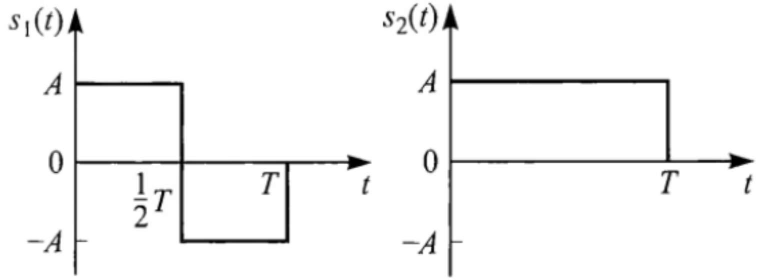
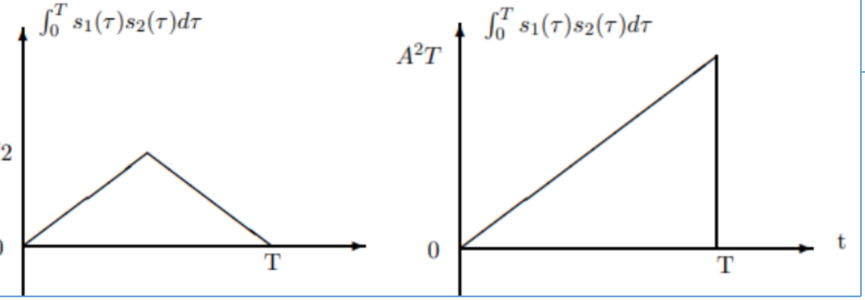
1. $r_l(t) = s_i(t) + z(t); 0 \le t \le T; i = 1,2$



(d). Suppose the receiver is implemented by means of two cross-correlators (multipliers followed by integrators) in parallel. Sketch the output of each integrator as a function of time for the interval $0 \le t \le T$ when the transmitted signal is $s_2(t)$.

<SOL>



2.(24%) Consider a signal detector with an input $r = s + n$, where the transmit signal equals to +M or –M with equal probability.

(b.) (8%) If the noise variable n is characterized by the Laplacian PDF as

$$p(n) = \frac{1}{\sqrt{2\sigma^2}} e^{-|n|\frac{\sqrt{2}}{\sigma}}$$

Determine the probability of error as a function of the parameters M and σ.

<SOL>

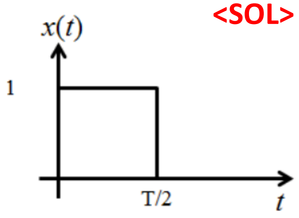
$$\begin{aligned} P_e &= \frac{1}{2}\int_{-\infty}^{0} \frac{1}{\sqrt{2\sigma^2}} e^{-|r-M|\frac{\sqrt{2}}{\sigma}} dr + \frac{1}{2}\int_{0}^{\infty} \frac{1}{\sqrt{2\sigma^2}} e^{-|r+M|\frac{\sqrt{2}}{\sigma}} dr \\ &= \frac{\sqrt{2}}{4\sigma}\int_{-\infty}^{-M} e^{-|x|\frac{\sqrt{2}}{\sigma}} dx + \frac{\sqrt{2}}{4\sigma}\int_{M}^{\infty} e^{-|y|\frac{\sqrt{2}}{\sigma}} dy \\ &= \frac{1}{2} e^{-\frac{\sqrt{2}}{\sigma}M} \end{aligned}$$

5. (20%) A binary communication scheme uses two equiprobable signals $s_1(t)$, $s_2(t)$ where $s_1(t) = x(t), 0 \le t \le T$, $s_2(t) = x(t - T/2), 0 \le t \le T$, and $x(t)$ is shown as follows. The power spectral density of the noise is $N_0/2$

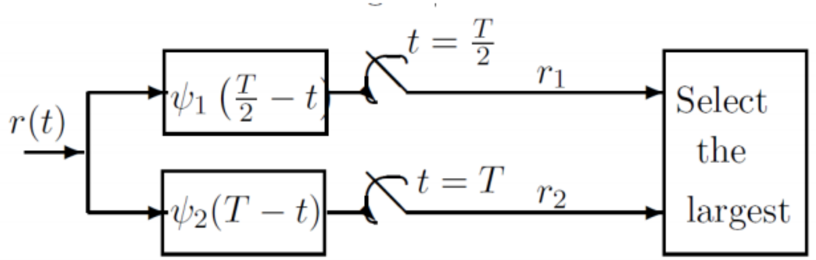
(a) (7%) Design an optimal matched filter receiver for this system. Carefully label the diagram and determine all the required parameters.

