

Theorem (2.3.40a). *Let f be the function $f : A \implies B$. Let S , and T be subsets of B . $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$.*

Proof. By the definition for the inverse image of the set $(S \cup T)$ under the function f^{-1} , we have $f^{-1}(S \cup T) = \{a \in A | f(a) \in (S \cup T)\}$. Then equivalently, $f^{-1}(S \cup T) \equiv \{a \in A | f(a) \in S\} \cup \{a \in A | f(a) \in T\}$. This is the formal definition for $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$. ■