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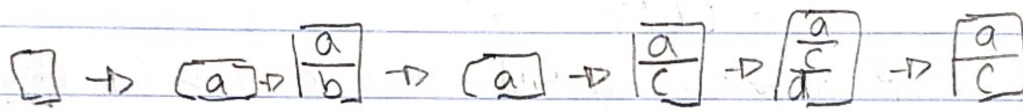
CS 411
HW 1

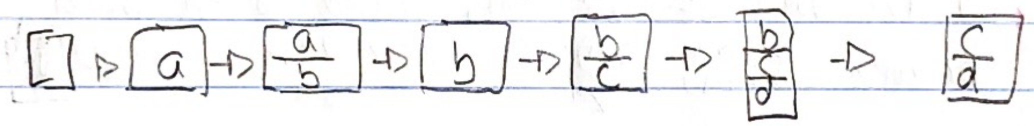
1.3 1b). No the algorithm is not stable. If you add 35' to the end of the list, 35' comes before 35 in the sorted list. The algorithm assumes A[j] is greater than but not equal to A[i], if it's not less than A[i].

1c) No it isn't in place. Two new arrays are allocated to accomodate sorting, S[] and Count[].

1.4 3a.)



b.)



2.1 7a) an input size of 1000 would yield 1 billion and and the 500 input size yields 125 million. Therefore the input size of 1000 would take 8 times as many multiplications.

b.) $T(n) = T(N)$

$$\frac{1}{3}n^3 = 10^{-3}\frac{1}{3}(N)^3$$

$$n^3 \approx 10^{-3}(N)^3$$

$$\frac{1}{10^{-3}} = \frac{(N)^3}{n^3}$$

$$\left(\frac{N}{n}\right)^3 = 10^3 \rightarrow \frac{N}{n} = \sqrt[3]{10^3} = \boxed{10}$$

The systems will increase by a factor of 10.

2.1/8, 2.1/9, 2.2/2, 2.3/1, 2.3/5, 2.4/1
2.4/4

2.1/8 a.) The function increases in value by 2 because
$\log_2 4n = \log_2 4 + \log_2 n = 2 + \log_2 n$.

b.) $n^{1/2} \to (4n)^{1/2} = 2\sqrt{n}$, increases twofold.
c.) $n \to 4n$, increases fourfold.
d.) n^2 increases sixteenfold because
$n^2 \to (4n^2) = 16n^2$
e.) n^3 increases 64-fold because $n \to (4n)^3 = 64n^3$
f.) The value of 2^n gets raised to the third
because $\frac{f(4n)}{f(n)} = \frac{(2^n)^4}{2^n} = \frac{2^n \cdot 2^n \cdot 2^n \cdot 2^n}{2^n} = (2^n)^3$

2.1/9) a.) Same, because $n(n+1) = n^2 + n \approx n^2$, within a constant multiple
b.) lower, because $100n^2 \approx n^2 < n^3$, within a constant multiple
c.) Same, because $\ln n = \log_e n$ and all logarithmic functions have the same order of growth within a constant multiple.
d.) higher, because $\log_2^2 n = \log_2 n \cdot \log_2 n$, which is greater than $\log_2 n^2 = 2\log_2 n$, within a constant multiple.
e.) Same. $2^{n-1} = \frac{1}{2} \cdot 2^n \approx 2^n$.
f.) lower. $(n-1)!$ will always have a lower order of growth than $n!$.

2.2/2, 2.3/1, 2.3/5, 2.4/1, 2.4/4

1)

2.2/2 a.) $\frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n \in O(n^3)$, $\boxed{T}$ b.) $\frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n \in O(n^2)$ $\boxed{T}$

c.) $\frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n \notin \Theta(n^3)$ $\boxed{F}$ d.) $\frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n \in \Omega(n)$, $\boxed{T}$

2.3/1 a.) $1+3+5+7+\ldots+999 = \sum_{i=1}^{500}(2i-1) = 2\left(\frac{500 \cdot 501}{2}\right) - 500 = \boxed{250{,}000}$

b.) $2+4+8+16+\ldots+1024 = \sum_{i=1}^{10}(2^i - 1) = 2046$, $2 \cdot \frac{2^{n+1}-1}{2-1} = \boxed{2046}$

c.) $\sum_{i=3}^{n+1} 1 = (n+1)-3+1 = n+1-2 = \boxed{n-1}$

d.) $\sum_{i=3}^{n+1} i = \frac{(n+1)(n+2)}{2} - 3 = \frac{n^2+3n+2}{2} - \frac{6}{2} = \boxed{\frac{n^2+3n-4}{2}}$

e) $\sum_{i=0}^{n-1} i(i+1) = \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} i$

f.) $\sum_{j=1}^{n} 3^{j+1} = 3\left[\sum_{j=1}^{n} 3^j\right] = 3\left[\sum_{j=0}^{n} 3^j - 1\right] = 3\left[\frac{3^{n+1}-1}{2} - 1\right]$

$= \boxed{\frac{3^{n+2}-9}{2}}$

g.) $\sum_{i=1}^{n}\sum_{j=1}^{n} ij = \sum_{i=1}^{n} i \sum_{j=1}^{n} j = \sum_{i=1}^{n} i\left(\frac{n(n+1)}{2}\right) = \frac{n(n+1)}{2}\sum_{i=1}^{n} i$

$= \frac{n(n+1)}{2} \cdot \frac{n(n+1)}{2} = \boxed{\frac{n^2(n+1)^2}{4}}$

h.) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \sum_{i=1}^{n}\left(\frac{1}{i} - \frac{1}{i+1}\right) = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \ldots + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= 1 - \frac{1}{n+1} = \boxed{\frac{n}{n+1}}$

2.3/5, 2.4/1, 2.4/4

2.3/5.) a.) It's finding the difference between the largest and Smallest values in an array.

b.) Comparison

c.) $C(n) = \sum_{i=1}^{n} 2 = 2(n-1)$ times

d.) $C(n) = 2(n-1) \in \Theta(n)$, linear.

e.) The algorithm does comparison twice, It would be better if there was one if-else bock rather than two if statements.

