In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
import skimage as skimage
import scipy

from scipy.sparse import csr_array
from scipy.sparse import lil_array
from scipy.sparse.linalg import lsqr

# note we imported the sparse least squares function
```

In [2]:
```python
# take some images with different exposure times.
# make sure your camera (phone) does not move between exposures. I used a ph

# images
Ims = []
# exposure times
dts = []

Ims.append(plt.imread('imgs/0.005.jpg'))
dts.append(0.005)

Ims.append(plt.imread('imgs/0.02.jpg'))
dts.append(0.02)

Ims.append(plt.imread('imgs/0.1.jpg'))
dts.append(0.1)

Ims.append(plt.imread('imgs/1.jpg'))
dts.append(1)

# Ims.append(plt.imread('1.7.jpeg'))
# dts.append(1.7)

# Ims.append(plt.imread('2.0.jpeg'))
# dts.append(2.0)
```

In [3]:
```python
# resize the images to a reasonable scale
Imsr = []
for idx in range(len(Ims)):
    # note this will resize factor of 1/8 for each side and convert to type

    # "cheap" conversion from color to gray scale by taking the mean of rgb
    I = np.mean(Ims[idx],axis=2)
    # resize to 1/8 of each dimension
    Ir=skimage.transform.resize(I, (I.shape[0] // 8, I.shape[1] // 8), anti_
    Imsr.append(Ir)
    print(f"Imsr[{idx}].shape={Imsr[idx].shape} Imsr[{idx}].dtype={Imsr[idx]
    print(f"values from {np.min(Imsr[idx][:])} to {np.max(Imsr[idx][:])}")

    # convert to uint8 (forcing a value from 0..255) just to have discrete p
```

```
    Imsr[idx]=Imsr[idx].astype("uint8")
    print(f"Imsr[{idx}].shape={Imsr[idx].shape} Imsr[{idx}].dtype={Imsr[idx]
    print(f"values in {np.min(Imsr[idx][:])} .. {np.max(Imsr[idx][:])}")
    print(f"dts[{idx}]={dts[idx]}")
```

```
Imsr[0].shape=(378, 504) Imsr[0].dtype=float64 values from 0.0 to 125.032616
98218526
Imsr[0].shape=(378, 504) Imsr[0].dtype=uint8 values in 0 .. 125
dts[0]=0.005
Imsr[1].shape=(378, 504) Imsr[1].dtype=float64 values from 0.004179065255576
94 to 250.5974586594087
Imsr[1].shape=(378, 504) Imsr[1].dtype=uint8 values in 0 .. 250
dts[1]=0.02
Imsr[2].shape=(378, 504) Imsr[2].dtype=float64 values from 0.028511227486451
4 to 255.0
Imsr[2].shape=(378, 504) Imsr[2].dtype=uint8 values in 0 .. 255
dts[2]=0.1
Imsr[3].shape=(378, 504) Imsr[3].dtype=float64 values from 0.164486891265720
25 to 255.0
Imsr[3].shape=(378, 504) Imsr[3].dtype=uint8 values in 0 .. 255
dts[3]=1
```

In [4]:

```
# simple version of optimization

# solving for entries in v
# first 256 entries are for g(0) ... g(255) then next h*w entries are for pi
#
# Z_ij is the pixel value at location i in image j
# dt_j is the exposure time for image j
# g(z) is the energy x that the sensor receives (exposure) to produce pixel
#      (the exposure is the irradiance R times exposure time)
#
#  x = R*dt
#  for pixel i in image j
#  x_ij = R_ij*dt_j
#
#  Z_ij = f(x_ij)  # the pixel value Z_ij comes from mapping the exposure th
#  g() is the inverse of f()
#
#  so if Z_ij = f(x_ij) we want g(Z_ij)=x_ij (=R_ij*dt_j)
#
#  taking logs of everything:
#
# g(Z_ij) = ln(R_i)+ln(dt_j) # pixel i,image j with time dt_j
# g(Z_ij) - ln(R_i) = ln(dt_j)
#
# making this into a constraint
#
# Each pixel i in imag j gives one row of A and one entry in b
# position: 0...... Z_ij ...255 0 1 2 ... i*w+j ... h*w
# coefficient: 0 0 ... 1.......0 0 .......... -1 ......0
#
# entry in b: ln(dt_j)
```

