Theorem (2.4.43). *The set of all finite bit strings is countable.*

Proof. Let $\{a_{n-1}\}$ be the sequence of bits for any finite bit string a(base-2) of length n. The unique base-2 expansion for $\{a_{n-1}\}$ is the integer

$$a(\text{base-10}) = \sum_{i=0}^{n-1} a_i 2^i$$

Also, this integer can be converted to the unique base-2 bit string for a(base-10) by

$$a(\text{base-2}) = \sum_{i=0}^{n-1} \left[a(\text{base-10}) (\text{mod } 2^{i+1})\right] 10^i$$

Since an invertible function exists between each finite bit string and some positive integer, there exists, a one-to-one correspondence between $\mathbb{Z}$ and the set of all finite bit strings. Thus, the cardinality for the set of all finite bit strings is $\aleph_0$, and the set of all finite bit strings is countable, by definition. ∎