If I break a stick of unit length into 2 pieces, what is the average size of the smaller piece?What is the average ratio of the smaller length to the larger length?

We only require one break to obtain 2 fragments. Say the break occurs at $x \in [0,1]$. The break is just as likely to be in the left hand half as the right. If the break occurs within $[0,1/2]$ then the smallest length is $x$, otherwise it is $1-x$. Therefore, Let $L$ be random variable, the length of the smallest piece, then

$$L = \begin{cases} x & \text{for } x \in [0,1/2] \\ 1-x & \text{for } x \in [1/2,1] \end{cases}$$

Therefore

$$\begin{aligned} \mathbb{E}(L) &= \int_0^{1/2} x\,dx + \int_{1/2}^1 (1-x)\,dx \\ &= [1/2x^2]_0^{1/2} + [x - 1/2x^2]_{1/2}^1 \\ &= \frac{1}{8} + \left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{8}\right) = \frac{2}{8} = \frac{1}{4} \end{aligned}$$

To answer the second part, when $x > 1/2$ the smallest length is given by $1-x$, and vice versa. Therefore

$$E = \int_{1/2}^1 \frac{1-x}{x}\,dx + \int_0^{1/2} \frac{x}{1-x}\,dx$$

is the expected average ratio. Regarding the second integral

$$\begin{aligned} &I = \int_0^{1/2} \frac{x}{1-x}\,dx \\ &\text{by making the change of variable} \quad x \to 1-y \implies dx = -dy \\ &\text{with limits } x = 0 \to y = 1, \quad x = 1/2 \to y = 1/2 \\ &I = \int_1^{1/2} -\frac{1-y}{y}\,dy = \int_{1/2}^1 \frac{1-y}{y}\,dy \end{aligned}$$

Therefore

$$E = 2\int_{1/2}^1 \frac{1-x}{x}\,dx = 2[\log x - x]_{1/2}^1 = 2[(0-1) - (\log 2 - 1/2)] = 2\log 2 - 1 \approx 0.39$$