(2) Given $u_{n+1} = 6 + \frac{2}{5}u_n$

given $u_1 = 2$

a) We simply use the definition given, with $n=1, n=2$.

$n=1$: $u_2 = u_{1+1} = 6 + \frac{2}{5}u_1 = 6 + \frac{4}{5} = \frac{6\times5+4}{5} = \underline{\underline{\frac{34}{5}}}$

$n=2$: $u_3 = u_{2+1} = 6 + \frac{2}{5}u_2 = 6 + \frac{2}{5}\left(\frac{34}{5}\right)$

$= \frac{6\times25 + 2\times34}{25} = \underline{\underline{\frac{218}{25}}}$

b) Result: For any sequence defined iteratively [such as $u_{n+1} = a + bu_n$, or $u_{n+2} = -u_{n+1} + au_n$ etc]

If we are told, or know, that "u_n tends to a limit L as $n \to \infty$"

Then, we may replace ALL 'u' terms with 'L', and solve for L

Since u_n tends to limit 'L' as n tends to infinity we replace u_{n+1} and u_n by L.

$u_{n+1} = 6 + \frac{2}{5}u_n \implies L = 6 + \frac{2}{5}L$

Solving for L:

$L(1-\frac{2}{5}) = 6$

$L(\frac{3}{5}) = 6$

$\Rightarrow L = \frac{6\times5}{3} = \underline{\underline{10}}$

④ Binomial expansions are very important...!!

a) The mark-scheme allows for the answer with "no method" shown. This is because some students will know/ be taught the binomial expansion earlier. See aside.

Without binomial:

$$\left(1-\frac{1}{x^2}\right)^3 = \left(1-\frac{1}{x^2}\right)\left(1-\frac{1}{x^2}\right)\left(1-\frac{1}{x^2}\right)$$

Do in stages

$$= \left(1-\frac{1}{x^2}\right)\left(1-\frac{2}{x^2}+\frac{1}{x^4}\right)$$

$$= \left(1-\frac{2}{x^2}+\frac{1}{x^4}-\frac{1}{x^2}+\frac{2}{x^4}-\frac{1}{x^6}\right)$$

Collect terms:

$$= \left(1-\frac{3}{x^2}+\frac{3}{x^4}-\frac{1}{x^6}\right)$$

So $p=-3$, $q=3$.

With binomial.

Pascals triangle.

$$\begin{array}{ccccccccc} &&&&1&&&& \\ &&&1&&1&&& \\ &&1&&2&&1&& \leftarrow (a+b)^2 = 1a^2+2ab+1b^2 \\ &1&&3&&3&&1& \leftarrow (a+b)^3 = 1a^3+3a^2b+3ab^2+1b^3 \\ 1&&4&&6&&4&&1 \quad \ldots \text{ So on.} \end{array}$$

Now just have to think about 'sign' as my 'b' here is negative $\left(-\frac{1}{x^2}\right)$. From here I can see right away $p=-3$ and $q=3$

Take $\left[a=1 \text{ and } b=-\frac{1}{x^2}\right]$ So $1a^3=1$, $3a^2b=3\left(-\frac{1}{x^2}\right)=-\frac{3}{x^2}$

$3ab^2=3\left(-\frac{1}{x^2}\right)^2=\frac{3}{x^4}$

$1b^3=\left(-\frac{1}{x^2}\right)^3=-\frac{1}{x^6}$.

6) i) The question is trying to help us integrate term-by-term. It's much easier to integrate a sum than it is a product.

$$\int \left(1-\frac{1}{x^2}\right)^3 dx = \int 1 - \frac{3}{x^2} + \frac{3}{x^4} - \frac{1}{x^6}\, dx$$

$$= \int 1 - 3x^{-2} + 3x^{-4} - x^{-6}\, dx$$

$$= x - \frac{3}{-1}x^{-1} + \frac{3}{-3}x^{-3} - \frac{x^{-5}}{-5}$$

$$= x + 3x^{-1} - x^{-3} + \frac{x^{-5}}{5} + C$$

↑ Don't forget

ii)

$$\int_{\frac{1}{2}}^{1} \left(1-\frac{1}{x^2}\right)^3 dx$$ use previous results.

$$= \left[x + 3x^{-1} - x^{-3} + \frac{x^{-5}}{5}\right]_{\frac{1}{2}}^{1}$$

$$= \frac{16}{5} - \left(\frac{1}{2} + 6 - 8 + \frac{32}{5}\right) = \frac{16}{5} - \frac{49}{\cancel{8}10}$$

$$= \frac{32-49}{10}$$

$$= -\frac{17}{10}$$