

Theorem (3.2.8b). *Let f be the function defined by $f(x) = 3x^5 + (\log x)^4$. $f(x)$ is $\mathcal{O}(x^5)$.*

Proof. Let g be the function defined by $g(x) = x^5$. If $x \geq 2$, then

$$f(x) = 3x^5 + (\log x)^4 \leq 3x^5 + 4x \leq 3x^5 + x^5 = 4x^5.$$

Therefore $|f(x)| \leq 4|g(x)|$, for all $x > 2$, and $f(x)$ is $\mathcal{O}(x^5)$ with constant witnesses $C = 4$, and $k = 2$. ■