Theorem (2.2.36)**.** *Let A, and B be sets.* $A \oplus B = (A - B) \cup (B - A)$

Proof. Let x be an element in $A \oplus B$. Then by the definition for symmetric difference $[(x \in A) \wedge (x \notin B)] \vee [(x \notin A) \wedge (x \in B)]$. Because logical conjunction is associative, the statement is equivalent to $[(x \in A) \wedge (x \notin B)] \vee [(x \in B) \wedge (x \notin A)]$. According to the definition for set difference, and by the definition for set union, it follows that x is an element in $(A - B) \cup (B - A)$.

Proving the converse, suppose x were an element in $(A - B) \cup (B - A)$. The logical definition being $[(x \in A) \wedge (x \notin B)] \vee [(x \in B) \wedge (x \notin A)]$. By the associative law for logical conjunction the following statement is equivalent $[(x \in A) \wedge (x \notin B)] \vee [(x \notin A) \wedge (x \in B)]$. Since this is the definition for symmetric difference, x is an element in $A \oplus B$.

Since $A \oplus B \subseteq (A - B) \cup (B - A)$ and $(A - B) \cup (B - A) \subseteq A \oplus B$ it immediately follows from the definition of set equality that $A \oplus B = (A - B) \cup (B - A)$. ■