Theorem (3.2.2f). *Let f be the function defined by $f(x) = \lfloor x \rfloor \lceil x \rceil$. $f(x)$ is $\mathcal{O}(x^2)$.*

Proof. Let g be the function defined by $g(x) = x^2$. By the properties for floor functions, $\lfloor x \rfloor \leq x$. By the properties for ceiling functions, $\lceil x \rceil < (x + 1)$. $(x + 1) \leq 2x$, for all $x \geq 1$, and clearly it must be the case that $\lceil x \rceil \leq 2x$. Multiplying these inequalities we get $\lfloor x \rfloor \lceil x \rceil \leq x \cdot 2x$, for all $x \geq 1$. For all $x > 1$, by the definitions for f and g, we have the definition of big-O, $|f(x)| \leq 2|g(x)|$. Therefore, $f(x)$ is $\mathcal{O}(x^2)$ with constant witnesses $C = 2$, and $k = 1$. ∎