MATH 2271- **Ordinary Differential Equations**

Lab Assignment (Dr. A. Sankar-Ramkarran)

Note:

- Please submit by **4:00 p.m**. on **Monday 25th March, 2019** in the Department of Mathematics and Statistics in the **MATH 2271** box.

- Be sure to include all working done by hand (this may also be typed), all Matlab code (functions, scripts) and the Matlab command window calls and graphs generated.

1. Consider the initial value problem

$$y^{'} + 2\,y = 2 - e^{-4x}\,,\ y(0) = 1$$

   (a) Using an appropriate analytical method obtain the solution,

$$y \;=\; 1 + \frac{1}{2}e^{-4x} - \frac{1}{2}e^{-2x}$$

   Please show all working (by hand or typed). [5 mks]

   (b) Using part (a), state the value of $y(0.3)$ (round off to 3 d.p.) [1 mk]

   Consider finding a numerical solution for the ODE over the interval $[0, 0.5]$

   (c) Using **The Euler Method** and considering step size $h = 0.1$ over the interval $[0, 0.5]$, obtain the approximate value of $y(0.3)$. Please show all working (by hand or typed) (work with 5 d.p. where appropriate and round off final answer to 3 d.p). [5 mks]

   (d) Write a general *Matlab function* **EulerFirst** which uses The Euler Method to find the numerical solution of the general first order initial value problem:

$$\frac{dy}{dx} = f(x, y)$$

   over the interval $[a,\, a + l]$, $l \in \mathbb{R}^{+}$ with step size h. Note:

   - The **input** variables for your function should be a, l, h, y_0 and f
   - **EulerFirst** should **output** the variables x and y (do not include any plot commands in this function).
   - Remember to save this function file as **EulerFirst.m**
   - In your submission, also include the necessary *Matlab function file* **f.m** [9 mks]

   (e) Write a script file **Script1** to overlay three numerical solutions for different step sizes, $h = 0.1$, $h = 0.01$ and $h = 0.001$. Note:

   - Label the axes appropriately and include a suitable title.
   - Include an appropriate legend for each of the curves ($h = 0.1$, $h = 0.01$ and $h = 0.001$). [5 mks]

   (f) Show the plot obtained by running **Script1** in the *Command Window* [5 mks]

   (g) Is The Euler Method underestimating or overestimating the exact solution? Give a reason for your answer. [2 mks]

   (h) Using the *Matlab function* **dfield9** (provided), generate the direction field and plot the **particular solution** of this IVP. In the display window show $0 \le x \le 0.5$ and $0 \le y \le 1$. Please include **both** the dfield9 Setup and the direction field as figures. [5 mks]

2. Consider the initial value problem

$$y'' + 4y = t^2 + 3e^t;$$
$$y(0) = 0, \; y'(0) = 2$$

(a) Using an appropriate analytical method obtain the solution,

$$y \;=\; \frac{7}{10}\,sin(2t) - \frac{19}{40}\,cos(2t) + \frac{1}{4}\,t^2 - \frac{1}{8} + \frac{3}{5}\,e^t$$

Please show all working (by hand or typed). [15 mks]

(b) Write the IVP as a **system of two first order** ODEs with a corresponding inital condition for each ODE. Please show all working (by hand or typed). [4 mks]

(c) Write the Matrix form of the **system** in (b). Please show all working (by hand or typed). [4 mks]

(d) Write a general *Matlab function* **EulerSecond** which uses The Euler Method to find and plot the numerical solution for the IVP over the interval $[a, a + l]$, $l \in \mathbb{R}^+$ with step size h by considering the system from (b). Include suitable code to label the axes. In your submission also include any other *Matlab function file(s)* utilized. [10 mks]

(e) Considering the interval $[0, 5]$ with a step size of 0.001, write the command which must be run in the *Command Window* to find a numerical solution. [1 mk]

(f) Show the plot obtained from running (e). [2 mks]

3. Consider the following system of first order ODEs

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix}' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

(a) Write the three first order ODEs with their corresponding inital condition of the given system.
[Note: The exact solutions are: $u = 2e^x$; $v = -3e^x + 4e^x cos(2x) + e^x sin(2x)$;
$w = 2e^x + 4e^x sin(2x) - e^x cos(2x)$.] [6 mks]

(b) Using the *Matlab function* **EulerSys** (provided) overlay plots of the numerical solutions for u, v and w over the interval $[0, 2]$, using a step size $h = 0.001$
Please ensure to provide:
- Any other *Matlab function file(s)* utilized to obtain the solution. [5 mks]
- The command line call used to obtain the solution. [1 mk]
- The plot obtained. [1 mk]

4. Consider the homogeneous system of first order ordinary differential equations

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$$

(a) Using an appropriate analytical method obtain the general solution,

$$\mathbf{x} = C_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + C_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$$

where C_1 and C_2 are arbitrary constants. [8 mks]

(b) Using the *Matlab function* **pplane8** (provided), generate the phase portrait for this linear system. You must illustrate the direction field arrows, the equilibrium point and at least 10 solution curves of your choice.
Please include **both** the pplane8 Setup and the phase portrait as figures. [4 mks]

(c) Comment on the phase portrait from (b). [2 mks]

Total:100 mks

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