

Theorem (3.2.7d). *Let f be the function defined by $f(x) = \frac{x^4+5\log x}{x^4+1}$. $f(x)$ is $\mathcal{O}(1)$.*

Proof. Let g be the function defined by $g(x) = 1$. If $x \geq 1$, then

$$\left(\frac{x^4+5\log x}{x^4+1}\right) = \left(\frac{x^4}{x^4+1} + \frac{5\log x}{x^4+1}\right) \leq \left(1 + \frac{5\log x}{x^4+1}\right) \leq \left(1 + \frac{5x}{x^4}\right) \leq \left(1 + \frac{5}{x^3}\right).$$

The function $1 + \frac{5}{x^3}$ is strictly decreasing with respect to $g(x)$, and never exceeding 2. Therefore, $|f(x)| \leq 2|g(x)|$, for all $x > 1$. Thus, $f(x)$ is $\mathcal{O}(1)$ with constant witnesses $C = 2$, and $k = 1$. ■