V47 Temperature dependence of molar heat of copper

Abstract

In this experiment, different models for explaining the temperature dependence of the molar heat of crystalline solids are compared. The experiment allows to measure the molar heat of a homogeneous metal sample as a function of temperature. The results are to be used to determine the Debye temperatur Θ_D.

References

[1] Rudolf Gross and Achim Marx, *Festkörperphysik*, De Gruyter (2022)

[2] J. B. Ketterson, *The physics of solids*, Oxford (2016) p.48 - 58

[3] Vimal Kumar Jain, *Solid state physics*, Springer (2022), pp 191 - 213

[4] Thomas A. Hahn, J. Applied Physics **41**, pp. 5096

[5] Landolt-Börnstein, 6. Aufl. Band II, 4.Teil p.478

Preparation

On the basis of the literature you should be able to answer the following guiding questions:

- How is the heat capacity of a solid defined?

- Why does the heat capacity differ depending on wether the pressure or the volume is kept constant? Why is the difference between these heat capacities relatively small in the solid, but very pronounced in the ideal gas?

- What is the classical expectation of the molar heat of copper? Where does the effective number of 6 dregrees of freedom per atom come from?

- What does the Einstein model for the molar heat of a solid include? What temperature dependence does the model predict for heat capacity? Expect (consider realistic phonon dispersions in the solids) a good modelling using the Einstein model?

- What does the phonon dispersion of a solid with a monoatomic base look like schematically? How can you determine the speed of sound from this?

- What does the Debye model for the molar heat of a solid include? What temperature dependence does the model predict for the heat capacity?

- How do you qualitatively correct the difference between the experimentally measured heat capacity at constant pressure and the theoretically modelled heat capacity at constant volume?
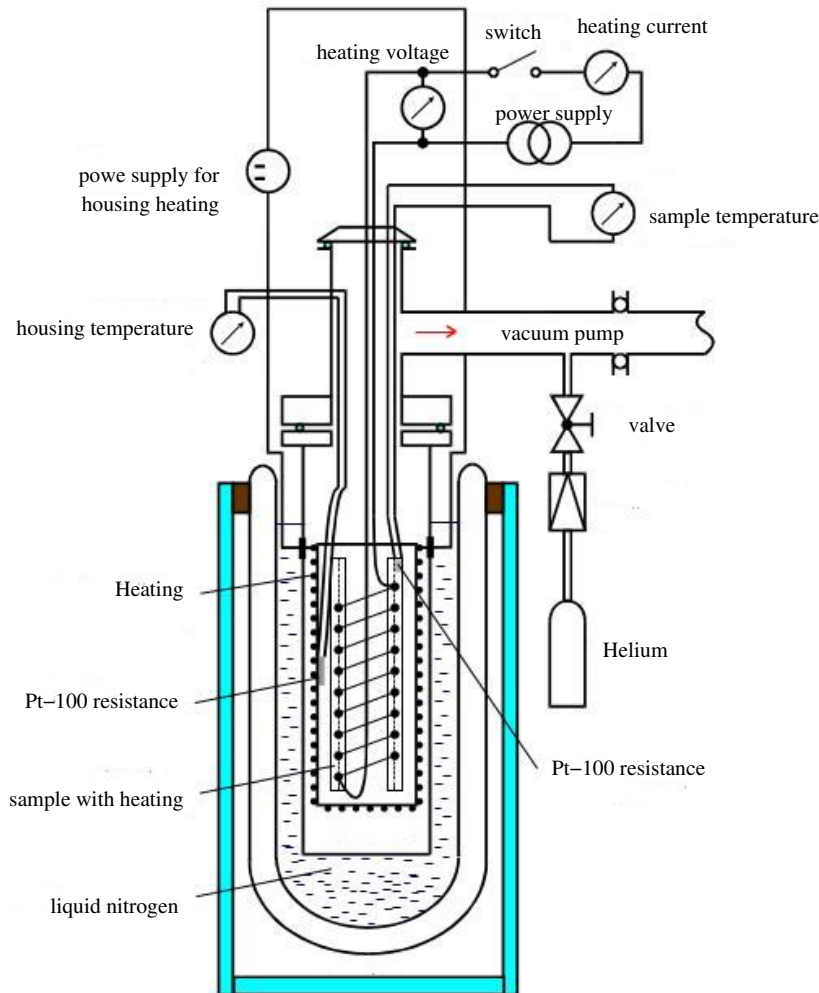
Figure 1: Experimental setup

Experimental setup

The measurement is carried out with the apparatus shown in Fig.1. First evacuate the recipient, then fill it with helium at barometric pressure and cool it down to approx. 80 K by filling the Dewar surrounding the recipient with liquid nitrogen. When the final temperature is reached after approx 1 h, the vacuum pump is switched on again and the internal pressure is reduced to as low as possible. A noticeable reduction in pressure can be achieved by keeping the recipient constantly at nitrogen temperature.

During the actual measuring process, a defined electrical energy E is applied to the cooled sample via heating coil and the temperature increase ΔT is measured. From the mass of the sample (m = 342 g), its molecular weight M, ΔT and E, one can calculate the molar heat C_p of the sample material. To determine the supplied energy E, the voltage U applied to the

heating coil, the current I flowing through it and the heating duration time t must be known. E should be chosen so that ΔT is between 7 degrees and 11 degrees.

