Theorem (2.3.67c)**.** *Let A be a set with universal set U. Let f_A be the characteristic function $f_{\overline{A}} : U \implies \{0, 1\}$. $f_{\overline{A}}(x) = 1 - f_A(x)$.*

Proof. Let x be an element in A. Then clearly $x \notin \overline{A}$. By the definition for characteristic functions $f_{\overline{A}}(x) = 0$, and $f_A(x) = 1$. It follows immediately that $f_{\overline{A}}(x) = 1 - f_A(x)$.

Now suppose $(x \notin A) \wedge (x \in \overline{A})$ By the definition for characteristic functions that is $f_{\overline{A}}(x) = 1$, and $f_A(x) = 0$. It follows immediately that $f_{\overline{A}}(x) = 1 - f_A(x)$. ∎