The Design of Sum-of-Sinusoids Channel Simulators Using the Iterative Nonlinear Least Square Approximation Method

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***Abstract*—In this paper, we introduce an iterative nonlinear least square approximation (INLSA) as an effective method for the design of sum-of-sinusoids (SOS) channel simulators. The proposed method determines the parameters of the simulation model iteratively by minimizing the Euclidean norm, which serves as a metric for measuring the fitting error. We compare the performance of the INLSA method with that of the method of exact Doppler spread (MEDS) and the L_p-norm method (LPNM). The performance comparisons will be carried out in terms of the goodness of fit to the autocorrelation functions (ACFs) corresponding to the Jakes and the Gaussian power spectral densities (PSDs). The obtained results indicate that the INLSA method is very precise in both cases. The results also show that the proposed method is better suited than the MEDS and the LPNM method to emulate the desired statistical properties of Gaussian shaped PSDs. Owing to its simplicity and excellent performance, the INLSA method is a powerful tool for designing channel simulation platforms required for the performance analysis of mobile communication systems.**

***Index Terms*—mobile fading channels, channel simulators, isotropic scattering, sum-of-sinusoids principle, iterative optimization.**

I. Introduction

The achievable data rates of wireless communication systems are heavily dependent on the propagation environment in which the system operates. Thus, a thorough investigation of mobile radio channels is of crucial importance for the optimization and performance evaluation of wireless systems. The interest of researchers was therefore devoted to the design of channel simulators enabling an accurate emulation of wireless propagation channels. The Rice's sum-of-sinusoids (SOS) principle [1], [2] for the modeling of coloured Gaussian processes has been widely employed for designing simulation models for fading channels [3]–[9]. The main idea behind the SOS principle is to approximate coloured Gaussian processes by a finite sum of sinusoidal waveforms controlled by the model parameters, namely the gains, the Doppler frequencies, and the phases. Making use of the deterministic channel modeling approach [10] and appropriately computing the SOS model parameters, one can efficiently design channel simulators having the desired statistical properties.

Several methods for computing the SOS model parameters can be found in the literature [11]–[14]. In [11] and [12], the authors presented the method of equal distances and the mean-square-error method, respectively. The common characteristic of both methods is that the distances between the two adjacent Doppler frequencies are equal. These two methods merely differ in the way of computing the Doppler coefficients. Due to the equidistance property of the Doppler frequencies, both methods have one major disadvantage, namely that the designed SOS process is periodical. Another well-known method is the method of equal areas presented in [11], [12]. The decisive disadvantage of this method is that it requires a relatively large number of sinusoids to approximate accurately the desired statistics in case of Gaussian shaped PSDs. The method of exact Doppler spread (MEDS), which was first introduced in [13], provides an excellent approximation of the autocorrelation function (ACF) corresponding to the Jakes PSD. The L_p-norm method (LPNM) [13], [14] is one of the widely used methods for computing the SOS channel model parameters. The LPNM is predicated on optimization techniques and demonstrates an excellent performance with respect to the approximation of the ACF corresponding to both the Jakes and the Gaussian PSDs. However, the determination of the model parameters using the LPNM is relatively time consuming. The development of new accurate and computationally efficient methods is therefore desirable for the performance analysis of wireless communication systems.

In this paper, we propose a simple and efficient iterative method for design of the SOS channel simulators. The proposed iterative nonlinear least square approximation (INLSA) method is based on the expectation maximization approach [15] and computes the model parameters by iteratively fitting the ACF of the simulation model to that of the reference model. The proposed method involves optimization techniques to determine the model parameters. In contrast to the LPNM method, however, the optimization procedure used in the INLSA method is carried out iteratively, which avoids a multidimensional search in determining the model parameters. This kind of iterative procedure reduces significantly the computational cost, while maintaining the same high level of fitting accuracy as that of the LPNM. In order to demonstrate the performance of the INLSA method, we implemented the algorithm in MATLAB and compared the new method with two well-known methods, namely the MEDS and the LPNM. We focused on an isotropic scattering scenario and considered the ACFs corresponding to the Jakes and the Gaussian PSDs as reference ACFs. The comparisons of all three methods are made with respect to the fitting accuracy and parameter computation time.

