4040-849 Optimization Methods

Programming Assignment 1 Report

Christopher Wood

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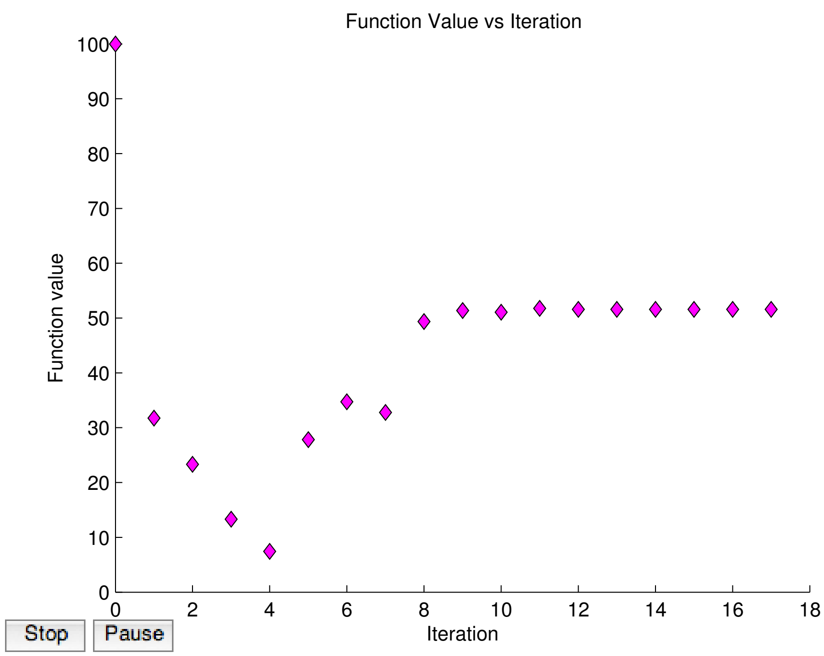
Problem 1.

Solution.

Solving the optimization problem using the *fmincon* function with the interior-point algorithm yielded the following data for each iteration.



The values of the objective function and design variables at each iteration are shown in Table 1. In addition, the final values of the objective function, design variables, and constraints after convergence of the interior-point algorithm are shown in the Table 2.

Problem 2-a.

Solution.

Based on the problem description, we seek to optimize the energy stored within the flywheel based on its design specifications, including the maximum permissable mass ($m = 70$ kg), radius ($r = 0.5$ m), rotational speed ($\omega = 3000$ rpm), stress ($\sigma_{max} = 140x10^5$ Pa), density ($\rho = 8000$ kg/m^3), and Poissons ratio ($v = 0.3$). We are given the equation for the energy of the flywheel as

Iteration	Function Value	X1	X2	X3	X4
0	100	0	0	0	0.149
1	31.74371	-0.118	0.0719	4.9217	-1.5518
2	23.33018	-0.313	0.226	4.6407	-2.3259
3	13.32189	-2.0674	2.1062	2.765	-1.1165
4	7.450429	-2.7524	2.1686	2.4142	-0.5794
5	27.82769	-1.7929	1.5429	3.1812	-0.434
6	34.71776	-1.3829	1.0559	3.0505	-0.9692
7	32.77882	-2.3277	0.4378	2.0451	-1.2382
8	49.36069	-1.5979	0.4646	1.8595	-0.9361
9	51.36701	-1.3108	0.6688	1.9072	-0.9095
10	51.06516	-1.0413	0.8928	2.0933	-0.797
11	51.76738	-1.0257	0.8771	2.0421	-0.8189
12	51.58357	-1.0465	0.8699	2.0499	-0.8145
13	51.58272	-1.0467	0.8717	2.0495	-0.8136
14	51.58265	-1.0494	0.8737	2.0483	-0.8112
15	51.58273	-1.049	0.8736	2.0484	-0.8113
16	51.58233	-1.049	0.8736	2.0485	-0.8113
17	51.58233	-1.049	0.8736	2.0485	-0.8113

Table 1: Objective function and design variable values for each iteration during the interior-point algorithm.

Objective Function	X1	X2	X3	X4	Constraint 1	Constraint 2	Constraint 3
51.5823	-1.049	0.8736	2.0485	-0.8113	-92.3447	0	0

Table 2: Final objective function, design variable, and constraint values after convergence of the interior-point algorithm.

follows:

$$
\begin{aligned}
E &= \frac{1}{2} I \omega^2 \\
&= \frac{1}{4} m r^2 \omega^2
\end{aligned}
$$

Now, treating the flywheel radius r and width w as design variables, we can re-write E in terms of r and w by making the following observations.

1. E is directly proportional to ω, so we will be able to store the maximum energy only when ω is at its maximum value (that is, 3000 rpm $= 100\pi$ rad/s).

