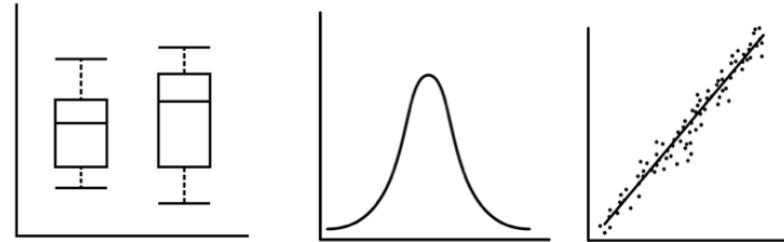


# PSYC 2300

## Introduction to Statistics



### Lecture 11: Differences Between Two Groups (Part II)

# Outline for today

- Review parts of last class
- The Dependent samples  $t$ -test
- Effect size for the Dependent samples  $t$ -test
- Dependent Samples  $t$ -tests in JASP
  - Download [Stats Class 13 Dataset \(Dependent-Samples T-Test\).jasp](#)



# Review

# Review: One-sample $t$ -test

**One-sample  $t$ -test statistic**

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{s_{\bar{x}}}$$

**Estimated standard error**

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

- Used when we **don't know the true standard deviation** in the population
- We can use the estimated standard error,  $s_{\bar{x}}$ , in place of the true standard error,  $\sigma_{\bar{x}}$

# Review: one sample t-test

As the name implies, we have **one sample** to compare to the population mean under the null hypothesis,  $H_0$ . For example:

*Does lie detection accuracy differ significantly from chance?*

- We collect  $n = 40$  students, they watch videos of liars and truth-tellers, and we compute their accuracy performance as  $\bar{x} = 0.56$  and  $s = 0.12$ 
  - The null hypothesis is  $H_0 : \mu = 0.50$  which would represent chance performance
  - The alternative hypothesis is  $H_a : \mu \neq 0.50$

$$\text{One sample } t\text{-test: } t_{\bar{x}} = \frac{0.56 - 0.50}{\frac{0.12}{\sqrt{40}}} = \frac{0.06}{.01897367} = 3.16$$

