4040-849 Optimization Methods

Programming Assignment 1 Report

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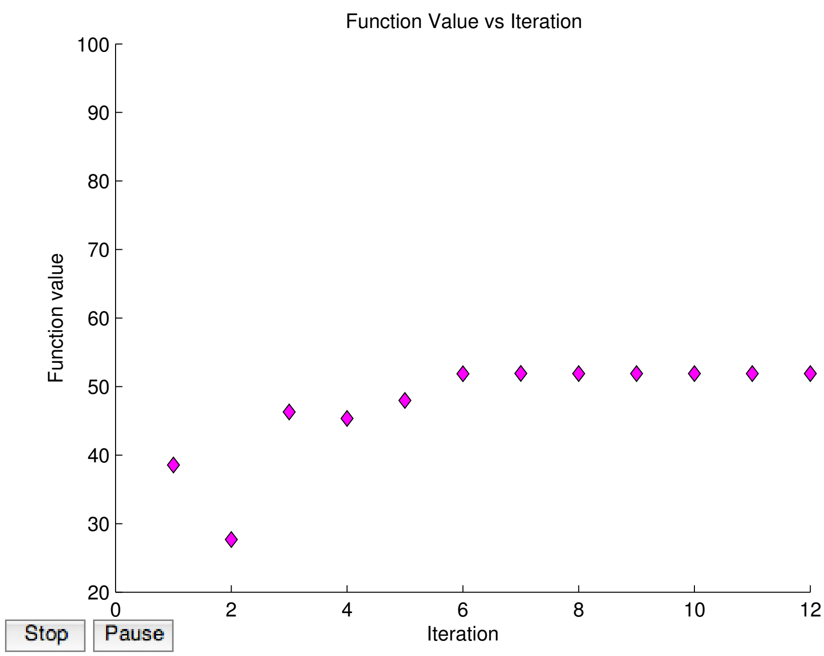
April 28, 2012

Problem 1.

Solution.

Solving the optimization problem using the *fmincon* function with the interior-point algorithm yielded the following data for each iteration.



The values of the objective function and design variables at each iteration are shown in Table 1. In addition, the final values of the objective function, design variables, and constraints after convergence of the interior-point algorithm are shown in the Table 2. From these tables, we can see that the optimal value of the function is 51.8983, which is obtained when $X^T = [0.0869, 0.6598, 1.8950, -1.4445]$.

Problem 2-a.

Solution.

Based on the problem description, we seek to optimize the energy stored within the flywheel based on its design specifications, including the maximum permissable mass ($m = 70$ kg), radius ($r = 0.5$ m), rotational speed ($\omega = 3000$ rpm), stress ($\sigma_{max} = 140 \times 10^5$ Pa), density ($\rho = 8000$ kg/m^3), and Poissons ratio ($v = 0.3$). We are given the equation for the energy of the flywheel as

Iteration	Function Value	X1	X2	X3	X4
0	100	0	0	0	1.0e-07 * 0.1490
1	38.56622	0.5758	0.8903	3.2829	-0.9824
2	27.67708	0.2329	1.1938	3.2614	-2.1305
3	46.30828	0.0242	0.1565	2.2417	-1.7951
4	45.35767	0.3914	0.5560	2.0194	-1.8260
5	47.97002	0.2306	0.6055	2.0312	-1.6103
6	51.88175	0.1067	0.6481	1.8819	-1.4587
7	51.92273	0.0782	0.6694	1.9020	-1.4333
8	51.89738	0.0865	0.6589	1.8953	-1.4447
9	51.89867	0.0871	0.6601	1.8948	-1.4445
10	51.89868	0.0869	0.6598	1.8950	-1.4445
11	51.89828	0.0869	0.6598	1.8950	-1.4445
12	51.89828	0.0869	0.6598	1.8950	-1.4445

Table 1: Objective function and design variable values for each iteration during the interior-point algorithm.

Objective Function	X1	X2	X3	X4	Constraint 1	Constraint 2	Constraint 3
51.5823	0.0869	0.6598	1.8950	-1.4445	-91.1129	0	0

Table 2: Final objective function, design variable, and constraint values after convergence of the interior-point algorithm.

follows:

$$
\begin{aligned}
E &= \frac{1}{2} I \omega^2 \\
&= \frac{1}{4} m r^2 \omega^2
\end{aligned}
$$

Now, treating the flywheel radius r and width w as design variables, we can re-write E in terms of r and w by making the following observations.

1. E is directly proportional to ω, so we will be able to store the maximum energy only when ω is at its maximum value (that is, 3000 rpm $= 3000 \frac{\text{rev}}{\text{min}} (\frac{1\text{min}}{60\text{s}})(\frac{2\pi\text{rad}}{1\text{rev}}) = 100\pi$ rad/s).

2. The mass of the flywheel can be derived using the equation for density ($\rho = \frac{m}{V}$), where the volume V of the flywheel is equal to that of a cylinder with radius r and height w ($V = \pi r^2 w$). Thus, after some algebraic maniupation, we determine the following:

$$m = \pi r^2 w \rho$$

Now, with these two observations, we can treat ω at its maximal value of 100π rad/s and substitute m with the expression $m = \pi r^2 w \rho$, since $\rho = 8000$ kg/m^3 is a fixed value and does not change.

