(6)

Curve C: $y = \dfrac{x^3 + \sqrt{x}}{x}$, $x > 0$

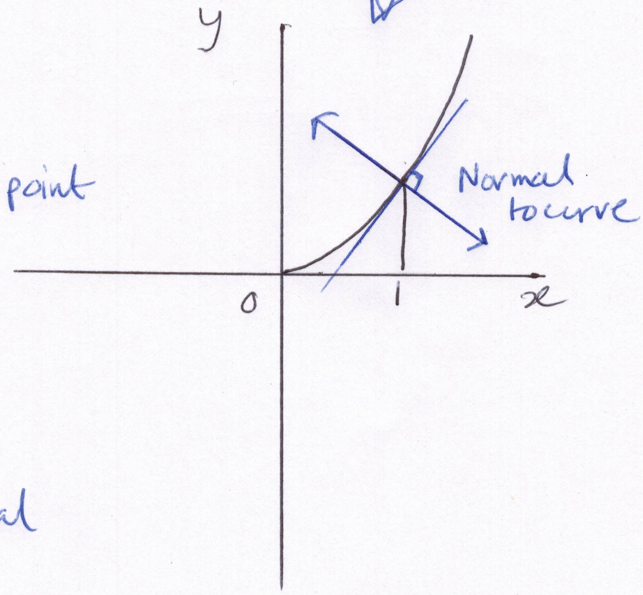
Only 1-side

a) $y = \dfrac{x^3 + \sqrt{x}}{x} = \dfrac{x^3}{x} + \dfrac{x^{\frac{1}{2}}}{x} = x^2 + x^{-\frac{1}{2}}$

So $p = 2$, $q = -\frac{1}{2}$

b)

i) $\dfrac{dy}{dx} = 2x - \frac{1}{2}x^{-\frac{3}{2}}$



ii) Normal to curve, at a point is perpendicular to the tangent at that point.

If m_t is gradient of tangent and m_n is gradient of normal

then $m_t \times m_n = -1$

$\Rightarrow m_n = \dfrac{-1}{m_t}$

Step 1 First find coordinates of point of interest:

$x_1 = 1$, $y_1 = \dfrac{1+1}{1} = 2$.

Step 2 Find gradient of tangent at $x = x_1 = 1$.

$\left.\dfrac{dy}{dx}\right|_{x=1} = 2(1) - \dfrac{1}{2} = \dfrac{3}{2} = m_t$

Step 3 Find gradient of normal: $m_t \times m_n = -1$

$\rightarrow m_n = -\dfrac{2}{3}$

(6) Step 4 use equation for a line to find equation for normal:

$$y - y_1 = m_n(x - x_1)$$

(x_1, y_1) is a point on the line.

$$\Rightarrow \underline{\underline{y - 2 = -\tfrac{2}{3}(x-1)}}$$

Can leave answer like this often questions ask for a particular form:

$$y - 2 = -\tfrac{2}{3}(x-1)$$

$$\Rightarrow 3(y-2) = -2(x-1)$$

$$\Rightarrow 3y - 6 - 2 + 2x = 0$$

$$\Rightarrow \underline{\underline{3y + 2x - 8 = 0}}$$

Same thing, just rearranged.

c)

i) $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2x - \tfrac{1}{2}x^{-\frac{3}{2}}\right)$$

$$= 2 + \tfrac{3}{4}\underline{x^{-\frac{5}{2}}}$$

ii) A maximum point (or min point) must satisfy

$\frac{dy}{dx} = 0$. Solving this $\frac{dy}{dx} = 2x - \frac{1}{2}x^{-\frac{3}{2}} = 0$

$$\Rightarrow 4x = x^{-\frac{3}{2}} = \frac{1}{x^{3/2}}$$

$$\Rightarrow 4x x^{\frac{3}{2}} = 1$$

$$\Rightarrow x^{\frac{5}{2}} = \frac{1}{4}$$

use this in $\frac{d^2y}{dx^2}$

But then $$\frac{d^2y}{dx^2} = 2 + \frac{3}{4}\frac{1}{x^{\frac{5}{2}}} = 2 + \frac{3}{4} \times 4 = 2 + 3 = \underline{\underline{5 > 0}}$$

(6)

c) ii) So $\frac{d^2y}{dx^2} > 0$ for all stationary points

and a maximum point must satisfy

$$\frac{d^2y}{dx^2} \leq 0.$$

So curve C has no maximum points.