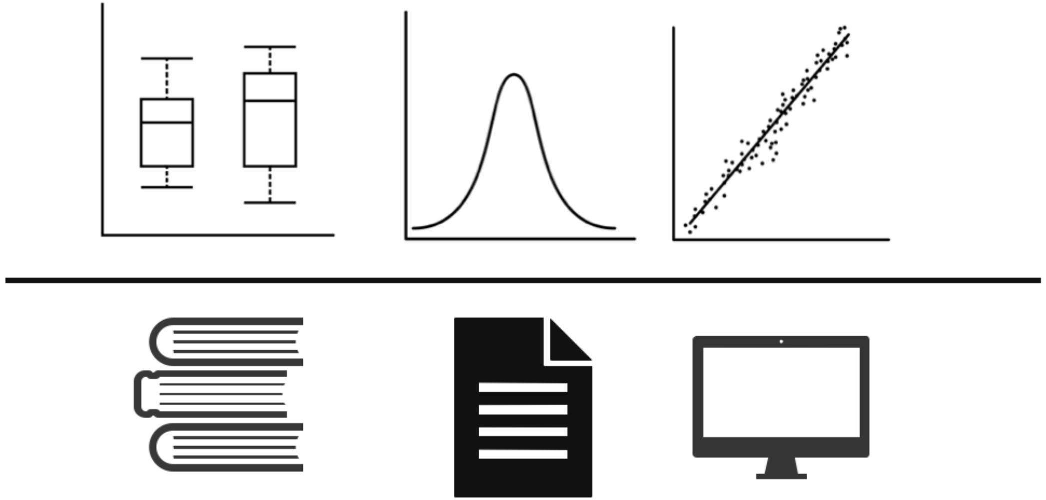
PSYC 2300

Introduction to Statistics



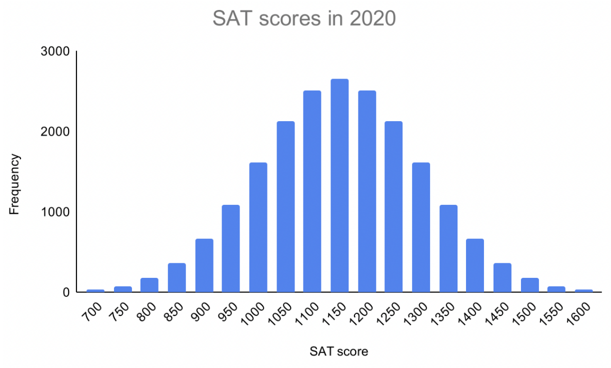
Lecture 06: Probability and the Normal Curve

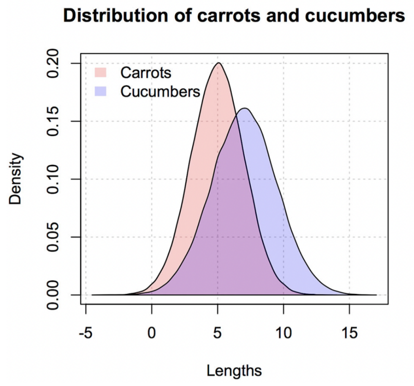
Outline for today

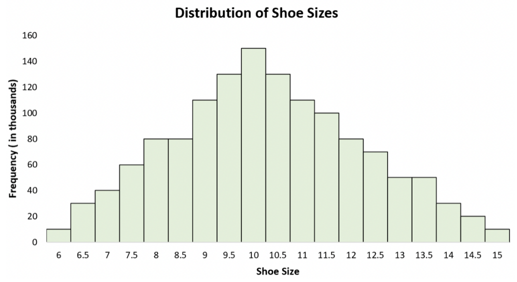
- Understanding the normal curve
- Standardization via *z*-scores
- *z*-score practice problem
    - Have ✎ and 🗎 ready
- Computing *z*-scores in JASP
    - Download `Stats Class 7 Dataset (Z-Scores).jasp`

