Theorem (2.4.19). *Let $\{a_n\}$ be a sequence of real numbers.*

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$$

.

Proof.

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = (a_n - a_{n-1}) + (a_{n-1} - a_{n-2}) + \cdots + (a_1 - a_0)$$

By associativity for addition from the field axioms for real numbers, that is

$a_n + (-a_{n-1} + a_{n-1}) + (-a_{n-2} + a_{n-2}) + (-a_{n-3} + a_{n-3}) + \cdots + (-a_1 + a_1) + -a_0$

Clearly the inner terms cancel out. Thus,

$$\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$$

■