Exhaustive Search Algorithms for 3-SAT

Team Proposal Document

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1 Problem Description

For our project we choose 3-SAT (or, more formally, 3-CNF-SAT), which is an NP-complete decision problem [1]. 3-SAT takes as input a 3-CNF Boolean formula and returns YES if the formula is satisfiable, and NO otherwise. A 3-CNF formula ϕ_n on n variables is expressed as the conjunction (Boolean AND) of arbitrarily many clauses, where each clause is the disjunction (Boolean OR) of exactly three literals (a literal is a Boolean variable or its negation). A formula ϕ_n is said to be satisfiable if and only if there exists an assignment of truth values to the n variables such that substituting them into the literals of ϕ will cause it to evaluate to true. Expressed as a formal language, we have 3-SAT = $\{\langle\phi\rangle : \phi$ is a satisfiable 3-CNF formula$\}$ (i.e. the set of all 3-CNF formula strings that are accepted by the 3-SAT language if they are satisfiable).

2 Exhaustive Search Algorithm for 3-SAT

An exhaustive search algorithm for solving the 3-SAT problem with input ϕ_n iterates over all 2^n configurations of variable truth values, and for each configuration, assigns the truth value of each variable to the appropriate literal in ϕ_n, and then evaluates ϕ_n to determine if it is true. If for every possible variable configuration ϕ_n does not evaluate to true, then the exhaustive 3-SAT algorithm returns NO. Otherwise, some satisfiable truth value configuration must exist, and so the exhaustive 3-SAT algorithm returns YES.

3 Programs and Performance Metrics

The sequential and parallel programs we will deliver will take as input a 3-CNF formula ϕ_n, encoded using the DIMACS CNF format [2], and output a Boolean truth value indicating whether or not the formula is satisfiable. The 3-CNF formula will be entered at the command line or it will be read from a file to facilitate our experiments. Based on the 3-SAT problem, the only restriction is that each clause must have exactly three literals. Therefore, our programs will enable the number of clauses and the number of variables to be parameters defined in the DIMACS CNF format. An example of the 3-CNF formula $(x_1 \vee x_2 \vee \neg x_3)$, which has one clause and three variables, encoded using DIMACS is shown below.

```
p cnf 3 1
1 2 -3 0
```

There are two main parts of the programs that implement the exhaustive search algorithm outlined in Section 2. First, the program must read in the 3-CNF formula ϕ_n and set up the appropriate data structures. Second, the program must traverse and assign all possible configurations of variable truth values into ϕ_n to then check for satisfiability. Both the sequential and parallel programs will share the first part, which is an inherently sequential task, so as to set up the shared data structures containing the 3-CNF formula.

The second part of the program can be done sequentially or in parallel. In a parallel program, this part is an instance of an agenda parallel problem, where each task evaluates a single truth value assignment for a variable configuration to ϕ_n and returns the Boolean result. At the end of the program we are only concerned with the summary of the task results (i.e. whether or not one task returned true). Furthermore, since there are no sequential dependencies that exist between each agenda task, we can divvy up the execution of these 2^n tasks among 2^n virtual processors. When implemented, we will clump the execution of many tasks together on a single processor because we will not have 2^n processors available for computation for large n.

Finally, since 3-SAT is both an interesting problem in academia and often arises in the industry, we will be measuring the metrics of speedup and sizeup as functions of the program running time T, problem size N, and number of processors K. In addition, we will also measure the efficiency and sizeup efficiency metrics. To compute the speedup and efficiency metrics, we will measure the execution time of the sequential ($T_{seq}(N,K), K = 1$) and parallel ($T_{par}(N,K)$) programs. We will also experimentally determine the sequential fraction of the program using these execution time values. Similarly, to compute the sizeup and sizeup efficiency metrics, we will measure the problem size for the sequential ($N_{seq}(T,K)$) and parallel ($N_{seq}(T,K)$) programs. However, as recognized by Gustafson, the First Problem Size Law for $N(T,K)$ is merely an approximation made under the assumption that the sequential fraction of the program execution time is constant for all problem sizes N, which is not necessarily true. Therefore, we will compute the sizeup and sizeup efficiency by measuring $N(T,K)$ under the Second Problem Size Law [8].

4 Literature Survey and Additional Resources

As part of the graduate requirement, we will analyze [3], [4], and [5] in our literature survey. In addition, we will use the information and resources made available on the International SAT Competition web site [7], as well as the book by Drechsler et al. on advanced SAT solving techniques [6].

References

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