Let $R(n)$ be a random draw of integers between 0 and $n-1$ (inclusive). I repeatedly apply R, starting at 10^{100}. What is the expected number of repeated applications until I get zero.

Answer: Approach as a Markov chain. Starting at n, we can transition to each one of the states $(n-1)$, $(n-2)$,...,2, 1 and the absorbing state 0. The question states that draws of the integers are random (uniform implied). Hence each of the transitions above occurs with equal probability, $1/n$. Similarly, if in state $n-1$, then transitions to one of $\{n-2, n-3, ..., 2, 1, 0\}$ occur with equal probability $1/(n-1)$, and so on. Hence we can construct the transition matrix P,
Where the columns and rows are labelled with states according to the order

$$\begin{array}{cc}
\begin{array}{cccccc} 1 & 2 & 3 & \dots & n & 0 \end{array} & \\
\begin{pmatrix}
\dots & \dots & \dots & \dots & \dots & \dots \\
\dots & & & \dots & & \\
\dots & \dots & \dots & \dots & & \dots \\
\vdots & & & & & \vdots \\
 & & \vdots & \dots & & \\
\dots & & & \dots & & \\
\dots & & & \dots & \dots & \dots
\end{pmatrix} &
\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ n-1 \\ n \\ 0 \end{array}
\end{array}$$

$$P = \left(\begin{array}{ccccc|c}
0 & 0 & 0 & \dots & 0 & 1 \\
1/2 & 0 & 0 & \dots & 0 & 1/2 \\
1/3 & 1/3 & 0 & \dots & 0 & 1/3 \\
\vdots & & & & & \vdots \\
1/(n-1) & 1/(n-1) & 1/(n-1) & \dots & 0 & 1/(n-1) \\
1/n & 1/n & 1/n & \dots & 0 & 1/n \\
\hline
0 & 0 & 0 & \dots & 0 & 1
\end{array}\right) = \left(\begin{array}{c|c} Q & * \\ * & 1 \end{array}\right)$$

and the absorbing state 0 has been partitioned. The upper left partition is labelled as matrix Q, and we form the fundamental matrix N as

$$N = (I - Q)^{-1}$$

According to standard theory the duration of the chain starting in state i until entering the absorbed state (0) is given by the i-th component of the vector t,

$$t = Nc, \; c = (1, 1, ..., 1)^T$$

Hence since we are interested in starting in state n and finishing in state 0, we are interested in the n-th component of t,

$$t = Nc = \begin{pmatrix}
1 & 0 & 0 & 0 & \dots & 0 \\
1/2 & 1 & 0 & 0 & \dots & 0 \\
1/2 & 1/3 & 1 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & 1 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
1/2 & 1/3 & 1/4 & 1/5 & \dots & 1
\end{pmatrix}
\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

So that $t_n = 1 + 1/2 + 1/3 + 1/4 + ... + 1/n = H_n \sim \log n$. Therefore the number of steps (function iterations) until hitting zero, from $n = 10^{100}$ is $H_{10^{100}} \sim 100 \log 10$.