Theorem (2.3.50)**.** *Let x be a real number.*
$\lfloor -x \rfloor = -\lceil x \rceil$, *and* $\lceil -x \rceil = -\lfloor x \rfloor$.

Proof. By the properties of ceiling functions,
$\lceil x \rceil = n \iff (n-1) < x \leq n$. Multiplying every side of this inequality by -1 yields $(-n+1) > -x \geq -n$. By the properties of floor functions, this means that $\lfloor -x \rfloor = -n$. And of course since $-1 \times \lceil x \rceil = -n$, we have $-n = \lfloor -x \rfloor = -\lceil x \rceil$.

By the properties of floor functions,
$\lfloor x \rfloor = n \iff n \leq x < (n+1)$. Multiplying every side of this inequality by -1 yields $-n \geq -x > (-n-1)$. By the properties of ceiling functions, this means that $\lceil -x \rceil = -n$. And of course since $-1 \times \lfloor x \rfloor = -n$, we have $-n = \lceil -x \rceil = -\lfloor x \rfloor$. ∎