HTML hw1 solution

學號:B10502166 姓名:謝銘倫 系級:電機三

1.

Self-supervised Learning for Medical Image Analysis:

Medical imaging is a vital tool for diagnosis and treatment planning in modern healthcare. However, annotating medical images, like X-rays or MRI scans, requires domain expertise and is time-consuming. Self-supervised learning offers a solution to this data annotation bottleneck. Using this approach, the model is pre-trained on a large volume of unlabeled medical images. The learning task might involve predicting a part of an image from another part or rotating images to a specific orientation. Once the model has learned meaningful features from this task, it can be fine-tuned on a smaller set of labeled data. This approach significantly reduces the need for extensive labeled datasets. By using self-supervised learning, we can more efficiently utilize the vast amounts of unlabeled medical images available. This can potentially enhance diagnostic accuracy and lead to better patient outcomes.

2.

Yes, I agree with the answer. Reinforcement learning can indeed be used to solve the shortest path problem in a maze. In this approach, a person or player can be chosen as the agent, and the maze can be treated as the environment. By properly setting rewards, such as giving higher rewards for fewer returning times or for finding the shortest path, the agent can learn to navigate the maze more efficiently.

The concept of reinforcement learning involves training an agent to make decisions based on the feedback it receives from its environment. In the case of a maze problem, the agent's goal is to find the shortest path from a starting point to a target destination. By assigning rewards to certain actions or states, the agent can learn which actions lead to a more favorable outcome.

For example, if the agent takes a path that leads to a dead end and has to backtrack, it can receive a negative reward. On the other hand, if the agent finds a shorter path or reaches the target destination, it can receive a positive reward. Through a process of trial and error, the agent learns to navigate the maze by maximizing its expected rewards.

By using reinforcement learning, we can tackle the shortest path problem in a maze without explicitly programming the optimal solution. Instead, the agent learns from its interactions with the environment and gradually improves its decision-making abilities. This approach can be particularly useful in scenarios where the optimal solution is unknown or difficult to compute.

In summary, reinforcement learning offers a promising approach to solving the shortest path problem in a maze. By treating the maze as an environment and setting appropriate rewards, we can train an agent to navigate the maze efficiently and find the shortest path.

3.

It is true that ChatGPT does not disapprove of any possibilities in this problem.
Off-the-shelf algorithms that are already proven to be the fastest or most optimal may not necessarily be improved by machine learning techniques. However, for problems that have not been fully proven or optimized, such as sorting problems, machine learning algorithms like AlphaDev can potentially discover algorithms with fewer instructions than human benchmarks, as discussed in the mentioned article.

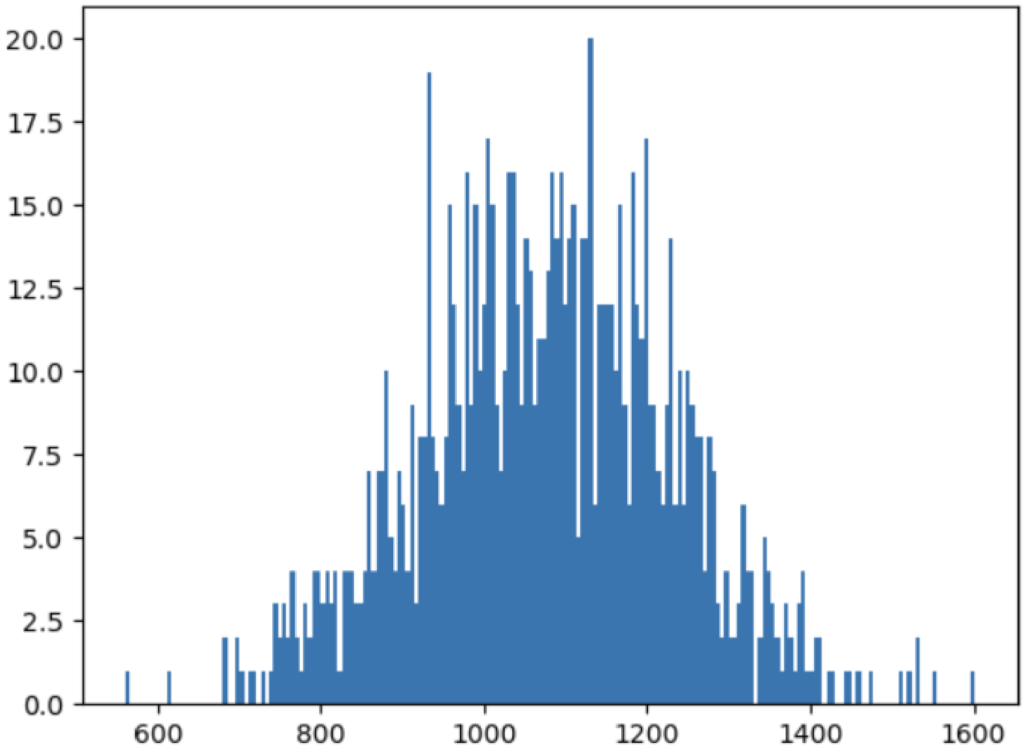
Machine learning algorithms, including reinforcement learning, have the ability to learn from data and adapt to different scenarios. They can explore and discover patterns or approaches that may not have been explicitly programmed. This allows them to potentially find more efficient solutions or even outperform existing algorithms in certain cases.

