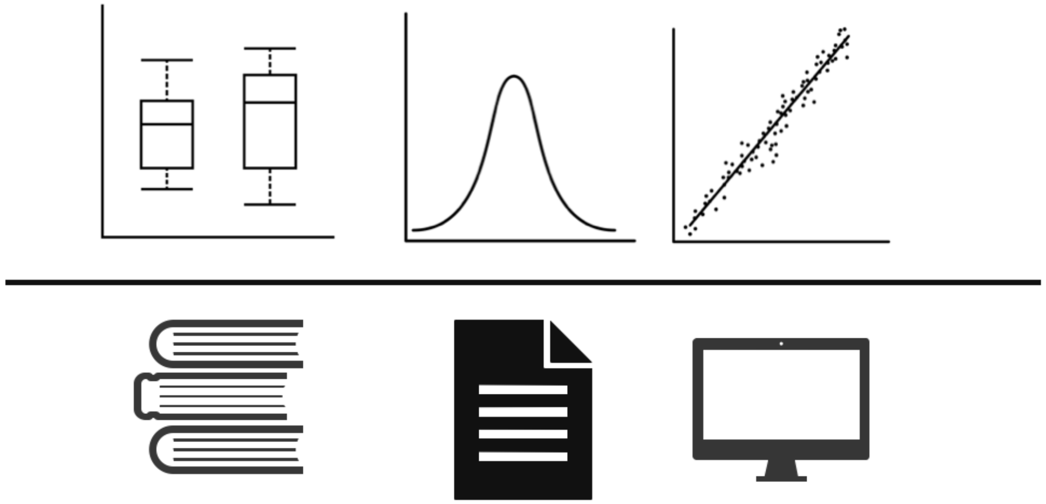
PSYC 2300

Introduction to Statistics

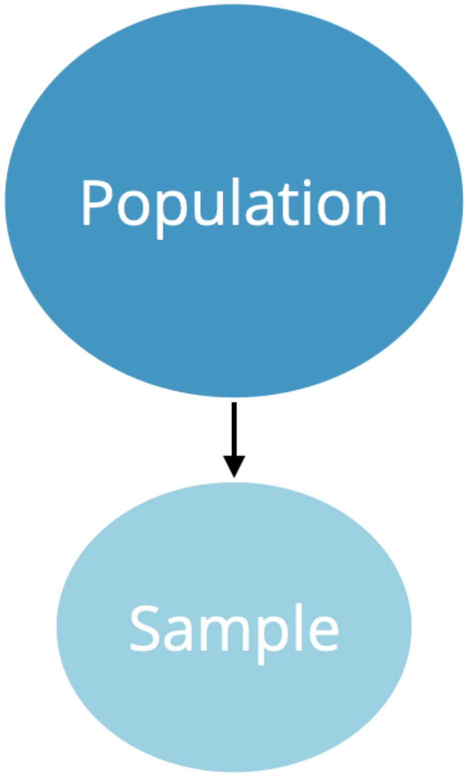


Lecture 12: Differences Between Many Groups

Outline for today

- Review parts of last class
- The One-Way ANOVA
- Effect size for the One-Way ANOVA
- One-Way ANOVAs in JASP
    - Download `Stats Class 14 Dataset (One-way ANOVA).jasp`

Review



μ

$\sigma = ?$

$\bar{x}$

s

t-tests: one sample

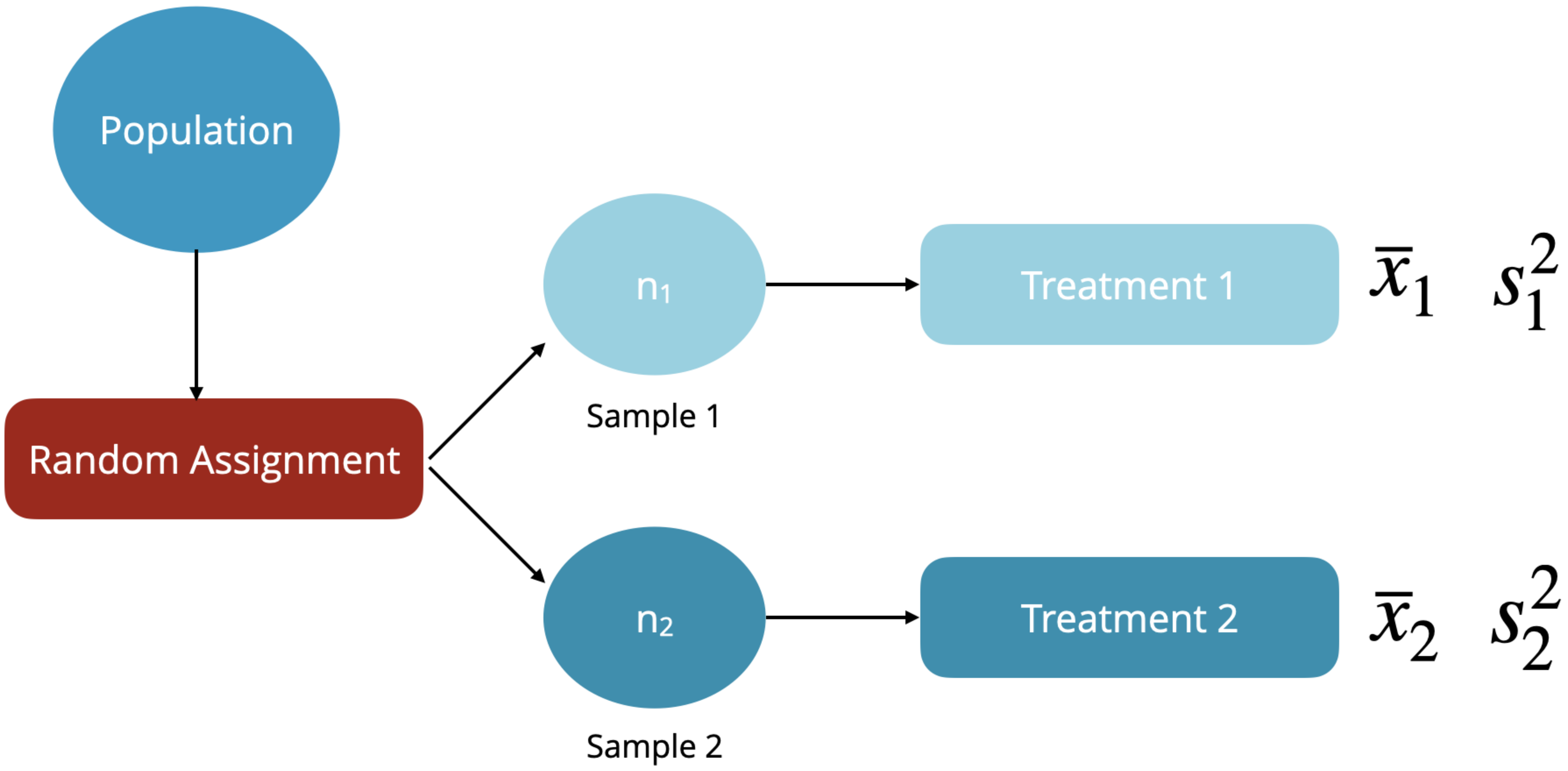
We have one sample, but we don't know the population standard deviation or population variance

