ME EN 2450 Assignment HW 3a

Name: Christopher Wall

I declare that the assignment here submitted is original except for source material explicitly acknowledged.

I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained in the University website.

Christopher Wall	9/23/2024
Name	Date
	u1467634
Signature	Student ID

Score

Total: ____/20

Q1:

(4 pts) Consider the following matrices

$$[A] = \begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix}, \qquad [B] = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}, \qquad [C] = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix},$$

$$[D] = \begin{bmatrix} 9 & 4 & 3 & -6 \\ 2 & -1 & 7 & 5 \end{bmatrix}, \qquad [E] = \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix}, \qquad [F] = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 7 & 3 \end{bmatrix}, \qquad [G] = \begin{bmatrix} 7 & 6 & 4 \end{bmatrix}$$

(a) What are the dimensions of each matrix?

(b) Identify the square, column, and row matrices.

(c) What are the values of the elements: a_{12}, b_{23}, d_{32}, f_{12}, and g_{12}? If any indices are invalid, report NaN. Assume rows and columns start at index 1.

(d) Perform the following operations:

(a) $[E] + [B]$

(b) $[A] \times [F]$

(c) $[B] - [E]$

(d) $7 \times [B]$

(e) $[E] \times [B]$

(f) $\{C\}^{\mathrm{T}}$ **<= please review Slide 22 of Lecture08**

(g) $[B] \times [A]$

(h) $[D]^{\mathrm{T}}$

Q2

(6 pts) Given the system of equations

$$-2.2x_1 + 20x_2 = 240$$
$$-1x_1 + 8.7x_2 = 87$$

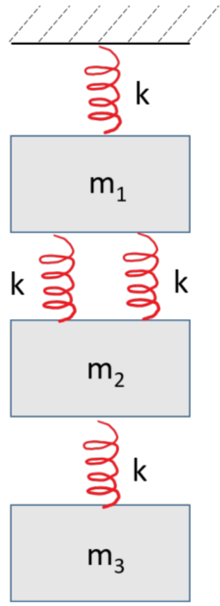
(a) Compute the determinant. Is the system singular?

(b) Graph the equations and approximate the unknowns.

(c) Solve by hand using the elimination of unknowns (i.e. what you knew before taking this class).

Q3

(6 pts) Use a linear system to solve for the displacement of each mass in the illustration. Let $k = 10\frac{kg}{s^2}$ and $g = 9.81\frac{m}{s^2}$



a Write a system of equations for the illustrated masses and springs. Hint: Sum forces in the vertical direction for each mass, m_i (where $i = 1...3$). A system of 3 equations (force balance) and 3 unknowns (displacement of each mass) will result.

b Convert the system into the standard (i.e. natural) form Ax = b.
Clearly list all entries of matrix A, vector x, and vector b as numbers (No unit)

Note: You do NOT need to solve this linear system.

Use $m_1 = 2kg$, $m_2 = 3kg$, and $m_3 = 2.5kg$.

Q4

(4 pts.) The following statement can be rigorously proven in linear algebra:

Given any square matrix **A**:
The linear system **Ax** = **b** has a unique solution vector **x** if and only if det(**A**) ≠ 0.

However, most proofs presented in typical math textbooks might not give students an "intuitive" understanding. Your task here is to use library resources / open internet / anything else you can find to gain a deeper grasp and better mental picture of this fact for yourself.

Recommended video: https://www.youtube.com/watch?v=Ip3X9LOh2dk

What to turn in: Please summarize your understanding into a short paragraph of 2 - 3 sentences.

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A=3x2 B=3x3 C=3x1 D=2x4 E=3x3
F=2x3 G=1x3

(a) What are the dimensions of each matrix?

