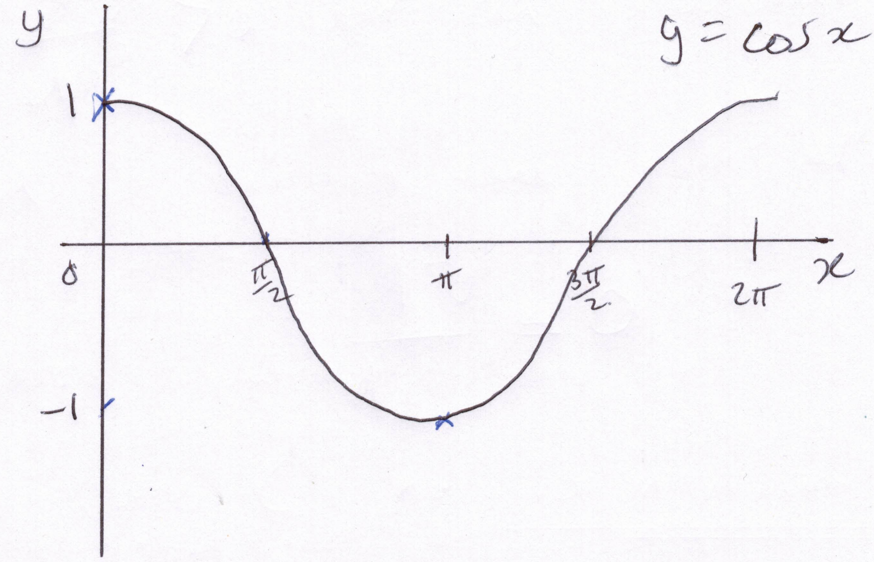
7

a)



Before we sketch: recall: $\cos(0) = 1$, $\cos(\frac{\pi}{2}) = 0$
$\cos(\pi) = -1$

b)

i) Given $\sin^2\theta = \cos\theta(2 - \cos\theta)$

Recall: $\sin^2\theta + \cos^2\theta \equiv 1$ (for <u>any</u> θ)

$$\sin^2\theta = 2\cos\theta - \cos^2\theta$$

$$\Rightarrow \sin^2\theta + \cos^2\theta = 2\cos\theta$$

$$\Rightarrow 1 = 2\cos\theta, \quad \text{using } \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

Always good idea to signal to examiner that you know what you are doing / method.

(7)

b)

ii) Solve $\sin^2 2x = \cos 2x(2-\cos 2x)$

This is testing if you are confident that $\sin^2\theta + \cos^2\theta = 1$ for <u>ANY</u> θ.

It is testing the '<u>ANY</u>' part.

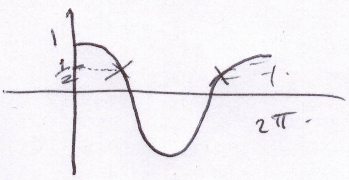
Let $2x = \theta$. (form the same as in 6i).

Then $\sin^2\theta = \cos\theta(2-\cos\theta)$

We already know this equation implies

$\cos\theta = \frac{1}{2}$.

Use graph in a) or knowledge of 'special' points.



so 2 solutions to $\cos\theta = \frac{1}{2}$ in $0 \le \theta \le 2\pi$.

$\cos\frac{\pi}{3} = \frac{1}{2}$ and (by symmetry)

$\cos\left(2\pi - \frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$.

Remember we need solutions for x, not θ.

So $\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

[Could also have numerical answers here].

So $\underline{\underline{x = 0.524, 2.62.}}$ (3.s.f)