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if A[i] < minval
        minval ← A[i]
else
        maxval ← A[i]
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Thus the efficiency class would be:

$$\sum_{i=1}^{n} 1 = n \in \Theta(n)$$

2.4/1 a) $x(n) = x(n-1) + 5$ for $n > 1$, $x(1) = 0$

$\to [x(n-2)+5]+5 = x(n-2)+10$

$[x(n-3)+5]+10 = x(n-3)+15$

B.S. $x(n) = x(n-i) + 5i$

$t-i=1$ $M(t) = M(t-(t-1)) + 5(t-1)$

$t-1=i$ $= M(1) + 5(t-1)$

$M(t) = 0 + 5(t-1) = \boxed{5(t-1)}$

So, $\boxed{5(n-1)}$

b. $x(n) = 3x(n-1)$; $n > 1$, $x(1) = 4$

$\rightarrow 3(3x(n-2))$

$\rightarrow 3(3(3x(n-3)))$

$\vdots$

$x(n) = 3^i x(n-i)$

$\rightarrow i = n-1$

So $M(t) = 3^{n-1} x(n-(n-1))$

$= 3^{n-1} x(1) = \boxed{4 \cdot 3^{n-1}}$

C.) $x(n) = x(n-1) + n$; $n > 0$, $x(0) = 0$

$k=2$ $[x(n-1-1) + (n-1)] + n = [x(n-2) + (n-1)] + n$

$k=3$ $\rightarrow [x(n-1-2) + (n-1-1) + (n-1)] + n = x(n-3) + (n-2) + (n-1) + n$

$\vdots$

$k=i$ $= x(n-i) + (n-i+1) + (n-i+2) + \dots (n-i+(i-2)) + (n-i+(i-1))$
$+ (n-i+i)$

$x(n) = x(n-i) + (n-i+1) + (n-i+2) + \dots + n$

$n=i$ $= x(0) + 1 + 2 + 3 + \dots + n = \boxed{\frac{n(n+1)}{2}}$

d.) $x(n) = x(n/2) + n$; $n > 1$, $x(1) = 1$, $n = 2^k$

$x(n) = x(2^{k-1}) + 2^k$

$= [x(2^{k-1-1}) + 2^{k-1}] + 2^k = x(2^{k-2}) + 2(^{k-1}) + 2^k$

$= [x(2^{k-2-1}) + 2^{k-1-1} + 2^{k-1}] + 2^k = x(2^{k-3}) + (2^{k-2}) + 2^{k-1} + 2^k$

$\vdots$

$= x(2^{k-i}) + 2^{k-i+1} + 2^{k-i+2} + \dots + 2^k$

let $i=k$, then $= x(2^{k-k}) + 2^{k-k+1} + 2^{k-k+2} + \dots + 2^k$

$= x(1) + 2^1 + 2^2 + \dots + 2^k$

$= 1 + 2 + 4 + \dots + 2^k = 2^{k+1} - 1 = 2 \cdot 2^k - 1 = \boxed{2n-1}$

$x(n) = x(n/3)+1 \quad ; \quad n>1, \; x(1)=1 \quad n=3^K$

$K=1 \quad = x(3^{K-1})+1$

$2 \quad = [x(3^{K-1-1})+1]+1 = x(3^{K-2})+2$

$3 \quad = [x(3^{K-1-2})+2]+1 = x(3^{K-3})+3$

$\vdots$

$= x(3^{K-i})+i$

$K=i$ Then $x(3^{K-K})+K = x(1)+K$

$\boxed{= 1 + \log_3 n}$

2.4/4 a) $Q(n) = Q(n-1)+2n-1 \quad ; \quad n>1 \quad Q(1)=1$

$Q(2) = Q(1)+(2)(2)-1 = 1+4-1 = 4$

$Q(3) = Q(2)+2(3)-1 = 4+6-1 = 9$

$Q(4) = Q(3)+2(4)-1 = 9+8-1 = 16$

So $\boxed{Q(n)=n^2}$, $Q(1) = 1^2 = 1$

b.) $M(n) = M(n-1)+1 \; ; M(1)=0, \; n>1 \; 2n-1$

$= [M(n-2)+1]+1$

$\rightarrow [M(n-3)+2]+1$

$\vdots$

let $i=n-1$ $\quad M(n-(n-1))+(n-1) \rightarrow M(1)+n-1 = \boxed{n-1}$

c.) $C(n) = C(n-1)+3 \quad ; \quad n>1 \quad C(1)=0$

$\rightarrow [C(n-2)+3]+3 = C(n-2)+6$

$\rightarrow [C(n-3)+6]+3 = C(n-3)+9$

$\vdots$

let $i=n-1$ $\quad \rightarrow C(n-i)+3i$

$C(n-(n-1))+3(n-1) = C(1)+3(n-1) = 0+3(n-1)$

$= \boxed{3(n-1)}$