**Theorem** (2.3.30). *Let $f$ and $f \circ g$ be injective functions. $g$ is injective.*

*Proof.* By the contrapositive. Suppose that $g$ were not injective. Then by the definition for injective functions we have the following universally quantified statement, with the domain of discourse being the domain of $g$, $\neg\forall a\forall b((g(a) = g(b)) \implies (a = b))$. Because $f = f$, this statement is logically equivalent to $\neg\forall a\forall b((f(g(a)) = f(g(b))) \implies (a = b))$. By the definition for the compositions of functions we can also draw this equivalence, $\neg\forall a\forall b((f \circ g)(a) = (f \circ g)(b)) \implies (a = b))$. That is, it is not the case that $f \circ g$ is injective, by the definition for injective functions. So it follows directly from the negation of the statement "$g$ is injective," that $f \circ g$ is not injective. Thus, if $f$ and $(f \circ g)$ are injective functions, then $g$ is indeed injective. ∎