DEPARTMENT OF COMPUTER SCIENCE, ROCHESTER INSTITUTE OF TECHNOLOGY

# Narrowing Bounds for Edge Folkman Numbers

Independent Study Proposal

Christopher Wood
Advisor: Professor Stanisław Radziszowski

February 23, 2013

## 1 BACKGROUND

Edge Folkman numbers, first introduced by Folkman in 1960 (TODO: CITE), are concerned with the study of graphs in which a monochromatic coloring of a particular subgraph always exists. We write $G \rightarrow (a_1, ..., a_k; p)^e$ if for ever edge coloring of an undirected simple graph $G$ not containing $K_p$, there exists a monochromatic $K_{a_i}$ in color $i$ for some $i \in \{1, ..., k\}$. The edge Folkman number is defined as $F_e(a_1, ..., a_k) = \min\{|V(G)| : G \rightarrow (a_1, ..., a_k; p)^e\}$. In 1970 Folkman proved that for all $k > \max(s, t)$, edge- and vertex- Folkman numbers $F_e(s, t; k$ and $F_v(s, t; k)$ exist. Prior to this, Erdos and Hajnal pose the problem of finding $F_e(3, 3; 4)$, which can be informally stated as the following:

*What is the order of the smallest $K_4$-free graph for which any 2-coloring of its edges must contain at least one monochromatic triangle?*

This is equivalent to finding the smallest $K_4$-free graph that is not the union of two triangle-free graphs. Since the proposition of this problem, there has been a significant amount of work towards aimed at narrowing the upper and lower bounds of $F_e(3, 3; 4)$. Table **??** enumerates the work on this problem and leads us to the current state of the field.
concerned with the study of graphs in which a monochromatic coloring of particular subgraphs always exists.

## 2 Proposed Work

TODO: lower bound, pushing towards 20 using large-scale computations TODO: upper bound attacking G127, and subgraph extensions using 3-sat solvers

## 3 Outcomes and Deliverables

Publication-ready paper and more in-depth progress report for the entire project