(b) Identify the square, column, and row matrices. Square: B, E   Col: C   Row: G

(c) What are the values of the elements: a_{12}, b_{23}, d_{32}, f_{12}, and g_{12}? If any indices are invalid, report NaN. Assume rows and columns start at index 1. $a_{12} = 7$ $d_{32} =$ NaN $g_{12} = 6$
$b_{23} = 7$ $f_{12} = 0$

(d) Perform the following operations:

(a) $[E] + [B]$

(b) $[A] \times [F]$

(c) $[B] - [E]$

(d) $7 \times [B]$

(e) $[E] \times [B]$

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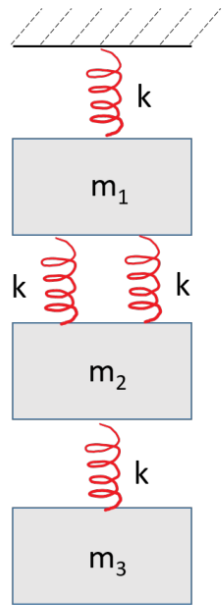

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Q1

$$[E]+[B] = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$$

$$[A]\times[F] = \begin{bmatrix} 19 & 49 & 25 \\ 5 & 14 & 7 \\ 21 & 42 & 23 \end{bmatrix}$$

$$[B]-[E] = \begin{bmatrix} 3 & -2 & -1 \\ -6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}$$

$$7 \times B = \begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 49 \\ 14 & 0 & 42 \end{bmatrix}$$

$$[E]\times[B] = \begin{bmatrix} 4+5+16 & 3+10+0 & 7+35+32 \\ 28+2+6 & 21+4+0 & 49+14+12 \\ 16+0+12 & 12+0+0 & 28+0+24 \end{bmatrix} = \begin{bmatrix} 25 & 13 & 74 \\ 36 & 25 & 75 \\ 28 & 12 & 52 \end{bmatrix}$$

$$\{C\}^T = \begin{bmatrix} 3 & 6 & 1 \end{bmatrix}$$

$$[B]\times[A] = \begin{bmatrix} 16+3+35 & 28+6+42 \\ 4+2+35 & 7+4+42 \\ 8+0+20 & 14+0+24 \end{bmatrix} = \begin{bmatrix} 54 & 76 \\ 41 & 53 \\ 28 & 38 \end{bmatrix}$$

