

**Theorem (2.3.21).** *Let  $f$  be the function  $f: \mathbb{R} \Rightarrow \mathbb{R}$ , such that  $\forall x((x \in \mathbb{R}) \Rightarrow (f(x) > 0))$ . Let  $g$  be the function  $g: \mathbb{R} \Rightarrow \mathbb{R}$  defined by  $g(x) = 1/f(x)$ .  $f(x)$  is strictly decreasing if and only if  $g(x)$  is strictly increasing.*

*Proof.* Suppose there exist real numbers  $x$  and  $y$  such that  $x < y$ , and suppose that  $f(x) > f(y)$ .  $f$  is a strictly decreasing real-valued function by definition. It follows that  $g(x) = 1/f(x) < 1/f(y) = g(y)$ , which is the definition for strictly increasing real-valued functions.

Conversely, suppose there exist real numbers  $x$  and  $y$  such that  $x < y$ , and suppose that  $g(x) < g(y)$ .  $g$  is a strictly increasing real-valued function by definition. It follows that  $f(x) = 1/g(x) > 1/g(y) = f(y)$ , which is the definition for strictly decreasing real-valued functions.

Thus,  $f(x)$  is strictly decreasing if and only if  $g(x)$  is strictly increasing. ■