

Theorem (3.2.16). *If f is a function such that $f(x)$ is $\mathcal{O}(x)$, then $f(x)$ is $\mathcal{O}(x^2)$.*

Proof. If $f(x)$ is $\mathcal{O}(x)$, then there exists constant witnesses C and k such that $|f(x)| \leq C|x|$, for all $x > k$. Clearly, $C|x| \leq C|x^2|$. Thus, $|f(x)| \leq C|x^2|$, for all $x > k$. It follows that $f(x)$ is $\mathcal{O}(x^2)$. ■