Theorem (3.2.8a). Let f be the function defined by $f(x) = 2x^2 + x^3 \log x$. f(x) is $\mathcal{O}(x^4)$.

Proof. Let g be the function defined by $g(x) = x^4$. If $x \ge 1$, then

$$f(x) = (2x^2 + x^3 \log x) \le (2x^2 + x^4) \le (2x^4 + x^4) = 3x^4.$$

Thus, $|f(x)| \leq 3|g(x)|$, for all x > 1, and therefore f(x) is $\mathcal{O}(x^4)$ with constant witnesses C = 3, and k = 1.