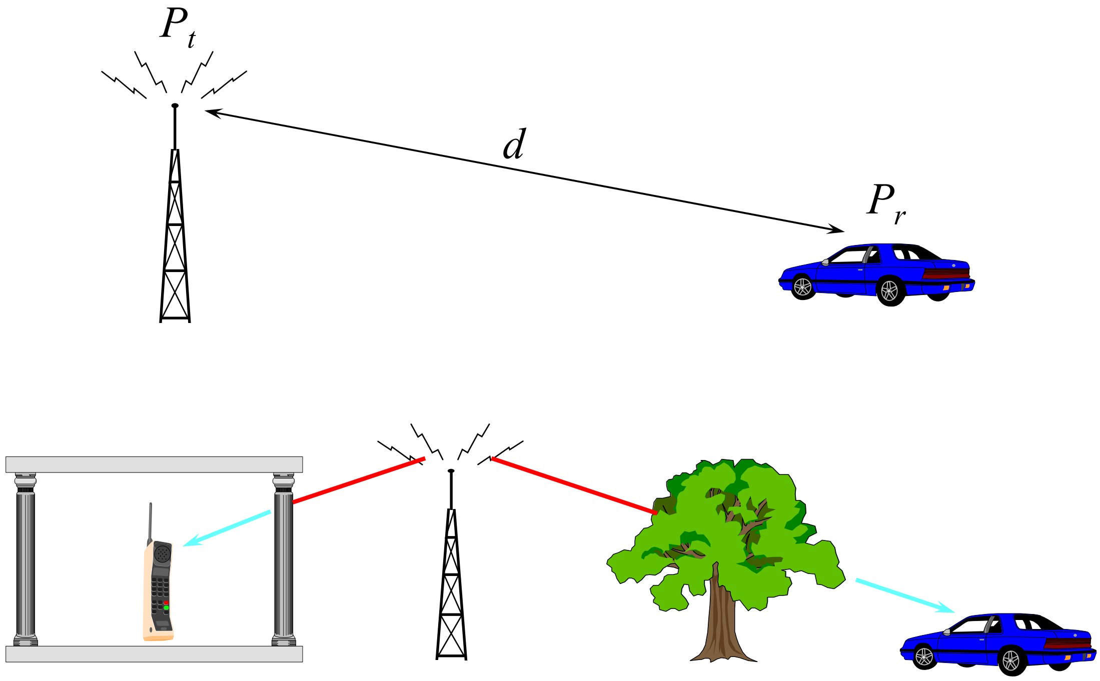
無線通訊系統
(Wireless Communications Systems)

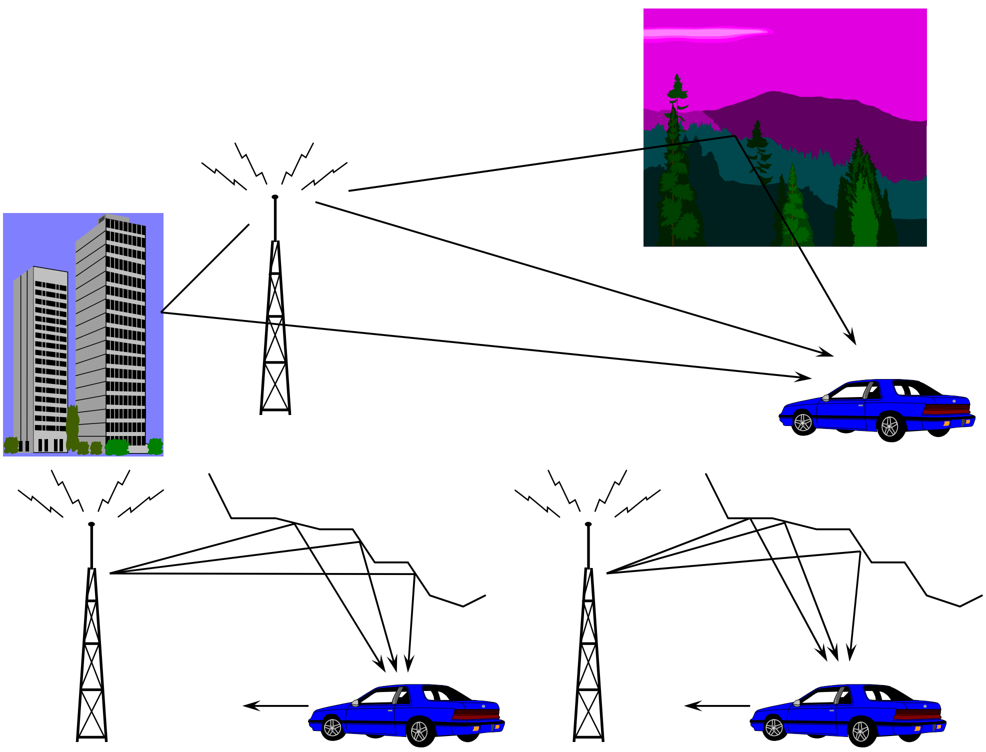
國立清華大學電機系暨通訊工程研究所
蔡育仁
台達館 821 室
Tel: 62210
E-mail: yrtsai@ee.nthu.edu.tw

Chapter 2
Propagation Effects

Path Loss and Shadowing

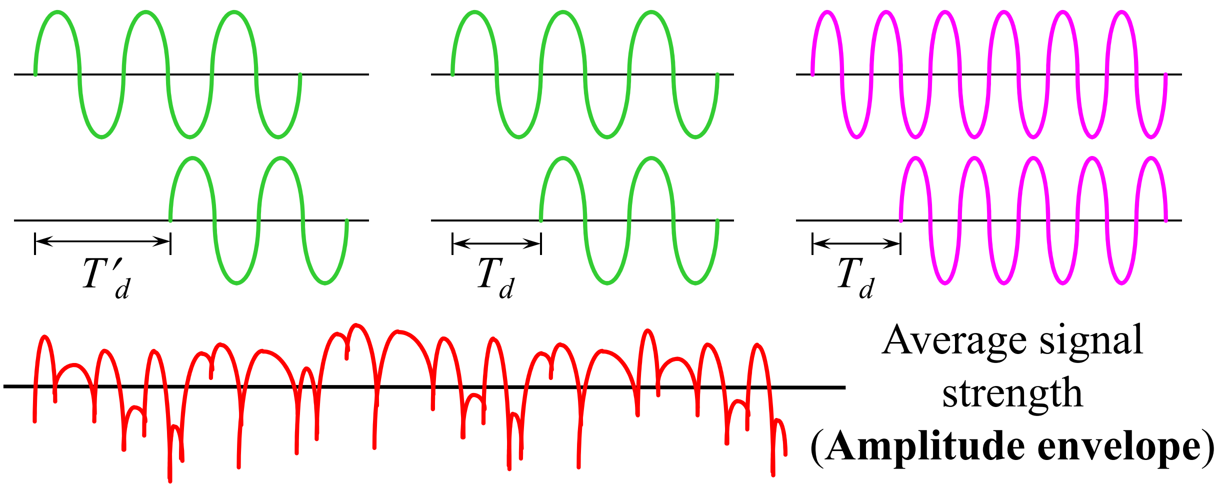


Multipath Propagation

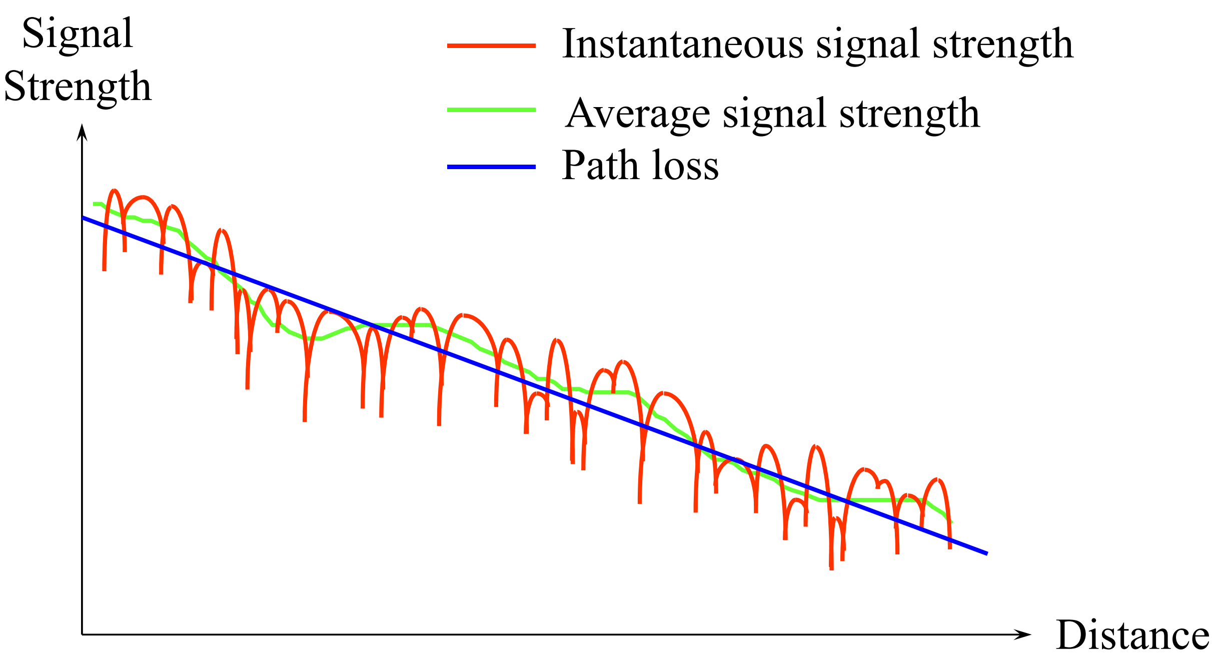


Fast Multipath Fading

- The variation of propagation channel results in the change of the received signal strength
- For the same propagation environment, **different frequency components** may experience different fading characteristics



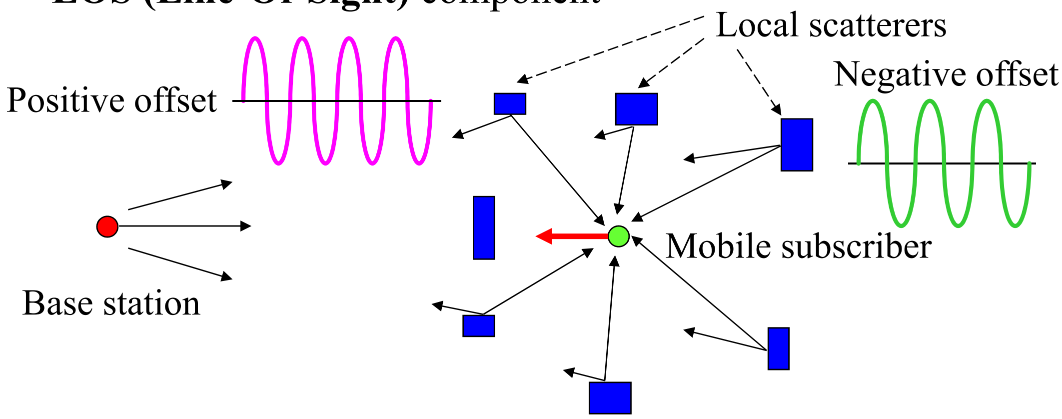
Radio Propagation



Propagation Modeling

Radio Propagations

- **Reciprocity Theorem :** If a propagation path exists, it carries energy equally well in **both directions**
- An MS in a typical macrocellular environment is usually surrounded by local scatterers
  - The plane waves arrive from many directions without a direct **LOS (Line-Of-Sight)** component



Radio Propagations

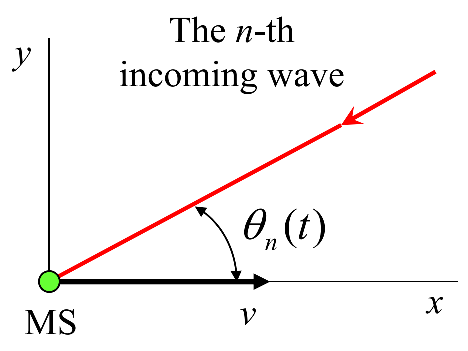
- **MS** in a **macrocellular** system: **isotropic scattering**
  - The arriving plane waves arrive from **all directions** with **equal probability**
  - In general, no direct LOS path exists between an MS and the BS
- **BS** in a **macrocellular** system: relatively free from local scatterers
  - The plane waves tend to arrive from **one general direction**
  - The cell radius is from 0.5km to several kilometers
- In a **microcellular** environment:
  - The BS antennas are only moderately elevated above the local scatterers
  - The cell radius is from 100m to several hundred meters
  - **A direct LOS path** may exist between an MS and the desired BS

Doppler (Frequency) Shift

- **Doppler (frequency) shift** is introduced for a mobile user
  - MS velocity: v
  - The **incidence angle** of the incoming wave: $\theta_n(t)$

$$f_{D,n}(t) = f_m \cos\theta_n(t) \quad \text{Hz}$$

  - where $f_m = v/\lambda_c$ and λ_c is the wavelength



Multipath Fading Channel

- Consider the transmission of the band-pass signal $s(t)$:

$$s(t) = \Re\left\{ \tilde{s}(t) e^{j2\pi f_c t} \right\}$$

  – $\tilde{s}(t)$ is the complex envelope and f_c is the carrier frequency

- The received band-pass signal is:

$$r(t) = \Re\left\{ \tilde{r}(t) e^{j2\pi f_c t} \right\}$$

  – $\tilde{r}(t)$ is the complex envelope

$$\boxed{\alpha_n(t);\ \tau_n(t);\ f_{D,n}(t)}$$

$$r(t) = \Re\left\{ \sum_{n=1}^{N} \alpha_n(t)\, e^{j2\pi\left(f_c + \underline{f_{D,n}(t)}\right)\left(t - \underline{\tau_n(t)}\right)}\ \tilde{s}(t - \tau_n(t)) \right\}$$

$$= \Re\left\{ \sum_{n=1}^{N} \alpha_n(t)\, e^{-j2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]}\ \tilde{s}(t - \tau_n(t)) e^{j2\pi f_c t} \right\}$$

$$= \Re\left\{ \tilde{r}(t) e^{j2\pi f_c t} \right\}$$

Multipath Fading Channel

- The received complex low-pass signal (for an N-path channel):

$$\tilde{r}(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]}\ \tilde{s}(t - \tau_n(t))$$

  – $\alpha_n(t)$ is the amplitude gain and $\tau_n(t)$ is the time delay

$$\Rightarrow \tilde{r}(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)}\ \tilde{s}(t - \tau_n(t))$$

  – The phase associated with the n-th path is

$$\phi_n(t) = 2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]$$

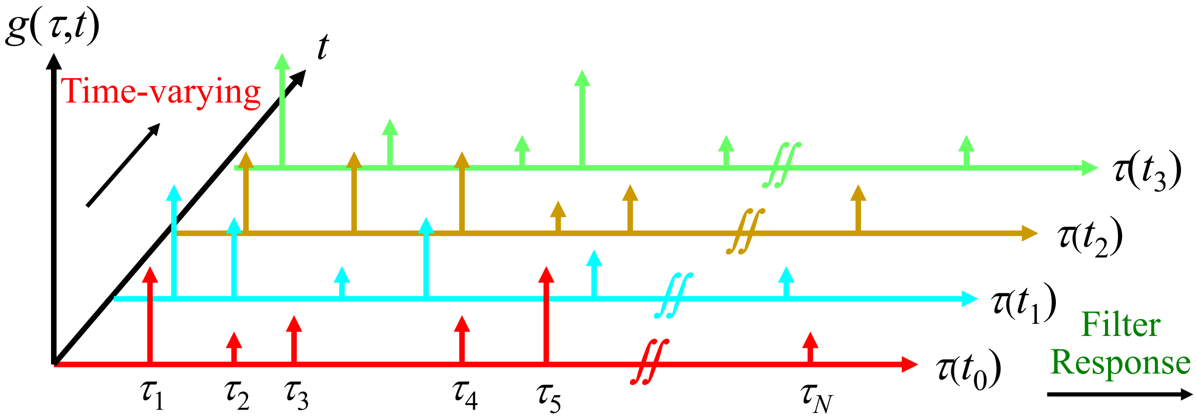
- The phase can be regarded as a **uniformly random** phase
  – Since $f_c \times \tau_n(t) >> 1$

Channel Modeling

- The channel is modeled as a time-variant linear filter

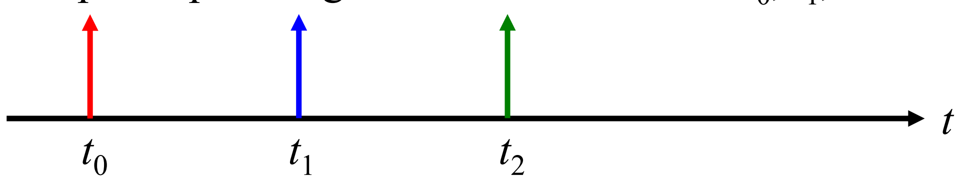
$$g(\tau,t)=\sum_{n=1}^{N}\alpha_n(t)e^{-j\phi_n(t)}\delta(\tau-\tau_n(t))$$

  – A small change in path delay $\tau_n(t)$ causes a large change in phase $\phi_n(t)$ (due to a very large $f_c+f_{D,n}(t)$)
  – Random **amplitude and phase** for each received path

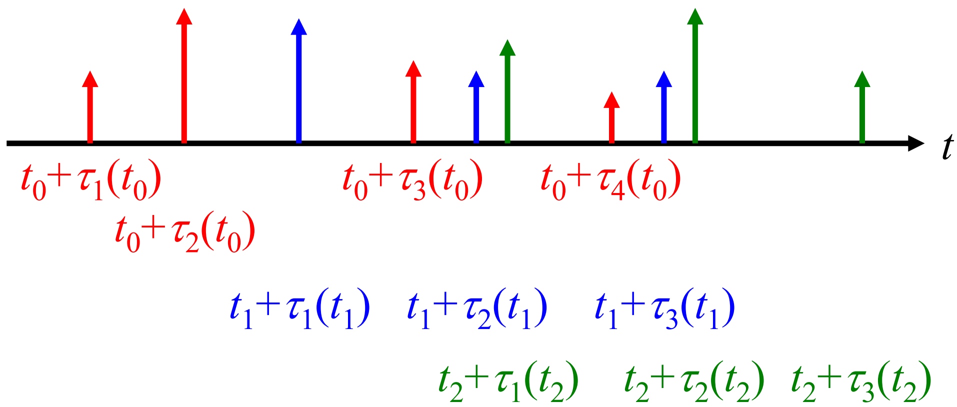


Channel Modeling

- If multiple impulse signals are transmitted at t_0, t_1, …



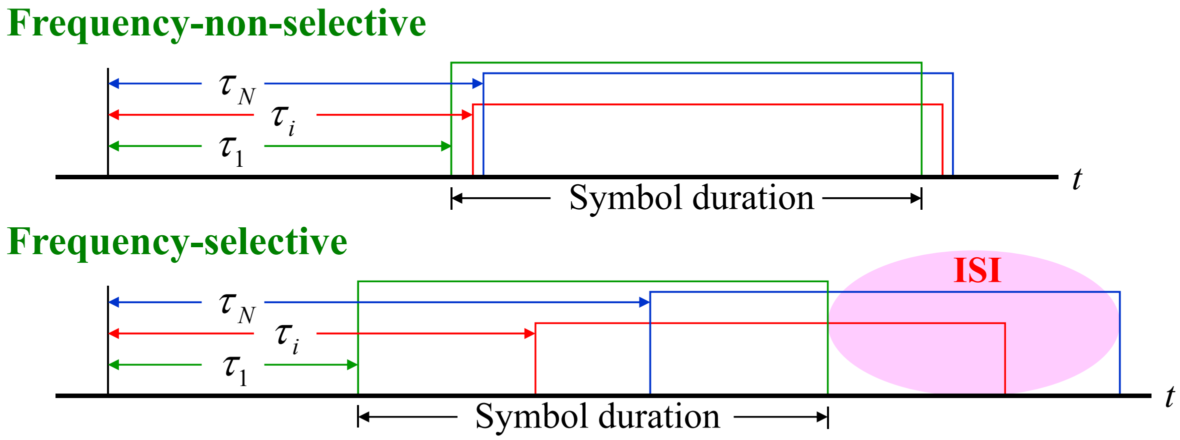
- The signal received at a receiver is



Frequency-Selective & -Non-Selective Fading

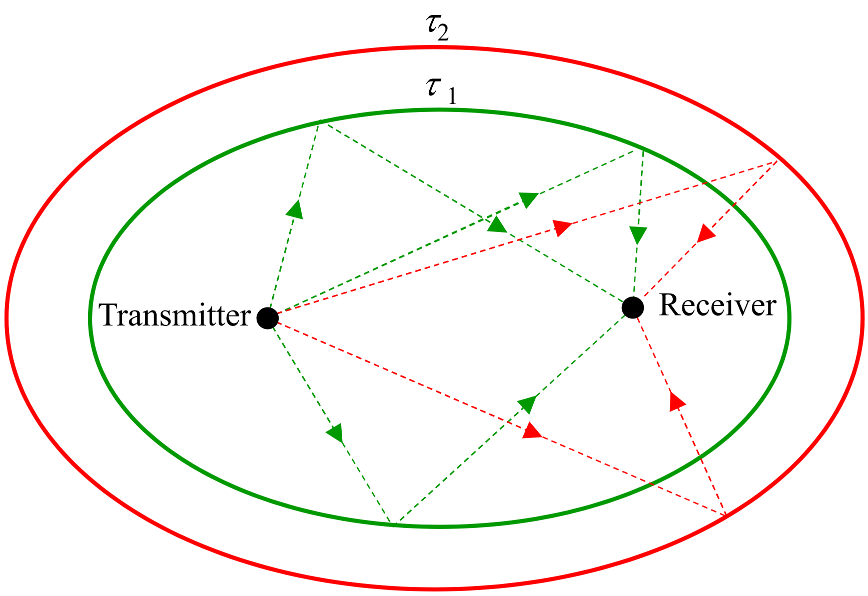
- **Frequency-non-selective:** if the differential of path delays $\tau_i - \tau_j$ are small compared to the duration of a modulated symbol, τ_n are all approximately equal to $\hat{\tau}$

$$g(\tau,t) \cong \sum_{n=1}^{N} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \hat{\tau}) = g(t)\delta(\tau - \hat{\tau})$$



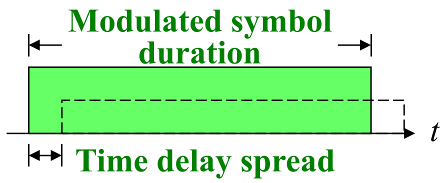
Frequency-Selective & -Non-Selective Fading

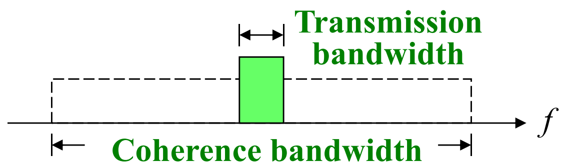
- **Frequency-selective:** if the differential of path delays $\tau_i - \tau_j$ are comparable to the duration of a modulated symbol



Frequency-Non-Selective Multipath Fading

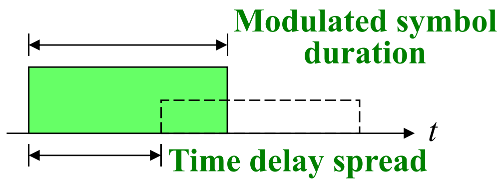
- **Frequency-non-selective multipath fading:**
  - Narrow-band transmission
  - Signal bandwidth << coherence bandwidth
  - The inverse of the signal bandwidth >> time spread of the propagation path delay
  - Modulated symbol duration >> time spread of the propagation path delay
  - All frequency components experience **the same random attenuation and a linear phase shift**
  - Very little or no distortion ⇒ **no ISI**, do not need equalization

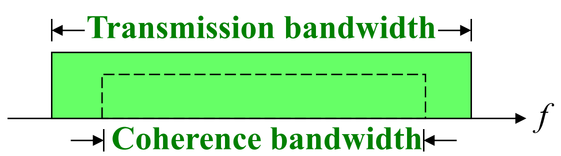




Frequency-Selective Multipath Fading

- **Frequency-selective multipath fading:** Question
  - Wide-band transmission
  - Signal bandwidth >≈ coherence bandwidth
  - The inverse of the signal bandwidth ≈< the time spread of the propagation path delay
  - Modulated symbol (or chip) duration ≈< time spread of the propagation path delay
  - Different frequency components may experience **different random attenuation and a non-linear phase shift**
  - Significant distortion ⇒ ISI, **equalization or RAKE is need**





Frequency-Non-Selective (Flat) Multipath Fading

Frequency-Non-Selective Multipath Fading

- At any time t, the random phase $\phi_n(t)$ may result in the **constructive** or **destructive** addition of the N components
- If the differential of path delays $\tau_i - \tau_j$ is small compared to the duration of a modulated symbol, for all $i \neq j$, all the path delays are approximately equal to $\hat{\tau}$
- Since the carrier frequency is very high, small differences in the path delays will correspond to **large differences** in $\phi_n(t)$

  $\Rightarrow$ The received signal still experiences fading
- The channel impulse response can be approximated as

$$g(\tau,t) = \sum_{n=1}^{N} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t)) \approx g(t)\delta(\tau - \hat{\tau})$$

- The corresponding channel transfer function is

Impulse response

$$T(t,f) = \mathbb{F}\{g(t,\underline{\tau})\} = \mathbb{F}\{g(t)\delta(\tau - \hat{\tau})\} = \underline{g(t)} e^{-j2\pi f\hat{\tau}}$$

Frequency-Non-Selective Multipath Fading

- The amplitude response is $|T(t, f)| = |g(t)|$
- **All frequency components** in the received signal are subject to the **same complex gain** $g(t)$
  - The phase is linear with respect to $f \Rightarrow$ **constant delay** for all f $\Rightarrow$ **no distortion**
- The received signal is said to exhibit <u>**flat fading**</u>
  - It holds for the corresponding frequency components only, i.e., the frequency components in the transmission bandwidth

Doppler Power Spectrum

Received Signal Correlation

- By assuming the transmission of an **unmodulated carrier**
- The received band-pass signal is

$$r(t) = \mathrm{Re}\left\{ \tilde{r}(t) e^{j2\pi f_c t} \right\} \qquad \boxed{\tilde{s}(t) = 1}$$

$$\tilde{r}(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)} \tilde{s}(t - \tau_n(t)) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)}$$

$$= g_I(t) + j g_Q(t)$$

  – where

$$\underline{g_I(t) = \sum_{n=1}^{N} \alpha_n(t) \cos\phi_n(t)} \qquad \underline{g_Q(t) = -\sum_{n=1}^{N} \alpha_n(t) \sin\phi_n(t)}$$

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

- The band-pass signal can be expressed as

$$r(t) = g_I(t)\cos 2\pi f_c t - g_Q(t)\sin 2\pi f_c t$$

Received Signal Correlation

- It is assumed that these random processes are all wide sense stationary (WSS)
  – $f_{D,n}(t) = f_{D,n}$, $\alpha_n(t) = \alpha_n$, and $\tau_n(t) = \tau_n$
- The autocorrelation of $r(t)$: (for an arbitrary **time difference τ**)

$$\phi_{rr}(\boldsymbol{\tau}) = E\left[r(t) r(t+\boldsymbol{\tau})\right]$$

$$= E\left[g_I(t) g_I(t+\boldsymbol{\tau})\right]\cos 2\pi f_c \boldsymbol{\tau} - E\left[g_Q(t) g_I(t+\boldsymbol{\tau})\right]\sin 2\pi f_c \boldsymbol{\tau}$$

$$= \phi_{g_I g_I}(\boldsymbol{\tau})\cos 2\pi f_c \boldsymbol{\tau} - \phi_{g_Q g_I}(\boldsymbol{\tau})\sin 2\pi f_c \boldsymbol{\tau}$$

$$\boxed{\begin{aligned} \phi_{g_I g_I}(\boldsymbol{\tau}) &= \phi_{g_Q g_Q}(\boldsymbol{\tau}); \\ \phi_{g_I g_Q}(\boldsymbol{\tau}) &= -\phi_{g_Q g_I}(\boldsymbol{\tau}) \end{aligned}}$$

$$\boxed{\begin{aligned} \sin x \sin y &= \tfrac{1}{2}\left[\cos(x-y) - \cos(x+y)\right] \\ \cos x \cos y &= \tfrac{1}{2}\left[\cos(x-y) + \cos(x+y)\right] \\ \sin x \cos y &= \tfrac{1}{2}\left[\sin(x+y) + \sin(x-y)\right] \\ \cos x \sin y &= \tfrac{1}{2}\left[\sin(x+y) - \sin(x-y)\right] \end{aligned}}$$

Received Signal Correlation

- According to $\phi_n(t) = 2\pi\left[\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right]$, $\tau_n(t) \approx \hat{\tau}$ and $g_I(t) = \sum_{n=1}^{N} \alpha_n(t)\cos\phi_n(t)$, we have

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = E\left[g_I(t)g_I(t+\boldsymbol{\tau})\right] = \Omega_p \times E_{\hat{\tau},\theta_n}\left[\cos\phi_n(t)\cos\phi_n(t+\boldsymbol{\tau})\right]$$

$$= \frac{\Omega_p}{2}\left\{E_{\hat{\tau},\theta_n}\left[\overset{x-y}{\underbrace{\cos 2\pi f_{D,n}\boldsymbol{\tau}}_{\text{A constant for } \hat{\tau}}}\right] + E_{\hat{\tau},\theta_n}\left[\overset{x+y}{\cos 2\pi[2(f_c + f_{D,n})\hat{\tau} - 2f_{D,n}t - f_{D,n}\boldsymbol{\tau}]}\right]\right\}$$

$$= \frac{\Omega_p}{2}E_{\theta_n}\left[\cos\left(2\pi f_m \boldsymbol{\tau}\cos\theta_n\right)\right] + 0 \quad \left(\because f_c\hat{\tau} >> 1 \text{ and } f_{D,n}(t) = f_m\cos\theta_n(t)\right)$$

$\phi_n(t)$ is uniformly distributed over [-π, π]

  - where the total received envelope power is

$$\Omega_p = E\left[g_I^2(t)\right] + E\left[g_Q^2(t)\right] = \sum_{n=1}^{N} E\left[\alpha_n^2\right]$$

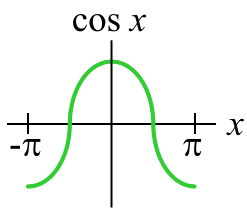
- Similarly, we have

$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = E_{\tau,\theta_n}\left[g_I(t)g_Q(t+\boldsymbol{\tau})\right] = \frac{\Omega_p}{2}E_{\theta_n}\left[\sin\left(2\pi f_m \boldsymbol{\tau}\cos\theta_n\right)\right]$$

Received Signal Correlation – IS

- For **isotropic scattering (IS)**: θ_n is uniformly distributed over $[-\pi, \pi]$

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi} \overbrace{\cos(2\pi f_m \boldsymbol{\tau}\cos\theta)}^{\text{Even function}}\, d\theta$$



$$= \frac{\Omega_p}{2}\frac{1}{\pi}\int_{0}^{\pi} \cos(2\pi f_m \boldsymbol{\tau}\sin\theta)\, d\theta = \frac{\Omega_p}{2}J_0(2\pi f_m \boldsymbol{\tau})$$

