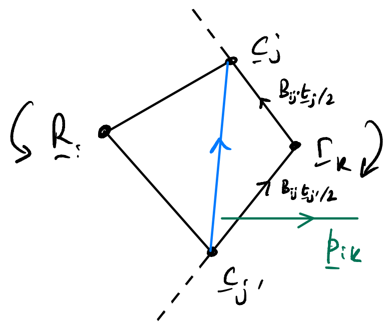
A single peripheral kite



$\underline{p}_{ik} = \sum_j \epsilon_i b_{ij} \underline{c}_j^p A_{jk}$ is the outward normal to the link between adjacent peripheral edge centroids.

The external force on the kite is

$$-\frac{p_{ext}}{2}\left(\underline{n}_{ij} + \underline{n}_{ij'}\right) \qquad \underline{n}_{ij} = -b_{ij}\,\epsilon_i\, \underline{t}_{ij}$$

But $\frac{1}{2}\sum_j b_{ij} \underline{t}_j^p A_{jk}$ is the vector $-\sum_j b_{ij} \underline{c}_j^p A_{jk}$ (connecting $\underline{c}_{j'}$ to $\underline{c}_j$)

So $\frac{1}{2}(\underline{n}_{ij} + \underline{n}_{ij'}) = \underline{p}_{ik}$ (after multiplying by ϵ_i)

The external force on the kite is $-p_{ext}\, \underline{p}_{ik}$.

The rotated internal force maps to $\underline{h}_j - \underline{h}_{j'}$. The rotated external force maps to $\underline{h}_{j'} - \underline{h}_j$.

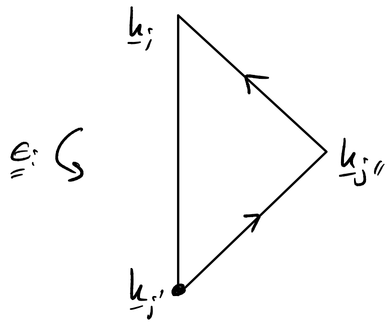
So $\underline{h}_{j'} - \underline{h}_j = \pm p_{ext}(\underline{c}_{j'} - \underline{c}_j)$ (sign depends on $\pm\epsilon$ rotation of force)

Now partition the external force into contributions on individual cells,

$$-\tfrac{1}{2} p_{ext}\, \underline{n}_{ij} \quad \text{and} \quad -\tfrac{1}{2} p_{ext}\, \underline{n}_{ij'}$$

After rotation, these are $-\frac{1}{2} p_{ext}\, \underline{t}_j$ and $-\frac{1}{2} p_{ext}\, \underline{t}_{j'}$

So the rotated force balance on the kite matches the triangle bounded by $\underline{c}_j$, $\underline{c}_{j'}$ and $\underline{r}_k$, but scaled by $\pm p_{ext}$.



where

$$\underline{h}_j - \underline{h}_{j'} = \pm p_{ext}(\underline{c}_j - \underline{c}_{j'})$$

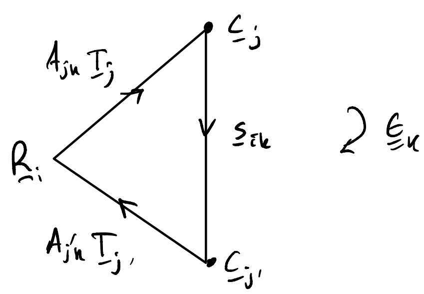
$$\underline{h}_{j''} - \underline{h}_{j'} = \pm p_{ext}(\underline{t}_{j'}/2)\, b_{ij'}$$

$$\underline{h}_j - \underline{h}_{j''} = \pm p_{ext}(\underline{t}_j/2)\, b_{ij}$$

and $(\underline{h}_j - \underline{h}_{j'}) + (\underline{h}_{j'} - \underline{h}_{j''}) + (\underline{h}_{j''} - \underline{h}_j) = \pm p_{ext}\left[\underline{c}_j - \underline{c}_{j'} - \frac{b_{ij}\underline{t}_j}{2} - \frac{b_{ij'}\underline{t}_{j'}}{2}\right] = 0.$

This is congruent to the triangle

$$A_{j'k}\, \underline{T}_{j'} + A_{jk}\, \underline{T}_j + \underline{s}_{ik} = 0$$



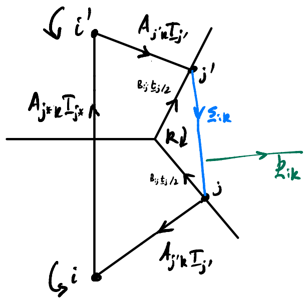
where $\underline{s}_{ik} = \sum_j b_{ij}\, \underline{c}_j^p A_{jk}$

So we can evaluate $\{\mathrm{CURL}^\vee \underline{h}\}_k$ as $\quad \dfrac{1}{\mathcal{E}_k}\Big(\sum_j A_{jk}\, \underline{T}_j \cdot \underline{h}_j \;+\; \underline{s}_{ik} \cdot \underline{h}_{j''}\Big)$

and $\{\mathrm{div}^\vee \underline{h}\}_k$ as $\quad \dfrac{1}{\mathcal{E}_k}\Big(\sum_j -A_{jk}\, \underline{\underline{\epsilon}}_k \underline{T}_j \cdot \underline{h}_j \;+\; \underline{p}_{ik} \cdot \underline{h}_{j''}\Big)$

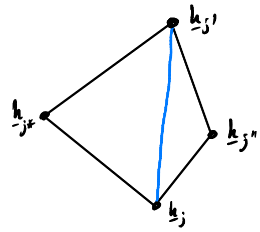
where $\underline{h}_{j''} = \underline{h}_j \mp p_{ext}\, B_j\, \underline{t}_j/2 = \underline{h}_{j'} \pm p_{ext}\, B_{j'}\, \underline{t}_{j'}/2.$

Two kites at a peripheral vertex



The external force on the kites is again $-p_{ext}\, \underset{\sim}{p}_{ik}$.

Again we use the two edges adjacent to vertex k to identify $\underline{h}_{j''}$, as



$$\underline{h}_{j''} = \underline{h}_j{}' \pm p_{ext}\left(B_{j'}\, \underline{t}_{j'}/2\right)$$

$$= \underline{h}_j \mp p_{ext}\left(B_j\, \underline{t}_j/2\right)$$

ensuring that $\underline{h}_j - \underline{h}_{j''} + \underline{h}_{j''} - \underline{h}_{j'} = \pm p_{ext}\left(B_j \underline{t}_j/2 + B_{j'} \underline{t}_{j'}/2\right) = \mp p_{ext}\, \underline{s}_{ik}$,

which is the stated form of $-p_{ext}\, \underline{p}_{ik}$.

Then $\{\mathrm{CURL}^\vee \underline{h}\}_k = \dfrac{1}{\mathcal{E}_k}\Big[\sum_j A_{jk}\, \underline{T}_j \cdot \underline{h}_j + \underline{s}_{ik} \cdot \underline{h}_{j''}\Big]$

and $\{\mathrm{div}^\vee \underline{h}\}_k = \dfrac{1}{\mathcal{E}_k}\Big[\sum_j -A_{jk}(\underline{\underline{\epsilon}}_k\, \underline{T}_j) \cdot \underline{h}_j + \underline{p}_{ik} \cdot \underline{h}_{j''}\Big]$