ME EN 2450 Assignment HW1a

Name: Christopher Wall

I declare that the assignment here submitted is original except for source material explicitly acknowledged.

I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website.

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Score

Presentation:	/2	<= Make sure your submission is clean, tidy, and well organized
Technical Content:	/10	
Total:	/12	

NOTE: All HW assignments need to be submitted to Gradescope.

Hard copy / Email submission will result in zero credit

Gradescope Access Code: 4J282R

Homework Formatting Tips
(These guidelines apply to this and all future HW assignments)

- Axes Labels and Legends on Plots:
  Please ensure that all your plots have clearly labeled axes and legends if applicable. This is crucial for understanding your visual representations. A plot without labeled axes does not mean anything!

- Avoid Printing and Scanning Code:
  Kindly refrain from printing out your code and then scanning it. Instead, export your code electronically in a readable format, such as a text document or pdf/html file. Codes that are printed then scanned are very hard to read and will result in reduced or zero credit.

- Proper Cropping of Screenshots:
  If you need to include screenshots in your submissions, please crop them to only contain the relevant regions. Including unnecessary parts of the screen makes it harder to assess your work efficiently. Also make sure that screenshots included are high quality and high resolution.

- Include Plots and Tables:
  It is imperative that you include the actual plots and tables with your submission. Merely providing the code used to generate them is not sufficient. Likewise, having the final plots or tables without the corresponding code is incomplete. Both components are necessary for a comprehensive evaluation of your work.

- Explain Your Process:
  It can be challenging for me to understand how you generated the plot or table. Please make sure to provide a brief explanation of your process in forms of comments, even if you encounter difficulties or errors in your code.

Exercise 1

A cylindrical storage tank, with cross-sectional area A, contains liquid at a depth y, defined such that $y = 0$ when the tank is half full. Liquid is withdrawn from the tank at a volume flow rate of $Q_{\text{out}} = \alpha\,(1+y)^{3/2}$, which depends on the depth of water in the tank. At the same time, liquid is replenished at a volume flow rate of $Q_{\text{in}} = 2Q\,\text{sin}^2\,(t)$. The differential equation describing the rate of change of the depth of the water in the tank is

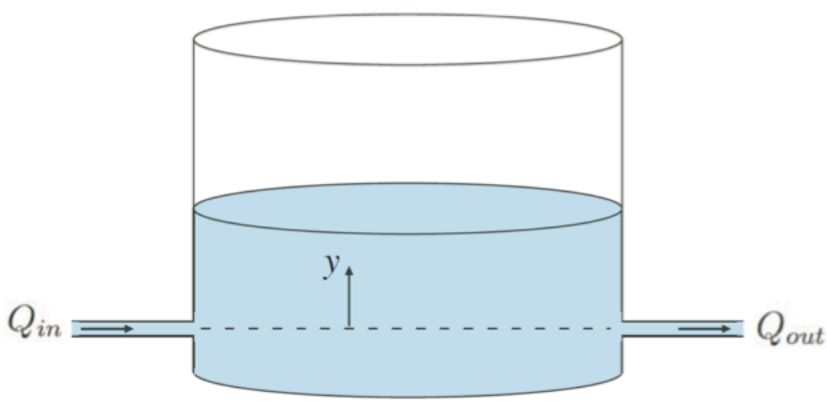
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2\frac{Q}{A}\,\text{sin}^2\,(t) - \frac{\alpha\,(1+y)^{3/2}}{A}.$$

Consider parameter values of $A = 850\;m^2$, $Q = 325\;m^3/s$, and $\alpha = 200\;m^{3/2}\;/s$. Assume that the initial condition is $y\,(t=0) = 2\,m$.

