Theorem (2.3.46)**.** *Let x be a real number, and let m be an integer.* $\lceil x+m \rceil = \lceil x \rceil + m$.

Proof. By the definition of the ceiling function, we have the follow tautology.
$\lceil x \rceil = \lceil x \rceil \iff (\lceil x \rceil - 1) < x \leq \lceil x \rceil$.
Adding the integer m to every side of this inequality gives the following resultant tautology, by definition,
$\lceil x+m \rceil = \lceil x \rceil + m \iff (\lceil x \rceil + m) - 1 < x + m \leq \lceil x \rceil + m$. ■