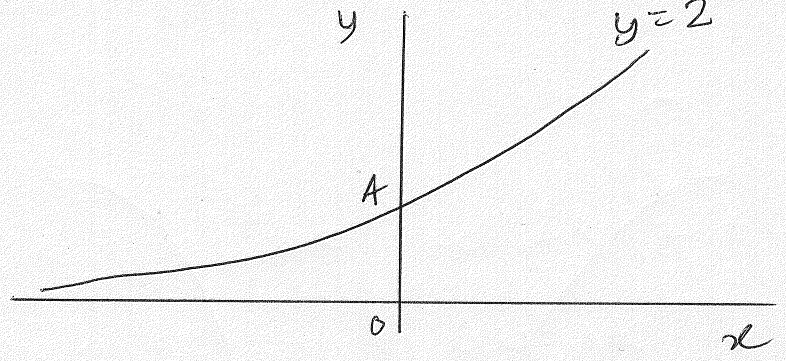
(8)

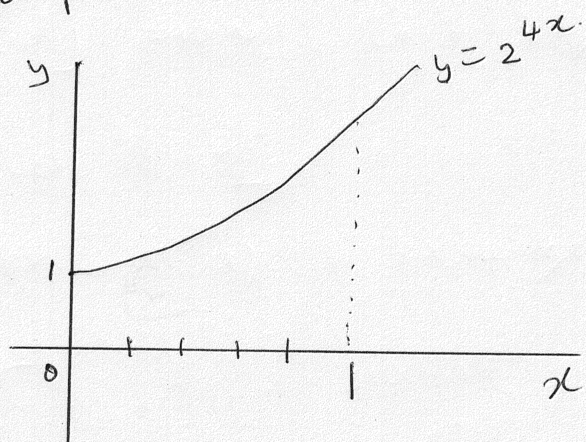


a) y-intercept happens when $x=0$. Plug this in:

$$y = 2^{4\times 0} = 2^0 = 1 \quad \text{(anything to power '0' is 1.)}$$

So $A = (0, 1)$

b) Draw picture (or use one above)



6 ordinates are

x	0	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	1
y	1	$2^{\frac{4}{5}}$	$2^{\frac{8}{5}}$	$2^{\frac{12}{5}}$	$2^{\frac{16}{5}}$	2^4

~~h~~ ~~First ordi~~ $h = \dfrac{\text{last ordinate} - \text{first ordinate}}{\text{number of strips}} = \dfrac{1-0}{5}$

$= \dfrac{1}{5}$

So $\displaystyle\int_0^1 2^{4x}\,dx \approx \tfrac{1}{2}h\left(y_0 + y_1 + 2\left(y_2 + y_3 + y_4 + y_5\right)\right)$

$= \frac{1}{10}\left(1 + 16 + 2\left(1.7411 + 3.0314 + 5.2780 + 9.1896\right)\right)$

$= \frac{1}{10}\left(17 + 2 \times 19.2402\right) = \frac{1}{10}\left(17 + 38.4803\right) = 5.55$ (2dp)

(8)

c) The transformation that maps

$y = 2^{4x}$ to $y = 2^{4x-3}$

[There are 2-ways to describe this transformation ...

Start with transformed equation and see how it relates to original one:]

$y = 2^{4x-3} = 2^{4x} \times 2^{-3}$ (law of indices).

$= \left(\frac{1}{2^3}\right) 2^{4x} = \frac{1}{8} 2^{4x}$

This is a <u>stretch</u>, parallel to the <u>y-axis direction</u>, with <u>scale factor</u> <u>$\frac{1}{8}$</u>.

[Need these words for full marks.]

<u>ALTERNATIVE</u>

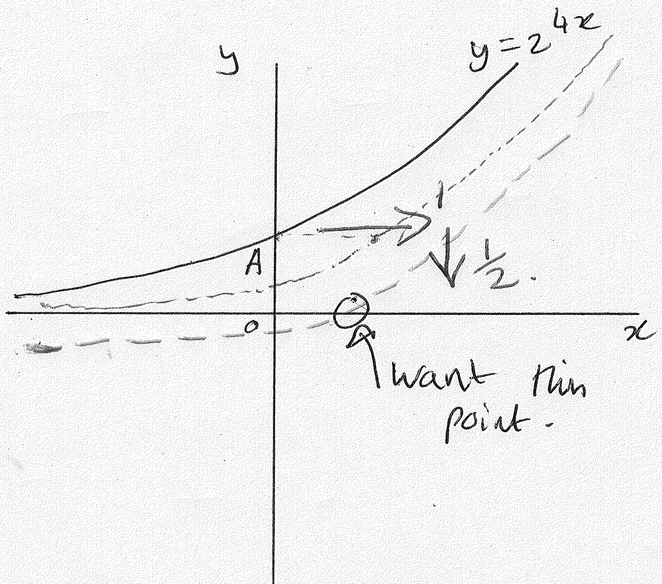
Let $f(x) = 2^{4x}$, try $f(x+c) = 2^{4(x+c)}$
$= 2^{4x+4c}$

So if $4c = -3 \Rightarrow c = \frac{-3}{4}$, then

$f(x - \frac{3}{4}) = 2^{4x-3}$ and this is a TRANSLATION in the direction of x-axis. $\begin{bmatrix} \frac{3}{4} \\ 0 \end{bmatrix}$.

