ME 2450 Assignment HW1b

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Submit this assignment as a single PDF file to **gradescope**.

I declare that the assignment here submitted is original except for source material explicitly acknowledged.

I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website.

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Homework Formatting Tips
(These guidelines apply to this and all future HW assignments)

- Axes Labels and Legends on Plots:
  Please ensure that all your plots have clearly labeled axes and legends if applicable. This is crucial for understanding your visual representations. A plot without labeled axes does not mean anything!

- Avoid Printing and Scanning Code:
  Kindly refrain from printing out your code and then scanning it. Instead, export your code electronically in a readable format, such as a text document or pdf/html file. Codes that are printed then scanned are very hard to read and will result in reduced or zero credit.

- Proper Cropping of Screenshots:
  If you need to include screenshots in your submissions, please crop them to only contain the relevant regions. Including unnecessary parts of the screen makes it harder to assess your work efficiently. Also make sure that screenshots included are high quality and high resolution.

- Include Plots and Tables:
  It is imperative that you include the actual plots and tables with your submission. Merely providing the code used to generate them is not sufficient. Likewise, having the final plots or tables without the corresponding code is incomplete. Both components are necessary for a comprehensive evaluation of your work.

- Explain Your Process:
  It can be challenging for me to understand how you generated the plot or table. Please make sure to provide a brief explanation of your process in forms of comments, even if you encounter difficulties or errors in your code.

Q1 (2 pt + bonus 1 pt)

Express the *unsigned* binary number 11111111 in base-10. (Show your work)

1 bonus point: Write a piece of computer code that takes as input a base-2 integer and prints its base-10 representation. Prove that it's working by running 3 other (not 11111111) base-2 numbers as input and showing printed output. Include a screenshot of the outputs, AND your code, in your submission.

Q2 (2pt + bonus 1 pt)

Express the base-10 number 11111111 as a *signed* binary number. How many bits are required to do so? (Show your work)

1 bonus point: Write a fully commented computer code that takes as input a base-10 integer and prints its base-2 representation. Prove that it's working by running 3 other (not 11111111) base-10 numbers as input and showing printed output. Include a screenshot of the outputs, AND your code, in your submission.

Q3 (5pt)

Consider the function

$$f(x) = \frac{5x}{(1-2x^2)^2}$$

a) Evaluate the function at x = 0.423 using 3-digit arithmetic with chopping. Report the value obtained and the true relative error. (Turn in hand calculations only. No computer code is required.)

b) Repeat part (a) except using 4-digit arithmetic with chopping.

HINT: Consider each arithmetic operation separately. First, calculate the numerator, i.e., $5x = (5) * (0.423)$and chop the answer to specified number of significant digits. Then, calculate $x^2 = (0.423) * (0.423)$and chop the answer to the specific number of significant digits, and so forth. After each operation, the resulting answer can only have 3 significant digits for part (a), and 4 significant digits for part (b).

Q4 (6 pts)

The quantity e^{-5} may be determined using the following two different formulas

$$\text{Formula 1 : } e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$\text{Formula 2 : } e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots}$$

a) Write a *fully commented* computer code to evaluate formulas 1 and 2 using 1–20 terms. Copy and paste your code as text and include it in your pdf submission.

b) In your code, calculate the *true* relative error (ϵ_t) and *approximate* relative error (ϵ_a) of each calculation.

c) Report your results in a table that looks something similar to the one shown below. (Be sure to clearly label the columns of the table) *Hint: While it is acceptable for you to manually make the table, it will save you a lot of time if you can make the computer to generate the table.*

d) Comment on which formula has the least error.

	Formula 1			Formula 2		
terms	value	ϵ_t (%)	ϵ_a (%)	value	ϵ_t (%)	ϵ_a (%)
1						
.						
.						
.						
20						

Q5 (8 pts)

Recall the cylindrical storage tank from Assignment HW 1a:

Solve the cylindrical-tank IVP using your implementation of Euler's method from Assignment 1a using, $h = 1, 0.1, 0.01, 0.001$. Note, this will result in 4 different numerical approximations (one for each h) of the solution, $y(t)$.

a) At a 1-second interval (i.e. at $t = 0, 1, 2, \dots, 10$ seconds) compute the approximate relative error between each refinement of h. This will result in three data sets: the approximate error between $h = 1 \to 0.1$, $h = 0.1 \to 0.01$, and $h = 0.01 \to 0.001$. Plot all three of these data sets together. Provide a statement regarding what information each data set provides.

b) Plot the approximate relative errors at $t = 2$ as a function of h. Provide a statement regarding what information this data set provides.