This paper has the following structure. In Section II, we discuss the reference model. In Section III, we present the deterministic simulation model. Section IV describes the proposed parameter computation method. Numerical examples and results are presented in Section V. Finally, Section VI concludes this paper.

II. The Reference Model

In this section, we discuss the reference model, which serves as a target for the parameter computation method. We consider a small-scale frequency-nonselective Rayleigh fading channel as the reference model. In this case, we can model the complex channel gain in the equivalent complex baseband as a zero-mean complex Gaussian process, denoted by

$$\mu(t) = \mu_1(t) + j\mu_2(t) \tag{1}$$

where $\mu_1(t)$ and $\mu_2(t)$ are the real-valued Gaussian processes, each having a variance of σ_0^2.

A typical and often assumed shape for the Doppler PSD $S_{\mu\mu}(f)$ of $\mu(t)$ is known as the Jakes PSD, which is defined as

$$S_{\mu\mu}(f) = \begin{cases} \frac{2\sigma_0^2}{\pi f_{\max}\sqrt{1-(f/f_{\max})^2}}, & |f| \le f_{\max}, \\ 0, & |f| > f_{\max}, \end{cases} \tag{2}$$

where $f_{\max}$ denotes the maximum Doppler frequency. From (2), it follows that the corresponding ACF $r_{\mu\mu}(\tau)$ of $\mu(t)$ is given by

$$r_{\mu\mu}(\tau) = 2r_{\mu_i\mu_i}(\tau) = 2\sigma_0^2 J_0(2\pi f_{\max}\tau), \ i = 1,2 \tag{3}$$

where $J_0(\cdot)$ denotes the zeroth order Bessel function of the first kind. Another widely used Doppler PSD $S_{\mu\mu}(f)$ is the Gaussian PSD

$$S_{\mu\mu}(f) = \frac{2\sigma_0}{f_c}\sqrt{\frac{\ln 2}{\pi}}e^{-\ln 2\left(\frac{f}{f_c}\right)^2} \tag{4}$$

where f_c is the 3-dB-cut-off frequency. The inverse Fourier transform of the Gaussian PSD results in the ACF

$$r_{\mu\mu}(\tau) = 2r_{\mu_i\mu_i}(\tau) = 2\sigma_0^2 e^{-\left(\pi\frac{f_c}{\sqrt{\ln 2}}\tau\right)^2}, \ i = 1,2. \tag{5}$$

III. The Deterministic Simulation Model

In this section, we first introduce the deterministic SOS simulation model, and subsequently, we review two different parameter computation methods. Let us start by presenting two SOS processes $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ in the following form

$$\tilde{\mu}_i(t) = \sum_{n=1}^{N_i} c_{i,n}\cos(2\pi f_{i,n}t + \theta_{i,n}), \quad i = 1,2 \tag{6}$$

where N_i denotes the number of sinusoids. The quantities $c_{i,n}$, $f_{i,n}$, and $\theta_{i,n}$ are the simulation model parameters called the gains, the Doppler frequencies, and the phases, respectively. The parameters $c_{i,n}$, $f_{i,n}$, and $\theta_{i,n}$ have to be determined in the simulation set-up phase and then they are kept constant during the whole simulation. From the fact that all the parameters of the SOS process $\tilde{\mu}_i(t)$ are constant, it follows that $\tilde{\mu}_i(t)$ is a deterministic process. The probability density function (PDF) of the deterministic process $\tilde{\mu}_i(t)$ is defined as

$$\tilde{p}_{\mu_i}(x) = 2\int_0^\infty \prod_{n=1}^{N_i} J_0(2\pi c_{i,n}\nu)\cos(2\pi\nu x)d\nu. \tag{7}$$

The ACF of the deterministic process $\tilde{\mu}_i(t)$ can be computed as

$$\tilde{r}_{\mu_i\mu_i}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\tilde{\mu}_i(t)\tilde{\mu}_i(t+\tau)dt. \tag{8}$$

