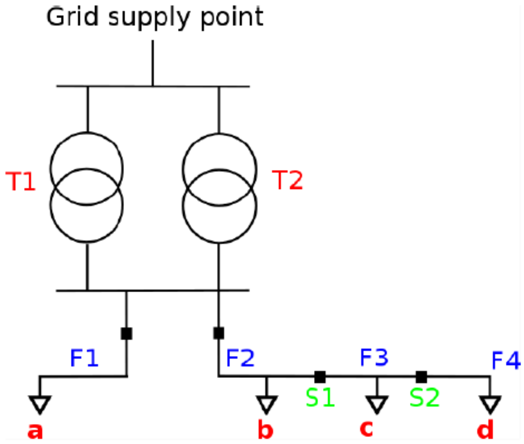
Distribution system reliability analysis

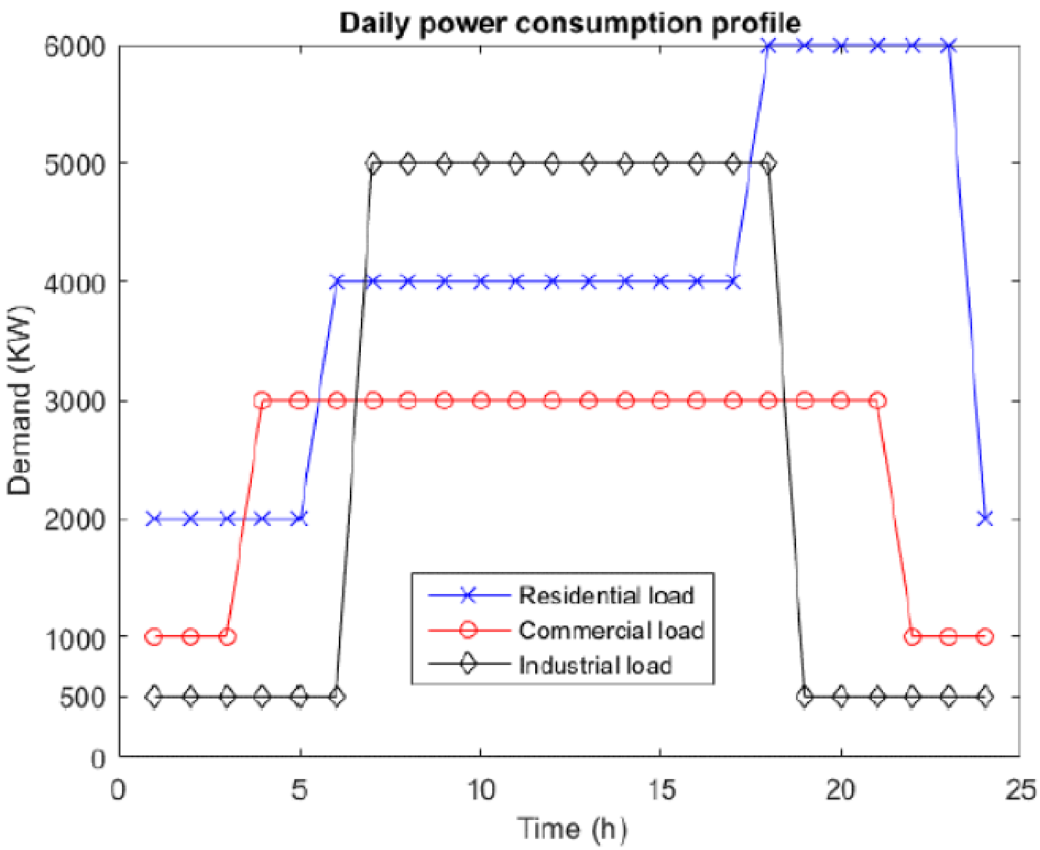
The reliability of the following power distribution system will be studied with three methods:

1. **Analytical**
2. **State-sampling Monte Carlo**
3. **Time-sequential Monte Carlo**

The last two will be performed in matlab.



