Theorem (2.2.23)**.** *Let A, B, and C be sets. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, such that set union is distributive over set intersection.*

Proof. Let x be an element in $A \cup (B \cap C)$. The logical definition being $(x \in A) \vee [(x \in B) \wedge (x \in C)]$. By the logical law for distribution of disjunction over conjunction we have $[(x \in A) \vee (x \in B)] \wedge [(x \in A) \vee (x \in C)]$. By definition, x is an element in $(A \cup B) \cap (A \cup C)$.

Proving the converse, let x be an element in $(A \cup B) \cap (A \cup C)$. The logical definition being $[(x \in A) \vee (x \in B)] \wedge [(x \in A) \vee (x \in C)]$. By the logical law for distribution we can factor out the term $x \in A$ over the conjunction. Thus, $(x \in A) \vee [(x \in B) \wedge (x \in C)]$, and x is an element in $A \cup (B \cap C)$ by definition.

Since $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$, it follows immediately from the definition of set equality that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Therefore, set union is indeed distributive over set intersection. ∎