**Theorem (2.4.21a).** *The summation of odd numbers from 1 to n is $n^2$.*

*Proof.* The summation of odd numbers from 1 to $n$ is given by,

$$\sum_{k=1}^{n} 2k - 1$$

by the definition for odd numbers. The identity of $2k - 1$ is the difference of squares $k^2 - (k-1)^2$. This identity can be demonstrated by the statement

$$k^2 - (k-1)^2 = [k + (k-1)][k - (k-1)] = (2k-1)[k + (-k+1)] = (2k-1)1$$

So the summation of odd numbers from 1 to $n$ is the telescoping summation

$$\sum_{k=1}^{n} k^2 - (k-1)^2$$

By Theorem 2.4.19, that is $n^2 - 0^2 = n^2$. Thus,

$$\sum_{k=1}^{n} 2k - 1 = n^2$$

and indeed the summation of odd numbers from 1 to $n$ is $n^2$. ■