Doing this substitution yields the following expression for the energy of the flywheel:

$$
\begin{aligned}
E &= \frac{1}{4}mr^2\omega^2 \\
&= \frac{1}{4}\pi r^4 w\rho\omega^2 \\
&= \frac{1}{4}(8000\pi(100\pi)^2 r^4 w)
\end{aligned}
$$

Now that we have identified the objective function that we must optimize, we must establish the constraints on the design variables r and w, which are enumerated below:

1. From the problem description we are told that $r \leq 0.5$ m.

2. From the problem description we are told that $m \leq 70$ kg, so replacing m with our previously derived expression $\pi r^2 w\rho$, we know that $\pi r^2 w\rho \leq 70$ kg, where $\rho = 8000$ kg/m^3.

3. Based on the distortion energy theory of failure that is used to derive the maximal tangential and radial stresses, and assuming that each of these stresses are equal ($\sigma_t = \sigma_r = \frac{1}{2}\rho(3+v)\omega^2 r^2$), we know the following:

$$
\begin{aligned}
\sigma_t^2 + \sigma_r^2 - \sigma_t\sigma_r &= 2\sigma_t^2 - \sigma_t^2 \\
&= \sigma_t^2 \\
&= (\frac{1}{2}\rho(3+v)\omega^2 r^2)^2 \\
&\leq \sigma_{max}^2
\end{aligned}
$$

   Again, using the fact that $\sigma_{max} = (140 \times 10^6)^2$ Pa and $\omega = 100\pi$ rad/s in this maximal case (as well as $v = 0.3$, $\rho = 8000$ kg/m^3, and $\sigma_{max} = 140 \times 10^6$ Pa), we can conclude the following:

$$
\frac{1}{4}(8000^2(3.3)^2(100\pi)^4 r^4) \leq (140 \times 10^6)^2
$$

   This can be further reduced by taking the square root of both sides of this expression, which yields the following constraint:

$$
\frac{1}{2}(8000(3.3)(100\pi)^2 r^2) \leq (140 \times 10^6)
$$

Now, realizing that by maximizing the energy we are minimizing the negation of the energy equation, we can write the problem formally as follows.

Minimize

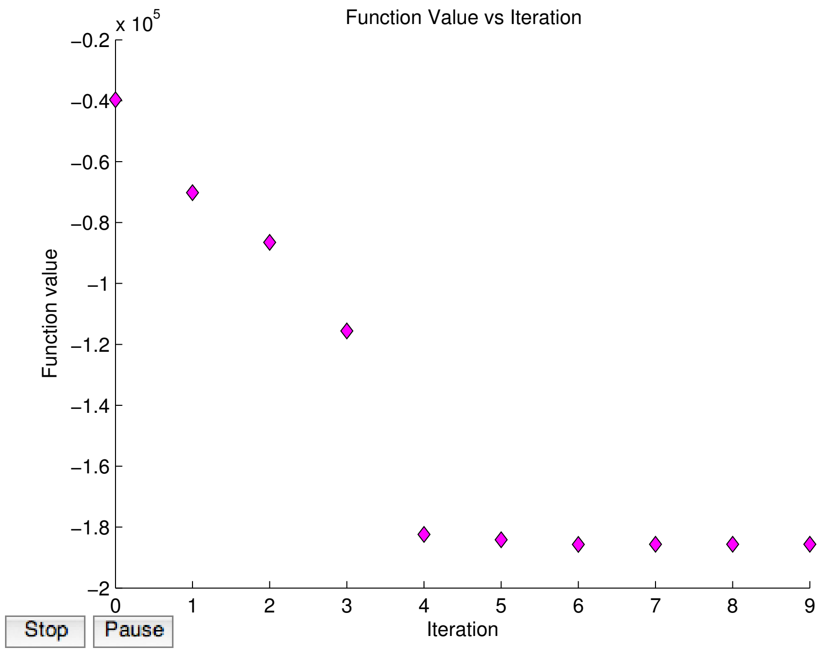
$$
E' = -E = -\frac{1}{4}(8000\pi(100\pi)^2 r^4 w)
$$

subject to the nonlinear constraints:

1. $r - 0.5 \leq 0$
2. $8000\pi r^2 w - 70 \leq 0$
3. $\frac{1}{2}(8000(3.3)(100\pi)^2 r^2) - (140 \times 10^6) \leq 0$

Problem 2-b.

Solution. Solving this optimization problem using the *fmincon* function with the interior-point algorithm yielded the following data for each iteration.



The values of the energy function, radius, and width at each iteration of the interior-point algorithm are shown in Table 3. In addition, the final values of the energy, radius, and width, and constraints after convergence of the interior-point algorithm are shown in the Table 4. It is important to note that since we needed to negate the energy (objective) function to convert it into an appropriate minimization problem for use with the interior-point algorithm, the actual optimal energy value is 185606.1, which is obtained when the radius and width are equal to 0.3278 m and 0.0259 m, respectively.

Iteration	Function Value	Radius	Width
0	-39688.03	0.2	0.04
1	-70200.94	0.2013	0.069
2	-86527.45	0.2395	0.0424
3	-115583.3	0.3154	0.0188
4	-182378.1	0.324	0.0267
5	-184122.4	0.326	0.0263
6	-185674.5	0.3278	0.0259
7	-185607.6	0.3278	0.0259
8	-185606.1	0.3278	0.0259
9	-185606.1	0.3278	0.0259

Table 3: Objective function and design variable values for each iteration during the interior-point algorithm.

Objective Function	Radius	Width	Constraint 1	Constraint 2	Constraint 3
-185606.1	0.3278	0.0259	-0.1722	0	-0.7586

Table 4: Final objective function, design variable, and constraint values after convergence of the interior-point algorithm.