(8)

a) $y = 2^{4x}$ translated by $\begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$ to give $g(x)$.



Must use function notation to make it clear:

let $f(x) = 2^{4x}$

Then $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is represented by $f(x-1)$

and $\begin{bmatrix} 0 \\ -\frac{1}{2} \end{bmatrix}$ is represented by $-\frac{1}{2}$ outside of function

So $g(x) = f(x-1) - \frac{1}{2}$

Require $g(x) = 0 \Rightarrow f(x-1) - \frac{1}{2} = 0$

$\Rightarrow f(x-1) = \frac{1}{2}$.

$\Rightarrow 2^{4(x-1)} = \frac{1}{2} = 2^{-1}$

(Take logs)

$\Rightarrow \log(2^{4(x-1)}) = 4(x-1)\log(2) = \log(2^{-1})$

$= -\log(2)$

(8)

d)

$$\text{so} \quad 4(x-1) = \frac{-\cancel{\log(2)}}{\cancel{\log(2)}} = -1$$

$$\Rightarrow \quad x = 1 - \tfrac{1}{4} = \tfrac{3}{4}.$$

$$\text{So} \quad \underline{Q = (\tfrac{3}{4}, 0)}$$

e)

i) $\log_a k = 3\log_a 2 + \log_a 5 - \log_a 4$

Show $k = 10$. [Questions want us to use log-laws.]

$$\log_a k = (\log_a 2^3 + \log_a 5) - \log_a 4$$

$$= \log_a (2^3 \times 5) - \log_a 4$$

$$= \log_a \left[\frac{2^3 \times 5}{2^2} \right]$$

$$= \log_a [2 \times 5]$$

$$\log_a k = \log_a 10$$

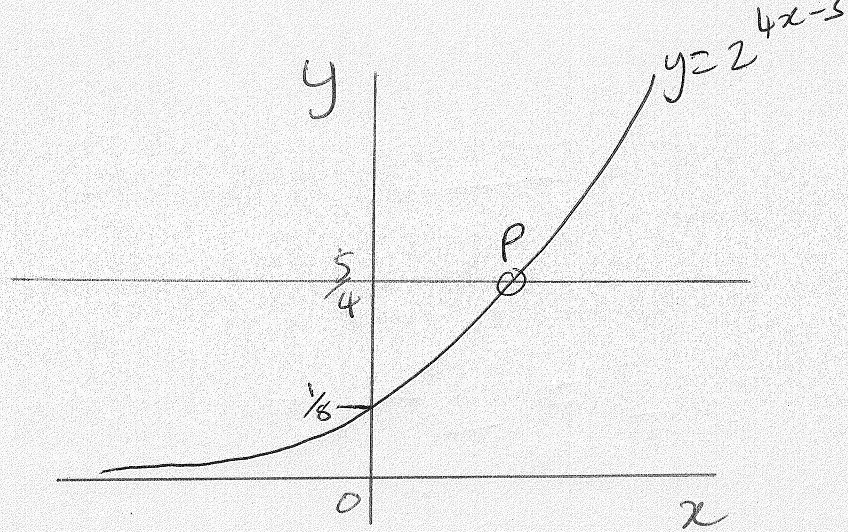
$$\therefore \underline{k = 10}$$

When working with logs use 'powers' where you can

(8)

e)

ii)



[We have to use part e) i) answer and change of log base formula.]

Change of log-base formula:

$$\frac{\log_b(x)}{\log_b(y)} = \log_y(x).$$

To find intersection of 2-curves - we equate them:

$$2^{4x-3} = \frac{5}{4}$$

[use brackets so not to get muddled.]

Take logs: $\log_a(2^{4x-3}) = \log_a(5/4)$

$$\Rightarrow (4x-3)\log_a(2) = \log_a(5) - \log_a(4)$$

$$\Rightarrow 4x\log_a(2) - 3\log_a(2) = \log_a(5) - \log_a(4)$$

$$\Rightarrow 4x\log_a(2) = 3\log_a(2) + \log_a(5) - \log_a(4)$$

$$= \log_a 10 \quad (\text{by part e)i)})$$

[Use change of base formula.]

$$\Rightarrow 4x = \frac{\log_a(10)}{\log_a(2)} = \frac{1}{\left(\frac{\log_a(2)}{\log_a(10)}\right)} = \frac{1}{\log_{10}(2)}$$

$$\Rightarrow x = \frac{1}{4\log_{10}(2)}$$