Theorem (3.2.9). *Let f be the function defined by f(x)* $= x^2 + 4x + 17$*. f(x) is* $\mathcal{O}(x^3)$*, but* x^3 *is not* $\mathcal{O}(f(x))$*.*

Proof. Let g be the function defined by $g(x) = x^3$. $f(x)$ is $\mathcal{O}(x^2)$, by the theorem that states that a polynomial of degree n is $\mathcal{O}(x^n)$. Therefore $f(x)$ is $\mathcal{O}(g(x))$.

If $x \geq 2$, then $f(x) \leq x^2 + 4x^2 + x^2 = 6x^2$. By the definition of big-O, if $g(x)$ is $\mathcal{O}(f(x))$, then $|x^3| \leq |f(x)| \leq 6x^2$. That is, $x^3 \leq 6x^2$, and $x \leq 6$. Clearly it is not the case that $g(x)$ is $\mathcal{O}(f(x))$. ■