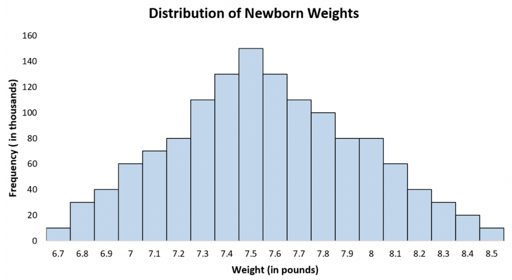
Normal Distribution

Normal Distribution



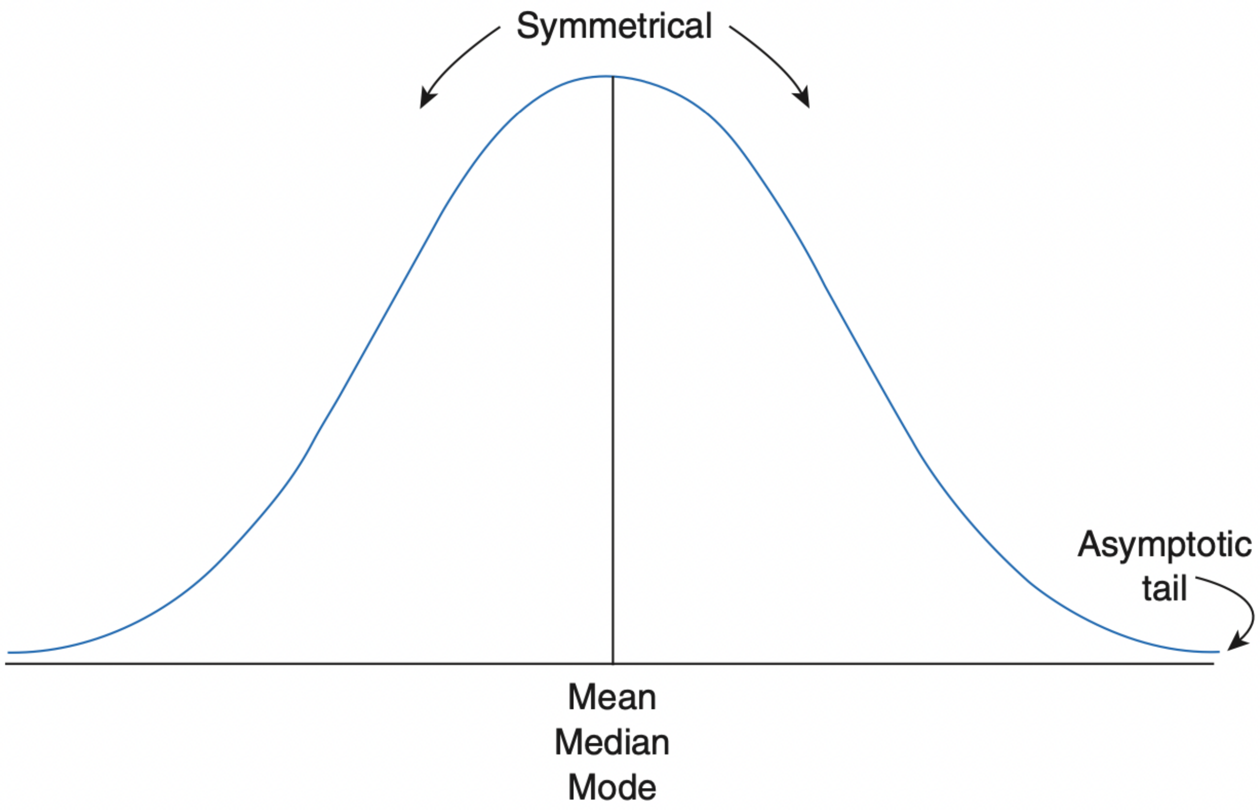






The normal distribution comes up everywhere

Normal Distribution

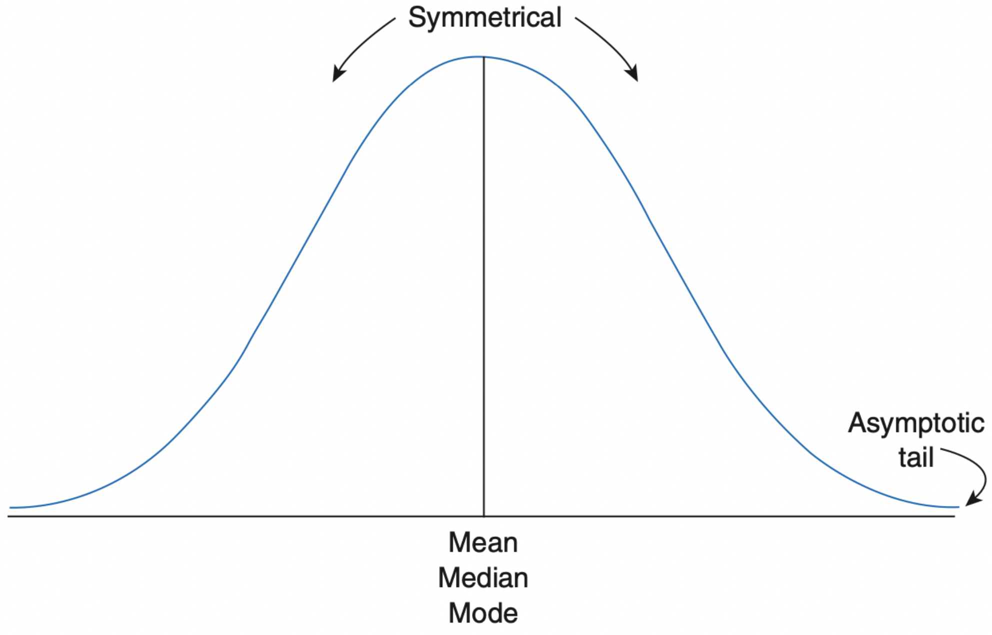


Also called the *Gaussian* distribution or *bell-curve* distribution

Normal Distribution

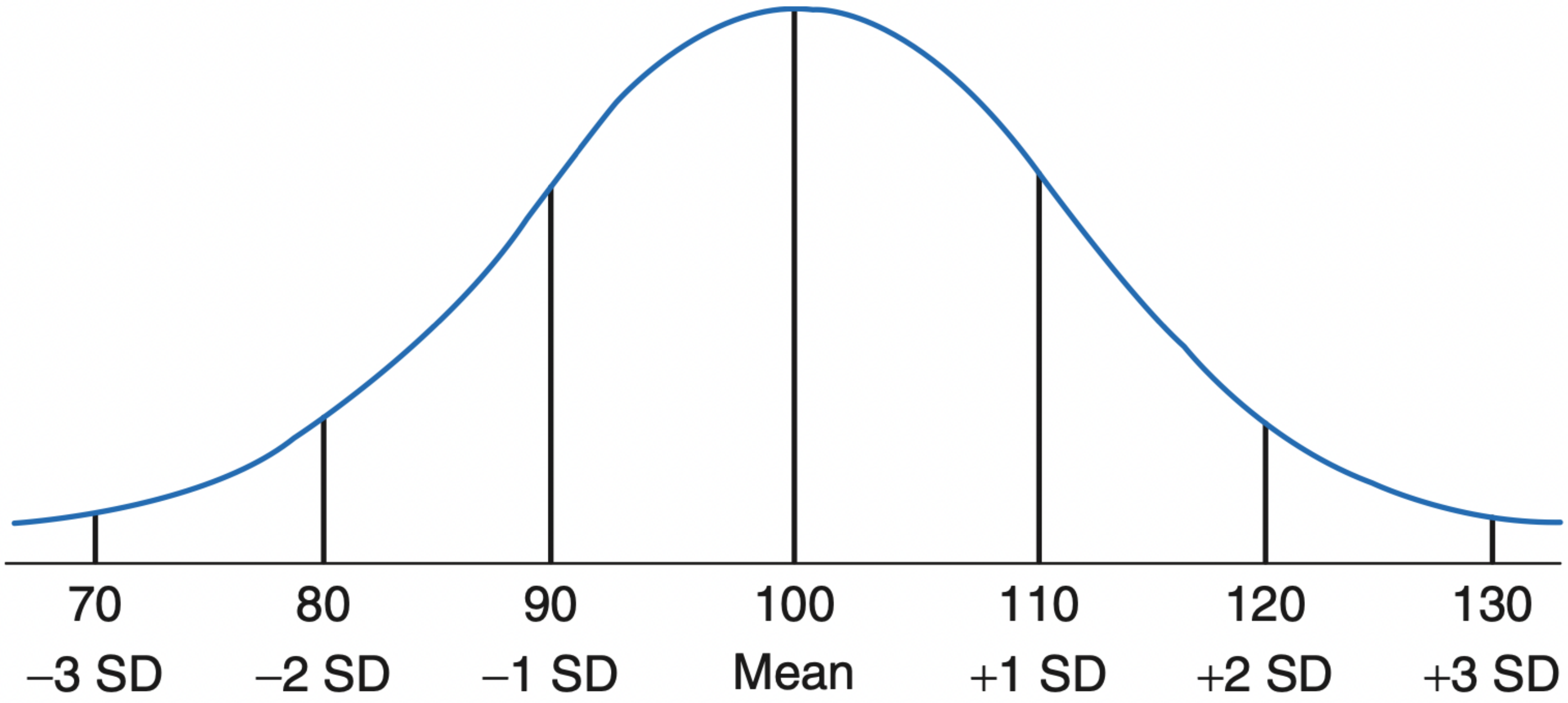
Three Central Characteristics

- Mean = Median = Mode
- Symmetrical around the mean
- Asymptotic tails: they approach but never touch the *x*-axis



Normal Distribution

We know a great deal about the normal distribution, particularly the probabilities associated with possible outcomes in a normal distribution



Probability

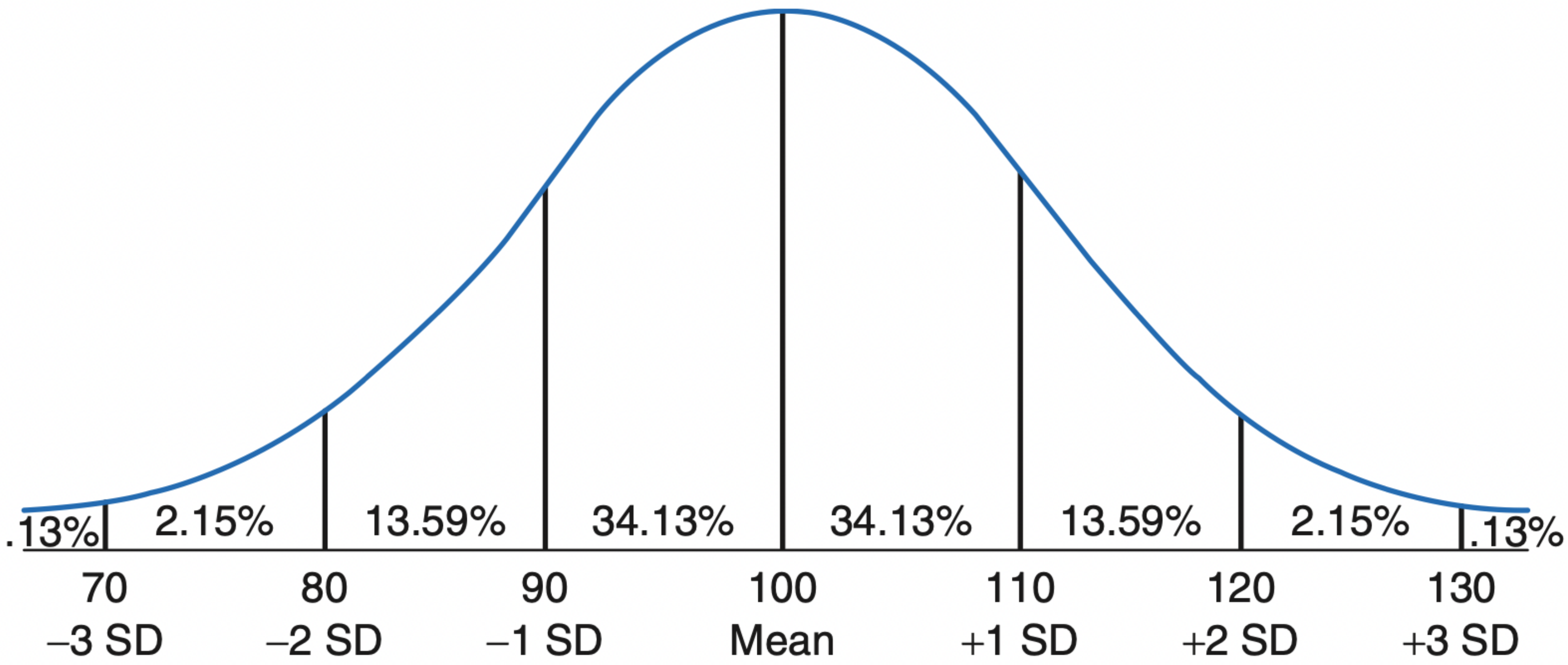
Probability is a huge topic that extends far beyond the limits of introductory statistics, and we do not attempt to examine it all here

Probability: For a situation in which several different outcomes are possible, the probability for any specific outcome is defined as a *fraction* or a *proportion* of all the possible outcomes. If the possible outcomes are identified as A, B, C, D, and so on, then

$$probability\,of\ A = \frac{number\,of\,outcomes\,classified\,as\ A}{total\,number\,of\,possible\,outcomes}$$

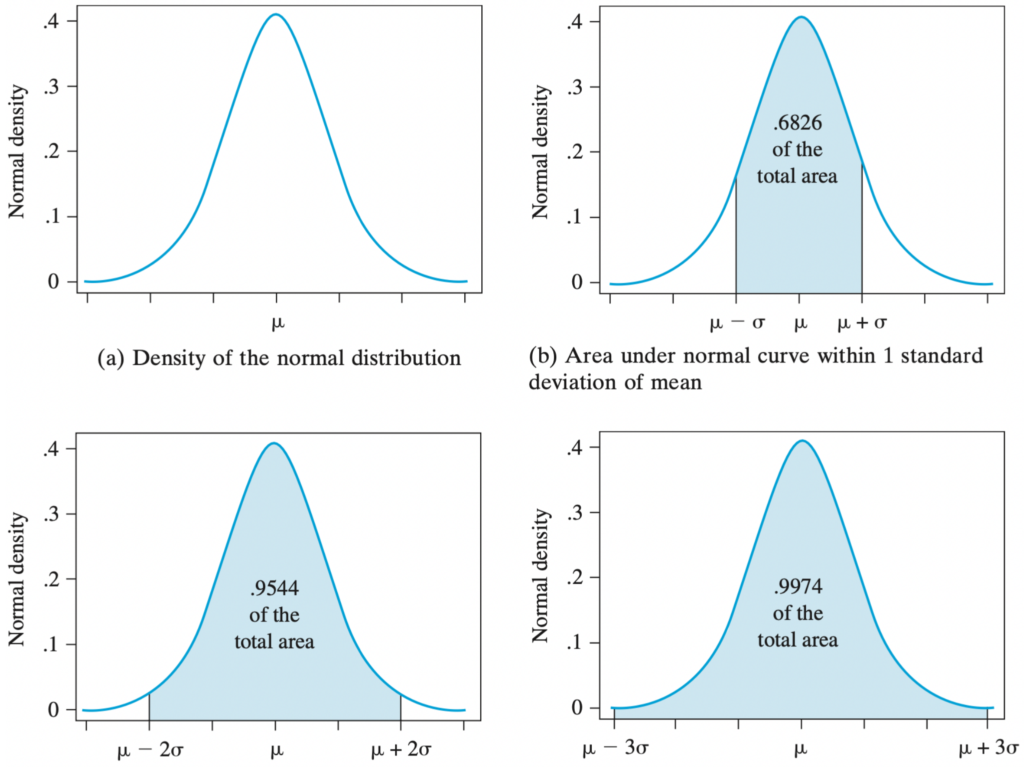
Normal Distribution

Approximate probabilities under the curve



Normal Distribution

- 68.26% is within $\pm$ **one** standard deviation of the mean
- 95.44% is within $\pm$ **two** standard deviations of the mean
- 99.74% is within $\pm$ **three** standard deviations of the mean



(a) Density of the normal distribution

(b) Area under normal curve within 1 standard deviation of mean

(c) Area under normal curve within 2 standard deviations of mean

(d) Area under normal curve within 3 standard deviations of mean

Also known as the "*68-95-99.7* rule"

Normal Distribution

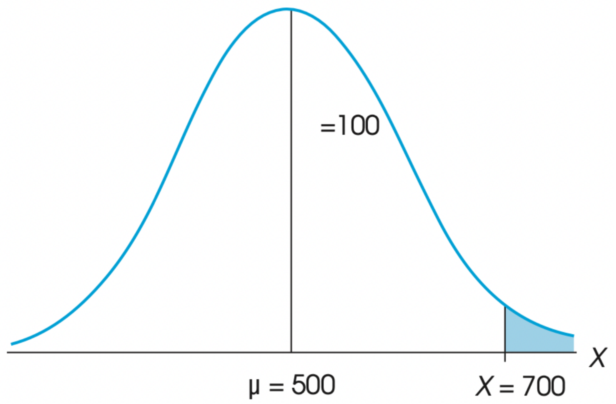
SAT scores is normal with a mean of $\mu = 500$ and a standard deviation of $\sigma = 100$

What is the probability of randomly selecting an individual from this population who has an SAT score greater than $x_i = 700$? In other words, $p(x_i > 700) =$?