(b) (7%) Determine the error probability for this communication system.

(c) (6%) Show that the receiver can be implemented using only one matched filter.



<SOL> (a) The optimal receiver is shown in the figure



where $\psi_1(t) = \begin{cases} \sqrt{\frac{2}{T}}, & 0 \le t < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$ $\quad \psi_2(t) = \begin{cases} \sqrt{\frac{2}{T}}, & \frac{T}{2} \le t < T \\ 0, & \text{otherwise} \end{cases}$

<SOL> (b) This is a binary equiprobable system, we can use $P_e = Q\left(\frac{d}{\sqrt{2N_0}}\right)$

$$\begin{aligned} d^2 &= \|s_1 - s_2\|^2 \\ &= \int_{-\infty}^{\infty} (s_1(t) - s_2(t))^2\, dt \\ &= T \end{aligned}$$

Therefore, $P_e = Q\left(\sqrt{\frac{T}{2N_0}}\right)$

<SOL>

(c) The signal waveform $\psi_1(\frac{T}{2} - t)$ matched to $\psi_1(t)$ is exactly the same with the signal waveform $\psi_2(T - t)$ matched to $\psi_2(t)$. That is

$$\psi_1\left(\frac{T}{2} - t\right) = \psi_2(T - t) = \psi_1(t) = \begin{cases} \sqrt{\frac{2}{T}}, & 0 \le t < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$

Thus, the optimal receiver can be implemented by using just one filter with impulse response $\psi_1(t)$ followed by two samplers which sample the output of the matched filter at t = T/2 and t = T to produce random variables r_1 and r_2 , respectively.

2. (15%) Consider a sequence of n binary random variables$X_1, X_2, \ldots, X_n$. Each sequence with an even number of 1's has probability $2^{-(n-1)}$ and each sequence with an odd number of 1's has probability 0. Find the mutual informations $I(X_1;X_2)$, $I(X_2;X_3|X_1)$;..... ; $I(X_{n-1}; X_n|X_1;\ldots X_{n-2})$.

<SOL> Consider a sequence of n binary random variables $X_1, X_2, \ldots, X_n$. Each sequence of length n with an even number of 1's is equally likely and has probability $2^{-(n-1)}$.

Any $n-1$ or fewer of these are independent. Thus, for $k \leq n-1$,

$$I(X_{k-1}; X_k|X_1, X_2, \ldots, X_{k-2}) = 0.$$

However, given $X_1, X_2, \ldots, X_{n-2}$, we know that once we know either X_{n-1} or X_n we know the other.

$$\begin{aligned} I(X_{n-1}; X_n|X_1, X_2, \ldots, X_{n-2}) &= H(X_n|X_1, X_2, \ldots, X_{n-2}) - H(X_n|X_1, X_2, \ldots, X_{n-1}) \\ &= 1 - 0 = 1 \text{ bit.} \end{aligned}$$

3. (20%) Let X be a geometrically distributed random variable, i.e.,

$$P(X = k) = p(1-p)^{k-1}, k = 1, 2, 3, \ldots$$

(1) Find the entropy of X.

(2) Given that X > K, where K is a positive integer, what is the entropy of X?

<(1)>

$$\begin{aligned} H(X) &= -\sum_{k=1}^{\infty} p(1-p)^{k-1} \log_2 p(1-p)^{k-1} \\ &= -\sum_{k=1}^{\infty} p(1-p)^{k-1} \log_2 p - \sum_{k=1}^{\infty} p(1-p)^{k-1} \log_2 (1-p)^{k-1} \\ &= -p\log_2 p \sum_{k=1}^{\infty} (1-p)^{k-1} - p\log_2 (1-p) \sum_{k=1}^{\infty} (k-1)(1-p)^{k-1} \\ &= -p\log_2 p \frac{1}{1-(1-p)} - p\log_2(1-p)\frac{1-p}{(1-(1-p))^2} \\ &= -\log_2 p - \frac{1-p}{p}\log_2(1-p) \end{aligned}$$

<(2)> If $k \leq K$, clearly $P(X = k \mid X > K) = 0$.

