Theorem (2.3.68)**.** *Let f be a function $f : A \implies B$, where A and B are finite sets, and $|A| = |B|$. f is injective if and only if f is surjective.*

Proof. Direct form by the contrapositive. Suppose the negation of the statement given by the definition for surjective functions, $\exists y \forall x(f(x) \neq y)$. This statement can only be true if either $|A| < |B|$ (contradicting the hypothesis,) or $\exists x \exists y((f(x) = f(y)) \wedge (x \neq y))$. Since contradiction is $\perp$ by the law for logical negation, by the identity law for logical disjunction, f is defined as not injective. Thus, if f is injective, then f is surjective.

Converse form by the contrapositive. Suppose the negation of the statement given by the definition for injective functions, $\exists x \exists y((f(x) = f(y)) \wedge (x \neq y))$. This statement can only be true if either $|A| > |B|$ (contradicting the hypothesis,) or $\exists y \forall x(f(x) \neq y)$. Since contradiction is $\perp$ by the law for logical negation, by the identity law for logical disjunction, f is defined as not surjective. Thus, if f is surjective, then f is injective.

$\therefore \forall x \forall y((f(x) = f(y)) \implies (x = y)) \iff \forall y \exists x(f(x) = y)$, whenever f is a function $f : A \implies B$, where A and B are finite sets, and $|A| = |B|$. ■