  - $J_0(x)$ is the zero-order Bessel function of the first kind

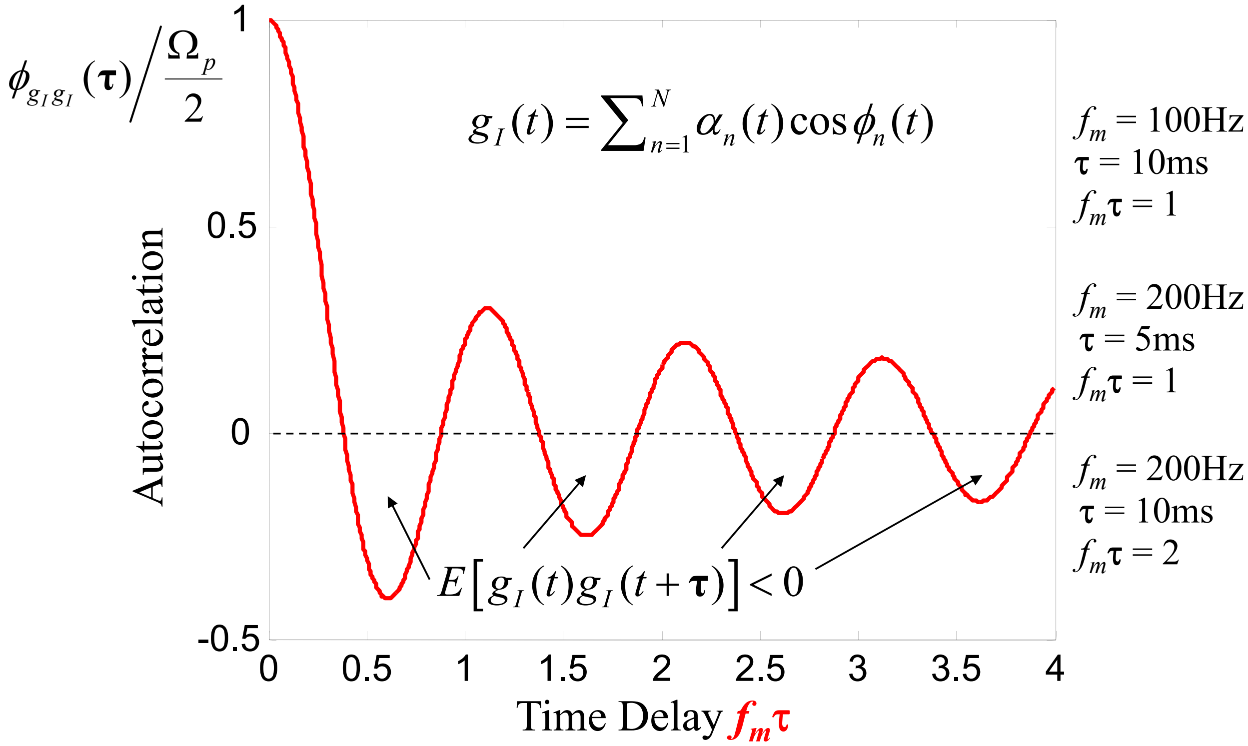
Odd function

$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin(2\pi f_m \boldsymbol{\tau}\cos\theta)\, d\theta = 0$$

$$\boxed{\because \sin(x) = -\sin(-x)}$$

- $g_I(t)$ and $g_Q(t)$ are uncorrelated

Received Signal Correlation – IS



Received Signal Spectrum – IS

- The power spectral density of $g_I(t)$ is

$$S_{g_I g_I}(f) = \mathbb{F}\{\phi_{g_I g_I}(\tau)\} = \begin{cases} \dfrac{\Omega_p}{2\pi f_m} \dfrac{1}{\sqrt{1-(f/f_m)^2}} & |f| \le f_m \\ 0 & \text{otherwise} \end{cases}$$

- The received complex envelope of $r(t)$ is

$$\tilde{r}(t) = g(t) = g_I(t) + j g_Q(t)$$

$$\Rightarrow \phi_{gg}(\tau) = \frac{1}{2} E\left[g^*(t) g(t+\tau)\right] = \phi_{g_I g_I}(\tau) + j\phi_{g_I g_Q}(\tau)$$

- The power spectral density of $g(t)$ (**Doppler power spectrum**) is

$$S_{gg}(f) = S_{g_I g_I}(f) + j S_{g_I g_Q}(f)$$

Received Signal Spectrum – IS

- For the received band-pass signal $r(t)$, we have

$$\phi_{rr}(\boldsymbol{\tau}) = \Re\,[\phi_{gg}(\boldsymbol{\tau})e^{j2\pi f_c \tau}]$$

- Since $\phi_{g_I g_Q}(\boldsymbol{\tau}) = 0$, we have the PSD of $r(t)$ as

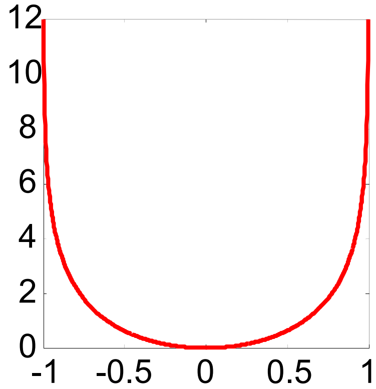
$$\begin{aligned} S_{rr}(f) &= \frac{1}{2}\left[S_{gg}(f-f_c) + S_{gg}(-f-f_c)\right] \\ &= \frac{1}{2}\left[S_{g_I g_I}(f-f_c) + S_{g_I g_I}(-f-f_c)\right] \\ &= \frac{\Omega_p}{4\pi f_m}\frac{1}{\sqrt{1-\left(\left|f-f_c\right|/f_m\right)^2}}, \qquad \left|f-f_c\right| \le f_m \end{aligned}$$

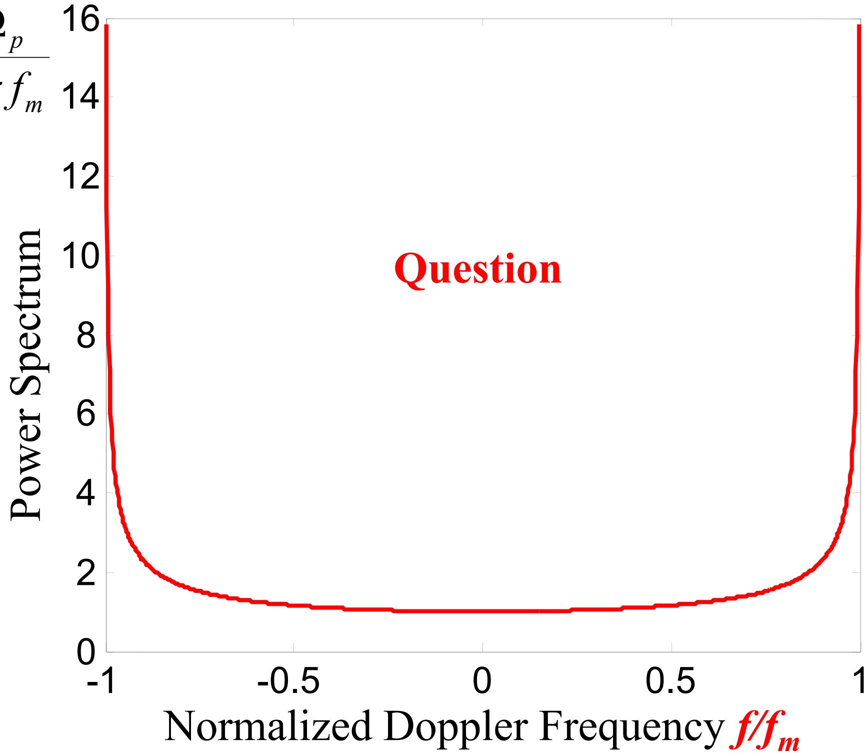
- $S_{rr}(t)$ is limited to $\left|f-f_c\right| \le f_m$

Received Signal Spectrum – IS

$$S_{g_I g_I}(f) \Big/ \frac{\Omega_p}{2\pi f_m}$$

(dB) **In dB scale**



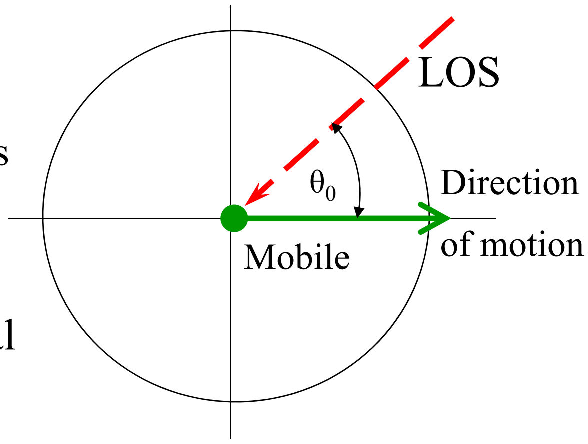


Received Signal Spectrum with LOS – IS

- If an **LOS** or a **strong specular component** is present in the received signal and arrives at angle θ_0: **Ricean／Rician fading**

$$S_{gg}(f) = \begin{cases} \dfrac{1}{K+1}\dfrac{\Omega_p}{2\pi f_m}\dfrac{1}{\sqrt{1-(f/f_m)^2}} + \dfrac{K}{K+1}\dfrac{\Omega_p}{2}\delta(f - f_m\cos\theta_0), & 0 \le |f| \le f_m \\ 0, & \text{otherwise} \end{cases}$$

  - where K is the <u>**Rice factor**</u>: the ratio of the power in the **specular** and **scatter** components of the received signal
- The PSD is the same as Fig. 2.4, except for an additional **discrete tone** at $f_c + f_m\cos\theta_0$

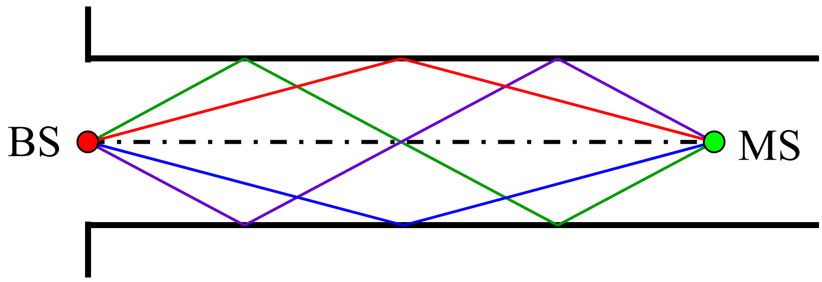


Received Signal Correlation (Microcellular)

- In microcellular environment, the plane waves may be <u>**channeled by the buildings**</u> along the streets and arrive at the receiver from just **one direction**
  - The scattering is <u>**non-isotropic**</u>

$$p(\theta) = \begin{cases} \dfrac{\pi}{4|\theta_m|}\cos\left(\dfrac{\pi}{2}\times\dfrac{\theta}{\theta_m}\right), & |\theta| \le |\theta_m| \le \pi/2 \\ 0, & \text{elsewhere} \end{cases}$$

  - θ_m: the directivity of the incoming waves



Received Signal Correlation (Microcellular)

- According to

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = \frac{\Omega_p}{2} E_{\theta_n}\left[\cos\left(2\pi f_m \boldsymbol{\tau} \cos\theta_n\right)\right]$$

$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = \frac{\Omega_p}{2} E_{\theta_n}\left[\sin\left(2\pi f_m \boldsymbol{\tau} \cos\theta_n\right)\right]$$

- We have

$$\phi_{g_I g_I}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\int_{-\pi}^{\pi} \cos(2\pi f_m \boldsymbol{\tau} \cos\theta)\underline{p(\theta)}\, d\theta \neq \frac{\Omega_p}{2} J_0(2\pi f_m \boldsymbol{\tau})$$

$$\phi_{g_I g_Q}(\boldsymbol{\tau}) = \frac{\Omega_p}{2}\int_{-\pi}^{\pi} \sin(2\pi f_m \boldsymbol{\tau} \cos\theta)\underline{p(\theta)}\, d\theta \neq 0$$

Fading Characteristics (Received Envelope/Power Distribution)

Rayleigh Fading

- **Rayleigh Fading:** the received complex low-pass signal is modeled as a complex Gaussian random process
  - $g_I(t)$ and $g_Q(t)$ are independent **zero-mean** Gaussian RVs
  - The received **complex envelope** $\alpha(t) = |\, g(t)\, |$ has a Rayleigh distribution

$$p_\alpha(x) = \frac{x}{\sigma^2}\exp\left[-\frac{x^2}{2\sigma^2}\right], \quad x \ge 0$$

$$\boxed{\begin{aligned} g_I(t) &= \sum_{n=1}^{N} \alpha_n(t)\cos\phi_n(t) \\ g_Q(t) &= -\sum_{n=1}^{N} \alpha_n(t)\sin\phi_n(t) \end{aligned}}$$

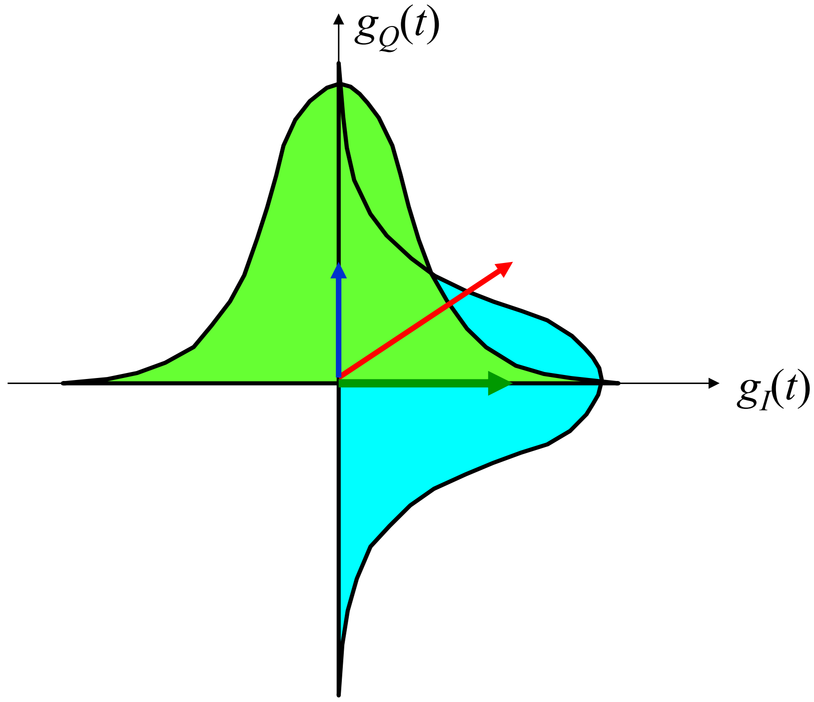
- The average power is

$$E[\alpha^2] = \Omega_p = 2\sigma^2$$

- The squared-envelope (power) $\alpha^2(t) = |\, g(t)\, |^2$ has an exponential distribution

$$p_{\alpha^2}(x) = \frac{1}{\Omega_p}\exp\left[-\frac{x}{\Omega_p}\right], \quad x \ge 0$$

Rayleigh Fading



Ricean Fading

- **Ricean Fading:** the received complex low-pass signal contains a LOS or a strong specular component
  - $g_I(t)$ and $g_Q(t)$ are independent Gaussian RVs with **non-zero mean** $m_I(t)$ and $m_Q(t)$ ($m_I(t)$ and $m_Q(t)$ depend on the LOS and θ_0)
  - The complex envelope $\alpha(t) = | g(t) |$ has a Ricean distribution

$$p_\alpha(x) = \frac{x}{\sigma^2}\exp\left[-\frac{x^2+s^2}{2\sigma^2}\right] I_0\left(\frac{xs}{\sigma^2}\right) \quad x \geq 0$$

Power of the LOS component

$$s^2 = m_I^2(t) + m_Q^2(t), \qquad K = s^2 / 2\sigma^2$$

  - The modified Bessel function of the first kind of zero order

$$I_0(x) = \int_0^{2\pi} \exp(x\cos\psi)\, d\psi \Big/ 2\pi$$

  - Rice factor $K = 0$: Rayleigh fading
  - Rice factor $K = \infty$: the channel does not exhibit fading

Ricean Fading

- The average power is

$$E[\alpha^2] = \Omega_p = s^2 + 2\sigma^2$$

$$s^2 = \frac{K\Omega_p}{K+1}, \qquad 2\sigma^2 = \frac{\Omega_p}{K+1}$$

- The squared-envelope $\alpha^2(t) = | g(t) |^2$ has a non-central chi-square distribution

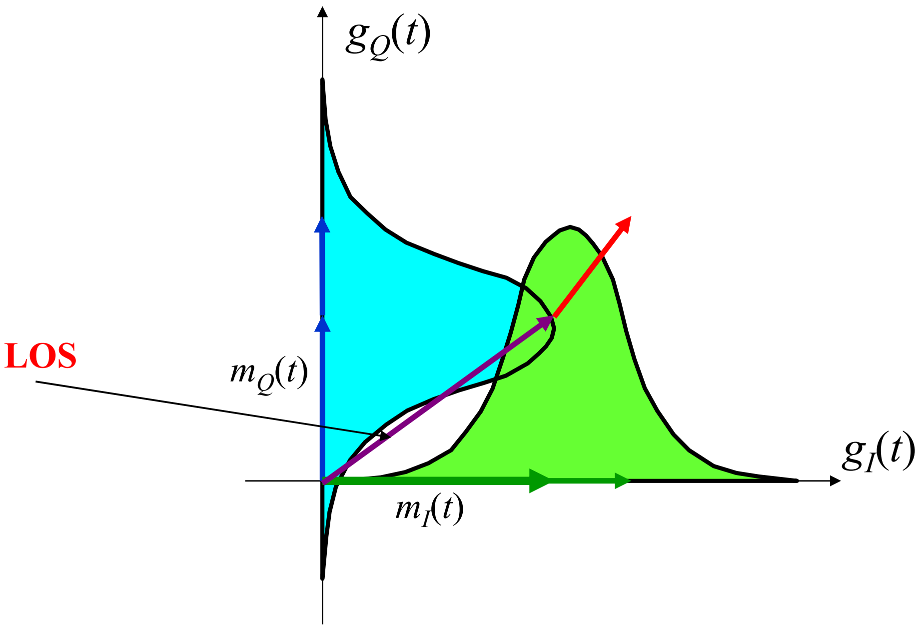
$$p_{\alpha^2}(x) = \frac{(K+1)}{\Omega_p}\exp\left[-K - \frac{(K+1)x}{\Omega_p}\right] I_0\left(2\sqrt{\frac{K(K+1)x}{\Omega_p}}\right), \quad x \geq 0$$

- The phase is not uniformly distributed over $[-\pi, \pi]$ for $K \neq 0$

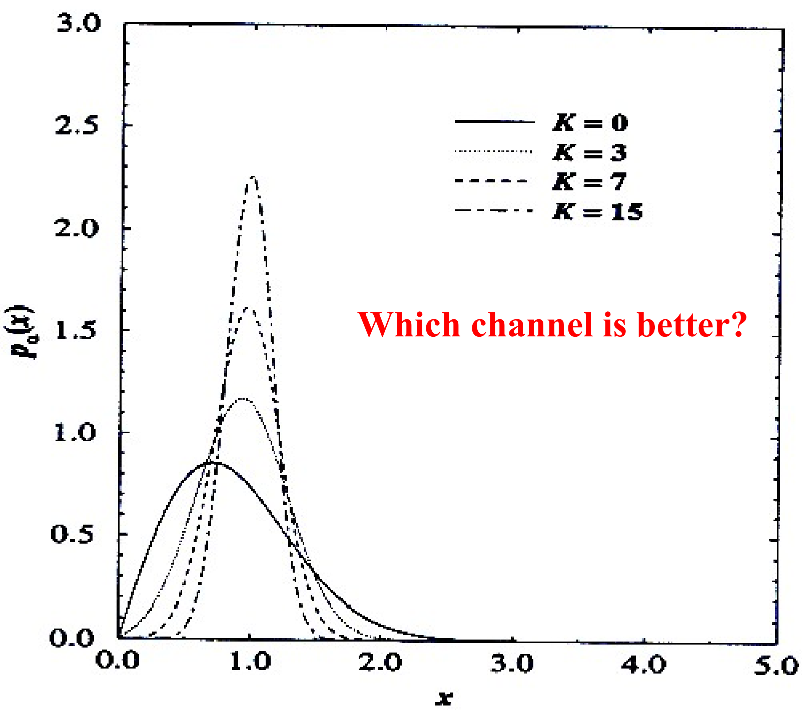
$$\phi(t) = \tan^{-1}\left(x_Q(t)/x_I(t)\right)$$

  - For $K = 0$: Rayleigh fading $p_\phi(x) = 1/2\pi, \quad -\pi \leq x \leq \pi$

Ricean Fading



Rayleigh and Ricean Distributions



Nakagami Fading

- **Nakagami Fading:** provides a closer match to some experimental data
  - The received complex envelope $\alpha(t) = |\, g(t)\, |$ has a Nakagami distribution

$$p_\alpha(x) = \frac{2m^m x^{2m-1}}{\Gamma(m)\Omega_p^m} \exp\left[-\frac{mx^2}{\Omega_p}\right], \quad m \geq \frac{1}{2}$$

    - where $\Gamma(m)$ is the Gamma function defined as

$$\Gamma(m) = \int_0^\infty u^{m-1} e^{-u}\, du$$
$$= (m-1)!, \quad \text{if } m \text{ is a positive integer}$$

  - m = 1: Rayleigh fading
  - m = 1/2: one-sided Gaussian
  - m = ∞: no fading

Nakagami Fading

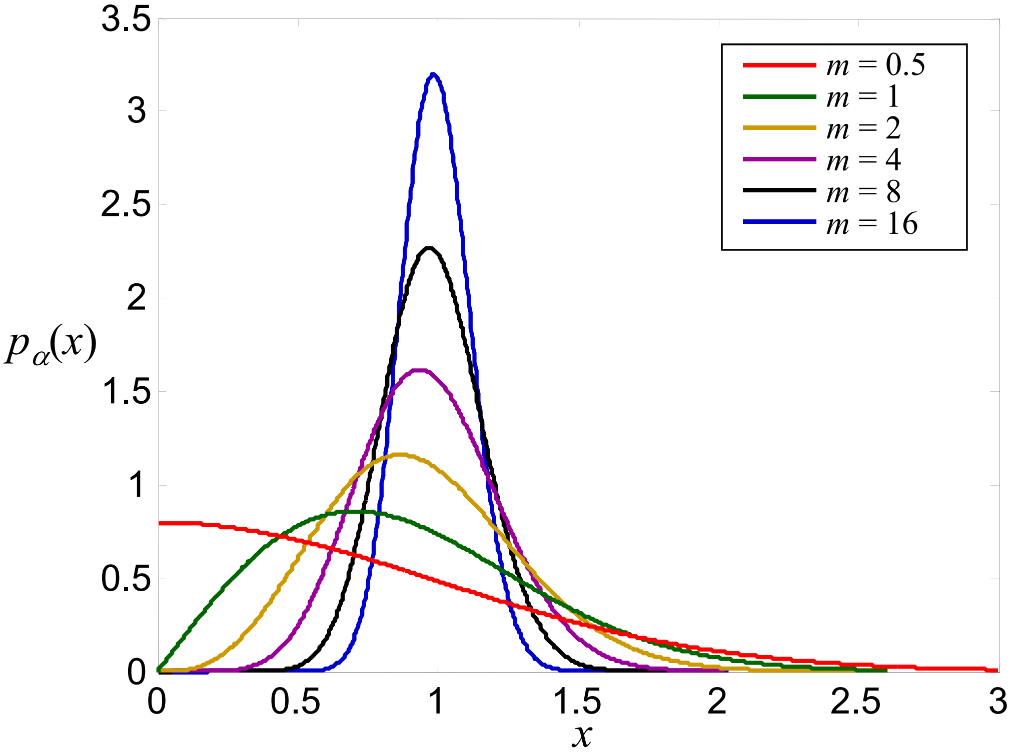
- Nakagami distribution can model fading conditions that are **either more or less** severe than Rayleigh fading
- Ricean fading can be closely approximated by

$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}} \qquad m \geq 1; \qquad m = \frac{(K+1)^2}{(2K+1)}$$

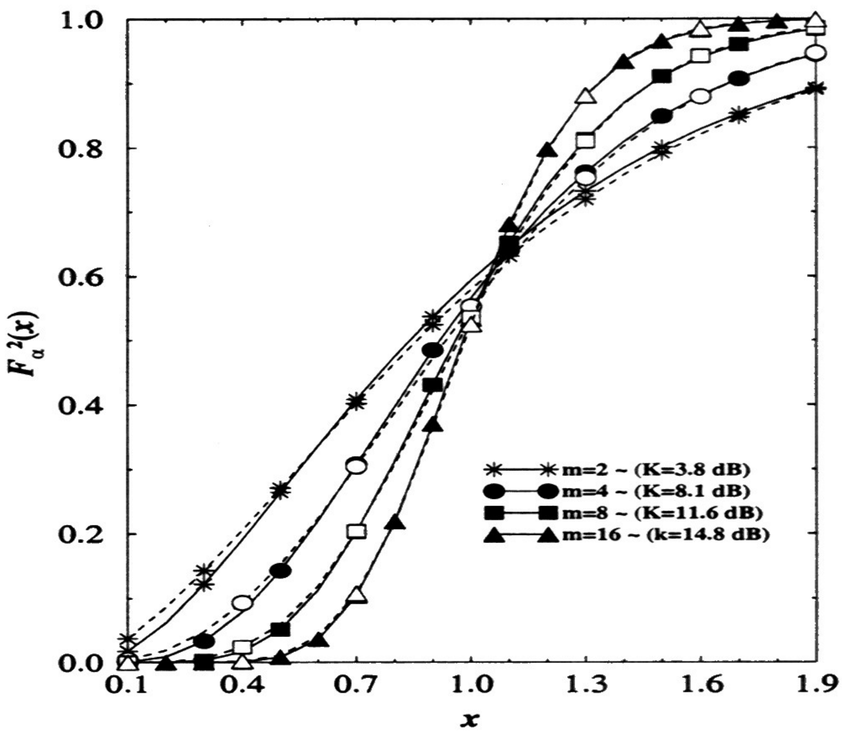
- The Nakagami distribution often leads to closed form analytical expressions
- The squared-envelope $\alpha^2(t) = |\, g(t)\, |^2$ has a Gamma density

$$p_{\alpha^2}(x) = \left(\frac{m}{\Omega_p}\right)^m \frac{x^{m-1}}{\Gamma(m)} \exp\left[-\frac{mx}{\Omega_p}\right]$$

Nakagami Fading



Nakagami and Ricean Distributions



Envelope Correlation

- The autocorrelation of the envelope $\alpha(t) = |g(t)|$:

$$\phi_{\alpha\alpha}(\tau) = E[\alpha(t)\alpha(t+\tau)] = \frac{\pi}{2}|\phi_{gg}(0)|\,F\left[-\frac{1}{2},-\frac{1}{2};1,\frac{|\phi_{gg}(\tau)|^2}{|\phi_{gg}(0)|^2}\right]$$

  – where $|\phi_{gg}(\tau)|^2 = \phi^2_{g_I g_I}(\tau) + \phi^2_{g_I g_Q}(\tau)$

$$= \phi^2_{g_I g_I}(\tau) \quad \text{(isotropic scattering)}$$

$$F\left[-\tfrac{1}{2},-\tfrac{1}{2};1,x\right] = 1 + \tfrac{1}{4}x + \tfrac{1}{64}x^2 + \cdots \text{(Hypergeometric Function)}$$

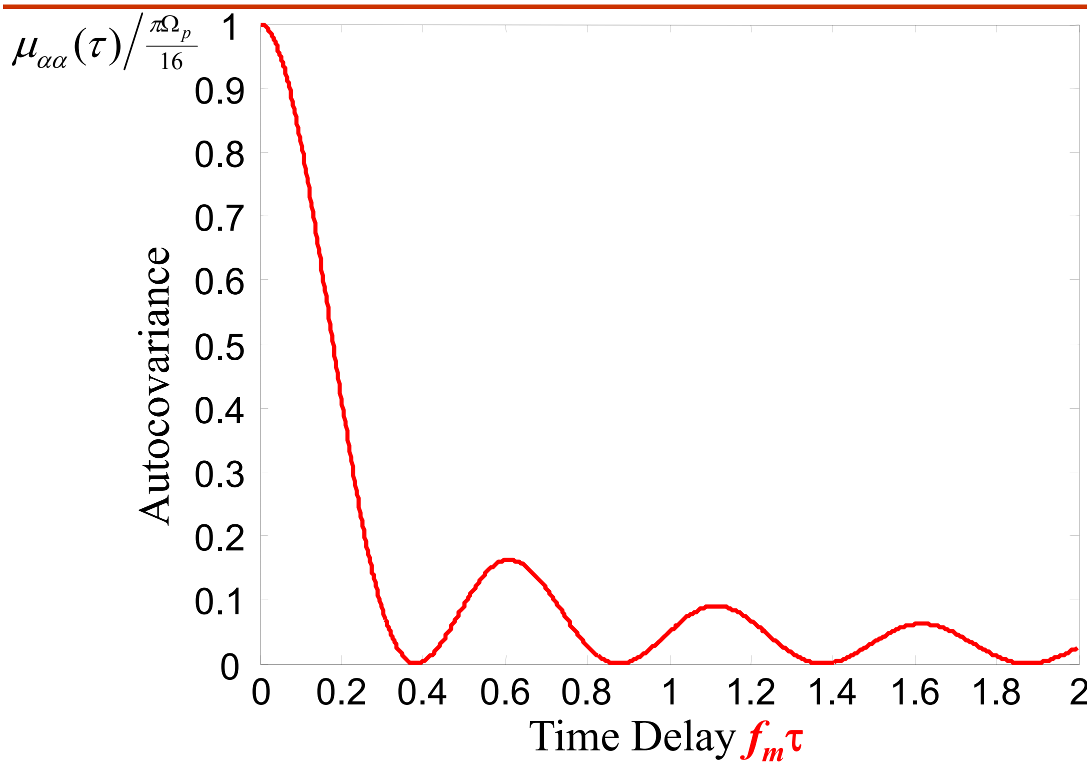
$$\phi_{\alpha\alpha}(\tau) \approx \frac{\pi}{2}|\phi_{gg}(0)|\left[1 + \frac{1}{4}\frac{|\phi_{gg}(\tau)|^2}{|\phi_{gg}(0)|^2}\right]$$

- The autocovariance function:

$$\mu_{\alpha\alpha}(\tau) = E[\alpha(t)\alpha(t+\tau)] - E[\alpha(t)]E[\alpha(t+\tau)]$$

$$= \frac{\pi}{8|\phi_{gg}(0)|}|\phi_{gg}(\tau)|^2 = \frac{\pi\Omega_p}{16}J_0^2(2\pi f_m\tau)$$

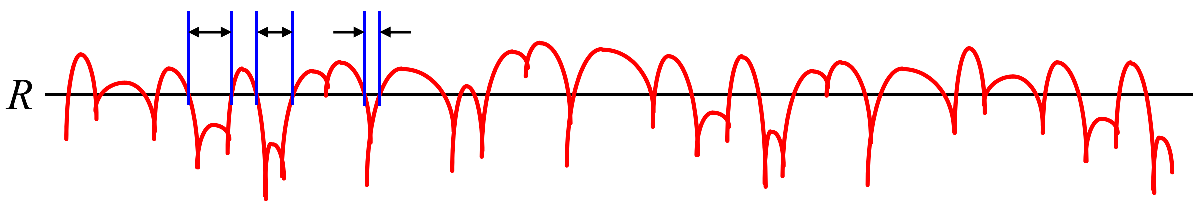
Envelope Correlation



Envelope Level Crossing Rate

The Impact of Multipath Fading

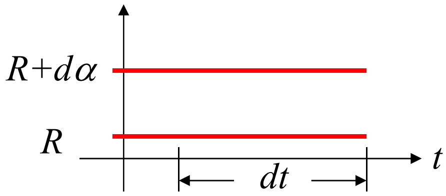
- The receive performance is severely degraded in a **deep fade region**.
  - For example, the received signal level is below a threshold *R*
- We care the following two things:
  - How often will deep fading occur?
    - **Envelope Level Crossing Rate**
  - How long will the deep fading last?
    - **Average Envelope Fade Duration**



Envelope Level Crossing Rate

- L_R: the rate at which the envelope crosses level R in the positive (or negative) going direction
- $\dot{\alpha} = |\dot{r}|$: the envelope slope
  - $\dot{\alpha}$ is either **positive** (for positive going direction) or **negative** (for negative going direction)
- $p(\alpha, \dot{\alpha})$: the join pdf of α and $\dot{\alpha}$
- dt: the observation time interval
- For given values of $\alpha = R$ and $\dot{\alpha}$, the probability is

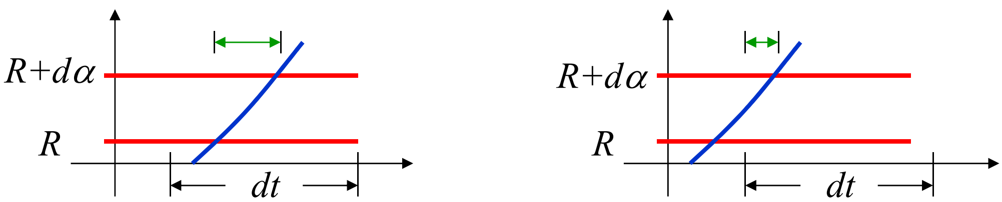
$$p(R,\dot{\alpha})\, d\alpha\, d\dot{\alpha}$$



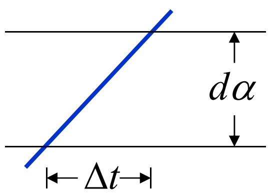
Envelope Level Crossing Rate

- The expected amount of time spent in the interval $(R, R+d\alpha)$ for given values of $\dot{\alpha}$ and dt is

$$p(R,\dot{\alpha})\, d\alpha\, d\dot{\alpha}\, dt$$



- The time required to cross the interval $d\alpha$ once for a given $\dot{\alpha}$ is $d\alpha / \dot{\alpha}$
  - The time spent in $(R, R+d\alpha)$ for one positive going direction cross



$$\dot{\alpha} = d\alpha / \Delta t$$

Envelope Level Crossing Rate

- The expected number of crossings of the envelope α within the interval $(R, R+d\alpha)$ for a given $\dot{\alpha}$ is

$$\left(p(R,\dot{\alpha})\, d\alpha\, d\dot{\alpha}\, dt\right) / \left(d\alpha / \dot{\alpha}\right) = \dot{\alpha}\; p(R,\dot{\alpha})\, d\dot{\alpha}\, dt$$

- The expected number of crossings in a time interval T for a given $\dot{\alpha}$ is

$$\int_0^T \dot{\alpha}\; p(R,\dot{\alpha})\, d\dot{\alpha}\, dt = \dot{\alpha}\; p(R,\dot{\alpha})\, d\dot{\alpha}\, T$$

- The excepted number of positive going direction crossings:

$$N_R = T\int_0^\infty \dot{\alpha}\; p(R,\dot{\alpha})\, d\dot{\alpha}$$

All slopes are counted.

