COM 5335 NETWORK SECURITY LECTURE 9 DIGITAL SIGNATURES

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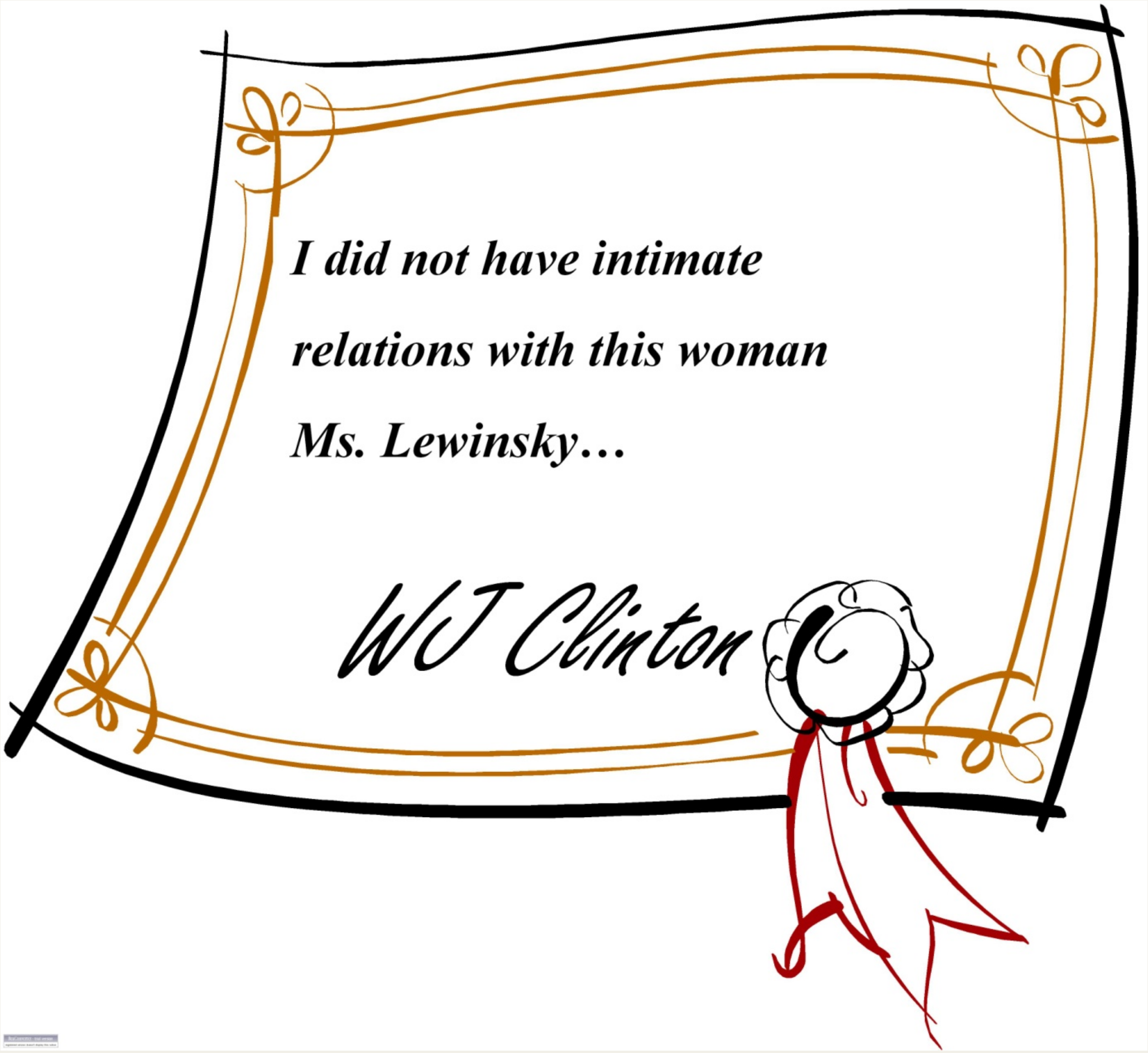
Outline

- Introduction
- RSA signature scheme
- ElGamal signature scheme
- Rabin public-key signature scheme
- DSA signature scheme (optional)
- Rabin one-time signature
- Arbitrated signature schemes

