B062 1111 羅証權

1. $\mu = 0$. $\sigma = 1$. $\sigma^2 = 1$

(a) $f_Z(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

(b) $P(-1 \le Z \le 1) = 2P(Z<1) - 1 = 2 \times 0.8413 - 1 = 0.6826$ #

(c) $P(-x \le Z \le x) = 0.95$

$2P(Z < x) - 1 = 0.95$

$P(Z < x) = 0.975$

$\Rightarrow x = 1.96$ # (查表)

(d) $Q = Z^2$. $f_Q(q) = \frac{1}{\sqrt{2}\,\Gamma(\frac{1}{2})} q^{-\frac{1}{2}} e^{-\frac{q^2}{2}} = \frac{1}{\sqrt{2\pi}} q^{-\frac{1}{2}} e^{\frac{-q^2}{2}}$

(e) $E[Q] = V = 1$

(f) $std[Q] = \sqrt{\sigma^2} = \sqrt{2V} = \sqrt{2}$

(g) $P(Q \le 1) = 0.6826$

2.

(a) $f_T(t) = \begin{cases} e^{-t} & .\ t>0 \\ 0 & .\ \text{else} \end{cases}$

$+t \quad e^{-t}$
$-1 \searrow -e^{-t}$
$+0 \searrow e^{-t}$

(b) $E[T] = \int_{-\infty}^{\infty} t e^{-t} dt = \int_0^{\infty} t e^{-t} dt = [-te^{-t} - e^{-t}]\Big|_0^{\infty} = 1$

(c) $std[T] = \sqrt{\frac{1}{\lambda^2}} = 1$

(d) $P(T>1) = 1 - e^{-1} = 0.3679$

(e) $f_{T_3}(t) = 3 \cdot e^{-t}$

(f) $E[T_3] = 3 \cdot \mu = 3$

(g) $std[T_3] = \sqrt{\beta^2} = 3$

(h) $P(T_3 > 3) \Rightarrow P(T>1) = 0.3679$

(i) $P(T_3 > 7) = 1 - F(7)$

$= 1 - \frac{1}{3}\int_0^7 e^{-\frac{t}{3}} dt = \frac{1}{e^{\frac{7}{3}}} = 0.097$

此值 > 0.05

所以在可接受範圍