

Theorem (3.2.14f). *Let f be the functions defined by $f(x) = x^3$, and let g be the function defined by $g(x) = \frac{x^3}{2}$. $f(x)$ is $\mathcal{O}(g(x))$.*

Proof. If $f(x)$ is $\mathcal{O}(g(x))$, then there exists constant witnesses C and k such that $|f(x)| \leq C|g(x)|$, for all $x > k$. Clearly, $x^3 \leq \left(2 \cdot \frac{x^3}{2}\right) = x^3$, for all $x > 0$. Thus, $f(x)$ is $\mathcal{O}(g(x))$ with constant witnesses $C = 2$, and $k = 0$. ■