

Theorem (2.3.21). *Let f be the function $f: \mathbb{R} \Rightarrow \mathbb{R}$, such that $\forall x((x \in \mathbb{R}) \Rightarrow (f(x) > 0))$. Let g be the function $g: \mathbb{R} \Rightarrow \mathbb{R}$ defined by $g(x) = 1/f(x)$. $f(x)$ is strictly decreasing if and only if $g(x)$ is strictly increasing.*

Proof. Suppose there exist real numbers x and y such that $x < y$, and suppose that $f(x) > f(y)$. f is a strictly decreasing real-valued function by definition. It follows that $g(x) = 1/f(x) < 1/f(y) = g(y)$, which is the definition for strictly increasing real-valued functions.

Conversely, suppose there exist real numbers x and y such that $x < y$, and suppose that $g(x) < g(y)$. g is a strictly increasing real-valued function by definition. It follows that $f(x) = 1/g(x) > 1/g(y) = f(y)$, which is the definition for strictly decreasing real-valued functions.

Thus, $f(x)$ is strictly decreasing if and only if $g(x)$ is strictly increasing. ■