COS4807 Assignment 4 - 737797

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Question 1

1 (i)

$\exists x(p(x) \rightarrow q(x)) \rightarrow (\exists xp(x) \rightarrow \exists xq(x))$
$\exists x(p(x) \rightarrow q(x)) \rightarrow (\exists yp(y) \rightarrow \exists zq(z))$
$\neg\exists x(\neg p(x) \vee q(x)) \vee (\neg\exists yp(y) \vee \exists zq(z))$
$\forall x\neg(\neg p(x) \vee q(x)) \vee (\forall y\neg p(y) \vee \exists zq(z))$
$\forall x(p(x) \wedge \neg q(x)) \vee (\forall y\neg p(y) \vee \exists zq(z))$
$\exists zq(z) \vee \forall x(p(x) \wedge \neg q(x)) \vee \forall y\neg p(y)$
$\exists z\forall x\forall y(q(z) \vee (p(x) \wedge \neg q(x)) \vee \neg p(y))$
$\exists z\forall x\forall y((q(z) \vee p(x) \vee \neg p(y)) \wedge (q(z) \vee \neg q(x) \vee \neg p(y)))$
$\forall x\forall y((q(a) \vee p(x) \vee \neg p(y)) \wedge (q(a) \vee \neg q(x) \vee \neg p(y)))$

$\{\{q(a), p(x), \neg p(y)\}, \{q(a), \neg q(x), \neg p(y)\}\}$

1 (ii)

$\forall x\forall y\neg(p(y) \leftrightarrow q(x))$
$\forall x\forall y\neg((p(y) \rightarrow q(x)) \wedge (q(x) \rightarrow p(y)))$
$\forall x\forall y\neg((\neg p(y) \vee q(x)) \wedge (\neg q(x) \vee p(y)))$
$\forall x\forall y(\neg(\neg p(y) \vee q(x)) \vee \neg(\neg q(x) \vee p(y)))$
$\forall x\forall y((p(y) \wedge \neg q(x)) \vee (q(x) \wedge \neg p(y)))$
$\forall x\forall y((p(y) \vee q(x)) \wedge (p(y) \vee \neg p(y)) \wedge (\neg q(x) \vee q(x)) \wedge (\neg q(x) \vee \neg p(y)))$

$\{\{p(y), q(x)\}, \{p(y), \neg p(y)\}, \{\neg q(x), q(x)\}, \{\neg q(x), \neg p(y)\}\}$

Which could be simplified to

$\{\{p(y), q(x)\}, \{\neg q(x), \neg p(y)\}\}$

1 (iii)

$\exists xp(x) \leftrightarrow \exists xq(x, x)$
$(\exists xp(x) \rightarrow \exists xq(x, x)) \wedge (\exists xq(x, x) \rightarrow \exists xp(x))$
$(\exists xp(x) \rightarrow \exists yq(y, y)) \wedge (\exists wq(w, w) \rightarrow \exists zp(z))$
$(\neg\exists xp(x) \vee \exists yq(y, y)) \wedge (\neg\exists wq(w, w) \vee \exists zp(z))$
$(\forall x\neg p(x) \vee \exists yq(y, y)) \wedge (\forall w\neg q(w, w) \vee \exists zp(z))$
$(\exists yq(y, y) \vee \forall x\neg p(x)) \wedge (\exists zp(z) \vee \forall w\neg q(w, w))$
$\exists y((q(y, y) \vee \forall x\neg p(x)) \wedge (\exists zp(z) \vee \forall w\neg q(w, w)))$
$\exists y\forall x((q(y, y) \vee \neg p(x)) \wedge (\exists zp(z) \vee \forall w\neg q(w, w)))$
$\exists y\forall x\exists z((q(y, y) \vee \neg p(x)) \wedge (p(z) \vee \forall w\neg q(w, w)))$
$\exists y\forall x\exists z\forall w((q(y, y) \vee \neg p(x)) \wedge (p(z) \vee \neg q(w, w)))$
$\forall x\exists z\forall w((q(a, a) \vee \neg p(x)) \wedge (p(z) \vee \neg q(w, w)))$
$\forall x\forall w((q(a, a) \vee \neg p(x)) \wedge (p(f(z)) \vee \neg q(w, w)))$

