + \cdots + (-a_1 + a_1) + -a_0$. Clearly the inner terms cancel out. Thus, $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$. ■