4005-800 ALGORITHMS

HOMEWORK 1

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PROBLEM 1.

Solution. TODO: explain reasoning here...

$$\begin{aligned}
2^{2^{n+1}} \\
2^{2^n} \\
(n+1)! \\
n! \\
TODO: continue
\end{aligned}$$

PROBLEM 2-a. *Using the definition of O, prove that $n = O(n^2)$.*

Solution. If $n \geq 1$, then $n^2 \geq n$. Further, $0^2 \geq 0$. Therefore, $n^2 \geq n$ for any $n \in \mathbb{N}$. Thus, $cn^2 \geq n$ when $n \geq 0$ and $c \geq 1$. Finally, by definition, this means that $n \in O(n^2)$, or simply $n = O(n^2)$.

PROBLEM 2-b. *Using the definition of O, prove that $n^k = O(n^{k'})$ if $k \leq k'$.*

Solution.

If $k \leq k'$, then we know that $n^k \leq n^{k'}$. Furthermore, dividing each term in this inequality by n^k yields the following new inequality,

$$1 \leq n^{k'-k},$$

where $n^{k'-k} \geq 1$ because $k \leq k'$.

PROBLEM 3. *Write a function fib that implements the recurrence relation for the Fibonacci numbers. What is the smallest n such that you notice fib running slowly?*

Solution.

PROBLEM 4.

Solution.

PROBLEM 5.

Solution.