$$[D]^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$$

Q2

$$-2.2x_1 + 20x_2 = 240$$

$$-x_1 + 8.7x_2 = 87$$

$$x_2 = \frac{87 + x_1}{8.7}$$

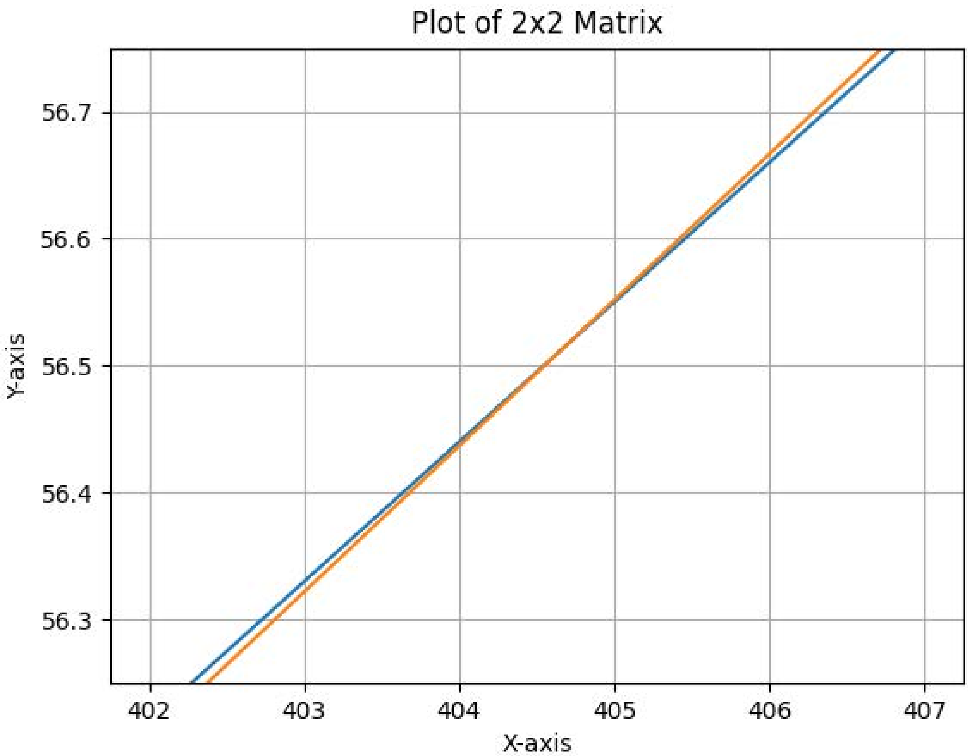
$$x_2 = 10 + \frac{1}{8.7}x_1$$

$$-2.2x_1 + 200 + \frac{20}{8.7}x_1 = 240$$

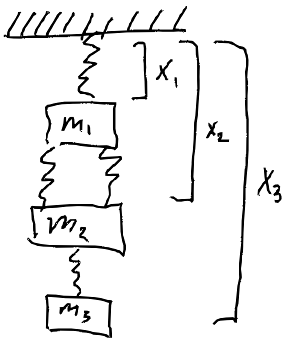
$$\boxed{\begin{aligned} x_1 &= \frac{40}{\frac{20}{8.7} - 2.2} = 404.651 \\ \hookrightarrow x_2 &= 56.512 \end{aligned}}$$

```
import numpy as np
import matplotlib.pyplot as plt

A = np.array([[-2.2, 20],
             [-1, 8.7]])

detA = np.linalg.det(A)

print(f'The determinant is {detA}')

#Because the determinant is non-zero the system is not singular
x1 = np.linspace(402, 407, 100)

#Equations solved for x2, where the second digit is the equation the x2 value is taken from
x21 = (240 + 2.2*x1)/20
x22 = (87 + x1)/8.7

# Create the plot
plt.plot(x1, x21)
plt.plot(x1, x22)
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.ylim((56.25, 56.75))
plt.title('Plot of 2x2 Matrix')

# Display the plot
plt.grid(True)
plt.show()

#There appears to be a solution at x1 = 404.5, x2 = 56.5
```

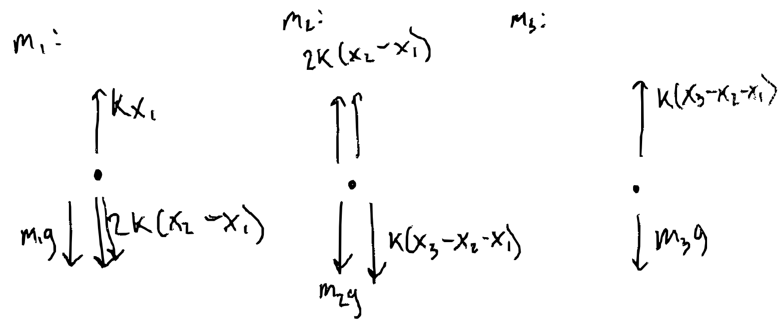
```
The determinant is 0.8600000000000001
```



Q3



FBDs:



equations: $kx_1 - 2k(x_2 - x_1) = m_1 g$

$$2k(x_2 - x_1) - k(x_3 - x_2 - x_1) = m_2 g$$

$$k(x_3 - x_2 - x_1) = m_3 g$$

$$3kx_1 - 2kx_2 - 0kx_3 = m_1 g$$

$$kx_1 + 3kx_2 - kx_3 = m_2 g$$

$$-kx_1 - kx_2 + kx_3 = m_3 g$$

$$\begin{bmatrix} 3 & -2 & 0 \\ 1 & 3 & -1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{g}{k} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$

units: $$\begin{bmatrix} 3 & -2 & 0 \\ 1 & 3 & -1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.96 \\ 2.94 \\ 2.45 \end{bmatrix} \quad \cancel{kg} \cdot \frac{\cancel{s^2}}{\cancel{kg}} \cdot \frac{m}{\cancel{s^2}} = m$$

no units

meters

Q4

I think of the determinant as a kind of measure of the linear independence of a matrix. This makes sense to me because for two vectors, the cross product is $|u||v|\sin\theta$, which gives the angle between scaled by the size of both vectors. The determinant also is a tool to provide information such as whether a matrix is invertible or what its scaling factor will be for transformations.