V64 Interferometry

Moritz Bosse
moritz.bosse@tu-dortmund.de

Linus Bölte
linus.bölte@tu-dortmund.de

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TU Dortmund – Fakultät Physik

Contents

1 Objective

The aim of this experiment is to determine the refractive index of air and glass using a Sagnac interferometer. Next to this the necessary prerequisit of aligning the interferometer will be done maximizing its contrast in order to achieve accurate measurements.

2 Theory

The underlying concept allowing measurements with an interferometer is the *interference* of two wavefronts. The interference effect relies upon the superposition principle, which permits the electric field vectors $\vec{E}$ of two electromagnetic waves to add up. This, however, only holds if the two waves fullfil a condition known as *coherence*. If they do not it is not the electric fields which add up but the intensity I, which is proportional to the electric field squared: $I \propto \vec{E}^2$.

2.1 Coherence

The most tangible definition of coherence is that the two waves need to have a constant phase relation. More precisely temporal and spatial coherence are distinguished, where the first is understood as the monochromaticity of both waves and the latter is measured as the phase difference of two spatially seperated points. Thus an incoherent source of light may be passed through a monochromator to achieve temporal coherence and a small slit for spatial coherence, allowing the arising elementary waves to interfere with each other. While an ideally coherent source may be theoretically characterised it rarely exists in reality, motivating the definition of a quantity known as the degree of coherence $\gamma_{1,2} \in [0, 1]$ given by

$$\gamma_{1,2}(\vec{r}_1, t_1; \vec{r}_2, t_2) = \frac{\langle E(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) \rangle}{\sqrt{\langle |E(\vec{r}_1, t_1)|^2 \rangle \langle |E(\vec{r}_2, t_2)|^2 \rangle}}. \tag{1}$$

The first-order degree of coherence is nothing but the amplitude to amplitude correlation of the two waves. A more practical way of quantifying coherence is the coherence length or time, which is defined as the time or path length it takes for the waves relative phase difference to exceed 2π.

2.2 Polarization

Considering that electromagnetic waves are transverse waves a direction is determined along which the electric or magnetic field oscillates. Depending on the relative location and direction of the different polarization states are assigned to a wave. If for example light is unpolarized there is no one oscillation direction but a manifold which does not allow to assign a certain polarization state. Passing unpolarized light through a polarization filter, allows to extract one oscillation direction leading to linearly polarized light at the angle the filter is sitting at. Elliptically polarized light is accquired by combining two linearly polarized waves with a phase offset. If there is a second linear polarizer after the first the intensity I transmitted through the second filter depends on their relative angle θ according to Malus Law

$$I = I_0 \cos^2(\theta). \tag{2}$$

Next to linear polarization filters, an unpolarized beam may also be polarized using a polarizing beam splitter cube (PBSC).

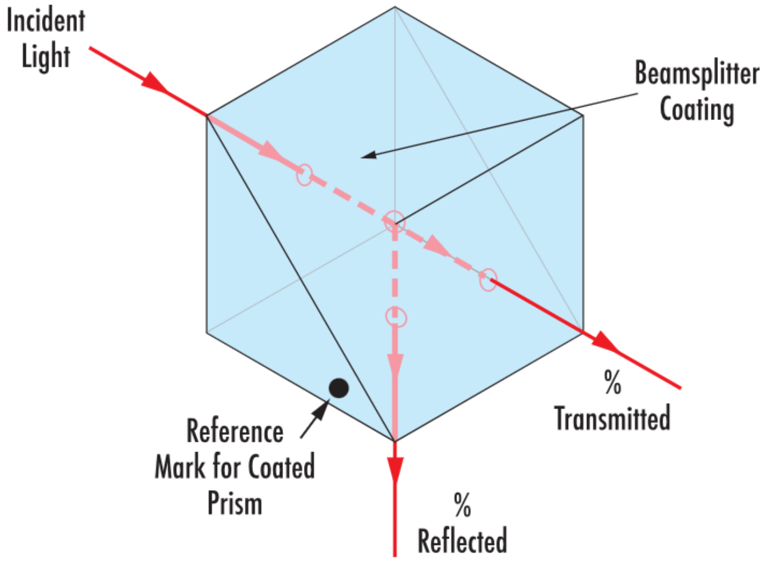

Figure 1: Polarizing Beamsplitter (PBSC), showing the incident light being split into an s- and p-polarized part at the dielectric coating on the diagonal of the cube [2].