If $k > K$, then $P(X = k \mid X > K) = \dfrac{P(X = k, X > K)}{P(X > K)} = \dfrac{p(1-p)^{k-1}}{P(X > K)}$

$$P(X > K) = \sum_{k=K+1}^{\infty} p(1-p)^{k-1} = p\frac{(1-p)^K}{1-(1-p)} = (1-p)^K$$

$$P(X = k \mid X > K) = \frac{p(1-p)^{k-1}}{(1-p)^K} = p(1-p)^{l-1}$$

where $l = k - K$.

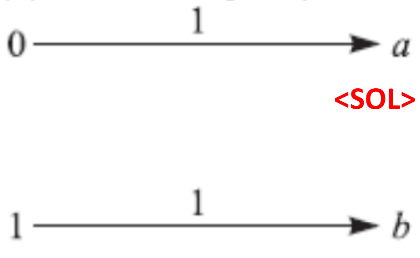
The conditional entropy is

$$\begin{aligned} H(X \mid X > K) &= -\sum P(X = k \mid X > K) \log_2 P(X = k \mid X > K) \\ &= -\sum_{l=1}^{\infty} p(1-p)^{l-1} \log_2 (p(1-p)^{l-1}) \\ &= -\sum_{l=1}^{\infty} p(1-p)^{l-1} \log_2 p - \sum_{l=1}^{\infty} p(1-p)^{l-1} \log_2 ((1-p)^{l-1}) \\ &= -p\log_2 p \frac{1}{1-(1-p)} - p\log_2(1-p)\frac{1-p}{(1-(1-p))^2} \\ &= -\log_2 p - \frac{1-p}{p}\log_2(1-p) \end{aligned}$$

Note that $P(X = k \mid X > K)$ is still geometrically distributed.

4. (20%)

(1) Find the capacity of channel 1. What input distribution achieves capacity?



Channel 1

<SOL>

$$[p(X)] = [p \quad 1-p]$$

$$[p(Y|X)] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, [p(X,Y)] = \begin{bmatrix} p & 0 \\ 0 & 1-p \end{bmatrix}$$

$$[p(Y)] = [p \quad 1-p] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [p \quad 1-p]$$

$$H(Y) = -plog_2 p - (1-p)log_2(1-p)$$

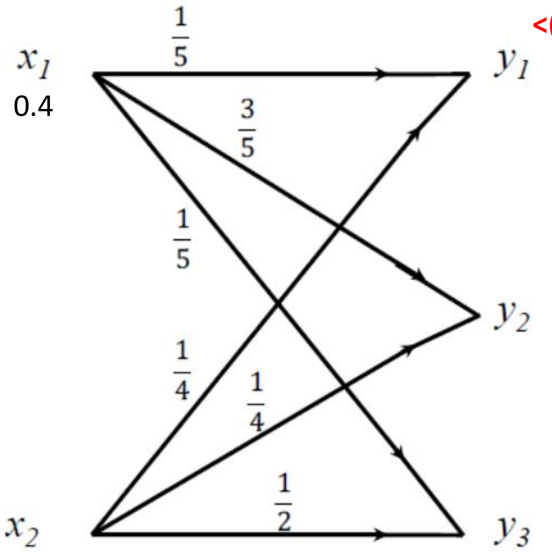
$$H(Y|X) = -plog_2(1) - (1-p)log_2(1) = 0$$

$$I(X;Y) = H(Y) - H(Y|X) = -plog_2 p - (1-p)log_2(1-p)$$

$$\frac{dI(X;Y)}{dp} = 0 \rightarrow -log_2 p + log_2(1-p) = 0 \rightarrow p = \frac{1}{2}$$

(1) If the channel output is y_1, what is the best decision that minimizes the error probability? Repeat for the cases where the channel output is y_2 and y_3.

(2) What is the overall error probability for the channel if the optimal decision scheme is used at the receiver?



