

**Theorem** (2.3.29b). *Let  $f$  be a function  $f : B \Rightarrow C$ , and let  $g$  be a function  $g : A \Rightarrow B$ . If both  $f$  and  $g$  are surjective, then  $f \circ g$  is surjective.*

*Proof.* Let  $C$  be the domain of discourse. By the hypothesis, and by the definition for surjective functions, the following universally quantified statement must be true,  $\forall c \exists b (f(b) = c)$ . Note that  $g(x)$  is in the domain of  $f$ , for every  $x$  in the domain of  $g$ , by the definition of  $g$ . It immediately follows from the general definition for functions that  $\forall c \exists a (f(g(a)) = c)$  must be a logically equivalent universal quantification. Since the composition of functions  $(f \circ g)(x)$  is defined by  $f(g(x))$ , it follows directly from the hypothesis that  $f \circ g$  is surjective, by the definition of surjective functions. ■