One-sample *t*-test statistic

$$t_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{s_{\bar{x}}}$$

Estimated standard error

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$


Population
Random Assignment
n_1
Sample 1
n_2
Sample 2
Treatment 1
Treatment 2
$\bar{x}_1 \quad s_1^2$
$\bar{x}_2 \quad s_2^2$

t-tests: independent samples

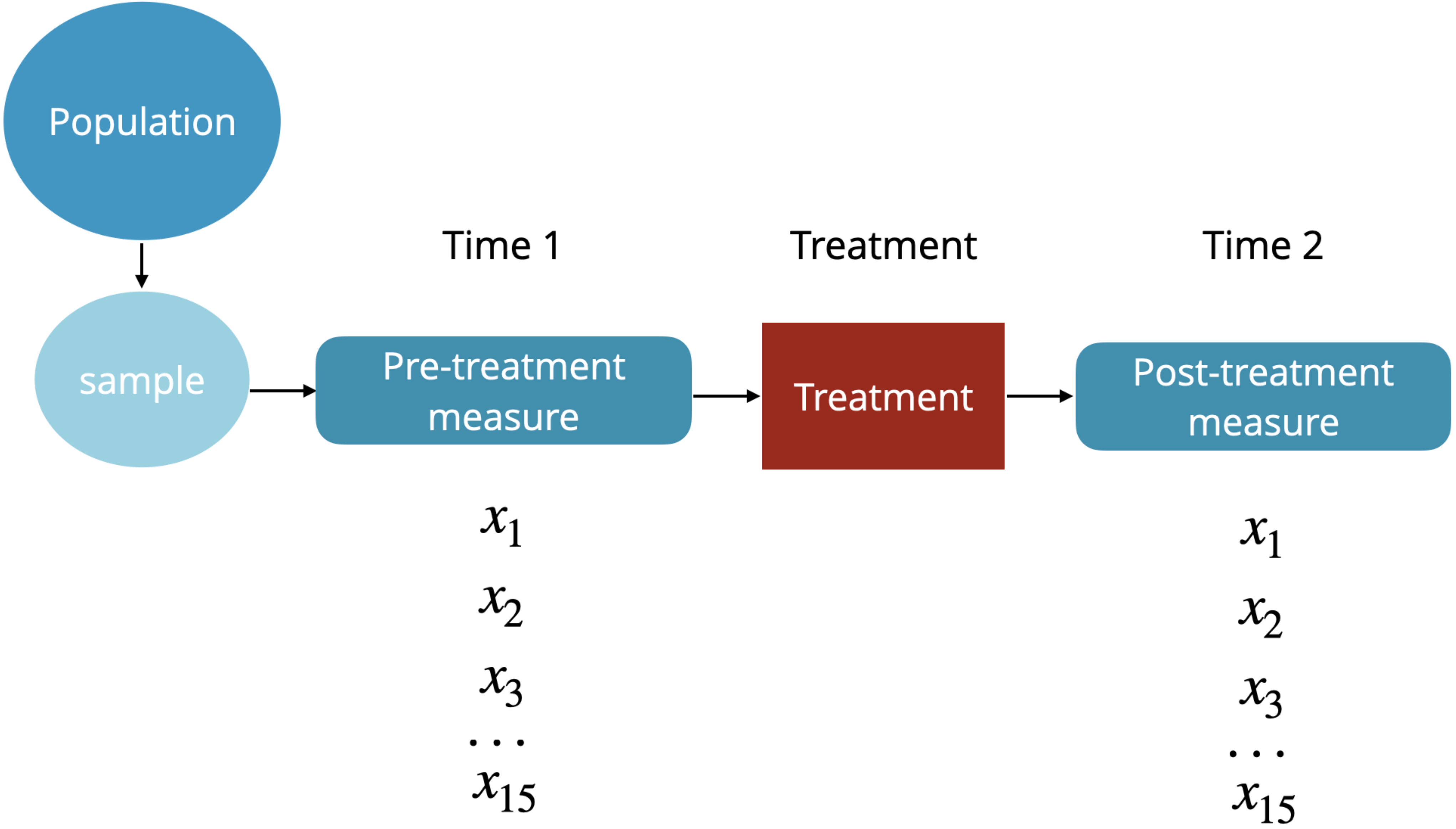
We have two independent samples

Independent samples *t*-test

$$t_{(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{s_{(\bar{x}_1 - \bar{x}_2)}}$$

Estimated standard error of the mean difference

$$s_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\left[\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}\right]\left[\frac{n_1 + n_2}{n_1 n_2}\right]}$$


Population
sample
Time 1
Treatment
Time 2
Pre-treatment measure
Treatment
Post-treatment measure
x_1
x_2
x_3
$\dots$
x_{15}
x_1
x_2
x_3
$\dots$
x_{15}

t-tests

We want to investigate whether some treatment made an effect compared to a baseline within the same individual(s)