$p = .0003 < \alpha$  so we reject the null hypothesis

# Review: Independent samples $t$ -test

Used when we have two independent samples

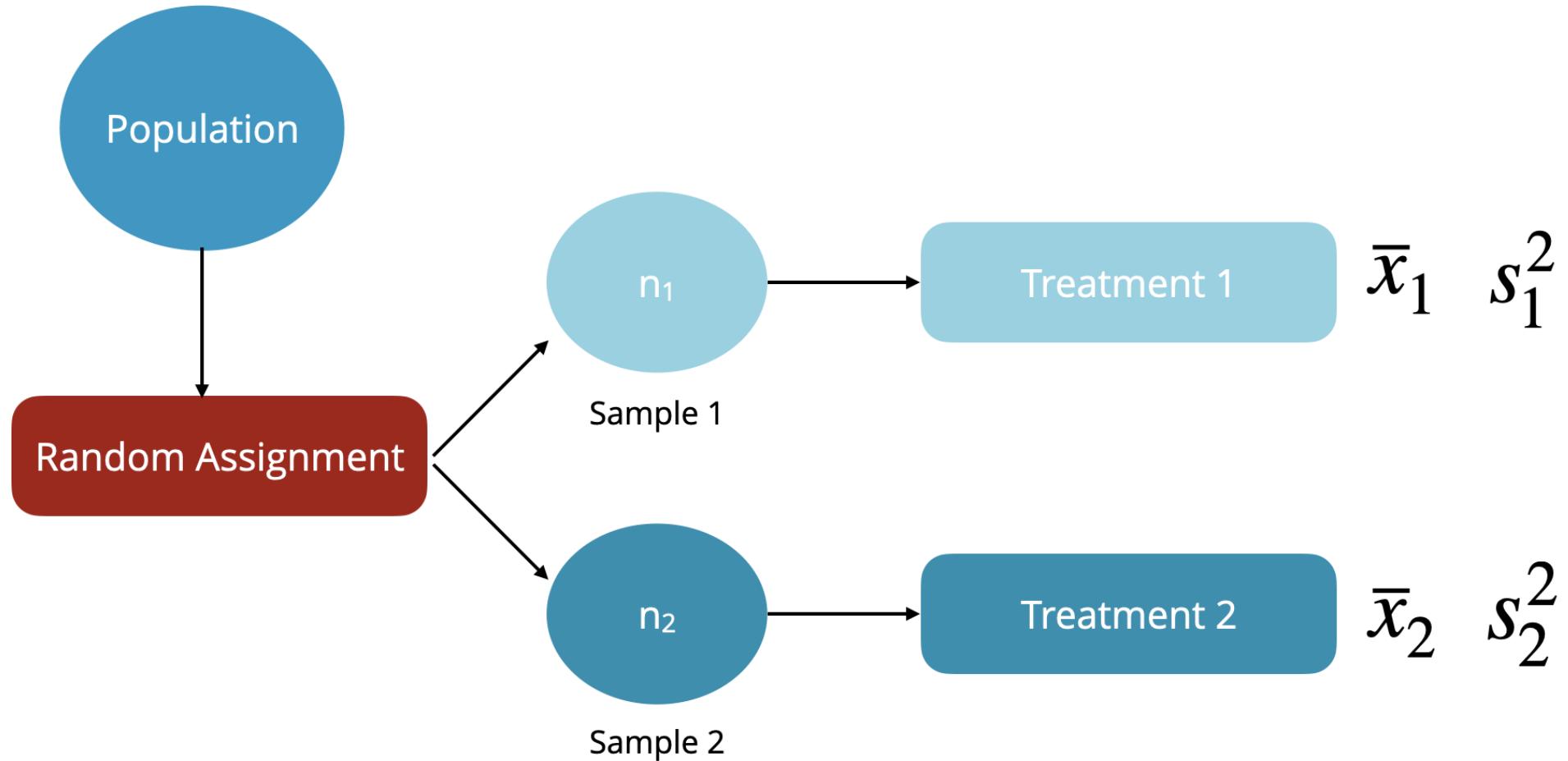
## Independent samples $t$ -test

$$t_{(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{s_{(\bar{x}_1 - \bar{x}_2)}}$$

## Estimated standard error of the mean difference

$$s_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\left[ \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \right] \left[ \frac{n_1 + n_2}{n_1 n_2} \right]}$$

# Independent Samples



# Review: Independent samples $t$ -test

As the name implies, we have **independent samples**, specifically two samples, to compare to their respective populations.

*Does getting bumped (vs. not getting bumped) in the hallway decrease compassion?*

**No hallway bump**

$$n_1 = 10$$

$$\bar{x}_1 = 5.5$$

$$s_1^2 = 2.5$$

**Hallway bump**

$$n_2 = 10$$

$$\bar{x}_2 = 8.5$$

$$s_2^2 = 3$$

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

$$t_{(\bar{x}_1 - \bar{x}_2)} = \frac{5.5 - 8.5}{0.74} = -4.054$$

$$p = 0.0008 < \alpha$$

Reject the null hypothesis

# *t*-test statistic

*General form of t-test statistics*

$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

Three types we'll learn:

- One sample *t*-test
- Independent samples *t*-test
- **Dependent samples *t*-test**

# Dependent samples *t*-test

# Independent vs. Dependent

- **Independent-samples** (called *between-subjects*) compare two *different* or unrelated groups of people
- **Dependent-samples** (called *within-subjects*) compare the *same* group of people to themselves

Put another way:

- A *t*-test for independent means indicates that two groups of different participants are being studied under two separate conditions
- A *t*-test for dependent means indicates that a single group of the same participants is being studied under two conditions

# Goal: Dependent samples *t*-test

- To investigate whether some treatment made an effect compared to a baseline within the same individual(s)
- The dependent-samples *t*-test asks whether the mean difference between each subject's pair of observed values is significantly different from 0

Put another way:

- Did *pre-treatment* scores differ from *post-treatment* scores?

# Example Experiment

## Research question

Does watching a TED Talk by Carol Dweck affect students' perceived self-efficacy?

*Self-efficacy* is an individual's belief or confidence in their ability to achieve goals



