(5) An infinite G.S has common ratio r

The question has already told us that $r<1$, since it say the series is infinite. In C2 they will not say this unless the series converges (i.e sum to infinity exists).

a)

$$a_1 \quad a_2 \quad \cdots \quad a_n \quad \cdots$$
$$a_1 \quad a_1 r \quad \cdots \quad a_1 r^{n-1} \quad \cdots$$

I write this so as I have 'n'th term available. It is easy to slip up in exams...

Given: $a_1 = 10$, $S_\infty = 50$

i) $S_\infty = \dfrac{a_1}{1-r} = 50 = \dfrac{10}{1-r}$

No thinking needed

$\Rightarrow \quad 1-r = \dfrac{10}{50} = \dfrac{1}{5} \quad \Rightarrow r = \dfrac{4}{5}.$

ii) find second term: $a_2 = a_1 r = 10 \times \frac{4}{5} = 8.$

b) For Arithmetic Series (A.S)

$$b_1 \quad b_2 \quad b_3 \quad \cdots \quad b_n$$
$$b_1 \quad b_1+d \quad b_1+2d \quad \quad b_1+(n-1)d$$

Equating G.S and A.S

G.S		A.S
a_1	$=$	b_1+3d
$a_1 r$	$=$	b_1+7d

But we know a_1, r so we plug these in

(5)

b)

i) $$10 = b_1 + 3d \quad -(1)$$
$$8 = b_1 + 7d \quad -(2)$$

And we have an easy simultaneous equation.

$$(2)-(1) \Rightarrow 8-10 = b_1 - b_1 + 7d - 3d$$
$$\Rightarrow -2 = 4d$$
$$\Rightarrow d = \frac{-2}{4} = -\frac{1}{2}$$

[Sense Check: Does the sign make sense? G.S is decreasing so $d<0$ seem to be sensible].

ii) We found 'd' for A.S so plug in to simultaneous to find b_1:

$$10 = b_1 + 3(-\tfrac{1}{2}) \Rightarrow 10 + \frac{3}{2} = b_1$$
$$\Rightarrow b_1 = \frac{23}{2}.$$

So in terms of the 'u' notation used in the question.

$$u_n = \frac{23}{2} + (n-1)(-\tfrac{1}{2}) = \frac{23}{2} - \frac{(n-1)}{2}$$

From formula book.

$$\sum_{n=1}^{40} u_n = \frac{1}{2}40(u_1 + u_{40})$$

(5)

b)

ii) $u_1 = \frac{23}{2}$, $u_{40} = \frac{23}{2} - \frac{(40-1)}{2} = \frac{23-39}{2} = \frac{-16}{2} = -8$

So $\sum_{n=1}^{40} u_n = \frac{1}{2}(40)\left(\frac{23}{2} - 8\right)$

$= \cancel{\tfrac{1}{2}}20\left(\frac{23-16}{2}\right) = \frac{20}{2}(23-16)$

$= 10(7) = 70.$