$\{\{q(a, a), \neg p(x)\}, \{p(f(z)), \neg q(w, w)\}\}$

Question 2

2(i)

$S_1 = \{\{q(a), p(x), \neg p(y)\}, \{q(a), \neg q(x), \neg p(y)\}\}$

$H_{S_1} = \{a\}$

$B_{S_1} = \{p(a), q(a)\}$

$M_{S_{1,1}} = \{q(a)\}$

$M_{S_{1,2}} = \{\}$

2(ii)

$S_2 = \{\{p(y), q(x)\}, \{\neg q(x), \neg p(y)\}\}$

There are no constants or nullary functions in S so we initialize the Herbrand universe with the arbitrary constant a.

$H_{S_2} = \{a\}$

$B_{S_2} = \{p(a), q(a)\}$

$M_{S_{2,1}} = \{p(a)\}$

$M_{S_{2,2}} = \{q(a)\}$

2(iii)

$S_3 = \{\{q(a, a), \neg p(x)\}, \{p(f(z)), \neg q(w, w)\}\}$

$H_{S_3} = \{a, f(a), f(f(a)), f(f(f(a))), \dots\}$

$B_{S_3} = \{p(a), p(f(a)), \dots, q(a, a), q(f(a), a), q(a, f(a)), q(f(a), f(a)), \dots\}$

$M_{S_{3,1}} = \{p(f(a)), p(f(f(a))), \dots\}$

$M_{S_{3,2}} = \{\}$

Question 3

3(i)

The mapping $\{x \leftarrow a, y \leftarrow f(a), z \leftarrow f(a)\}$ would unify the atoms $\{p(a, f(x), y), p(x, y, z)\}$

3(ii)

The atoms $\{p(x, f(a), x), p(a, y, y)\}$ are not unifiable because the variable y would need to map to both a and $f(a)$.

3(iii)

The atoms $\{p(f(x), f(y), x), p(y, z, f(z)\}$ are not unifiable because any substitutions lead to mutual recursion where x is a function of z, which is a function of y, which is again a function of x.

3(iv)

The atoms $\{p(f(x, b), g(y), f(a, x), p(z, g(z), y)\}$ are not unifiable because x would map to both a and b which are different constants.