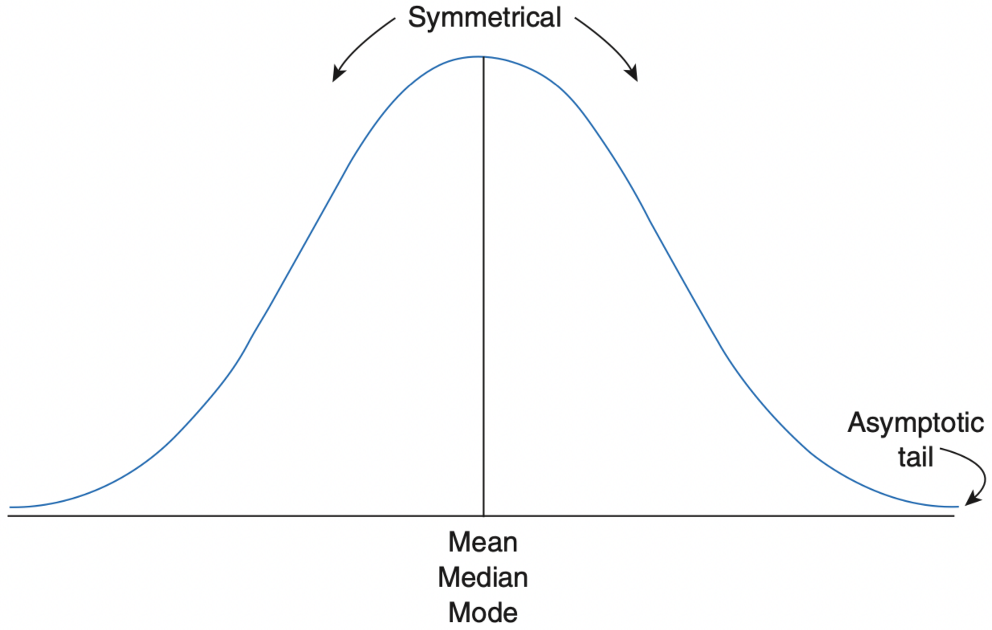
95.44% is within $\pm$ **two** standard deviations of the mean

$$100 - 95.44 = 4.56$$

$$\frac{4.56}{2} = 2.28$$



Normal Distribution



Also referred to as the *probability density function* of the normal distribution

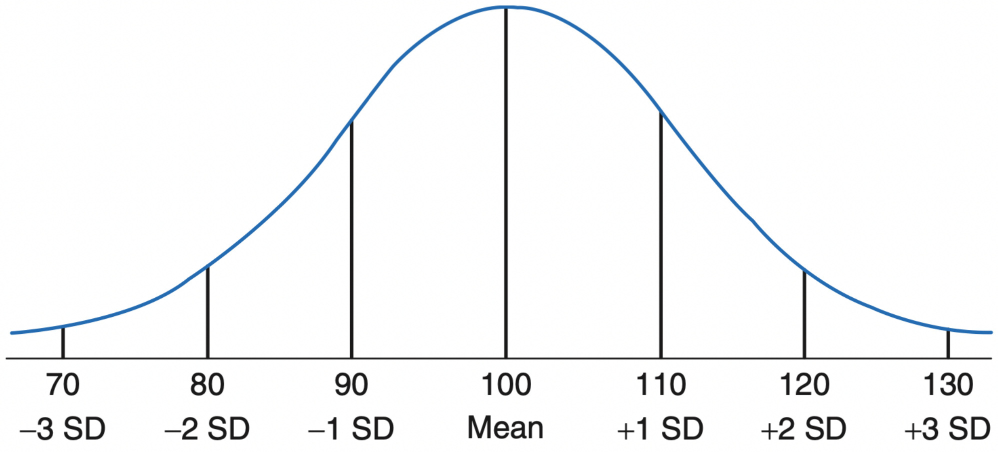
Normal Gaussian

$$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The exact shape of the normal distribution is specified by the above equation relating to each *x* value (score) with each *y* value (frequency)

Normal Distribution

Example *normal probability density function*



$$\mu = 100$$

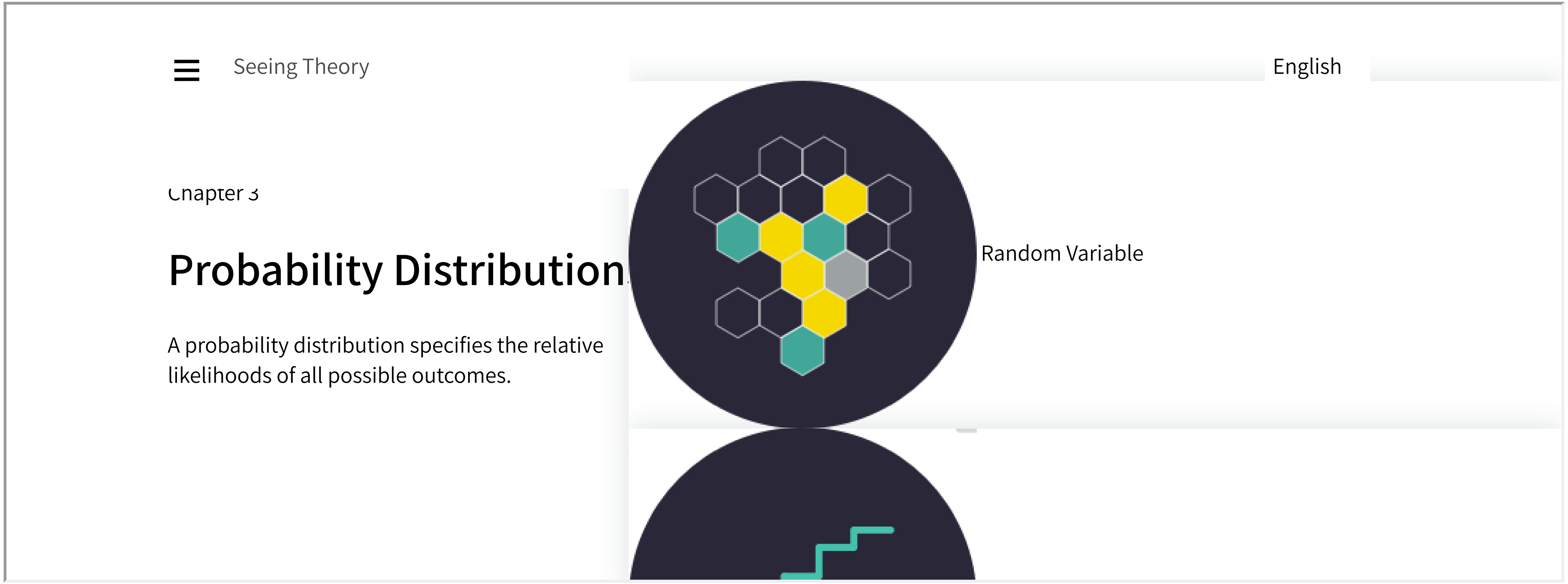
$$\sigma = 10$$

Normal Gaussian

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\frac{1}{\sqrt{2\pi 10^2}} e^{-\frac{(x-100)^2}{2*10^2}}$$

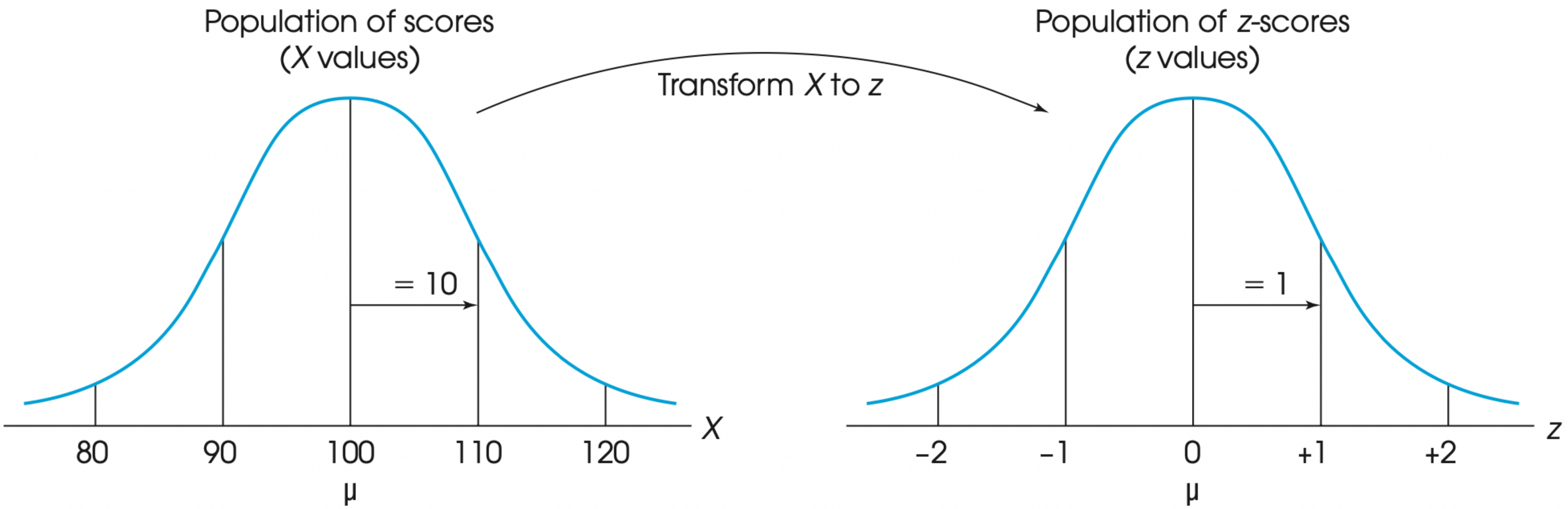
Normal Distribution



Link to visualizing the probability distribution function of the normal distribution

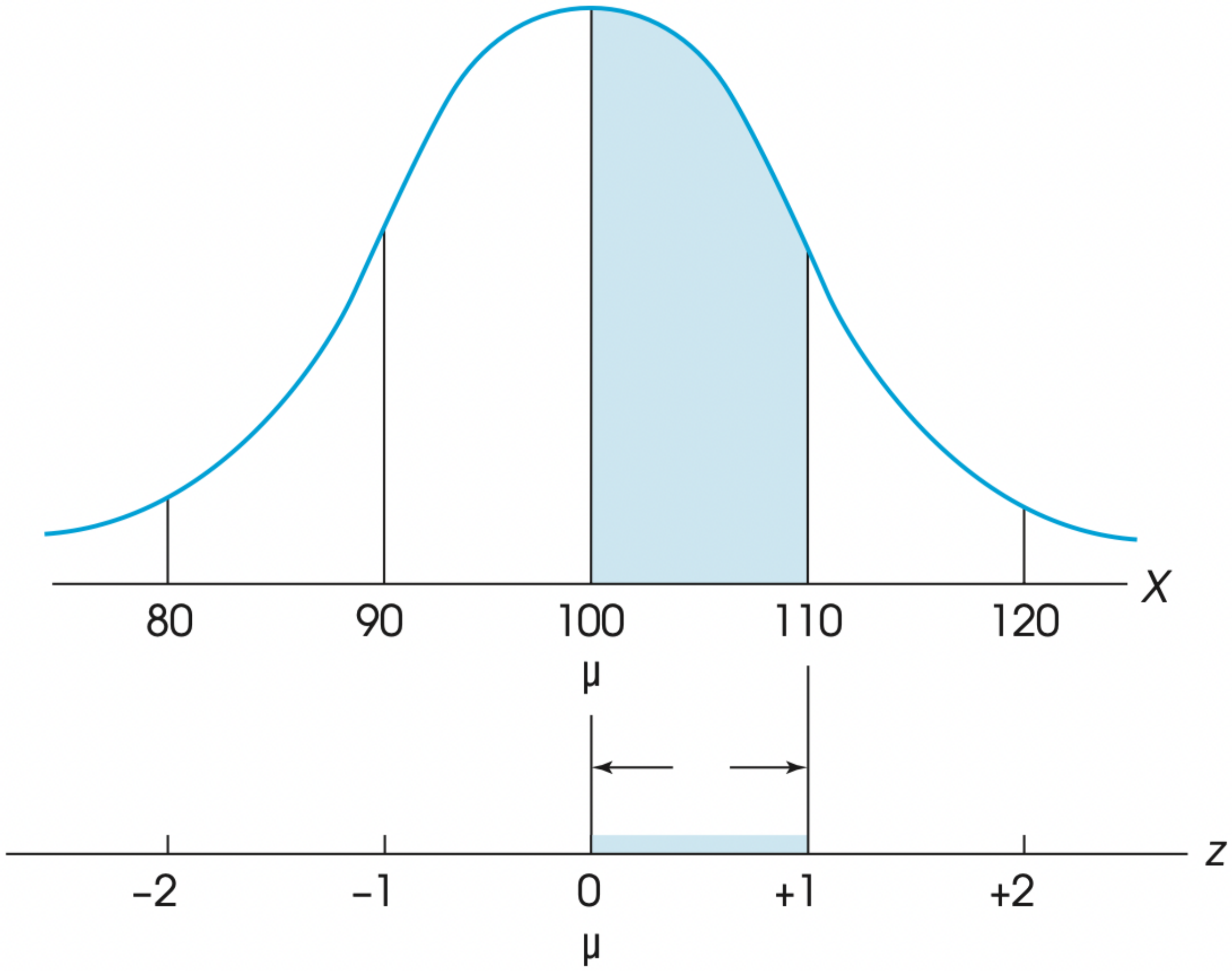
z-scores

Standardizing Scores



Standardized scores are comparable because they are standardized in units of standard deviations