It splits a beam into a s-polarized (orthogonal to the incident plane) and p-polarized (parallel to the incident plane) part.

2.3 Visibility

While the first-order correlation is a valid approach to determine the degree of coherence, a more practical approach is to determine the visibility or contrast. The visibility K is defined as follows

$$K = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}. \tag{3}$$

Here I_{max} and I_{min} are the highest and lowest intensities of a speckle pattern on a textured surface. With $K \in [0, 1]$, it translates to the degree of coherence directly. The superposition principle allows to define an equation for the interfering waves intensity

$$I \propto \langle |E_1 \cos(\omega t) + E_2 \cos(\omega t + \delta)|^2 \rangle. \tag{4}$$

Expanding the upper expression, dividing by $\cos^2(\omega t)$ and assuming a phase difference $\delta = 2\pi n$ or $\delta = (2n+1)\pi$ for the maximum and minimum intensity with $n \in \mathbb{N}$ allows to calculate the maximum I_{max} and minimum intensity I_{min} as follows

$$I_{\text{max/min}} \propto E_1^2 + E_2^2 \pm E_1 E_2. \tag{5}$$

Considering the electric field strengths dependence on the incoming lights polarization angle ϕ, due to the PBSC, their amplitude needs to be corrected

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = E_0 \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \end{pmatrix} = \sqrt{E_1 + E_2} \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \end{pmatrix}. \tag{6}$$

Inserting the above expression in equation 5 yields

$$I_{\text{max/min}} \propto I_0(1 \pm 2\sin(\phi)\cos(\phi)), \tag{7}$$

with I_0 being the input intensity. This equation now enables to get an expression for the visibility of an interferometer with a PBSC which depends on the input polarization angle

$$K = |2\sin(\phi)\cos(\phi)|. \tag{8}$$

2.4 Refractive indices of Glas and Gases

The refractive index n and the speed of light c in a medium depend on each other, leading to a different path length in relation to a light beam that did not propagate through the medium. Due to the interference effect discussed above a number M of interference maxima and minima are observed if the amount of medium in the beam path is varied, resulting in

$$M = \frac{\Delta\Phi}{2\pi}, \tag{9}$$

where $\Delta\Phi$ denotes the phase shift. This phase shift is given as

$$\Delta\Phi(\theta) = \frac{2\pi}{\lambda_{\text{vac}}} L \frac{n-1}{2n} \theta^2. \tag{10}$$

Here, L is thickness of the glass, λ_{vac} is the wavelength in vacuum and θ is the angle of the glass in the beam path. Considering that the used glass plates are installed at an angle of $\theta_0 = \pm 10°$, equation 10 is modified resulting in

$$\Delta\Phi(\theta) = \frac{2\pi}{\lambda_{\text{vac}}} L \frac{n-1}{2n} ((\theta + \theta_0)^2 + (\theta - \theta_0)^2). \tag{11}$$

Using equation 9 this yields the following expression for the refractive index of glass

$$n = \frac{1}{1 - \frac{M\lambda_{\text{vac}}}{2L\theta\theta_0}}. \tag{12}$$

Analogue to the calculation of the refractive index of glass the refractive index of a gas is done using the phase shift resulting from a chamber of gas with length L

$$\varDelta\varPhi = \frac{2\pi}{\lambda_{\text{vac}}}(n-1)L. \tag{13}$$

Using equation 9 this yields

$$n = \frac{M\lambda_{\text{vac}}}{L} + 1. \tag{14}$$

In addition to the experimental determination of the refractive index of a gas it is also calculated using the Lorentz-Lorenz law which connects the refractive index to the polarizibility of a gas:

$$\frac{n^2-1}{n^2+1} = \frac{Ap}{RT}. \tag{15}$$

Here, R is the universal gas constant, T is the ambient temperature, p is the pressure and A is the molrefraction value.