- The envelope level crossing rate:

$$L_R = \int_0^\infty \dot{\alpha}\; p(R,\dot{\alpha})\, d\dot{\alpha}$$

Envelope Level Crossing Rate

- For Ricean fading:

$$p(\alpha,\dot{\alpha}) = \sqrt{\frac{1}{2\pi b_2}}\exp\left\{-\frac{\dot{\alpha}^2}{2b_2}\right\} \times \frac{\alpha}{b_0}\exp\left\{-\frac{(\alpha^2+s^2)}{2b_0}\right\} I_0\left(\frac{\alpha s}{b_0}\right) = p(\dot{\alpha})p(\alpha)$$

  - where $b_2 = b_0(2\pi f_m)^2/2$ and $2b_0$ is the power of the scatter component of the received signal

- The envelope level crossing rate is

$$L_R = \sqrt{2\pi(K+1)}\, f_m \rho e^{-K-(K+1)\rho^2} I_0\left(2\rho\sqrt{K(K+1)}\right)$$

  - where $\rho = \frac{R}{\sqrt{\Omega_p}} = \frac{R}{R_{rms}}$
  - $\sqrt{\Omega_p} \triangleq R_{rms}$: the rms envelope level

Envelope Level Crossing Rate

- For Rayleigh fading ($K = 0$):

$$L_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

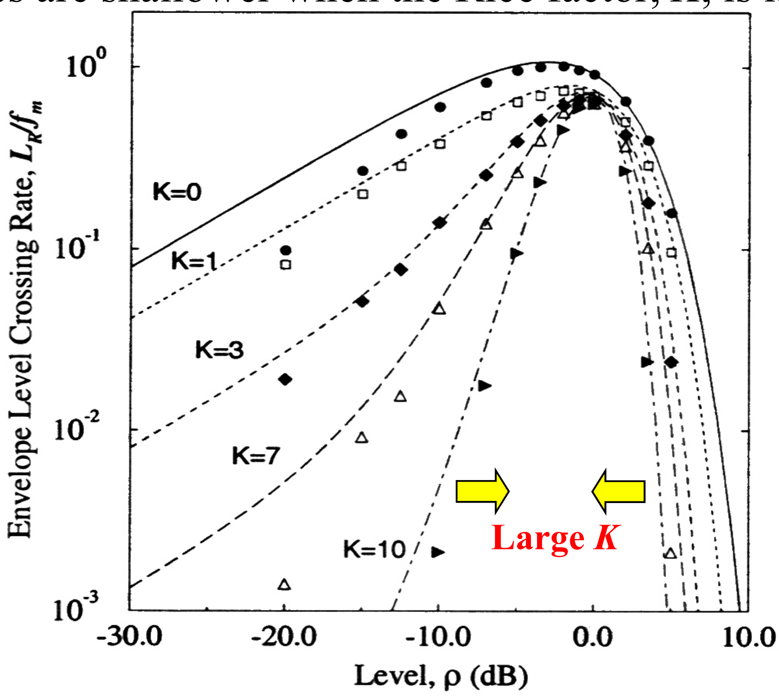
- Maximum LCR: around **ρ = 0 dB** (nearly independent of K)
  - For Rayleigh fading channel:

$$\frac{dL_R}{d\rho} = \sqrt{2\pi} f_m \left(e^{-\rho^2} - 2\rho^2 e^{-\rho^2}\right) = \sqrt{2\pi} f_m \left(1 - 2\rho^2\right) e^{-\rho^2} = 0$$

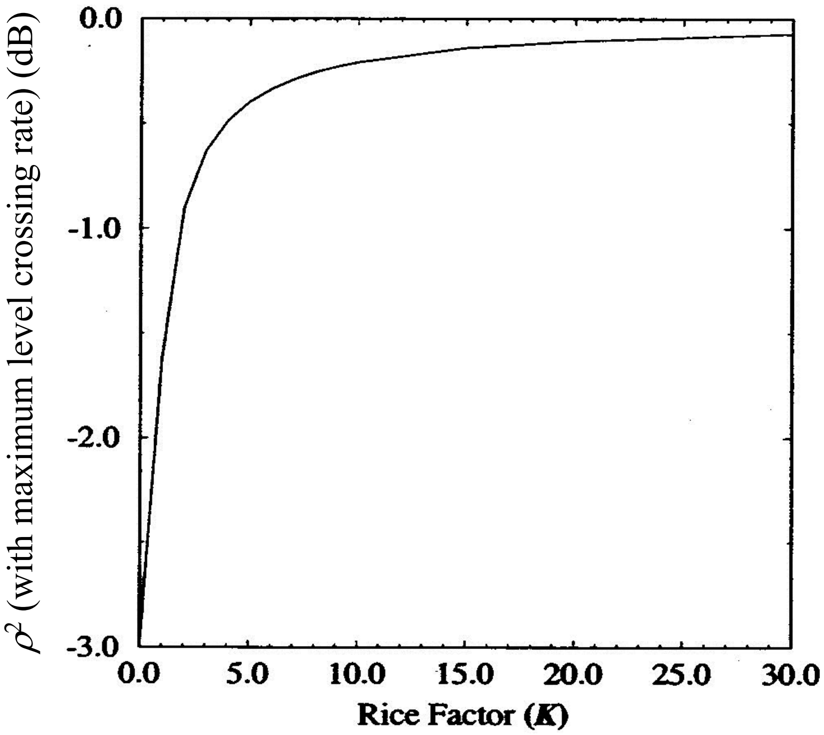
$$\Rightarrow \rho = 1/\sqrt{2}$$

Envelope Level Crossing Rate

- The fades are shallower when the Rice factor, K, is larger



Envelope Level Crossing Rate



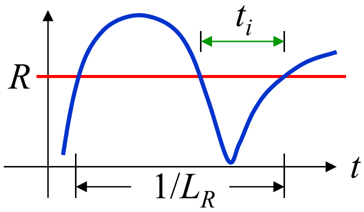
Average Envelope Fade Duration

Average Envelope Fade Duration

- Consider a very long observation time interval T
- The probability that the received envelope level is less than R can be expressed as

Can be obtained based on the envelope distribution, i.e., Rayleigh/Ricean distribution

$$P(\alpha \le R) = \frac{1}{T}\sum_i t_i$$



- The average envelope **fade duration** is

$$\bar{t} = \frac{1}{TL_R}\sum_i t_i = \frac{P(\alpha \le R)}{L_R}$$

  - $1/L_R$ is the mean time interval between two adjacent level-crossings
  - $P(\alpha \le R)$: the probability of $\alpha \le R$
  - Only one interval less than R in the $1/L_R$ duration

Average Envelope Fade Duration

- **Ricean:** $P(\alpha \le R) = \int_0^R p(\alpha) d\alpha = 1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)$
  - where $Q(a,b)$ is the Marcum Q function

$$Q(a,b) \triangleq \int_b^\infty \alpha \exp\left[-\tfrac{1}{2}\left(\alpha^2 + a^2\right)\right] I_0(a\alpha)\, d\alpha$$

- The average envelope fade duration is

$$\bar{t} = \frac{1 - Q\left(\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right)}{\sqrt{2\pi(K+1)}\, f_m \rho e^{-K-(K+1)\rho^2} I_0(2\rho\sqrt{K(K+1)})}$$

- **Rayleigh:** $P(\alpha \le R) = \int_0^R p(\alpha)\, d\alpha = 1 - e^{-\rho^2}$
- The average envelope fade duration is

$$\bar{t} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}, \qquad L_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

Average Envelope Fade Duration

- The average level crossing rate, zero crossing rate and average fade duration all depend on the velocity of MS
  - $f_m = v/\lambda_c$ and 1 mile = 1.609 km
  - Example:

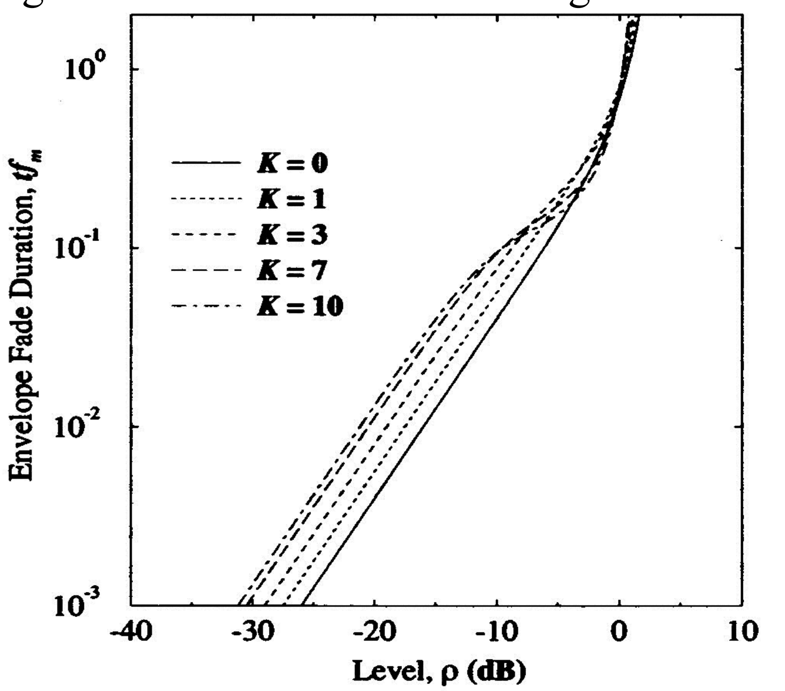
$$v = 60 \text{ mile/hr} = 97 \text{ km/hr} = 27 \text{ m/sec};$$
$$f_c = 900 \text{ MHz} \Rightarrow f_m = 81 \text{ Hz}$$

  - Rayleigh:

$$L_R = 74 \text{ fades/sec at } \rho = 0 \text{ dB}; \quad \bar{t} = 8.5 \text{ ms}$$
$$L_R = 2.0 \text{ fades/sec at } \rho = -20 \text{ dB}; \quad \bar{t} = 50\ \mu\text{s}$$

Average Envelope Fade Duration

- The average fade duration tends to be larger with the Rice factor K

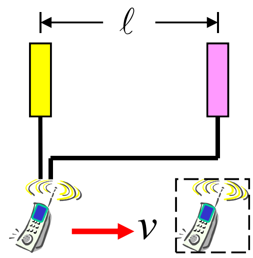


Spatial Correlation

Spatial Correlation

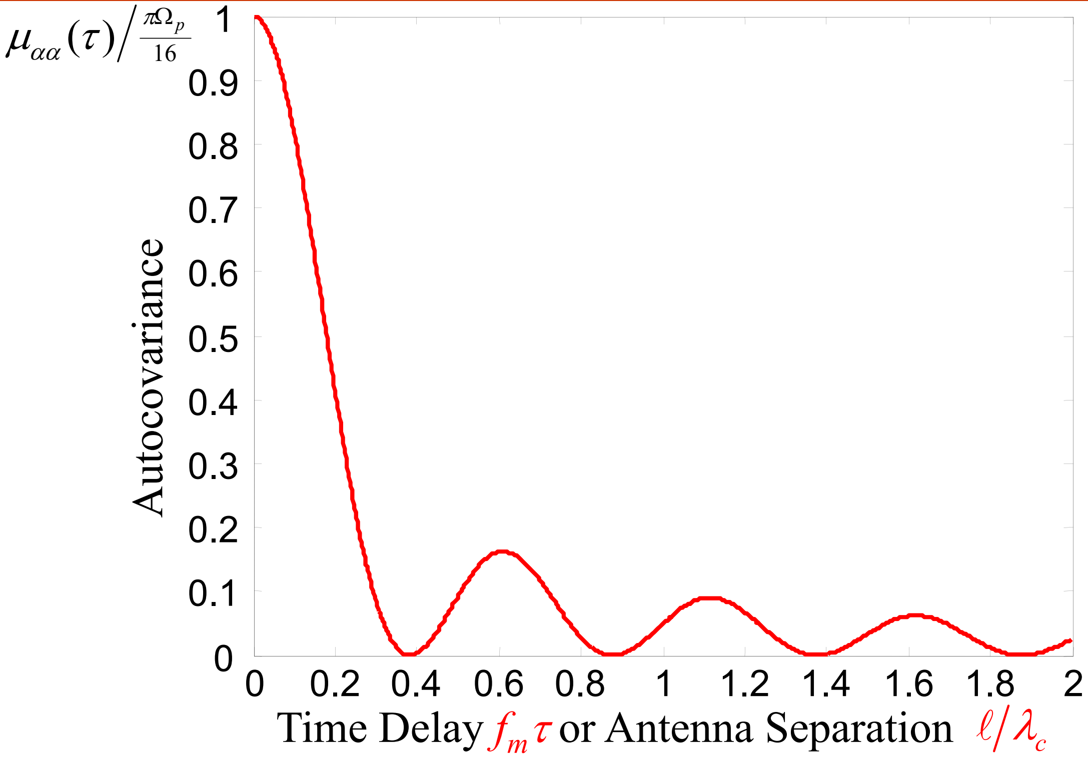
- **Diversity reception:** use two separate receiving antennas to provide uncorrelated diversity branches
- The antenna separation: ℓ
  - By distance-time transformation

$$\ell = v\tau, \quad \ell/\lambda_c = v\tau/\lambda_c = f_m\tau$$



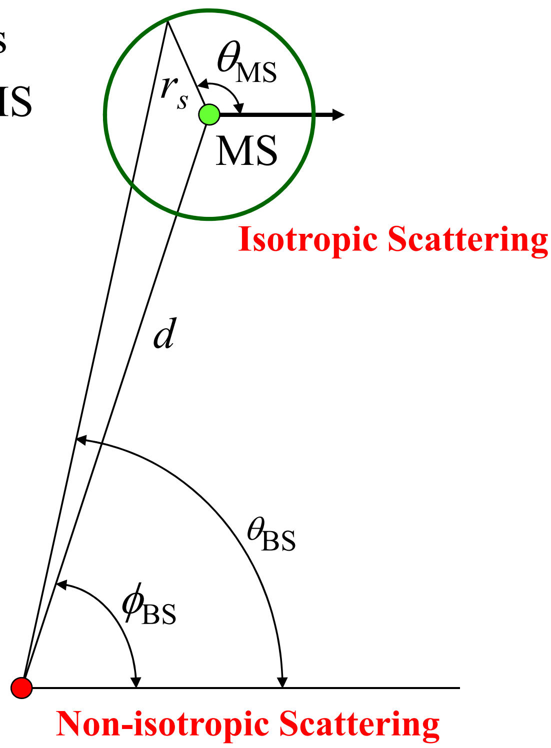
- For the case of **<u>isotropic scattering</u>**:
  - Autocorrelation: $\phi_{g_I g_I}(\ell) = \frac{\Omega_p}{2} J_0(2\pi\,\ell/\lambda_c)$
  - Autocovariance: $\mu_{\alpha\alpha}(\ell) = \frac{\pi\Omega_p}{16} J_0^2(2\pi\,\ell/\lambda_c)$
- The normalized envelope autocovariance is zero at $\ell = 0.38\lambda_c$
  - Less than 0.3 for $\ell > 0.38\lambda_c$

Spatial Correlation



Spatial Correlation

- r_s: the radius of primary scatterers
- d: the distance between BS and MS
- ϕ_{BS}: arriving angle

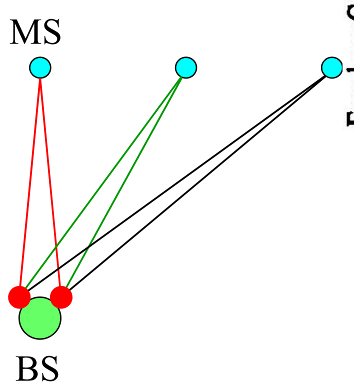


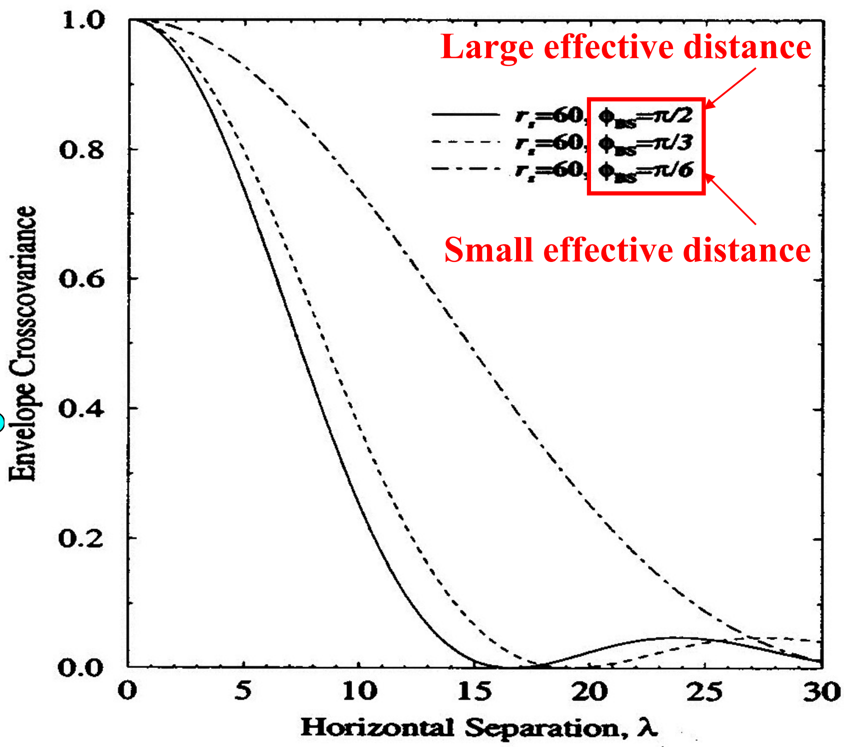
Spatial Correlation

- For an MS (isotropic scattering), the antenna elements should space about **a half-wavelength apart**
- For a BS, the antenna elements separate about **$20\lambda_c$** to obtain a correlation of about 0.7
  - The location of BS antennas is highly above the buildings
  - The arriving plane waves at the BS tend to be concentrated in **a narrow angle** of arrival (non-isotropic scattering)
  - The two antennas located at the BS will view the MS from only a slightly different angle
  - The spatial correlation is higher than isotropic scattering
- Another scheme of diversity reception: **polarization reception**

Spatial Correlation for BS

- Almost like a single incoming wave

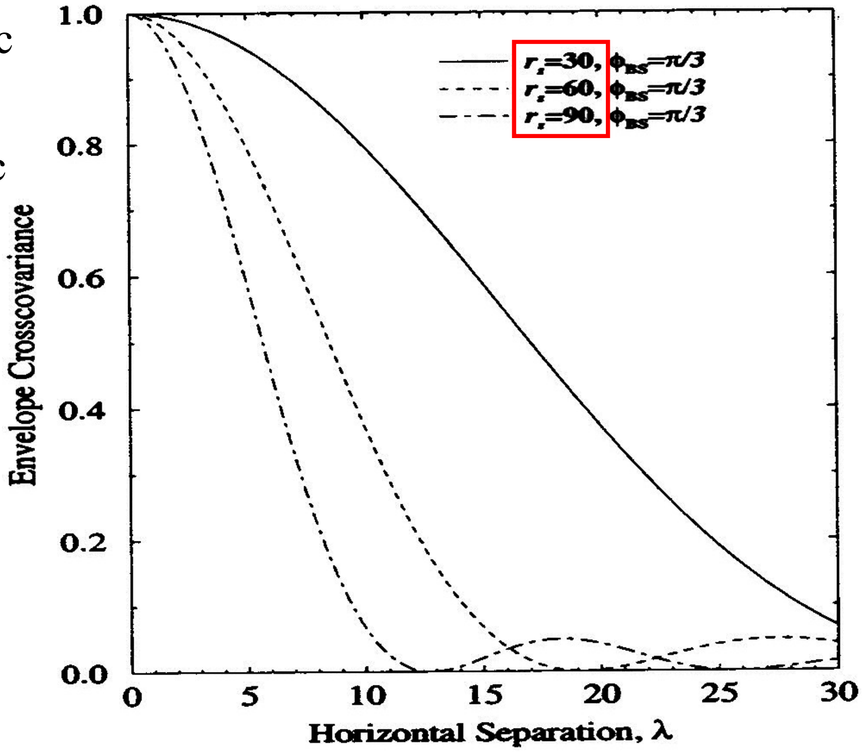




Spatial Correlation for BS

- $r_s\uparrow \Rightarrow$ isotropic scattering
- $d\downarrow \Rightarrow$ isotropic scattering

Question



Frequency-Selective Multipath Fading

Transmission Functions

- The multipath fading channels can be modeled as **time-variant linear filters**

$\Rightarrow$ Four transmission functions are used for representation

  - Input delay-spread function $g(\tau, t)$
  - Output Doppler-spread function $H(f, \nu)$
  - Time-variant transfer function $T(f, t)$
  - Delay Doppler-spread function $S(\tau, \nu)$

- The parameters:
  - t: time domain
  - f: frequency domain
  - τ: time delay
  - ν: Doppler frequency shift

Transmission Functions

- The time delay (delay spread) determines the channel frequency response
  - The time delay τ can be viewed as the impulse response of the filter $\Rightarrow$ corresponding to the frequency response of the filter
  - The distributions of τ and f vary with time t
  - τ relates to f in different domains
- The varying of time corresponds to the change in the scattering environment (the change of Doppler frequency shift)
  - t relates to ν in different domains
- $t \leftrightarrow \nu$
- $\tau \leftrightarrow f$

Transmission Functions

$$g(\tau, t) \underset{\tau \leftrightarrow f}{\overset{Fourier}{\Leftrightarrow}} T(f, t)$$

$$T(f, t) \underset{t \leftrightarrow v}{\overset{Fourier}{\Leftrightarrow}} H(f, v)$$

$$S(\tau, v) \underset{\tau \leftrightarrow f}{\overset{Fourier}{\Leftrightarrow}} H(f, v)$$

$$g(\tau, t) \underset{t \leftrightarrow v}{\overset{Fourier}{\Leftrightarrow}} S(\tau, v)$$

Classification of Channels

- Three channel types:
  - Wide Sense Stationary (WSS) channel
  - Uncorrelated Scattering (US) channel
  - Wide Sense Stationary Uncorrelated Scattering (WSSUS) channel

Wide Sense Stationary (WSS) Channel

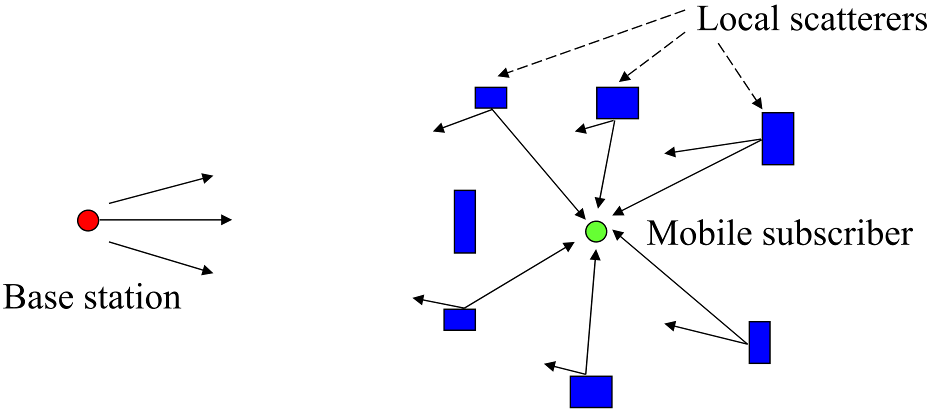
- The fading statistics **remain constant** over short periods of time
- The channel correlation functions depend on the **time difference** Δt
- $t \leftrightarrow v$: the fading characteristics are constant in the time domain $\leftrightarrow$ a delta function in the **correlation** of Doppler frequency shift
  - WSS channels give rise to scattering with **uncorrelated Doppler shifts**
  - The attenuations and phase shifts, associated with signal components **having different Doppler shifts**, are uncorrelated
- <u>**The fading statistics remain constant**</u>
- <u>**Signal components having different Doppler shifts are uncorrelated**</u>

Uncorrelated Scattering (US) Channel

- The attenuations and phase shifts, associated with the paths of **different delays**, are uncorrelated
- $\tau \leftrightarrow f$: the fading characteristics are uncorrelated (delta function) in the delay time domain $\leftrightarrow$ **constant** characteristics in the frequency domain
  - WSS in the frequency variable
  - The correlation functions depend on the frequency difference Δf
- <u>**WSS in the frequency variable**</u>
- <u>**Signal components having different delays are uncorrelated**</u>

WSSUS Channel

- Wide Sense Stationary Uncorrelated Scattering Channel
- The channel displays uncorrelated scattering in both the **time-delay** and **Doppler shift**
- Most of the radio channels can be modeled as WSSUS channels



Multipath Intensity Profile

- For WSSUS, the autocorrelation function of $g(\tau, t)$: $\phi_g(\Delta t; \tau)$
- Multipath intensity profile: For $\Delta t = 0$, $\phi_g(0;\tau) = \phi_g(\tau)$ shows the power profile
  - The average power at channel output of time delay τ
  - It can be viewed as the scattering function averaged over all Doppler shifts
- Average delay: $$\mu_\tau = \frac{\int_0^\infty \tau\, \phi_g(\tau)\, d\tau}{\int_0^\infty \phi_g(\tau)\, d\tau}$$
- RMS delay spread: $$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \phi_g(\tau)\, d\tau}{\int_0^\infty \phi_g(\tau)\, d\tau}}$$

Multipath Intensity Profile

- Middle profile: W_x
  - Contains x% of the total power in the profile

$$W_x = \tau_3 - \tau_1$$

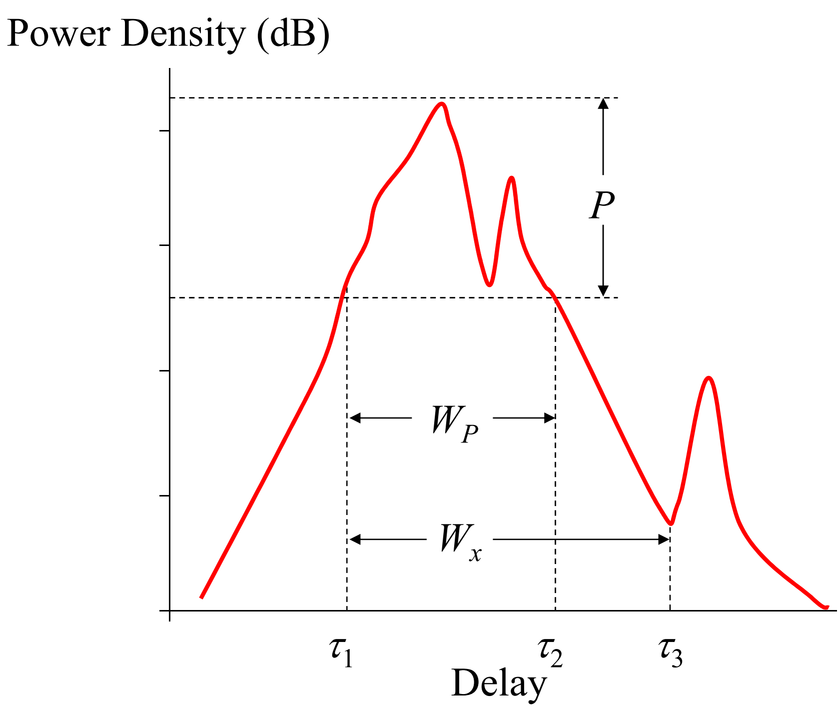
$$\int_0^{\tau_1} \phi_g(\tau)\, d\tau = \int_{\tau_3}^{\infty} \phi_g(\tau)\, d\tau$$

$$\int_{\tau_1}^{\tau_3} \phi_g(\tau)\, d\tau = x\% \int_0^{\infty} \phi_g(\tau)\, d\tau$$

- Difference in delay: W_P
  - The delay profile rises to a value P dB below the maximum value: τ_1
  - The delay profile drops to a value P dB below the maximum value: τ_2

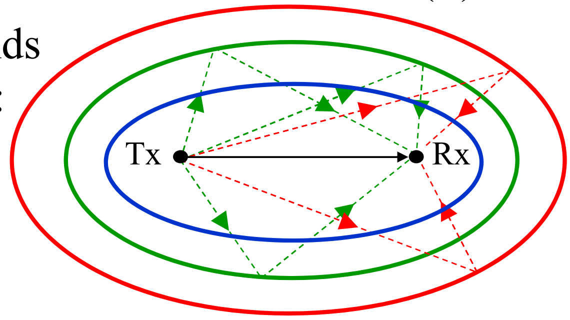
$$W_P = \tau_2 - \tau_1$$

Multipath Intensity Profile



Multipath Intensity Profile

- The power delay profiles play a key role in determining the need of an adaptive equalizer
- If the delay spread exceeds **<u>10% to 20%</u>** of the symbol duration
  - An adaptive equalizer is required
- Delay spread diminish (↓) with the decrease in cell size (↓)
- The delay spread strongly depends on the propagation environment:
  - Urban, suburban, open area
  - Macrocellular: 1 ~ 10 μs
  - In building: 30 ~ 60 *n*s



- The value of delay spread impacts on the transmission rate
  - Under the considerations of **complexity** and **performance**

Coherence Bandwidth

- For WSSUS, the autocorrelation function of $T(t, f)$ is $\phi_T(\Delta t; \Delta f)$: spaced-frequency spaced-time correlation function
- For $\Delta t = 0$, $\phi_T(0;\Delta f) = \phi_T(\Delta f)$ measures the frequency correlation of the channel (depending on the multipath intensity profile)
- **Coherence Bandwidth** B_c:
  - The smallest value of Δf for which $\phi_T(\Delta f)$ equals some suitable correlation coefficient, such as 0.5
- $\phi_g(\tau)$ and $\phi_T(\Delta f)$ are Fourier transform pair

$$B_c \propto \frac{1}{\sigma_\tau}$$

  - σ_τ: the rms delay spread

Coherence Bandwidth

- For frequency non-selective fading:
  - The transmission bandwidth ($1/T_b$) is smaller than B_c
  - The symbol duration $T_b >> \sigma_\tau$
- For frequency selective fading:
  - The transmission bandwidth ($1/T_b$) is larger than or equivalent to B_c
  - The symbol duration $T_b \cong \sigma_\tau$ or $T_b < \sigma_\tau$