# Example Experiment

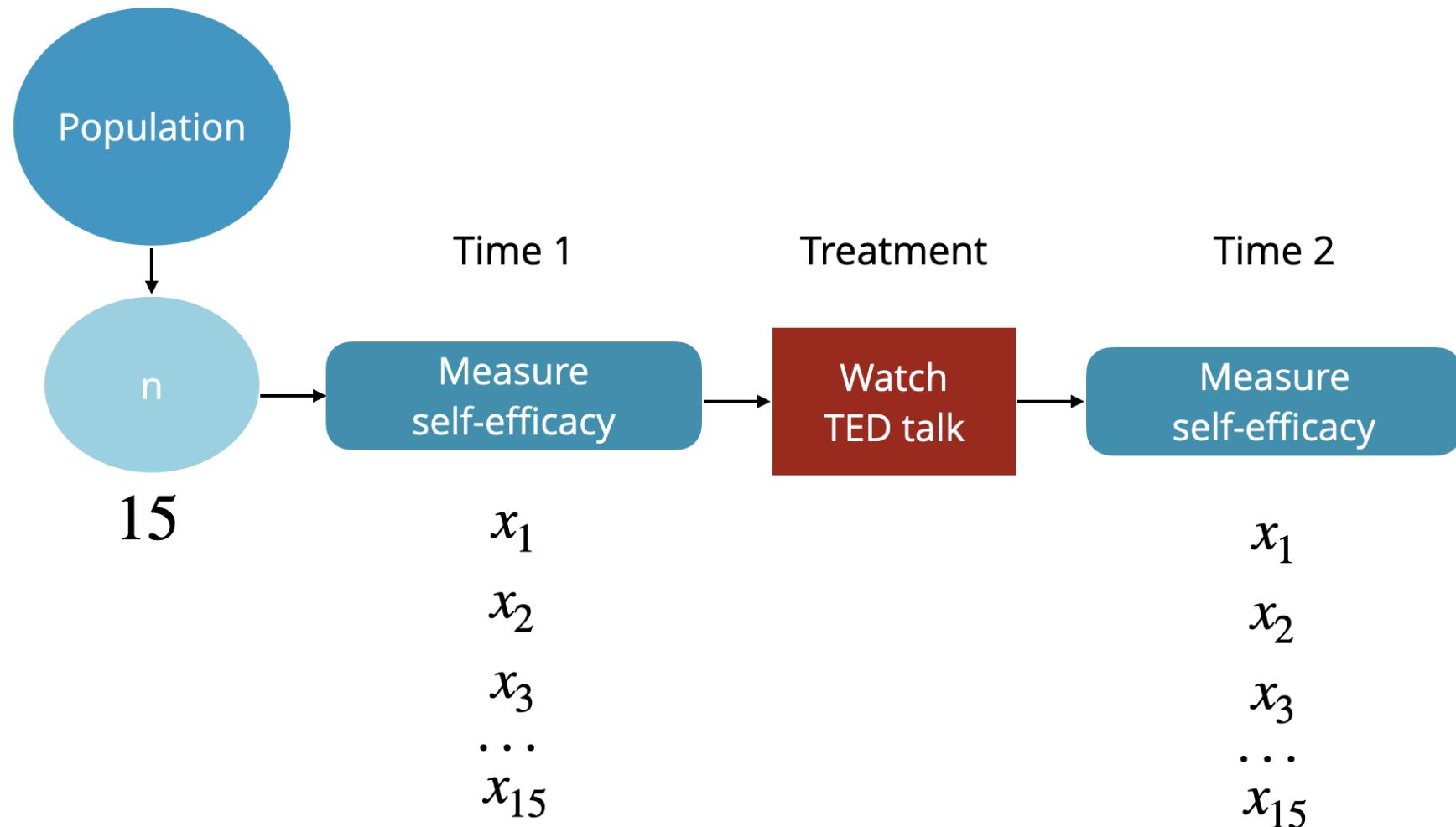
## Self-Efficacy Scale

*"Please rate the following on a scale of 1 (strongly disagree) to 14 (strongly agree)"*

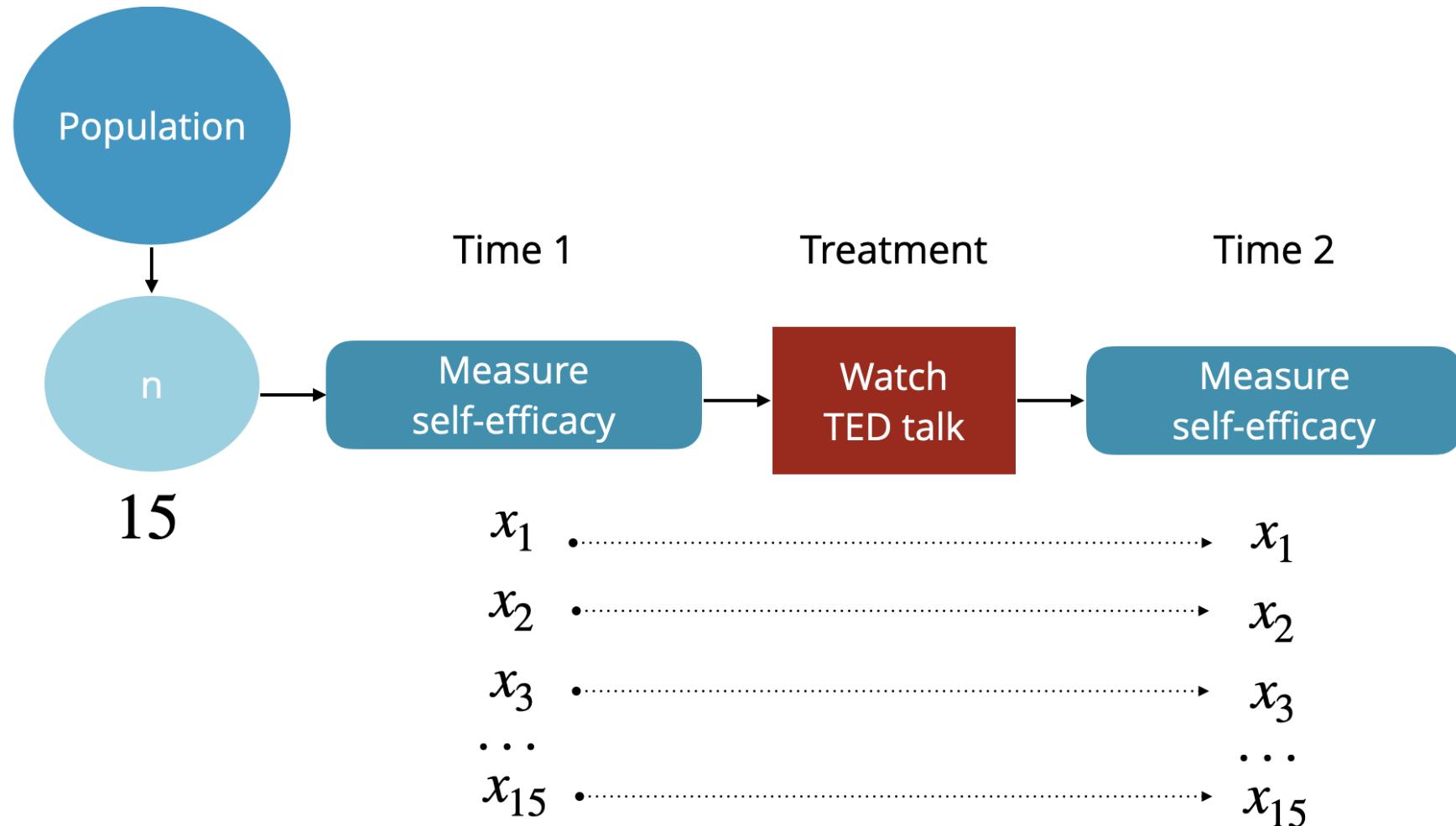
1. If I can't do a job the first time, I keep trying until I can.
2. When I have something unpleasant to do, I stick to it until I finish it.
3. Failure just makes me try harder.
4. I am a self-reliant person.