Substituting (6) in (8) results in the following expression for the ACF $\tilde{r}_{\mu_i\mu_i}(\tau)$

$$\tilde{r}_{\mu_i\mu_i}(\tau) = \sum_{n=1}^{N_i}\frac{c_{i,n}^2}{2}\cos(2\pi f_{i,n}\tau). \tag{9}$$

It should be noted that the deterministic processes $\tilde{\mu}_1(t)$ and $\tilde{\mu}_2(t)$ are uncorrelated iff $f_{1,n} \neq \pm f_{2,m}$ for all $n = 1,2,\ldots,N_1$ and $m = 1,2,\ldots,N_2$. For the purpose of comparison, we review in the next two subsections two methods (MEDS and LPNM) for computing the model parameters.

A. MEDS

The MEDS [13] has been especially developed for the Jakes PSD, for which it results in an excellent approximation of the corresponding ACF in (3). According to this method, the gains $c_{i,n}$ and Doppler frequencies $f_{i,n}$ are defined as

$$c_{i,n} = \sigma_0\sqrt{\frac{2}{N_i}} \tag{10}$$

and

$$f_{i,n} = f_{\max}\sin\left[\frac{\pi}{2N_i}\left(n - \frac{1}{2}\right)\right] \tag{11}$$

respectively, for all $n = 1,2,\ldots,N_i$ $(i = 1,2)$. If we apply the MEDS in connection with the Gaussian PSD, then the Doppler frequencies $f_{i,n}$ are computed by finding the zeros of

$$\frac{2n-1}{2N_i} - \text{erf}\left(\frac{f_{i,n}}{f_c}\sqrt{\ln 2}\right) = 0 \tag{12}$$

for all $n = 1,2,\ldots,N_i$ $(i = 1,2)$, where erf$(\cdot)$ is the error function. The gains $c_{i,n}$ remain unchanged, i.e., they are computed according to (10).

B. LPNM

The LPNM [13], [14] is based on the idea that the sets of model parameters $\{c_{i,n}\}$ and $\{f_{i,n}\}$ are obtained by minimizing the following L_p-norm

$$E^{(p)}_{r_{\mu_i\mu_i}} := \left\{ \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} |r_{\mu_i\mu_i}(\tau) - \tilde{r}_{\mu_i\mu_i}(\tau)|^p \right\}^{1/p} \tag{13}$$

where $\tau_{\max}$ defines the time-lag interval [0, $\tau_{\max}$] over which the approximation is carried out. In the LPNM, we do not impose any boundary conditions on the model parameters $c_{i,n}$ and $f_{i,n}$. With reference to the L_p-norm in (13), the ACF $\tilde{r}_{\mu_i\mu_i}(\tau)$ of the deterministic SOS process $\tilde{\mu}_i(t)$ needs to be fitted as close as possible to a desired ACF $r_{\mu_i\mu_i}(\tau)$ of the reference model $\mu_i(t)$. It should be noted that the determination of the sets of model parameters constitutes a multidimensional optimization problem.

IV. The Proposed Parameter Computation Method

This section focuses on the description of the proposed INLSA algorithm for computing the parameters of the SOS simulation model. In the following, we consider the ACF $r_{\mu_i\mu_i}(\tau)$ of the reference model at the discrete time-lags τ_k, where $\tau_k = k\Delta\tau$, $k = 0, 1, \ldots, K$, and $\Delta\tau$ is the time-lag sampling interval. In this case, the ACF $r_{\mu_i\mu_i}(\tau)$ can be represented in the form of a discrete ACF $r_i[\tau_k]$. Also, by using (9), we can compute the discrete ACF $\tilde{r}_i[\tau_k]$ of the simulation model at the discrete time-lags τ_k. In order to determine the set of model parameters $\mathcal{P} = \{c_{i,n}, f_{i,n}\}$ such that the discrete ACF $\tilde{r}_i[\tau_k]$ of the simulation model approximates the discrete ACF $r_i[\tau_k]$ of the reference model, we need to minimize the following squared Frobenius norm

$$E(\mathcal{P}) = \left\| r_i[\tau_k] - \tilde{r}_i[\tau_k] \right\|_2^2. \tag{14}$$