Standardizing Scores



Developing z-scores

Two hypothetical students in my class:

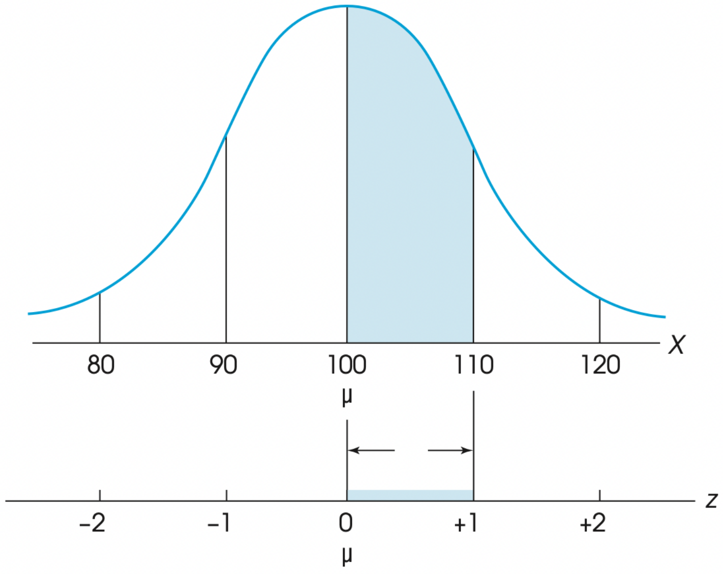
Name	**Quiz 1 *raw* scores**	**Quiz 2 *raw* scores**
Jessie	76	88
Sam	81	85

Who's doing better in the course?

We don't know from the raw scores alone

Developing z-scores

***z*-score**: describes the position of a raw score in terms of its distance from the mean, when measured in *standard deviation units*. The *z*-score is positive if the value lies above the mean, and negative if it lies below the mean



Developing z-scores

Step 1: Calculate the mean for Quiz 1, μ_1, and Quiz 2, μ_2

Name	Quiz 1 *raw* scores	Quiz 2 *raw* scores
Jessie	76	88
Sam	81	85

	Quiz 1	Quiz 2
μ	81.40	87.90

Developing z-scores

Step 2: subtract raw scores from the mean (*deviation scores*)

Name	Quiz 1 *raw* scores	Quiz 2 *raw* scores
Jessie	76 - μ_1	88 - μ_2
Sam	81 - μ_1	85 - μ_2

	Quiz 1	Quiz 2
μ	81.40	87.90

- Subtract raw scores from Quiz 1, x_1, from the mean of Quiz 1, μ_1
- Subtract raw scores from Quiz 2, x_2, from the mean of Quiz 2, μ_2

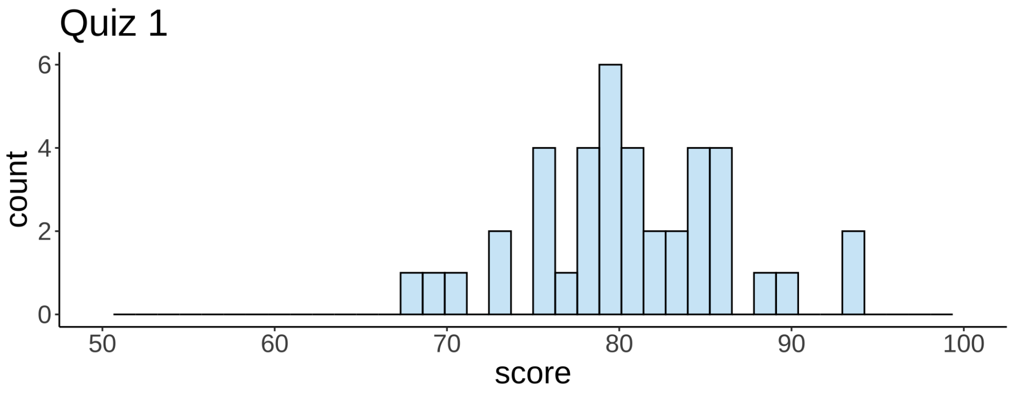
Developing z-scores

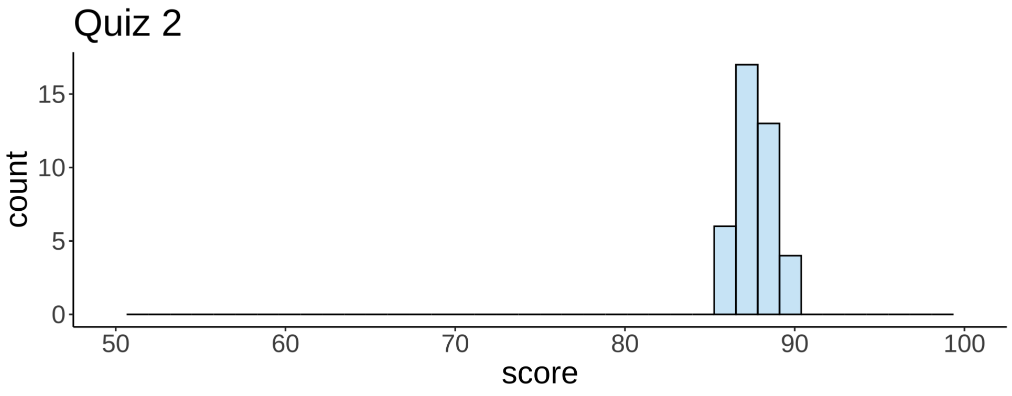
Name	Quiz 1 *deviation* scores	Quiz 2 *deviation* scores
Jessie	-5.40	0.10
Sam	-0.40	-2.90

	Quiz 1	Quiz 2
μ	81.40	87.90

We now know how far away each score is from the mean, but we still have one more thing about distributions to consider

Developing z-scores





$$\mu = 81.40$$

$$\sigma = 8.40$$

$$\mu = 87.90$$

$$\sigma = 1.10$$

The variability for each Quiz is different

Developing z-scores

Name	Quiz 1 *deviation* scores	Quiz 2 *deviation* scores
Jessie	-5.40	0.10
Sam	-0.40	-2.90

	Quiz 1	Quiz 2
μ	81.40	87.90
σ	8.40	1.10

- We need to consider the standard deviation, σ, of each of the Quizzes

Developing z-scores

Step 3: divide each deviation score, $deviation_i$, by their respective standard deviations

Name	Quiz 1 *deviation* scores	Quiz 2 *deviation* scores
Jessie	$\frac{-5.40}{\sigma_1}$	$\frac{0.10}{\sigma_2}$
Sam	$\frac{-0.40}{\sigma_1}$	$\frac{-2.90}{\sigma_2}$

	Quiz 1	Quiz 2
μ	81.40	87.90
σ	8.40	1.10

Developing z-scores

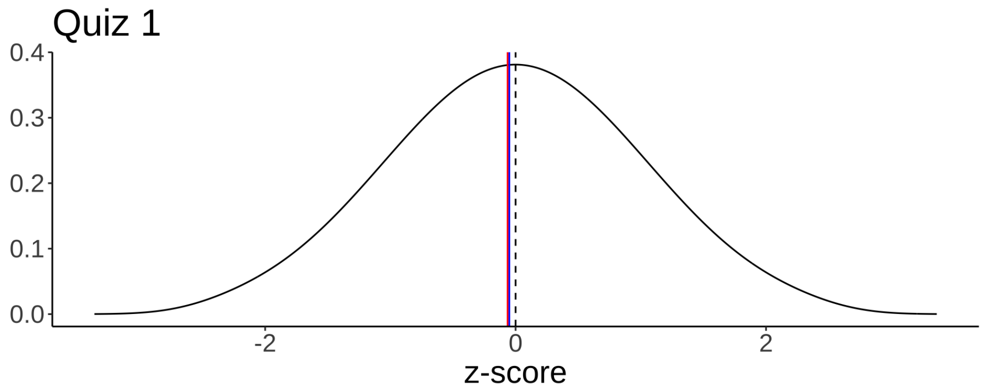
Name	Quiz 1 *z*-scores	Quiz 2 *z*-scores
Jessie	-0.64	0.09
Sam	-0.05	-2.64

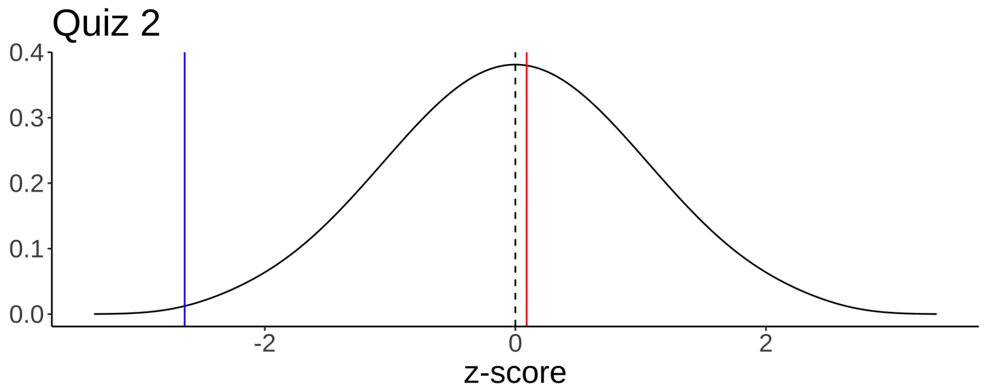
	Quiz 1	Quiz 2
μ	81.40	87.90
σ	8.40	1.10

On **Quiz 1**, Jessie and Sam scored similarly.

On **Quiz 2**, Jessie did well whereas Sam did not.

Developing z-scores





Name	Quiz 1 *z*-scores	Quiz 2 *z*-scores
Jessie	-0.64	0.09
Sam	-0.05	-2.64

Jessie in red
Sam in blue

Developing z-scores

The result of the steps we just did is called calculating *z*-scores

Z-score

$$z_i = \frac{x_i - \mu}{\sigma}$$

$$z_i = \frac{deviation_i}{\sigma}$$

The number of standard deviations a particular score deviates from its corresponding mean

z-scoring is useful because a *z*-score of 1 *means the same thing in any distribution*

