The 3-*SAT* Decision Problem
Towards a Parallel Search Implementation

Team Satisfaction

Christopher Wood, Eitan Romanoff, Ankur Bajoria

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Agenda

Boolean Satisfiability

Boolean satisfiability is an *NP*-complete decision problem defined as:

$$SAT : \phi \rightarrow \{YES, NO\}$$

Input: Boolean formula ϕ_n on n variables.

$$\phi_5 = (x_1 \vee x_2 \vee \neg x_3) \vee \neg x_4 \wedge x_5 \wedge (\neg x_2 \vee x_3 \wedge (x_4 \wedge x_1))$$

Output: *YES* if there exists a truth assignment to the variables in ϕ_n such that it evaluates to true, *NO* otherwise.

$$\phi_n \text{ is satisfiable} \Leftrightarrow SAT(\phi_n) = YES$$

3-$SAT \in NP$

- A special case of *SAT* that fixes the format of ϕ_n.
- Each input formula is in 3-*CNF* form:
  - The conjunction (Boolean AND) of arbitrarily many clauses, where each clause is the disjunction (Boolean OR) of exactly three literals (a Boolean variable or its negation).

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_3)$$

- *SAT* reduces to 3-*SAT*, so 3-$SAT \in NP$.

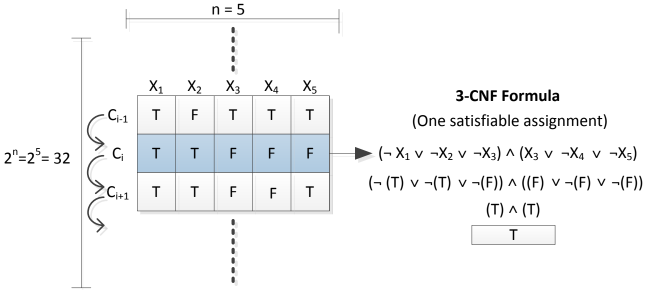
Exhaustive Search for 3-*SAT*

Input: 3-*CNF* formula ϕ_n on n variables, **Output:** YES or NO

$C \leftarrow FALSE^n$ (array of n False values, the initial configuration)
for $i = 0 \to 2^n - 1$ **do**
    $SAT \leftarrow TRUE$
    for all $clause \in \phi_n$ **do**
        if $evaluate(clause, C) = FALSE$ **then**
            $SAT \leftarrow FALSE$
        end if
    end for
    if $SAT = TRUE$ **then return** YES
    $C \leftarrow nextConfig(C)$
end for
return NO

Exhaustive Search for 3-*SAT* - A *Very Satisfiable* Example!

Input: $\phi_5 = (\neg X_1 \vee \neg X_2 \vee \neg X_3) \wedge (X_3 \vee \neg X_4 \vee \neg X_5)$
Output: Yes
Note: no early termination once a satisfiable solution is found



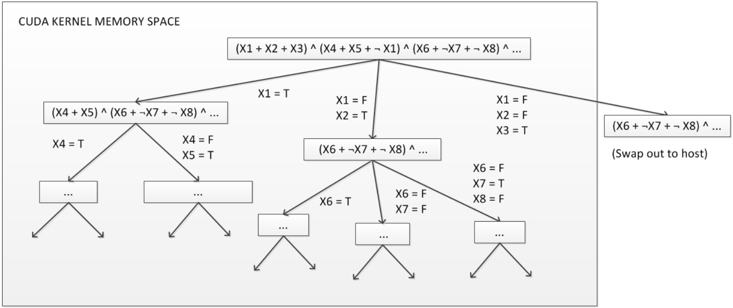
Evaluating a Clause

- Evaluating a clause depends on how ϕ_n and the variable truth assignments are stored.
    - `boolean[] variables` for variable assignments and `Literal[][3] formula` for ϕ_n.
    - A `Literal` has a variable ID and negated flag

```
for (int c = 0; c < numClauses; c++) {
    boolean clauseValue = false;
    for (int l = 0; l < 3 && clauseValue == false; l++) {
        if (formula[c][l].negated == true && !variables[formula[c][l].id])
            clauseValue = true;
        else if (formula[c][l].negated == false && variables[formula[c][l].id])
            clauseValue = true;
    }
    // Check the value of the clause now...
}
```

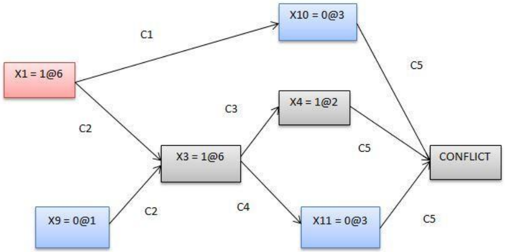
Recursive Divide and Conquer with CUDA

- Idea: Recursively search the variable assignment space
- Approach: A formulas is a “stack” of clauses that are reduced or popped off during assignment
- Implementation: Master-worker pattern for distributing formulas to individual CUDA kernels



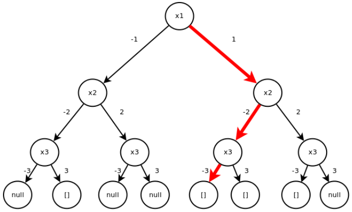
Advanced Clause Learning and Sharing with ManySAT