Dependent samples *t*-test

$$t_{\bar{x}_d} = \frac{\bar{x}_d}{s_{\bar{x}_d}}$$

Estimated standard error of the difference scores

$$s_{\bar{x}_d} = \sqrt{\frac{s_d^2}{n}}$$

Within-subjects designs

	Time 1		Time 2		
	Baseline self-efficacy		Post-treatment self-efficacy		Difference score
	x_1	$-$	x_1	$=$	x_{d_1}
	x_2	$-$	x_2	$=$	x_{d_2}
	x_3	$-$	x_3	$=$	x_{d_3}
	$\cdots$	$-$	$\ldots$	$=$	$\cdots$
	x_{15}	$-$	x_{15}	$=$	$x_{d_{15}}$

One-way ANOVA

One-way ANOVA

The problem: All our tests so far involve only one or two groups

Goal: To be able to measures differences along some dimension among participants in more than two groups

- The test we use to compare many groups is called the ANOVA, or the **AN**alysis **O**f **VA**riance

- Instead of z or t, ANOVAs use F distributions

ANOVA: Terminology

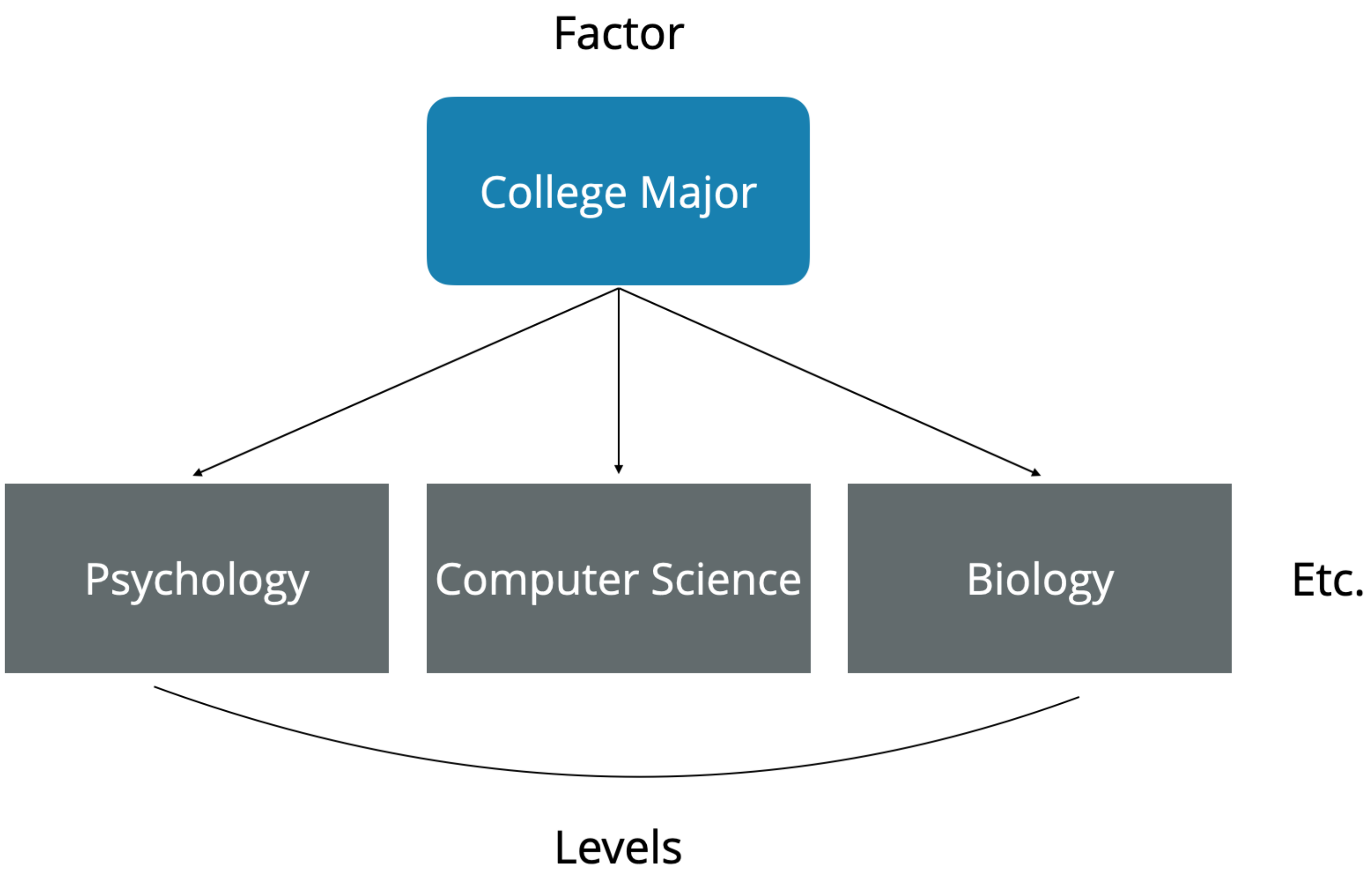
Factor: Independent variable(s) in the study

Level(s): Groups within each independent variable

Response: The dependent variable in the study

Today, we'll learn about *one-way ANOVAs*, which have only one factor

Example: One factor, several levels



ANOVA: What does it do?

- The ANOVA asks, “are there any significant differences among the different groups we’re testing?”
- Essentially, it investigates whether **any** of the population means of our groups differ
- The ANOVA is an **omnibus test**, meaning it tests whether there are *any* differences among groups
- It doesn’t tell which group(s) is(are) different; you need post-hoc tests for this (we’ll come back to this later)

You can also think of *onmibus test* as an 'overall' test

Statistical Hypotheses

Null Hypothesis H_0

There is no difference

The means are equal

$\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$

Alternative Hypothesis H_a

There is a difference

The means are not all equal

$k =$ the number of groups

Why not just use many *t*-tests?

