Theorem (1.6.3). *If χ is an even integer, then χ^2 is an even integer.*

Proof. By the definition for even numbers, there exists an integer η such that $\chi = 2\eta$. Hence,

$$\langle 2\eta \rangle^2 = 4\eta^2 = 2\langle 2\eta^2 \rangle$$

Integers are closed under multiplication. Thus, the factor $\langle 2\eta^2 \rangle$ is an integer. It follows that χ^2 is even, by the definition for even numbers. $\blacksquare$