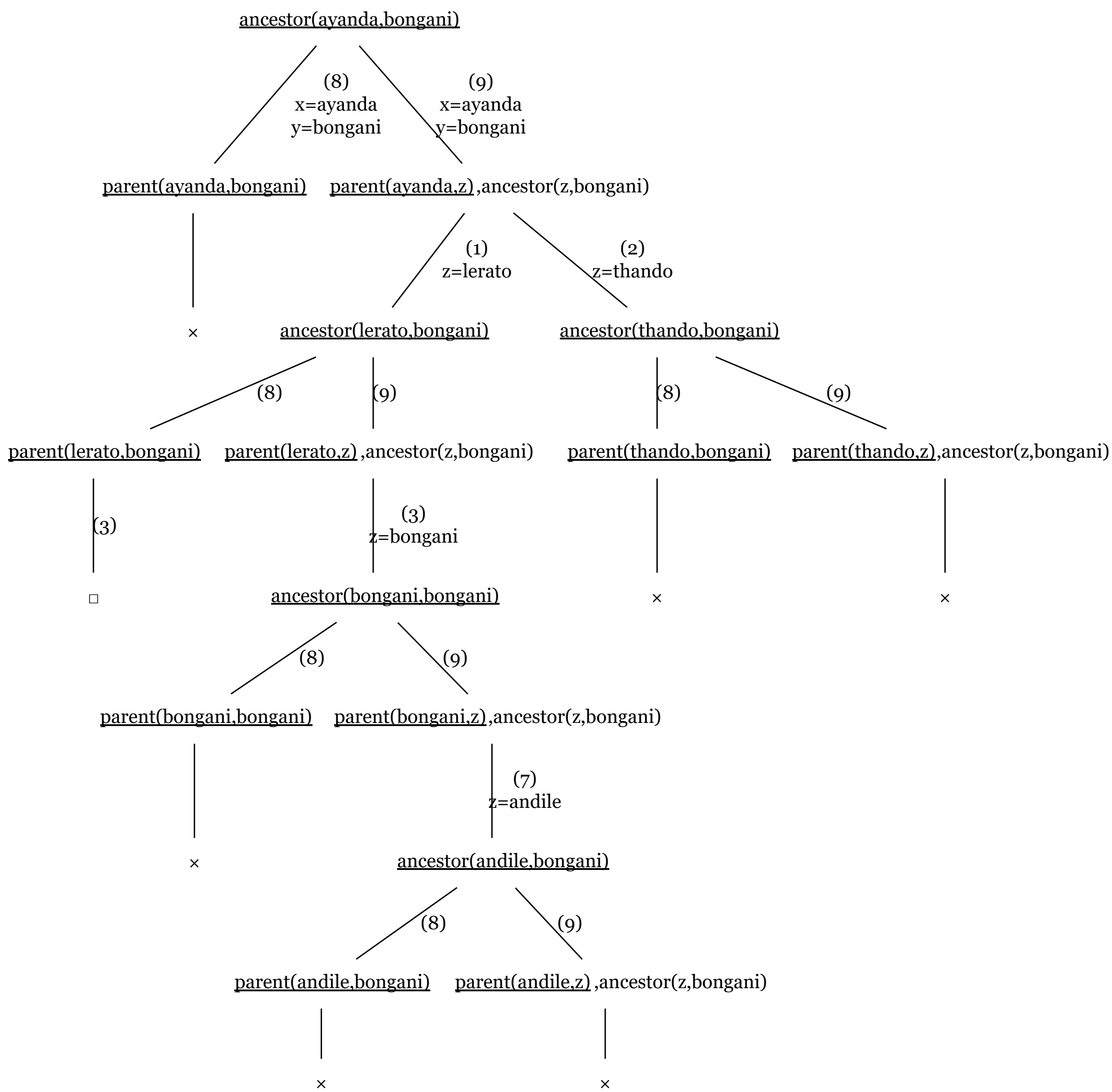
Question 4

1.	$\{p(a, f(x)), \neg q(x, f(a))\}$		
2.	$\{\neg p(y, y), r(a, y)\}$		
3.	$\{\neg p(x, y), \neg r(x, f(a))\}$		
4.	$\{p(x, f(y)), \neg q(f(y), x)\}$		
5.	$\{q(x, x)\}$		
6.	$\{p(a, f(f(a)))\}$	$x \leftarrow f(a)$	$1, 5$
7.	$\{\neg r(a, f(a))\}$	$x \leftarrow a, y \leftarrow f(f(a))$	$3, 6$
8.	$\{\neg p(f(a), f(a))\}$	$y \leftarrow f(a)$	$2, 7$
9.	$\{\neg q(f(a), f(a))\}$	$x \leftarrow f(a), y \leftarrow a$	$4, 8$
10.	$\{\square\}$	$x \leftarrow f(a)$	$5, 9$

Resolution on the given clauses has led to the empty clause so we conclude that the given set of clauses are not satisfiable.

