Theorem (3.2.19a). *Let f be the function defined by $f(n) = (n^2 + 8)(n + 1)$. $f(n)$ is $\mathcal{O}(n^3)$.*

Proof. $f(n)$ is the product of functions $(f'f'')(n)$ where $f'(n) = (n^2+8)$, and $f'' = (n + 1)$. Since a k^{th} degree polynomial is $\mathcal{O}(x^k)$, it follows that $f'(n)$ is $\mathcal{O}(n^2)$, and $f''(n)$ is $\mathcal{O}(n)$. The upper bound for a product of functions is the product of the bounding functions for each function occurring in the product of functions. Hence, the upper bound for $f(n)$ is $\mathcal{O}(n(n^2))$. This means that $f(n)$ is $\mathcal{O}(n^3)$. ∎