3 Measurement

For the measurements of the refractive indices of glass and air a Sagnac interferometer is chosen. Due to its stability compared to other interferometer designs like Michelson-Morley or Mach-Zender. This good stability is achieved because both interferometer arms are using the same beam path, see figure 2.

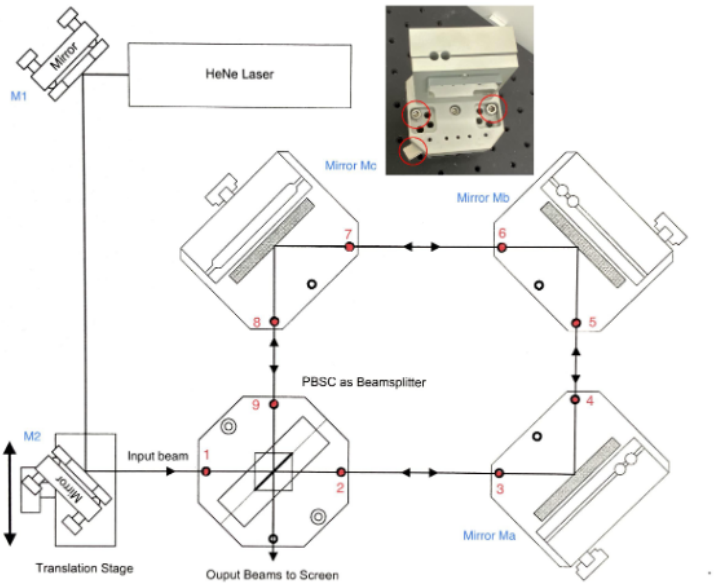

Figure 2: Experimental setup of the Sagnac interferometershowing the beam path from the HeNe-Laser through the PBSC [1].

The Helium-Neon (HeNe)-Laser used for the interferometer setup has a wavelength of $\lambda_{\mathrm{NeNe}} = 632{,}99\,\mathrm{nm}$. It is passed into the interferometer using two adjustable mirrors (M1 and M2), where the second is mounted on a linear stage, allowing to move the beam transversely without changing its angle. Additionaly a linear polarization filter is installed in between mirror M2 and the PBSC. The filter is needed to control the intensity of both the interferometer arms. If it is set at 45° relativ to the s- and p- polarized light both of them have the same intensity. After the PBSC both arms run antiparralel to each other through the interferometer. The output is then transmitted or reflected at the PBSC and passed through another polarization filter needed to isolate the parts of the beam which are able to interfere with each other. Before the intensity is measured the beam is split once more by another PBSC allowing to measure the s- and p-polarized parts individually and enabling the implementation of a differential amplifier, whose signal is used to count the number of maxima by the read-out electronics.

3.1 Alignment

The first step to aligning the used experimental setup is to center the beam on the mirrors M1 and M2 which are then used to align through the center of the PBSC. A beam walk allows to then align the beam through provided adjustment plates, which are mounted in the red holes shown in figure 2. After the first alignment it is necessary to look at the overlapping beams behind the interferomter on a screen. A polarization filter set at 45° allows to observe the interference pattern which has fringes in it, produced by roughly different beam paths and thus phase differences leading to interference. Eliminating the fringes then results in perfectly parallel beams along the entire interferometer. Shifting M2 on its linear stage then seperates the s- and p-polarized beams into two horizontally offset beams.

3.2 Measurement of the Contrast

After aligning the interferometer its contrast is measured. For this purpose a rotation glass pane holder is installed in the beam. To now determine the contrast as a function of the polarization direction of the input laser beam, the diode voltage of just one diode is measured. For each polarization direction in an intervall of $[0 - 180°]$ in 15° steps the diode voltage is recorded for the interference maximum and minimum. The interference maxima and minima are adjusted using the glass panes in the rotation holder. Afterwards the polarizer is set to the maximum contrast for the following measurements.

3.3 Measurement of refractive indices

For the measurement of refractive indices the number of maxima is measured using both diodes connected to the Modern Interferometry Controller. The read-out electronics then count the number of zero crossings hence the number of maxima or minima. Slowly rotating the glass holder in an intervall $[0°, 8°]$ while counting the zero crossings allows to measure the refractive index of glass. Analogue the refractive index of air is measured by installing a gas cell in one beam path with the length $L = (100{,}0 \pm 0{,}1)\,\mathrm{mm}$, slowly increasing the cell pressure from a vacuum and counting the zero crossings. These measurements are repeated five times.

