

**Theorem (2.3.40a).** *Let  $f$  be the function  $f : A \implies B$ . Let  $S$ , and  $T$  be subsets of  $B$ .  $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$ .*

*Proof.* By the definition for the inverse image of the set  $(S \cup T)$  under the function  $f^{-1}$ , we have  $f^{-1}(S \cup T) = \{a \in A | f(a) \in (S \cup T)\}$ . Then equivalently,  $f^{-1}(S \cup T) \equiv \{a \in A | f(a) \in S\} \cup \{a \in A | f(a) \in T\}$ . This is the formal definition for  $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$ . ■