Component	Fault process	Repair process
T1	MTTF=20 years	MTTR = 5 days
T2	MTTF=25 years	MTTR= 2 days
F1	MTTF = 6 years	k=3 , θ=3
F2	Failure rate = 0.1/year	k=5 , θ=2
F3	Failure rate= 0.1/year	k=5, θ=3
F4	Failure rate = 0.2/year	k=8 , θ=3



Important: Switches S1 and S2 have a 10% failure chance when an error occurs causing the error to propagate upstream.

1: MTTF, MTTR , MTBF.

We know that:

1. MTBF = MTTF + MTTR
2. $Availability \ = \ \frac{(uptime)}{(uptime + downtime)} \times \ 100\% \ = \frac{MTTF}{MTBF} \times \ 100\%$
3. For the Gamma distribution it is know that:
   $p(x) \ = \ \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}$
   $mean \ = \ k\theta$

- T1:
  - MTTF = $20 \ \times \ 8,760 \ = \ 175200$ hours
  - MTTR = $5 \ \times \ 24 \ = \ 120$ hours
  - MTBF = $175200 \ + \ 120 \ = \ 175320$
  - $Availability \ = \ \frac{175200}{175320} \times \ 100\% \ = \ 99.9316\%$
- T2:
  - MTTF = $25 \ \times \ 8,760 \ = \ 219000$ hours
  - MTTR = $2 \ \times \ 24 \ = \ 48$ hours
  - MTBF = $219000 \ + \ 48 \ = \ 219048$
  - $Availability \ = \ \frac{219000}{219048} \times \ 100\% \ = \ 99.9781\%$

- F1:
  - MTTF = $6 \ \times \ 8,760 \ = \ 52560$ hours
  - MTTR = $3 \ \times \ 3 \ = \ 9$ hours
  - MTBF = $52560 \ + \ 9 \ = \ 52569$
  - $Availability \ = \ \frac{52560}{52569} \times \ 100\% \ = \ 99.9829\%$

- F2:
  - MTTF = $\frac{1}{0.1} \times \ 8,760 \ = \ 87600$ hours
  - MTTR = $5 \ \times \ 2 \ = \ 10$ hours
  - MTBF = $87600 \ + \ 10 \ = \ 87610$

- $Availability\ =\ \frac{87600}{87610} \times\ 100\%\ \ = 99.9886\%$

- F3:
  - MTTF = $\frac{1}{0.1} \times\ \ 8,760\ \ = 87600$ hours
  - MTTR = $5\ \times\ \ 3\ \ =\ \ 15$ hours
  - MTBF = $87600\ +\ 15\ \ = 87615$
  - $Availability\ =\ \frac{87600}{87615} \times\ 100\%\ \ = 99.9829\%$

- F4:
  - MTTF = $\frac{1}{0.2} \times\ \ 8,760\ \ = 43800$ hours
  - MTTR = $8\ \times\ \ 3\ \ =\ \ 24$ hours
  - MTBF = $43800\ +\ 24\ \ = 43824$
  - $Availability\ =\ \frac{43800}{43824} \times\ 100\%\ \ = 99.9452\%$

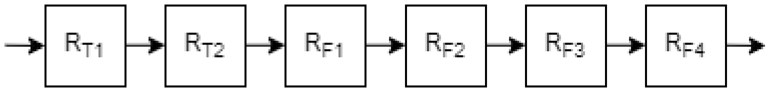
	MTTF (hours)	MTTR (hours)	MTBF (hours)	Availability
T1	175200	120	175320	99.9316%
T2	219000	48	219048	99.9781%
F1	52560	9	52569	99.9829%
F2	87600	10	87610	99.9886%
F3	87600	15	87615	99.9829%
F4	43800	24	43824	99.9452%

Notice: All numbers are rounded to 6 decimals. (4 for percentages)

2: Reliability block diagram & LOLP.