<(1)>

If the output is Y_1:

$$P(X=x_1|Y=y_1) = \frac{P(Y=y_1|X=x_1)P(X=x_1)}{P(Y=y_1)} = \frac{1/5*0.4}{1/5*0.4+1/4*0.6} = \frac{8}{23}$$

$$P(X=x_2|Y=y_1) = \frac{P(Y=y_1|X=x_2)P(X=x_2)}{P(Y=y_1)} = \frac{1/4*0.6}{1/5*0.4+1/4*0.6} = \frac{15}{23}$$

Thus the best decision is X_2.

y1 y2 以此類推

<(2)> The error rate when the input is X_1:

$$P_{e1} = \sum_{i=1}^{3} P(\text{decide } X_2|Y_i)P(Y_i|\text{send } X_1) = 1*\frac{1}{5}+0*\frac{3}{5}+1*\frac{1}{5} = \frac{2}{5}$$

The error rate when the input is X_2:

$$P_{e2} = \sum_{i=1}^{3} P(\text{decide } X_1|Y_i)P(Y_i|\text{send } X_2) = 0*\frac{1}{4}+1*\frac{1}{4}+0*\frac{1}{2} = \frac{1}{4}$$

$$P_e = P(\text{send } X_1)P_{e1} + P(\text{send } X_2)P_{e2} = 0.4*\frac{2}{5}+0.6*\frac{1}{4} = 0.31$$

1. (20%) Consider sending a QPSK modulated symbol over AWGN channel with four equiprobable signals,

$$s_i(t) = A_0\cos(2\pi f_c t + i\pi/2),\ 0 \le t \le T,\ i = 0, 1, 2, 3$$

where f_c is the carrier frequency. The channel is AWGN with noise power spectral density of $N_0/2$.

(a) (5%) Find the symbol error probability of this system in terms of A_0, T, and N_0.

<(a)>

$$Si(t) = A\cos(2\pi f_c t + i\frac{\pi}{2})$$

$$|Si(t)| = \frac{1}{\sqrt{2}} A_0\sqrt{T}$$

$$\therefore d\min^2 = (\sqrt{2}\,|Si(t)|)^2 = A_0{}^2 T$$

$$BER(P_b) = Q(\frac{d_{min}/2}{\sqrt{N_o/2}})$$

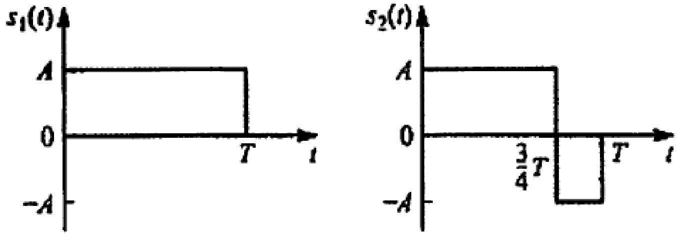
$$P_e = 1-(1-P_b)^2 = 2P_b = 2Q(\frac{A_0\sqrt{T}}{\sqrt{2N_o}})$$

QPSK 的訊號頻寬為 $\frac{2}{T}$

2. (21%) The following Figure shows two equivalent low-pass signals. They are used to transmit a binary information sequence. The transmitted signals, which are equally probable, are corrupted by additive zero-mean white Gaussian noise (AWGN) with an equivalent low-pass representation $z(t)$ and an autocorrelation function $R_Z(\tau)=E[z^*(t)z(t+\tau)]=2N_0\delta(\tau)$

(b) (7%) Please calculate the probability of a binary digit error if coherent detection is employed at the receiver.

(c) (7%) Please calculate the probability of a binary digit error if non-coherent detection is employed at the receiver.



<(b)>

$$d_{\min}=\sqrt{\int_0^T (s_1(t)-s_2(t))^2\,dt}=A\sqrt{T}$$

$$\sigma_c^2=R_z(0)=2N_0 \qquad \sigma^2=\frac{\sigma_c^2}{2}=N_0$$

$$p_e=Q\left(\frac{A\sqrt{T}/2}{\sigma}\right)=Q\left(\frac{A\sqrt{T}/2}{\sqrt{N_0}}\right)=Q\left(\frac{A\sqrt{T}}{2\sqrt{N_0}}\right)$$

<(c)>

The bit error probability for non-coherent detection is given by

$$P_{2,nc}=Q_1(a,b)-\frac{1}{2}e^{-(a^2+b^2)/2}I_0(ab)$$

Where $Q_1(.)$ is the generalized Marcum Q function

$$a=\sqrt{\frac{\varepsilon}{2N_0}\left(1-\sqrt{1-|\rho|^2}\right)}=\sqrt{\frac{\varepsilon}{2N_0}\left(1-\frac{\sqrt{3}}{2}\right)}$$

$$b=\sqrt{\frac{\varepsilon}{2N_0}\left(1+\sqrt{1-|\rho|^2}\right)}=\sqrt{\frac{\varepsilon}{2N_0}\left(1+\frac{\sqrt{3}}{2}\right)}$$

(g) (3%) From your knowledge of the signal characteristics, please give the probability of error for this binary communication system.