Experiment-wise alpha level: the total probability of a Type I error that is accumulated from all of the individual tests in the experiment. Typically, the experiment-wise alpha level is substantially greater than the value of alpha used for any one of the individual tests

- *Type I Error rate* increases drastically as you run additional analyses
- This is called **alpha escalation**
  - Each test has a risk of a Type I error, and the more tests you do, the more risk there is

Review: Decisions

Truth

Decision	There is no true effect H_0	There is an effect H_1
	Fail to reject H_0 **SPECIFICITY** $1-\alpha$	Fail to reject H_0 **TYPE II ERROR** β
	Reject H_0 **TYPE I ERROR** α	Reject H_0 **POWER** $1-\beta$

Building the *F*-statistic

$$F = \frac{Variance\ due\ to\ group\ differences}{Variance\ due\ to\ random\ chance}$$

Numerator

Variance between groups will be small if there are no differences between groups, leading to a small *F* statistic

Variance between groups will be large if there are differences between groups, leading to a large *F* statistic

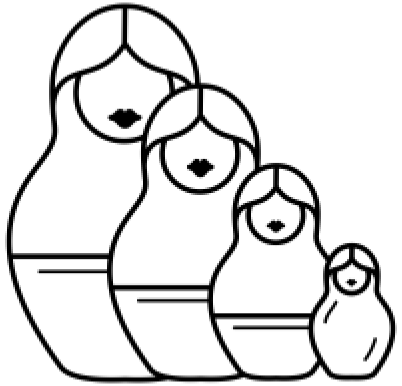
Denominator

Error/random chance differences among individuals within single groups

Building the *F*-statistic

- That is why is called the ANOVA: We analyze and compare **variance** in the dependent variable separately *between groups* and *within groups*

- There is, however, a lot of calculations needed



F-statistic: One-way ANOVA

$$F = \frac{MS_{between}}{MS_{within}}$$

$MS_{between}$: mean sum of squares between groups (variance between groups)

MS_{within}: mean sum of squares within groups (variance within groups)

F-statistic: One-way ANOVA

Mean sum of squares between groups

$$MS_{between} = \frac{SS_{between}}{df_{between}}$$

Mean sum of squares within groups

$$MS_{within} = \frac{SS_{within}}{df_{within}}$$

$$df_{between} = k - 1$$

$$df_{within} = nT - k$$

$k =$ number of groups
$nT =$ total number of participants across all groups
$df =$ degrees of freedom, an estimate of sample size

F-statistic: One-way ANOVA

Mean sum of squares between groups

$$MS_{between} = \frac{SS_{between}}{df_{between}}$$

Mean sum of squares within groups

$$MS_{within} = \frac{SS_{within}}{df_{within}}$$

$$SS_{between} = \sum \frac{(\sum x)^2}{n} - \frac{(\sum\sum x)^2}{nT}$$

$$SS_{within} = \sum\sum(x^2) - \sum \frac{(\sum x)^2}{n}$$

Sum of Squares: One-way ANOVA

For each group, first calculate		Afterwards, calculate across groups
n	Add up all the ns $\dashrightarrow$	nT
$\sum x$	Add up all the $\sum x$ $\dashrightarrow$	$\sum\sum x$
$\bar{x}$	Square the above value and divide by nT	$\frac{(\sum\sum x)^2}{nT}$
$\frac{(\sum x)^2}{n}$	Add up all the $\frac{(\sum x)^2}{n}$ $\dashrightarrow$	$\sum\frac{(\sum x)^2}{n}$
$\sum(x^2)$	Add up all the $\sum(x^2)$ $\dashrightarrow$	$\sum\sum(x^2)$

Sum of Squares: One-way ANOVA

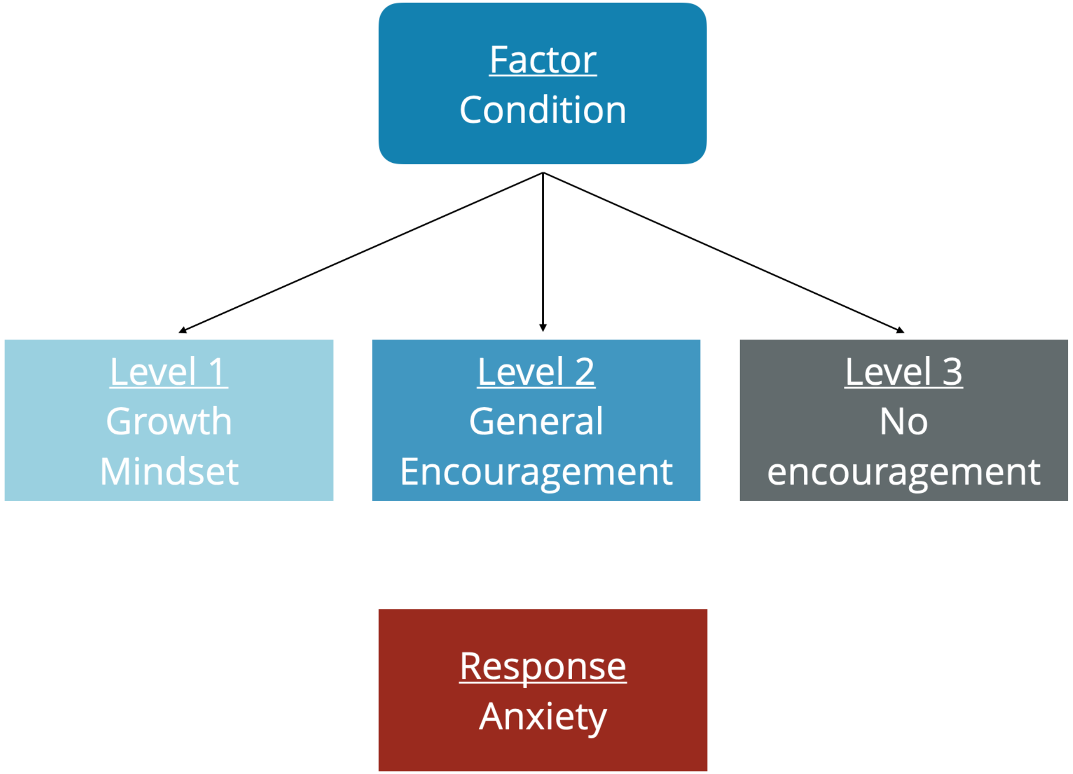
Once you have these values, plug them into the *SS* formulas

$$SS_{between} = \sum \frac{(\sum x)^2}{n} - \frac{(\sum \sum x)^2}{nT}$$

$$SS_{within} = \sum \sum (x^2) - \sum \frac{(\sum x)^2}{n}$$

Example Experiment

Research Question: Does teaching students about *growth mindsets* affect students' anxiety more than giving them *general encouragement* or *no encouragement*?



