

Theorem (2.2.36b). *Let f be the function $f : A \implies B$. Let S , and T be subsets of A . $f(S \cap T) \subseteq f(S) \cap f(T)$.*

Proof. Let a be an element in A such that $f(a) \in f(S \cap T)$. Hence, by the definition for the image of $(S \cap T)$ under the function f , $a \in (S \cap T)$. The set intersection is defined as $(a \in S) \wedge (a \in T)$. Of course $[f(a) \in f(S)] \wedge [f(a) \in f(T)]$. That is, $f(a) \in [f(S) \cap f(T)]$, and indeed $f(S \cap T) \subseteq f(S) \cap f(T)$. ■