Sherer et al. (1982)

# Within-subjects designs



# Within-subjects designs



# Within-subjects designs

Time 1

Time 2

Baseline  
self-efficacy

Post-treatment  
self-efficacy

Difference  
score

$x_1$

—

$x_1$

=

$x_{d_1}$

$x_2 \quad x_2$

$x_3 \quad x_3$

...

$x_{15} \quad x_{15}$

# Within-subjects designs

Time 1

## Time 2

## Baseline self-efficacy

# Post-treatment self-efficacy

## Difference score

$$x_1 - x_1 = x_{d_1}$$

$$x_2 - x_2 = x_{d_2}$$

$$\begin{matrix} x_3 & x_3 \\ \cdots & \cdots \\ x_{15} & x_{15} \end{matrix}$$

# Within-subjects designs

Time 1	Time 2	
Baseline self-efficacy	Post-treatment self-efficacy	Difference score
$x_1$	$-$	$x_1$
$=$		$x_{d_1}$
$x_2$	$-$	$x_2$
	$=$	$x_{d_2}$
$x_3$	$-$	$x_3$
	$=$	$x_{d_3}$
$\dots$	$-$	$\dots$
	$=$	$\dots$
$x_{15}$	$-$	$x_{15}$
	$=$	$x_{d_{15}}$

# Statistical Hypotheses

**Null hypothesis  $H_0$**

$$\mu_d = 0$$

There is no difference

**Alternative hypothesis  $H_a$**

$$\mu_d \neq 0$$

There is a difference

# Building the *t*-test statistic

*General form of t-test statistics*

$$t = \frac{\text{sample statistic} - \text{population parameter}}{\text{estimated standard error of statistic}}$$

# Building the *t*-test statistic

*General form of dependent samples t-test*

$$t = \frac{\text{sample difference score mean} - \text{population difference score mean}}{\text{estimated standard error of statistic}}$$

$\bar{x}_1 - \bar{x}_2$  = sample difference score mean

$\mu_1 - \mu_2$  = population difference score mean

Note:

$\bar{x}_1$  is the same as  $\bar{x}_{\text{pre-treatment}}$

$\bar{x}_2$  is the same as  $\bar{x}_{\text{post-treatment}}$

# Building our t-test statistic

*General form of dependent samples t-test*

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\text{estimated standard error of statistic}}$$

$$\bar{x}_1 - \bar{x}_2 = \bar{x}_d$$

$$\mu_1 - \mu_2 = \mu_d$$

# Building our t-test statistic

*General form of dependent samples t-test*

$$t = \frac{\bar{x}_d - \mu_d}{\text{estimated standard error of statistic}}$$

Under the null hypothesis,  $H_0, \mu_d = 0$

# Building our t-test statistic

*General form of dependent samples t-test*

$$t = \frac{\bar{x}_d}{\text{estimated standard error of statistic}}$$

# Dependent samples *t*-test

## Dependent samples *t*-test

$$t_{\bar{x}_d} = \frac{\bar{x}_d}{s_{\bar{x}_d}}$$

$\bar{x}_d$  is the mean of the difference scores

$s_{\bar{x}_d}$  is the standard error of the mean difference

# Dependent samples *t*-test

**Estimated standard error of the difference scores**

$$s_{\bar{x}_d} = \sqrt{\frac{s_d^2}{n}}$$

$s_d^2$  is the variance of the difference scores

$n$  is the size of the sample (the number of difference scores)

# Dependent samples *t*-test

Dependent samples *t*-test

$$t_{\bar{x}_d} = \frac{\bar{x}_d}{s_{\bar{x}_d}}$$

Standard error of the difference scores

$$s_{\bar{x}_d} = \sqrt{\frac{s_d^2}{n}}$$

# Practice Activity

Research question: Does cognitive behavioral therapy reduce symptoms of depression?

participant	pre_treatment	post_treatment
1	5	2
2	3	3
3	6	2
4	9	4

## Dependent samples *t*-test

$$t_{\bar{x}_d} = \frac{\bar{x}_d}{s_{\bar{x}_d}}$$

## Standard error of the difference scores

$$s_{\bar{x}_d} = \sqrt{\frac{s_d^2}{n}}$$

# Practice Activity

participant	pre_treatment	post_treatment	difference_score
1	5	2	3
2	3	3	0
3	6	2	4
4	9	4	5

$\bar{x}_d = 3$  mean of the difference scores

$s_d^2 = 4.67$  variance of the difference scores

# Practice Activity

$$t_{\bar{x}_d} = \frac{\bar{x}_d}{s_{\bar{x}_d}}$$

$$t_{\bar{x}_d} = \frac{3}{\sqrt{\frac{s_d^2}{n}}}$$

$$t_{\bar{x}_d} = \frac{3}{\sqrt{\frac{4.67}{4}}} = \frac{3}{1.080509} = 2.78$$

# Effect size for dependent samples $t$ -test

# Effect size for dependent samples *t*-test

Effect size for dependent samples *t*-test

$$d_{\bar{x}_d} = \frac{\bar{x}_d}{s_d}$$

$\bar{x}_d$  is the mean of the difference scores

$s_d$  is the standard deviation of the difference scores (**NOT** standard error)