During the measurement, it is important that the electrical energy E is only used to heat the sample. Energy losses through convection, heat radiation and heat conduction must be eliminated. Therefore, the recipient is evacuated and it is ensured that the copper cylinder that completely surrounds the sample has the same temperature as the sample. The copper cylinder has its own heating coil and temperature measuring device for this purpose. The thermometer used here are Pt-100 resistors whose resistance is a molecular function of the temperature. The exact data can be found in the table below. Linear interpolation is possible within a temperature intervall of 10 degrees. It is possible to calculate T via the equation

$$T = 0.00134\,R^2 + 2.296\,R - 243.02 \tag{1}$$

T [$^\circ C$]	-200	-190	-180	-170	-160	-150	-140	-130	-120
R [Ω]	18.44	22.71	27.03	31.28	35.48	39.65	43.80	47.93	52.04

T [$^\circ C$]	-110	-100	-90	-80	-70	-60	-50	-40	-30
R [Ω]	56.13	60.20	64.25	68.28	72.29	76.28	80.25	84.21	88.17

T [$^\circ C$]	-20	-10	0	+10	+20	+30	+40
R [Ω]	92.13	96.07	100.00	103.90	107.79	111.67	115.54

Table 1: Temperature of Pt-100 thermometer as a function of resistance R.

Measurements and analysis

- Measure the molar heat C_p of copper as a function of temperature in the range of approx. 80 K to 300 K.

- Calculate C_V with the help of the formula

$$C_p - C_V = 9\,\alpha^2\,\kappa\,V_0\,T \tag{2}$$

and plot these quantities in a linear diagram against T. The values for α can get by extrapolation from the following table[4].

T [K]	60	80	90	100	120	140	150	163
α [$10^{-6}/deg$]	5.40	8.21	9.33	10.33	11.97	13.20	13.68	14.19

T [K]	180	200	240	250	280	295	300	323
α [$10^{-6}/deg$]	14.73	15.18	15.81	15.98	16.39	16.56	16.61	16.86

Table 2: Linear coefficient of expansion α of copper as a function of temperature[4].

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- Plot the universal Debye curve $C_V = f(\frac{\Theta_D}{T})$ which is given in the following table. Fit the measured (C_V, T)-pairs by choosing a suitable debye temperature Θ_D in the plottet Debye curve. For this purpose, only measured values up to $T_{max} = 170\ K$ are taken into account. What value for Θ_D is obtained?

Work instruction for table 3: the table is intended to make it easier for you to convert experimentally observed heat capacities into values for $\frac{\Theta_D}{T}$. Example: if you determine $C_v = 9.998\ J/mol\ K$ (numer is written in bold), the corresponding value is $\frac{\Theta_D}{T} = 6.3$, i.e. the left column indicates the digit before the decimal point, the row position the first decimal place.

$\frac{\Theta_D}{T}$	0	1	2	3	4	5	6	7	8	9
0	24.9430	24.9310	24.8930	24.8310	24.7450	24.6340	24.5000	24.3430	24.1630	23.9610
1	23.7390	23.4970	23.2360	22.9560	22.6600	22.3480	22.0210	21.6800	21.3270	20.9630
2	20.5880	20.2050	19.8140	19.4160	19.0120	18.6040	18.1920	17.7780	17.3630	16.9470
3	16.5310	16.1170	15.7040	15.2940	14.8870	14.4840	14.0860	13.6930	13.3050	12.9230
4	12.5480	12.1790	11.8170	11.4620	11.1150	10.7750	10.4440	10.1190	9.8030	9.4950
5	9.1950	8.9030	8.6190	8.3420	8.0740	7.8140	7.5610	7.3160	7.0780	6.8480
6	6.6250	6.4090	6.2000	**5.9980**	5.8030	5.6140	5.4310	5.2550	5.0840	4.9195
7	4.7606	4.6071	4.4590	4.3160	4.1781	4.0450	3.9166	3.7927	3.6732	3.5580
8	3.4468	3.3396	3.2362	3.1365	3.0403	2.9476	2.8581	2.7718	2.6886	2.6083
9	2.5309	2.4562	2.3841	2.3146	2.2475	2.1828	2.1203	2.0599	2.0017	1.9455
10	1.8912	1.8388	1.7882	1.7393	1.6920	1.6464	1.6022	1.5596	1.5184	1.4785
11	1.4400	1.4027	1.3667	1.3318	1.2980	1.2654	1.2337	1.2031	1.1735	1.1448
12	1.1170	1.0900	1.0639	1.0386	1.0141	0.9903	0.9672	0.9449	0.9232	0.9021
13	0.8817	0.8618	0.8426	0.8239	0.8058	0.7881	0.7710	0.7544	0.7382	0.7225
14	0.7072	0.6923	0.6779	0.6638	0.6502	0.6368	0.6239	0.6113	0.5990	0.5871
15	0.5755	0.5641	0.5531	0.5424	0.5319	0.5210	0.5117	0.5020	0.4926	0.4834

Table 3: Numerical values of the Debye function for R = 8.31439 (J/mol deg)[5]. Molar heat C_v in [J/mol].

- Calculate ω_D and Θ_D for copper from the requirement

$$\int z(\omega)d\omega = 3\,N_L \tag{3}$$

Compare the result with the value obtained from the fit. ($v_{long} = 4.7 km/h$ and $v_{trans} = 2.26 km/h$)