In the context of the shortest path problem in a maze, reinforcement learning can be a valuable approach. By learning from interactions with the environment, an agent can adapt its decision-making process to find the shortest path. This flexibility and adaptability make machine learning algorithms suitable for problems where the optimal solution is not well-defined or when the problem itself is subject to change.

While off-the-shelf algorithms may be the preferred choice for certain well-established problems, machine learning can offer advantages in scenarios where optimization through learning is desired or where existing algorithms have limitations. It is important to consider the specific problem, available data, and the potential benefits and trade-offs between different approaches.

Overall, machine learning, including reinforcement learning, can complement traditional algorithms and provide new possibilities for solving complex problems.

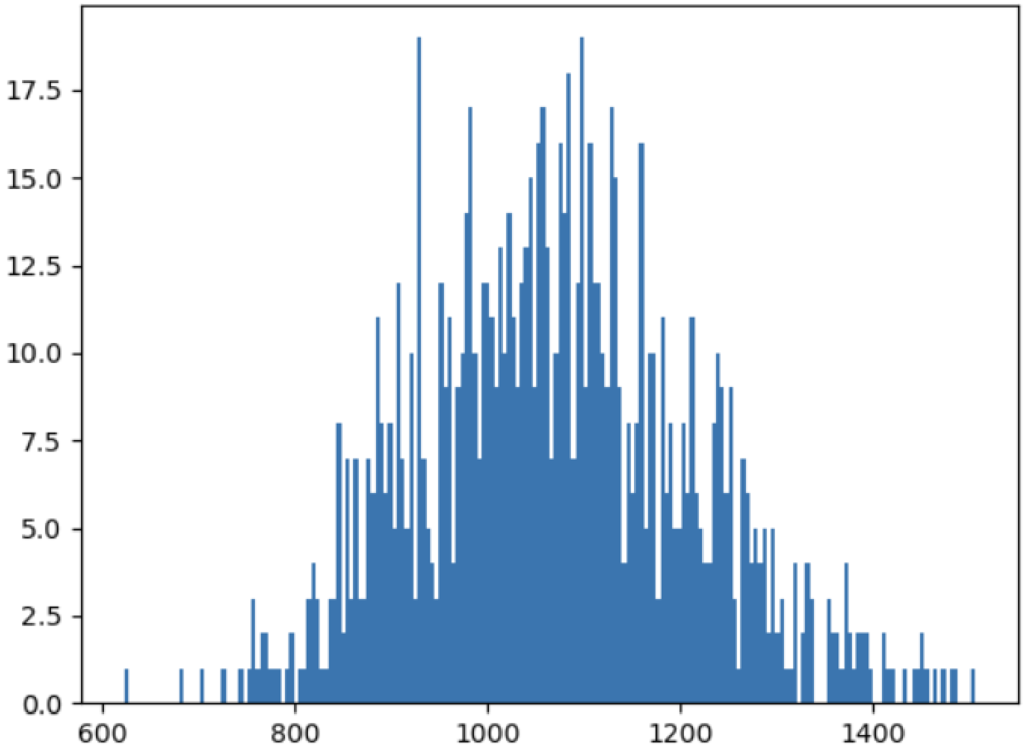
9.



execute: pla_p9.py
get: PLA_P9.png

median:
1079.5

10.