Doppler Spread and Coherence Time

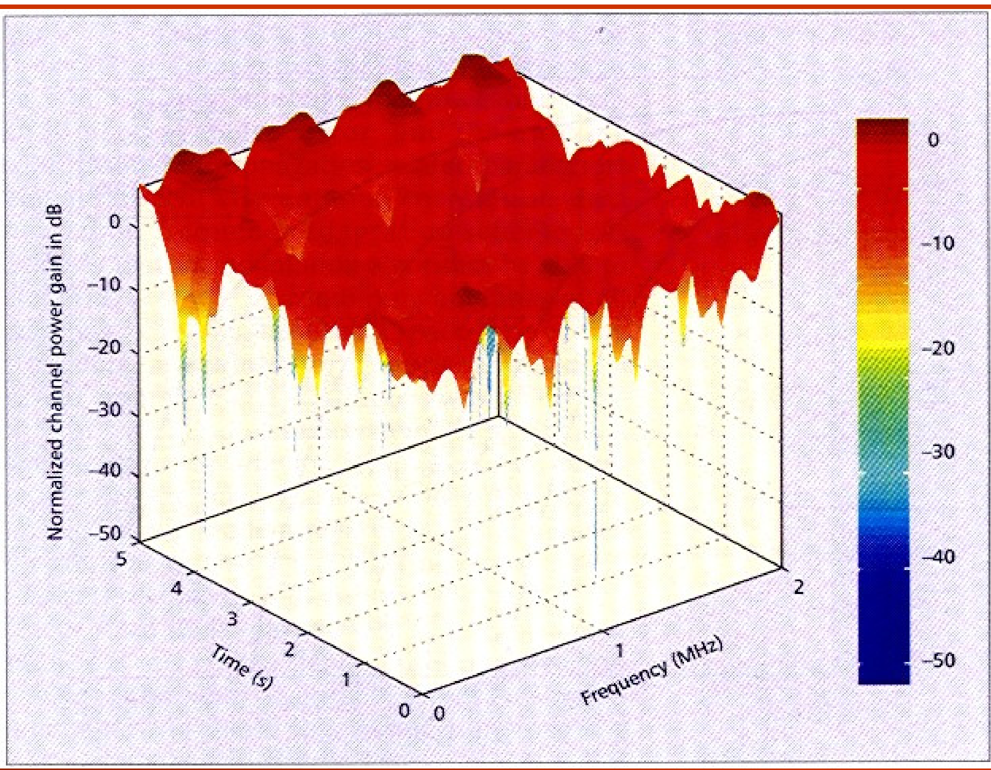
- For WSSUS, the autocorrelation function of $H(\nu, f)$: $\phi_H(\nu\,; \Delta f)$
- Doppler power spectral density: For $\Delta f = 0$, $\phi_H(\nu\,; 0) = \phi_H(\nu)$ shows the power density
  - The average power at the channel output as a function of Doppler frequency ν
- **Doppler Spread** B_d:
  - The range of values over which $\phi_H(\nu)$ is significant
- $\phi_H(\nu)$ and $\phi_T(\Delta t)$ are Fourier transform pair
  - The inverse of the Doppler spread B_d gives a measure of the coherence time T_c

$$T_c \approx \frac{1}{B_d}$$

Doppler Spread and Coherence Time

- **Coherence time** (corresponding to the average fade duration):
  - Can be used to evaluate the performance of coding and interleaving techniques
  - Coding and interleaving ⇒ <u>**time diversity**</u>
- The duration of interleaving should much larger than the coherence time
- The Doppler spread and the coherence time depend directly on the **velocity** of a moving MS

Fading Channel **Question**



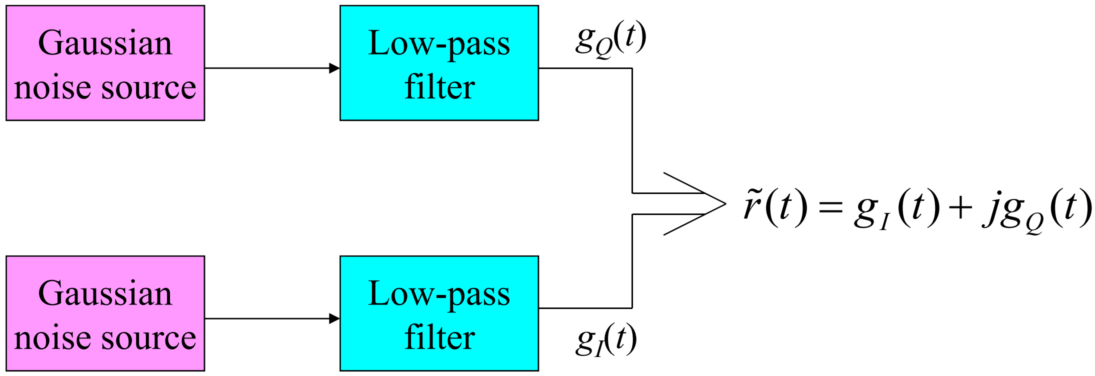
Laboratory Simulation

Simulation of Multipath-Fading Channels

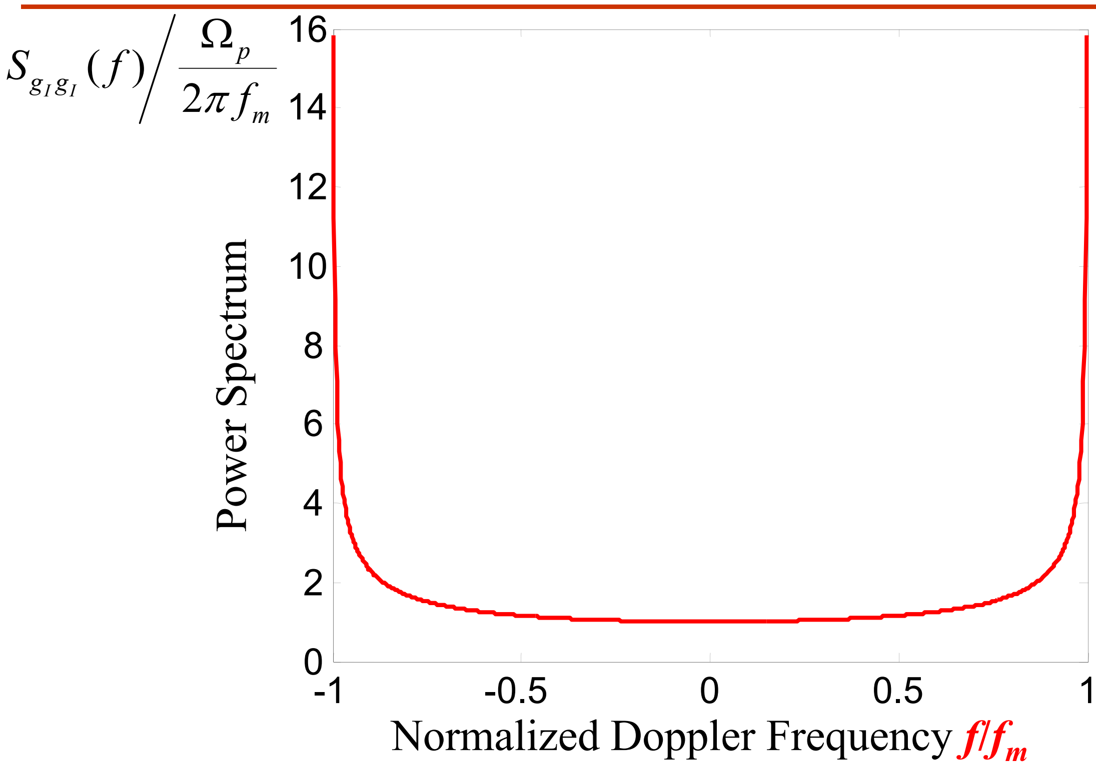
- The multipath fading channel simulator:
  - Filtered Gaussian Noise Method
  - Jakes' Method
  - Wide-band multipath-fading channels

Filtered Gaussian Noise Method

- Gaussian noise sources:
  - Zero mean: Rayleigh fade envelope
  - Non-zero mean: Ricean fade envelope
  - The two different noise sources must have the same PSD
- Low-pass filter: the output PSD should have the actual Doppler PSD of the multipath fading channel



Doppler Spectrum of Multipath Fading Channel



Filtered Gaussian Noise Method

- In order to approximate the Doppler spectrum of the multipath fading channel, a **high order filter** is required
  - $\Rightarrow$ Long impulse response
  - $\Rightarrow$Significantly increase the run times
- Advantage: different paths are uncorrelated (if the Gaussian noise sources are uncorrelated)
- Disadvantage: hard to provide correct autocorrelation (a high order filter is required)
- If the noise sources have power spectral densities of $\Omega_p/2$ and the low-pass filters have transfer function $H(f)$
  - We have $S_{g_I g_I}(f) = S_{g_Q g_Q}(f) = \frac{\Omega_p}{2}\left|H(f)\right|^2$

$$S_{g_I g_Q}(f) = S_{g_Q g_I}(f) = 0$$

Filtered Gaussian Noise Method

- Let $g_{I,k} \equiv g_I(kT)$ and $g_{Q,k} \equiv g_Q(kT)$ represent the real and imaginary parts of the complex envelope at epoch k, where T is the simulation step size
- Using a **first-order** low-pass digital filter

$$(g_{I,k+1}, g_{Q,k+1}) = \zeta(g_{I,k}, g_{Q,k}) + (1-\zeta)(w_{1,k}, w_{2,k})$$

  - where $w_{1,k}$ and $w_{2,k}$ are **independent** zero-mean Gaussian random variables

$$E\left[g_{I,k}g_{I,k}\right] = \zeta^2 E\left[g_{I,k-1}g_{I,k-1}\right] + (1-\zeta)^2\sigma^2 \Rightarrow \sigma_{g_I}^2 = \sigma_{g_Q}^2 = \frac{1-\zeta}{1+\zeta}\sigma^2$$

  - σ^2: the variance of $w_{1,k}$ and $w_{2,k}$, and

$$\phi'_{g_I g_I}(n) = \phi'_{g_Q g_Q}(n) = E[g_{I,k}g_{I,k+n}] = \frac{1-\zeta}{1+\zeta}\sigma^2\zeta^{|n|}$$

Auto-correlation

Cross-correlation $\longrightarrow$ $\phi'_{g_I g_Q}(n) = \phi'_{g_Q g_I}(n) = 0$

Filtered Gaussian Noise Method

- The values of σ^2 and ζ should be specified
- For isotropic scattering, the ideal auto-correlation is

$$\phi_{g_I g_I}(n) = \frac{\Omega_p}{2} J_0(2\pi f_m nT)$$

$$\boxed{u[n] = \begin{cases} +1, & n \ge 0 \\ 0, & n < 0 \end{cases}}$$

- Taking DFT on $\phi'_{g_I g_I}(n)$

Different

$$\phi'_{g_I g_I}(n) = \frac{1-\zeta}{1+\zeta}\sigma^2 \zeta^{|n|} = \frac{1-\zeta}{1+\zeta}\sigma^2 \left(\zeta^n u[n] + \zeta^{-n} u[-n] - \delta[n]\right)$$

$$\zeta^n u[n] \xleftrightarrow{\mathbb{F}} \frac{1}{1-\zeta e^{-j2\pi f t}},\ \zeta^{-n} u[-n] \xleftrightarrow{\mathbb{F}} \frac{1}{1-\zeta e^{j2\pi f t}},\ \delta[n] \xleftrightarrow{\mathbb{F}} 1$$

$$S'_{g_I g_I}(f) = \mathbb{F}\left\{\phi'_{g_I g_I}(n)\right\} = \frac{(1-\zeta)^2 \sigma^2}{1+\zeta^2 - 2\zeta \cos 2\pi f T}$$

Filtered Gaussian Noise Method

$$S'_{g_I g_I}(f) = \frac{(1-\zeta)^2 \sigma^2}{1+\zeta^2 - 2\zeta \cos 2\pi f T}$$

- Set the 3 dB point of $S'_{g_I g_I}(f)$ to $f_m/4$, $\boxed{S'_{g_I g_I}(f_m/4) = S'_{g_I g_I}(0)/2}$ we have

$$\zeta^2 - 2\zeta\left(2 - \cos(\pi f_m T/2)\right) + 1 = 0$$

$$\zeta = 2 - \cos(\pi f_m T/2) - \sqrt{(2-\cos(\pi f_m T/2))^2 - 1}$$

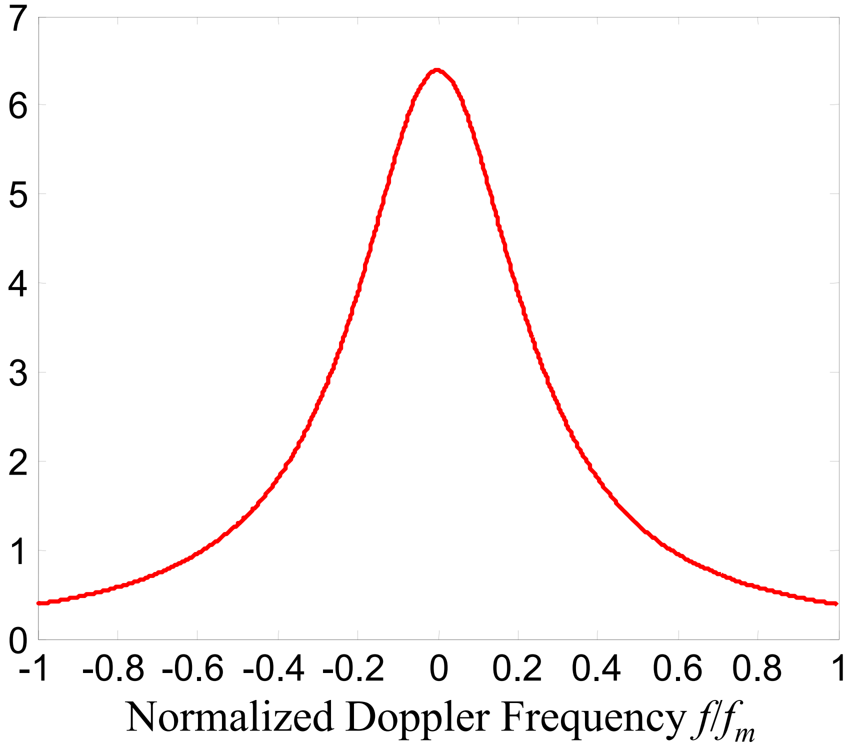
- To normalized the mean square envelope to Ω_p

$$\sigma_{g_I}^2 = \frac{1-\zeta}{1+\zeta}\sigma^2 = \frac{\Omega_p}{2} \Rightarrow \sigma^2 = \frac{1+\zeta}{(1-\zeta)}\frac{\Omega_p}{2}$$

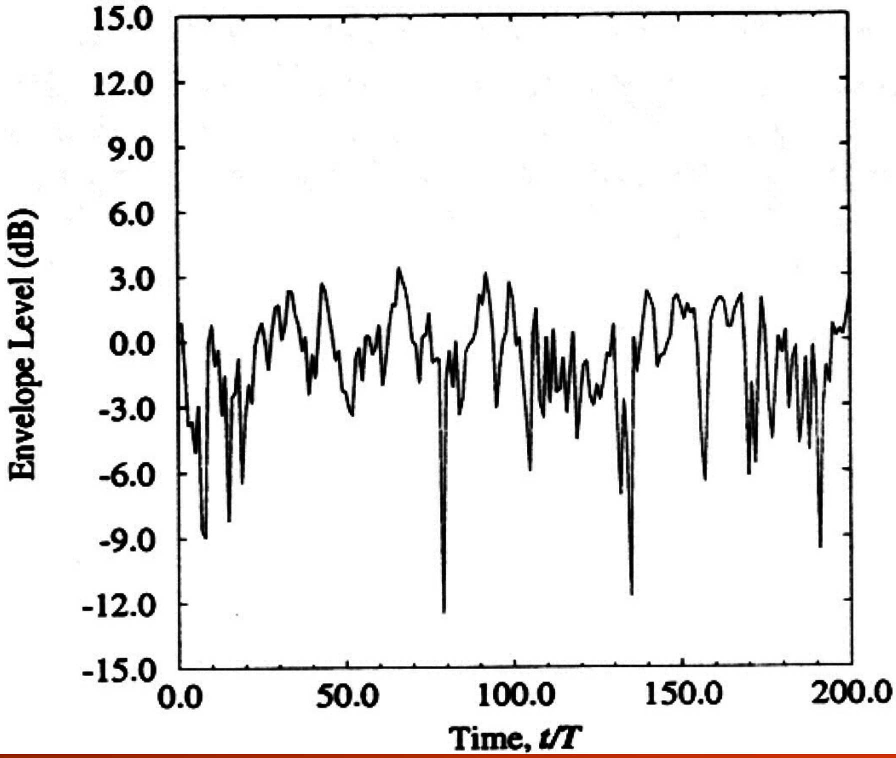
Filtered Gaussian Noise Method

$S'_{g_I g_I}(f)$

Obtained by Filtered Gaussian Noise method



Filtered Gaussian Noise Method



Sum of Sinusoids Method

- From

$$g(t) = \sum_{n=1}^{N} \alpha_n(t)\, e^{-j\phi_n(t)}$$

$$\phi_n(t) = 2\pi\left\{\left(f_c + f_{D,n}(t)\right)\tau_n(t) - f_{D,n}(t)t\right\}$$

- Assume that
  - The channel is stationary ($f_{D,n}(t) = f_{D,n}$, $\tau_n(t) = \tau_n$, $\alpha_n(t) = \alpha_n$)
  - Equal strength of multipath components ($\alpha_n = 1$, $\forall n$)

$$g(t) = \sum_{n=1}^{N} e^{j2\pi\left[f_m t\cos\theta_n - \left(f_c + f_m\cos\theta_n\right)\tau_n\right]} = \sum_{n=1}^{N} e^{j\left(2\pi f_m t\cos\theta_n + \hat{\phi}_n\right)}$$

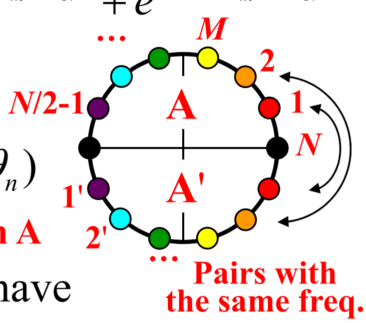
- For an isotropic scattering environment, we can assume that the **incident angles** are uniformly distributed

$$\theta_n = \frac{2\pi n}{N}, \quad n = 1, 2, \cdots, N$$

Sum of Sinusoids Method

- Choose $N/2$ to be an odd integer, we can rewrite $g(t)$ as

$$g(t) = \sum_{n=1}^{N/2-1}\Big[\underbrace{e^{j\left(2\pi f_m t\cos\theta_n + \hat{\phi}_n\right)}}_{\mathbf{A}} + \underbrace{e^{-j\left(2\pi f_m t\cos\theta_n + \hat{\phi}_{-n}\right)}}_{\mathbf{A'}}\Big] + e^{j\left(2\pi f_m t + \hat{\phi}_N\right)} + e^{-j\left(2\pi f_m t + \hat{\phi}_{-N}\right)}$$



- Some terms have the same freq. components

$$2\pi f_m t\cos(\theta_n + \pi) = -2\pi f_m t\cos\theta_n = 2\pi f_m t\cos(\pi - \theta_n)$$

Term $N/2+n$ — Term n in A' $\xleftrightarrow{\text{same freq.}}$ Term $N/2-n$ in A

- Combining the terms with the same freq., we have

$$g(t) = \sqrt{2}\sum_{n=1}^{M}\Big[e^{j\left(2\pi f_m t\cos\theta_n + \hat{\phi}_n\right)} + e^{-j\left(2\pi f_m t\cos\theta_n + \hat{\phi}_{-n}\right)}\Big] + e^{j\left(2\pi f_m t + \hat{\phi}_N\right)} + e^{-j\left(2\pi f_m t + \hat{\phi}_{-N}\right)}$$

Maintain the same total power

$$M = \frac{1}{2}\left(\frac{N}{2} - 1\right)$$

- For the same freq. components, the phases are now set to be the same $\Rightarrow$ Correlation is introduced into the phases

Sum of Sinusoids Method

$$g(t)=\sqrt{2}\sum_{n=1}^{M}\left[e^{j(2\pi f_m t\cos\theta_n+\hat{\phi}_n)}+e^{-j(2\pi f_m t\cos\theta_n+\hat{\phi}_{-n})}\right]+e^{j(2\pi f_m t+\hat{\phi}_N)}+e^{-j(2\pi f_m t+\hat{\phi}_{-N})}$$

- If we further adopt the constraint that $\hat{\phi}_n=-\hat{\phi}_{-n}$, we have

$$\boxed{\begin{aligned}\cos(x+y)&=\cos(x)\cos(y)-\sin(x)\sin(y)\\ \sin(x+y)&=\sin(x)\cos(y)+\cos(x)\sin(y)\end{aligned}}$$

$$\boxed{2\pi f_n t=2\pi f_m t\cos\theta_n}$$

$$\begin{aligned}g(t)&=g_I(t)+jg_Q(t)\\ &=\sqrt{2}\left\{\left[2\sum_{n=1}^{M}\cos\beta_n\cos\underline{2\pi f_n t}+\sqrt{2}\cos\alpha\cos 2\pi f_m t\right]\right.\\ &\qquad\left.+j\left[2\sum_{n=1}^{M}\sin\beta_n\cos 2\pi f_n t+\sqrt{2}\sin\alpha\cos 2\pi f_m t\right]\right\}\end{aligned}$$

  - where $\alpha=\hat{\phi}_N=-\hat{\phi}_{-N},\quad \beta_n=\hat{\phi}_n=-\hat{\phi}_{-n}$
- Only **(*M*+1)** independent frequency oscillators are required
  - There are (M+1) different frequencies

Sum of Sinusoids Method

- Considering the channel statistics

$$E\left[g_I^2(t)\right]=2\sum_{n=1}^{M}\cos^2\beta_n+\cos^2\alpha=M+\cos^2\alpha+\sum_{n=1}^{M}\cos 2\beta_n$$

$$E\left[g_Q^2(t)\right]=2\sum_{n=1}^{M}\sin^2\beta_n+\sin^2\alpha=M+\sin^2\alpha-\sum_{n=1}^{M}\cos 2\beta_n$$

$$E\left[g_I(t)g_Q(t)\right]=2\sum_{n=1}^{M}\sin\beta_n\cos\beta_n+\sin\alpha\cos\alpha$$

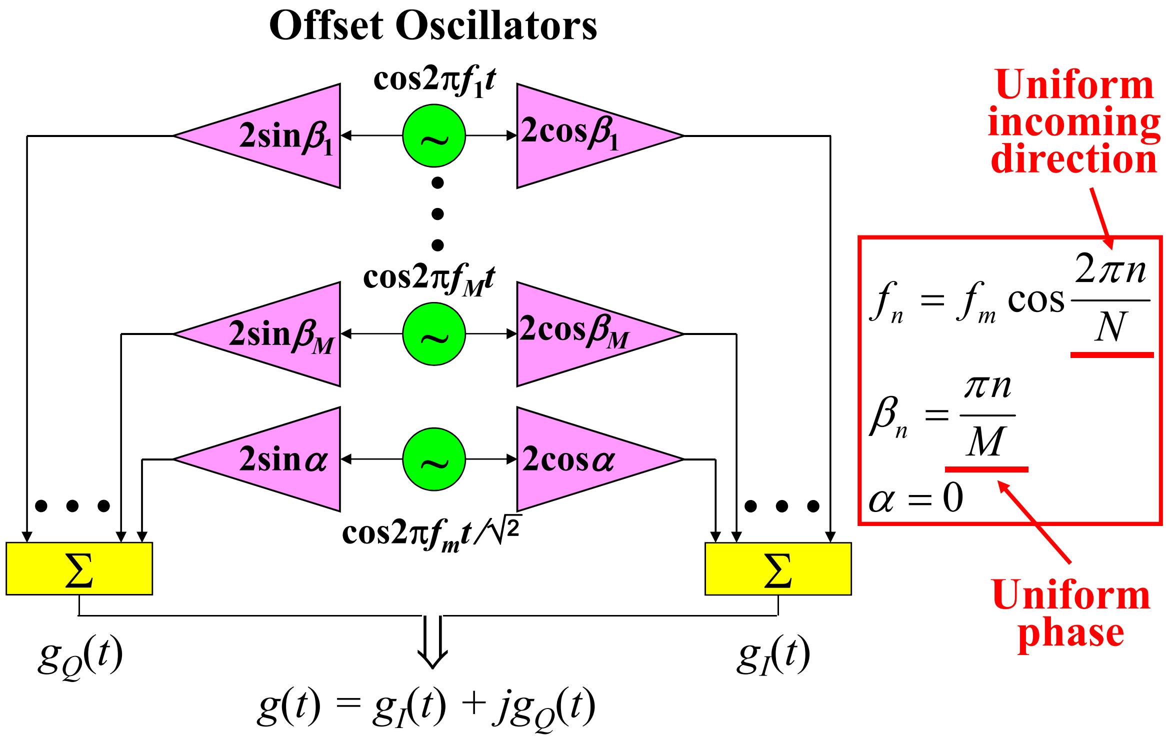
- It is desirable that

$$E\left[g_I^2(t)\right]=E\left[g_Q^2(t)\right],\quad E\left[g_I(t)g_Q(t)\right]=0$$

- Choose the parameters $\beta_n=\dfrac{\pi n}{M},\quad \alpha=0$

$$E\left[g_I^2(t)\right]=M+1,\quad E\left[g_Q^2(t)\right]=M,\quad E\left[g_I(t)g_Q(t)\right]=0$$

Sum of Sinusoids Method



$$f_n = f_m \cos\frac{2\pi n}{N}$$

$$\beta_n = \frac{\pi n}{M}$$

$$\alpha = 0$$

Sum of Sinusoids Method

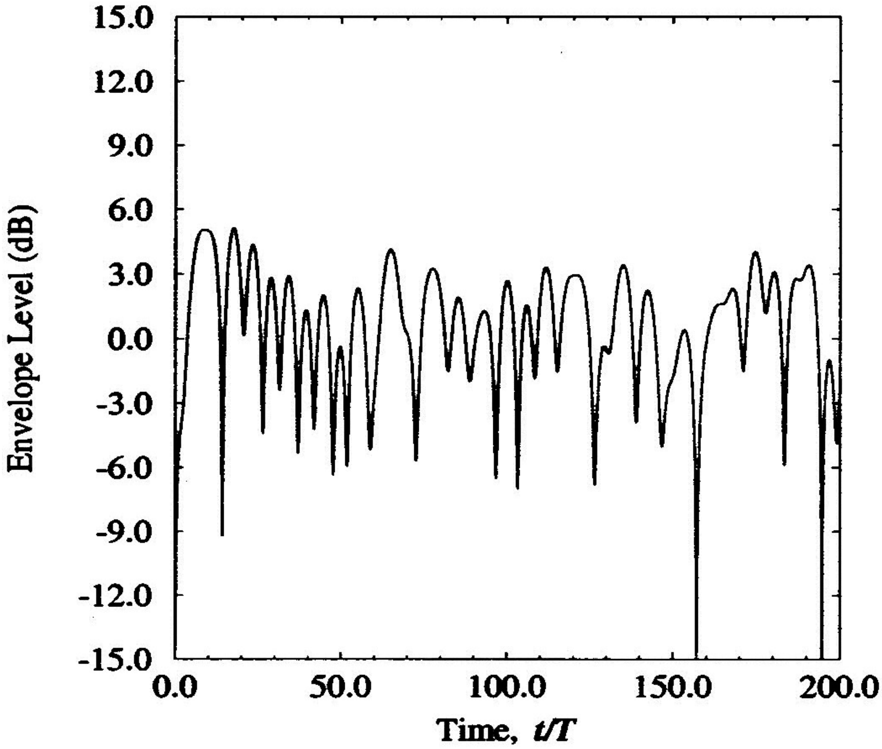
- If the last term of $g_I(t)$ is ignored, we have

$$E\left[g_I^2(t)\right]=M,\quad E\left[g_Q^2(t)\right]=M,\quad E\left[g_I(t)g_Q(t)\right]=0$$

  – when $\beta_n = \frac{\pi n}{M},\quad \alpha = 0$
- Advantage: the **autocorrelation** of inphase and quadrature components reflect an **isotropic scattering** environment with a reasonable complexity
- The channel model output is a **deterministic process**
  – No random number generator is applied

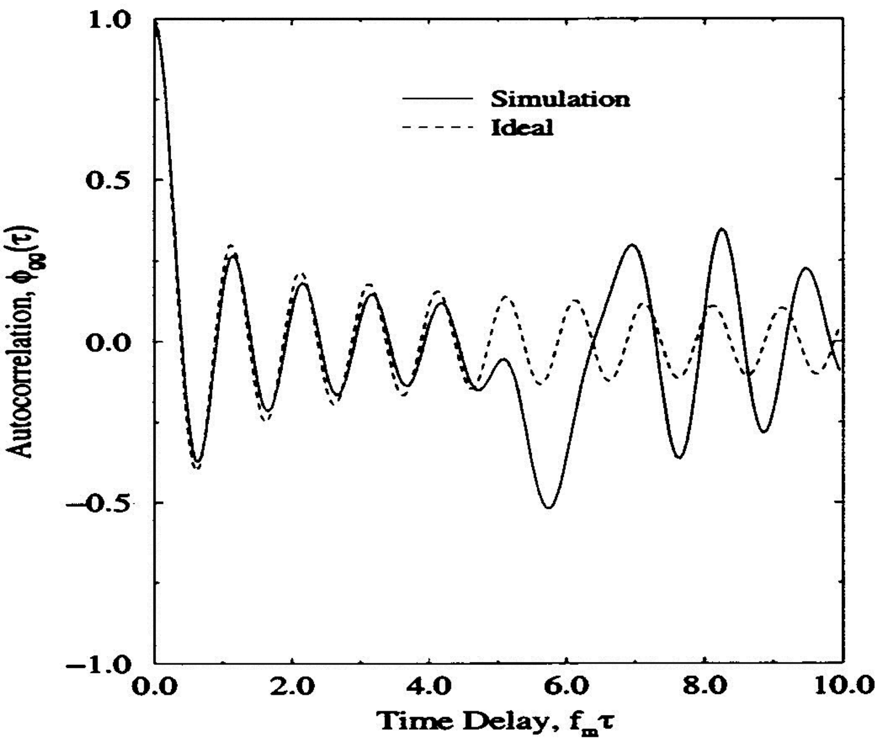
Sum of Sinusoids Method

- $M = 8$



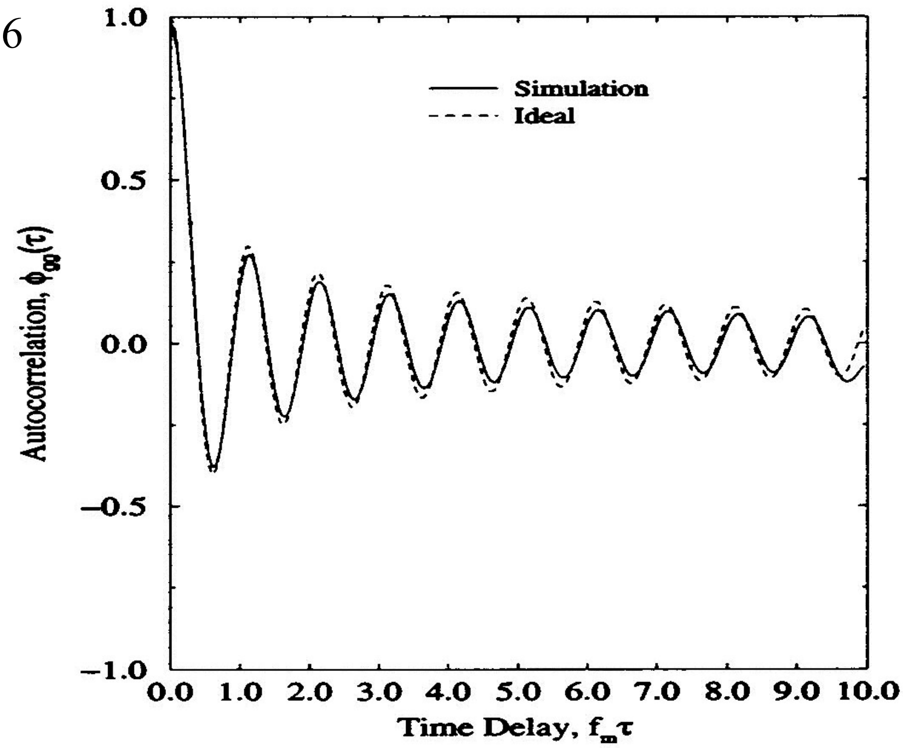
Sum of Sinusoids Method

- $M = 8$



Sum of Sinusoids Method

- $M = 16$



Wide-Band Multipath-Fading Channels

- For wide-band communication systems, the time-domain resolution is increased and multiple paths can be resolved
- τ–spaced model:
  – Model the channel by a tapped delay line
  – Assume a number of discrete paths at different delays

$$\tilde{r}(t) = \sum_{i=1}^{\ell} g_i(t)\tilde{s}(t-\tau_i)$$

  – $g_i(t)$ and τ_i are the tap gain and delay of the i-th path

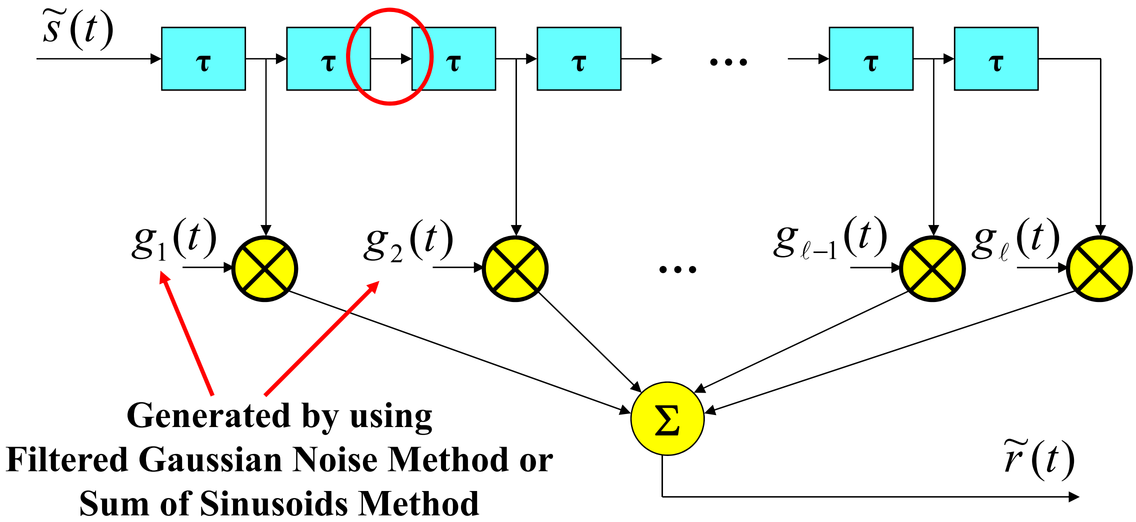
$$g(t,\tau) = \sum_{i=1}^{\ell} g_i(t)\delta(t-\tau_i)$$