2. The mass of the flywheel can be derived using the equation for density ($\rho = \frac{m}{v}$), where the volume of the flywheel is equal to that of a cylinder with radius r and height w. Thus, after algebraic maniupation, we determine the following:

$$m = \pi r^2 w \rho$$

Now, with these two observations, we can treat ω at its maximal value of 100π rad/s and substitute m with the expression $m = \pi r^2 w \rho$, since ρ is a fixed value and does not change. Doing this substitution yields the following expression for the energy of the flywheel:

$$
\begin{aligned}
E &= \frac{1}{4} \pi r^4 w \rho \omega^2 \\
&= \left(\frac{8000\pi (100\pi)^2}{4} \right) r^4 w
\end{aligned}
$$

Now that we have identified the objective that we must optimize, we must establish the constraints on the design variables r and w, which are enumerated below:

1. From the problem description we are told that $r \leq 0.5$ m.

2. From the problem description we are told that $m \leq 70$ kg, so replacing m with our previously derived expression $\pi r^2 w \rho$, we know that $\pi r^2 w \rho \leq 70$ kg. Solving this in terms of r and w yields $r^2 w \leq \frac{70}{8000\pi}$ (since $\rho = 8000$ kg/m^3).

3. Based on the distortion energy theory of failure that is used to derive the maximal tangential and radial stresses, and assuming that each of these stresses are equal ($\sigma_t = \sigma_r$), we know that

$$\sigma_t^2 + \sigma_r^2 - \sigma_t \sigma_r \leq \sigma_{max}^2,$$

can be reduced to

$$\frac{1}{2} \rho (3 + v) \omega^2 r^2 \leq \sigma_{max}^2.$$

Again, using the fact that $\omega = 100\pi$ rad/s in this maximal case (as well as $v = 0.3$, $\rho = 8000$ kg/m^3, and $\sigma_{max} = 140$ x 10^6 Pa), we can conclude the following:

$$\frac{8000(3.3)(100\pi)^2}{2} r^2 \leq (140x10^6)^2$$

Now, realizing that by maximizing the energy we are minimizing the negation of the energy equation, we can write the problem formally as follows.

Minimize

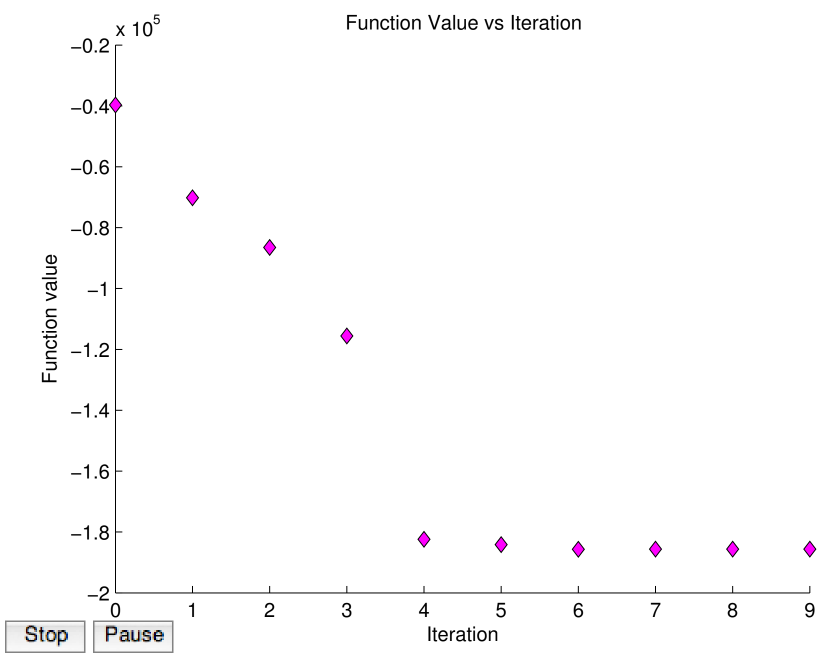
$$E' = -E = -\Big(\frac{8000\pi(100\pi)^2}{4}\Big) r^4 w$$

subject to the nonlinear constraints:

1. $r - 0.5 \leq 0$
2. $r^2 w - \frac{70}{8000\pi} \leq 0$
3. $\frac{8000(3.3)(100\pi)^2}{2} r^2 - (140 \text{ x } 10^6)^2 \leq 0$

Problem 2-b.

Solution. Solving this optimization problem using the *fmincon* function with the interior-point algorithm yielded the following data for each iteration.



The values of the energy function, radius, and width at each iteration of the interior-point algorithm are shown in Table **??**. In addition, the final values of the energy, radius, and width, and constraints after convergence of the interior-point algorithm are shown in the Table **??**. It is important to note that since we needed to negate the energy (objective) function to convert it into an appropriate minimization problem for use with the interior-point algorithm, the actual optimal energy value is 185606.1.

Iteration	Function Value	Radius	Width
0	-39688.03	0.2	0.04
1	-70200.94	0.2013	0.069
2	-86527.45	0.2395	0.0424
3	-115583.3	0.3154	0.0188
4	-182378.1	0.324	0.0267
5	-184122.4	0.326	0.0263
6	-185674.5	0.3278	0.0259
7	-185607.6	0.3278	0.0259
8	-185606.1	0.3278	0.0259
9	-185606.1	0.3278	0.0259

Table 3: Objective function and design variable values for each iteration during the interior-point algorithm.

Objective Function	Radius	Width	Constraint 1	Constraint 2	Constraint 3
-185606.1	0.3278	0.0259	-0.1722	0	-0.0038

Table 4: Final objective function, design variable, and constraint values after convergence of the interior-point algorithm.