

Theorem (3.2.14e). *Let f be the function defined by $f(x) = x^3$, and let g be the function defined by $g(x) = 3^x$. $f(x)$ is $\mathcal{O}(g(x))$.*

Proof. If $f(x)$ is $\mathcal{O}(g(x))$, then there exists constant witnesses C and k such that $x^3 \leq C \cdot 3^x$, for all $x > k$. If $C = 1$ and $k = 1$, x^3 is a decreasing function with respect to 3^x . Thus, $f(x)$ is $\mathcal{O}(g(x))$. ■