Owing to the computational complexity, the direct minimization of the model error $E(\mathcal{P})$ in (14) is intractable. Therefore, the proposed method utilizes an iterative approach to determine the model parameters. The proposed INLSA method starts with $N_i = 1$ and an arbitrarily chosen initial value for $f^{(0)}_{i,1}$. At every iteration q ($q = 0, 1, 2, \ldots$), the algorithm computes the complete set of model parameters $\mathcal{P}$. The parameter computation procedure is carried out separately for each sinusoid l ($l = 1, 2, \ldots, N_i$) by using the auxiliary error function $y^{(q)}_l[\tau_k]$ as follows:

$$y^{(q)}_l[\tau_k] = r_i[\tau_k] - \frac{1}{2} \sum_{n=1, n\neq l}^{N_i} \left(c^{(q)}_{i,n}\right)^2 \cos(2\pi f^{(q)}_{i,n}\tau_k) \tag{15}$$

$$c^{(q+1)}_{i,l} = \arg\min_{c_{i,l}} \left\| y^{(q)}_l[\tau_k] - \frac{1}{2} c^2_{i,l} \cos(2\pi f^{(q)}_{i,l}\tau_k) \right\|_2^2 \tag{16}$$

$$f^{(q+1)}_{i,l} = \arg\min_{f_{i,l}} \left\| y^{(q)}_l[\tau_k] - \frac{1}{2} \left(c^{(q+1)}_{i,l}\right)^2 \cos(2\pi f_{i,l}\tau_k) \right\|_2^2. \tag{17}$$

In this algorithm, the iterative computation of the set of parameters $\mathcal{P}$ continues as long as the relative change in the model error $E(\mathcal{P})$ is larger than a predefined threshold level ε. When the threshold level ε is reached, the iteration stops, and the number of propagation paths N_i is increased by one, i.e., $N_i + 1 \rightarrow N_i$. The initial values for $c^{(0)}_{i,N}$ and $f^{(0)}_{i,N}$ are set to zero. After this, the iterative parameter computation procedure is carried out again starting from (15). This process is repeated until no perceptible progress can be observed by increasing N_i or a given maximum number of paths is reached. Further simplifications can be made by using the closed-form solution for the minimization problem in (16) as

$$c^{(q+1)}_{i,l} = \sqrt{\frac{\left\{\mathbf{y}^{(q)}_{i,l}\right\}^T \mathbf{p}^{(q)}_{i,l}}{\left\{\mathbf{p}^{(q)}_{i,l}\right\}^T \mathbf{p}^{(q)}_{i,l}}} \tag{18}$$

where the operator $\{\cdot\}^T$ designates the transpose. The symbol $\mathbf{y}^{(q)}_{i,l}$ refers to the column vector containing the stacked values of the auxiliary error function $y^{(q)}_l[\tau_k]$. The column vector $\mathbf{p}^{(q)}_{i,l}$ contains the stacked values of the samples $\cos(2\pi f^{(q)}_{i,l}\tau_k)$. The important characteristics of the proposed method is the iterative optimization approach used to determine the model parameters. The comparison of the proposed method to the MEDS and LPNM is demonstrated in the next section.

V. Numerical Results

This section discusses the performance of the proposed INLSA method. We compare the proposed method to the methods discussed in Section III. The comparisons are made with respect to their capabilities to approximate the Jakes and Gaussian ACFs.