<(g)>

$$d_{\min}=2\sqrt{T}$$

$$\sigma_z^2=2N_0$$

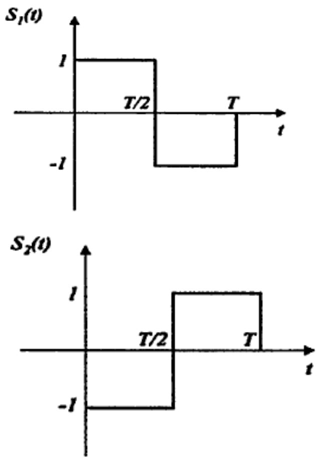
$$p_e=Q\left(\frac{d_{\min}/2}{\sigma_z}\right)=Q\left(\frac{\sqrt{T}}{\sqrt{2N_0}}\right)$$

$$R_Z(\tau)=E[z^*(t)z(t+\tau)]=2N_0\delta(\tau)$$

3. (21%) Two equivalent low-pass signals $s_1(t)$ and $s_2(t)$ are used to transmit a binary sequence over an additive white Gaussian noise channel.

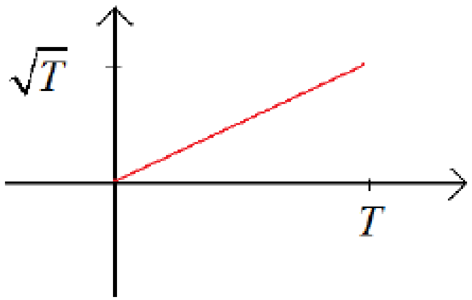
where $z(t)$ is a zero-mean Gaussian random noise with autocorrelation function

(e) (3%) Suppose the receiver is implemented by means of the correlator(s) (multipliers followed by integrators). Please sketch the output of each integrator as a function of time for the interval $0\le t\le T$ when the transmitted signals are $s_1(t)$ and $s_2(t)$ respectively.

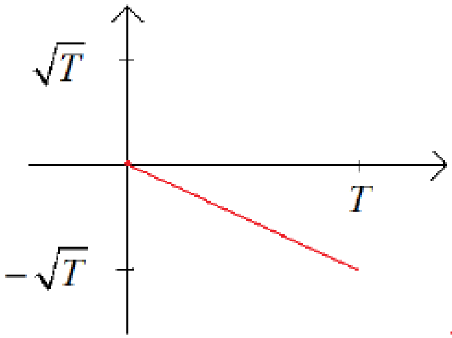


<(e)>

$$\int_0^T S_1(\tau)S_1(\tau)\,d\tau$$



$$\int_0^T S_1(\tau)S_2(\tau)\,d\tau$$

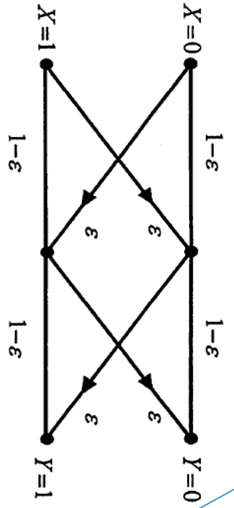


4. **(24%) Consider a relay transmission system that employs cascading the binary symmetric channels with the same cross-over error probability as follows.**

(a) **(8%) Determine the error probability at the destination.**

(b) **(8%) Determine $P(X=0)$ and $P(X=1)$ that maximizes the capacity,**

(c) **(8%) Calculate the capacity of this channel.**



<(a)>

$P(X=0,Y=0)=(1-\varepsilon)(1-\varepsilon)+\varepsilon\cdot\varepsilon=1-2\varepsilon+2\varepsilon^2$

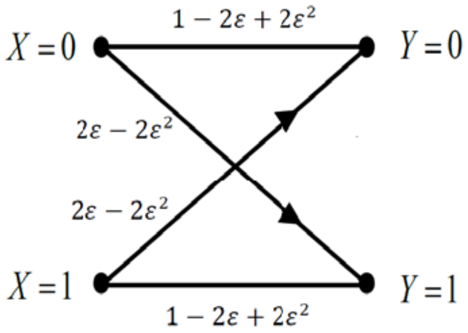
$P(X=0,Y=1)=(1-\varepsilon)\varepsilon+\varepsilon(1-\varepsilon)=2\varepsilon-2\varepsilon^2$

