

Theorem (2.3.36a). *Let f be the function $f : A \implies B$. Let S , and T be subsets of A . $f(S \cup T) = f(S) \cup f(T)$.*

Proof. By the definition for the image of a set $(S \cup T)$ under the function f we have $f(S \cup T) = \{t | \exists s \in S \cup T (t = f(s))\} \equiv \{f(s) | s \in (S \cup T)\}$. Since $\{f(s) | s \in (S \cup T)\}$ is a set, from this we can write $f(S \cup T) \equiv \{f(s) | (s \in S)\} \cup \{f(s) | (s \in T)\}$. The right-hand side of this equivalence is the set $f(S) \cup f(T)$, by the definition for the image of a set S or T under the function $f \therefore f(S \cup T) = f(S) \cup f(T)$. ■