4040-849 Optimization Methods

Written Assignment 2

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PROBLEM 1-a.

Solution.

Making the substitution of $f(\lambda)$ for $\frac{\tau_{zy}}{p_max}$, where $\lambda = \frac{z}{b}$, we get a simplified equation that can be simplified as follows.

$$
\begin{aligned}
f(\lambda) &= -\frac{1}{2}\left[-\frac{1}{\sqrt{1+\lambda^2}} + \left(2 - \frac{1}{1+\lambda^2}\right)\sqrt{1+\lambda^2} - 2\lambda\right] \\
&= -\frac{1}{2}\left[-\frac{1}{\sqrt{1+\lambda^2}} + 2\sqrt{1+\lambda^2} - \frac{\sqrt{1+\lambda^2}}{1+\lambda^2} - 2\lambda\right] \\
&= \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2} + \frac{0.5\sqrt{1+\lambda^2}}{1+\lambda^2} + \lambda \\
&= \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda
\end{aligned}
$$

Therefore, as shown, we can reduce the problem of finding the location of the maximum shear stress for $v_1 = v_2 = 3$ reduces to maximizing the function shown below:

$$
f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda
$$

PROBLEM 1-b.

Solution.

$$
f'(x) = -f(x) = -\frac{0.5}{\sqrt{1+x^2}} + \sqrt{1+x^2}\left(1 - \frac{0.5}{1+x^2}\right) - x
$$

J	A_1	B_1	L_1	L_2^*
4	5	6	1	1
7	8	9	1	1

PROBLEM 1-c.

Solution.

$$f(x) = \frac{0.5}{\sqrt{1+x^2}} - \sqrt{1+x^2}\left(1 - \frac{0.5}{1+x^2}\right) + x$$

$$f'(x) = \frac{x\left(-x^2 - 2.\right)}{\left(x^2+1\right)^{3/2}} + 1$$

$$f''(x) = \frac{x^2 - 2.}{\sqrt{x^2+1}\left(x^2+1.\right)^2}$$

PROBLEM 1-d.

Solution.