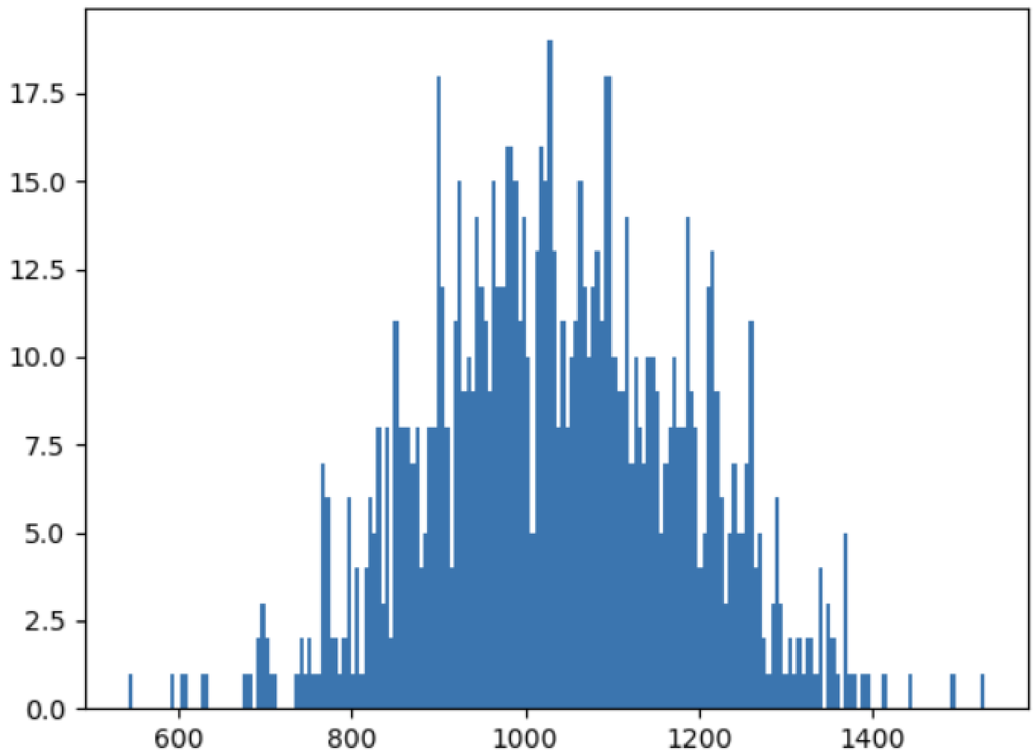


execute: pla_p10.py
get: PLA_P10.png

median: 1065.0

Findings:
The result of problem 10 is equivalent(very similar) to problem 9, which means the we will get the same w_PLA after running the PLA no matter problem 10 scale up x_n to 11.26

11.

