Theorem (2.2.20)**.** *Let A, and B be sets.* $(A \cap B) \cup (A \cap \overline{B}) = A$.

Proof. Let x be an element in $(A \cap B) \cup (A \cap \overline{B})$. By definition then, we have $[(x \in A) \wedge (x \in B)] \vee [(x \in A) \wedge (x \notin B)]$. By the logical law of distribution for conjunction over disjunction we can factor the term $(x \in A)$ out on the conjunction. Thus, $(x \in A) \wedge [(x \in B) \vee (x \notin B)]$. By the logical law of negation we have $(x \in A) \wedge T$ which is equivalent to $x \in A$ by the identity law of logic.

Proving the converse, let x be an element in A. That is, $(x \in A)$. By the logical law of identity, $(x \in A) \wedge T \equiv (x \in A)$. Since $(x \in B) \vee (x \notin B)$ is true by the logical law of negation, we can, by logical equivalence, construct the following statement while retaining the logical identity for the statement $(x \in A)$; that is $(x \in A) \wedge [(x \in B) \vee (x \notin B)]$. By the logical law of distribution for conjunction over disjunction we have the definition of our original expression, $[(x \in A) \wedge (x \in B)] \vee [(x \in A) \wedge (x \notin B)]$. Hence, $x \in [(A \cap B) \cup (A \cap \overline{B})]$.

Because $(A \cap B) \cup (A \cap \overline{B}) \subseteq A$ and $A \subseteq (A \cap B) \cup (A \cap \overline{B})$ it follows from the definition of set equivalence that $(A \cap B) \cup (A \cap \overline{B}) = A$. ■