

# 4040-849 OPTIMIZATION METHODS

## PROGRAMMING ASSIGNMENT 1 REPORT

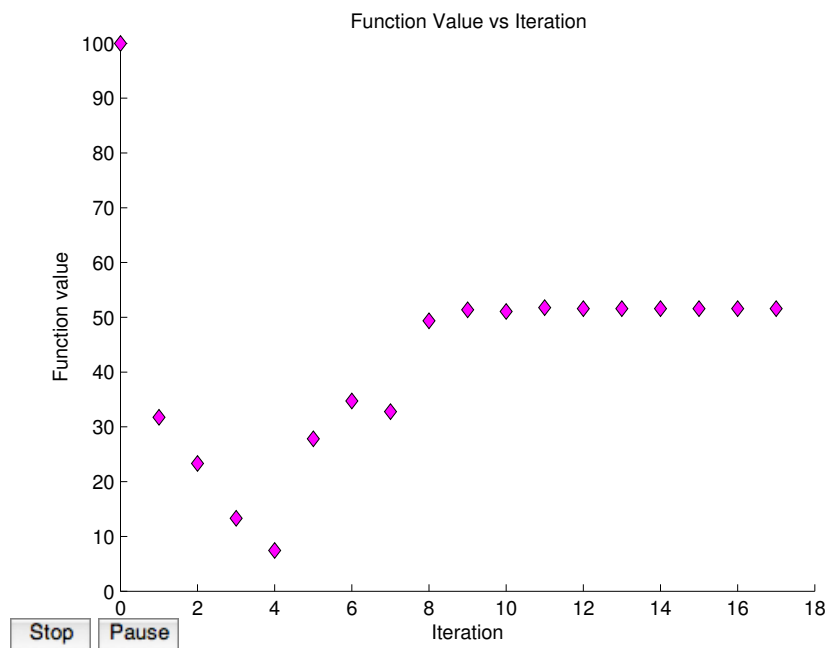
Christopher Wood

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### Problem 1.

Solution.

Solving the optimization problem using the *fmincon* function with the interior-point algorithm yielded the following data for each iteration.



The values of the objective function and design variables at each iteration are shown in the table below.

The final values of the objective function, design variables, and constraints after convergence are shown in the table below.

### Problem 2-a.

Solution.

Iteration	Function Value	X1	X2	X3	X4
0	100	0	0	0	0.149
1	31.74371	-0.118	0.0719	4.9217	-1.5518
2	23.33018	-0.313	0.226	4.6407	-2.3259
3	13.32189	-2.0674	2.1062	2.765	-1.1165
4	7.450429	-2.7524	2.1686	2.4142	-0.5794
5	27.82769	-1.7929	1.5429	3.1812	-0.434
6	34.71776	-1.3829	1.0559	3.0505	-0.9692
7	32.77882	-2.3277	0.4378	2.0451	-1.2382
8	49.36069	-1.5979	0.4646	1.8595	-0.9361
9	51.36701	-1.3108	0.6688	1.9072	-0.9095
10	51.06516	-1.0413	0.8928	2.0933	-0.797
11	51.76738	-1.0257	0.8771	2.0421	-0.8189
12	51.58357	-1.0465	0.8699	2.0499	-0.8145
13	51.58272	-1.0467	0.8717	2.0495	-0.8136
14	51.58265	-1.0494	0.8737	2.0483	-0.8112
15	51.58273	-1.049	0.8736	2.0484	-0.8113
16	51.58233	-1.049	0.8736	2.0485	-0.8113
17	51.58233	-1.049	0.8736	2.0485	-0.8113

Objective Function	X1	X2	X3	X4	Constraint 1	Constraint 2	Constraint 3
51.5823	-1.049	0.8736	2.0485	-0.8113	-92.3447	0	0

Based on the problem description, we seek to optimize the energy stored within the flywheel based on its design specifications, including the maximum permissible mass ( $m = 70$  kg), radius ( $r = 0.5$  m), rotational speed ( $\omega = 3000$  rpm), stress ( $\sigma_{max} = 140 \times 10^5$  Pa), density ( $\rho = 8000$  kg/m<sup>3</sup>), and Poissons ratio ( $\nu = 0.3$ ). We are given the equation for the energy of the flywheel as follows:

$$\begin{aligned}
E &= \frac{1}{2} I \omega^2 \\
&= \frac{1}{4} m r^2 \omega^2
\end{aligned}$$

Now, treating the flywheel radius  $r$  and width  $w$  as design variables, we can re-write  $E$  in terms of  $r$  and  $w$  by making the following observations.