Group Calculations

For each group, first calculate		Afterwards, calculate across groups
n	Add up all the ns	nT
$\sum x$	Add up all the $\sum x$	$\sum\sum x$
$\bar{x}$	Square the above value and divide by nT	$\dfrac{(\sum\sum x)^2}{nT}$
$\dfrac{(\sum x)^2}{n}$	Add up all the $\dfrac{(\sum x)^2}{n}$	$\sum\dfrac{(\sum x)^2}{n}$
$\sum(x^2)$	Add up all the $\sum(x^2)$	$\sum\sum(x^2)$

Group Calculations: Group 1

Growth Mindset

Growth	Growth^2
6	36
6	36
5	25
8	64
7	49
6	36
3	9
4	16

$$n_1 = 8$$

$$\bar{x}_1 = 5.625$$

$$\sum x_1 = 45$$

$$\frac{(\sum x_1)^2}{n_1} = \frac{45^2}{8} = 253.125$$

$$\sum (x_1^2) = 271$$

Group Calculations: Group 2

General Encouragement

General	$General^2$
8	64
7	49
6	36
5	25
6	36
7	49
8	64
6	36

$$n_2 = 8$$

$$\bar{x}_2 = 6.625$$

$$\sum x_2 = 53$$

$$\frac{(\sum x_2)^2}{n_2} = \frac{53^2}{8} = 351.125$$

$$\sum (x_2^2) = 359$$

Group Calculations: Group 3

No Encouragement

None	$None^2$
9	81
8	64
9	81
9	81
7	49
8	64
6	36
9	81

$$n_3 = 8$$

$$\bar{x}_3 = 8.125$$

$$\sum x_3 = 65$$

$$\frac{(\sum x_3)^2}{n_3} = \frac{65^2}{8} = 528.125$$

$$\sum (x_3^2) = 537$$

Example Experiment

We now have all the group statistics we need to compute the across group statistics

Growth	**General**	**None**
$n_1 = 8$	$n_2 = 8$	$n_3 = 8$
$\bar{x}_1 = 5.625$	$\bar{x}_2 = 6.625$	$\bar{x}_3 = 8.125$
$\sum x_1 = 45$	$\sum x_2 = 53$	$\sum x_3 = 65$
$\dfrac{(\sum x_1)^2}{n_1} = 253.125$	$\dfrac{(\sum x_2)^2}{n_2} = 351.125$	$\dfrac{(\sum x_3)^2}{n_3} = 528.125$
$\sum (x_1^2) = 271$	$\sum (x_2^2) = 359$	$\sum (x_3^2) = 537$

Calculate Terms Across Groups

For each group, first calculate		Afterwards, calculate across groups
n	Add up all the ns	nT
$\sum x$	Add up all the $\sum x$	$\sum\sum x$
$\bar{x}$	Square the above value and divide by nT	$\frac{(\sum\sum x)^2}{nT}$
$\frac{(\sum x)^2}{n}$	Add up all the $\frac{(\sum x)^2}{n}$	$\sum\frac{(\sum x)^2}{n}$
$\sum(x^2)$	Add up all the $\sum(x^2)$	$\sum\sum(x^2)$

Calculate Terms Across Groups

Calculate all the terms needed for $SS_{between}$ and SS_{within}

$$nT = n_1 + n_2 + n_3 = 8 + 8 + 8 = 24$$

$$\sum\sum x = \sum x_1 + \sum x_2 + \sum x_3 = 45 + 53 + 65 = 163$$

$$\frac{(\sum\sum x)^2}{nT} = \frac{163^2}{24} = 1107.04$$

$$\sum\frac{(\sum x)^2}{n} = \frac{(\sum x_1)^2}{n_1} + \frac{(\sum x_2)^2}{n_2} + \frac{(\sum x_3)^2}{n_3} = 253.125 + 351.125 + 528.125 = 1132.38$$

$$\sum\sum(x^2) = \sum(x_1^2) + \sum(x_2^2) + \sum(x_3^2) = 271 + 359 + 537 = 1167$$

Calculating $SS_{between}$ and SS_{within}

$$SS_{between} = \sum \frac{(\sum x)^2}{n} - \frac{(\sum \sum x)^2}{nT}$$

$$SS_{within} = \sum \sum (x^2) - \sum \frac{(\sum x)^2}{n}$$

$$\frac{(\sum \sum x)^2}{nT} = 1107.04 \qquad \sum \frac{(\sum x)^2}{n} = 1132.38 \qquad \sum \sum (x^2) = 1167$$

