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**PROBLEM Becky's Vroom Vroom Dilemma**. *Let $C_1$ and $C_2$ be two cars travelling on the highway. If both cars are moving at an equivalent constant velocity and are at a distance of $d$ apart, is it possible for the separation between $C_1$ and $C_2$ to ever change from $d$?*

$$\begin{bmatrix} I & \vec{0} \\ A & -B \end{bmatrix} \begin{bmatrix} U \\ 1 \end{bmatrix} = \begin{bmatrix} U \\ \vec{0} \end{bmatrix}$$

**Solution**.

Let $v_1$ and $v_2$ be the velocity of cars $C_1$ and $C_2$, respectively, and $d_{0,1}$ and $d_{0,2}$ initial displacement scalar values for $C_1$ and $C_2$ at time $t = 0$. The distance of both cars can be modeled using the following equation,

$$d_i(t) = (v_i t) + d_{0,i},$$

where $i$ is the index of the car $C_1$ or $C_2$. Since the initial separation between the cars is $d$, and by treating the initial displacement value as 0, we know $d_{0,1} = 0$ and $d_{0,2} = d$. Also, we know that the cars are travelling at the same velocity $v$, so $v_1 = v_2 = v$. Therefore, we know the following:

$$d_1(t) = v_1 t + d_{0,1} = v_1 t = vt$$

$$d_2(t) = v_2 t + d_{0,2} = v_2 t + d = vt + d$$

If we anaylze $d_1$ and $d_2$ as $t \to \infty$, we can determine the separation of the cars. However, notice that the difference between $d_1$ and $d_2$ will always correspond to the difference between $C_1$ and $C_2$ at time $t$. We now compute the difference as follows:

$$d_2(t) - d_1(t) = (vt + d) - vt = d$$

Therefore, since $d_2(t) - d_1(t) = d$ is indepdent of time, we know that the difference will always equal $d$ for all instances of time. Thus, the separation between the cars will never change.