Theorem (3.2.22f)**.** *Let f be the function defined by $f(x) = \lceil \frac{x}{2} \rceil$. $f(x)$ is $\Theta(x)$.*

Proof. If $x \geq 1$ then $|\lceil \frac{x}{2} \rceil| \leq |x|$, for all x. So $f(x)$ is $\mathcal{O}(x)$ with constant witnesses $C = 1$ and $k = 1$. By the properties for ceiling functions, $\lceil x \rceil \geq x$. This of course means that $\lceil \frac{x}{2} \rceil \geq \frac{1}{2}x$. If $x \geq 1$, then $|\lceil \frac{x}{2} \rceil| \geq \frac{1}{2}|x|$. Thus, $f(x)$ is $\Omega(x)$ with constant witnesses $C = \frac{1}{2}$ and $k = 1$. It follows immediately that $f(x)$ is $\Theta(x)$. ∎