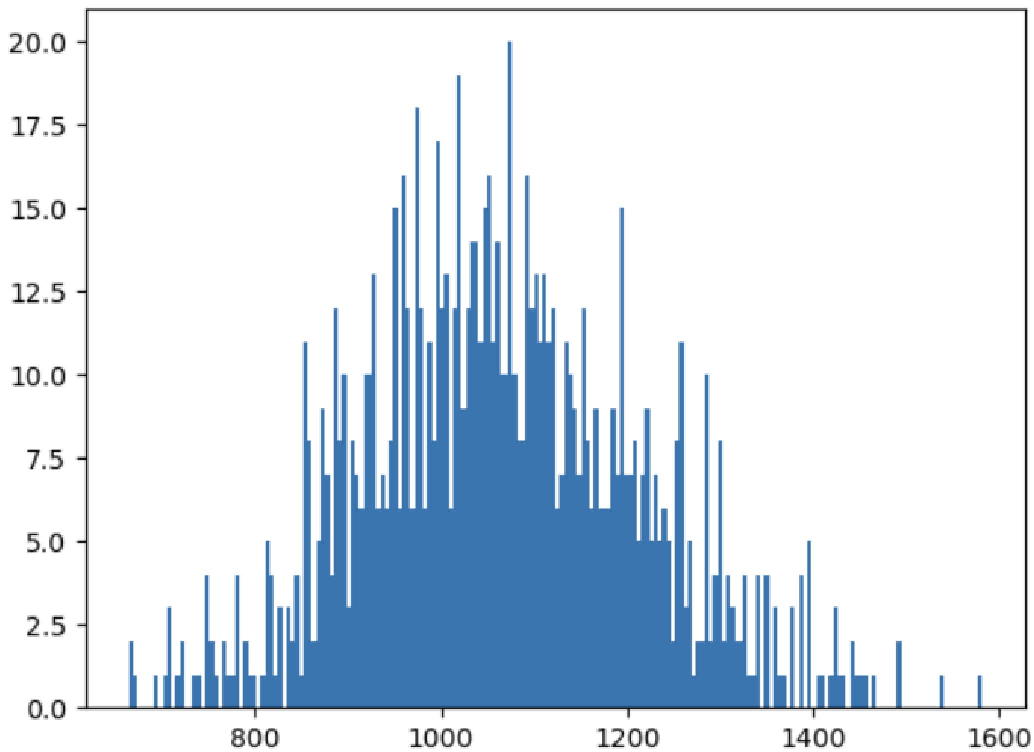


execute: pla_p11.py
get: PLA_P11.png

median:
1056.0

Findings:
The result of problem 11 is equivalent(very similar) to problem 9, which means the we will get the same w_PLA after running the PLA no matter problem 11 set the bias term to x_0=11.26

12.



execute: pla_p12.py
get: PLA_P12.png

median: 1072.5

Findings:

The result of problem 12 is equivalent(very similar) to problem 9, which means the we will get the same w_PLA after running the PLA no matter problem 12 keeps correcting the same example.

HTML hw #1

1. ~~Self-supervised learning teaches machines using self-generated hints from the data itself, which is ideal for tasks like image colorization.~~

2. ~~I agree, using reinforcement learning, agents can learn and navigate mazes, optimizing paths through rewards and penalities, that is, by trail and error.~~

3. ~~ChatGPT provided a broad view on ML's limits; Deepmind's success in sorting is specific, exceptional case in ML-driven algorithm improving.~~

4. By PLA, $W(t+1) = W(t) + y_n(t)\, X_n(t)$

Consider $n=0$ $\left(W_0 = 0, X_0 = 1 \right)$, We check the first term of $W(t)$ with $W_t[0]$

$$W_0(t+1) = W_0(t) + y_n(t) \cdot 1$$

$$\Rightarrow W_0(t+1) = W_0(t) + y_n(t)$$

Given $W_0(0) = 0$ by definition, When "mistake" occurs $\left(\text{sign}(W_n^T X_n) \neq y_n(t) \right)$

$W_{t+1} = W_n + y_n X_n \Rightarrow$ For $W_t[0]$ term $\left(X_0 = 1, w_0 = 0 \right)$

with mistake $T_+ \Rightarrow W_{t+1}[0] = W_t[0] + T_+(+1) = W_t[0] + T_+$

with mistake $T_- \Rightarrow W_{t+1}[0] = W_t[0] + T_-(-1) = W_t[0] - T_-$

Combine all the update for ① ②

$$W_t[0] = W_0 \underset{(W_0=0)}{=} 0 + T_+ + (-T_-) = \underline{\underline{T_+ - T_-}} \ \#$$

5.

To find upper bound of PLA, we consider R, ρ, and find them

$$R^2 = \max \|X\|^2 , \quad \rho = \min_n y_n \frac{W_f^T}{\|W_f\|} X_n$$

$$T \le \frac{R^2}{\rho^2}$$

- For R : $\because X \in R^{d+1}$, and x contains m words and one bias term ($X_0 = 1$)

$$\therefore \|X\| = \sqrt{m+1} \qquad \underbrace{[1\,0\,1\,0\,1\,0\,1 \cdots\ 1]}$$

X_0

$d+1$ terms with $m+1$ terms are 1.

$$\Rightarrow R^2 = m+1$$

- For ρ, (the smallest change while mistake)

given $f(x) = z_+(x) - z_-(x) - 0.5$, for difference of $\operatorname{sign}(z_+(x) - z_-(x))$ and 0.5 is $\gamma = 0.5$

$$\therefore T \le \left(\frac{m+1}{(0.5)^2}\right) \Rightarrow T \le 4(m+1)$$

- Given there are d words

$$M_{total} \le 4(m+1) \cdot d = 4d(m+1)$$

- The bias term X_0 can also contribute to additional mistake

which is $M_{bias} = 4(m+1) \cdot 1 = 4(m+1)$

$$\therefore M_{final} \le M_{total} + M_{bias} = 4d(m+1) + 4(m+1) = (4d+4)(m+1)$$

which is proved the upperbound can be $(4d+1)(m+1)$ #

6.