Developing z-scores

Z-score

$$z_i = \frac{x_i - \mu}{\sigma}$$

x score from z-score

$$x_i = \mu + z_i\sigma$$

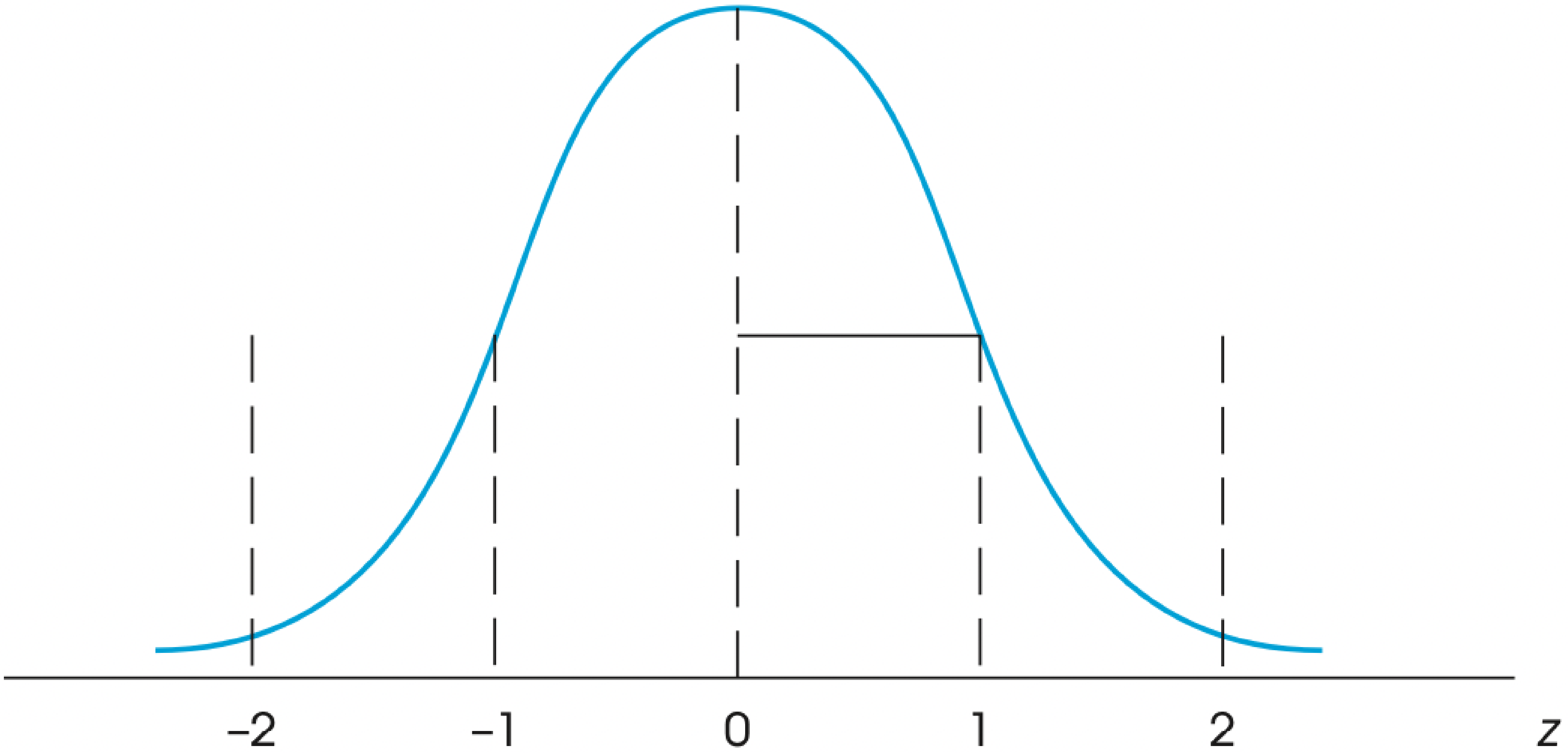
Example: Your IQ *z*-score is 1.4 and the distribution of IQ is $\mu = 100$ and $\sigma = 15$

$$x_i = 100 + (1.4)(15) = 121$$

Uses of z-scores

- **(1)** Describing scores in distributions with a single number
- **(2)** Equating and rescaling entire distributions
  - Z-scoring an entire distribution (or dataset) gets you a new distribution with a mean of 0, and a standard deviation of 1
  - But, the distribution **maintains** its shape
- **(3)** Comparing scores from non-equivalent datasets

Uses of z-scores



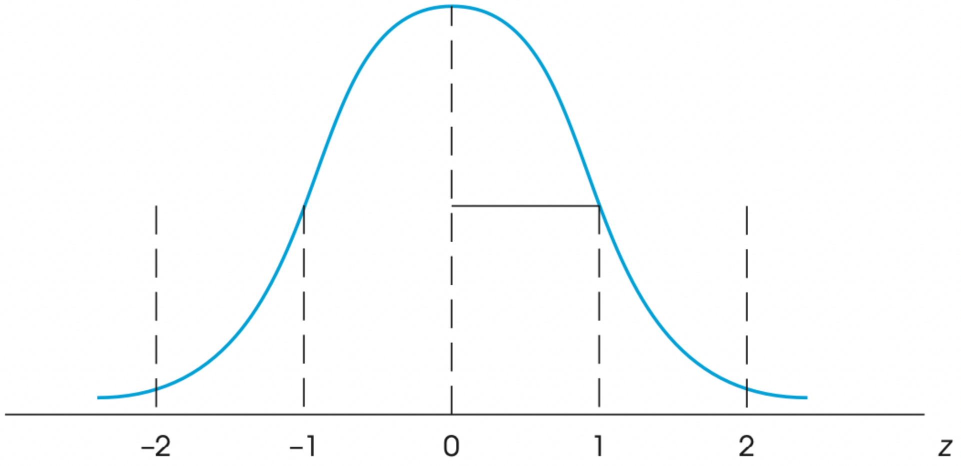
Uses of z-scores

Z-scoring a distribution that is already normal yields the **unit normal distribution**

$$\mu = 0$$

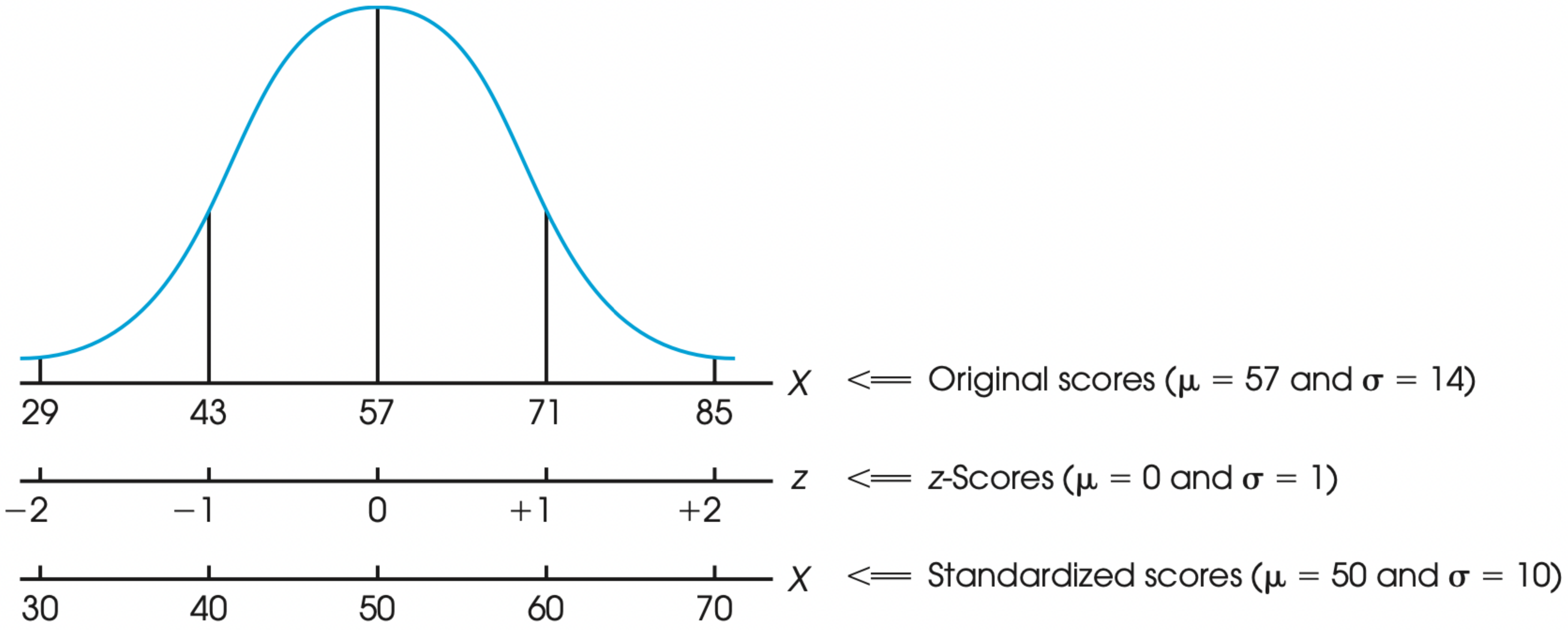
$$\sigma = 1$$

$$\frac{1}{\sqrt{2\pi 1^2}} e^{-\frac{(x-0)^2}{2*1^2}}$$



Uses of z-scores

Equating and rescaling entire distributions



z-score Practice

z-score Practice

Suppose, for example, Alex received a score of $x_1 = 60$ on a psychology exam and a score of $x_2 = 56$ on a biology test.

Suppose the psychology scores had $\mu_1 = 50$ and $\sigma_1 = 10$, and the biology scores had $\mu_2 = 48$ and $\sigma_2 = 4$.

Compute the *z*-score for each class and from the *z*-score, determine which class Alex did better in.

