Theorem (2.2.17)**.** *Let A, B, and C be sets.* $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$.

Proof. Let x be an element in $\overline{A \cap B \cap C}$. By the definitions for set complementation and logical negation we have $x \notin (A \cap B \cap C) \equiv \neg(x \in A \cap B \cap C)$. By the definition of set intersection that is $\neg[(x \in A) \wedge (x \in B) \wedge (x \in C)]$. Using DeMorgans law (from logic) we can distribute the logical negation across the conjunctions giving us the expression $\neg(x \in A) \vee \neg(x \in B) \vee \neg(x \in C)$. Carrying out those logical negations on each respective term, and again by the definition for set complementation we have $(x \in \overline{A}) \vee (x \in \overline{B}) \vee (x \in \overline{C})$. This is the definition of $\overline{A} \cup \overline{B} \cup \overline{C}$. Hence, $\overline{A \cap B \cap C} \subseteq \overline{A} \cup \overline{B} \cup \overline{C}$.

Now suppose the converse case, where x is an element of $\overline{A} \cup \overline{B} \cup \overline{C}$. As already stated in the previous paragraph the definition for this expression is $(x \in \overline{A}) \vee (x \in \overline{B}) \vee (x \in \overline{C})$. By the definitions for complementation and logical negation that is $\neg(x \in A) \vee \neg(x \in B) \vee \neg(x \in C)$. Using DeMorgans law (from logic) we can factor out the logical negations such that $\neg[(x \in A) \wedge (x \in B) \wedge (x \in C)]$. Which, by the arguments given in the first paragraph we know is equivalent to $\overline{A \cap B \cap C}$. Thus, $\overline{A} \cup \overline{B} \cup \overline{C} \subseteq \overline{A \cap B \cap C}$.

It immediately follows from the definition of set equivalence that $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ ∎