```python
#
#  We want to solve for v so that Av=b
#  or the v that minimizes |Av-b|^2


[h,w]=Imsr[0].shape
# total how many images in Ismr
l = len(Imsr)

#num of rows total number of image * total number of pixels
r = l * h * w

#num of columns 256 for 0 to 255 pixels and h*w for total number of pixels
c = 256 + h * w


# setup rows of A and entries in b
#sparse matrix
A = lil_array((r + 1, c))
b = np.zeros(r + 1)

row = 0
check = 0
for j in range(l):
    for i in range(h):
        for k in range(w):

            Z_ij = Imsr[j][i, k]
            # position: 0...... Z_ij ...255 0 1 2 ... i*w+j ... h*w
            # coefficient: 0 0 ... 1.......0 0 .......... -1 ......0
            A[row, Z_ij] = 1
            A[row, i * w + k + 256] = -1
            # entry in b: ln(dt_j)
            b[row] = np.log(dts[j])
            row += 1

# add a constraint that "fixes" g(128) to be say 4 or such
A[row, 128] = 1
b[row] = np.log(4)
```

In [5]:

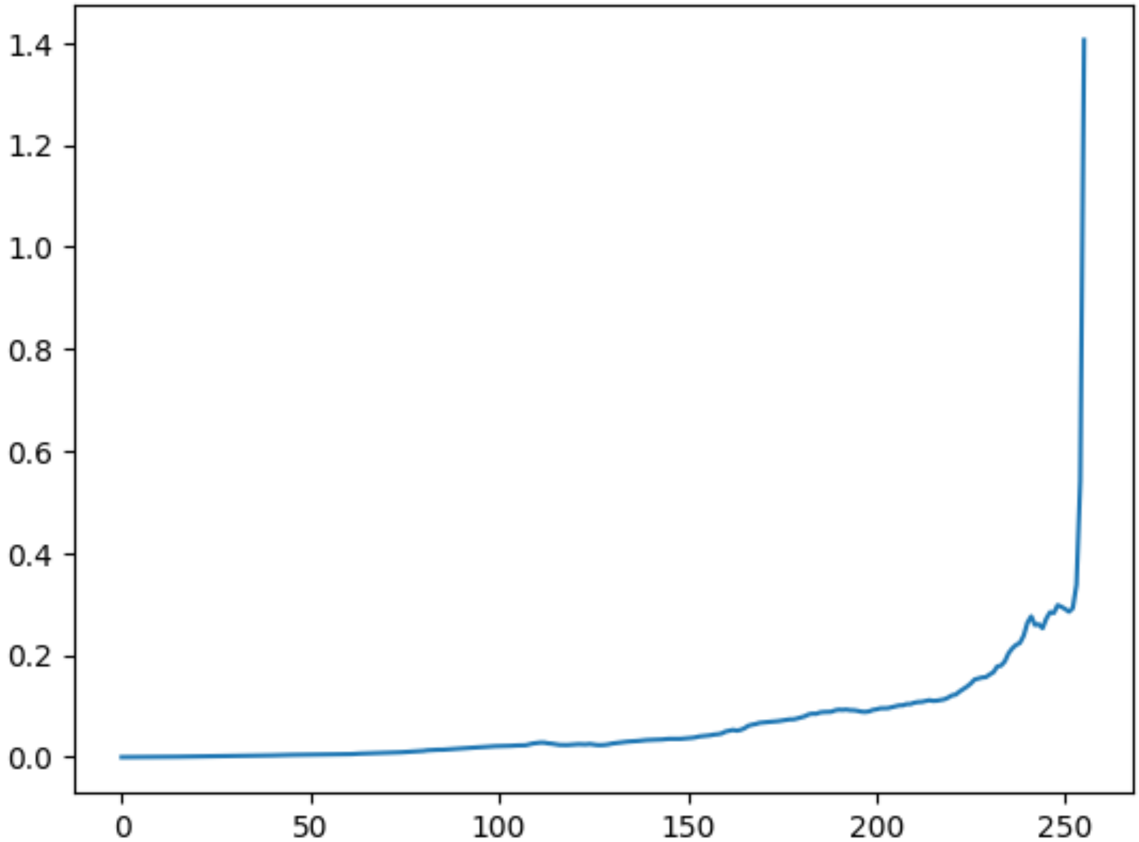
```python
#solve for least squares solution
Acsr=csr_array(A)# convert a to a csrarray before calling least squares
soln = lsqr(Acsr,b,atol=1e-07, btol=1e-07)
v = soln[0]
```

In [14]:

```python
plt.plot(np.exp(v[0:256]))
```

Out[14]:

```
[<matplotlib.lines.Line2D at 0x2b597f820>]
```



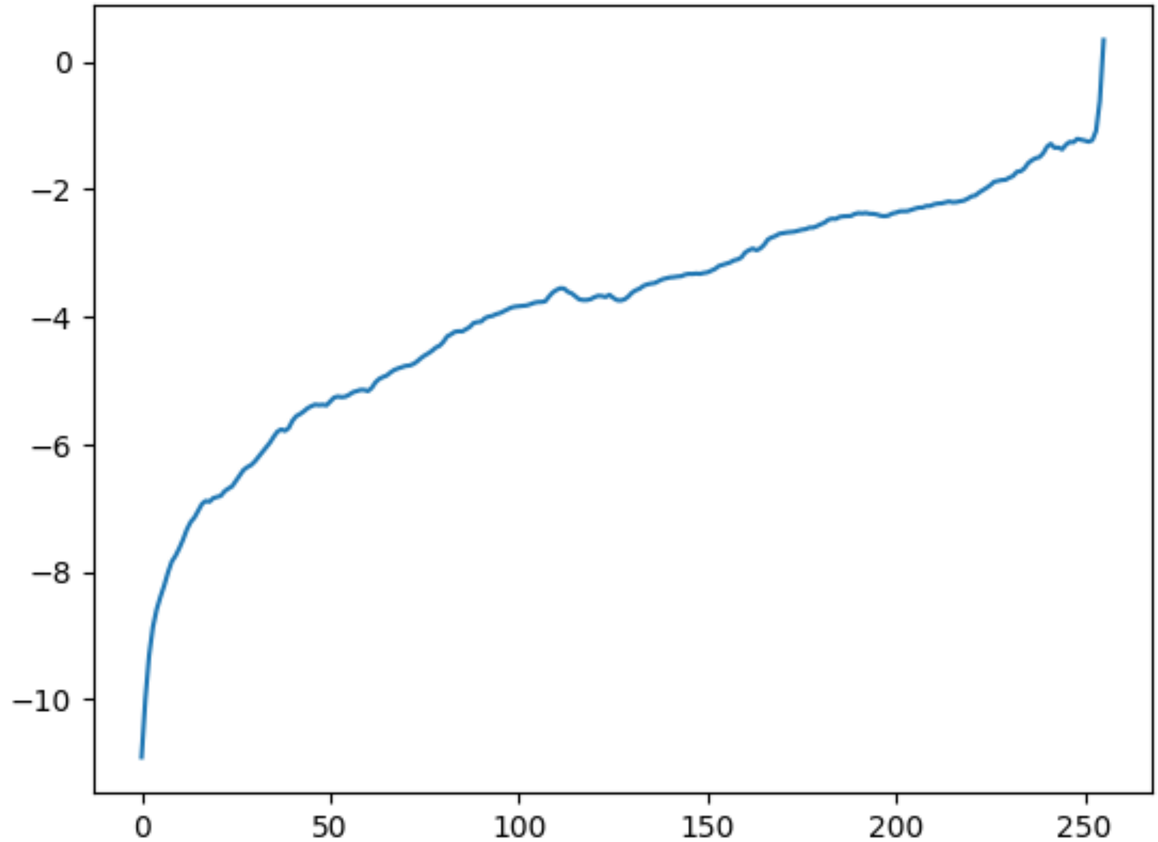
In [15]:
```
# Adding some bells and whistles to the optimization setup
#
#
# solving for entries in v
# first 256 entries are for g(0) ... g(255) then next h*w entries are for pi
# g(x) = ln(R_i)+ln(dt_j) # pixel i,image j with time dt_j
# g(x) - ln(R_i) = ln(dt_j)
#
# weighted version where weight depends on the pixel value (can care less ab
# w(R_i)g(x) - q(R_i)ln(R_i) = w(R_i)ln(dt_j)
#
# also add regularization so that g(x) tends to be smooth
#
# l(g(i+1)-g(i)) - l(g(i)-g(i-1)) = 0
# lg(i)-2g(i+1)-g(i+2) = 0
#
# where l is some weight on this constraint
#
# remember to add a constraint that "fixes" g(128) to be say 4 or such