Z-score

$$z_i = \frac{x_i - \mu}{\sigma}$$

z-score Practice

Psychology

$$x_1 = 60$$

$$\mu_1 = 50$$

$$\sigma_1 = 10$$

$$z_1 = \frac{60-50}{10} = 1.00$$

Biology

$$x_2 = 56$$

$$\mu_2 = 48$$

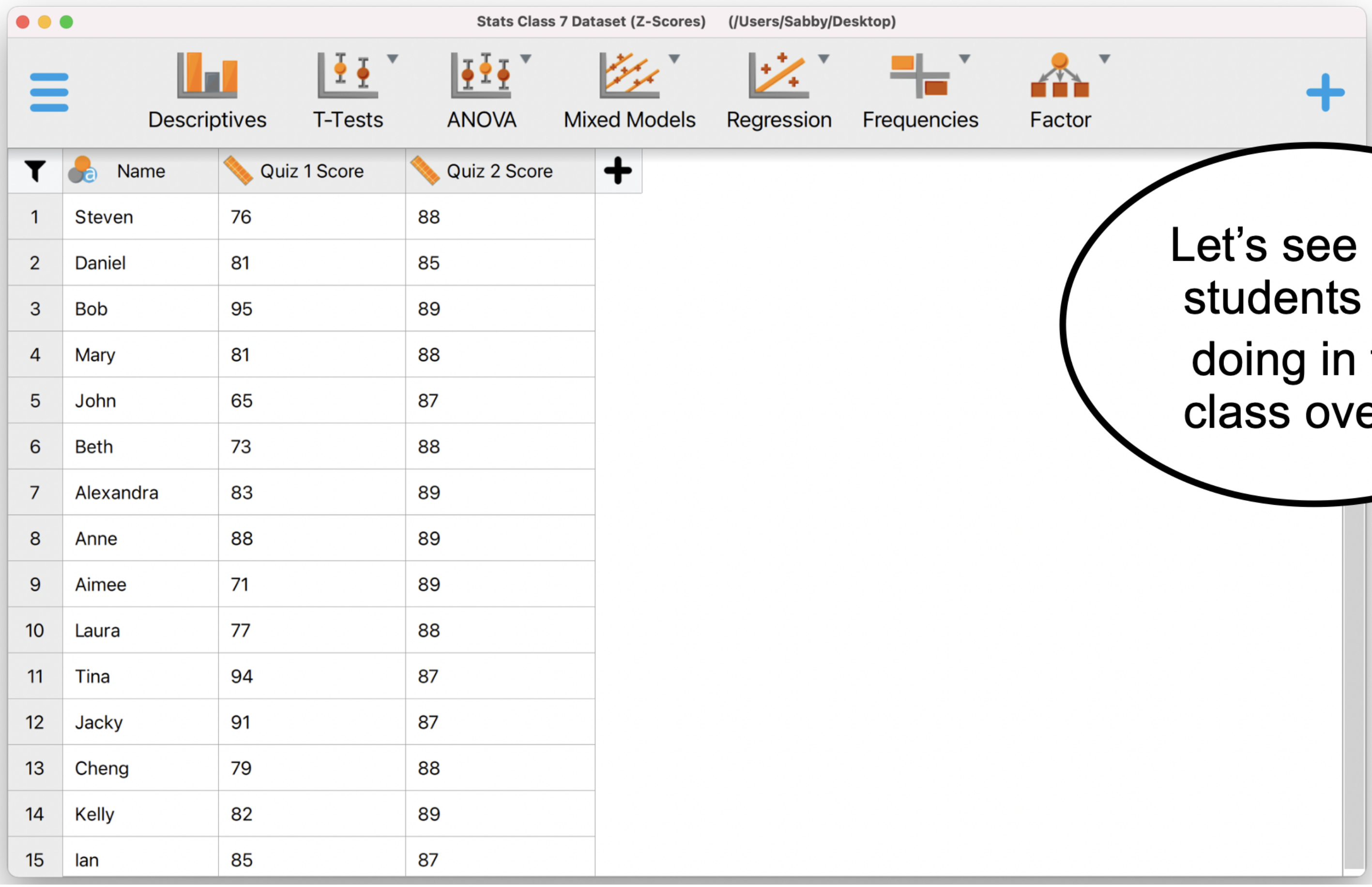
$$\sigma_2 = 4$$

$$z_2 = \frac{56-48}{4} = 2.00$$

Alex did better in biology compared to psychology

Computing *z*-scores in JASP

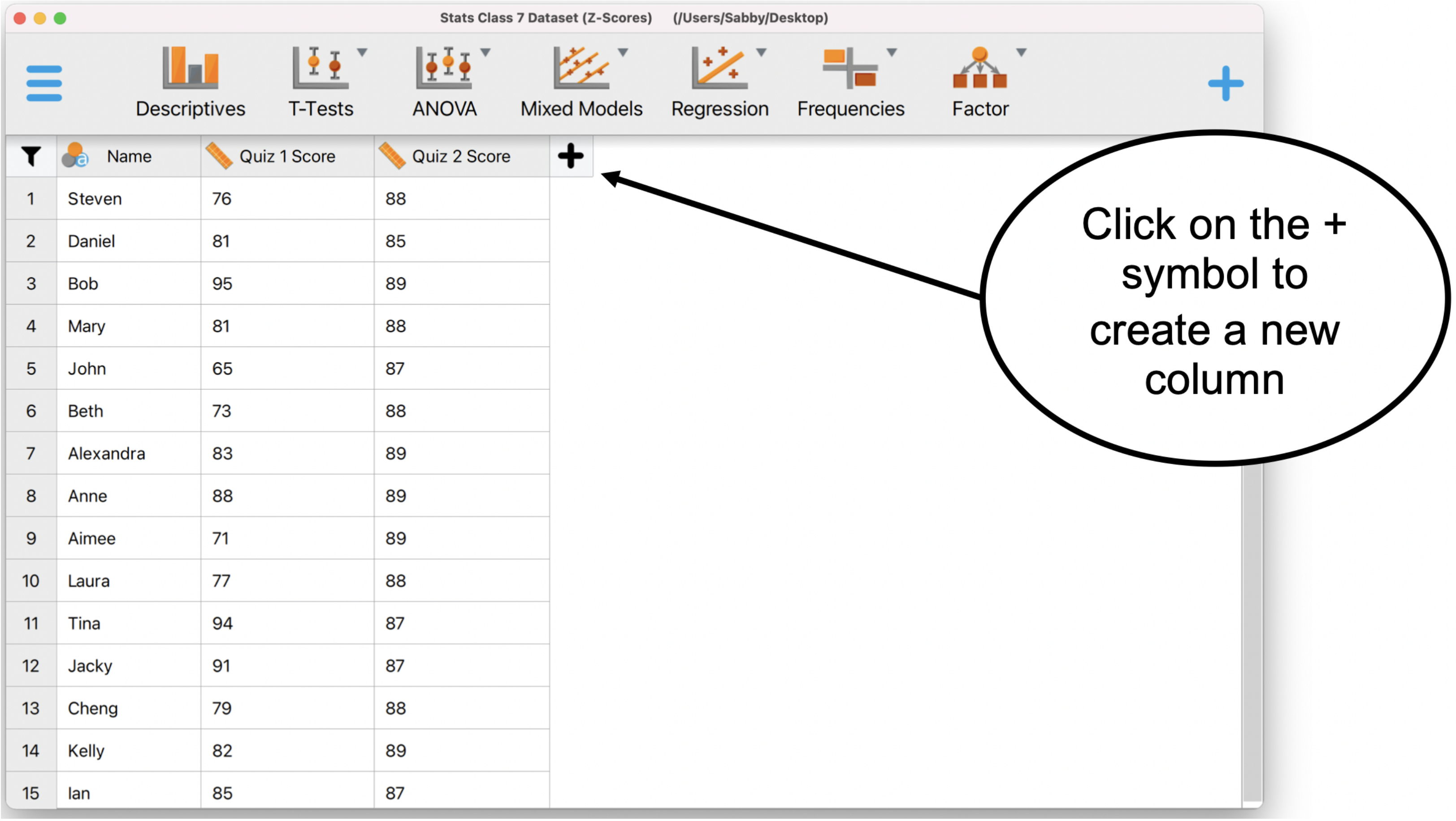
JASP: z-scores



	Name	Quiz 1 Score	Quiz 2 Score
1	Steven	76	88
2	Daniel	81	85
3	Bob	95	89
4	Mary	81	88
5	John	65	87
6	Beth	73	88
7	Alexandra	83	89
8	Anne	88	89
9	Aimee	71	89
10	Laura	77	88
11	Tina	94	87
12	Jacky	91	87
13	Cheng	79	88
14	Kelly	82	89
15	Ian	85	87

Let's see how students are doing in the class overall

JASP: z-scores



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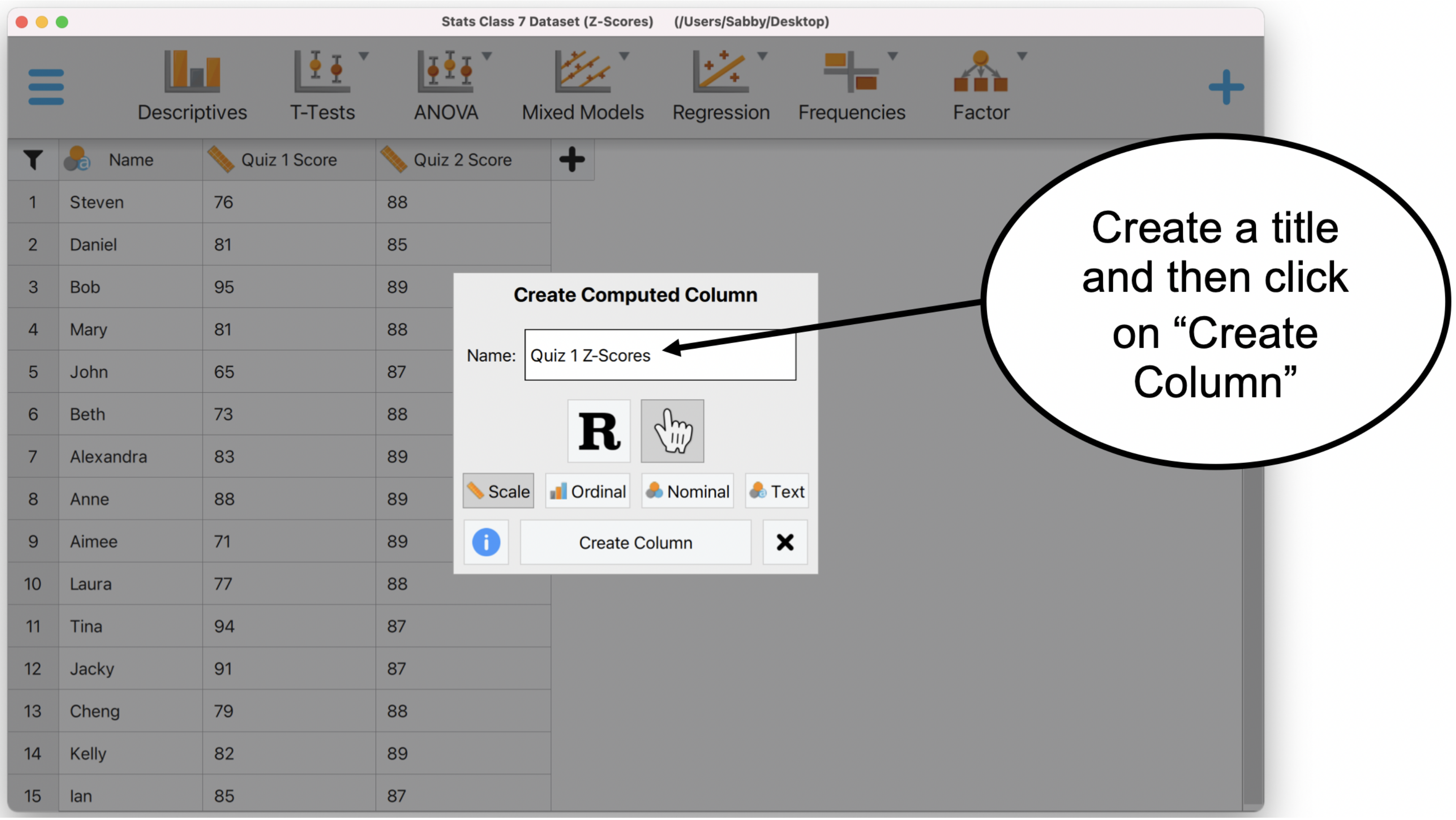
JASP: z-scores