For no Loss of load, we need all six components to work simultaneously, in order to supply every load. This means that the reliability blocks will be in series.

$R_{total} = R_{T1} \times R_{T2} \times R_{F1} \times R_{F2} \times R_{F3} \times R_{F4}$



$Availability = A_{T1} \times A_{T2} \times A_{F1} \times A_{F2} \times A_{F3} \times A_{F4} =$
$= 0.999316 \times 0.999781 \times 0.999829 \times 0.999886 \times 0.999829 \times 0.999452$
$Availability = 0.998094$
$Unavailability = 1 - Availability = 0.1906\%$

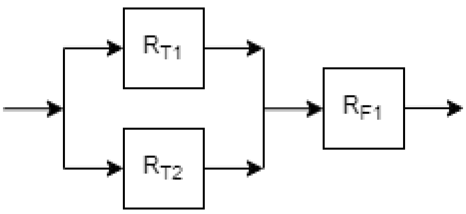
The loss of load probability is the unavailability of the system and therefore:

$LOLP = 0.1906\%$

3: Reliability block diagrams & Unavailabilities.

From the schematic of the power supply system it is clear that if feeder F1 gets damaged, not only load b but also c and d will lose power since they are connected in series. Similarly when F2 fails, load d will lose power too.

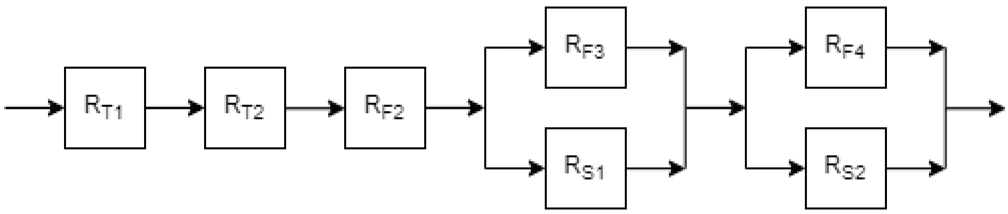
- For load **a**, we need at least one of the transformers to work and at the same time the feeder F1 to work.
  $R_a = \left(R_{T1} || R_{T2}\right) \times R_{F1}$



$Availability = \left(A_{T1} || A_{T2}\right) \times A_{F1} = \left(1 - \left(1 - A_{T1}\right)\left(1 - A_{T2}\right)\right) \times A_{F1}$
$= (1 - (1 - 0.999316)(1 - 0.999781)) \times 0.999829 = 0.99982885023$
$Unavailability = 1 - Availability = 0.0171\%$

- For load **b**, we need both transformers to work and at the same time the feeder F2 to work. Also, the feeder F3 must not fail simultaneously with the switch S1 which means those two are connected in parallel in the RBD.

  $R_b = R_{T1} \times R_{T2} \times R_{F2} \times (R_{F3}||R_{S1}) \times (R_{F4}||R_{S2})$



  $Availability = A_{T1} \times A_{T2} \times A_{F2} \times (A_{F3}||A_{S1}) \times (A_{F4}||A_{S2})$

  $\left(A_{T1} \times A_{T2} \times A_{F2}\right) = 0.999316 \times 0.999781 \times 0.999886 = 0.998983$

  $A_{F3}||A_{S1} = 1 - \left(\left(1 - A_{F3}\right)\left(1 - A_{S1}\right)\right) = 0.999983$

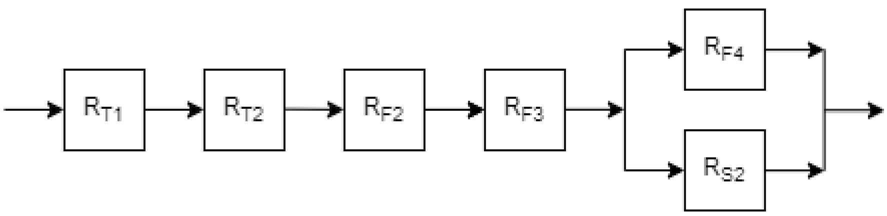
  $\left(A_{F4}||A_{S2}\right) = 1 - \left(\left(1 - A_{F2}\right)\left(1 - A_{S2}\right)\right) = 0.999945$

  $Availability = 0.998983 \times 0.999983 \times 0.999945 = 0.998911$

  $Unavailability = 1 - Availability \simeq 0.1089\%$

- For load **c**, we need both transformers to work and at the same time the feeders F2,F3 to work. Also, the feeder F4 must not fail simultaneously with the switch S2.

