

**Theorem (1.6.4).** *The additive inverse of an even number is an even number.*

*Proof.* Let  $\chi$  be an even number. There exists an integer  $\eta$  such that  $\chi = 2\eta$ , by the definition for even numbers. The additive inverse for  $\chi$  is,

$$-1\langle\chi\rangle = -1\langle 2\eta\rangle$$

By commutativity of multiplication that is,

$$-1\langle 2\eta\rangle = 2\langle -\eta\rangle$$

Since integers are closed under multiplication, the factor  $\langle -\eta\rangle$  is an integer. It follows that the additive inverse of  $\chi$  is an even number, by the definition for even numbers. ■