ME EN 2450 Assignment HW 4

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I declare that the assignment here submitted is original except for source material explicitly acknowledged.

I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website.

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Name	Date
Signature	Student ID

Q1: Differential Equation: Initial Value Problem (IVP)

A cylindrical storage tank, with base area A, contains liquid at a height y, defined such that $y = 0$ when the tank is half full. Liquid is withdrawn from the tank at a constant rate of $Q_{\text{out}} = Q$. At the same time, liquid is replenished at a volume flow rate of $Q_{\text{in}} = 3Q\,\sin^2(t)$. The differential equation describing the rate of change of the height of the water in the tank is

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 3\frac{Q}{A}\sin^2(t) - \frac{Q}{A}.$$

a) (4 points) Classify the above differential equation:

ODE / PDE ?

Which order?

Linear / nonlinear ?

Homogeneous / Non-homogeneous ?

b) (4 points) Assume that the initial condition is $y\,(t = 0) = 0$. For the parameter values of $A = 1250$, $Q = 450$, use Euler's method to solve for y **by hand** (i.e no coding), from $t = 0$ to $t = 1$ with $h = 0.25$.

c) (10 points) With the same initial conditions and parameters as above, but with a different step size $h = 0.05$, write a Python/Matlab code to: (i) List the numerical solution as a table; (ii) Plot the numerical solution as a curve with clearly labeled axes. Submit all your code, the table, and the plot.

d) (2 points) Use your code to make another plot with step size $h = 0.001$. Submit the plot with clearly labeled axes.

e) (4 points) With the same initial conditions and parameters as above, but with a different step size $h = 0.5$, use Heun's method to solve **by hand** (i.e no coding).

NOTE: Unless otherwise indicated, please always use radian for all trigonometry functions in this course.

Hints:
You can probably re-use the code you wrote for HW1a.
You results in each of (b) - (e) are not supposed to be the same. Make sure you understand why they are different.

Q2: ODE Order Reduction

(a) (3 points) Convert the following ODE into a system of 1st-order ODEs in the standard form. Show your steps.

$$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - 4y = e^{2x}$$

(b) (3 points) Convert the following ODE into a system of 1st-order ODEs in the standard form. Show your steps.

$$\frac{d^5y}{dx^5} + y = \cos(3x)$$

$$\frac{\partial y}{\partial t} = 3\frac{Q}{A}\sin^2(t) - Q/A$$

a) ODE
first order
Linear
non homogenous

b) $t=0$, $y=0$ $A=1250$ $Q=450$
for $t=0$ to $t=1$ $h=0.25$

$$\frac{\partial y}{\partial t} = 3\frac{450}{1250}\sin^2(0) - \frac{450}{1250} = -0.36$$
$$y_{.25} = 0 - .36(.25) = -0.09$$

$$\frac{\partial y}{\partial t} = 3\frac{450}{1250}\sin^2(.25) - \frac{450}{1250} = -.29389$$

$$y_{.5} = -.09 - .29389(.25) = -.16347$$

$$\frac{\partial y}{\partial t} = 3\frac{450}{1250}\sin^2(.5) - \frac{450}{1250} = -0.1176$$

$$y_{.75} = -.16347 - 0.11172(.25) = -0.19141$$

$$\frac{\partial y}{\partial t} = 3\frac{450}{1250}\sin^2(.75) - \frac{450}{1250} = 0.1418$$

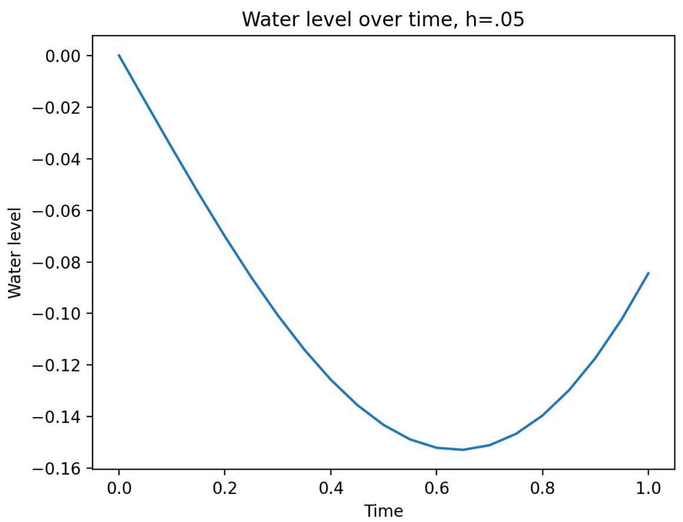
$$y_1 = -0.19141 + 0.1418(.25) = \boxed{-0.15596}$$