  $R_c = R_{T1} \times R_{T2} \times R_{F2} \times R_{F3} \times (R_{F4}||R_{S2})$



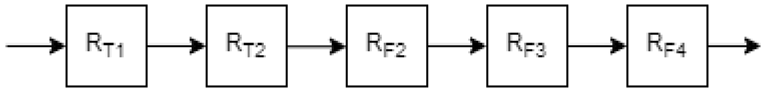
  $Availability = A_{T1} \times A_{T2} \times A_{F2} \times A_{F3} \times (A_{F4}||A_{S2})$

  $\left(A_{T1} \times A_{T2} \times A_{F2} \times A_{F3}\right) = 0.998983 \times 0.999829 = 0.998812$

  $\left(A_{F4}||A_{S2}\right) = 0.999945$

  $Availability = 0.998812 \times 0.999945 = 0.998757$

  $Unavailability = 1 - Availability \simeq 0.1243\%$

- For load **d**, we need both transformers to work and at the same time the feeders F2, F3 and F4 to function.
  $R_b = R_{T1} \times R_{T2} \times R_{F2} \times R_{F3} \times R_{F4}$
  

  $Availability\ = A_{T1} \times A_{T2} \times A_{F2} \times A_{F3} \times A_{F4}$
  $Availability\ =\ 0.998812\ \times\ 0.999452\ =\ 0.998265$
  $Unavailability\ =\ 1\ -\ Availability\ \simeq\ 0.1735\%$

4: CML

To calculate the *cml* (customer minutes lost), we need first to find the minutes that each load will be out of power per year. There are around 525,948 minutes per year.

$Loss_a\ =\ 0.000171\ \times\ 525948\ \simeq\ 89.9\ minutes/year$

$Loss_b\ =\ 0.001089\ \times\ 525948\ \simeq\ 572.8\ minutes/year$

$Loss_c\ =\ 0.001243\ \times\ 525948\ \simeq\ 653.8\ minutes/year$

$Loss_d\ =\ 0.001735\ \times\ 525948\ \simeq\ 912.6\ minutes/year$

The total *cml* is the average loss of minutes per customer and therefore each loss will be weighted by the corresponding customer proportion.
$P_{total}\ =\ 3100$
$P_a = P_b = P_d\ =\ 1000/3100\ \simeq\ 0.3226$
$P_c\ =\ 100/3100\ \simeq\ 0.03226$
$cml = P_a \times Loss_a + P_b \times Loss_b + P_c \times Loss_c + P_d \times Loss_d$
$=\ 0.3226\ \times\ 89.9\ +\ 0.3226\ \times\ 572.8\ +\ 0.03226\ \times\ 653.8\ +\ 0.3226\ \times\ 912.6$

$CML\ \simeq\ 529.3$ *Minutes lost per year per customer*

5: State-sampling Monte Carlo

To define the states of this system we must first define the state variables. There are six components that can be either working or not, so this gives us six binary variables: $v_{T1}, v_{T2}, v_{F1}, v_{F2}, v_{F3}, v_{F4}$.
Each of those is 1 if the corresponding component is functioning and 0 if it is not. However in order to take into account the probabilities of S1 and S2 without using redundant variables, we will assign 3 possible values to v_{F3}, v_{F4}: 0 for feeder failure while the corresponding switch also fails, 1 for feeder failure without switch failure and 2 for normal function. State 111122 means that everything is functioning while state 000000 means that everything has failed simultaneously.

