Theorem (2.2.15)**.** *Let A and B be sets.* $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

Proof. Let x be an element in $\overline{A \cup B}$. By the definition of set complementation we have $\neg[x \in (A \cup B)]$. By the definition of set union, $\neg[(x \in A) \vee (x \in B)]$. Applying DeMorgans law (from logic) to the logical operations we get $\neg(x \in A) \wedge \neg(x \in B) \equiv (x \in \overline{A}) \wedge (x \in \overline{B})$. This is the definition of $x \in (\overline{A} \cap \overline{B})$. Therefore $\overline{A \cup B} \subseteq (\overline{A} \cap \overline{B})$.

Now suppose x were an element in $\overline{A} \cap \overline{B}$. Then by definition $(x \in \overline{A}) \wedge (x \in \overline{B}) \equiv \neg(x \in A) \wedge \neg(x \in B)$. By DeMorgans law (from logic) we have $\neg[(x \in A) \vee (x \in B)]$. Since this is the definition for set union it follows that $\neg[x \in (A \cup B)]$. Finally, applying the definition of set complementation we arrive at $x \in \overline{A \cup B}$. Therefore $(\overline{A} \cap \overline{B}) \subseteq \overline{A \cup B}$.

Because $\overline{A \cup B} \subseteq (\overline{A} \cap \overline{B})$ and $(\overline{A} \cap \overline{B}) \subseteq \overline{A \cup B}$ the sets are equivalent by definition. That is, $\overline{A \cup B} = (\overline{A} \cap \overline{B})$. Thereby proving DeMorgans law for sets, that the complement of the union of two sets is equivalent to the intersection of those set complements. ■