Digital Signature



Definitions

- Digital signature - a data string which associates a message with some originating entity
- Digital signature generation algorithm – a method for producing a digital signature
- Digital signature verification algorithm – a method for verifying whether a digital signature is authentic
- Digital signature scheme - consists of a signature generation algorithm and an associated verification algorithm

Digital Signature

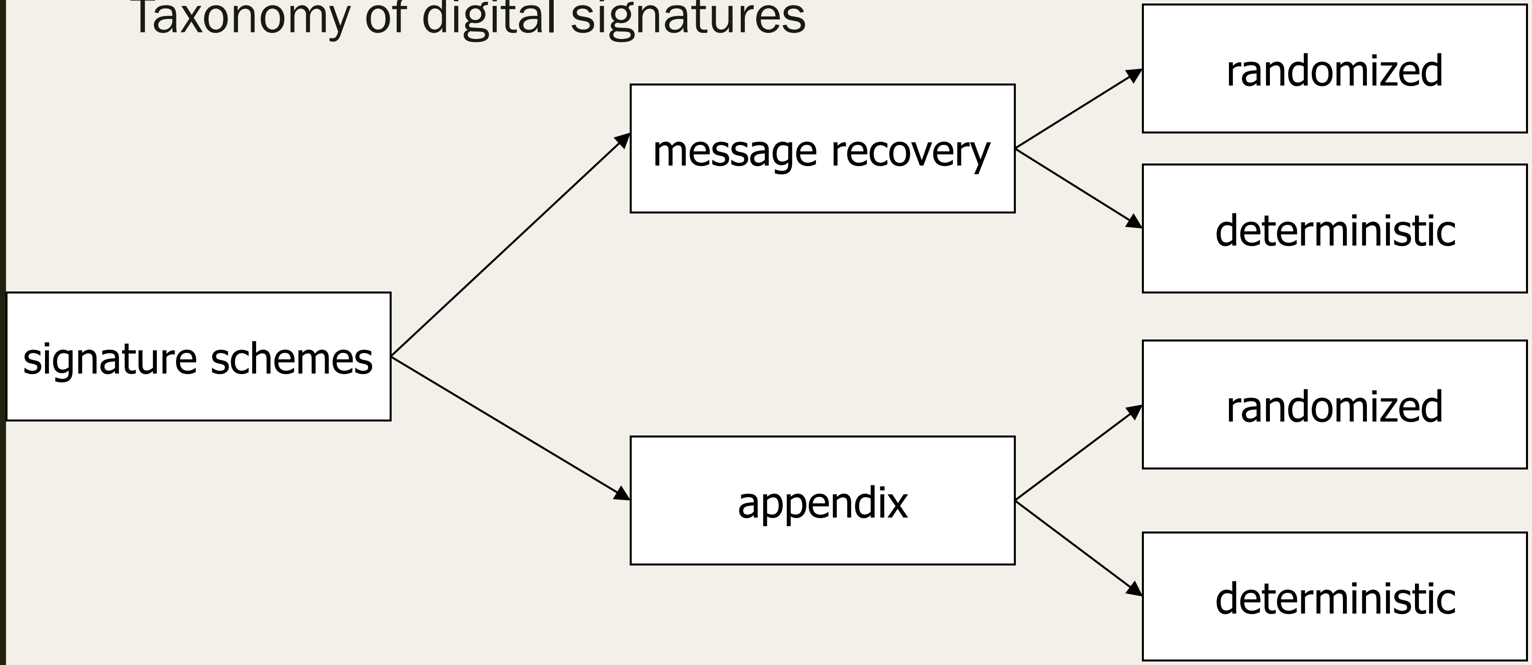
- Digital Signatures can provide
- Authentication
- Data Integrity
- Non-Repudiation
- Timestamping

Desirable Properties

- It should be efficient to compute by the signer
- It should be easy to verify by everybody
- It should be computationally infeasible for an attacker to forge the signature

Types of Digital Signatures

Taxonomy of digital signatures



Schemes w/ Appendix

- Requires the message as input to verification algorithm
- Rely on cryptographic hash functions rather than customized redundancy functions
- Schemes of this type
  - *ElGamal, DSA, Schnorr etc.*