$$SS_{between} = 1132.38 - 1107.04 = 25.34$$

$$SS_{within} = 1167 - 1132.38 = 34.62$$

Calculating $df_{between}$ and df_{within}

$$df_{between} = k - 1$$

$$df_{within} = nT - k$$

$k =$ number of groups
$nT =$ total sample size
$df =$ degrees of freedom

$$df_{between} = k - 1 = 3 - 1 = 2$$

$$df_{within} = nT - k = 24 - 3 = 21$$

Calculating $MS_{between}$ and MS_{within}

$$MS_{between} = \frac{SS_{between}}{df_{between}}$$

$$MS_{within} = \frac{SS_{within}}{df_{within}}$$

$$SS_{between} = 1132.38 - 1107.04 = 25.34$$

$$df_{between} = k - 1 = 3 - 1 = 2$$

$$MS_{between} = \frac{25.34}{2} = 12.67$$

$$SS_{within} = 1167 - 1132.38 = 34.62$$

$$df_{within} = nT - k = 24 - 3 = 21$$

$$MS_{within} = \frac{34.62}{21} = 1.64$$

Calculating the *F* statistic

$$F = \frac{MS_{between}}{MS_{within}}$$

$$MS_{between} = \frac{25.34}{2} = 12.67$$

$$MS_{within} = \frac{34.62}{21} = 1.64$$

$$F = \frac{12.67}{1.64} = 7.70$$

$$F = \frac{MS_{between}}{MS_{within}}$$

$$MS_{between} = \frac{SS_{between}}{df_{between}}$$

$$MS_{within} = \frac{SS_{within}}{df_{within}}$$

$$SS_{between} = \sum \frac{(\sum x)^2}{n} - \frac{(\sum \sum x)^2}{nT}$$

$$SS_{within} = \sum \sum (x^2) - \sum \frac{(\sum x)^2}{n}$$

$$df_{between} = k - 1$$

$$df_{within} = nT - k$$

Effect size for the One-way ANOVA

Effect size: One-way ANOVA

The effect size for a one-way ANOVA is represented by *Eta squared*, η^2, not Cohen's *d*

Effect size for One-way ANOVA

$$\eta^2 = \frac{SS_{between}}{SS_{Total}}$$

$$SS_{total} = SS_{between} + SS_{within} = \sum\sum(x^2) - \frac{(\sum\sum x)^2}{nT}$$

Effect size: One-way ANOVA

Getting the total sum of squares

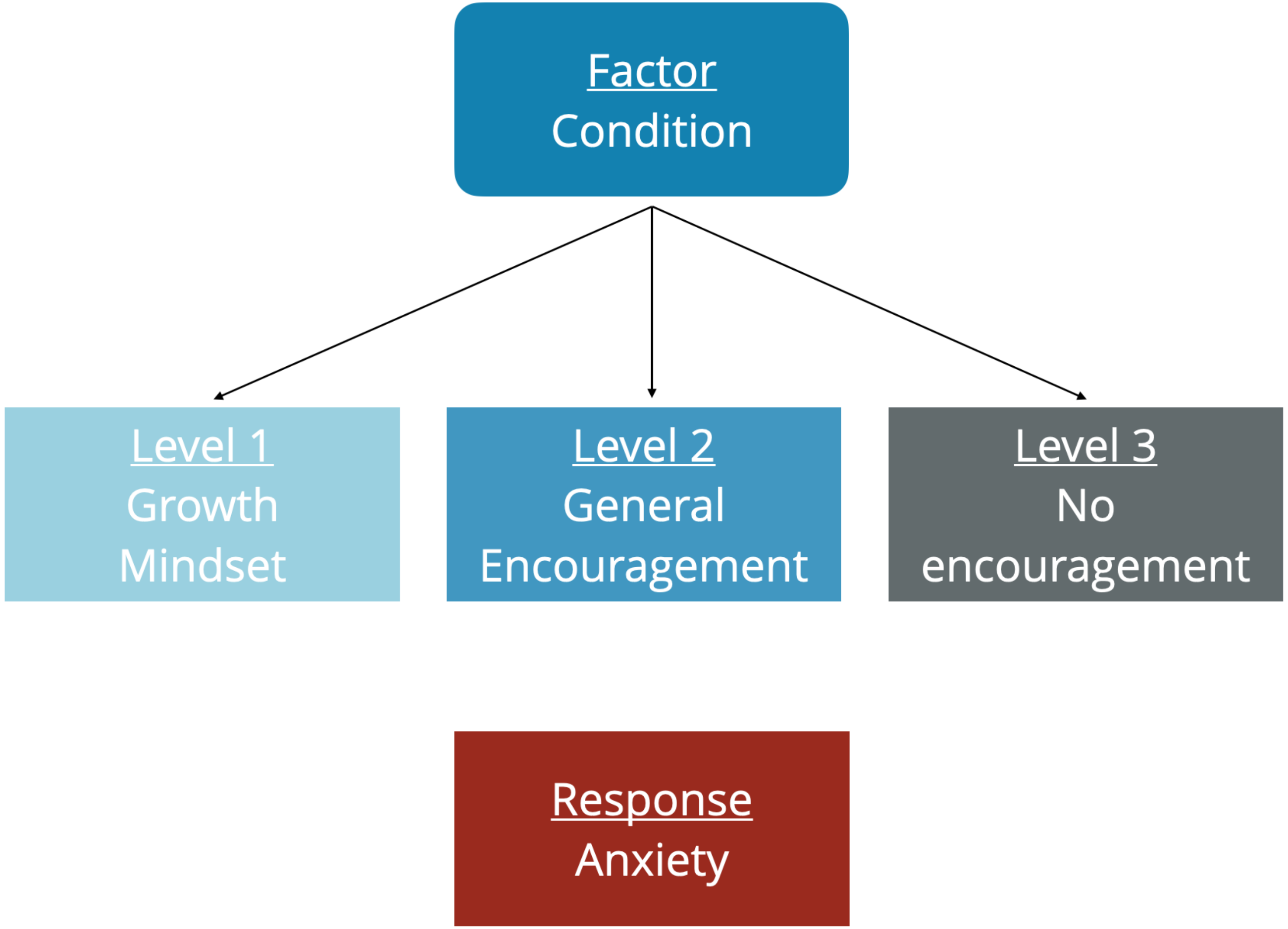
$$SS_{total} = SS_{between} + SS_{within}$$