1.  $E$  is directly proportional to  $\omega$ , so we will be able to store the maximum energy only when  $\omega$  is at its maximum value (that is,  $3000$  rpm =  $100\pi$  rad/s).
2. The mass of the flywheel can be derived using the equation for density ( $\rho = \frac{m}{v}$ ), where the volume of the flywheel is equal to that of a cylinder with radius  $r$  and height  $w$ . Thus, after algebraic manipulation, we determine the following:

$$m = \pi r^2 w \rho$$

Now, with these two observations, we can treat  $\omega$  at its maximal value of  $100\pi$  rad/s and substitute  $m$  with the expression  $m = \pi r^2 w \rho$ , since  $\rho$  is a fixed value and does not change. Doing this substitution yields the following expression for the energy of the flywheel:

$$\begin{aligned} E &= \frac{1}{4} \pi r^4 w \rho \omega^2 \\ &= \left( \frac{8000\pi(100\pi)^2}{4} \right) r^4 w \end{aligned}$$

Now that we have identified the objective that we must optimize, we must establish the constraints on the design variables  $r$  and  $w$ , which are enumerated below:

1. From the problem description we are told that  $r \leq 0.5$  m.
2. From the problem description we are told that  $m \leq 70$  kg, so replacing  $m$  with our previously derived expression  $\pi r^2 w \rho$ , we know that  $\pi r^2 w \rho \leq 70$  kg. Solving this in terms of  $r$  and  $w$  yields  $r^2 w \leq \frac{70}{8000\pi}$  (since  $\rho = 8000$  kg/m<sup>3</sup>).
3. Based on the distortion energy theory of failure that is used to derive the maximal tangential and radial stresses, and assuming that each of these stresses are equal ( $\sigma_t = \sigma_r$ ), we know that

$$\sigma_t^2 + \sigma_r^2 - \sigma_t \sigma_r \leq \sigma_{max}^2,$$

can be reduced to

$$\frac{1}{2} \rho (3 + \nu) \omega^2 r^2 \leq \sigma_{max}^2.$$

Again, using the fact that  $\omega = 100\pi$  rad/s in this maximal case (as well as  $\nu = 0.3$ ,  $\rho = 8000$  kg/m<sup>3</sup>, and  $\sigma_{max} = 140 \times 10^6$  Pa), we can conclude the following:

$$\frac{8000(3.3)(100\pi)^2}{2} r^2 \leq (140 \times 10^6)^2$$

Now, realizing that by maximizing the energy we are minimizing the negation of the energy equation, we can write the problem formally as follows.

Minimize

$$E' = -E = -\left( \frac{8000\pi(100\pi)^2}{4} \right) r^4 w$$

subject to the nonlinear constraints:

1.  $r - 0.5 \leq 0$

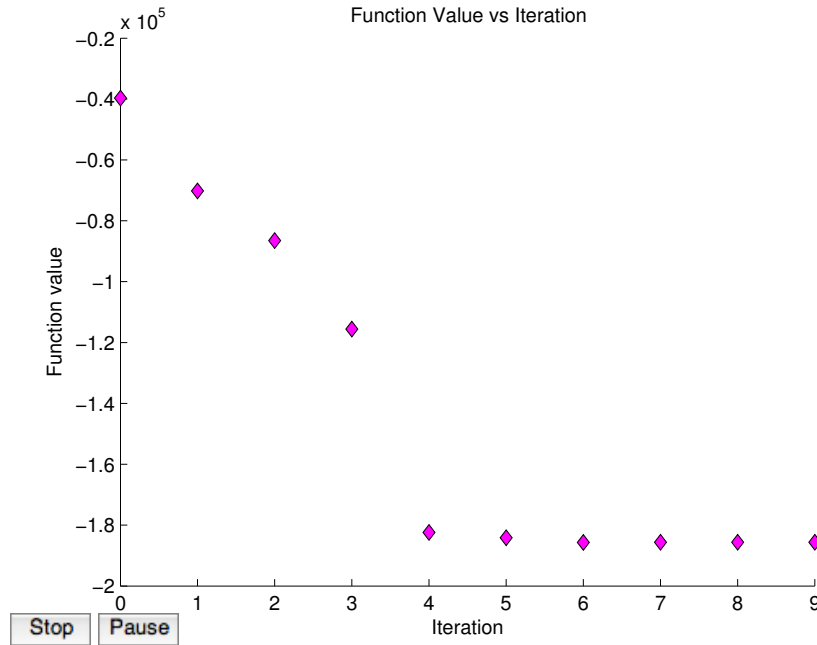
Iteration	Function Value	Radius	Width
0	-39688.03	0.2	0.04
1	-70200.94	0.2013	0.069
2	-86527.45	0.2395	0.0424
3	-115583.3	0.3154	0.0188
4	-182378.1	0.324	0.0267
5	-184122.4	0.326	0.0263
6	-185674.5	0.3278	0.0259
7	-185607.6	0.3278	0.0259
8	-185606.1	0.3278	0.0259
9	-185606.1	0.3278	0.0259

$$2. \quad r^2 w - \frac{70}{8000\pi} \leq 0$$

$$3. \quad \frac{8000(3.3)(100\pi)^2}{2} r^2 - (140 \times 10^6)^2 \leq 0$$

**Problem 2-b.**

Solution. Solving this optimization problem using the *fmincon* function with the interior-point algorithm yielded the following data for each iteration.



The values of the objective function and design variables at each iteration are shown in the table below.

Objective Function	Radius	Width	Constraint 1	Constraint 2	Constraint 3
-185606.1	0.3278	0.0259	-0.1722	0	-0.0038

The final values of the objective function, design variables, and constraints after convergence are shown in the table below.