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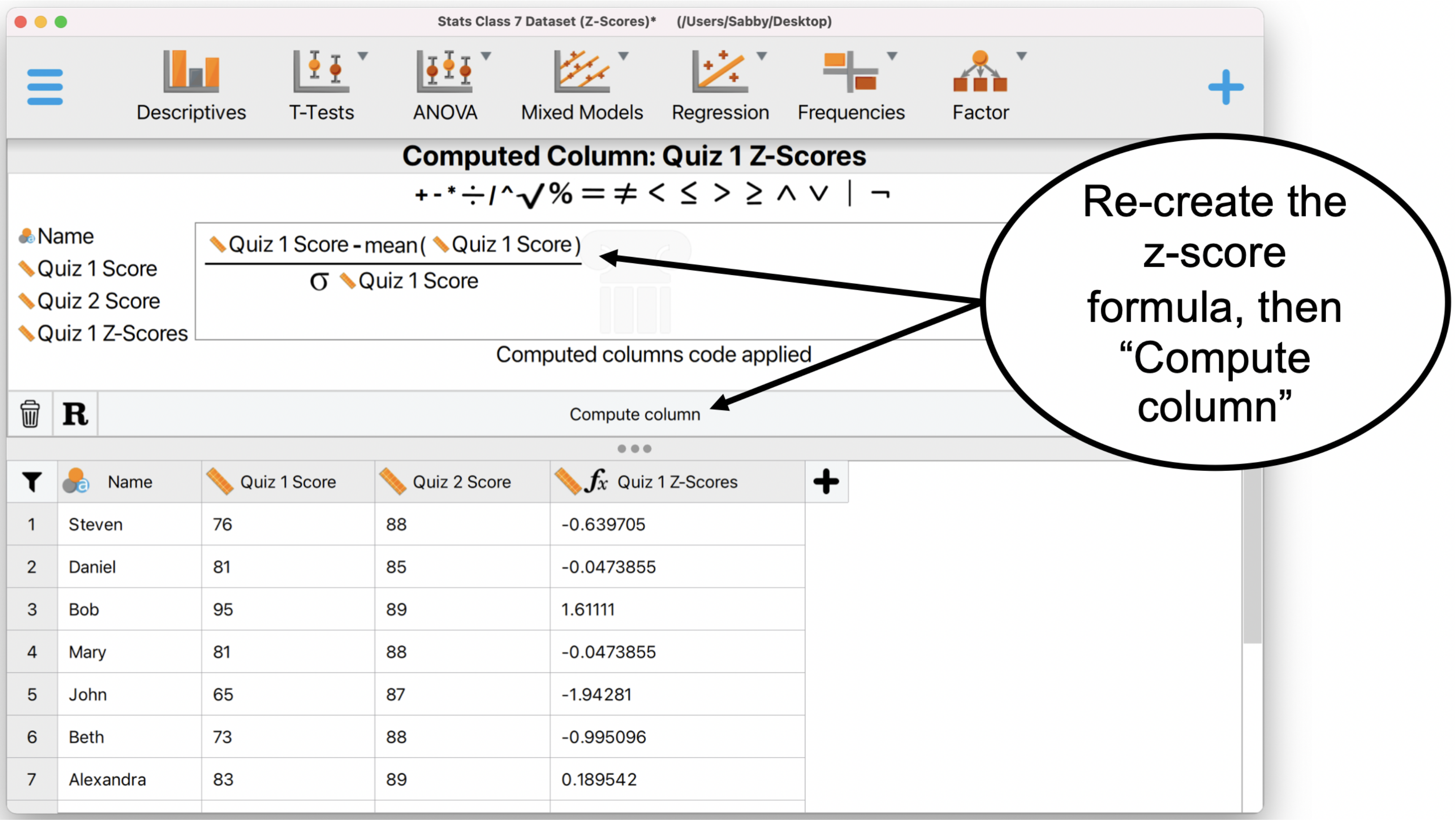
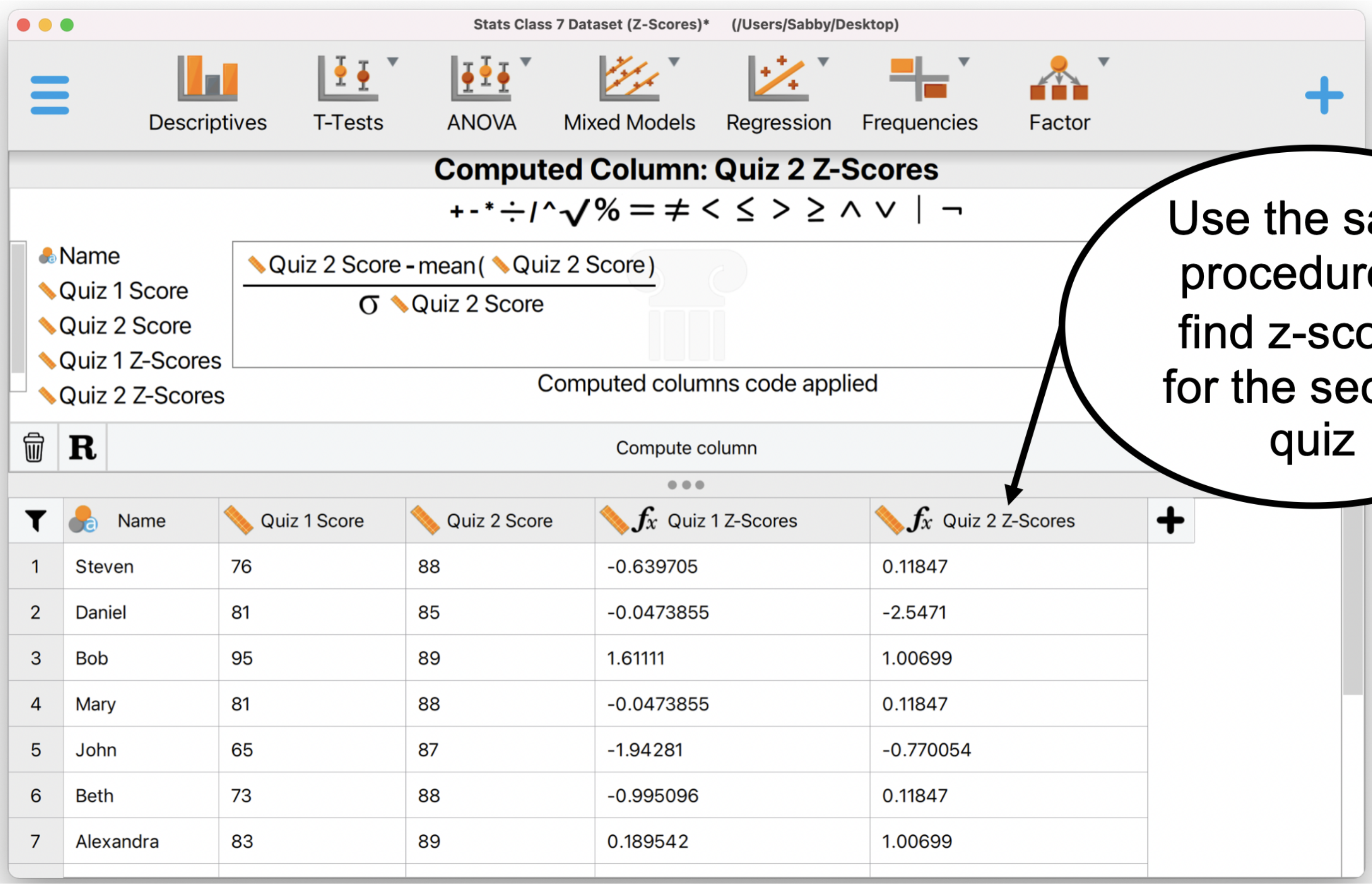
JASP: z-scores



JASP: z-scores



	Name	Quiz 1 Score	Quiz 2 Score	Quiz 1 Z-Scores	Quiz 2 Z-Scores
1	Steven	76	88	-0.639705	0.11847
2	Daniel	81	85	-0.0473855	-2.5471
3	Bob	95	89	1.61111	1.00699
4	Mary	81	88	-0.0473855	0.11847
5	John	65	87	-1.94281	-0.770054
6	Beth	73	88	-0.995096	0.11847
7	Alexandra	83	89	0.189542	1.00699

Use the same procedure to find z-scores for the second quiz

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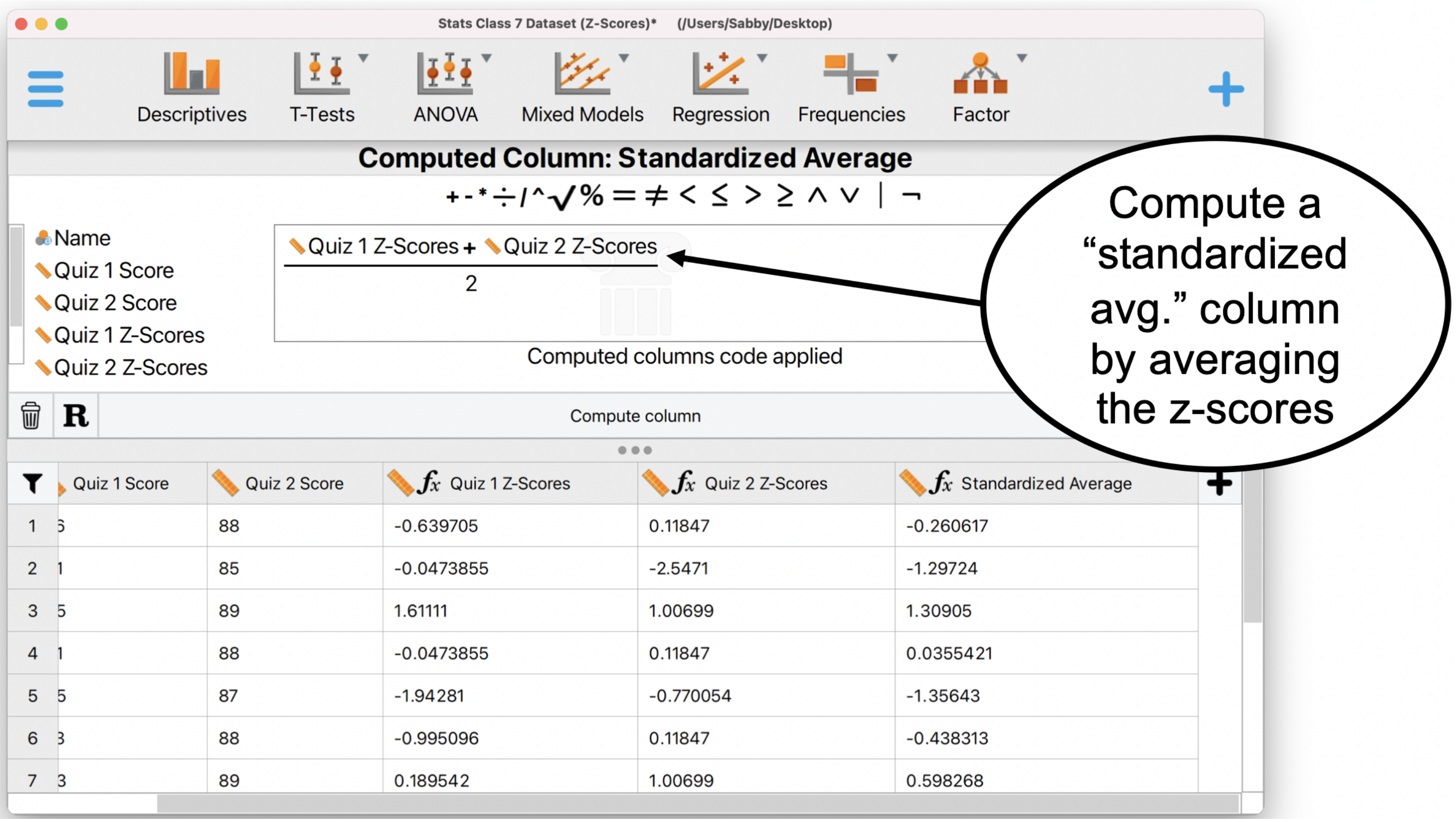
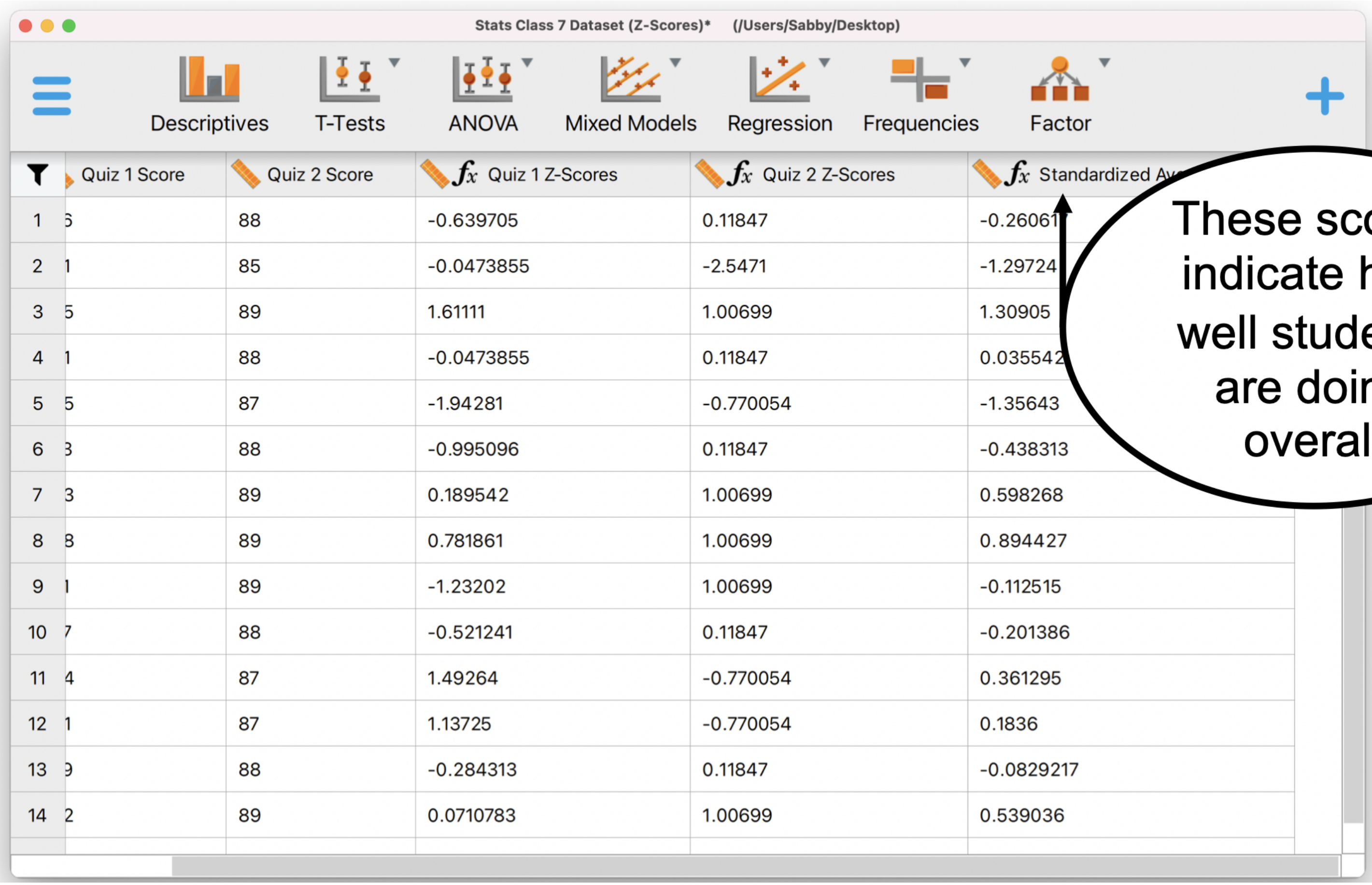
JASP: z-scores



