Theorem (2.3.45). Let x be a real number. $(x-1) < \lfloor x \rfloor \le x \le \lceil x \rceil < (x+1)$.

Proof. Notice that $\epsilon = x - \lfloor x \rfloor$, so $(0 \le \epsilon < 1)$. It is important to note too, that multiplying this inequality by -1 on every side yields $(0 \ge -\epsilon > -1) = (-1 < -\epsilon \le 0)$. Finally, note that $\sigma = \lceil x \rceil - x$, so $(0 \le \sigma < 1)$. But these inequalities together state that $-1 < -\epsilon \le 0 \le \sigma < 1$. Since this inequality is true, by adding x to every side we find that the following statement is also true: $(x-1) < \lfloor x \rfloor \le x \le \lceil x \rceil < (x+1)$.