

Theorem (3.2.22b). *Let f be the function defined by $f(x) = 3x + 7$. $f(x)$ is $\Theta(x)$.*

Proof. If $x \geq 7$ then $f(x) = |3x+7| \leq 4|x|$. Thus, $f(x)$ is $\mathcal{O}(x)$ with constant witnesses $C = 4$ and $k = 7$. Obviously, $f(x) = |3x+7| \geq 1|x|$, for all $x \geq 1$. Thus, $f(x)$ is $\Omega(x)$ with constant witnesses $C = 1$, and $k = 1$. Since $f(x)$ is both $\mathcal{O}(x)$ and $\Omega(x)$, it follows that $f(x)$ is $\Theta(x)$. ■