COM5180 Assignment #2
due Jun 22$^{\text{th}}$ (Mon) 23:59

You may type your answer as a pdf file or scan your hand-written solutions. Submit on iLMS by the deadline.

1. Prove Theorem 1 in Note#6.
2. Prove or disprove the following statement. If $A = LDM^*$ is an LDM decomposition of a nonsingular self-adjoint matrix A over $\mathbb{C}$. Then $L = M$.
3. (a) Derive the Householder transformation matrix H over $\mathbb{C}$. (b) State and prove/disprove the corresponding 6 properties of H. (c) State and prove the complex version of Note #8's Theorem 2.
4. State and explain with proof how complex-valued Givens QR works.
5. Prove or disprove the following statement. $\mathbb{F}_4$ (the finite field of 4 elements) is isomorphic to $\mathbb{Z}_4$ (i.e. $\{0, 1, 2, 3\}$ with (mod 4)- addition as well as multifiplication).
6. Find the multiplicative inverse of $x^5 + x^3 + 1$ in $\mathbb{F}_{64}$ represented by $\mathbb{F}_2[x]/(x^6 + x + 1)$.
7. Prove that an Euclidean domain D must be a *principal ideal domain*, i.e., if I is an ideal of D then $I = (a) \stackrel{\text{def}}{=} \{ar | r \in D\}$ for some $a \in D$.