  – The tap gain and tap delay vectors

$$\mathbf{g}(t) = \left(g_1(t),\, g_2(t),\, \cdots,\, g_\ell(t)\right)$$
$$\boldsymbol{\tau} = \left(\tau_1,\, \tau_2,\, \cdots,\, \tau_\ell\right)$$

Wide-Band Multipath-Fading Channels

- The path delays are multiples of some small number τ



Multiple Faded Envelopes

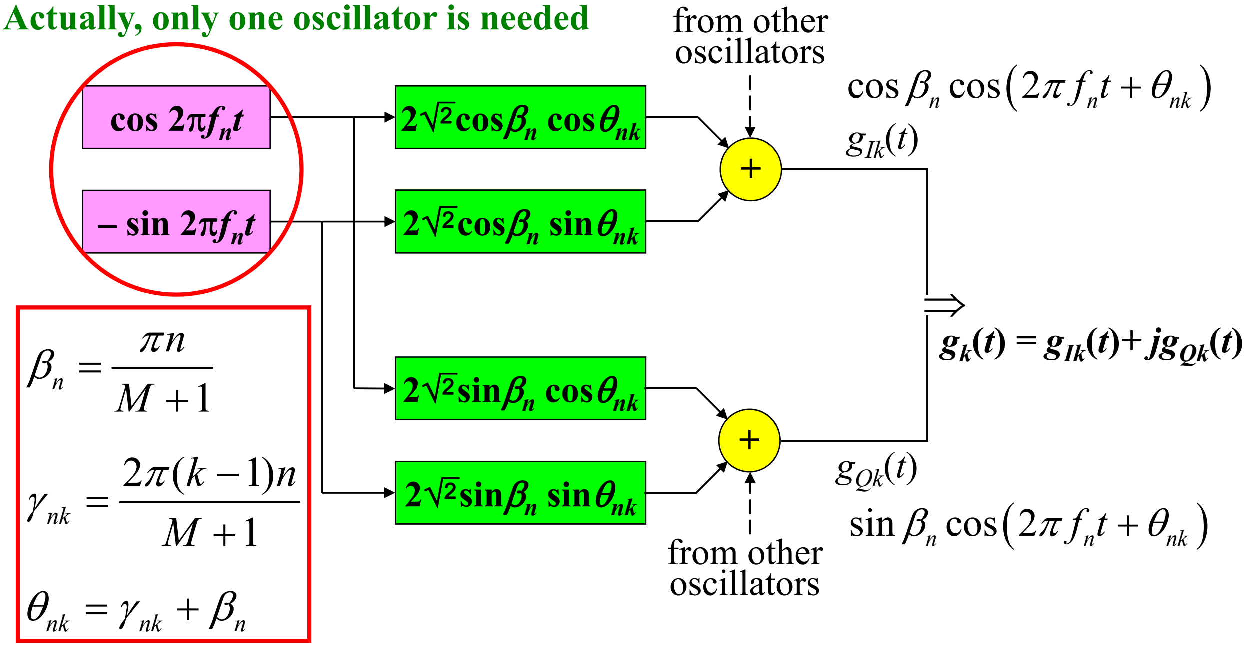
- In many cases, it is desirable to generate multiple envelopes with **uncorrelated fading** (i.e., different paths with resolvable delays)
  - Generate up to ***M*** fading envelopes by using the same ***M*** frequency oscillators
- Give the n-th oscillator, $1 \le n \le M$, an additional phase shift $\theta_{nk} = \gamma_{nk} + \beta_n$, $1 \le k \le M$, where k is the index of fading envelopes
- An additional constraint: the multiple faded envelopes should be **uncorrelated**
  - Choose appropriate values of γ_{nk} and β_n
- The k-th fading envelope is (ignore the last term of $g_I(t)$)

$$g_k(t) = 2\sqrt{2}\sum_{n=1}^{M}\left(\cos\beta_n + j\sin\beta_n\right)\underline{\cos\left(2\pi f_n t + \theta_{nk}\right)}$$

Multiple Faded Envelopes

- By using two quadrature frequency oscillators

Actually, only one oscillator is needed



Multiple Faded Envelopes

- Choose the parameters with the objective yielding uncorrelated waveforms

$$\beta_n = \frac{\pi n}{M+1}, \quad \gamma_{nk} = \frac{2\pi(k-1)n}{M+1}, \quad n = 1, 2, \cdots, M$$

- Significant cross-correlation between the different generated fading envelopes (**without modification**)
- A modification that uses orthogonal **Walsh-Hadamard** codewords to decorrelate the fading envelopes is applied
  - $A_k(n)$: the k-th row of Hadamard matrix $\mathbf{H}_M$
  - **$A_k(n)$: +1 ("0") or –1 ("1")**

$$g_k(t) = 2\sqrt{2}\sum_{n=1}^{M} A_k(n)\left(\cos\beta_n + j\sin\beta_n\right)\cos\left(2\pi f_n t + \theta_{nk}\right)$$

Multiple Faded Envelopes

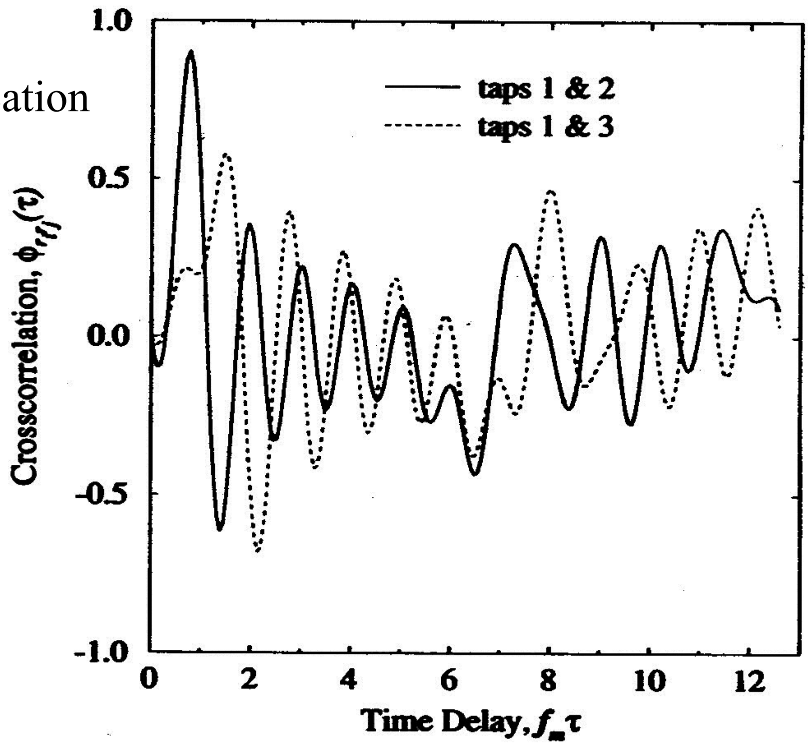
- Walsh-Hadamard codes:
  - It is an **orthogonal code set**
  - The cross-correlation between different codes is zero
- The code period of Walsh codes must be a power of 2
  - The code length must be 2, 4, 8, 16, …

$$H_1 = [0]; \; H_{2^n} = \begin{bmatrix} H_{2^{(n-1)}} & H_{2^{(n-1)}} \\ H_{2^{(n-1)}} & \overline{H_{2^{(n-1)}}} \end{bmatrix};$$

$$H_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \; H_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}; \; H_8 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix};$$

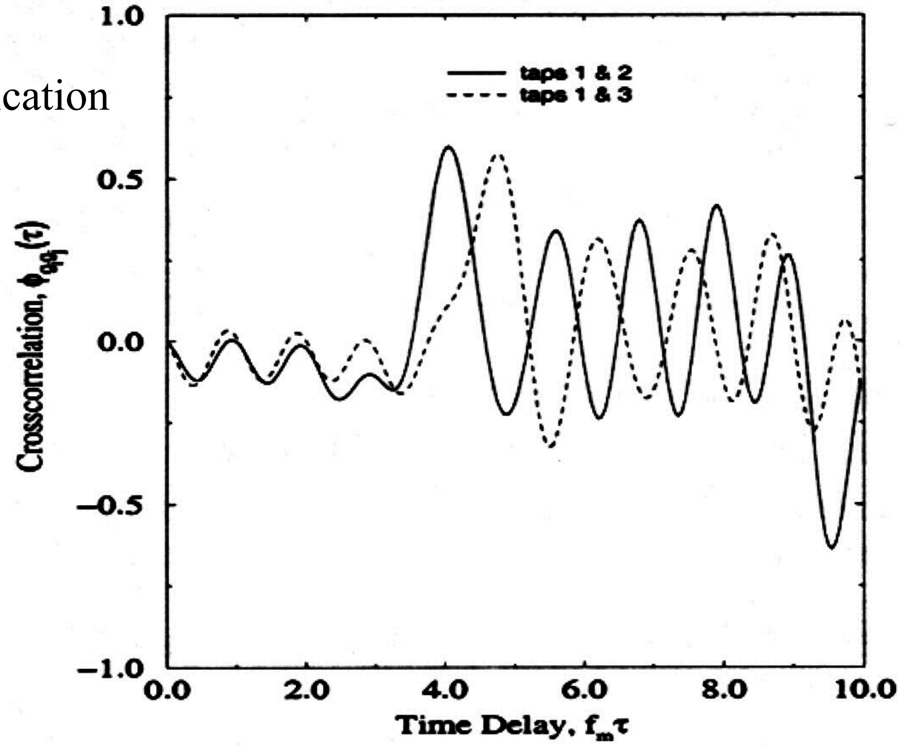
Multiple Faded Envelopes

- $M = 8$
- Without modification



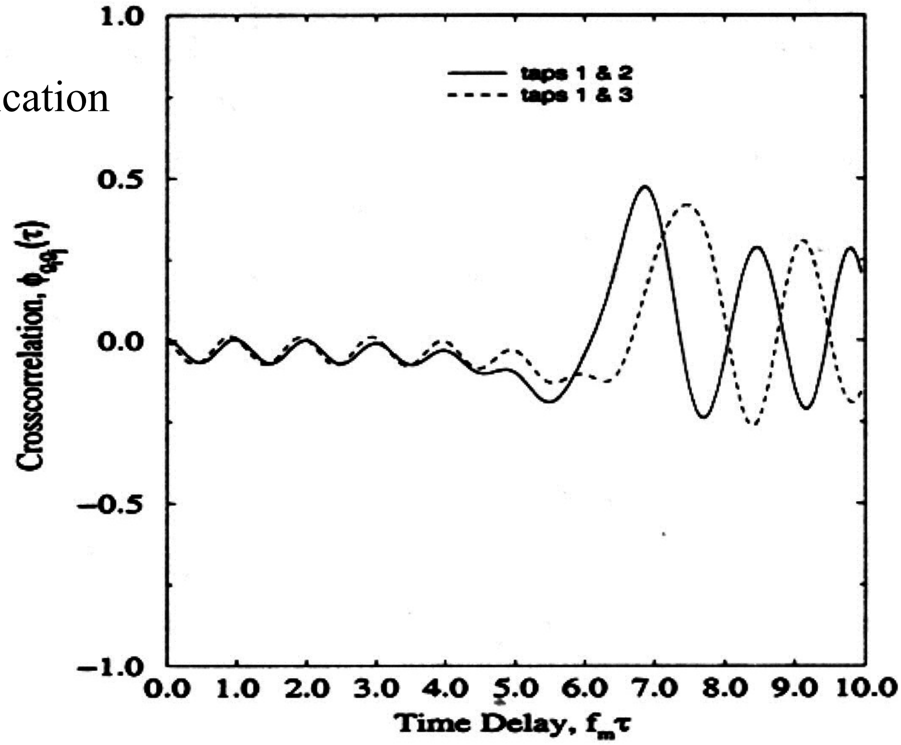
Multiple Faded Envelopes (Modified)

- $M = 8$
- With modification



Multiple Faded Envelopes (Modified)

- $M = 16$
- With modification



Shadowing

Shadowing

- Ω_v: the mean envelope level, $\Omega_v=E[\alpha(t)]$
  - where $\alpha(t)$ is Rayleigh or Ricean distributed
  - The local mean: averaged over a few wavelengths
- Ω_p: the mean squared envelope level, $\Omega_p=E[\alpha^2(t)]$
- Ω_v and Ω_p are random variables due to shadow variations that caused by
  - **Macrocell:** large terrain features (buildings, hills)
  - **Microcell:** small objects (vehicles, human)
- Ω_v and Ω_p follow the log-normal distributions

Linear Scale

$$p_{\Omega_v}(x)=\frac{2}{x\sigma_\Omega\xi\sqrt{2\pi}}\exp\left[-\frac{(10\log_{10}x^2-\mu_{\Omega_v(\mathrm{dBm})})^2}{2\sigma_\Omega^2}\right]$$

$$p_{\Omega_p}(x)=\frac{1}{x\sigma_\Omega\xi\sqrt{2\pi}}\exp\left[-\frac{(10\log_{10}x-\mu_{\Omega_p(\mathrm{dBm})})^2}{2\sigma_\Omega^2}\right]$$

Shadowing

– where ξ = ln10/10 and

$$\mu_{\Omega_v(\text{dBm})} = 30 + 10E[\log_{10}\Omega_v^2]; \quad \mu_{\Omega_p(\text{dBm})} = 30 + 10E[\log_{10}\Omega_p]$$

- $\Omega_{v(\text{dBm})}$ and $\Omega_{p(\text{dBm})}$ have the Gaussian densities
  – The mean is determined by the propagation **path loss**

dB Scale

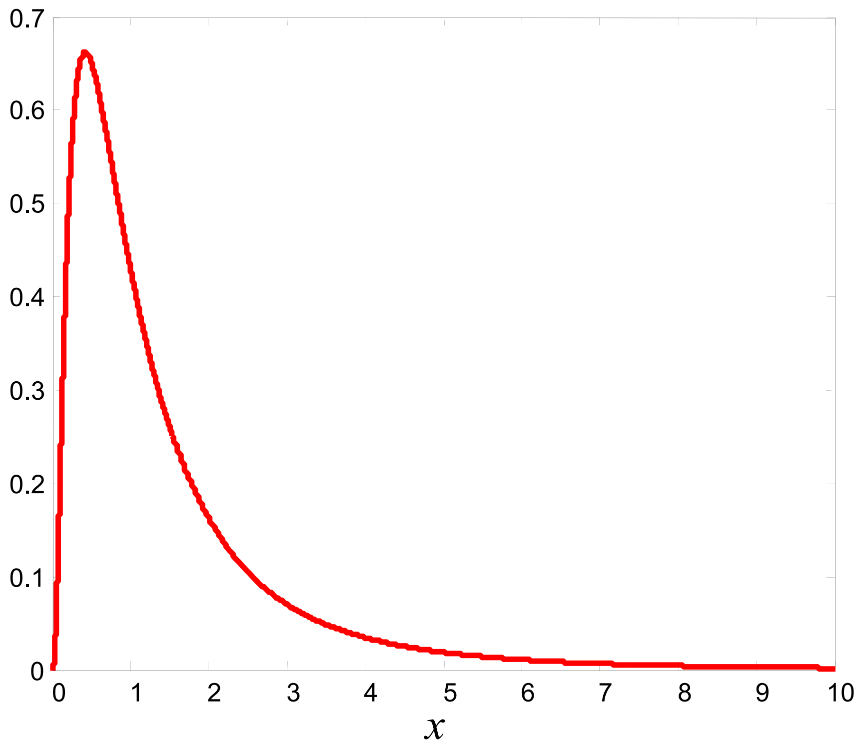
$$p_{\Omega_v(\text{dBm})}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp\left[-\frac{(x-\mu_{\Omega_v(\text{dBm})})^2}{2\sigma_\Omega^2}\right]$$

$$p_{\Omega_p(\text{dBm})}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp\left[-\frac{(x-\mu_{\Omega_p(\text{dBm})})^2}{2\sigma_\Omega^2}\right]$$

- The standard deviation of log-normal shadowing ranges:
  – Macrocell: **5 ~ 12 dB** with typical value σ_Ω = 8 dB
    - σ_Ω increases slightly with frequency ($\sigma_{1.8\text{GHz}} = \sigma_{900\text{MHz}}$+0.8dB)
  – Microcell: **4 ~ 13 dB**

Shadowing

Log-normal pdf



Simulation of Shadowing

- A shadow simulator should account the spatial correlation
- One simple model: the log-normal shadowing is modeled as
  - A Gaussian white noise process
  - Filtered with a first-order low-pass filter

$$\Omega_{k+1(\mathrm{dBm})} = \zeta\, \Omega_{k(\mathrm{dBm})} + (1-\zeta) v_k$$

  - k: the location index
  - ζ: control the spatial correlation of the shadowing
  - v_k: a zero-mean Gaussian random variable, $\phi_{vv}(n) = \tilde{\sigma}^2 \delta(n)$
- The spatial autocorrelation function:

$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(n) = \frac{1-\zeta}{1+\zeta} \tilde{\sigma}^2 \zeta^{|n|}$$

$$\sigma_\Omega^2 = \phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(0) = \frac{1-\zeta}{1+\zeta} \tilde{\sigma}^2$$

Simulation of Shadowing

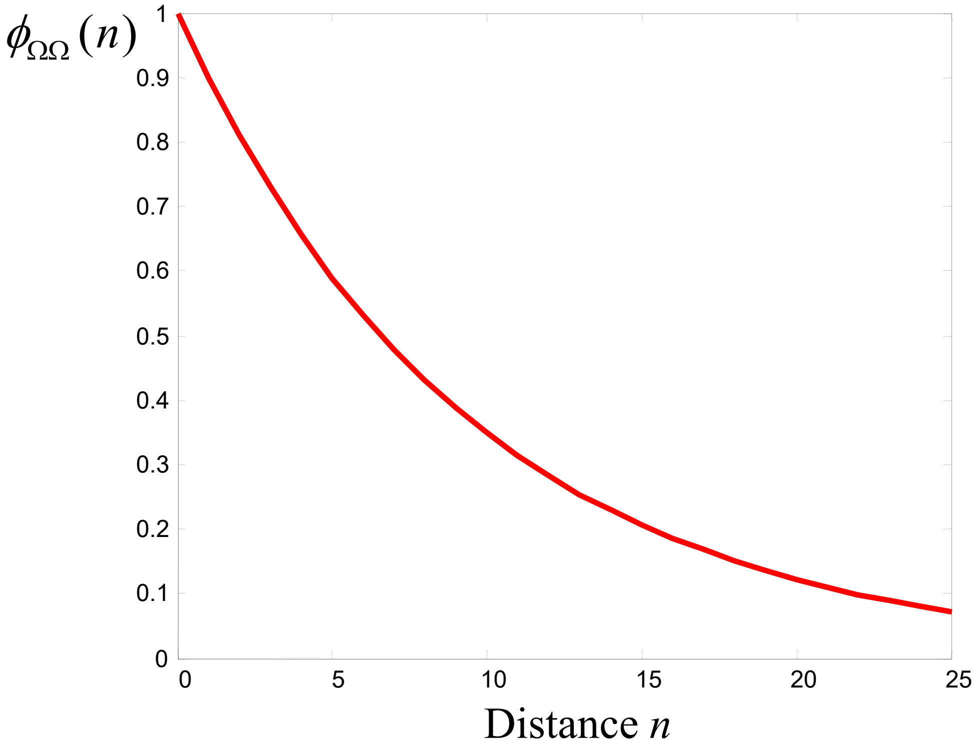
$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(n) = \sigma_\Omega^2 \zeta^{|n|}$$

- This approach generates shadows that decorrelated **exponentially with distance**
- If an MS is traveling with velocity v, the envelope is sampled for every T seconds, and ζ_D is the shadow correlation of spatial distance D m
  - Time difference $kT \Rightarrow$ spatial distance vkT

$$\phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(k) \equiv \phi_{\Omega_{(\mathrm{dBm})}\Omega_{(\mathrm{dBm})}}(kT) = \sigma_\Omega^2 \zeta_D^{(vT/D)|k|}$$

  - $\zeta = \zeta_D^{(vT/D)}$
- Suburban 900 MHz: $\sigma_\Omega \approx 7.5$ dB with corr. 0.82 (100m)
- Microcell 1700 MHz: $\sigma_\Omega \approx 4.3$ dB with corr. 0.3 (10m)

Simulation of Shadowing



Path Loss Models

Path Loss Models

- Free space: the received signal power

$$\mu_{\Omega_p} = \Omega_t G_T G_R \left(\frac{\lambda_c}{4\pi d} \right)^2$$

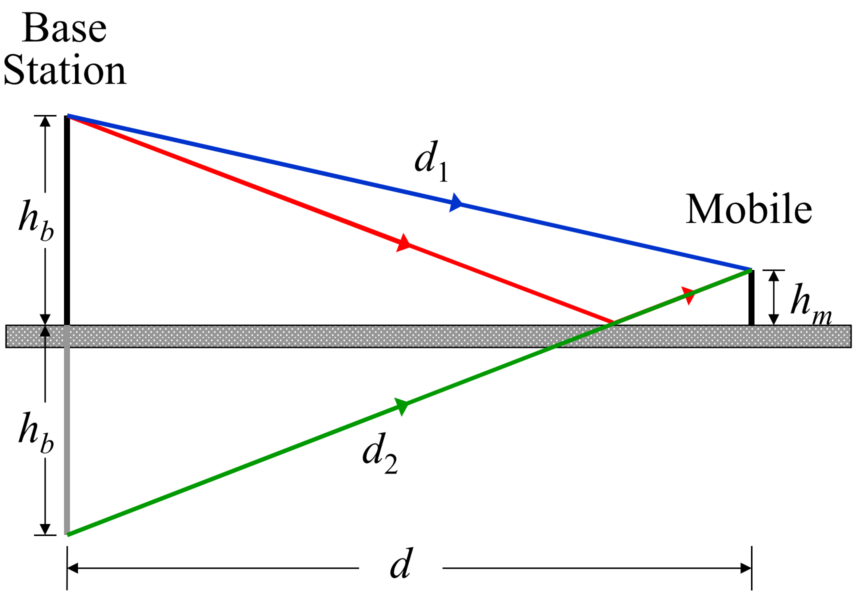
$$\mu_{\Omega_p (dB)} = 10\log_{10} \left(\Omega_t G_T G_R \left(\frac{\lambda_c}{4\pi d} \right)^2 \right)$$

$$\mu_{\Omega_p (dB)} = 10\log_{10}(\Omega_t G_T G_R / 16\pi^2) + 20\log_{10}\lambda_c - 20\log_{10} d$$

  - Ω_t: the transmission power
  - G_T and G_R: the transmitter and receiver antenna gains
  - λ_c: the wavelength
  - d: the radio path length

Path Loss Models

- Mobile radio environment (Two-ray model)



Path Loss Models

$$P_r = P_t(\tfrac{\lambda_c}{4\pi d})^2 \left|1+\alpha_v e^{j\Delta\phi}\right|^2$$

$$= P_t(\tfrac{\lambda_c}{4\pi d})^2 \left|1-\cos\Delta\phi - j\sin\Delta\phi\right|^2$$

$$= P_t(\tfrac{\lambda_c}{4\pi d})^2 \times 2(1-\cos\Delta\phi)$$

$$= P_t(\tfrac{\lambda_c}{4\pi d})^2 \times 4\sin^2\tfrac{\Delta\phi}{2}$$

α_v : reflection coefficient
$\alpha_v = -1$, for mobile environment
$\Delta\phi$: phase difference

$$\Delta\phi = \tfrac{2\pi}{\lambda_c}\Delta d, \text{ and } \Delta d = d_2 - d_1$$

$$d_1 = \sqrt{(h_b - h_m)^2 + d^2}, \text{ and } d_2 = \sqrt{(h_b + h_m)^2 + d^2}$$

$$d_2^2 - d_1^2 = 2d_1\Delta d + \Delta d^2 = 4h_b h_m$$

$$\left(\because \begin{cases} d_1 \approx d \\ \Delta d^2 \approx 0 \end{cases} \text{for } d >> 0\right)$$

$$\Rightarrow \Delta d \approx 2h_b h_m / d, \quad \Delta\phi = \tfrac{4\pi h_b h_m}{\lambda_c d}$$

$$\Rightarrow P_r = P_t(\tfrac{\lambda_c}{4\pi d})^2 \times 4\sin^2(\tfrac{2\pi h_b h_m}{\lambda_c d})$$

Path Loss Models

- The received signal power is

$$\mu_{\Omega_p} = 4\Omega_t\left(\frac{\lambda_c}{4\pi d}\right)^2 G_T G_R \sin^2\left(\frac{2\pi h_b h_m}{\lambda_c d}\right)$$

- When $d >> h_b\ h_m$, $\sin x \approx x$

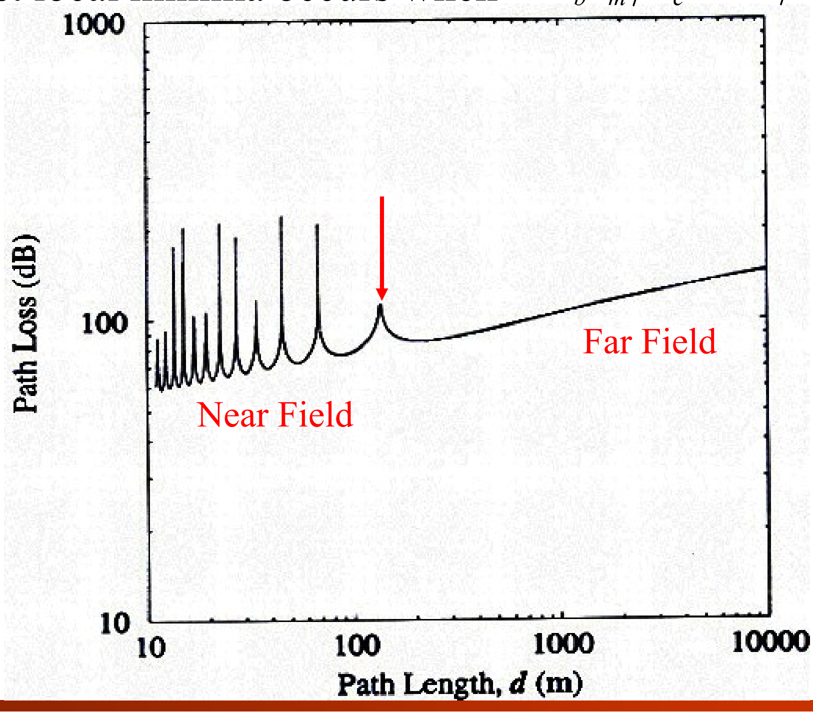
$$\mu_{\Omega_p} = \Omega_t G_T G_R\left(\frac{h_b h_m}{d^2}\right)^2$$

- The differences to the free space model are:
  - The path loss is not frequency dependent
  - The signal power decays with the 4th power of the distance
- The path loss is independent of Ω_t, G_T, and G_R

$$L_{p(\mathrm{dB})} = 10\log_{10}\left\{\frac{\Omega_t G_T G_R}{\mu_{\Omega_p}}\right\} = -10\log_{10}\left\{4\left(\frac{\lambda_c}{4\pi d}\right)^2 \sin^2\left(\frac{2\pi h_b h_m}{\lambda_c d}\right)\right\} \text{ dB}$$

Path Loss Models

- The last local minima occurs when $2\pi h_b h_m / \lambda_c d = \pi/2$



Path Loss in Macrocells

- The path loss models used in macrocell applications are **empirical** models
  - Obtained by curve fitting the experimental data
- For 900 MHz cellular systems, the most common used path loss model is
  - **Okumura-Hata's model**
    - Empirical data was collected by Okumura (in Tokyo)
    - Modeled by Hata

Okumura-Hata's Model

- f_c: 150~1500MHz, d: >1Km, h_b: 30~200m, h_m: 1~10m

$$L_{p(\mathrm{dB})} = \begin{cases} A + B\log_{10}(d) & \text{for urban area} \\ A + B\log_{10}(d) - C & \text{for suburban area} \\ A + B\log_{10}(d) - D & \text{for open area} \end{cases}$$

– where

$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$
$$C = 5.4 + 2[\log_{10}(f_c/28)]^2$$
$$D = 40.94 + 4.78[\log_{10}(f_c)]^2 - 18.33\log_{10}(f_c)$$

$$a(h_m) = \begin{cases} [1.1\log_{10}(f_c) - 0.7]h_m - [1.56\log_{10}(f_c) - 0.8], \text{ for medium or small city} \\ \begin{cases} 8.28[\log_{10}(1.54h_m)]^2 - 1.1, \text{ for } f_c \le 200 \text{ MHz} \\ 3.2[\log_{10}(11.75h_m)]^2 - 4.97, \text{ for } f_c \ge 400 \text{ MHz} \end{cases}, \text{ for large city} \end{cases}$$

Okumura-Hata's Model

- Another empirical model published by the CCIR:

$$L_{p(\mathrm{dB})} = A + B\log_{10}(d) - E$$

– where

$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$
$$E = 30 - 25\log_{10}(\text{\% of area covered by buildings: 1~100})$$
$$a(h_m) = [1.1\log_{10}(f_c) - 0.7]h_m - [1.56\log_{10}(f_c) - 0.8]$$

- The parameter E accounts for the degree of urbanization
  - $E = 0$ when the area is covered by approximately 16 % of buildings

Okumura-Hata's Model

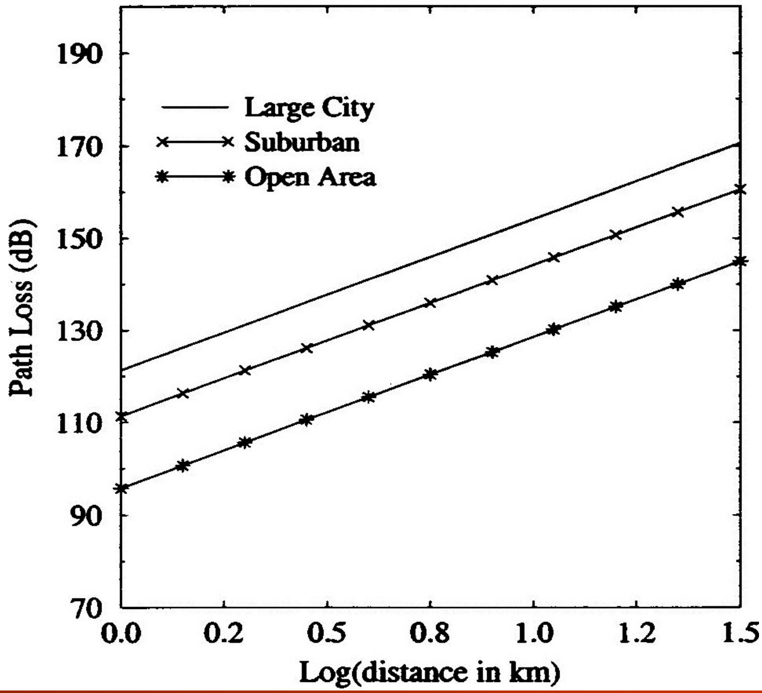
- **Another expression**
- f_c: 150~1500MHz, d: >1Km, h_b: 30~200m, h_m: 1~10m
- $P_O = A + B \log(d)$
  $= [69.55 + 26.16 \log(f_c) – 13.82 \log(h_b)]$
  $+ [44.9\text{-}6.55\log(h_b)] \log(d)$
- $a(h_m)$:
  - Large city:
    - $f_c < 200$MHz: $a(h_m) = 8.28 [\log(1.54\, h_m)]^2 – 1.1$
    - $f_c > 400$MHz: $a(h_m) = 3.2 [\log(11.75\, h_m)]^2 – 4.97$
  - Medium or Small city:
    - $a(h_m) = [1.1 \log(f_c) – 0.7]h_m – [1.56\log(f_c) – 0.8]$

