

1. Let  $\alpha$  be the root of  $m(x) = x^2 + 4$  in  $\mathbb{F}_7[x]$ .

$$m(\alpha) = 0 = \alpha^2 + 4 \Rightarrow \alpha^2 = -4 \equiv 3 \pmod{7}.$$

$$\Rightarrow \alpha^4 = 9 \pmod{7} \equiv 2 \pmod{7} \Rightarrow \alpha^8 = 4 \pmod{7} \Rightarrow \alpha^{16} = 16 \pmod{7} \equiv 2 \pmod{7}.$$

$$\Rightarrow \{\alpha^2, \alpha^4, \alpha^8\} \text{ are primitive elements.}$$

$$\Rightarrow \text{ord}(\alpha^2) = 3.$$

2.  $(1 \ 1)$  represents  $x+1$  and  $(2 \ 2)$  represents  $2x+2$ .

$$x+a \text{ for } a=0, 1, \dots, 48.$$

$$x^2+x+a \text{ for } a=0, \dots, 48 \text{ except } 0, 7, 42.$$

$$x^2+a \text{ for } a=0, \dots, 48 \text{ except } 0, 13, 36, 25, 24, 40, 48.$$

3.  $x+a$  for  $a=0, \dots, 48$

$$x^2+x+a \text{ for } a=0, \dots, 48 \text{ except } 0, 7, 42.$$

$$x^2+a \text{ for } a=0, \dots, 48 \text{ except } 0, 13, 24, 25, 36, 40, 48.$$

4.  $x+1, x, x+2, x+3, x+4, x+5, x+6$ .

$$x^2+x+1, x^2+x+2, x^2+x+3, x^2+x+4, x^2+x+5, x^2+x+6.$$

$$x^2+1, x^2+2, x^2+4, x^2+5$$