The 3-*SAT* Decision Problem Exhaustive Search Implementations

Team Satisfaction

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Agenda

1. Problem Statement
2. Exhaustive Search Algorithm
3. Sequential Program Demo
4. Questions

Boolean Satisfiability

Boolean satisfiability is an *NP*-complete decision problem defined as:

$$SAT : \phi \rightarrow \{YES, NO\}$$

Input: Boolean formula ϕ_n on n variables.

$$\phi_5 = (x_1 \vee x_2 \vee \neg x_3) \vee \neg x_4 \wedge x_5 \wedge (\neg x_2 \vee x_3 \wedge (x_4 \wedge x_1))$$

Output: *YES* if there exists a truth assignment to the variables in ϕ_n such that it evaluates to true, *NO* otherwise.

$$\phi_n \text{ is satisfiable} \Leftrightarrow SAT(\phi_n) = YES$$

3-$SAT \in NP$

- A special case of *SAT* that fixes the format of ϕ_n.
- Each input formula is in 3-*CNF* form:
  - The conjunction (Boolean AND) of arbitrarily many clauses, where each clause is the disjunction (Boolean OR) of exactly three literals (a Boolean variable or its negation).

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee \neg x_3)$$

- *SAT* reduces to 3-*SAT*, so 3-$SAT \in NP$.

Exhaustive Search for 3-*SAT*

Input: 3-*CNF* formula ϕ_n on n variables, **Output:** YES or NO

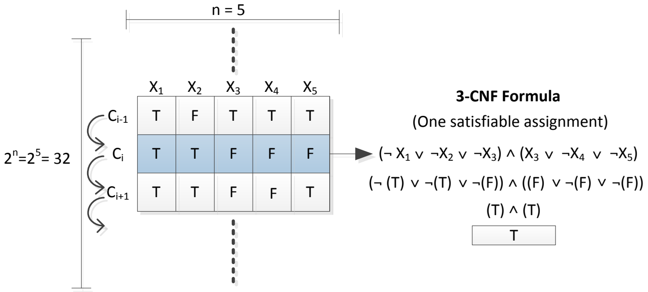
$C \leftarrow FALSE^n$ (array of n False values, the initial configuration)
for $i = 0 \to 2^n - 1$ **do**
  $SAT \leftarrow TRUE$
  for all $clause \in \phi_n$ **do**
    if $evaluate(clause, C) = FALSE$ **then**
      $SAT \leftarrow FALSE$
    end if
  end for
  if $SAT = TRUE$ **then return** YES
  $C \leftarrow nextConfig(C)$
end for
return NO

Exhaustive Search for 3-*SAT* - A *Very Satisfiable* Example!

Input: $\phi_5 = (\neg X_1 \vee \neg X_2 \vee \neg X_3) \wedge (X_3 \vee \neg X_4 \vee \neg X_5)$

Output: Yes

Note: no early termination once a satisfiable solution is found



Evaluating a Clause

- Evaluating a clause depends on how ϕ_n and the variable truth assignments are stored.
  - `boolean[] variables` for variable assignments and `Literal[][3] formula` for ϕ_n.
  - A `Literal` has a variable ID and negated flag

```
for (int c = 0; c < numClauses; c++) {
    boolean clauseValue = false;
    for (int l = 0; l < 3 && clauseValue == false; l++) {
        if (formula[c][l].negated == true && !variables[formula[c][l].id])
            clauseValue = true;
        else if (formula[c][l].negated == false && variables[formula[c][l].id])
            clauseValue = true;
    }
    // Check the value of the clause now...
}
```

The Sequential Solver

Demo time!

$$\phi_5 = (\neg X_1 \vee \neg X_2 \vee \neg X_3) \wedge (X_3 \vee \neg X_4 \vee \neg X_5)$$

Questions?

Fire away!