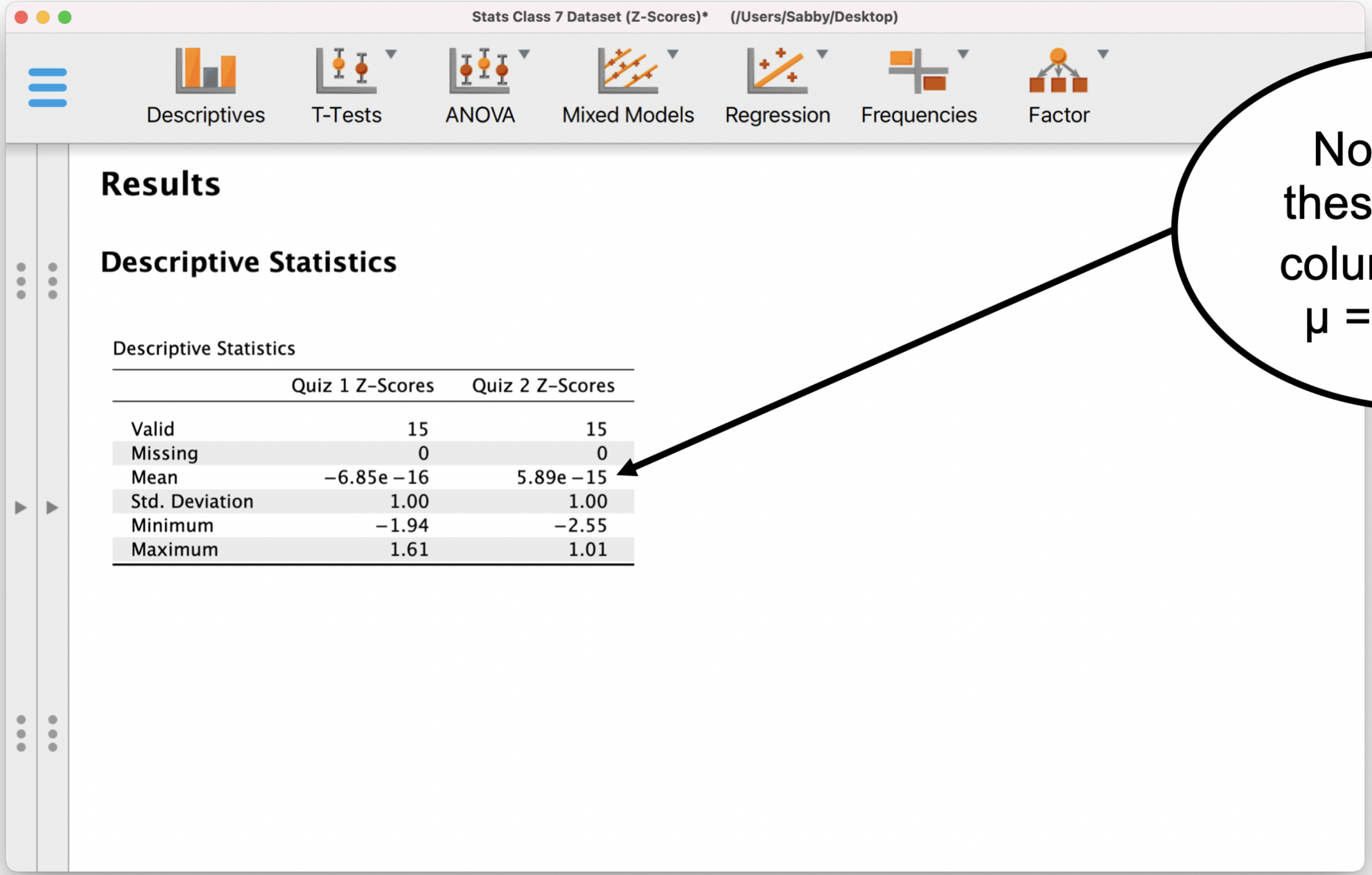
JASP: z-scores



	Quiz 1 Score	Quiz 2 Score	Quiz 1 Z-Scores	Quiz 2 Z-Scores	Standardized Av
1	6	88	-0.639705	0.11847	-0.26061
2	1	85	-0.0473855	-2.5471	-1.29724
3	5	89	1.61111	1.00699	1.30905
4	1	88	-0.0473855	0.11847	0.035542
5	5	87	-1.94281	-0.770054	-1.35643
6	3	88	-0.995096	0.11847	-0.438313
7	3	89	0.189542	1.00699	0.598268
8	8	89	0.781861	1.00699	0.894427
9	1	89	-1.23202	1.00699	-0.112515
10	7	88	-0.521241	0.11847	-0.201386
11	4	87	1.49264	-0.770054	0.361295
12	1	87	1.13725	-0.770054	0.1836
13	9	88	-0.284313	0.11847	-0.0829217
14	2	89	0.0710783	1.00699	0.539036

These scores indicate how well students are doing overall

JASP: z-scores



Descriptive Statistics

	Quiz 1 Z-Scores	Quiz 2 Z-Scores
Valid	15	15
Missing	0	0
Mean	−6.85e −16	5.89e −15
Std. Deviation	1.00	1.00
Minimum	−1.94	−2.55
Maximum	1.61	1.01

Notice that these z-score columns have $\mu = 0$, $\sigma = 1$

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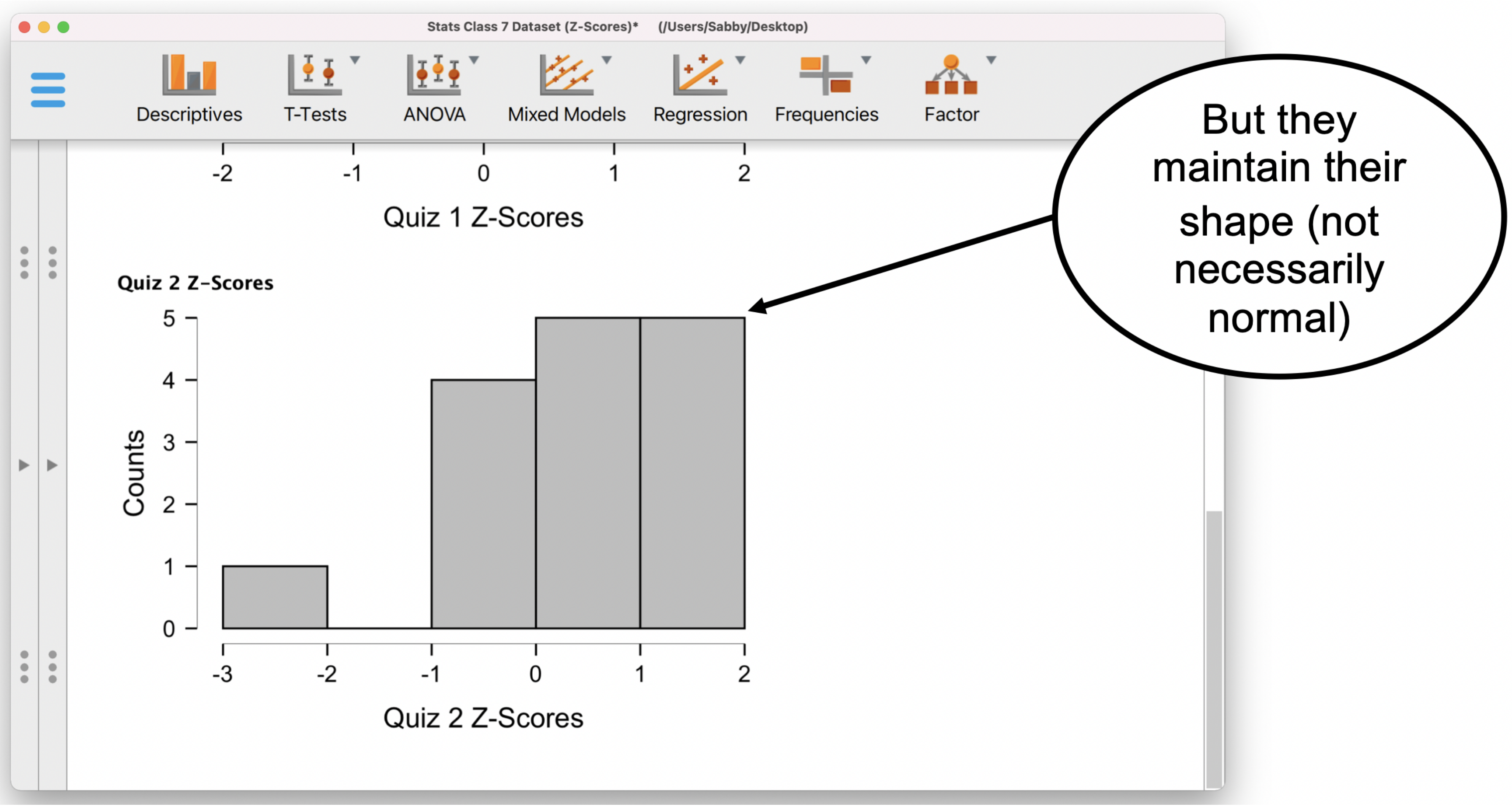
JASP: z-scores



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Next time

Lecture

- Introduction to Statistical Significance

Reading

- Chapter Eight