# Dependent samples $t$ -test in JASP

# Example Experiment

**Research question:** Does watching a TED Talk by Carol Dweck affect students' perceived self-efficacy?

**Null hypothesis  $H_0$**

$$\mu_d = \mu_1 - \mu_2 = 0$$

There is no difference

Students' perceived self-efficacy *before* watching the TED talk **will not differ** from their self-efficacy *after* watching the TED talk.

**Alternative hypothesis  $H_a$**

$$\mu_d = \mu_1 - \mu_2 \neq 0$$

There is a difference

Students' perceived self-efficacy *before* watching the TED talk **will differ** from their self-efficacy *after* watching the TED talk.

# JASP: dependent samples $t$ -test

The dependent-samples  $t$ -test is also called the “paired samples”  $t$ -test, so let’s select that option under the “T-Tests” menu

The screenshot shows the JASP software interface. At the top, there is a menu bar with various statistical analysis options: Descriptives, T-Tests, ANOVA, Mixed Models, Regression, Frequencies, Factor, and Reliability. Below the menu bar is a toolbar with icons for each of these categories. A blue arrow points from the text in the callout box to the "T-Tests" button in the toolbar. The main window displays a dataset titled "Stats Class 13 Dataset (Dependent Samples T-Test) (/Users/Sabby/Desktop)". The dataset has three columns: Subject ID, Post-Treatment Self-Efficacy, and Efficacy Score. The "T-Tests" menu is open, showing three sub-options: Classical, Bayesian, and Nonparametric. The "Classical" option is selected, and its sub-options are visible: Independent Samples T-Test, Paired Samples T-Test, and One Sample T-Test. The "Paired Samples T-Test" option is highlighted with a gray background.

Subject ID	Post-Treatment Self-Efficacy	Efficacy Score
1	11	11
2	7	7
3	10	10
4	6	6
5	7	7
6	9	9
7	11	11
8	11	10
9	4	6
10	7	4
11	7	13
12	6	7
13	5	5
14	11	13
15	10	10

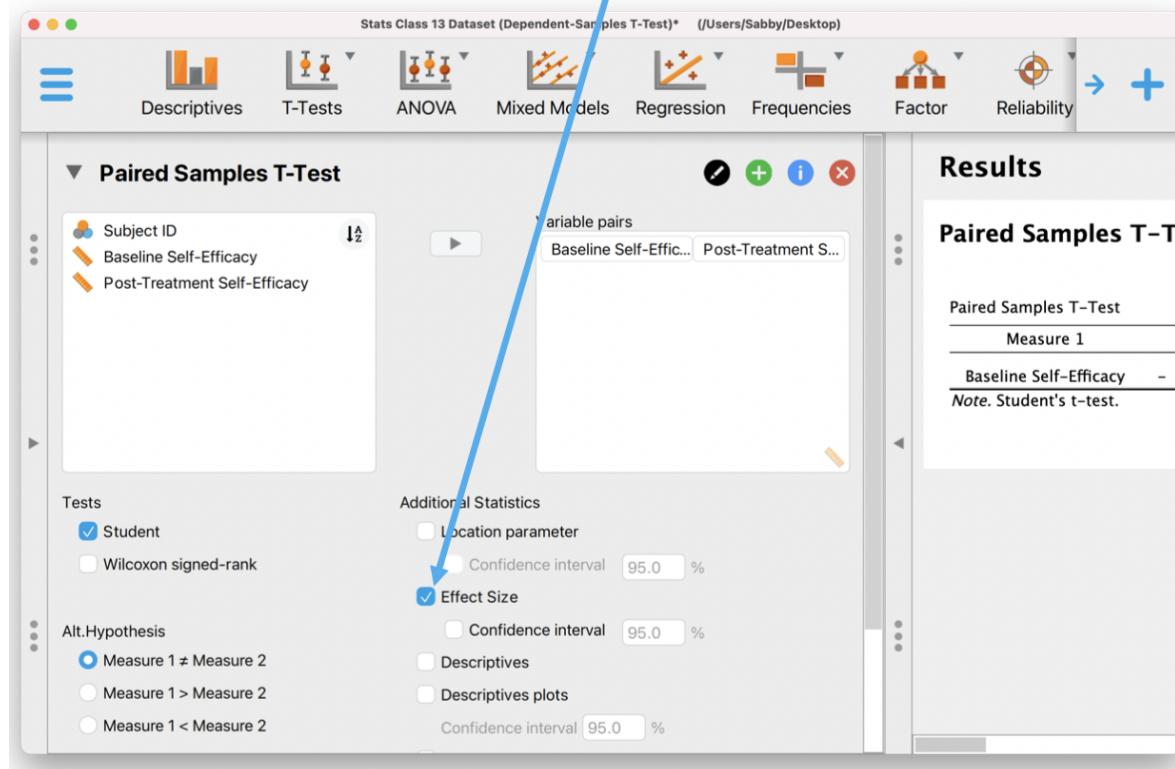
# JASP: dependent samples $t$ -test

Select the “before” and “after” variables into the “Variable pairs” box; JASP will automatically assume these are pairs of observed values