4 Results

4.1 Determination of the Contrast

The data taken according to section 3.2 are found in table 1. Using equation 3 the contrast is calculated. An error estimation is done by assigning each of the measured intensities an uncertainty of $\sigma_I = 0{,}05\,\mathrm{V}$. The gaussian error propagation then yields the following expression for the contrasts uncertainty

$$\sigma_K = \sqrt{\left(\frac{2 * I_{\mathrm{min}}\sigma_I}{(I_{\mathrm{min}} + I_{\mathrm{max}})^2}\right)^2 + \left(\frac{2 * I_{\mathrm{max}}\sigma_I}{(I_{\mathrm{min}} + I_{\mathrm{max}})^2}\right)^2}.$$

The calculated values are shown in figure 3, together with a theoretical curve given by equation 8.

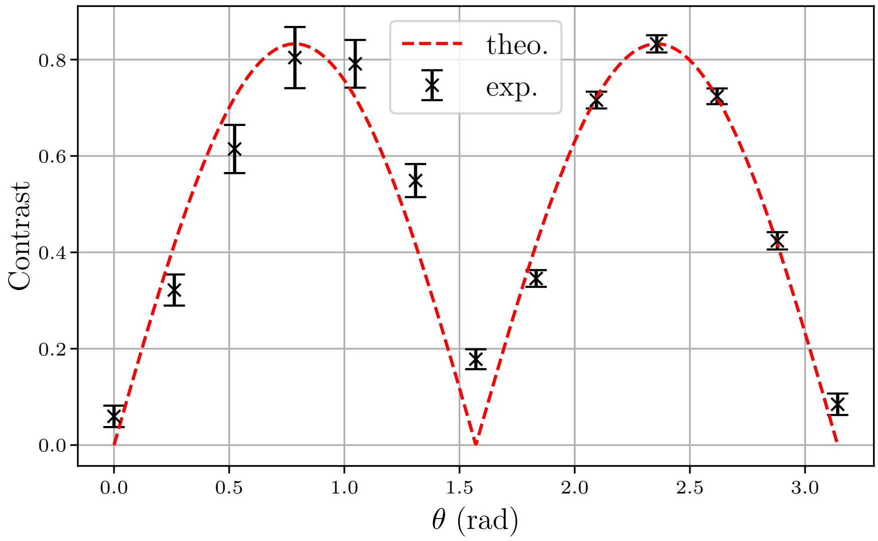

Figure 3: Comparison of the theoretical curve and experimental values for the contrast of the used Sagnac interferometer against the input laser lights polarization angle.

The maximum contrast $K = 0{,}83 \pm 0{,}02$ was measured at 135° and used for the further measurements.

4.2 Measurements of refractive indices

The data taken according to section 3.3 are found in tables 2 and 3 respectively for the measurement with glass and air. From the number of maxima produced by a given phase shift calculated from equation 9 and the phase shift induced by a glass pane given by equation 10 an expression for the number of maxima in dependence of the refractive index is derived

$$M(n) = \frac{2L}{\lambda_{\text{vac}}} * \frac{n-1}{n}\theta_0\theta$$

After taking the mean of the number of measured maxima for each angle over all series of measurement the refractive index of glass follows from a linear fit of the equation above. Here, the uncertainty of the number of maxima is estimated by there standard deviation. This linear fit yields the determined value of the refractive index of glass