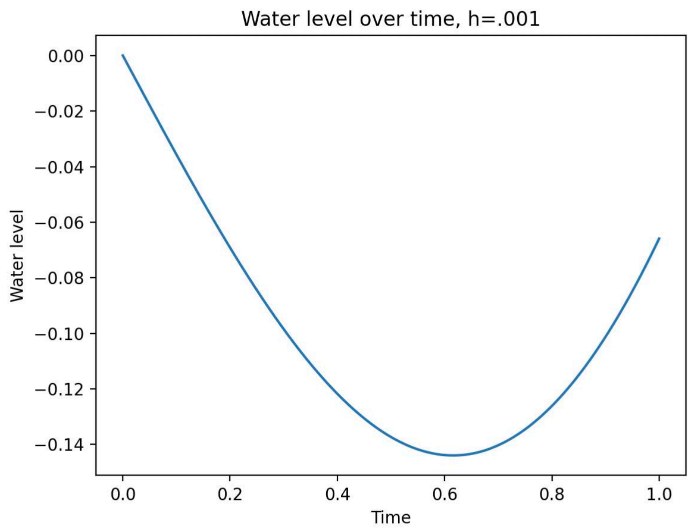
e) $t=0$, $y=0$ $A=1250$ $Q=450$
$t=0 \rightarrow 1$ $h=.5$

$$\frac{\partial y}{\partial t} = 3\frac{450}{1250}\sin^2(0) - \frac{450}{1250} = -0.36$$
$$y_{.5} = 0 - .36(.5) = -0.18$$
$$\frac{\partial y}{\partial t} = 3\frac{450}{1250}\sin^2(.5) - \frac{450}{1250} = -0.11176$$
$$y_1 = -.18 - .1176(.5) = \boxed{-.12412}$$

```python
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

#Euler's method

#Forcing function:

def f(x):
    Q = 450
    A = 1250
    return 3 * (Q/A) * np.sin(x)**2 -(Q/A)

def eulers( t0, y0, tf, h, function):
    tValues = []
    yValues = [y0]
    guess = y0
    for i in np.arange(t0, tf, h):
        tValues.append(i)
        guess = guess + function(i)*h
        yValues.append(guess)
    tValues.append(tf)
    data = {
        ('Time'): tValues,
        ('Level'): yValues
    }
    return guess, data

final, data = eulers(0, 0, 1, .05, f)
df = pd.DataFrame(data)
print(df)
plt.figure()
plt.plot(data['Time'], data['Level'])
plt.xlabel('Time')
plt.ylabel('Water level')
plt.title('Water level over time, h=.05')
plt.show()

final, data = eulers(0, 0, 1, .001, f)
plt.figure()
plt.plot(data['Time'], data['Level'])
plt.xlabel('Time')
plt.ylabel('Water level')
plt.title('Water level over time, h=.001')
plt.show()
```

	Time	Level
0	0.00	0.000000
1	0.05	-0.018000
2	0.10	-0.035865
3	0.15	-0.053327
4	0.20	-0.070121
5	0.25	-0.085990
6	0.30	-0.100684
7	0.35	-0.113968
8	0.40	-0.125619
9	0.45	-0.135430
10	0.50	-0.143214
11	0.55	-0.148802
12	0.60	-0.152049
13	0.65	-0.152833
14	0.70	-0.151055
15	0.75	-0.146644
16	0.80	-0.139554
17	0.85	-0.129766
18	0.90	-0.117287
19	0.95	-0.102152
20	1.00	-0.084424





Q2

$$\frac{\partial^3 y}{\partial x^3} - 4\frac{\partial^2 y}{\partial x^2} + 6\frac{\partial y}{\partial x} - 4y = e^{2x}$$

a)

$$z = \frac{\partial y}{\partial x}$$

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial z}{\partial x} + 6z - 4y = e^{2x}$$

$$\alpha = \frac{\partial z}{\partial x}$$

$$\frac{\partial \alpha}{\partial x} - 4\alpha + 6z - 4y = e^{2x}$$

$$\boxed{\begin{gathered}\frac{\partial \alpha}{\partial x} - 4\alpha + 6z - 4y = e^{2x} \\ \alpha = \frac{\partial z}{\partial x} \quad z = \frac{\partial y}{\partial x}\end{gathered}}$$

b)

$$\frac{\partial^5 y}{\partial x^5} + y = \cos(3x)$$

$$z = \frac{\partial y}{\partial x} \qquad \frac{\partial^4 z}{\partial x^4} + y = \cos(3x$$

$$a = \frac{\partial z}{\partial x} \qquad \frac{\partial^3 a}{\partial x^3} + y = \cos(3x)$$

$$b = \frac{\partial a}{\partial x} \qquad \frac{\partial^2 b}{\partial x^2} + y = \cos(3x)$$

$$c = \frac{\partial b}{\partial x} \qquad \frac{\partial c}{\partial x} + y = \cos(3x)$$

$$\boxed{\left\{\begin{aligned} &\frac{\partial c}{\partial x} + y = \cos(3x) \\ &c = \frac{\partial b}{\partial x} \quad b = \frac{\partial a}{\partial x} \\ &a = \frac{\partial z}{\partial x} \quad z = \frac{\partial y}{\partial x}\end{aligned}\right\}}$$