# make a new version of A and b with these weights and constraints


# Assume Imsr is a list of image arrays and dt is a list of exposure times
[h, w] = Imsr[0].shape
l = len(Imsr)

# Total number of variables and equations, plus regularization terms and the
```

```python
c = 256 + h * w
r = l * h * w + 256 - 2 + 1  # Additional -2 for the first and last g(x) reg

A = lil_array((r, c))
b = np.zeros(r)

# Function to define weights based on pixel value
def weight(z):
    if z <= 5 or z >= 250:
        return 1  # Lower weight for extreme values
    else:
        return 10  # Higher weight for mid-range values

# Populate A and b with weighted equations
row = 0
for j in range(l):
    for i in range(h):
        for k in range(w):
            Z_ij = Imsr[j][i, k]
            w_ij = weight(Z_ij)
            A[row, Z_ij] = w_ij  # Apply weight to g(Z_ij)
            A[row, 256 + i*w + k] = -w_ij  # Apply weight to -ln(R_i)
            b[row] = w_ij * np.log(dts[j])  # Apply weight to ln(dt_j)
            row += 1

# Regularization for smoothness of g(x)
l = 50  # Weight for regularization terms
for i in range(1, 255):  # Skip the first and last pixel value for this cons
    A[row, i-1] = l
    A[row, i] = -2 * l
    A[row, i+1] = l
    row += 1

# Constraint for fixing g(128)
A[row, 128] = 1
b[row] = np.log(4)
```

In [16]:
```python
Acsr=csr_array(A)
soln = lsqr(Acsr,b,atol=1e-07, btol=1e-07)
v=soln[0]
```

In [17]:
```python
plt.plot(v[0:256]) # note plotting g(z) here not exp(g(z))
```

Out[17]:
```
[<matplotlib.lines.Line2D at 0x2b5b27280>]
```



In [18]:

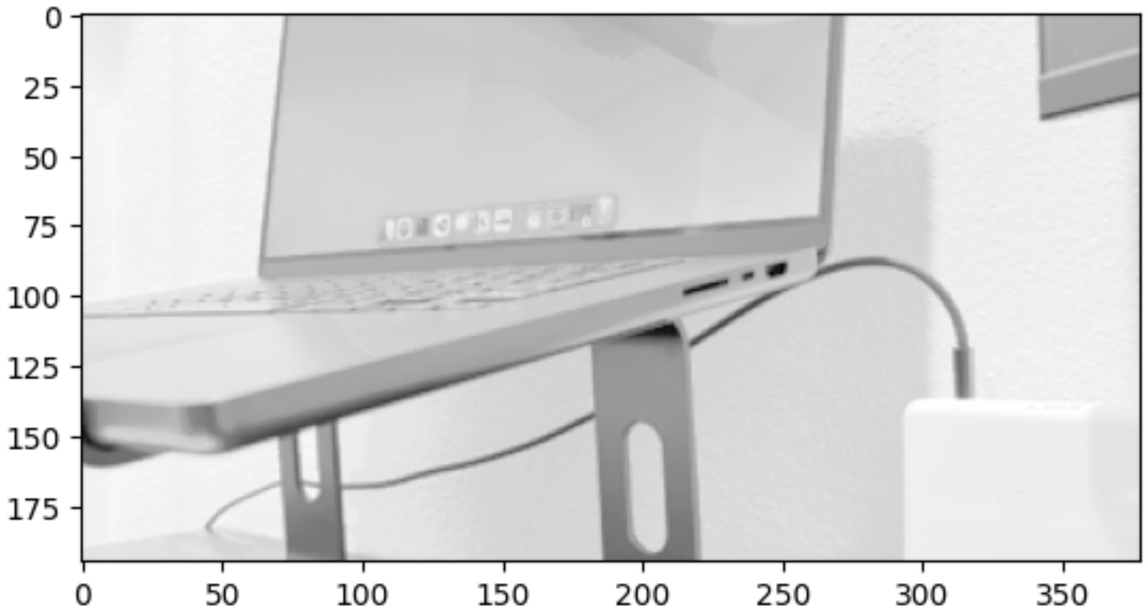
```
f=plt.figure()
f.set_size_inches(10, 6)
ax = f.add_subplot(1,5,1)
ax.imshow(np.fliplr(np.transpose(np.reshape(v[256:],(h,w)))),cmap="gray")
ax.axis("off")
for i in range(len(Imsr)):
    ax = f.add_subplot(1,5,i+2)
    ax.imshow(np.fliplr(np.transpose(np.log(Imsr[i]+1))),cmap="gray")
    ax.axis("off")
```