Okumura-Hata's Model

- **Distance correction factor**:
  - $d < 20$Km: $cr(d) = 0$
  - $d > 20$Km: $cr(d) = (d – 20)[0.31081 + 0.1865\log(h_b/100)]$
  - $d > 64.36$Km: $cr(d) = (d – 20)[0.31081 + 0.1865 \log(h_b/100)] – 0.174(d – 64.36)$
- Environment correction factor:
  - Urban area: $ce(f_c) = 0$
  - Suburban area: $ce(f_c) = – 2[\log(f_c/28)]^2 – 5.4$
  - Open area: $ce(f_c) = – 4.78[\log(f_c)]^2 + 18.33 \log(f_c) – 40.94$
- $\mathbf{P_L = P_O - a(h_m) + cr(d) + ce(f_c)}$ **dB**

Okumura-Hata's Model

- f_c = 900 MHz, h_b = 70 m, h_m = 1.5 m



Path Loss in Outdoor Macro-/Micro-cells

- For the PCS microcellular systems operating in 1800-2000 MHz frequency bands, the three common used path loss models are
  - COST231-Hata model (Macrocellular)
  - COST231-Walfish-Ikegami model (Macrocellular/Microcellular)
  - Two-slope model (Microcellular)

COST231-Hata Model

- Extend Okumura-Hata model for 1500-2000 MHz range
- f_c: 1500~2000 MHz, d: 1~20 Km, h_b: 30~200m, h_m: 1~10m

$$L_{p(\text{dB})} = A + B\log_{10}(d) + C$$

Okumura-Hata's Model
$$A = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$

$$A = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_b) - a(h_m)$$
$$B = 44.9 - 6.55\log_{10}(h_b)$$
$$C = \begin{cases} 0 & \text{medium city and suburban areas with moderate tree density} \\ 3 & \text{for metropolitan centers} \end{cases}$$

- Good accuracy for a path length larger than 1 km
- Should not be used for smaller ranges
  - The path loss becomes **highly dependent** upon the local topography

COST231-Walfish-Ikegami Model

- f_c: 800~2000 MHz, d: 0.02~5 km, h_b: 4~50 m, h_m: 1~3 m
- The model can be applied for the cases that BS antennas are either **above or below** the roof tops
- The model is not very accurate when the BS antennas are **about the same height** as the roof tops
- The path loss for **LOS propagation** in a street canyon:

$$L_p = \underline{42.6} + 26\log_{10}(d) + 20\log_{10}(f_c), \quad d \geq 20\text{m}$$

  - First constant: free-space path loss at a distance of 20 m
  - Parameters: distance d (km) and carrier frequency f_c (MHz)

COST231-Walfish-Ikegami Model

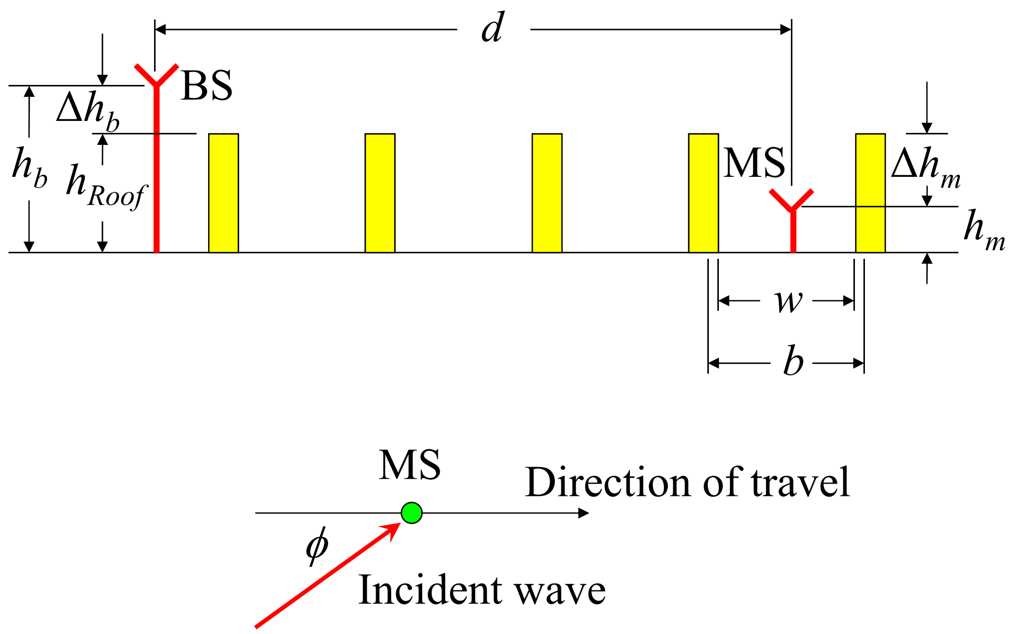
- The path loss for **non-LOS propagation**:

$$L_{p(\mathrm{dB})} = \begin{cases} L_o + L_{rts} + L_{msd} & \text{for } L_{rts} + L_{msd} \geq 0 \\ L_o & \text{for } L_{rts} + L_{msd} < 0 \end{cases}$$

- L_p is expressed in terms of the following parameters:

h_b = BS antenna height, $4 \leq h_b \leq 50$ (m)
h_m = MS antenna height, $1 \leq h_m \leq 3$ (m)
h_{Roof} = roof heights of buildings (m)
$\Delta h_b = h_b - h_{Roof}$ = height of BS relative to rooftops (m)
$\Delta h_m = h_{Roof} - h_m$ = height of MS relative to rooftops (m)
w = width of streets (m)
b = building separation (m)
ϕ = road orientation with respect to the direct radio path (degree)

COST231-Walfish-Ikegami Model



COST231-Walfish-Ikegami Model

– where L_o is the **free-space loss**

$$L_o = 32.4 + 20\log_{10}(d) + 20\log_{10}(f_c)$$

– L_{rts} is the **roof-to-street** diffraction and scatter loss

$$L_{rts} = -16.9 - 10\log_{10}(w) + 10\log_{10}(f_c) + 20\log_{10}\Delta h_m + L_{ori}$$

$$L_{ori} = \text{orientation loss} = \begin{cases} -10 + 0.354(\phi), & 0 \le \phi \le 35^\circ \\ 2.5 + 0.075(\phi - 35^\circ), & 35^\circ \le \phi \le 55^\circ \\ 4.0 - 0.114(\phi - 55^\circ), & 55^\circ \le \phi \le 90^\circ \end{cases}$$

– L_{msd} is the **multi-screen diffraction** loss

$$L_{msd} = L_{bsh} + k_a + k_d \log_{10}(d) + k_f \log_{10}(f_c) - 9\log_{10}(b)$$

$$L_{bsh} = \text{shadowing gain} = \begin{cases} -18\log_{10}(1 + \Delta h_b) & h_b > h_{Roof} \\ 0 & h_b \le h_{Roof} \end{cases}$$

COST231-Walfish-Ikegami Model

$$k_a = \begin{cases} 54, & h_b > h_{Roof} \\ 54 - 0.8\Delta h_b, & d \ge 0.5 \text{ km and } h_b \le h_{Roof} \\ 54 - 0.8\Delta h_b d / 0.5, & d < 0.5 \text{ km and } h_b \le h_{Roof} \end{cases}$$

$$k_d = \begin{cases} 18, & h_b > h_{Roof} \\ 18 - 15\Delta h_b / h_{Roof}, & h_b \le h_{Roof} \end{cases}$$

$$k_f = -4 + \begin{cases} 0.7(f_c / 925 - 1), & \text{medium city and suburban} \\ 1.5(f_c / 925 - 1), & \text{metropolitan area} \end{cases}$$

- If no data for buildings and roads:
  - b = 20~50 m, $w = b/2$, $\phi = 90^\circ$ and h_{Roof} = 3 × number of floors + roof (m), where roof = 3 (m) (pitched) or 0 (m) (flat)
- The model is best for $h_b >> h_{Roof}$
- The model is poor for $h_b << h_{Roof}$ and $h_b \approx h_{Roof}$

Two-Slope Model (Street Microcells)

- For a range less than 500m and the antenna height less than 20m

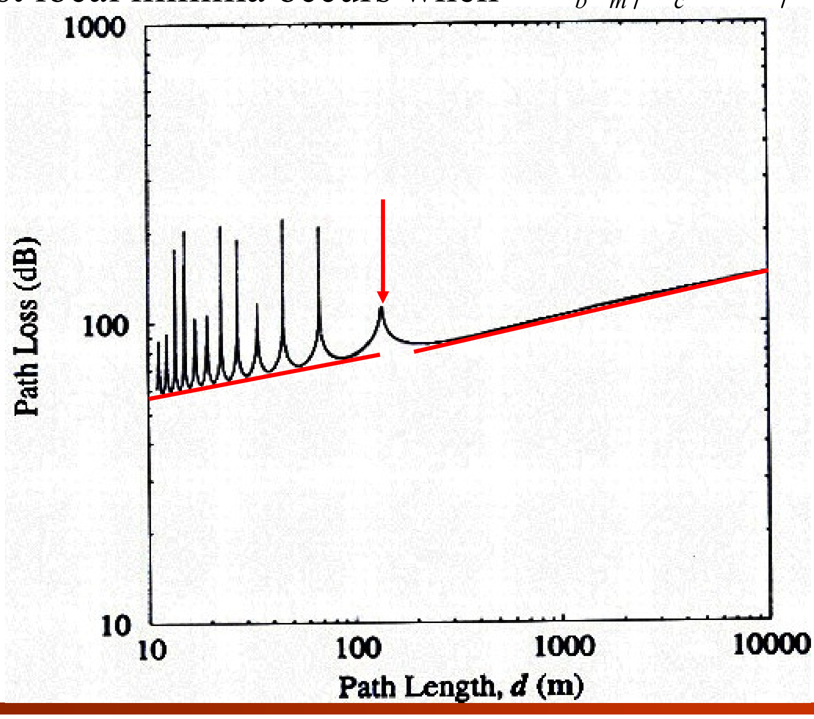
$$\mu_{\Omega_p} = \frac{k\Omega_t}{d^a(1+d/g)^b}$$

$$\begin{aligned}
\mu_{\Omega_p} &= 10\log_{10}(k\Omega_t) - 10\log_{10}(d^a(1+d/g)^b) \quad \text{(dBm)} \\
&= 10\log_{10}(k\Omega_t) - 10\log_{10} g^a - 10\log_{10}(d/g)^a - 10\log_{10}(1+d/g)^b \\
&= 10\log_{10}(k\Omega_t) - 10a\log_{10} g - 10a\log_{10}(d/g) - 10b\log_{10}(1+d/g) \\
&\approx \begin{cases} 10\log_{10}(k\Omega_t) - 10a\log_{10} d, & \text{if } d \ll g \\ 10\log_{10}(k\Omega_t) - 10a\log_{10} g - 10(a+b)\log_{10}(d/g), & \text{if } d \gg g \end{cases}
\end{aligned}$$

- When close into the BS: free-space propagation $\Rightarrow a = 2$
- At larger distance: inverse-fourth power law $\Rightarrow b = 2$

Two-ray Path Loss Models

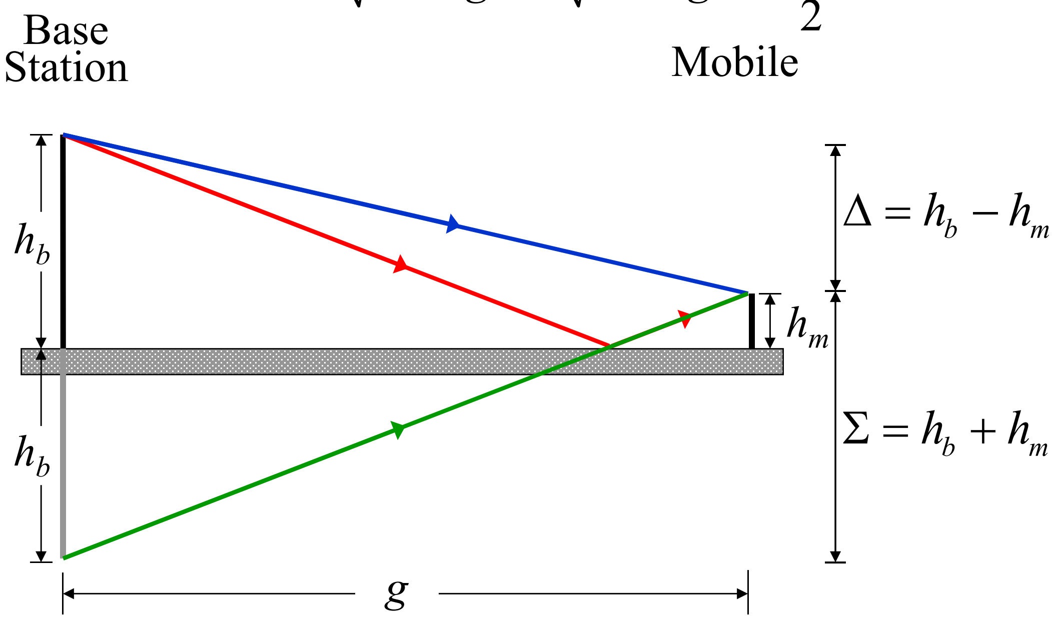
- The last local minima occurs when $2\pi h_b h_m / \lambda_c d = \pi/2$



Two-ray Path Loss Models

- The first Fresnel Zone

$$\Delta d = \sqrt{\Sigma^2 + g^2} - \sqrt{\Delta^2 + g^2} = \frac{\lambda_c}{2}$$



Two-Slope Model (Street Microcells)

- We set $\Sigma = h_b + h_m$ and $\Delta = h_b - h_m$
- Find the break-point g

$$\sqrt{\Sigma^2 + g^2} - \sqrt{\Delta^2 + g^2} = \frac{\lambda_c}{2}$$

$$\sqrt{\Sigma^2 + g^2} = \sqrt{\Delta^2 + g^2} + \frac{\lambda_c}{2}$$

$$\Sigma^2 + g^2 = \Delta^2 + g^2 + \lambda_c\sqrt{\Delta^2 + g^2} + (\tfrac{\lambda_c}{2})^2$$

$$\left[\Sigma^2 - \Delta^2 - (\tfrac{\lambda_c}{2})^2\right]^2 = (\lambda_c)^2(\Delta^2 + g^2)$$

$$(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2)(\tfrac{\lambda_c}{2})^2 + (\tfrac{\lambda_c}{2})^4 = (\lambda_c)^2 g^2$$

$$g = \frac{1}{\lambda_c}\sqrt{(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2)(\tfrac{\lambda_c}{2})^2 + (\tfrac{\lambda_c}{2})^4}$$

Two-Slope Model (Street Microcells)

- For conventional environments, break point g = 150 ~ 300m

$$g = \frac{1}{\lambda_c}\sqrt{\left(\Sigma^2 - \Delta^2\right)^2 - 2(\Sigma^2 + \Delta^2)\left(\tfrac{\lambda_c}{2}\right)^2 + \left(\tfrac{\lambda_c}{2}\right)^4}$$

- For high frequency ($\lambda_c^2 << (\Sigma^2 - \Delta^2)^2$):

$$g \approx \frac{1}{\lambda_c}\sqrt{\left(\Sigma^2 - \Delta^2\right)^2} = \frac{\Sigma^2 - \Delta^2}{\lambda_c} = \frac{4h_b h_m}{\lambda_c}$$

Different environments and different MS antenna heights

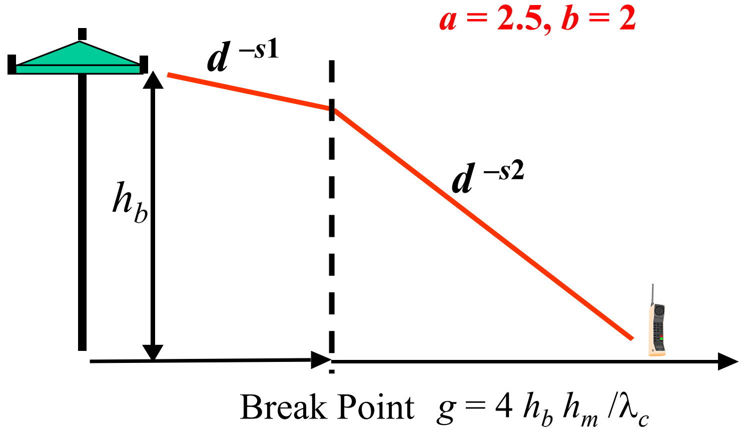
BS Antenna Height (m)	a	b	g (m)
5	2.30	-0.28	148.6
9	1.48	0.54	151.8
15	0.40	2.10	143.9
19	-0.96	4.72	158.3

Two-Slope Model (Street Microcells)

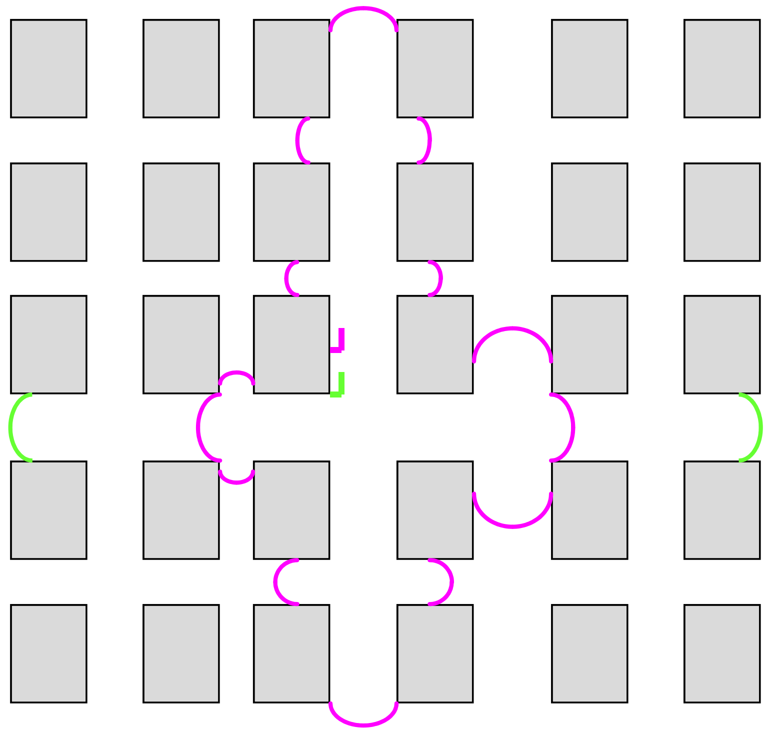
- JTC model (microcell model)
  - $d_{bp} = (4\, h_b\, h_m)/\lambda_c$, (break point)

$$L_{P(\mathrm{dB})} = \begin{cases} 38.1 + 25\log_{10}(d), & d < d_{bp} \\ 38.1 + 25\log_{10}(d_{bp}) + 45\log_{10}(d/d_{bp}), & d > d_{bp} \end{cases}$$

***a* = 2.5, *b* = 2**



Street Microcells Coverage

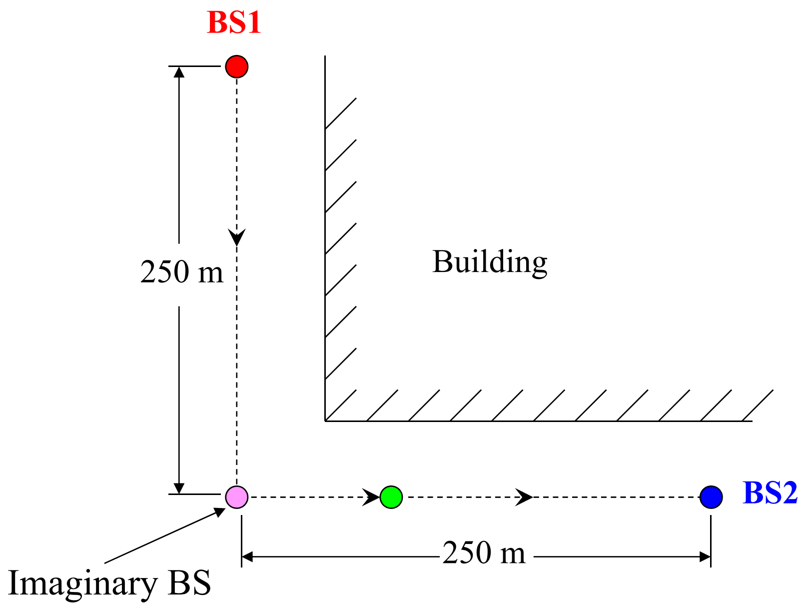


Corner Effect (Street Microcells)

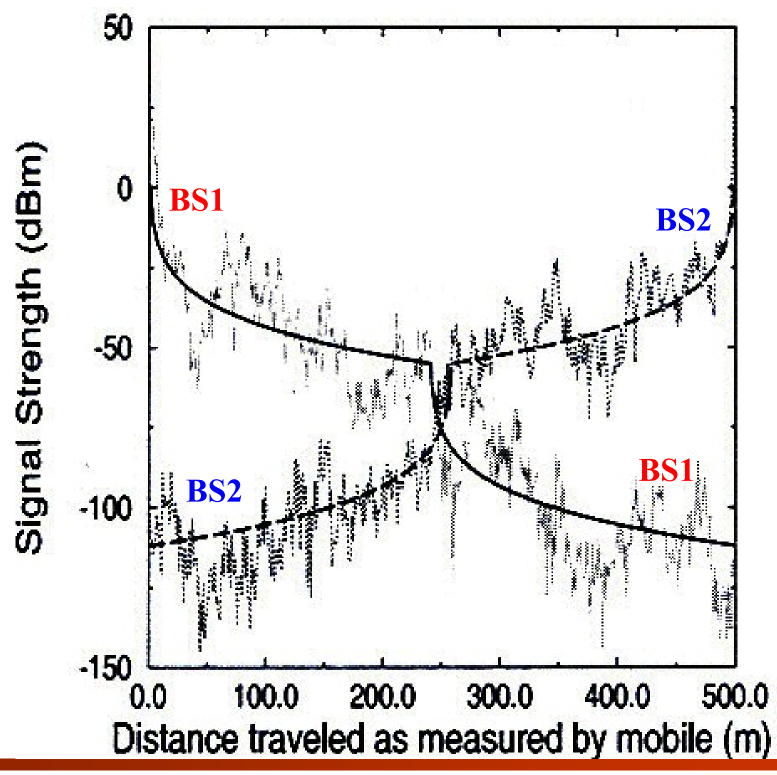
- Corner Effect: street microcells with NLOS propagation
  - The average received signal strength drops by **25~30 dB** over a distance as small as 10 m to 50 m
- The NLOS propagation is modeled as:
  - A LOS propagation from an **virtual transmitter** located at corner
  - The transmit power is equal to the received power at corner from BS

$$\mu_{\Omega_p} = \begin{cases} \dfrac{k\Omega_t}{d^a(1+d/g)^b}, & d \le d_c \\ \dfrac{k\Omega_t}{d_c^a(1+d_c/g)^b} \cdot \dfrac{1}{(d-d_c)^a(1+(d-d_c)/g)^b}, & d > d_c \end{cases}$$

Corner Effect (Street Microcells)



Corner Effect (Street Microcells)



Path Loss in Indoor Microcells

- The path loss and shadowing characteristics for indoor environments **vary greatly** from one building to another

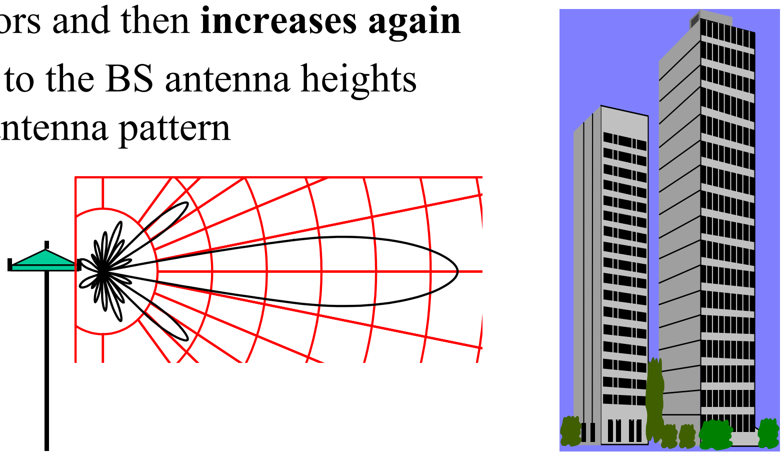
Building	Frequency (MHz)	β	σ_Ω (dB)
Retail Stores	914	2.2	8.7
Grocery Stores	914	1.8	5.2
Office, hard partition	1500	3.0	7.0
Office, soft partition	900	2.4	9.6
Office, soft partition	1900	2.6	14.1

- Floor loss:
  - One floor: 15 ~ 20 dB
  - Up to 4 floors: additional 6 ~ 10 dB/floor
  - 5 or more floors: increase only a few dB for each additional floor

Path Loss in Indoor Microcells

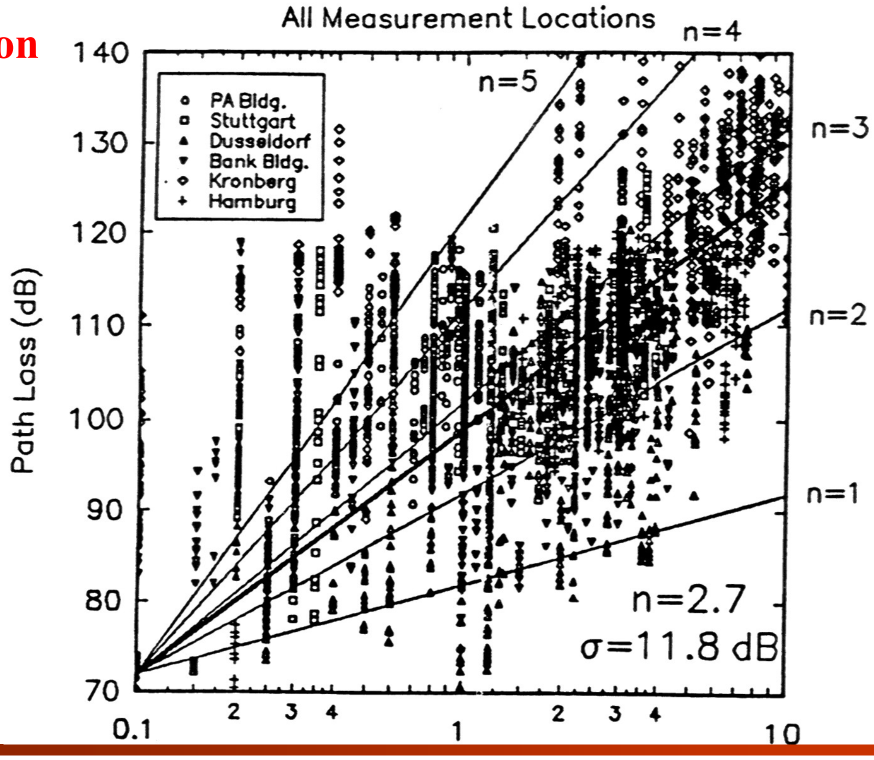
- Building penetration loss:
  - Decreases with the increase in frequency
  - Typical values: 16.4, 11.6 and 7.6 dB at 441 MHz, 896.5 MHz and 1400 MHz
  - Decreases by about 2 dB/floor from ground level up to about 9~15 floors and then **increases again**

  $\Rightarrow$ It is due to the BS antenna heights and the antenna pattern



Path Loss & Shadowing

Question



Some Stochastic Channel Models

Stochastic Channel Model (SCM)

- **SCM** is a **parametric model** for the **delay spread functions**
- **Requirements for SCMs:**
  - **Completeness**: SCMs must reproduce all effects that impact on the performance of communication systems
  - **Accuracy**: SCMs must accurately describe these effects.
  - **Simplicity/low complexity**: Each effect must be described by a simple model.
- Good **SCMs** can
  - Guarantee simulation scenarios close to reality
  - Enable theoretical study of some particular system aspects and performance
  - Be used to simulate the channel in Monte Carlo simulations with acceptable computational effort

COST 207 Channel Models

Channel Models Proposed by COST

- **COST**: European COoperation in Science and Technology
- **COST-207**: Digital Land Mobile Radiocommunications (1988)
  - Channel models for GSM 900 systems
- **COST-231**: Evolution of Land Mobile Radio (including personal) Communication (1996)
  - Channel models for GSM 1800 systems
- **COST-259**: Wireless Flexible Personalized Communications (2000)
  - Channel models for DECT, UMTS and HIPERLAN 2
- **COST-273**: Towards Mobile Broadband Multimedia Networks (2005)
  - Channel models for UMTS and WLAN
  - **MIMO** channel models

COST 207 Channel Models

- Normalized delay-Doppler scattering function:

$$S_n(\tau,\nu) \triangleq S(\tau,\nu)/P \Rightarrow \int S_n(\tau,\nu)\, d\tau\, d\nu = 1$$

  - where P is the total received power
- We can decompose $S_n(\tau,\nu)$ as

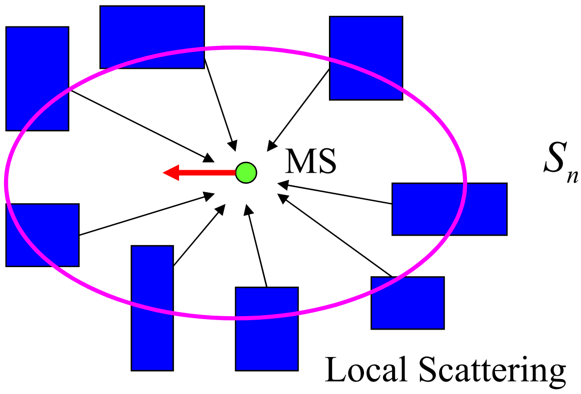
$$S_n(\tau,\nu) = \underbrace{S_n(\tau)}_{\text{Normalized delay scattering function}} \times \underbrace{S_n(\nu|\tau)}_{\text{Delay-dependent normalized Doppler scattering function}}$$

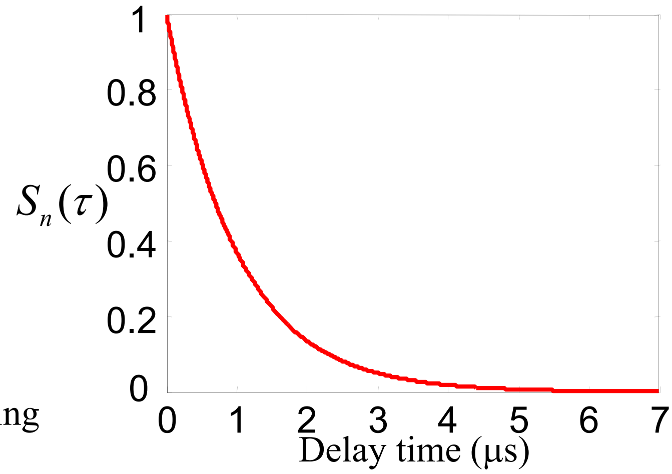
- The COST 207 models are specified by the two functions:
  - $S_n(\tau)$: scattering amplitude of the channel in terms of the time delay τ
  - $S_n(\nu|\tau)$: scattering amplitude of the channel in terms of Doppler frequency ν, given the time delay τ

COST 207 (Normalized Delay Scattering Fun.)