$$SS_{between} = \sum \frac{(\sum x)^2}{n} - \frac{(\sum \sum x)^2}{nT}$$

$$SS_{within} = \sum \sum (x^2) - \sum \frac{(\sum x)^2}{n}$$

$$SS_{total} = \sum \sum (x^2) - \frac{(\sum \sum x)^2}{nT}$$

Example Experiment



Calculating $SS_{between}$, SS_{within}, and SS_{total}

$$SS_{between} = \sum \frac{(\sum x)^2}{n} - \frac{(\sum \sum x)^2}{nT}$$

$$SS_{within} = \sum \sum (x^2) - \sum \frac{(\sum x)^2}{n}$$

$$\frac{(\sum \sum x)^2}{nT} = 1107.04$$

$$\sum \frac{(\sum x)^2}{n} = 1132.38$$

$$\sum \sum (x^2) = 1167$$

$$SS_{between} = 1132.38 - 1107.04 = 25.34$$

$$SS_{within} = 1167 - 1132.38 = 34.62$$

$$SS_{total} = SS_{between} + SS_{within}$$

$$SS_{total} = 25.34 + 34.62 = 59.96$$

Effect size: One-way ANOVA

Effect size for One-way ANOVA

$$\eta^2 = \frac{SS_{between}}{SS_{Total}}$$

$$SS_{between} = 1132.38 - 1107.04 = 25.34$$

$$SS_{total} = 25.34 - 34.62 = 59.96$$

$$\eta^2 = \frac{25.34}{59.96} = 0.42$$

Effect Size Interpretation for Eta-Squared

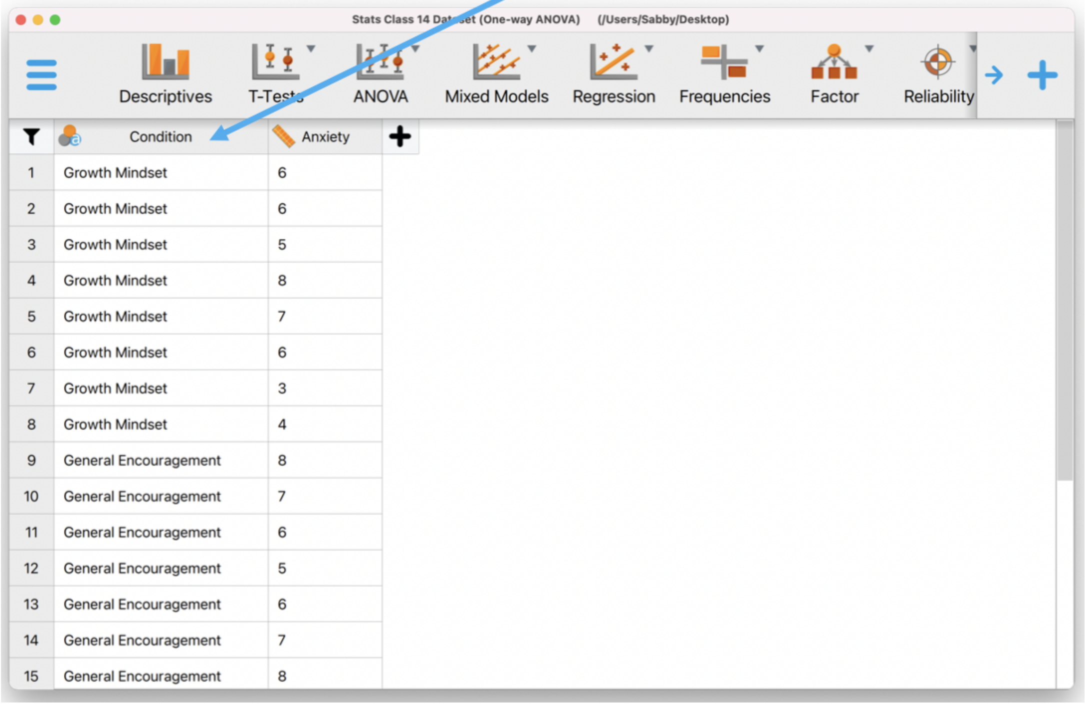
Eta squared: the proportion of the total variability in the dependent variable that is accounted for by variation in the independent variable. It is the ratio of the between groups sum of squares to the total sum of squares

- A “small” effect size is about .01
- A “medium” effect size is about .06
- A “large” effect size is about .14

One-way ANOVA in JASP

JASP: One-way ANOVA

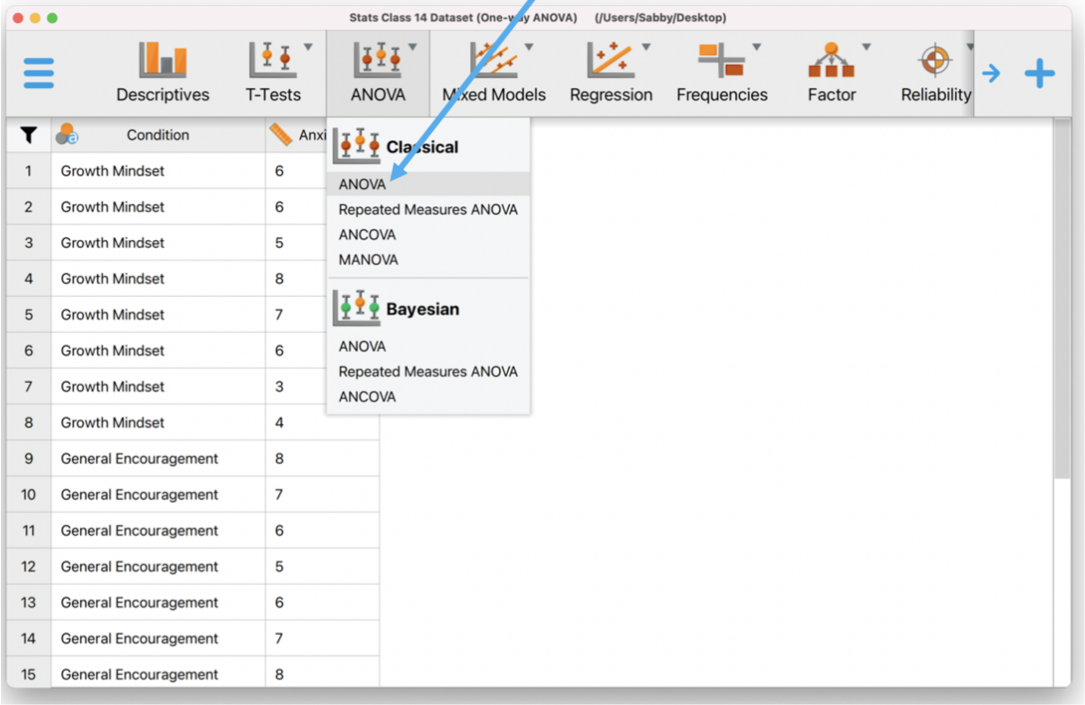
Here we have the same dataset for which we computed the One-Way ANOVA by hand in class. Let's find F using JASP!



