Theorem (3.2.12). *$x \log x$ is $\mathcal{O}(x^2)$, but x^2 is not $\mathcal{O}(x \log x)$.*

Proof. $\log x \leq x$, for all x in the domain of logarithmic functions. Thus, it follows that $x \log x \leq x^2$, for all $x > 1$. Therefore $x \log x$ is $\mathcal{O}(x^2)$ with constant witnesses $C = 1$, and $k = 1$.

If x^2 were $\mathcal{O}(x \log x)$, then $x^2 \leq C \cdot x \log x$, for all $x > 1$. But $C \cdot x \log x$ is a strictly decreasing function with respect to x^2. So this is impossible for the unbounded domain of x. Therefore, x^2 is not $\mathcal{O}(x \log x)$. ■