Theorem (3.2.11)**.** *Let f be the function defined by f(x)* $= 3x^4 + 1$*, and let g be the function defined by g(x)* $= \frac{x^4}{2}$*. f(x) is* $\Theta(g(x))$*.*

Proof. If $x \geq 1$, then $x^4 \leq 6x^4$. Dividing both sides by two, $\frac{x^4}{2} \leq 3x^4$. Of course $|\frac{x^4}{2}| \leq |3x^4 + 1|$, for all $x > 1$. Therefore $f(x)$ is $\Omega(g(x))$ with constant witnesses $C = 1$, and $k = 1$.

If $x \geq 1$, then $6x^4 + 2 \leq 8x^4$. Multiplying both sides by $\frac{1}{2}$,
$3x^4 + 1 \leq 8 \cdot \frac{x^4}{2}$. Of course $|3x^4 + 1| \leq 8|\frac{x^4}{2}|$, for all $x > 1$. So $f(x)$ is $\mathcal{O}(g(x))$ with constant witnesses $C = 8$, and $k = 1$.

Because $f(x)$ is $\Omega(g(x))$, and $f(x)$ is $\mathcal{O}(g(x))$, it follows that $f(x)$ is $\Theta(g(x))$. ■