- Idea: Exploit heuristics for particular types of formulas
- Approach: Run multiple sequential solvers in parallel and reduce the result together
- Implementation: Each solver uses different restart policies, literal selection strategies, and shared conflict-driven clause learning



Intelligent Literal Decisions and Advanced Data Structures for DPLL

- Idea: Enhance literal selection to more effectively traverse the search space
- Approach: Use trie data structures to build “guided paths” that indicate whether a branch has been visited or not
- Implementation: Distribute guided paths across different worker processes using a master-worker pattern



(x1 ^ ~x2 ^ ~x3)

Parallel Program Characteristics

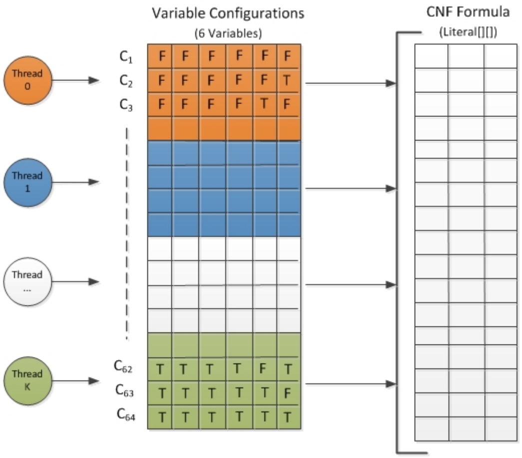
Our design goals included:

- Evenly divide the computation among different threads
- Minimize (or remove) conlficts for shared variables

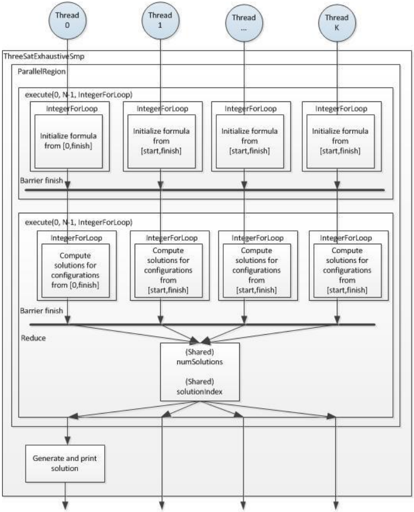
Our parallel design strategy:

- *Result parallelism* for exhaustive program and *agenda parallelism* for decision program
- Split the evaluation of each variable configuration among every thread
- Reduce the final result into the main thread
- Balance the load using a guided schedule

Computation Partition Strategy



Thread Synchronization

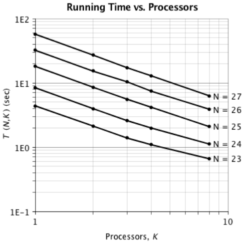


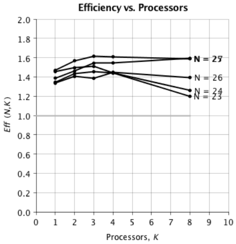
Action!

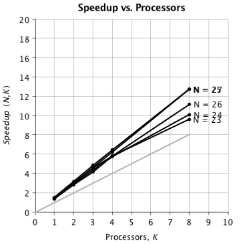
Demo time!

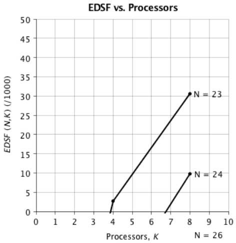
$$\phi_5 = (\neg X_1 \vee \neg X_2 \vee \neg X_3) \wedge (X_3 \vee \neg X_4 \vee \neg X_5)$$

Speedup Metrics (Exhaustive)

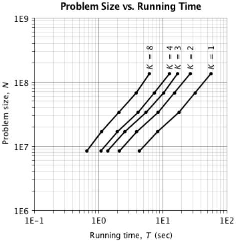


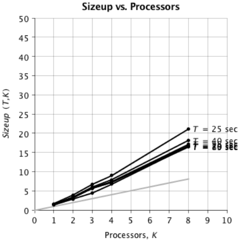


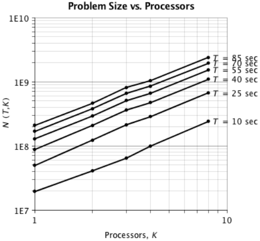


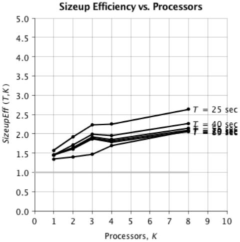


Sizeup Metrics (Exhaustive)









Performance Observations

- We achieved *superlinear* speedups and sizeups using a guided schdule with varying the number of variables
- Exaustive 3-SAT problems have implicit unbalanced loads
  - A guided schedule yielded the best results for speedup and sizeup
  - A dynamic schedule caused a *significant* amount of overhead
- Our parallel programs achieve better performance when the number of variables was varied:
  - The problem size $N = f(N_v, N_c) = 2^{N_v} \times N_c$

Lessons Learned

- If the problem size is a function of *multiple* variables, experiments should only change one of such variables to gather valid performance data
- A guided schedule yielded the most balanced load for our computation partition strategy
- Exploiting the cache and JVM can yield *extremely* good speedup and sizeup efficiencies

Future Work

- Implement more advanced heuristics for literal selection
- Strive for wider splits of the configuration search space tree among multiple processes
- Experiment with different data structures to see what's the most optimal

Questions?

Fire away!