1

JASP: One-way ANOVA

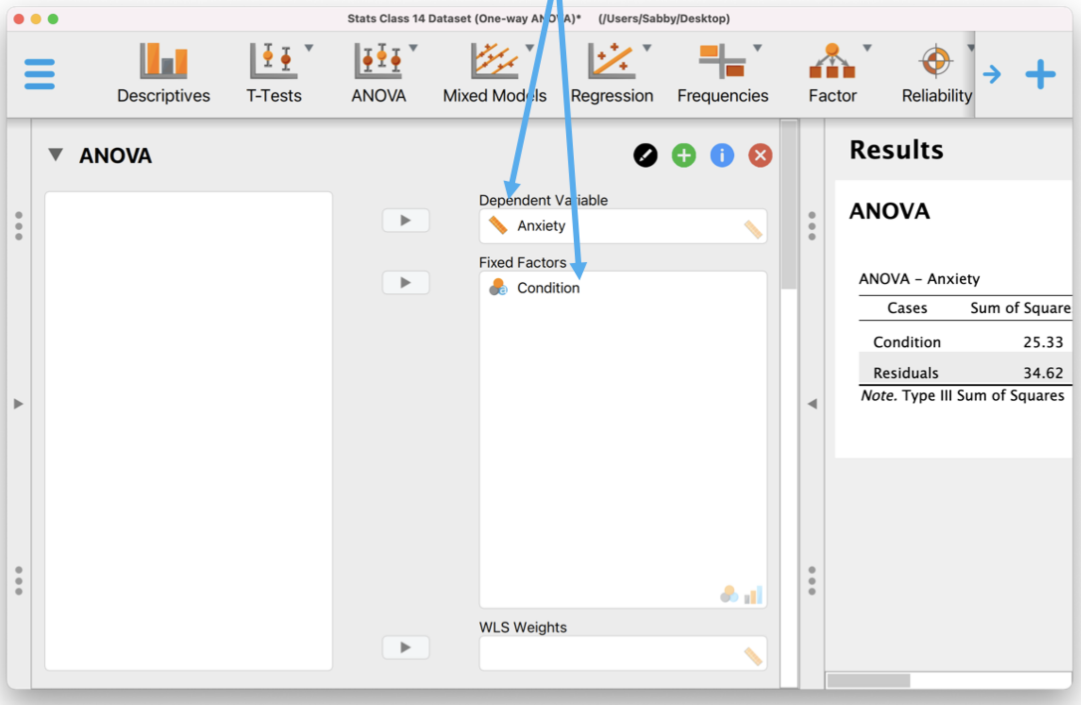
Start by clicking on "ANOVA" and simply selecting "ANOVA," which allows for both One-Way and Factorial ANOVAs.



2

JASP: One-way ANOVA

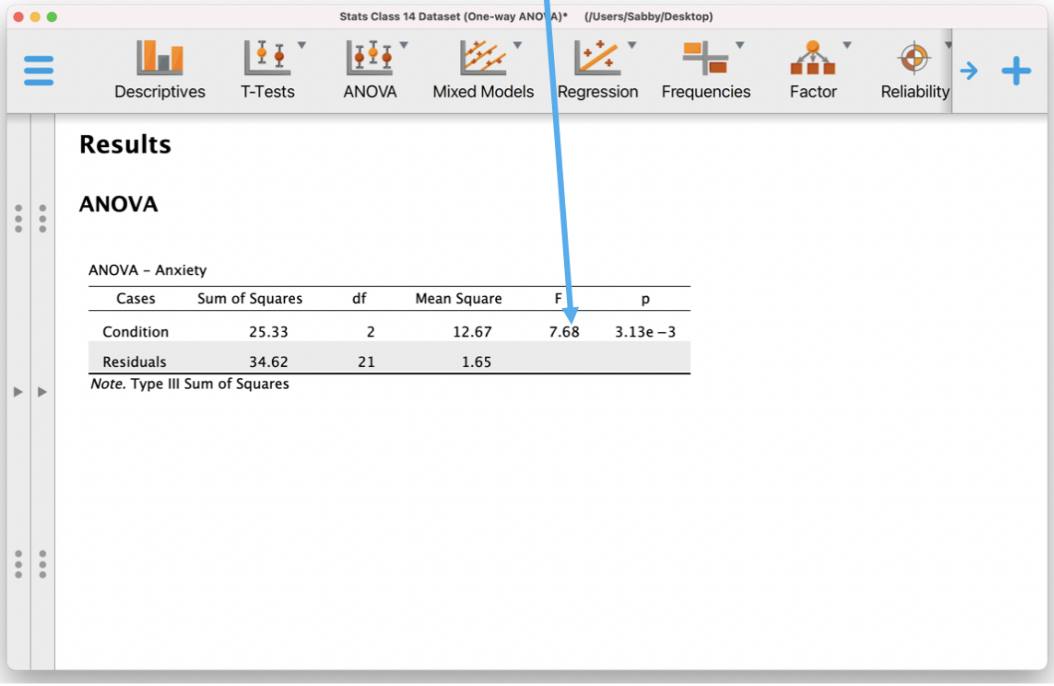
Place Anxiety (our response) in the "Dependent Variable" box and Condition (our factor) in the "Fixed Factors" box.



3

JASP: One-way ANOVA

And that's it! Notice our F-ratio is 7.70, which is what we calculated by hand earlier. Quite a bit easier using software, right? ☺



Results

ANOVA

ANOVA – Anxiety

Cases	Sum of Squares	df	Mean Square	F	p
Condition	25.33	2	12.67	7.68	3.13e −3
Residuals	34.62	21	1.65		

Note. Type III Sum of Squares

4

Next time

Lecture

- Differences between many factors

Reading

- Chapter 13

Quiz 4

- Due tonight 2/16/2022 11:59pm MT
    - Covers Ch.11-12