Theorem (2.4.36). *A subset of a countable set is countable.*

Proof. Let A and B be sets such that $A \subseteq B$. B is countable by the hypothesis, and so by the definition for countability $|B| \leq \aleph_0$. By the definition of subset, $|A| \leq |B| \therefore |A| \leq \aleph_0$ and it follows that the subset of a countable set is countable. ■