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Narrowing the Edge Folkman Number Bounds

Independent Study Proposal

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1 BACKGROUND

Edge Folkman numbers, first introduced by Folkman in 1970 [1], are concerned with the study of graphs in which a monochromatic coloring of a particular subgraph always exists. We write $G \to (a_1, ..., a_k; p)^e$ if for ever edge coloring of an undirected simple graph G not containing K_p, there exists a monochromatic K_{a_i} in color i for some $i \in \{1, ..., k\}$. The edge Folkman number is defined as $F_e(a_1, ..., a_k) = \min\{|V(G)| : G \to (a_1, ..., a_k; p)^e\}$. In 1970 Folkman proved that for all $k > \max(s, t)$, edge- and vertex- Folkman numbers $F_e(s, t; k$ and $F_v(s, t; k)$ exist. Prior to this, Erdos and Hajnal pose the problem of finding $F_e(3, 3; 4)$, which can be informally stated as the following [2]:

What is the order of the smallest K_4-free graph for which any 2-coloring of its edges must contain at least one monochromatic triangle?

This is equivalent to finding the smallest K_4-free graph that is not the union of two triangle-free graphs. Since the proposition of this problem, there has been a significant amount of work towards aimed at narrowing the upper and lower bounds of $F_e(3, 3; 4)$. Table 1.1 enumerates the work on this problem and leads us to the current state of the field.

2 PROPOSED WORK

The current lower bound for $F_e(3, 3; 4)$ stands at 19 (TODO: CITE). A significant step forward would be to push this bound to 20 using massive computations. Naturally, it is infeasible to

Table 1.1: History of $F_e(e, e; 4)$

Year	Bounds	Who	Ref.
1967	any?	Erdős-Hajnal	[2]
1970	exist	Folkman	[1]
1972	≥ 10	Lin	[3]
1975	$\leq 10^{10}$?	Erdős offers $100 for proof	
1986	$\leq 8 \times 10^{11}$	Frankl-Rődl	[4]
1988	$\leq 3 \times 10^{9}$	Spencer	[5]
1999	≥ 16	Piwakowski et al (implicit)	[6]
2007	≥ 19	Radziszowski-Xu	[7]
2008	≤ 9697	Lu	[8]
2008	≤ 941	Dudek-Rődel	[9]
2012	≤ 786	Lange et al	TODO
2012	≤ 100?	Garaham offers $100 for proof	

enumerate all possible graphs on 20 vertices and check to see if the arrowing property does not hold.
TODO: lower bound, pushing towards 20 using large-scale computations TODO: upper bound attacking G127, and subgraph extensions using 3-sat solvers

3 Outcomes and Deliverables

Publication-ready paper and more in-depth progress report for the entire project

References

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