Schemes w/ Message Recovery

- The original message is not required as input to the verification algorithm
- The original message can be recovered from the signature itself
- Schemes of this type
  - *RSA, Rabin, Nyberg-Rueppel*

Breaking a Signature Scheme

- Total Break: private key is comprimised
- Selective forgery: adversary can create a valid signature on a preselected message
- Existential forgery: adversary can create a valid signature with no control over the message

Types of Attacks

- Key-only
  - *adversary knows only the public key*
- Message attacks
  - *Known-message attack: adversary has signatures for a set of messages which are known to the adversary but not chosen by him*
  - *Chosen-message attack: adversary obtains valid signatures from a chosen list of his choice (non adaptive)*
  - *Adaptive chosen-message attack: adversary can use the signer as an oracle*

RSA Signature Scheme

- Key generation n, p, q, e, d
- Sign
  - *Compute $m_r = R(m)$, where R is an* ***invertible redundancy function*** *that maps a message into the signature space*
  - *Compute $s = m_r^d \bmod n$*
  - *The signature for m is s*
- Verify
  - *Obtain authentic public key (n, e)*
  - *Compute $m_r = s^e \bmod n$*
  - *Verify that $m_r \in M_r$*
  - *Recover $m = R^{-1}(mr)$*

RSA Signature (cont'd)

- Attacks
  - *Integer factorization*
  - *Homomorphic property*
- Reblocking problem
  - *If signatures are encrypted different modulus sizes can render the message unrecoverable*
- Importance of the redundancy function
  - *ISO/IEC 9796*

RSA Signature (cont'd)

- Performance (p, q are k-bit primes)
  - *Signature* $O(k^3)$
  - *Verification* $O(k^2)$
- Bandwidth
  - *Bandwidth is determined by R. For example, ISO/IEC 9796 maps k-bit messages to 2k-bit elements in MS for a 2k-bit signature (efficiency of ½)*

ElGamal Signature Scheme

- Key generation: p, α, a, y=α^a mod p
- Signature Generation
  - *Select random k, $1 \leq k \leq p-1$, gcd(k, p-1)=1*
  - *Compute r = α^k mod p*
  - *Compute k^{-1} mod (p-1)*
  - *Compute s = k^{-1} * (h(m) - ar) mod (p-1), where h is a one-way hash function.*
  - *Signature is (r,s)*

ElGamal Signature (cont'd)

- Signature Verification
  - *Verify* $1 \le r \le p\text{-}1$
  - *Compute* $v_1 = y^r r^s \bmod p$
  - *Compute* $h(m)$ *and* $v_2 = \alpha^{h(m)} \bmod p$
  - *Accept iff* $v_1 = v_2$

$$s \equiv k^{-1}\{h(m) - ar\} \pmod{p-1}$$

$$ks \equiv h(m) - ar \pmod{p-1}$$

$$\alpha^{h(m)} \equiv \alpha^{ar+ks} \equiv (\alpha^a)^{\mathrm{r}} \mathrm{r}^{\mathrm{s}} \pmod{p}$$

ElGamal Signature (cont'd)

- Security (based on DLP)
  - *Index-calculus attack: p should be large*
  - *Pohlig-Hellman attack: p-1 should not be smooth*
  - *Weak generators: If $p \equiv 1 \bmod 4$, $\alpha | p-1$, DL can be broken for subgroup S of order α. Forgeries are then possible*

ElGamal Signature (cont'd)

- In addition…
  - *k must be unique for each message signed*
    - $(s_1\text{-}s_2)k=(h(m_1)\text{-}h(m_2))\text{mod }(p\text{-}1)$
  - *An existential forgery attack can be mounted if a hash function is not used*

ElGamal Signature (cont'd)

- Performance
  - *Signature Generation*
    - One modular exponentiation
    - One Euclidean Algorithm
    - Both can be done offline
  - *Verification*
    - Three modular exponentiations
- Generalized ElGamal Signatures

Rabin Public-Key Signature Scheme

- Signature: $s = m^{1/2} \bmod n$
  - *Using any sqrt*
- Verification: $m = s^2 \bmod n$
- About ¼ of the messages have sqrts.
- If a message doesn't have a sqrt, then it has to be slightly modified by doing proper padding.

