Theorem (3.2.23f). *Let f be the function defined by f(x)* $= \lfloor x \rfloor \lceil x \rceil$.
f(x) is $\Theta(x^2)$.

Proof. $x - \lfloor x \rfloor = \epsilon$, and $x - \epsilon = \lfloor x \rfloor$. $\lceil x \rceil - x = \sigma$, and $x + \sigma = \lceil x \rceil$. Thus, $f(x) = (x-\epsilon)(x+\sigma) = x^2 + x\sigma - x\epsilon - \epsilon\sigma = x^2 + x(\sigma - \epsilon) - \epsilon\sigma$. Now the following inequality can be established $f(x) = x^2 + x(\sigma - \epsilon) - \epsilon\sigma \leq x^2 + x^2 - x \leq 2x^2$. This means, $|f(x)| \leq 2|x^2|$, for all $x \in \mathbb{R}$. Thus, $f(x)$ is $\mathcal{O}(x^2)$ with constant witnesses $C_1 = 2$ and any $k \in \mathbb{R}$.

Now certainly $2x^2 \geq x^2$, and obviously $2x^2 + 2x(\sigma - \epsilon) - 2\epsilon\sigma \geq x^2$. Thus, $x^2 + x(\sigma - \epsilon) - \epsilon\sigma \geq \frac{1}{2} \cdot x^2$, for all $x \in \mathbb{R}$. It trivially follows that $\frac{1}{2}|x^2| \leq |f(x)|$, for all $x \in \mathbb{R}$. So $f(x)$ is $\Omega(x^2)$ with constant witnesses $C_2 = \frac{1}{2}$, and any $k \in \mathbb{R}$.

Since we have $C_2|x^2| \leq |f(x)| \leq C_1|x^2|$, for $x > k$, by definition it follows that $f(x)$ is $\Theta(x^2)$. ∎