- **Typical urban non-hilly area (TU)**:

$$S_n(\tau) \propto \begin{cases} \exp(-\tau); & 0 \le \tau \le 7\mu s \\ 0; & \text{elsewhere} \end{cases}$$

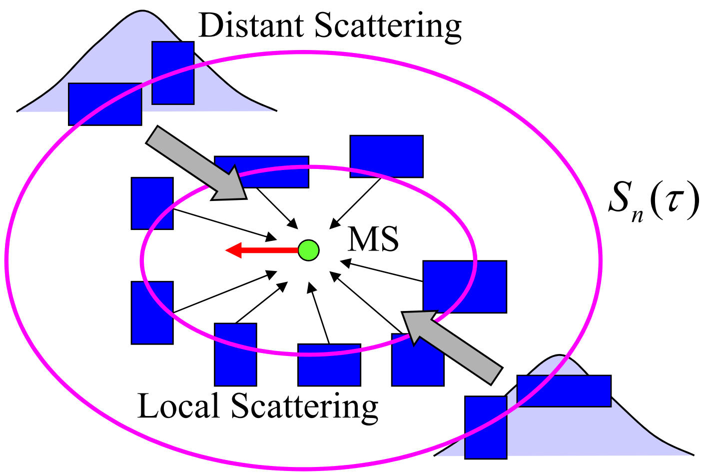


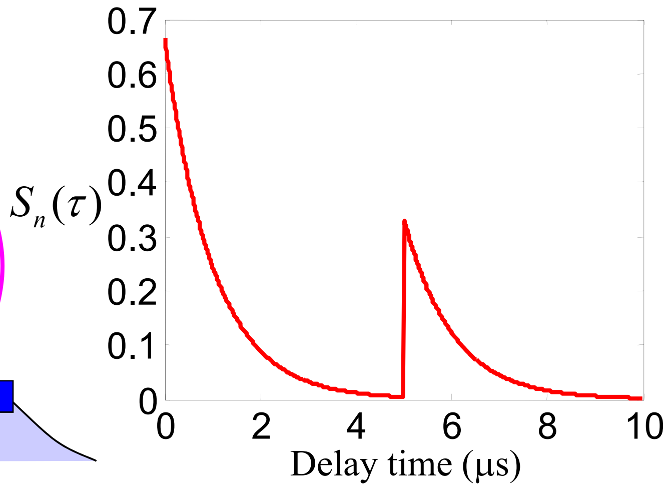


COST 207 (Normalized Delay Scattering Fun.)

- **Typical bad urban hilly area (BU)**:

$$S_n(\tau) \propto \begin{cases} \exp(-\tau); & 0 \le \tau \le 5\mu s \\ 0.5\exp(5-\tau); & 5\mu s \le \tau \le 10\mu s \\ 0; & \text{elsewhere} \end{cases}$$

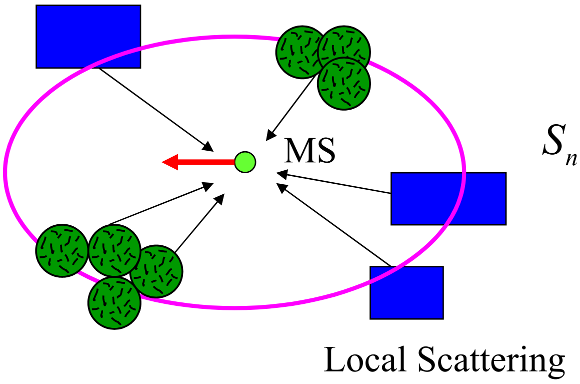


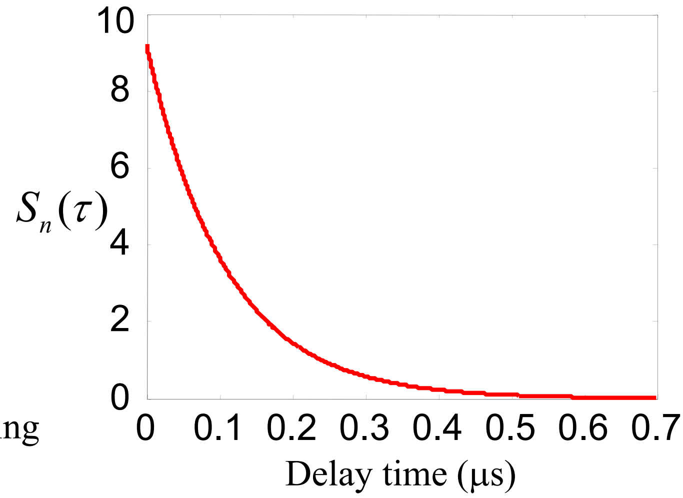


COST 207 (Normalized Delay Scattering Fun.)

- **Typical rural non-hilly area (RA)**:

$$S_n(\tau) \propto \begin{cases} \exp(-9.2\tau); & 0 \le \tau \le 0.7\mu s \\ 0; & \text{elsewhere} \end{cases}$$

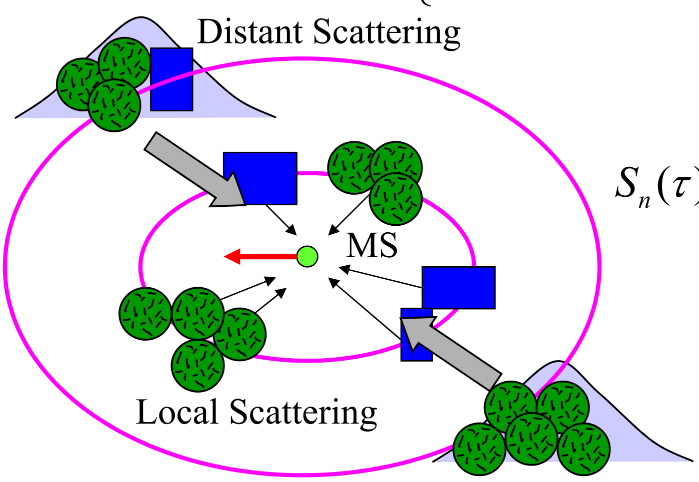


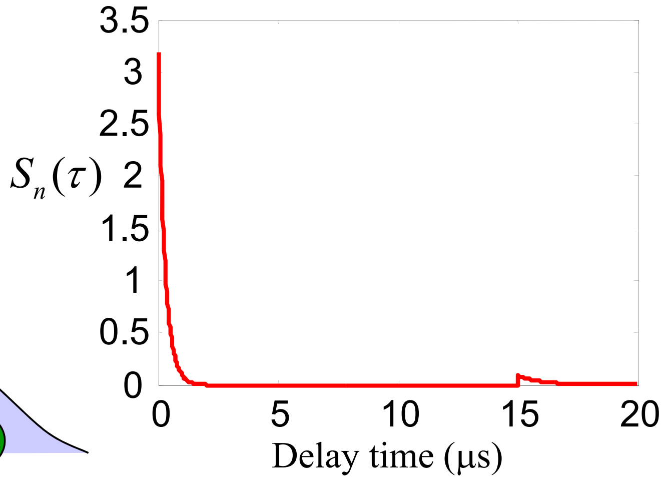


COST 207 (Normalized Delay Scattering Fun.)

- **Typical hilly terrain (HT)**:

$$S_n(\tau) \propto \begin{cases} \exp(-3.5\tau); & 0 \le \tau \le 2\mu s \\ 0.1\exp(15-\tau); & 15\mu s \le \tau \le 20\mu s \\ 0; & \text{elsewhere} \end{cases}$$

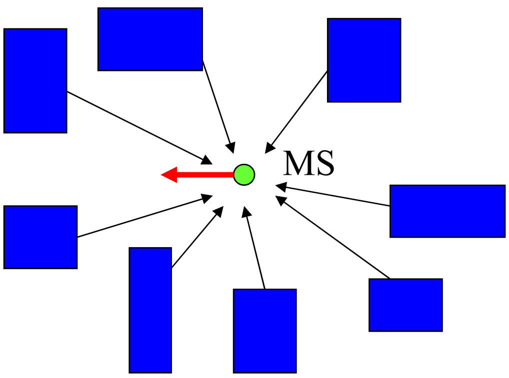


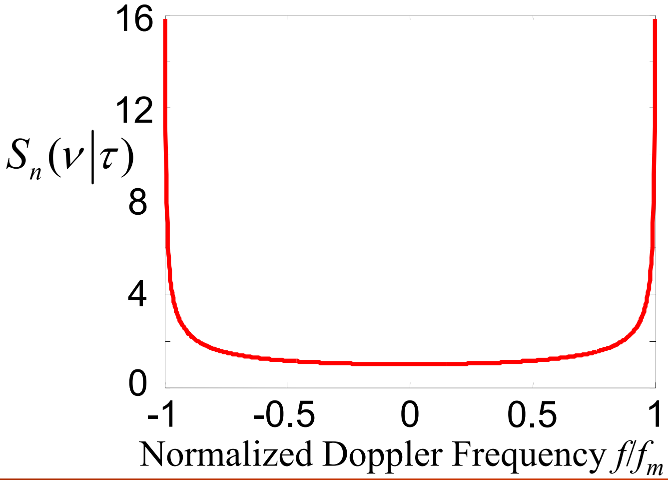


COST 207 (Normalized Doppler Scattering Fun.)

- **Classical Doppler spectrum (CLASS)**: isotropic scattering ($\tau \le 0.5\mu s$)

$$S_n(\nu|\tau) \propto \begin{cases} \dfrac{1}{\pi f_m} \times \dfrac{1}{\sqrt{1-(\nu/f_m)^2}}; & |\nu| \le f_m \\ 0; & \text{elsewhere} \end{cases}$$



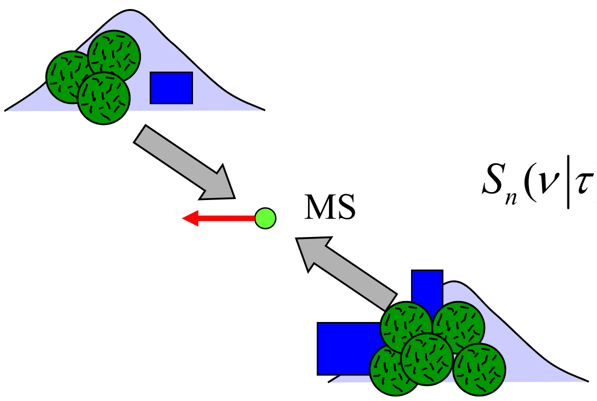


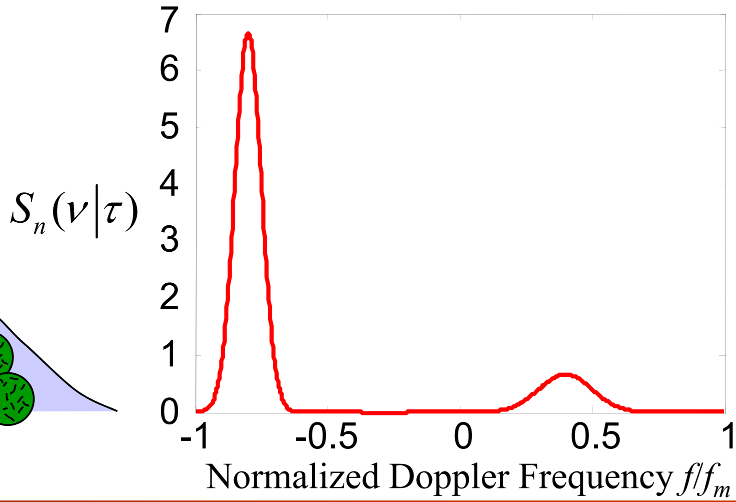
COST 207 (Normalized Doppler Scattering Fun.)

- **Gaussian 1 Doppler spectrum (GAUS1)**: non-isotropic scattering ($0.5\mu s \le \tau \le 2\mu s$)

$$S_n(\nu|\tau) \propto G(\nu; a_1, -0.8 f_m, 0.05 f_m) + G(\nu; a_2, 0.4 f_m, 0.1 f_m)$$

$$G(\nu; a, \nu_1, \nu_2) = a \times \exp(-(\nu-\nu_1)^2 / 2\nu_2^2), \quad a_2/a_1 = -10 \text{ dB}$$



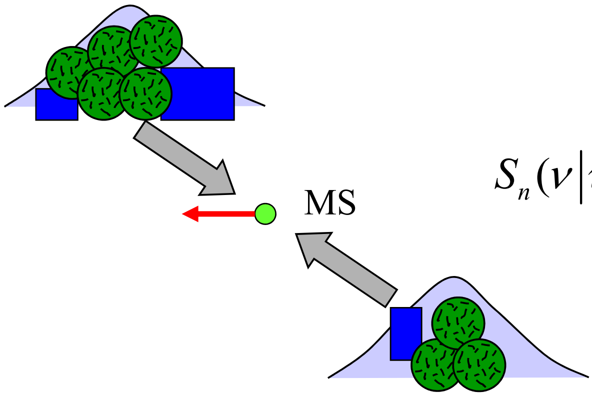


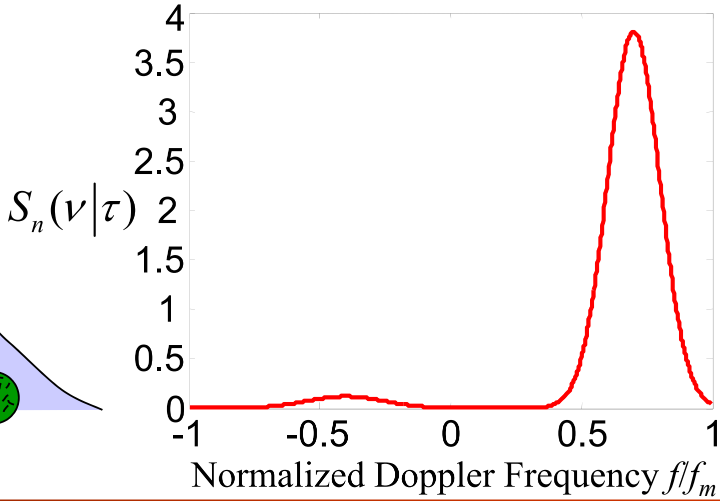
COST 207 (Normalized Doppler Scattering Fun.)

- **Gaussian 2 Doppler spectrum (GAUS2)**: non-isotropic scattering (2μs ≤ τ)

$$S_n(\nu|\tau) \propto G(\nu; a_1, 0.7f_m, 0.1f_m) + G(\nu; a_2, -0.4f_m, 0.15f_m)$$

$$G(\nu; a, \nu_1, \nu_2) = a \times \exp(-(\nu - \nu_1)^2 / 2\nu_2^2), \quad a_2/a_1 = -15 \text{ dB}$$

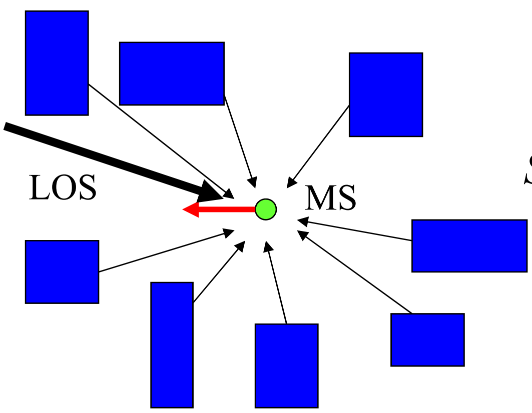


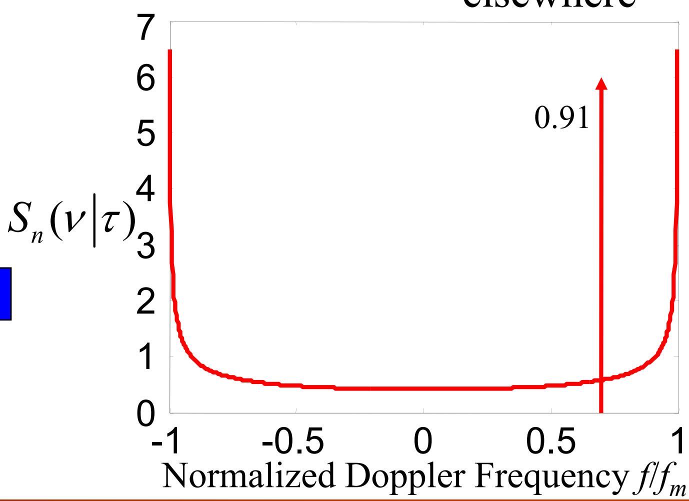


COST 207 (Normalized Doppler Scattering Fun.)

- **RICE Doppler spectrum (RICE)**: isotropic scattering with LOS

$$S_n(\nu|\tau) \propto \begin{cases} \dfrac{0.41}{2\pi f_m} \times \dfrac{1}{\sqrt{1-(\nu/f_m)^2}} + 0.91\delta(\nu - 0.7f_m); & |\nu| \le f_m \\ 0; & \text{elsewhere} \end{cases}$$





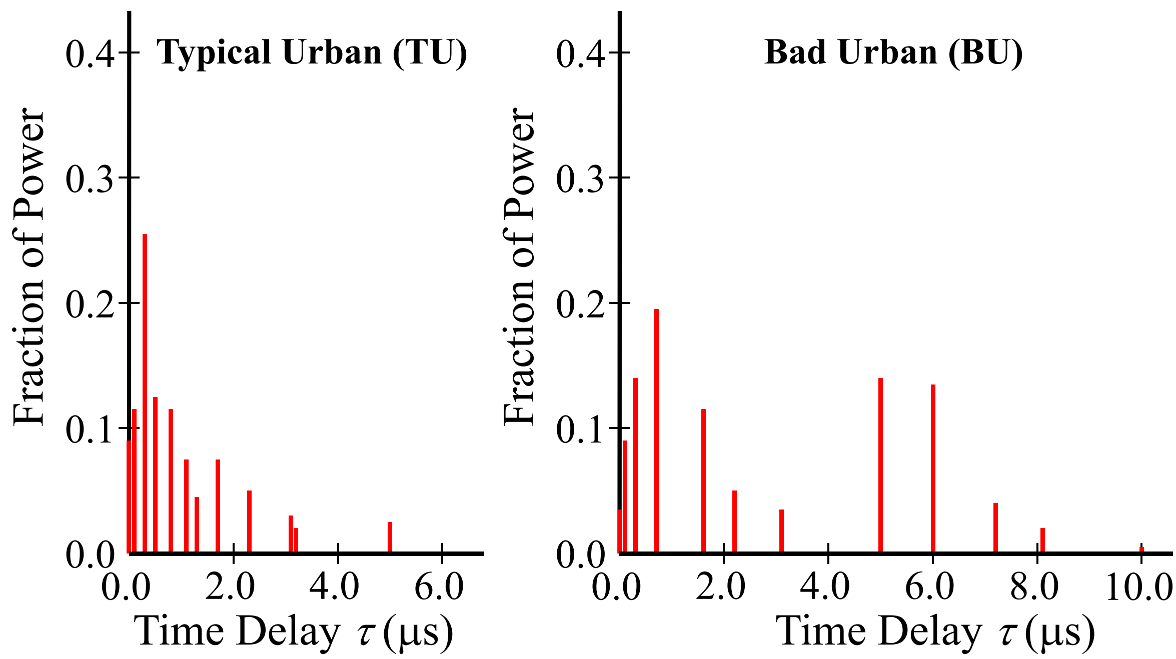
COST 207 Channel Models

- **Typical urban (TU)** ($\sigma_\tau = 1.0$ μs) and **bad urban (BU)** ($\sigma_\tau = 2.5$ μs) power delay profiles

Typical Urban (TU)			Bad Urban (BU)		
Delay (μs)	Fractional Power	Doppler	Delay (μs)	Fractional Power	Doppler
0.0	0.092	CLASS	0.0	0.033	CLASS
0.1	0.115	CLASS	0.1	0.089	CLASS
0.3	0.231	CLASS	0.3	0.141	CLASS
0.5	0.127	CLASS	0.7	0.194	GAUS1
0.8	0.115	GAUS1	1.6	0.114	GAUS1
1.1	0.074	GAUS1	2.2	0.052	GAUS2
1.3	0.046	GAUS1	3.1	0.035	GAUS2
1.7	0.074	GAUS1	5.0	0.140	GAUS2
2.3	0.051	GAUS2	6.0	0.136	GAUS2
3.1	0.032	GAUS2	7.2	0.041	GAUS2
3.2	0.018	GAUS2	8.1	0.019	GAUS2
5.0	0.025	GAUS2	10.0	0.006	GAUS2

COST 207 Channel Models

- **12-rays**



COST 207 Channel Models

- Typical rural (non-hilly) area (RA) ($\sigma_\tau = 0.1$ μs)

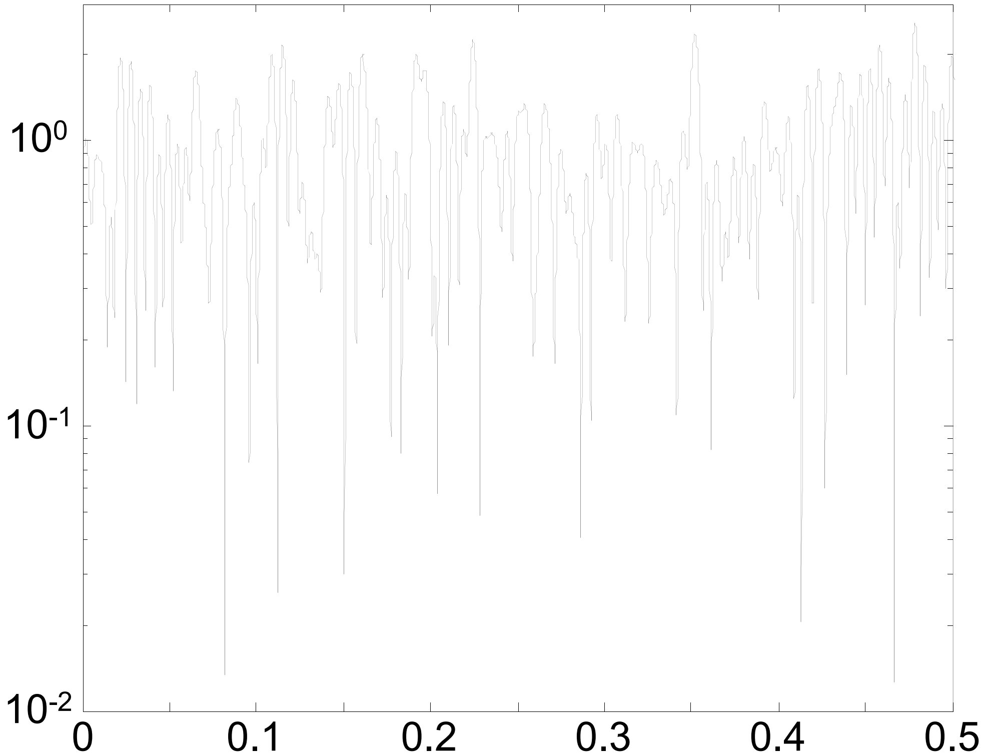
Typical rural (non-hilly) area (RA)		
Delay (μs)	Fractional Power	Doppler
0.0	0.602	RICE
0.1	0.241	CLASS
0.2	0.096	CLASS
0.3	0.036	CLASS
0.4	0.018	CLASS
0.5	0.006	CLASS

COST 207 Channel Models

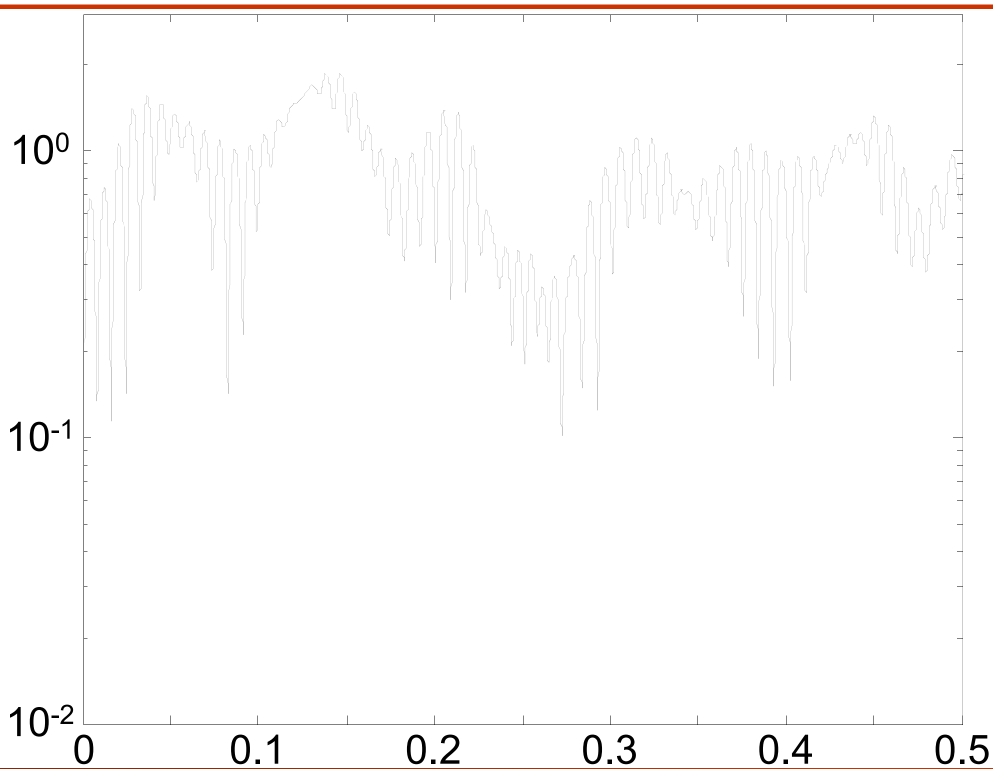
- Typical hilly terrain (HT) ($\sigma_\tau = 5.0$ μs)

Typical hilly terrain (HT)			Reduced hilly terrain (HT)		
Delay (μs)	Fractional Power	Doppler	Delay (μs)	Fractional Power	Doppler
0.0	0.026	CLASS	0.0	0.413	CLASS
0.1	0.042	CLASS	0.1	0.293	CLASS
0.2	0.066	CLASS	0.3	0.145	CLASS
0.3	0.105	CLASS	0.5	0.074	CLASS
0.4	0.263	GAUS1	15.0	0.066	GAUS2
0.5	0.263	GAUS1	17.2	0.008	GAUS2
1.0	0.105	GAUS1			
1.1	0.042	GAUS2			
1.2	0.034	GAUS2			
1.3	0.026	GAUS2			
1.4	0.016	GAUS2			
1.5	0.011	GAUS2			

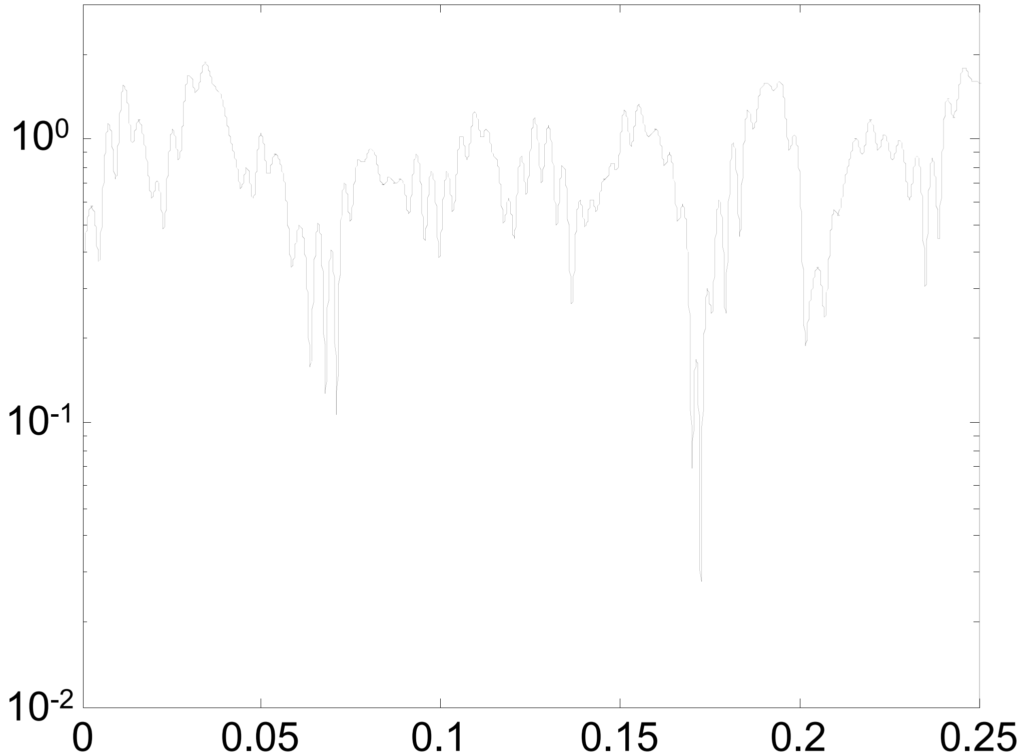
Time-domain Fading Gain – CLASS



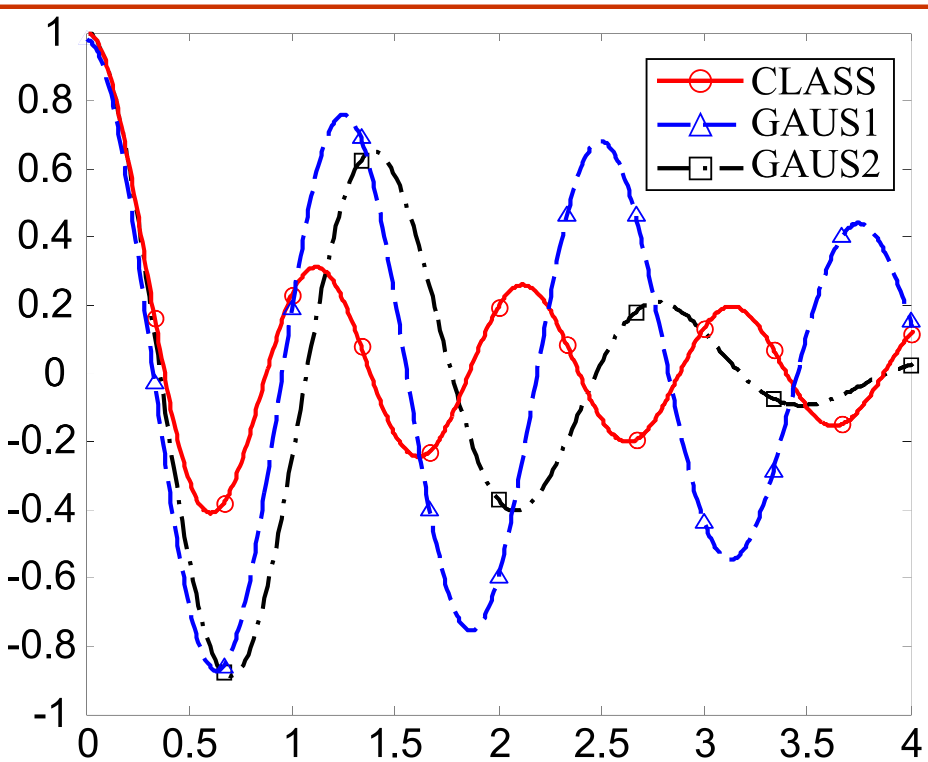
Time-domain Fading Gain – GAUS1



Time-domain Fading Gain – GAUS2



Comparison in Auto-Correlation



ITU Channel Models

ITU Channel Models

- **Recommendation ITU-R M.1225** – Guidelines for Evaluation of Radio Transmission Technologies (RTTs) for IMT-2000
- For the terrestrial environments, the propagation effects are divided into three distinct types of model.
  - **Mean path loss**
  - **Slow variation** about the mean due to shadowing and scattering
  - **Rapid variation** in the signal due to multipath effects
- The slow variation is considered to be log-normally distributed. This is described by the standard deviation.
- The rapid variation is characterized by the **channel impulse response**.

ITU Channel Models – Path Loss

- **Path loss model for indoor office test environment**
- The indoor path loss model (dB) is derived from the COST 231 indoor model.

$$L = 37 + 30\log_{10}R + 18.3n^{\left(\frac{n+2}{n+1} - 0.46\right)}$$

  - R: the transmitter-receiver separation (m)
  - n: the number of floors in the path
- L shall in no circumstances be less than **free space loss**.
- A log-normal shadow fading standard deviation of **12 dB** can be expected.

ITU Channel Models – Path Loss

- **Path loss model for outdoor to indoor and pedestrian test environment**

$$L = 40\log_{10}R + 30\log_{10}f + 49$$

  - R: the base station – mobile station separation (km)
  - f: carrier frequency of 2000 MHz for IMT-2000 band application
- L shall in no circumstances be less than **free space loss**.
- This model is valid for **non-line-of-sight (NLOS)** case only.
- Log-normal shadow fading with a standard deviation of **10 dB** for outdoor users and **12 dB** for indoor users is assumed.
- The average building penetration loss is **12 dB** with a standard deviation of **8 dB**.

