

Theorem (2.2.18c). *Let A , B , and C be sets. $(A - B) - C \subseteq (A - C)$.*

Proof. Let x be an element in $(A - B) - C$. Note that $A - B \equiv A \cap \overline{B}$ and $(A \cap \overline{B}) - C \equiv (A \cap \overline{B}) \cap \overline{C}$. By the associative laws and the commutative laws for the intersection of sets we have, $x \in (A \cap \overline{C}) \cap \overline{B}$. By definition that is $[(x \in A) \wedge (x \in \overline{C})] \wedge (x \notin B)$. Or rather, $x \in (A - C) \wedge (x \notin B)$. By logical identity $x \in (A - C)$. Since $x \in [(A - B) - C] \implies x \in (A - C)$, $(A - B) - C \subseteq (A - C)$. ■