```
/var/folders/x9/gt3hm8v90sqg1lhhj3y63ndw0000gn/T/ipykernel_16663/3258305814.
py:8: RuntimeWarning: divide by zero encountered in log
  ax.imshow(np.fliplr(np.transpose(np.log(Imsr[i]+1))),cmap="gray")
```

In [19]:

```
da_Im=np.fliplr(np.transpose(np.reshape(v[256:],(h,w))))
plt.imshow(da_Im[145: 340,:],cmap="gray")
```

Out[19]: <matplotlib.image.AxesImage at 0x2b59412a0>

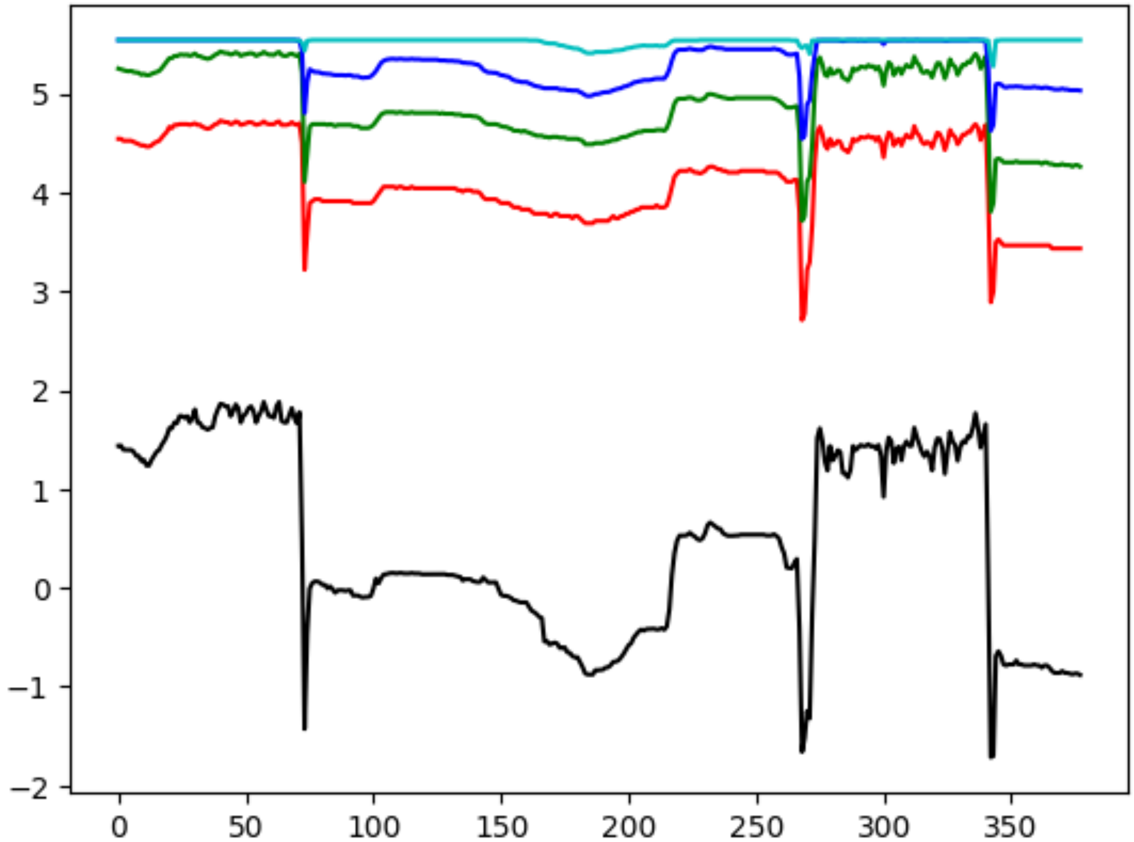


In [20]:

```
# show one horixontal line (bottom of image above) in each exposure and the
# note that the hdr image has detail in both the darker and the lighter area
# exposures lack detail in one (e.g. no dark detail in red) or the other (no
plt.plot(da_Im[145,:],'k')
plt.plot(np.flipud(np.log(Imsr[0][:,145])),color='r')
plt.plot(np.flipud(np.log(Imsr[1][:,145])),color='g')
plt.plot(np.flipud(np.log(Imsr[2][:,145])),color='b')
plt.plot(np.flipud(np.log(Imsr[3][:,145])),color='c')
# plotted logs to make it easier to see variations
```

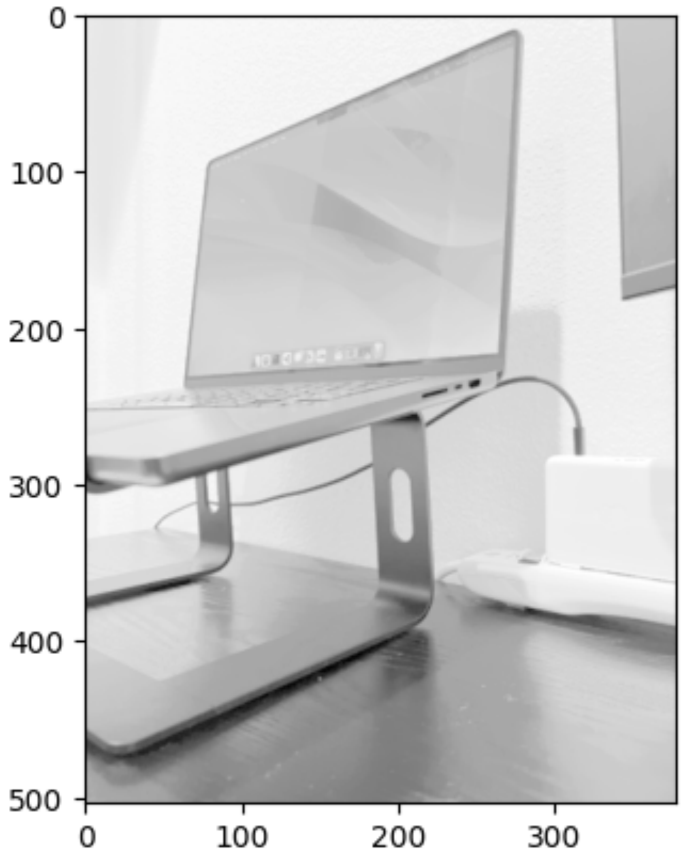
Out[20]:

```
[<matplotlib.lines.Line2D at 0x2b58d6800>]
```



In [21]:
```
test_im=np.fliplr(np.transpose(np.reshape(v[256:],(h,w))))
plt.imshow(test_im, cmap = 'gray')
```

Out[21]: <matplotlib.image.AxesImage at 0x2b4390040>



In [ ]: