Theorem (3.2.7c)**.** *Let f be the function defined by* $f(x) = \frac{x^4 + x^2 + 1}{x^3 + 1}$. *f(x) is* $\mathcal{O}(x)$.

Proof. Let g be the function defined by $g(x) = x$. If $x \geq 1$, then

$$\left(\frac{x^4 + x^2 + 1}{x^3 + 1} \right) = \left(\frac{x^4}{x^3 + 1} + 1 \right) \leq \left(\frac{x^4}{x^3} + 1 \right) = (x + 1).$$

If $x \geq 1$, then $x + 1 \leq 2 \cdot g(x)$. Thus, $|f(x)| \leq 2|g(x)|$, for all $x > 1$. Therefore $f(x)$ is $\mathcal{O}(x)$ with constant witnesses $C = 2$, and $k = 1$. $\blacksquare$