It is difficult to obtain an analytical solution in this case. Instead, we can obtain an *approximate* solution using Euler's method as follows

$$y_{i+1} = y_i + \left(2\frac{Q}{A}\,\text{sin}^2\,(t_i) - \frac{\alpha\,(1+y_i)^{3/2}}{A}\right)h,$$

where y_i is the water level at the current time t_i and y_{i+1} is the predicted water level at the next time step.



a) Write out by hand an estimate of the water level y at time $t = 1.5$ sec using Euler's method with a step size of $h = 0.5$ sec. (3 pts)

b) Implement Euler's method in a ***fully commented*** computer code to solve for the depth of water as a function of time from $t = 0$ to 10 sec with a step size of 0.5 sec. Show all numerical results in a nicely formatted table. Print out your code and attach it to your assignment. (3 pts)

c) Plot the solution (y versus t) for $0 \leq t \leq 10$. Be sure to clearly label all axes in your plot. (2 pts)

d) Assuming no forcing function exists here, list ALL of the following for the mathematical model (i.e. the ODE) in this problem (2 pts):
Dependent variable(s)
Independent variable(s)
System parameter(s)

$$\frac{\partial y}{\partial t} = 2\frac{Q}{A}\sin^2(t) - \frac{\alpha(1+y)^{3/2}}{A}$$

Euler's $\quad y_{i+1} = y_i + \left(2\frac{Q}{A}\sin^2(t_i) - \frac{\alpha(1+y_i)^{3/2}}{A}\right)h$

a) @ $t=0 \quad y = 2m$

Consts $\begin{cases} A = 850\ m^2 \\ Q = 325\ m^3/s \\ \alpha = 200\ m^{3/2}/s \end{cases}$

$$y_{0.5} = 2 + \left(2\left(\frac{325}{200}\right)\sin^2(0) - \frac{200(1+2)^{3/2}}{850}\right)(0.5)$$

$y_{0.5} = 1.388688\ m$

$$y_1 = y_{.5} + \left(2\,\frac{(325)}{200}\sin^2(0.5) - \frac{200(1+y_{.5})^{3/2}}{850}\right)h$$

$y_1 = 1.042241\ m$

$$y_{1.5} = y_1 + \left(2\,\frac{325}{200}\sin^2(0.5) - \frac{200(1+y_1)^{3/2}}{850}\right)h$$