$X_n = (1, X_n^{orig})$ $\quad W(t+1) = W(t) + y_n(t) X_n(t)$

$X_n' = (-1, X_n^{orig})$ $\quad W_0 = 0$

Consider hypothesis: $h(x) = \text{sign}\left[\sum_{i=1}^{d} W_i X_i - \text{threshold}\right]$

$$= \text{sign}\left[\sum_{i=1}^{d} W_i X_i + \underbrace{(-\text{threshold})}_{W_0 = 0} \cdot \underbrace{X_0}_{(+1 \text{ for } X_n,\ -1 \text{ for } X_n')}\right]$$

$$= \text{sign}\left[\sum_{i=0}^{d} W_i X_i\right]$$

$$= \text{sign}\left[W^T X\right]$$

$\because$ We define threshold $W_0 = 0$, no matter $X_0 = +1$ or $X_0 = -1$, it won't affect the value of inner product $W^T X$ (which means $\text{sign}(w_{PLA}^T X) = \text{sign}(w_{PLA}'^T X)$)

However, for PLA update rule $W(t+1) = W(t) + y_n(t) X_n(t)$

For $X_0 = 1$, $W(t+1) = W(t) + y_n(t)\begin{bmatrix} 1 \\ x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} = W(t) + y_n(t)\begin{bmatrix} 1 \\ X_n^{orig}(t) \end{bmatrix}$

$$= \begin{bmatrix} W_0(t) + y_n(t) \\ W_n(t) + y_n(t) X_n^{orig}(t) \end{bmatrix}$$

For $X_0 = -1$, $W(t+1) = W(t) + y_n(t)\begin{bmatrix} -1 \\ x_1 \\ x_2 \\ x_n \end{bmatrix} = W(t) + y_n(t)\begin{bmatrix} -1 \\ X_n^{orig}(t) \end{bmatrix}$

$$= \begin{bmatrix} W_0(t) - y_n(t) \\ W_n(t) + y_n(t) X_n^{orig}(t) \end{bmatrix}$$

Hence, if we sum up from $t=0$ to $t=T$ when terminates

For $x_0=1$,

$$W_{PLA}=\begin{bmatrix}\sum_{i=0}^{T} y_n(i)\\ \sum_{i=0}^{T} y_n(i)\,x_n^{orig}(i)\end{bmatrix}$$

For $x_0=-1$

$$W'_{PLA}=\begin{bmatrix}-\sum_{i=0}^{T} y_n(i)\\ \sum_{i=0}^{T} y_n(i)\,x_n^{orig}(i)\end{bmatrix}$$

Consider without $W_{PLA}[0]$ and $W'_{PLA}[0]$ term, We can find Same result from W_{PLA} and W'_{PLA}, which is $\sum_{i=0}^{T} y_n(i)\,x_n^{orig}(i)$

$\Rightarrow$ They are "equivalent" with same hypothesis #

7. the bound of PLA is $\left(\frac{R}{\rho}\right)^2$

$$R^2 = \max_n \|x_n\|^2, \quad \rho = \min_n y_n \frac{W_f^T}{\|w_f\|} X_n$$

updated by Norman

$\longrightarrow$

$$Z_n \leftarrow \frac{X_n}{\|X_n\|}, \quad \rho_z = \min_n \frac{y_n W_f Z_n}{\|w_f\|}$$

By $W_f^T w_{t+1} \geq W_f^T (w_t + y_n Z_n)$

$$\Rightarrow W_f^T W_{t+1} \geq w_f^T w_t + \underbrace{\min_n y_n W_f^T Z_n}_{\rho_z}$$

$$w_f^T W_1 \geq W_f^T w_0 + \rho_z$$

$$w_f^T W_2 \geq w_f^T W_1 + \rho_z$$

$$\vdots$$

$$+)\ w_f^T w_T \geq w_f^T W_{T-1} + \rho_z$$

$$W_f^T W_T \geq W_f^T \underbrace{w_0}_{(=0)} + T \cdot \rho_z = T \cdot \rho_z$$

