

**Theorem (3.2.1e).** *Let  $f$  be the function defined by  $f(x) = \lfloor x \rfloor$ .  $f(x)$  is  $\mathcal{O}(x)$ .*

*Proof.* Let  $g$  be the function defined by  $g(x) = x$ . The floor function of  $x$  is less than  $x$  by the properties for floor functions. So  $|\lfloor x \rfloor| \leq |x|$  is true for all  $x \in \mathbb{R}$ . Therefore,  $|f(x)| \leq 1|g(x)|$ , for all  $x \in \mathbb{R}$ . It follows from the definition of big-O notation that  $f(x)$  is  $\mathcal{O}(x)$  with constant witnesses  $C = 1$ , and any  $k \in \mathbb{R}$ . ■