Rabin Signature (cont’d)

- In practice, one might try to append a small amount of random bits and hope that it has a sqrt.
- On average, two such attempts would be enough.
- Such method has no guarantee of success, so a deterministic method would be preferable.

Modified Rabin Signature Scheme

- To overcome this problem, a modified Rabin signature scheme is designed.
- It provides a deterministic method for associating messages with elements in the signing space such that computing s square root (or something close to it) is always possible.
- Details of modified Rabin signature scheme is beyond the scope of this course.

Rabin Signature Pros & Cons

- Advantage:
  - *Verification is extremely fast*
  - *Cracking the signature scheme is provable as hard as doing factorization.*
- Disadvantage: like Rabin encryption, one system can only generate signature for a single user.

DSA Signature

- DSA Algorithm : key generation
  - *select a prime q of 160 bits*
  - *Choose* $0 \leq t \leq 8$, *select* $2^{511+64t} < p < 2^{512+64t}$ *with* $q | p-1$
  - *Select g in* Z_p^*, *and* $\alpha = g^{(p-1)/q} \bmod p$, $\alpha \neq 1$ $(ord(\alpha)=q)$
  - *Select* $1 \leq a \leq q-1$, *compute* $y = \alpha^a \bmod p$
  - *public key* (p,q,α,y), *private key a*

DSA Signature (cont'd)

- DSA signature generation
  - *Select a random integer k, 0 < k < q*
  - *Compute* $r=(\alpha^k \bmod p) \bmod q$
  - *Compute* $k^{-1} \bmod q$
  - *Compute* $s=k^{-1} * (h(m) + ar) \bmod q$
  - *Signature =* (r, s)

DSA Signature (cont'd)

- DSA signature verification
  - *Verify $0<r<q$ and $0<s<q$, if not, invalid*
  - *Compute $w= s^{-1} \bmod q$*
  - *Compute $u_1=w*h(m) \bmod q$, $u_2=r*w \bmod q$*
  - *Compute $v = (\alpha^{u_1}y^{u_2} \bmod p) \bmod q$*
  - *Valid iff $v=r$*

$$h(m) \equiv -ar + ks \pmod q$$
$$wh(m) + arw \equiv k \pmod q$$
$$u_1 + au_2 \equiv k \pmod q$$
$$\alpha^{u_1} y^{u_2} \bmod p \pmod q = \alpha^k \bmod p \pmod q$$

DSA Signature (cont'd)

- Security of DSA
  - *two distinct DL problems: Z_p*, cyclic subgroup order q*
- Parameters:
  - *q~160bits, p 768~1Kb, p,q, α can be system wide*
- Probability of failure
  - *$Pr[s=0]= (1/2)^{160}$*

DSA Signature (cont’d)

- Performance
  - *Signature Generation*
    - One modular exponentiation
    - Several 160-bit operations (if p is 768 bits)
    - The exponentiation can be pre-computed
  - *Verification*
    - Two modular exponentiations

Nyberg-Rueppel Signature Scheme

- Can be regarded as an ElGamal signature scheme w/ message recovery
- Key generation:
  - *Same as DSA but no constraints on the sizes of p,q*
  - *Select g in* Z_p^**, and* $\alpha = g^{(p-1)/q} \bmod p$*,* $\alpha \neq 1$ *(*$ord(\alpha)=q$*)*
  - *Select, p,q w/* $q|(p-1\}$*,* $\alpha \in Z_p^*$*, (private key) a,* $y= \alpha^a$*.*

Nyberg-Rueppel Signature Scheme

- Sign message *m*
  - *Compute $m_r = R(m)$, where R is an invertable redundancy function*
  - *Select a random secret integer k s.t. $1 \leq k \leq p-1$*
  - *Compute $r \equiv \alpha^k \pmod{p}$, $e \equiv m_r r \pmod{p}$, and $s \equiv ae + k \pmod{q}$*
  - *Signature is (e,s)*
- Verify signature (*e,s*)
  - *Obtain authentic public key (p,q,α,y)*
  - *Verify that $0 < e < p$ and $0 \leq s \leq q$. If not, reject.*
  - *Compute $v \equiv \alpha^s y^{-e} \pmod{p}$ and $m \equiv ve \pmod{p}$*
  - *Recover $m = R^{-1}(m_r)$*