$P(X=1,Y=0)=\varepsilon(1-\varepsilon)+(1-\varepsilon)\varepsilon=2\varepsilon-2\varepsilon^2$

$P(X=1,Y=1)=\varepsilon\cdot\varepsilon+(1-\varepsilon)(1-\varepsilon)=1-2\varepsilon+2\varepsilon^2$

Let $P(X=0)=p$, $P(X=1)=1-p$

$P_e=p(2\varepsilon+2\varepsilon^2)+(1-p)(2\varepsilon+2\varepsilon^2)=2\varepsilon+2\varepsilon^2$



<(b)>

The capacity of this channel:

$C=\max\{I(X;Y)\}=\max\{H(Y)-H(Y|X)\}$

Since this channel is symmetric, the probabilities of sending 0 or 1 should be equal to maximize the capacity: $P(X=0)=P(X=1)=0.5$

Hence,

$$P(Y=0)=\frac{1}{2}(1-2\varepsilon+2\varepsilon^2)+\frac{1}{2}(2\varepsilon-2\varepsilon^2)=\frac{1}{2}$$

$$P(Y=1)=\frac{1}{2}(1-2\varepsilon+2\varepsilon^2)+\frac{1}{2}(2\varepsilon-2\varepsilon^2)=\frac{1}{2}$$

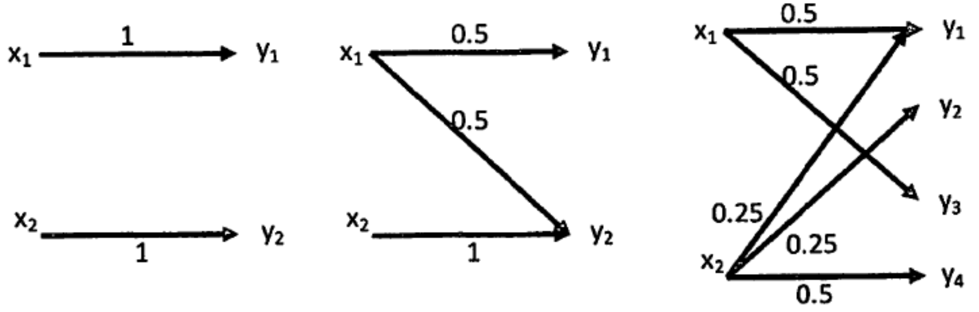
<(c)>

$$H(Y)=-\frac{1}{2}\log_2\frac{1}{2}+-\frac{1}{2}\log_2\frac{1}{2}=1$$

$$H(Y|X)=-\sum_y\sum_x p(x,y)\log_2 p(y|x)$$

$$=-(1-2\varepsilon+2\varepsilon^2)\log_2(1-2\varepsilon+2\varepsilon^2)-(2\varepsilon+2\varepsilon^2)\log_2(2\varepsilon+2\varepsilon^2)=H(2\varepsilon+2\varepsilon^2)$$

Therefore, the capacity is $C=\max\{H(Y)-H(Y|X)\}=1-H(2\varepsilon-2\varepsilon^2)$



(c) **(8%) Let C_1, C_2 and C_3 denote the capacity of channels (1), (2), and (3), respectively. Please compare the capacity of C_3 with $(C_1+C_2)/2$. Which is larger and explain your reason?**

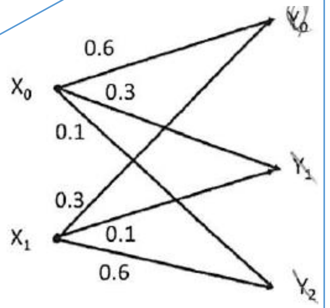
(c) Let p_i be the probability that achieves $C_i=\max_p\{I_i(X;Y)\},\forall i=1,2,3$

observation, we know that the inputs and outputs relation of channel(C) is combination of channel(A) and channel(B) with transition probabilities divide by 2 ,i.e.,

Since p_3 achieves C_3 ,if $C_3>\frac{1}{2}C_1+\frac{1}{2}C_2$, either $I_1(X;Y)|_{p=p3}>|C_1$ or

$I_2(X;Y)|_{p=p3}>|C_2$ which contradicts to the definition of capacity. $\Rightarrow C_3\le\frac{1}{2}C_1+\frac{1}{2}C_2$

We also know from (1) and (2) that $p_1\neq p_2$, and p_3 cannot achieve the capacity C_1 and C_2 at the same time, so we can conclude that $C_3<\frac{1}{2}C_1+\frac{1}{2}C_2$