The numerical simulation results presented in this section have been obtained by choosing $f_{\max}$ = 91 Hz and σ_0^2 = 1. In Fig. 1, we have depicted the curves for the ACFs $\tilde{r}_{\mu_i\mu_i}(\tau)$ and $r_{\mu_i\mu_i}(\tau)$ corresponding to the Jakes PSD, where the value 10 has been chosen for the number of sinusoids N_i. From Fig. 1, it can be observed that the proposed method, as well as the MEDS and LPNM provide an excellent fitting accuracy within the interval [0, $\tau_{\max}$], where $\tau_{\max} = N_i/(2f_{\max})$. For a better performance assessment of the proposed INLSA method, we have also plotted in Fig. 2 the curves for the following mean-square error

$$E_{r_{\mu_i\mu_i}} = \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} |r_{\mu_i\mu_i}(\tau) - \tilde{r}_{\mu_i\mu_i}(\tau)|^2. \tag{19}$$

In Fig. 2, we can observe which influence the number of sinusoids N_i has on the performance of the various methods. We conclude from the inspection of Fig. 2 that the proposed method results in a significantly lower mean-square error $E_{r_{\mu_i\mu_i}}$ compared to the MEDS.

In the following, we present the performance evaluations concerning the Gaussian PSD for N_i = 10 and $\tau_{\max} = N_i/(2\kappa_c f_c)$, where $f_c = \sqrt{\ln 2} f_{\max}$ and $\kappa_c = 2\sqrt{2/\ln 2}$. The corresponding ACFs $\tilde{r}_{\mu_i\mu_i}(\tau)$ and $r_{\mu_i\mu_i}(\tau)$ are illustrated

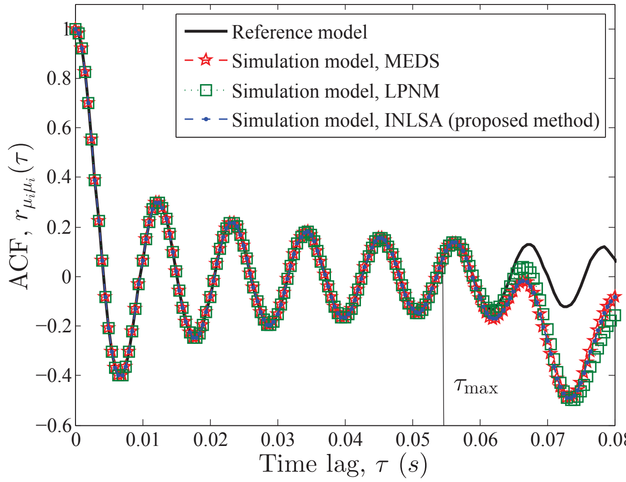

Fig. 1. The ACFs $r_{\mu_i\mu_i}(\tau)$ (reference model) and $\tilde{r}_{\mu_i\mu_i}(\tau)$ (simulation model) for different parameter computation methods (Jakes PSD).

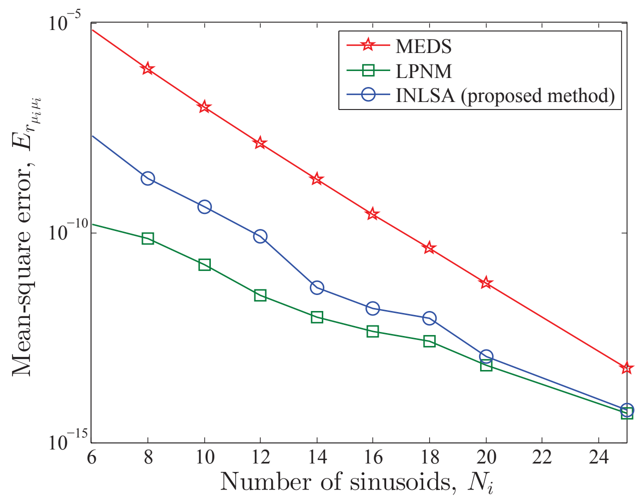

Fig. 2. Mean-square error $E_{r_{\mu_i\mu_i}}$ (Jakes PSD).