The screenshot shows the JASP software interface. At the top, there is a toolbar with various statistical analysis icons: Descriptives, T-Tests, ANOVA, Mixed Models, Regression, Frequencies, Factor, and Reliability. Below the toolbar, the main window has a sidebar on the left containing a tree view of analysis sections. The "Paired Samples T-Test" section is expanded, showing three variables: Subject ID, Baseline Self-Efficacy, and Post-Treatment Self-Efficacy. An arrow points from the text in the blue box above to the "Variable pairs" button next to the "Post-Treatment S..." variable. The main panel below the sidebar contains sections for "Tests" (with "Student" checked), "Additional Statistics" (with "Location parameter" checked), and "Alt.Hypothesis" (with "Measure 1 ≠ Measure 2" selected). To the right of the main panel is a "Results" pane which displays the "Paired Samples T-Test" results for "Measure 1" comparing "Baseline Self-Efficacy" and "Post-Treatment Self-Efficacy".

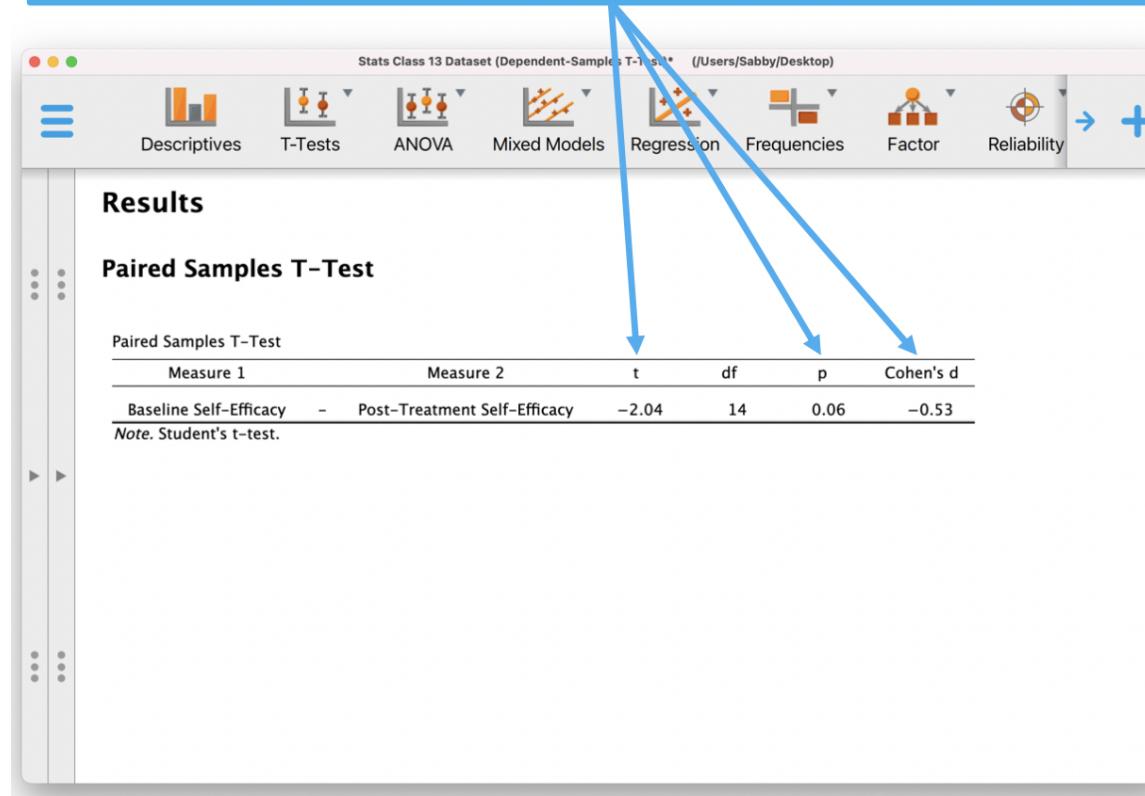
# JASP: dependent samples $t$ -test

Click on “Effect Size” to have JASP calculate your Cohen’s  $d$  effect size as well.



# JASP: dependent samples *t*-test

JASP gives you the dependent-samples *t*-test statistic, the *p*-value associated with this test, and the Cohen's *d* effect size



# JASP: dependent samples $t$ -test

A dependent-samples  $t$ -test is the same as a one-sample  $t$ -test of difference scores against 0. Let's see another way to do this!

The screenshot shows the JASP interface with a dataset titled "Stats Class 13 Dataset (Dependent-Samples T-Test)". The dataset contains three columns: "Subject ID", "Baseline Self-Efficacy", and "Post-Treatment Self-Efficacy". A blue arrow points from the text above to a modal dialog titled "Create Computed Column". In the dialog, the name "Difference Scores" is entered in the "Name:" field. Below the name are two icons: a black "R" and a hand cursor icon. Underneath the icons are four category buttons: "Scale" (selected), "Ordinal", "Nominal", and "Text". At the bottom of the dialog are three buttons: an info icon, "Create Column", and a close "X" button.

