

**Theorem (2.3.41).** *Let  $f$  be the function  $f : A \Longrightarrow B$ . Let  $S$  be a subset of  $B$ .  $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$ .*

*Proof.* By the definition for the inverse image of  $\overline{S}$  under the function  $f^{-1}$ , we have  $f^{-1}(\overline{S}) \equiv \{a \in A \mid f(a) \in \overline{S}\}$ . Factoring the complementation out from the right-hand side of the equivalence,  $f^{-1}(\overline{S}) \equiv \overline{\{a \in A \mid f(a) \in S\}}$ . But this statement is the negation of the formal definition for the inverse image of  $S$  under the function  $f^{-1}$ . In other words,  $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$ . ■