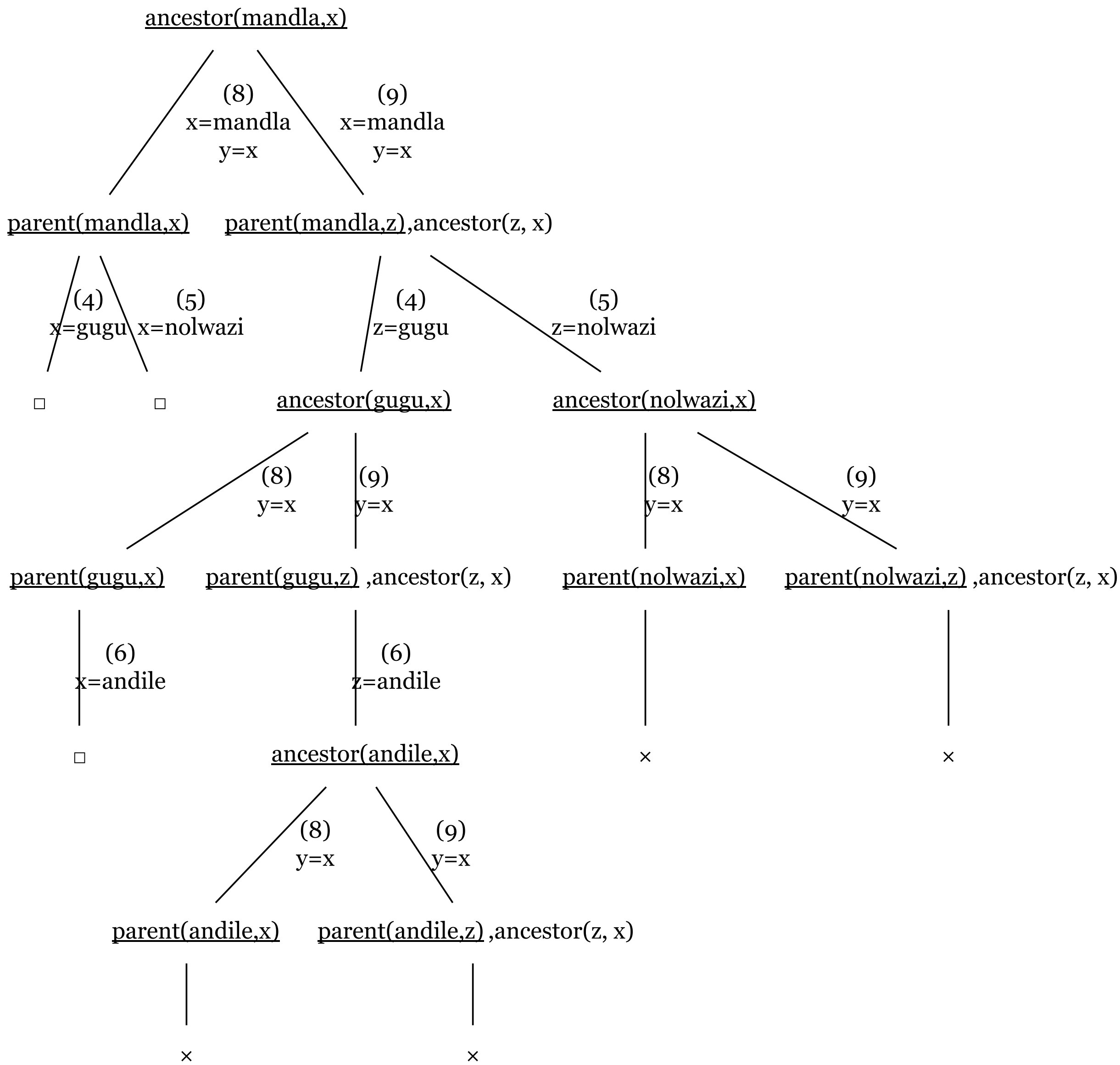
Question 5

5(i)



The SLD tree indicates that Ayanda is an ancestor of Bongani.

5(ii)



The SLD tree indicates that Mandla is the ancestor to three people because x resolves to Gugu, Nolwazi and Andile.