in Fig. 3. From Fig. 3, we observe an excellent agreement between the ACFs of the simulation model and the reference model when the proposed method is used. Regarding Fig. 3, one can also see that the ACF $\tilde{r}_{\mu_i\mu_i}(\tau)$ of the deterministic SOS process is considerably different from the reference ACF $r_{\mu_i\mu_i}(\tau)$ in case of the Gaussian PSD when the MEDS is used. Studying Fig. 4, it becomes obvious that the MEDS leads clearly to higher values of $E_{r_{\mu_i\mu_i}}$ in comparison with the proposed method and the LPNM. The obtained results so far, clearly states that the proposed method along with the LPNM demonstrates an exceptional performance in approximating the ACFs corresponding to both the Jakes and the Gaussian PSDs. This performance is achieved by including the gains $c_{i,n}$ and the Doppler frequencies $f_{i,n}$ into optimization procedure. However, aforementioned procedure leads to degradations with respect to the PDF of the SOS process. In order to demonstrate this problem we have plotted in Fig. 5 the curves for the mean-

TABLE I
PARAMETER COMPUTATION TIME (GAUSSIAN PSD)

Methods	N_i =10	N_i =20	N_i =30	N_i =40
LPNM	8.67 s	44.21 s	105.39 s	149.21 s
INLSA (proposed method)	0.73 s	3.43 s	5.91 s	11.2 s
MEDS	0.62 s	0.84 s	1.91 s	2.25 s

square error of the PDF

$$E_{p_{\mu_i}} = \int_{-\infty}^{\infty} |p_{\mu_i}(x) - \tilde{p}_{\mu_i}(x)|^2 \quad (20)$$

where $p_{\mu_i}(x)$ is the Gaussian distribution of the stochastic process $\mu_i(t)$. When studying Fig. 5, it becomes obvious that the LPNM has the largest mean-square error $E_{p_{\mu_i}}$. From Fig. 5, we also notice that the MEDS allows the lowest values of $E_{p_{\mu_i}}$ among the three methods. In contrast to the LPNM, the mean-square error $E_{p_{\mu_i}}$ of the INLSA method is still very good and close to that of the MEDS. In Fig. 6, the mean-square error $E_{r_{\mu_i\mu_i}}$ curves are presented for the case of $c_{i,n} = \sigma_0\sqrt{2/N_i}$ for all three methods, i.e., only the Doppler frequencies $f_{i,n}$ are optimized in the INLSA method and LPNM. By comparing Figs. 4 and 6, we can observe that fixing the gains reduces significantly the approximation quality of both the INLSA method and LPNM. Finally, we have illustrated the influence of the number of sinusoids N_i on the time consumed by the various methods in Table I. Obviously, among the three considered methods the MEDS requires the shortest time for computing the model parameters. The results presented in Table I also show that the proposed method has a significantly lower computational complexity than the LPNM. Thus, among the methods discussed, the INLSA method is the best choice for modeling the desired statistical properties of the Gaussian Doppler PSDs.

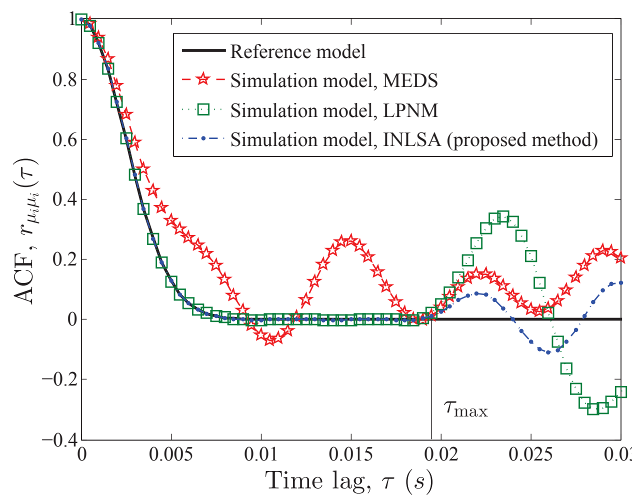

Fig. 3. The ACFs $r_{\mu_i\mu_i}(\tau)$ (reference model) and $\tilde{r}_{\mu_i\mu_i}(\tau)$ (simulation model) for different parameter computation methods (Gaussian PSD).