By $\|w_{t+1}\|^2 \leq \|w_t\|^2 + \underbrace{\max_n \|z_n\|^2}_{R_z^2 = 1 \ (\because \text{normalized})}$

$$\|W_1\|^2 \leq \|W_0\|^2 + R_z^2$$

$$\|W_2\|^2 \leq \|W_1\|^2 + R_z^2$$

$$\vdots$$

$$+)\ \|W_T\|^2 \leq \|W_{T-1}\|^2 + R_z^2 \quad (\|w_0\| = 0)$$

$$\|W_T\|^2 \leq \|W_0\|^2 + T \cdot R_z^2 \Rightarrow \|W_T\| \leq \sqrt{T} \cdot R_z = \sqrt{T} R_z$$

$$\because 1 \geq \frac{W_T^{f} \cdot W_T}{\|W_T\|^{f}\,\|W_T\|} \geq \frac{T \cdot \rho}{1 \cdot \sqrt{T} \cdot R_z} \Rightarrow T \leq \frac{R_z^2}{\rho_z^2} = \frac{1}{\rho_z^2}$$

$\therefore$ The upper bound is $\frac{1}{\rho_z^2}$ #

($\because$ z_n is normalized $\therefore R_z^2 = 1$)

8. Given τ-separable $\Leftrightarrow$ exist perfect W_f such that $y_n w_f^T x_n > \tau$

Initialization $W_0 = 0$,

By rule of update: $y_n w_f^T x_n > \min(y_n w_f^T x_n) > \tau \Rightarrow \rho > \tau$

$$w_T^f w_{t+1} = w_T^f(w_t + y_n x_n) = w_T^f w_t + y_n w_T^f x_n > w_T^f w_t + \tau$$

$$\|w_{t+1}\|^2 = \|w_t + y_n x_n\|^2$$
$$= \|w_t\|^2 + 2 y_n w_t^T x_n + \|y_n x_n\|^2$$
$$\leq \|w_t\|^2 + 2\tau + \|y_n x_n\|^2$$
$$= \|w_t\|^2 + 2\tau + \|x_n\|^2 \qquad \left(\begin{array}{l}\because y_n = \pm 1 \\ \therefore \|y_n\|^2 = 1\end{array}\right)$$
$$\leq \|w_t\|^2 + 2\tau + \underbrace{\max \|x_n\|^2}_{R^2}$$

For ① $\Rightarrow$
$$w_T^f w_1 \geq w_T^f w_0 + \tau \quad (w_0 = 0)$$
$$w_T^f w_2 \geq w_T^f w_1 + \tau$$
$$\vdots$$
$$w_T^f w_T \geq w_T^f w_{T-1} + \tau$$
$$\overline{w_T^f w_T \geq T \cdot \tau}$$

For ② $\Rightarrow$
$$\|w_1\|^2 \leq \|W_0\|^2 + 2\tau + R^2 \quad (W_0 = 0)$$
$$\|w_2\|^2 \leq \|w_1\|^2 + 2\tau + R^2$$
$$\vdots$$
$$+)\ \|W_T\|^2 \leq \|W_{T-1}\|^2 + 2\tau + R^2$$
$$\overline{\|W_T\|^2 \leq 2T \cdot \tau + T \cdot R^2}$$
$$\Rightarrow \|W_T\| \leq \sqrt{T}\sqrt{2\tau + R^2}$$

$$\because 1 \geq \frac{w_T^f w_T}{\|w_T^f\|\,\|w_T\|} \geq \frac{T \cdot \tau}{1 \cdot \sqrt{T}\sqrt{2\tau + R^2}} \Rightarrow T \leq \frac{2\tau + R^2}{\tau^2}$$

$\therefore$ T is Bounded #

Hence PAM halts in finite number of steps #

13. (bonus)

By normalizing, The upper bounds become $\frac{1}{\rho_z^2}$

However, the normalization can make the weight updates in PLA more consistent (make $\|z_n\|=1$) and potentially influence the geometry of the problem which may speed up convergence, but there's NO guarantee.

$\Rightarrow$ The effect of normalization on PLA's speed can vary based on specific circumstances.

#