Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y. Show that the inverse of the composition $f \circ g$ is given by $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

Proof. Since f:Y $\rightarrow$ Z, and g:X $\rightarrow$ Y, it follows from the definition of composition and from the definition of function that $\forall x\ \exists y\ \exists z$ (($x \in$ X, $y \in$ Y, $z \in$ Z) $\rightarrow$ (($f \circ g)(x) = f(y) = z$)). f and g are invertible, so $\forall x\ \exists y\ \exists z$ (($x \in$ X, $y \in$ Y, $z \in$ Z) $\rightarrow [(f^{-1}((f \circ g)(x)) = g(x) = y) \wedge (g^{-1}(y) = x)]$). Hence, $\forall z\ \exists x$ (($z \in$ Z, $x \in$ X) $\rightarrow g^{-1}(f^{-1}(z)) = x$), and by the definition of composition, $g^{-1}(f^{-1}) = g^{-1} \circ f^{-1}$.

$(f \circ g)^{-1}((f \circ g)) = \iota_x$. Since $\forall x$ ($\iota_x(x) = (g^{-1} \circ f^{-1})(x) = x$), it follows that $\iota_x = g^{-1} \circ f^{-1}$.

$\therefore\ (f \circ g)^{-1} = g^{-1} \circ f^{-1}$ ■