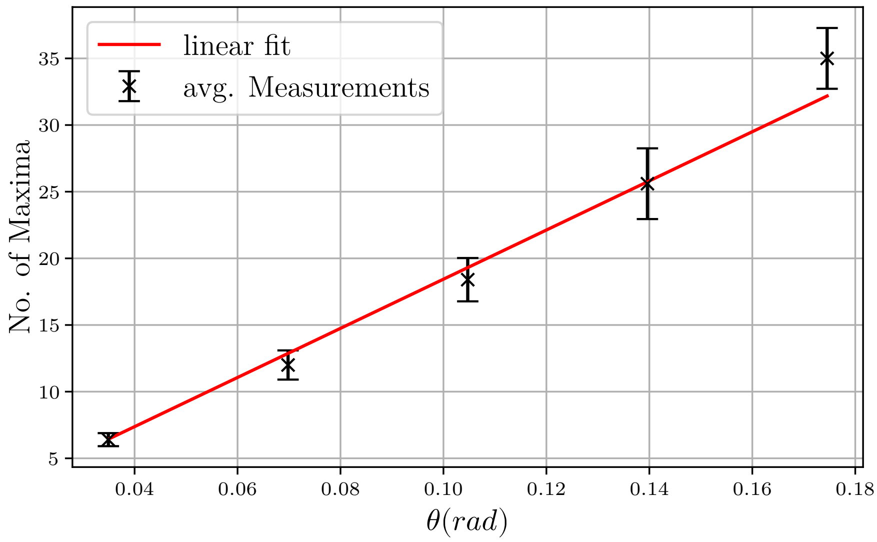

Figure 4: Number of measured maxima against the angle of the rotation holder with the glass panes, including a linear fit to determine the glasses refractive index.

$$n_{\text{glass}} = 1{,}50 \pm 0{,}02.$$

For the determination of the refractive index of air an analogue ansatz is chosen. Firstly the mean over the series of measurement from M_3 to M_6 is taken. M_1 and M_2 are excluded, because irregularities occured while they were recorded. With equation 14 the corresponding refractive indices are calculated, an error is estimated from the uncertainties on the cell length and the standard deviation of the number of maxima according to the gaussian error propagation

$$\sigma_n = \sqrt{\left(\frac{\lambda_{\text{vac}}}{L}\sigma_M\right)^2 + \left(-\frac{\lambda_{\text{vac}}M}{L^2}\sigma_L\right)}.$$

From these values the refractive index of air at standard atmosphere (15 °C, 1030 hPa)

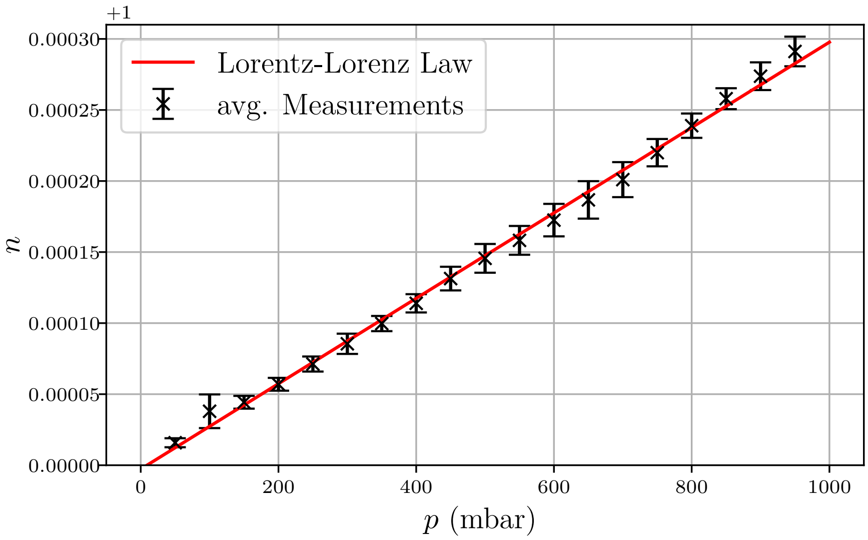

Figure 5: Experimental values of the refractive index of air plotted against the pressure of the installed gas cell and linear fit with the Lorentz-Lorenz Law.

are determined using the Lorentz-Lorenz Law 15. Equation 15, however, simplifies to

$$n \approx \frac{3}{2}\frac{Ap}{RT} + 1$$

for a refractive index $n \approx 1$. Fitting the experimentally determined values with a linear equation

$$n(p) = \frac{3}{2}\frac{p}{RT_0}a + b,$$

where $T_0 = 294{,}25\,\mathrm{K}$ is the ambient temperature while conducting the experiment. The fit yields

$$a = (4{,}90 \pm 0{,}07)\,\mathrm{mol/m^{-3}} \qquad b = 9{,}999\,97 \pm 0{,}000\,02$$