Based on the reliability blocks of A3 one can easily infer which loads are getting supplied successfully at each state. The probabilities of each binary variable being 0 or 1 are equal to the availability of the corresponding component. For the other two, the probabilities of the switches failing must also be taken into consideration.

To also take into account the demand, the states should also include information about the time of the day it is. Another variable t can be defined that will take (integer) values between 1 and 24 (inclusive).

The function *"return_value"* samples a random state based on the probabilities in the following table.

Variable	Pr (value = 0)	Pr (value = 1)	Pr (value = 2)
v_{T1}	0.000684	0.999316	-
v_{T2}	0.000219	0.999781	-
v_{F1}	0.000171	0.999829	-
v_{F2}	0.000114	0.999886	-

v_{F3}	0.0000171	0.0001539	0.999829
v_{F4}	0.0000548	0.0004932	0.999452

Function *"Is_Supplied"* takes a system state as input and returns info about which loads are getting power supply at that state.

6: LOL, CML & ENS

- For a single state, the LOL is a binary variable that is 0 if at least one load receives no supply and 1 otherwise. By sampling multiple states and averaging the value of LOL, one can estimate the LOLP.

- The CML is defined over the course of a year, and therefore to calculate it for a single state we have to assume the system stays at that state for a whole year. By sampling multiple states and averaging the CML for each we get the actual CML of the system.

- The ENS is defined as:

  $ENS = \sum_{j} L_{a(j)} U_{j}$ (kWh)

  Where $L_{a(j)}$ is the average load connected at point j and U_{j} is the annual outage time.

The *"evaluate_state"* function takes as input a state and returns the ENS and CML and LOL. It's important to notice that those values correspond to the case where the power system stays at the given state for a whole year.

7: LOLP, expected CML & EENS

The *“run_simulations”* function performs the State Sampling Monte Carlo simulation process and gives the following results. The Confidence intervals are calculated at 95% for 10^8 samples.

Metric	CI low	CI high
LOL	0.0019	0.0019
Expected CML	526	531
EENS	86677 kWh	87614 kWh

It is apparent that the values for LOL and Expected CML agree with the results obtained by the analytical solution.

8: Time-sequential Monte Carlo (Markov Model)

In this case we will use again the 6 variables of the state-sampling method that will define the states. Time will not be a state variable but will be taken into account separately during the simulation process. The chosen discretization step will be *1h*.
To define the Markov Model we need two things: The failure rates and the repair rates. Those numbers define the probabilities with which each component will be functioning or not in the next state. For this analysis we consider the failure and repair rates to be constant and will be expressed in $hours^{-1}$.

	MTTF (hours)	Failure rate λ ($hours^{-1}$)	MTTR (hours)	Repair rate μ ($hours^{-1}$)
T1	175200	0.00000570776	120	0.00833333333
T2	219000	0.00000456621	48	0.02083333333
F1	52560	0.00001902587	9	0.11111111111
F2	87600	0.00001141552	10	0.1
F3	87600	0.00001141552	15	0.06666666666
F4	43800	0.00002283105	24	0.04166666666

If a non-uniform distribution (like gamma) would be used for the repair process we would also have to take into account the time since the failure happened to infer the probability of the system getting repaired at any given moment.

- The *"step"* function takes as input the current state of the system and returns the next state, sampled according to the transition probabilities that depend on the values of μ and λ for each component.

- The *"yearly_evolution"* function performs 8766 hourly steps to simulate the evolution of the system over a year. It starts at time =

1 and initial state = [1,1,1,1,2,2] which means all components are functioning. It also takes a parameter “samples” as input that corresponds to the amount of years that will be simulated.

By running *yearly_evolution(2000)* the following results are obtained.

Metric	Average
Expected CML	523.3 minutes
EENS	86688 kWh

The estimations are pretty close to the numbers of the previous sections, given the small amount of total simulations conducted here. By increasing the amount of simulations we get even better results.