5. **(20%) A communication channel with binary input and ternary output alphabets is shown in the following figure.**

<(a)>

$H(Y)=-0.3log_2(0.3+0.3p)-(0.1+0.2p)log_2(0.1+0.2p)-(0.6-0.5p)log_2(0.6-0.5p)$

$\frac{d}{dp}I(X;Y)=0\rightarrow -0.3log_2(0.3+0.3p)-0.2log_2(0.1+0.2p)+0.5log_2(0.6-0.5p)=0$

$\rightarrow\frac{(0.6-0.5p)^{0.5}}{(0.3+0.3p)^{0.3}(0.1+0.2p)^{0.2}}=1\rightarrow p=0.5324$

2. (20%) Two equiprobable messages m_1 and m_2 are transmitted through a channel with input X and output Y relate by $Y = \alpha X + N$, where N is zero-mean AWGN with variance $N_0/2$ and α is a random variable independent of noise.

(b)(5%) Following (a), but $\alpha = \pm 1$ with equal probability. What is the optimal decision rule and the resulting error probability?

(c)(5%) Following (a), but $\alpha = 0$ or 1 with equal probability. What is the optimal decision rule and the resulting error probability?

(d)(5%) Assuming on-off signaling (i.e. X= 0 or A) and $\alpha = 0$ or 1 with equal probability. What is the optimal decision rule?

<(b)> Use ML detection, we make a decision in favor of A is

$$p(y\,|\,X=A) \overset{X=A}{>} P(y\,|\,X=-A),$$

$$\frac{1}{2}P(y\,|\,X=A,\alpha=1)+\frac{1}{2}P(y\,|\,X=A,\alpha=-1) \overset{X=A}{>} \frac{1}{2}P(y\,|\,X=-A,\alpha=1)+\frac{1}{2}P(y\,|\,X=-A,\alpha=-1).$$

$$\frac{1}{2}\frac{1}{\sqrt{2\pi}\sqrt{N_0/2}}e^{-\frac{(y-A)^2}{2*N_0/2}}+\frac{1}{2}\frac{1}{\sqrt{2\pi}\sqrt{N_0/2}}e^{-\frac{(y+A)^2}{2*N_0/2}} \overset{X=A}{>} \frac{1}{2}\frac{1}{\sqrt{2\pi}\sqrt{N_0/2}}e^{-\frac{(y+A)^2}{2*N_0/2}}+\frac{1}{2}\frac{1}{\sqrt{2\pi}\sqrt{N_0/2}}e^{-\frac{(y-A)^2}{2*N_0/2}}$$

Since both sides of inequality are equal, any received Y can be equally detected as A or – A, and the error probability $P_e = \frac{1}{2}$.

<(c)> When α=0 no information is transmitted and any decision is irrelevant. $P_{e,\alpha=0} = \frac{1}{2}$

When α=1, the threshold is zero, $P_{e,\alpha=1} = Q(\frac{A}{\sqrt{N_0/2}})$

$$P_e = P(\alpha=1)P_{e,\alpha=1} + P(\alpha=0)P_{e,\alpha=0} = \frac{1}{2}*Q(\frac{A}{\sqrt{N_0/2}})+\frac{1}{2}*\frac{1}{2} = \frac{1}{2}*Q(\frac{A}{\sqrt{N_0/2}})+\frac{1}{4}$$

<(d)> When α=0 no information is transmitted and any decision is irrelevant. $P_{e,\alpha=0} = \frac{1}{2}$

When α=1, use ML detection, we make a decision in favor of A is

$$P(y\,|\,X=A,\alpha=1) \overset{X=A}{>} P(y\,|\,X=0,\alpha=1),\quad \frac{1}{\sqrt{2\pi}\sqrt{N_0/2}}e^{-\frac{(y-A)^2}{2*N_0/2}} \overset{X=A}{>} \frac{1}{\sqrt{2\pi}\sqrt{N_0/2}}e^{-\frac{(y)^2}{2*N_0/2}},$$

$$e^{\frac{(-2yA+A^2)}{2*N_0/2}} \overset{X=A}{>} 1 = e^0 \Rightarrow \frac{2yA-A^2}{2*N_0/2} \overset{X=A}{>} 0 \Rightarrow y \overset{X=A}{>} \frac{A}{2}.$$

A binary communication system uses equiprobable signals $s_1(t)$ and $s_2(t)$

(b)(5%) Assuming that, at the receiver, the received signals are multiplied by $\sqrt{2}\cos(2\pi f_c t+\theta)$ by the demodulator and then passed through a LPF. The demodulator has a random phase ambiguity θ, $0 \le \theta \le \pi$ in carrier recovery, and employs the coherent detector, what is the resulting error probability in terms of θ?

