Theorem (2.4.21b)**.** *The summation of natural numbers from 1 to n is* $\frac{n(n+1)}{2}$.

Proof. By Theorem 2.4.21a we know that $\sum_{k=1}^{n} 2k - 1 = n^2$. This is the same as saying $-n + \sum_{k=1}^{n} 2k = n^2 \equiv \sum_{k=1}^{n} 2k = n^2 + n = n(n+1)$. We can factor the coefficient 2 out of the term of summation, $2 \sum_{k=1}^{n} k = n(n+1)$. And of course dividing both sides by 2 gives $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$. Thus, indeed, the summation of natural numbers from 1 to n is $\frac{n(n+1)}{2}$. $\blacksquare$