Theorem (2.3.49)**.** *Let n be an integer. If n is even, then $\lfloor \frac{n}{2} \rfloor = \frac{n}{2}$. If n is odd, then $\lfloor \frac{n}{2} \rfloor = \frac{(n-1)}{2}$.*

Proof. By cases.

(i) Since n is even, there exists an integer k such that $n = 2k$.
$\lfloor \frac{n}{2} \rfloor = \lfloor \frac{2k}{2} \rfloor = \lfloor k \rfloor = k$. Also, $\frac{n}{2} = \frac{2k}{2} = k$. So $k = \lfloor \frac{n}{2} \rfloor = \frac{n}{2}$.

(ii) Since n is odd, there exists an integer k such that $n = 2k + 1$.
$\lfloor \frac{n}{2} \rfloor = \lfloor \frac{2k+1}{2} \rfloor = \lfloor k + \frac{1}{2} \rfloor = k$. Also, $\frac{(n-1)}{2} = \frac{[(2k+1)-1]}{2} = \frac{2k}{2} = k$.
So, $k = \lfloor \frac{n}{2} \rfloor = \frac{(n-1)}{2}$. ■