From this the refractive index at standard atmosphere is calculated to be

$$n_{\mathrm{air}} = 1{,}0003 \pm 0{,}0002$$

The error was calculated from an error propagation with the uncertainties on the fit parameters

$$\sigma_n = \sqrt{\left(\frac{3}{2}\frac{p}{RT}\sigma_a\right)^2 + \sigma_b^2}$$

5 Discussion

The comparison between the polarisation angle and contrast exhibited a satisfactory agreement between the expected theoretical curve and the observed data. The maximum contrast ($K_0 = 0.83 \pm 0.02$), although not reaching the ideal contrast ($K = 1$), suggests a potential suboptimal alignment. The experiment yielded a refractive index for glass ($n_{\text{glass, exp}} = 1.50 \pm 2$) [3], slightly deviating by 1.3% from the theoretical value ($n_{\text{glass, theory}} \approx 1.52$) [3] for crownglas. It has to be noted however that the refractive indices for different glas vary, which means that because the glas type is not known an accurate theoretical value is not known. Considering general experimental uncertainties and the unknown composition of the glass panes used, the determined refractive index for glass is deemed sufficiently precise. Lastly, the refractive index of air under standard atmosphere conditions ($15^{\circ}\mathrm{C}, 1013\,\mathrm{hPa}$) was measured as $n_{\text{air, exp}} = 1.0003 \pm 0.00002$, exhibiting a deviation of ($\mathit{\Delta}_{\text{rel}}(n_{\text{air}}) < 0.001\%$) from the theoretical value ($n_{\text{air, theory}} = 1.000276$) [3]. This slight discrepancy could be attributed, for instance, to the influence of humidity in the air, which was not considered in the theoretical value.

6 Appendix

Table 1: Measurement of the polarization angle against the intensity minima and maxima.

$\phi(^\circ)$	$I_{\text{min}}(V)$	$I_{\text{max}}(V)$
0	1.50	1.69
15	0.78	1.52
30	0.32	1.34
45	0.14	1.29
60	0.19	1.63
75	0.53	1.82
90	1.43	2.05
105	1.41	2.90
120	0.71	4.29
135	0.43	4.71
150	0.74	4.62
165	1.23	3.04
180	1.46	1.73

Table 2: Measurements of the number of interference maxima against the angle of the glass pane.

$\theta(^\circ)$	M_1	M_2	M_3	M_4	M_5
2	7	6	7	6	6
4	12	12	13	10	13
6	19	18	21	16	18
8	26	29	27	21	25
10	36	37	37	31	34

Table 3: Measurements of the number of interference maxima against the pressure of the gas chamber filled with air.

p(mbar)	M_1	M_2	M_3	M_4	M_5	M_6
50	3	4	2	3	3	2
100	7	7	9	5	6	4
150	9	9	7	7	8	6
200	11	11	9	9	10	8
250	14	16	12	11	12	10
300	20	19	15	13	14	12
350	23	23	17	15	16	15
400	25	27	19	17	19	17
450	27	31	22	20	22	19
500	30	34	25	22	24	21
550	33	40	27	24	26	23
600	35	45	29	26	29	25
650	37	50	32	28	31	27
700	39	53	34	31	33	29
750	44	57	37	34	35	33
800	0	61	39	36	39	37
850	0	64	41	39	42	41
900	0	67	45	41	44	43
950	0	72	48	44	47	45

References

[1] Fakultät Physik, TU Dortmund. *V64 Interferometry*. 2023.

[2] Edmund Optics. *What are Beamsplitters?*. `https://www.edmundoptics.eu/knowledge-center/application-notes/optics/what-are-beamsplitters/`. Accessed on 2023-12-20.

[3] Spektrum der Physik. *Brechzahl*. `https://www.spektrum.de/lexikon/physik/brechzahl/1958#:~:text=Brechzahl%3A%20Absolute%20Brechzahlen%20verschiedener%20Materialien,einer%20Wellenl%C3%A4nge%20von%20589%20nm.&text=und%20damit%20auch%20dn%2Fd%CF%89,anomale%20Dispersion%20vor%20(Dispersion).`. Accessed on 2023-01-04.