<(b)> After demodulator, the received signal becomes:

$$r_1(t) = \sqrt{2}\cos(2\pi f_c t+\theta)\times\sqrt{2E_b}\varphi_1(t)\cos(2\pi f_c t)$$

$$= 2\sqrt{E_b}\varphi_1(t)\frac{1}{2}[\cos(4\pi f_c t+\theta)+\cos(\theta)] = \sqrt{E_b}\varphi_1(t)[\cos(4\pi f_c t+\theta)+\cos(\theta)]$$

Pass through a LPF → $r_1(t) = \sqrt{E_b}\varphi_1(t)\cos(\theta)$

Similar to $r_1(t)$, $r_2(t) = \sqrt{E_b}\varphi_2(t)\cos(\theta)$ $\quad P_b = Q\left(\frac{\frac{d_{min}}{2}}{\sqrt{\frac{N_0}{2}}}\right) = Q\left(\frac{\frac{\sqrt{2E_b}\cos\theta}{2}}{\sqrt{\frac{N_0}{2}}}\right) = Q(\sqrt{\frac{E_b}{N_0}}\cos\theta)$

(15%) Consider transmission of real valued X_i over the AWGN channel with channel bandwidth B as

$$Y_i(t) = X_i + N_i$$

where the noise $\{N_i\}$ are i.i.d. zero mean Gaussian random variables with power spectral density σ_n^2 and the input X_i is subject to the power constraint $E\left[X_i^2\right] \le P$. Let the transmission signal be modulated with M-ary signaling, and the energy per bit be denoted by E_b. Please derive and find the minimum E_b such that reliable communication is possible. (Note: The derivation is needed to earn the full credits.)

<SOL> For reliable communications $\rightarrow R_b \le C$

$$R_b = \frac{P}{E_b}$$

$$C = B \times \frac{1}{2}\log_2(1+\frac{P}{\sigma_n^2 \times B})$$

$$E_b \ge \frac{P}{C} \ge \frac{P}{\lim_{B\to\infty}\frac{1}{2}B\log_2\left(1+\frac{P}{\sigma_n^2 B}\right)} \cong \frac{P}{\frac{1}{2}\log_2 e\,\frac{P}{\sigma_n^2}} = 2\sigma_n^2\ln 2$$

Hence, the minimum E_b required to achieve reliable communication is $2\sigma_n^2\ln 2$.

(24%) The signal constellation for a communication system with 8 equiprobable symbols as shown in Fig. P.1. The channel is AWGN with noise power spectral density of $N_0/2$.

(1) (6%) Using the union bound, find a bound in terms of A and N_0 on the error probability for this channel.

(2) (6%) Determine the average SNR per bit for this channel.

(3) (6%) Express the bound found in (1) in terms of the average SNR per bit.

(4) (6%) Given the same average power, find and compare the error probability of a M-ary PSK system with $M = 8$.

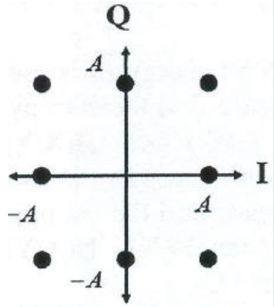


Fig. P1. The modulation constellations

<SOL>

$(a)\ d_{min} = A,\ P_e \leq 7Q(\frac{A}{\sqrt{2N_o}})$

$(b)\ E_{avg} = (2A^2 \times 4 + A^2 \times 4) \times \frac{1}{8} = \frac{3}{2}A^2,\ SNR = \frac{E_{avg}}{N_o} = \frac{3A^2}{2N_o}$

$(c)\ P_e \leq 7Q(\sqrt{\frac{E_b}{N_o}})$

$(d)\ d_{min} = \sqrt{\frac{3}{2}}A \times sin\frac{\pi}{8} \times 2 = 0.937A$

$P_{e,8PSK} \leq 7Q(0.937\sqrt{\frac{E_b}{N_o}}) \rightarrow P_{e,8QAM} < P_{e,8PSK}$