VI. SUMMARY AND CONCLUSIONS

In this paper, we described the INLSA method for designing SOS channel simulators. The INLSA method aims

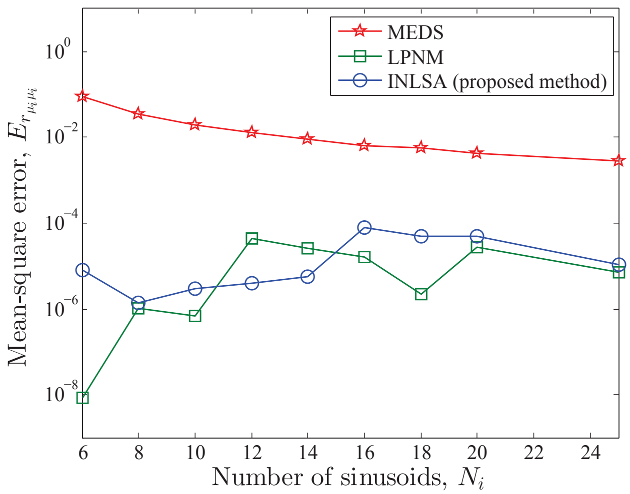

Fig. 4. Mean-square error $E_{r_{\mu_i\mu_i}}$ (Gaussian PSD).

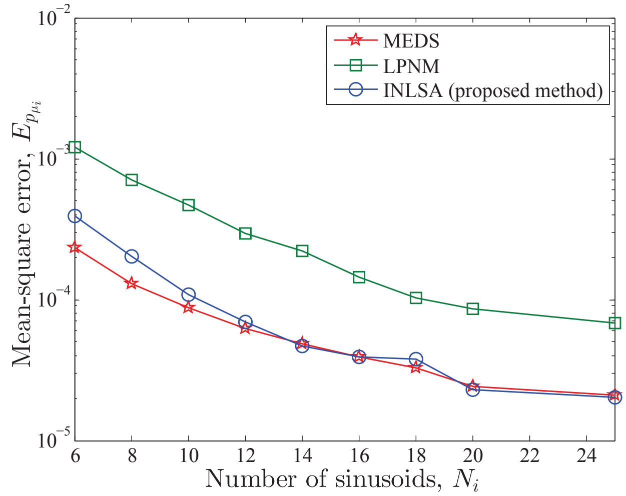

Fig. 5. Mean-square error $E_{p_{\mu_i}}$ (Gaussian PSD).

to determine the optimal set of model parameters by approximating iteratively a given ACF of the reference model. The performance analysis of the proposed method has been made by means of the accuracy of fitting the Jakes and the Gaussian ACFs. We have demonstrated that the INLSA method delivers an excellent performance in both cases. A performance comparison between the well-known LPNM and the proposed INLSA method with respect to the parameter computation time has shown that the proposed method is superior to the LPNM. Owing to the low complexity and the excellent performance, we conclude that the INLSA method is an excellent choice for designing SOS channel simulators, which are important for the design, optimization, and test of mobile communication systems.

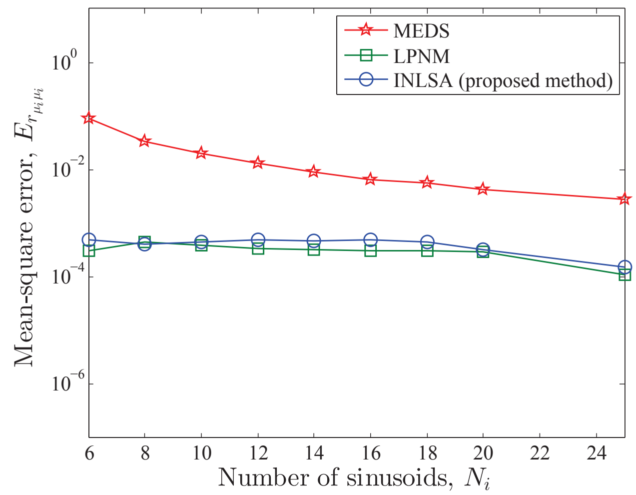

Fig. 6. Mean-square error $E_{r_{\mu_i\mu_i}}$, $c_{i,n} = \sigma_0\sqrt{2/N_i}$ with $\sigma_0 = 1$ (Gaussian PSD).

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