Subject ID	Baseline Self-Efficacy	Post-Treatment Self-Efficacy
1	11	13
2	7	7
3	10	
4	6	
5	7	
6	9	
7	11	
8	11	
9	4	
10	7	4
11	7	13
12	6	7
13	5	5
14	11	13
15	10	10

# JASP: dependent samples $t$ -test

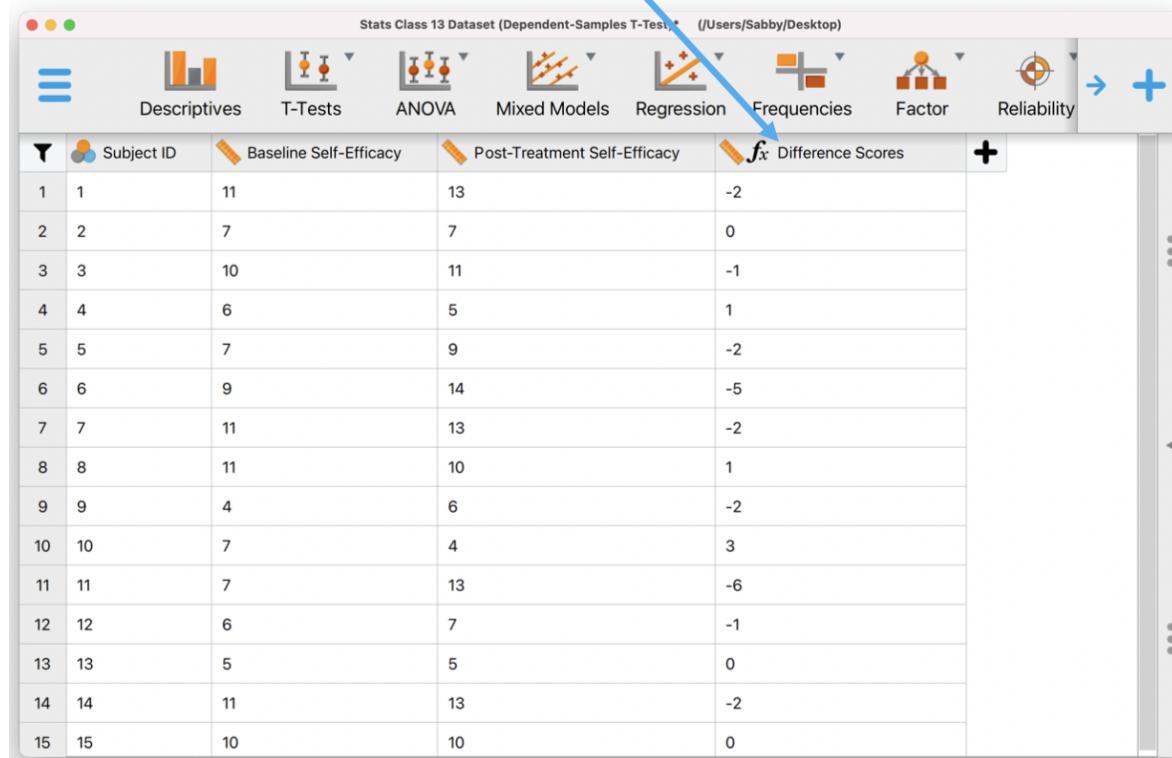
Subtract the post- and pre-scores from one another to create a “Difference Scores” variable. Then click “Compute column.”

The screenshot shows the JASP software interface with a blue callout box highlighting the 'Computed Column' dialog. The dialog title is 'Computed Column: Difference Scores'. It contains a formula bar with operators like +, -, \*, /, ^, %, =, ==, <, ≤, >, ≥, ∧, ∨, |, and -. Below the formula bar is a list of variables: Subject ID, Baseline Self-Efficacy, Post-Treat...f-Efficacy, and Difference Scores. The 'Difference Scores' variable is selected. A blue arrow points from the text above to this selection. Another blue arrow points from the formula bar down to the 'Compute column' button at the bottom of the dialog. The main JASP window shows a table with columns for Subject ID, Baseline Self-Efficacy, Post-Treatment Self-Efficacy, and Difference Scores. The first few rows of data are:

	Subject ID	Baseline Self-Efficacy	Post-Treatment Self-Efficacy	Difference Scores
1	1	11	13	
2	2	7	7	
3	3	10	11	
4	4	6	5	
5	5	7	9	
6	6	9	14	
7	7	11	13	

# JASP: dependent samples $t$ -test

Now we have a column of Difference Scores, which we can test against 0 to see if the average difference was different from 0.



	Subject ID	Baseline Self-Efficacy	Post-Treatment Self-Efficacy	Difference Scores
1	1	11	13	-2
2	2	7	7	0
3	3	10	11	-1
4	4	6	5	1
5	5	7	9	-2
6	6	9	14	-5
7	7	11	13	-2
8	8	11	10	1
9	9	4	6	-2
10	10	7	4	3
11	11	7	13	-6
12	12	6	7	-1
13	13	5	5	0
14	14	11	13	-2
15	15	10	10	0