$y_{1.5} = 0.9696214\ m$

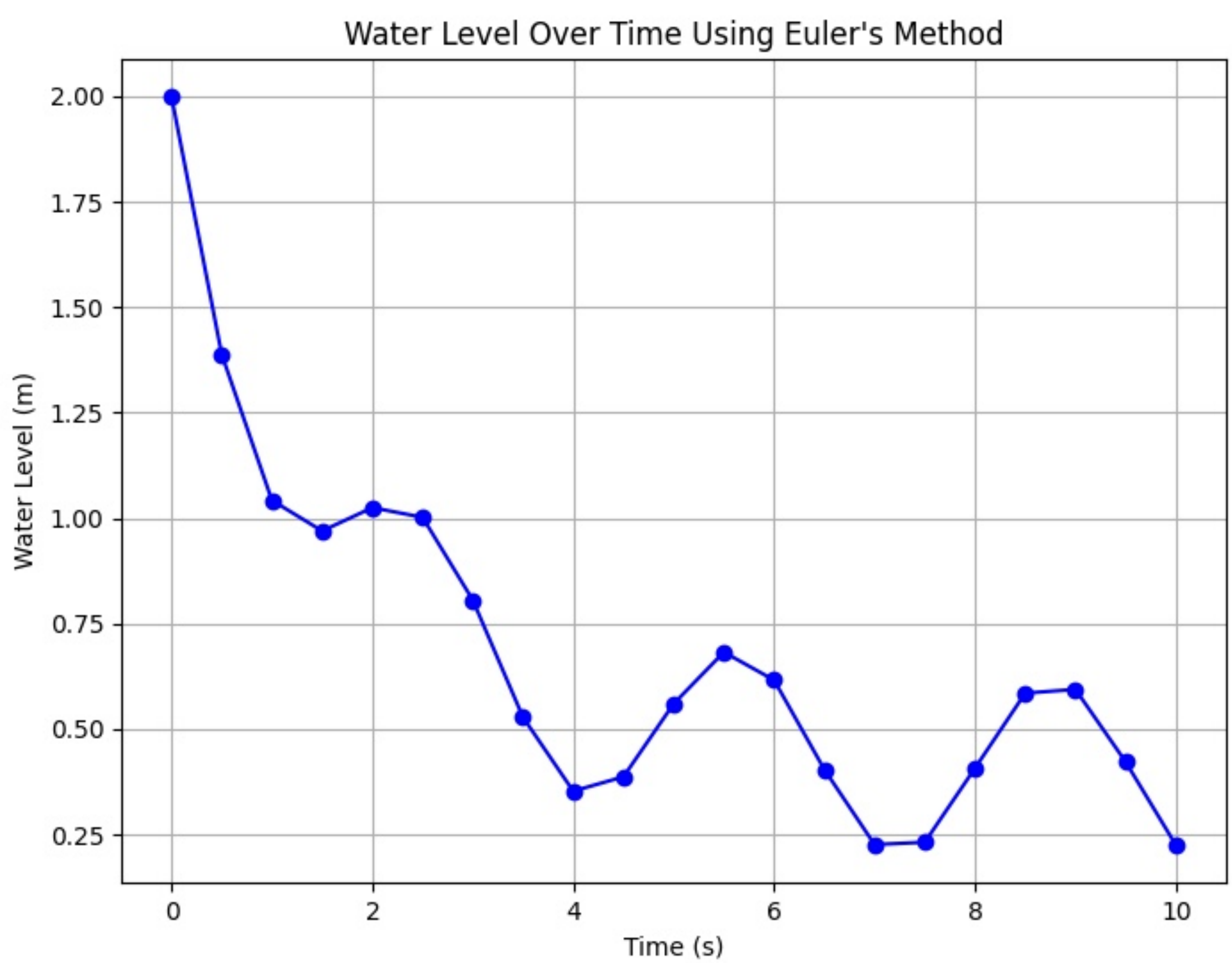
d) Dependent variables:
- water level

Independent variables:
- time

System parameters:
- cross section A
- Q - flow constant
- α - system constant

```
import math
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

t = 0           #time s
y = 2           #water level m
A = 850         #Area, m^2
Q = 325         #flow rate m^3/2
alpha = 200     #constant m^3/2 / s
h = .5          #step size s
maxTime = 10    #simulation limit s

#Store values to be produced then plotted from euler's
time_values = []
y_values = []

#defines dy/dt
def forcingfunc(time, ylevel):
    return 2 * (Q/A) * math.sin(time)**2  - ((alpha)/A * ((1 + ylevel)**(3/2)))

#loop through estimations
while t < maxTime:
    #print("Time: %2f, Y-level: %f " % (t, y))
    time_values.append(t)
    y_values.append(y)
    y = y + forcingfunc(t, y)*h
    t += .5

#appending last time
time_values.append(t)
y_values.append(y)
#print("Time: %2f, Y-level: %f " % (t, y))

# Plotting the water level over time
plt.figure(figsize=(8, 6))
plt.plot(time_values, y_values, marker='o', linestyle='-', color='b')
plt.title('Water Level Over Time Using Euler\'s Method')
plt.xlabel('Time (s)')
plt.ylabel('Water Level (m)')
plt.grid(True)
plt.show()

data = {
    ('Time') : time_values,
    ('Water level') : y_values
}

df = pd.DataFrame(data)
print(df)
```

	Time	Water level
0	0.0	2.000000
1	0.5	1.388688
2	1.0	1.042241
3	1.5	0.969621
4	2.0	1.024858
5	2.5	1.002016
6	3.0	0.805704
7	3.5	0.527855
8	4.0	0.352723
9	4.5	0.386621
10	5.0	0.559888
11	5.5	0.682272
12	6.0	0.615903
13	6.5	0.404095
14	7.0	0.226050
15	7.5	0.231372
16	8.0	0.407028
17	8.5	0.584934
18	9.0	0.593969
19	9.5	0.422153
20	10.0	0.224786



Water Level Over Time Using Euler's Method
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
Water Level (m)
0
2
4
6
8
10
Time (s)