Theorem (1.6.1). Let x and y be integers. If x and y are odd, then x + y is even.

Proof. By definition, there exists integers m and n such that x = 2m + 1 and y = 2n + 1. 2m + 1 + 2n + 1 = 2(m + n + 1). m + n + 1 is an integer k because the sum of integers is an integer x + y = 2k is even by definition.