# JASP: dependent samples $t$ -test

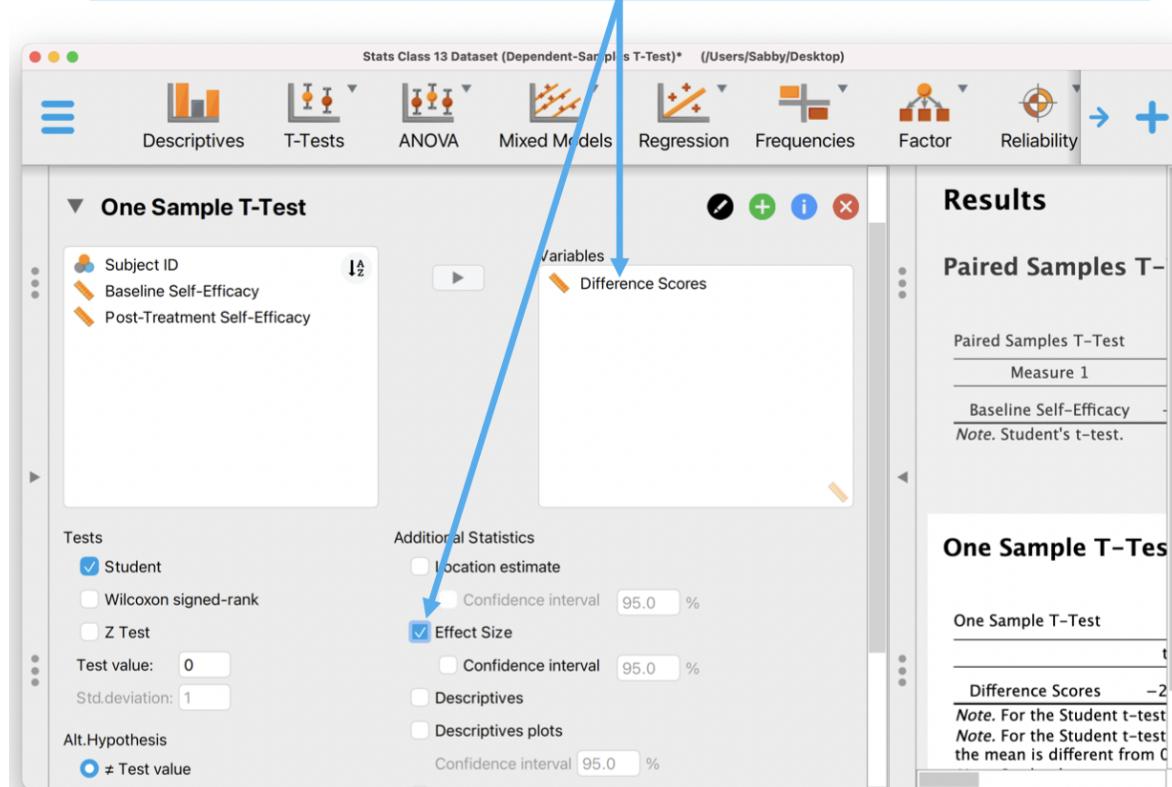
Select the one-sample t-test under the “T-Tests” menu.

The screenshot shows the JASP software interface with a blue box highlighting the "T-Tests" menu item in the top navigation bar. A blue arrow points from the text "Select the one-sample t-test under the ‘T-Tests’ menu." to the "T-Tests" button. Below the menu bar, there is a table with columns labeled "Subject ID", "Post-Treatment Self-Efficacy", and "Difference Scores". The "T-Tests" menu is open, displaying three options: "Classical", "Independent Samples T-Test", "Paired Samples T-Test", and "One Sample T-Test". Below the Classical option, another menu is open, showing "Bayesian", "Independent Samples T-Test", "Paired Samples T-Test", and "One Sample T-Test".

Subject ID	Post-Treatment Self-Efficacy	Difference Scores
1	11	-2
2	7	0
3	10	-1
4	6	1
5	7	-2
6	9	-5
7	11	-2
8	11	1
9	4	-2
10	7	3
11	7	-6
12	6	-1
13	5	0
14	11	-2
15	10	0

# JASP: dependent samples $t$ -test

Place the Difference Scores in the “Variables” box, and click on “Effect Size” to show the Cohen’s d effect size.



# JASP: dependent samples *t*-test

Notice that the results (e.g., the *t*-test statistic, the *p*-value, and the Cohen's *d* effect size) are identical for the two analyses!

The screenshot shows the JASP software interface with a blue header bar. Below the header is a toolbar with icons for Descriptives, T-Tests, ANOVA, Mixed Models, Regression, Frequencies, Factor, and Reliability. A blue box highlights the T-Tests icon. The main window displays two statistical analyses:

**Paired Samples T-Test**

Measure 1	Measure 2	<i>t</i>	df	<i>p</i>	Cohen's <i>d</i>
Baseline Self-Efficacy	- Post-Treatment Self-Efficacy	-2.04	14	0.06	-0.53

*Note.* Student's *t*-test.

**One Sample T-Test**

	<i>t</i>	df	<i>p</i>	Cohen's <i>d</i>
Difference Scores	-2.04	14	0.06	-0.53

*Note.* For the Student *t*-test, effect size is given by Cohen's *d*.  
*Note.* For the Student *t*-test, the alternative hypothesis specifies that the mean is different from 0.  
*Note.* Student's *t*-test.

# Next time

## Lecture

- Difference between many groups

## Reading

- Chapter 12

## Quiz 4

- Due Wednesday 2/16/2022 11:59pm  
MT
  - Covers Ch.11-12