ITU Channel Models – Path Loss

- **Path loss model for vehicular test environment**
- This model is applicable for the test scenarios in **urban** and **suburban** areas outside the high rise core

$$L = 40(1 - 4\times 10^{-3}\Delta h_b)\log_{10} R - 18\log_{10}\Delta h_b + 21\log_{10} f + 80\text{ dB}$$

  - R: the base station – mobile station separation (km)
  - f: carrier frequency of 2000 MHz for IMT-2000 band application
  - Δh_b: the base station antenna height (m), **measured from the average rooftop level**
- L shall in no circumstances be less than **free space loss**.
- This model is valid for **NLOS** case only.
- Log-normal shadow fading with std. 10 dB is assumed.
- The path loss model is valid for a range of Δh_b from **0 to 50 m**.

ITU Channel Models – Shadowing

- **Decorrelation length of the long-term fading**
- Due to the slow fading process versus distance Δx, adjacent fading values are correlated.
- Its normalized autocorrelation function $R(\Delta x)$ can be described by an exponential function

$$R(\Delta x) = \exp\left(-\frac{|\Delta x|}{d_{cor}}\ln 2\right)$$

  - d_{cor}: is the decorrelation length, which is dependent on the environment
  - d_{cor} is assumed to be 20 m

ITU Channel Models – Multipath Fading

- A channel impulse response model based on a tapped-delay line model is given. The model is characterized by
  - The **number of taps**
  - The **time delay** relative to the first tap
  - The **average power** relative to the strongest tap
  - The **Doppler spectrum** of each tap
- **Channel A** is the low delay spread case
- **Channel B** is the median delay spread case

The percentage of time the particular channel may be encountered with the associated r.m.s. delay

Test environment	Channel A		Channel B	
	r.m.s. (ns)	*P* (%)	r.m.s. (ns)	*P* (%)
Indoor office	35	50	100	45
Outdoor to indoor and pedestrian	45	40	750	55
Vehicular – high antenna	370	40	4 000	55

ITU Channel Models – Multipath Fading

- For each tap of the channels three parameters are given: the **time delay** relative to the first tap, the **average power** relative to the strongest tap, and the **Doppler spectrum** of each tap.
- **Indoor office test environment** tapped-delay-line parameters

Tap	Channel A		Channel B		Doppler spectrum
	Relative delay (ns)	Average power (dB)	Relative delay (ns)	Average power (dB)	
1	0	0	0	0	Flat
2	50	–3.0	100	–3.6	Flat
3	110	–10.0	200	–7.2	Flat
4	170	–18.0	300	–10.8	Flat
5	290	–26.0	500	–18.0	Flat
6	310	–32.0	700	–25.2	Flat

ITU Channel Models – Multipath Fading

- **Outdoor to indoor and pedestrian test environment** tapped-delay-line parameters

	Channel A		Channel B		Doppler spectrum
Tap	Relative delay (ns)	Average power (dB)	Relative delay (ns)	Average power (dB)	
1	0	0	0	0	Classic
2	110	–9.7	200	–0.9	Classic
3	190	–19.2	800	–4.9	Classic
4	410	–22.8	1200	–8.0	Classic
5	–	–	2300	–7.8	Classic
6	–	–	3700	–23.9	Classic

ITU Channel Models – Multipath Fading

- **Vehicular test environment, high antenna,** tapped-delay-line parameters

	Channel A		Channel B		Doppler spectrum
Tap	Relative delay (ns)	Average power (dB)	Relative delay (ns)	Average power (dB)	
1	0	0.0	0	–2.5	Classic
2	310	–1.0	300	0	Classic
3	710	–9.0	8900	–12.8	Classic
4	1090	–10.0	12900	–10.0	Classic
5	1730	–15.0	17100	–25.2	Classic
6	2510	–20.0	20000	–16.0	Classic

IEEE 802.16 Broadband Wireless Access Working Group

(Channel Models for Fixed Wireless Applications)

IEEE 802.16 Broadband Wireless Access

- **Channel Models for Fixed Wireless Applications** (2003)
  - A set of propagation models applicable to the multi-cell architecture with non-line-of-sight (NLOS) conditions is presented.
- Typically, the scenario is as follows:
  - Cells are < 10 km in radius
  - Under-the-eave/window or rooftop installed **directional** antennas (2 – 10 m) at the receiver
  - 15 – 40 m BTS antennas
  - High cell coverage requirement (80-90%)
- The wireless channel is characterized by:
  - Path loss (including shadowing), Multipath delay spread, Fading characteristics, Doppler spread, Co-channel and adjacent channel interference

IEEE 802.16 CMs – Path Loss

- For a given close-in reference distance d_0, the path loss is

$$PL_{(\text{dB})} = A + 10\gamma \log_{10}(d/d_0) + s, \quad \text{for } d > d_0$$

$$A = 20\log_{10}(4\pi d_0/\lambda), \quad \gamma = a - bh_b + c/h_b, \quad d_0 = 100m$$

  - **Category A:** hilly terrain with moderate-to-heavy tree densities
  - **Category B:** Intermediate path loss condition
  - **Category C:** mostly flat terrain with light tree densities
  - s: the shadowing effect, which follows lognormal distribution with the std. ranged between **8.2** and **10.6 dB**.

Model parameter	Terrain Type A	Terrain Type B	Terrain Type C
a	4.6	4	3.6
b	0.0075	0.0065	0.005
c	12.6	17.1	20

IEEE 802.16 CMs – Path Loss

- The above path loss model is based on published literature for frequencies close to **2 GHz** and for receive antenna heights close to **2 m**.
- In order to use the model for other frequencies and for **receive antenna heights** between **2 m and 10 m**, correction terms have to be included.

$$PL_{\text{modified}} = PL + \Delta PL_f + \Delta PL_h$$

  - The frequency (MHz) correction term: $\Delta PL_f = 6\log_{10}(f/2000)$
  - The receive antenna height correction term:
    - Categories A and B: $\Delta PL_h = -10.8\log_{10}(h_b/2)$
    - Category C: $\Delta PL_h = -20\log_{10}(h_b/2)$

IEEE 802.16 CMs – Multipath Delay Profile

- For directional antennas, the delay profile can be represented by **a spike-plus-exponential shape**. It is characterized by τ_{rms} (RMS delay spread) which is defined as

$$\tau_{\text{rms}} = \sqrt{\sum_j P_j \tau_j^2 - (\tau_{\text{avg}})^2}$$

- The delay profile is given by

$$P(\tau) = A\delta(\tau) + B\sum_{i=0}^{\infty} \exp(-i\Delta\tau/\tau_0)\delta(\tau - i\Delta\tau)$$

  - where A, B and $\Delta\tau$ are experimentally determined
- The delay spread model is of the following form $\boxed{\tau_{\text{rms}} = T_1 d^{\varepsilon} y}$
  - where d is the distance in km, T_1 is the median value of τ_{rms} at d = 1 km, ε is an exponent that lies between 0.5-1.0, and y is a lognormal variate.
  - 32° and 10° **directive antennas** reduce the median τ_{rms} values for omni-directional antennas by factors of 2.3 and 2.6, respectively.

IEEE 802.16 CMs – Fading Characteristics

- The narrow band received signal fading can be characterized by a **Ricean** distribution.
- A model for estimating the K-factor (in linear scale) is

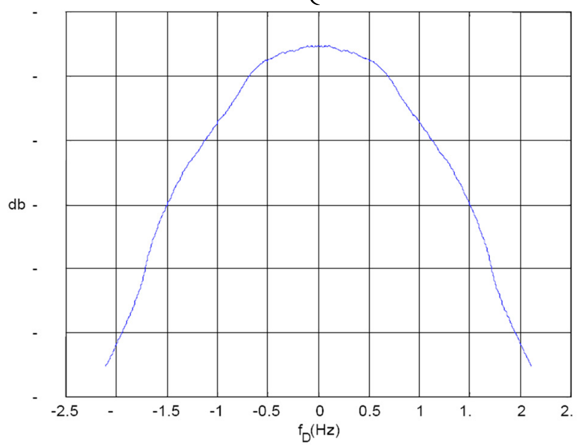
$$K = F_s F_h F_b K_o d^{\gamma} u$$

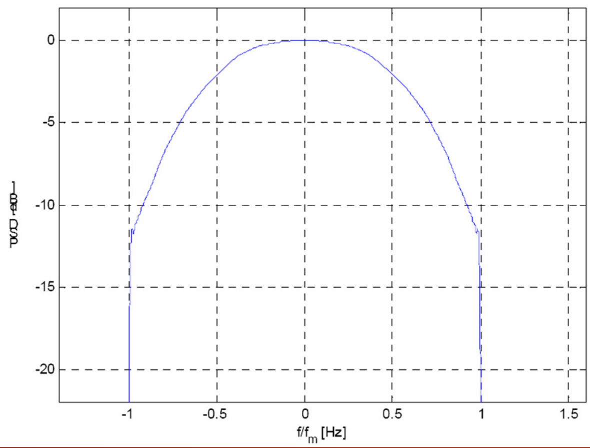
  - F_s is a **seasonal factor**, F_s = 1.0 in summer (leaves); 2.5 in winter (no leaves)
  - F_h is the receive antenna height factor, $F_h = (h/3)^{0.46}$ (h is the receive antenna height in meters)
  - F_b is the beam-width factor, $F_b = (b/17)^{-0.62}$ (b in degrees)
  - K_o and γ are regression coefficients, $K_o = 10$; $\gamma = -0.5$
  - u is a lognormal variable which has zero dB mean and a std. 8 dB

IEEE 802.16 CMs – Doppler Spectrum

- In fixed wireless channels the Doppler PSD of the scattering component is mainly distributed around $f = 0$ Hz.
  - A rounded shape is used as a rough approximation

$$S(f)=\begin{cases}1-1.72f_0^2+0.785f_0^4, & |f_0|\le 1\\ 0, & |f_0|>1\end{cases},\quad f_0=\frac{f}{f_m}$$





IEEE 802.16 CMs – Antenna Gain Reduction

- The gain due to the **directivity** can be reduced because of the **scattering**.
  - The effective gain is less than the actual gain.
- Denote ΔG_{BW} as the Antenna Gain Reduction Factor.
  - **Gaussian distributed** random variable (truncated at 0 dB, i.e., $\Delta G_{\mathrm{BW}} \ge 0$) with a mean ($\mu_{\mathrm{grf}}$) and std. ($\sigma_{\mathrm{grf}}$) given by

$$\mu_{\mathrm{grf}} = -(0.53+0.1I)\ln(\beta/360)+(0.5+0.04I)(\ln(\beta/360))^2$$
$$\sigma_{\mathrm{grf}} = -(0.93+0.02I)\ln(\beta/360)$$

  - β: the beam-width (in degrees); $I = 1$ (winter) or $I = -1$ (summer)
- In the link budget calculation, if G is the gain of the antenna (dB), the effective gain of the antenna equals $G - \Delta G_{\mathrm{BW}}$.
  - If a 20-degree antenna is used, the mean of $\Delta G_{\mathrm{BW}} \approx 7$ dB.

IEEE 802.16 CMs – Modified SUI CMs

- **Stanford University Interim (SUI)** channel models
- The parametric view of the SUI channels is summarized in the following tables.

SUI Channels	Terrain Type	Delay Spread	Doppler Spread	K-Factor
SUI-1	C	Low	Low	High
SUI-2	C	Low	Low	High
SUI-3	B	Low	Low	Low
SUI-4	B	Moderate	High	Low
SUI-5	A	High	Low	Low
SUI-6	A	High	High	Low

IEEE 802.16 CMs – SUI - 1 Channel Model

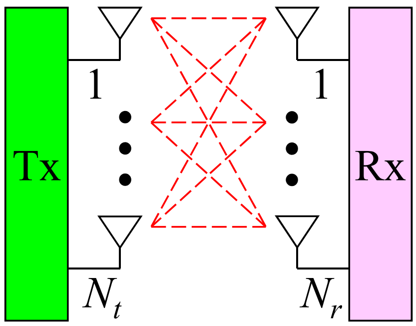
SUI - 1	Tap 1	Tap 2	Tap 3	Units
Delay	0	0.4	0.9	μs
Power (omni ant.) 90 % K-fact. 75 % K-fact.	0 4 20	-15 0 0	-20 0 0	dB
Power (30° ant.) 90 % K-fact. 75 % K-fact.	0 16 72	-21 0 0	-32 0 0	dB
Doppler	0.4	0.3	0.5	Hz

Antenna Correlation: $\rho_{ENV} = 0.7$
Gain Reduction Factor: $\Delta G_{BW} = 0$ dB
Normalization Factor:
$F_{omni} = -0.1771$ dB, $F_{30°} = -0.0371$ dB

Terrain Type: C
Omni antenna: $\tau_{RMS} = 0.111$ μs,
overall K: $K = 3.3$ (90%); $K = 10.4$ (75%)
30° antenna: $\tau_{RMS} = 0.042$ μs,
overall K: $K = 14.0$ (90%); $K = 44.2$ (75%)

MIMO Channel Models

MIMO Channel Models



- A MIMO (Multiple-Input and Multiple-Output) system is one that consists of multiple transmit and receive antennas.
- For a system consisting of N_t transmit and N_r receive antennas, the channel can be described by the $N_r \times N_t$ matrix.

$$\mathbf{G}(t,\tau)=\begin{bmatrix} g_{1,1}(t,\tau) & g_{1,2}(t,\tau) & \cdots & g_{1,N_t}(t,\tau) \\ g_{2,1}(t,\tau) & g_{2,2}(t,\tau) & \cdots & g_{2,N_t}(t,\tau) \\ \vdots & \vdots & & \vdots \\ g_{N_r,1}(t,\tau) & g_{N_r,2}(t,\tau) & \cdots & g_{N_r,N_t}(t,\tau) \end{bmatrix}$$

  – where $g_{q,p}(t,\tau)$ denotes the time-varying sub-channel impulse response between the pth transmit and qth receive antennas.

Analytical MIMO Channel Models

- **Analytical MIMO channel models** are most often used under quasi-static flat fading conditions.
- The time-variant channel impulses $g_{q,p}(t,\tau)$ for flat fading channels can be treated as complex Gaussian random processes under conditions of **Rayleigh** and **Ricean** fading.
- The various analytical models generate the MIMO matrices as realizations of complex Gaussian random variables having specified **means** and **correlations**.
- For Ricean fading, the channel matrix can be expressed as

$$\mathbf{G} = \sqrt{\frac{K}{K+1}}\bar{\mathbf{G}} + \sqrt{\frac{1}{K+1}}\mathbf{G}_S$$

  – $\bar{\mathbf{G}}$: is the LOS or specular component (a deterministic part)
  – $\mathbf{G}_S$: is the scatter component having zero-mean (a random part)

Analytical MIMO Channel Models

- The **simplest** MIMO model assumes that the entries of the matrix **G** are **independent and identically distributed (i.i.d.)** complex Gaussian random variables.
  – The **rich scattering** or **spatially white** environment.
  – It simplifies the performance analysis on MIMO channels.
  – However, **in reality** the sub-channels will be **correlated**, and the i.i.d. model will lead to optimistic results.
- Define $\mathbf{g} = \text{vec}\{\mathbf{G}\} = \left[\mathbf{g}_1^T, \mathbf{g}_2^T, \cdots, \mathbf{g}_{N_t}^T\right]^T$
  – where $\mathbf{G} = [\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_{N_t}]$ is the channel matrix
  – $\mathbf{g}$ is a zero-mean complex Gaussian random column vector of length $n = N_t \times N_r$
  – Its statistics are fully specified by the $n \times n$ covariance matrix $\mathbf{R_G} = \frac{1}{2} E[\mathbf{g}\mathbf{g}^H]$

Analytical MIMO Channel Models

- Hence, $\mathbf{g}$ is a multivariate complex Gaussian distributed vector
  - $\mathbf{g} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R_G})$
- If $\mathbf{R_G}$ is invertible, the probability density function of $\mathbf{g}$ is

$$f(\mathbf{g}) = \frac{1}{(2\pi)^n \det(\mathbf{R_G})} \exp\left[-\tfrac{1}{2}\mathbf{g}^H \mathbf{R_G}^{-1}\mathbf{g}\right], \quad \mathbf{g} \in C^n$$

- The covariance matrix $\mathbf{R_G}$ depends on the **propagation environments** and the **antenna configuration**.
- Given a covariance matrix $\mathbf{R_G}$, realizations of an MIMO channel can be generated by

$$\mathbf{G} = \text{unvec}\{\mathbf{g}\}, \quad \text{with } \mathbf{g} = \mathbf{R_G}^{1/2}\mathbf{w}$$

  - $\mathbf{R_G}^{1/2}$ is any matrix square root of $\mathbf{R_G}$; that is, $\mathbf{R_G} = \mathbf{R_G}^{1/2}(\mathbf{R_G}^{1/2})^H$
  - $\mathbf{w}$ is a length n vector where $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ (a white noise vector)

Kronecker Model

- The Kronecker model constructs the MIMO channel matrix $\mathbf{G}$ under the assumption that the spatial correlation at the **transmitter and receiver** is **separable**.
- This is equivalent to restricting the correlation matrix $\mathbf{R_G}$ to have the Kronecker product form $\mathbf{R_G} = \mathbf{R}_t \otimes \mathbf{R}_r$
  - where $\otimes$ is the Kronecker product
  - The $N_t \times N_t$ and $N_r \times N_r$ transmit and receive correlation matrices are $\mathbf{R}_t = \frac{1}{\sqrt{2}} E\left[\mathbf{G}^H\mathbf{G}\right], \quad \mathbf{R}_r = \frac{1}{\sqrt{2}} E\left[\mathbf{G}\mathbf{G}^H\right]$
- The Kronecker product of an $n \times n$ matrix $\mathbf{A}$ and an $m \times m$ matrix $\mathbf{B}$:

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{1,1}\mathbf{B} & \cdots & a_{1,n}\mathbf{B} \\ \vdots & \ddots & \vdots \\ a_{n,1}\mathbf{B} & \cdots & a_{n,n}\mathbf{B} \end{bmatrix}$$

Kronecker Model

- Given covariance matrices $\mathbf{R}_t$ and $\mathbf{R}_r$, realizations of an MIMO channel for the Kronecker model can be generated by

$$\mathbf{g} = \mathbf{R}_{\mathbf{G}}^{1/2}\mathbf{w} = (\mathbf{R}_t \otimes \mathbf{R}_r)^{1/2}\mathbf{w} \Rightarrow \underline{\mathbf{G} = \mathbf{R}_r^{1/2}\mathbf{W}\left(\mathbf{R}_t^{1/2}\right)^T}$$

  – where $\mathbf{W}$ is an $N_r \times N_t$ matrix consisting of i.i.d. zero mean complex Gaussian random variables (a white noise matrix)

- In general, the elements of the matrix $\mathbf{R}_\mathbf{G}$ represent correlations between the faded envelopes of the MIMO sub-channels:

$$\frac{1}{2}E\left[g_{q,p}g^*_{\tilde{q},\tilde{p}}\right] = \phi\left(q,p,\tilde{q},\tilde{p}\right)$$

  – A function of **four** sub-channel index parameters

- One important implication of the Kronecker property is **spatial stationarity**

$$\frac{1}{2}E\left[g_{q,p}g^*_{\tilde{q},\tilde{p}}\right] = \phi\left(q-\tilde{q},p-\tilde{p}\right)$$

Kronecker Model

- The **spatial stationarity** property implies that the sub-channel correlations are determined not by their position in the matrix **G**, but by their **difference in position**.
- In addition to the stationary property, the Kronecker product form also implies that

$$\frac{1}{2}E\left[g_{q,p}g^*_{\tilde{q},\tilde{p}}\right] = \phi_T\left(p-\tilde{p}\right)\cdot\phi_R\left(q-\tilde{q}\right)$$

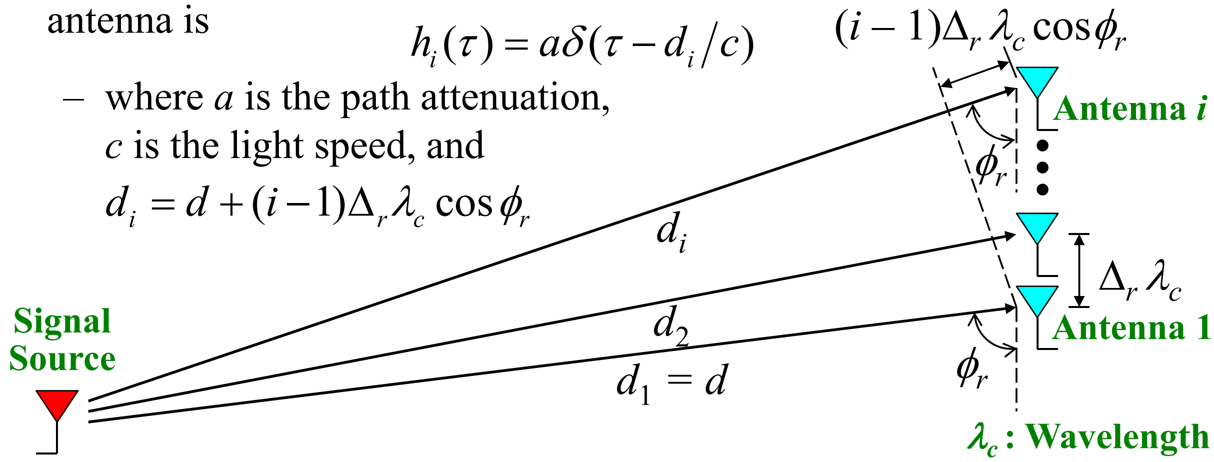
- This means that the correlation can be separated into two parts: **a transmitter part** and **a receiver part**, and both parts are **stationary**.

Angular-domain Model

- We assume that N_t transmit antennas and N_r receive antennas are placed in **uniform linear arrays** with the normalized (to $\boldsymbol{\lambda_c}$) antenna separations $\Delta_t << d$ and $\Delta_r << d$, respectively.
- Considering a **SIMO** channel with the incidence angle ϕ_r, the time impulse response between the signal source and the i-th receive antenna is

$$h_i(\tau) = a\delta(\tau - d_i/c)$$

  - where a is the path attenuation, c is the light speed, and $d_i = d + (i-1)\Delta_r \lambda_c \cos\phi_r$



Angular-domain Model

- Assume that $d_i/c << 1/W$, where W is the channel bandwidth
  - A **frequency non-selective fading** channel ($(d_i - d)/c << 1/W$)
- The channel gain at the i-th receive antenna is

$$g_i = a\exp(-j2\pi f_c d_i/c) = a\exp(-j2\pi d_i/\lambda_c)$$

  - where λ_c is the carrier wavelength and f_c is the carrier frequency
- For the considered **SIMO** channel, the received signal vector can be represented as $\mathbf{y} = \mathbf{g}x + \mathbf{w}$
  - where x is the transmit signal, $\mathbf{w}$ is the channel noise vector, and

$$\mathbf{g} = a\exp(-j2\pi d/\lambda_c)\begin{bmatrix} 1 \\ \exp[-j2\pi\Delta_r\Omega_r] \\ \exp[-j2\pi 2\Delta_r\Omega_r] \\ \vdots \\ \exp[-j2\pi(N_r-1)\Delta_r\Omega_r] \end{bmatrix}$$

$$\boxed{\Omega_r = \cos\phi_r}$$

Angular-domain Model

- Similarly, for a **MISO** channel with the radiation angle ϕ_t, , the received signal can be represented as

$$y = \tilde{\mathbf{g}}^T \mathbf{x} + w$$

  – where **x** is the transmitted signal vector, w is the channel noise, and

$$\tilde{\mathbf{g}} = a\exp(-j2\pi d/\lambda_c)\begin{bmatrix} 1 \\ \exp[-j2\pi\Delta_t\Omega_t] \\ \exp[-j2\pi 2\Delta_t\Omega_t] \\ \vdots \\ \exp[-j2\pi(N_t-1)\Delta_t\Omega_t] \end{bmatrix}$$

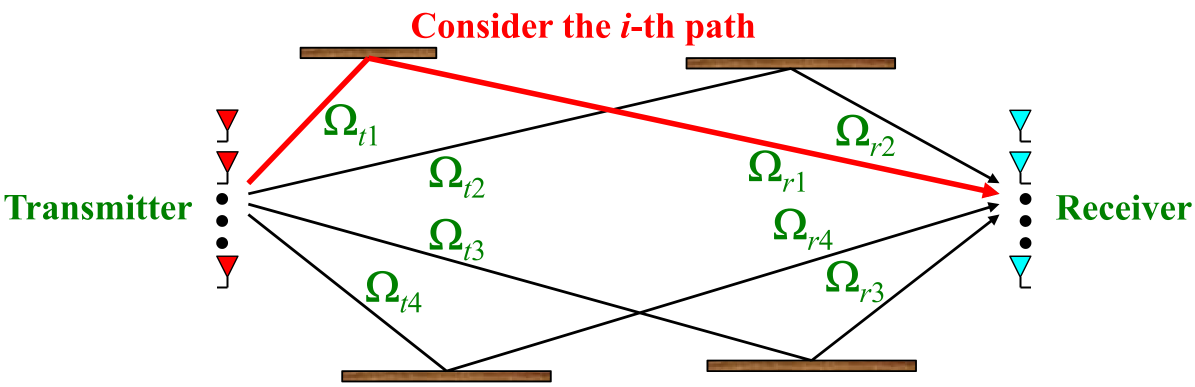
$$\boxed{\Omega_t = \cos\phi_t}$$

Angular-domain Model

- Consider a narrowband **MIMO** channel

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \mathbf{w}$$

  – where **G** is the **spatial-domain** MIMO channel model
- Suppose there be an arbitrary number of physical paths between the transmitter and the receiver.



Angular-domain Model

- The i-th path has an overall attenuation of a_i, making an angle of ϕ_{ti} ($\Omega_{ti} = \cos\phi_{ti}$) with the transmit antenna array and an angle of ϕ_{ri} ($\Omega_{ri} = \cos\phi_{ri}$) with the receive antenna array.
- The **channel matrix G** can be represented as

$$\mathbf{G} = \sum_i a_i \sqrt{N_t N_r} \exp(-j2\pi d^{(i)}/\lambda_c)\mathbf{e}_r(\Omega_{ri})\mathbf{e}_t(\Omega_{ti})$$

  – where $d^{(i)}$ is the distance between transmit antenna 1 and receive antenna 1 along **the *i*-th path**, and

$$\mathbf{e}_t(\Omega) \triangleq \frac{1}{\sqrt{N_t}} \begin{bmatrix} 1 \\ \exp[-j2\pi\Delta_t\Omega] \\ \exp[-j2\pi 2\Delta_t\Omega] \\ \vdots \\ \exp[-j2\pi(N_t-1)\Delta_t\Omega] \end{bmatrix}; \mathbf{e}_r(\Omega) \triangleq \frac{1}{\sqrt{N_r}} \begin{bmatrix} 1 \\ \exp[-j2\pi\Delta_r\Omega] \\ \exp[-j2\pi 2\Delta_r\Omega] \\ \vdots \\ \exp[-j2\pi(N_r-1)\Delta_r\Omega] \end{bmatrix}$$

Angular-domain Model

- Define the normalized total antenna length as

$$L_t = \Delta_t N_t \quad \text{and} \quad L_r = \Delta_r N_r$$

- Let $\mathbf{U}_t$ and $\mathbf{U}_r$ be the $N_t \times N_t$ and $N_r \times N_r$ unitary matrices

$$\mathbf{U}_t \triangleq \left[\mathbf{e}_t(0) \quad \mathbf{e}_t(1/L_t) \quad \cdots \quad \mathbf{e}_t((N_t-1)/L_t)\right]$$
$$\mathbf{U}_r \triangleq \left[\mathbf{e}_r(0) \quad \mathbf{e}_r(1/L_r) \quad \cdots \quad \mathbf{e}_r((N_r-1)/L_r)\right]$$

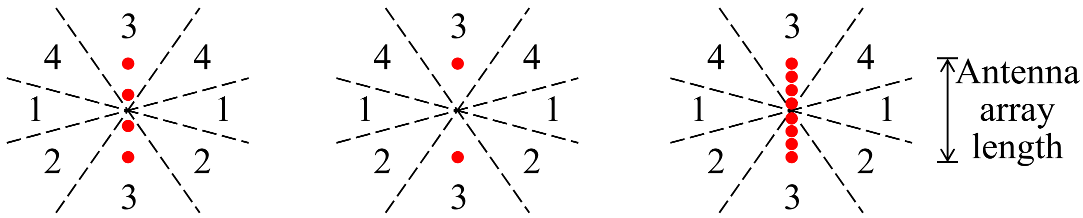
  – where the columns of the matrices are

$$\mathbf{e}_t(\Omega) \triangleq \frac{1}{\sqrt{N_t}} \begin{bmatrix} 1 \\ \exp[-j2\pi\Delta_t\Omega] \\ \exp[-j2\pi 2\Delta_t\Omega] \\ \vdots \\ \exp[-j2\pi(N_t-1)\Delta_t\Omega] \end{bmatrix}; \mathbf{e}_r(\Omega) \triangleq \frac{1}{\sqrt{N_r}} \begin{bmatrix} 1 \\ \exp[-j2\pi\Delta_r\Omega] \\ \exp[-j2\pi 2\Delta_r\Omega] \\ \vdots \\ \exp[-j2\pi(N_r-1)\Delta_r\Omega] \end{bmatrix}$$

- The column vectors form a set of **orthonormal** bases
  – Both $\mathbf{U}_t$ and $\mathbf{U}_r$ are **DFT matrices**

Angular-domain Model

- If the antenna separation is $\Delta_r = \Delta_t = 1/2$, each basis vector, $\mathbf{e}_t(k/L_t)$ or $\mathbf{e}_r(k/L_r)$, corresponds to **a single pair of main lobes** around the angles $\pm\cos^{-1}(k/L_r)$.
  - Provide the best angle-domain resolution
- If $\Delta_r = \Delta_t > 1/2$, some of the basis vectors have more than one pair of main lobes.
- If $\Delta_r = \Delta_t < 1/2$, some of the basis vectors have no main lobes.



$\Delta_r = \Delta_t = 1/2$ **$\Delta_r = \Delta_t > 1/2$** **$\Delta_r = \Delta_t < 1/2$**

Angular-domain Model

- Assume that the antenna separation is $\Delta_t = \Delta_r = 1/2$, each basis vector corresponds to a radiation angle or an incidence angle.
- The transformations $\mathbf{x}^{(a)} \triangleq \mathbf{U}_t^H \mathbf{x}$ and $\mathbf{y}^{(a)} \triangleq \mathbf{U}_r^H \mathbf{y}$ correspond to transferring the coordinates of the transmitted and received signals into **the angular-domain**. ($(\cdot)^H$: Hermitian operator)
- The received signal in the angular-domain is represented as

$$\begin{aligned}\mathbf{y}^{(a)} &= \mathbf{U}_r^H \mathbf{y} = \mathbf{U}_r^H \mathbf{G}\mathbf{x} + \mathbf{U}_r^H \mathbf{w} \\ &= \mathbf{U}_r^H \mathbf{G}\mathbf{U}_t \mathbf{x}^{(a)} + \mathbf{U}_r^H \mathbf{w} \\ &\triangleq \mathbf{G}^{(a)} \mathbf{x}^{(a)} + \mathbf{w}^{(a)}\end{aligned}$$

  - where the **angular-domain channel matrix** is

$$\mathbf{G}^{(a)} = \mathbf{U}_r^H \mathbf{G}\mathbf{U}_t$$

  - The angular-domain noise vector is

$$\mathbf{w}^{(a)} = \mathbf{U}_r^H \mathbf{w}$$

The noise power remains the same

Angular-domain Model

- Hence, different physical paths (different radiation angles and/or different incidence angles) approximately contribute to different entries in the angular-domain channel matrix $\mathbf{G}^{(a)}$.
  - The angular resolution depends on L_t (L_r), and Δ_t (Δ_r)
- Based on $\mathbf{G}^{(a)} = \mathbf{U}_r^H \mathbf{G} \mathbf{U}_t$, the (i, j)-th element is

$$g_{i,j}^{(a)} = \mathbf{e}_r^H\left((i-1)/L_r\right)\mathbf{G}\mathbf{e}_t\left((j-1)/L_t\right)$$

  - which is an element contributed by the path corresponding to the j-th radiation angle and the i-th incidence angle

$$\mathbf{G}^{(a)} = \begin{bmatrix} g_{1,1}^{(a)} & g_{1,2}^{(a)} & \cdots & g_{1,N_t}^{(a)} \\ g_{2,1}^{(a)} & g_{2,2}^{(a)} & \cdots & g_{2,N_t}^{(a)} \\ \vdots & \vdots & & \vdots \\ g_{N_r,1}^{(a)} & g_{N_r,2}^{(a)} & \cdots & g_{N_r,N_t}^{(a)} \end{bmatrix}$$

Angular-domain Model

- For an environment with limited angular spread at the receiver and/or the transmitter, many entries of $\mathbf{G}^{(a)}$ may be zero.
  - Significantly reduce the estimation and computation complexity