# Code for “Coexistence theory of mutualism” paper

## Figure 2

In[1165]:=
```
Clear["Global`*"]
```

### Scenario 1: Mutualisms influence plant coexistence even when commodities are not limiting

#### Scenario 1a: plants disproportionately reward the mutualistic partners upon which they rely

In[1166]:=
```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 0.9, e11 → 0.1, e12 → 0.1,
   e13 → 0.1, v11 → 0.8, v12 → 0.2, v13 → 0, τ11 → 0, τ12 → 0, τ13 → 0,
   b2 → 1, c21 → 1.05, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1,
   v21 → 0.36, v22 → 0.63, v23 → 0.01, τ21 → 0, τ22 → 0, τ23 → 0,
   β1 → 1, β2 → 1, bM3 → 1, δ1 → 1, δ2 → 1, δ3 → 1, μ11 → 3.2,
   μ21 → 1, μ12 → 1, μ22 → 5, μ13 → 0, μ23 → 1};

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta1}{\delta1} + \frac{e12\ v12\ \beta2}{\delta2} + \frac{e13\ v13\ bM3}{\delta3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta1}{\delta1} + \frac{e22\ v22\ \beta2}{\delta2} + \frac{e23\ v23\ bM3}{\delta3};$$

$$\alpha12 = \frac{1}{r1}\left(c12 + e11\ v11\ v21\ \tau21 + e12\ v12\ v22\ \tau22 + e13\ v13\ v23\ \tau23 - \left(\frac{e11\ v11\ v21\ \mu21}{\delta1} + \frac{e12\ v12\ v22\ \mu22}{\delta2} + \frac{e13\ v13\ v23\ \mu23}{\delta3}\right)\right);$$

$$\alpha21 = \frac{1}{r2}\left(c21 + e21\ v21\ v11\ \tau11 + e22\ v22\ v12\ \tau12 + e23\ v23\ v13\ \tau13 - \left(\frac{e21\ v21\ v11\ \mu11}{\delta1} + \frac{e22\ v22\ v12\ \mu12}{\delta2} + \frac{e23\ v23\ v13\ \mu13}{\delta3}\right)\right);$$

$$\alpha11 = \frac{1}{r1}\left(c11 + e11\ v11\ v11\ \tau11 + e12\ v12\ v12\ \tau12 + e13\ v13\ v13\ \tau13 - \left(\frac{e11\ v11\ v11\ \mu11}{\delta1} + \frac{e12\ v12\ v12\ \mu12}{\delta2} + \frac{e13\ v13\ v13\ \mu13}{\delta3}\right)\right);$$

$$\alpha 22 = \frac{1}{r2}\left(c22 + e21\, v21\, v21\, \tau 21 + e22\, v22\, v22\, \tau 22 + e23\, v23\, v23\, \tau 23 - \left(\frac{e21\, v21\, v21\, \mu 21}{\delta 1} + \frac{e22\, v22\, v22\, \mu 22}{\delta 2} + \frac{e23\, v23\, v23\, \mu 23}{\delta 3}\right)\right);$$

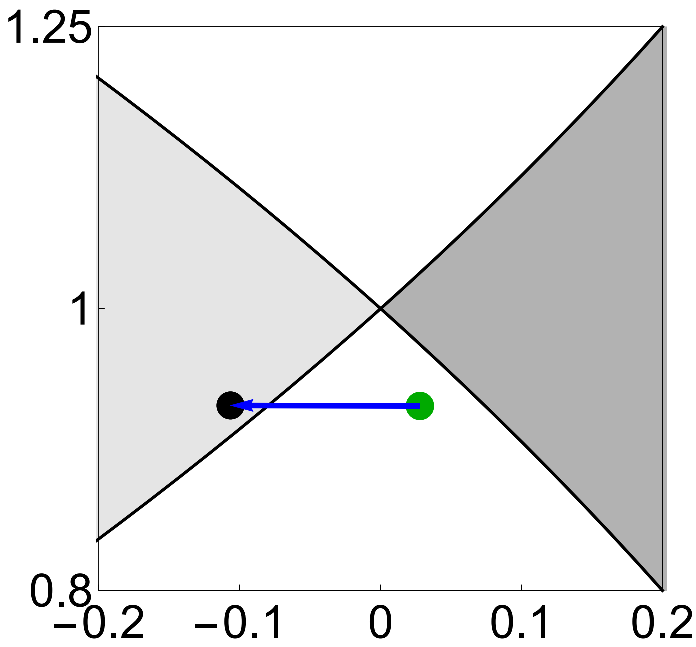
(* Niche and fitness difference *)

$$\rho = \sqrt{\frac{\alpha 12\, \alpha 21}{\alpha 11\, \alpha 22}};$$

$$\varkappa 2 \varkappa 1 = \sqrt{\frac{\alpha 12\, \alpha 11}{\alpha 21\, \alpha 22}};$$

```
(* Plots *)
(* Region plot showing interaction outcomes *)
RP1 =
  Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{-0.2, 0.2}, {0.8, 1.25}},
    Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
   LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
    FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
   AxesOrigin → {-1, Log[0.4]}, Frame → True, AspectRatio → 1,
   FrameTicks → {{{{Log[0.8], 0.8}, {Log[1], 1}, {Log[1.25], 1.25}}, None},
     {{-0.2, -0.1, 0, 0.1, 0.2}, None}}, FrameStyle →
    {{Black, Black}, {Black, Black}}, LabelStyle → Directive[FontSize → 24]];
(* Points and arrows *)
pS1a = Show[
   (* Resource competition *)
   ListPlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param},
    PlotMarkers → {Graphics[{EdgeForm[{Darker[Green], Thick}],
        FaceForm[Darker[Green]], Disk[]}], 0.05}],
   (* Competition and mutualism *)
   ListPlot[{{1 - ρ, Log[ϰ2ϰ1]} /. param}, PlotMarkers →
     {Graphics[{EdgeForm[{Black}], FaceForm[Black], Disk[]}], 0.05}], (* Arrow *)
   ListLinePlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param,
      {1 - ρ, Log[ϰ2ϰ1]} /. param}, PlotMarkers → None,
     PlotStyle → {Blue, Thickness[0.01]}] /. Line → Arrow];
Show[
 RP1,
 pS1a]
```

Out[1177]=




## Scenario 1b: partner 2 acquires greater mutualistic benefits from both plant species than does partner 1

In[1178]:=

```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 0.9, e11 → 0.1, e12 → 0.1,
   e13 → 0.1, v11 → 1.3, v12 → 0.7, v13 → 0, τ11 → 0, τ12 → 0, τ13 → 0,
   b2 → 1, c21 → 1.05, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1,
   v21 → 0.1, v22 → 1.9, v23 → 0.01, τ21 → 0, τ22 → 0, τ23 → 0,
   β1 → 1, β2 → 1, bM3 → 1, δ1 → 1, δ2 → 1, δ3 → 1, μ11 → 0.04,
   μ21 → 0.1, μ12 → 2, μ22 → 0.7, μ13 → 0, μ23 → 1};

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta1}{\delta1} + \frac{e12\ v12\ \beta2}{\delta2} + \frac{e13\ v13\ bM3}{\delta3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta1}{\delta1} + \frac{e22\ v22\ \beta2}{\delta2} + \frac{e23\ v23\ bM3}{\delta3};$$

$$\alpha12 = \frac{1}{r1}\left(c12 + e11\ v11\ v21\ \tau21 + e12\ v12\ v22\ \tau22 + e13\ v13\ v23\ \tau23 - \left(\frac{e11\ v11\ v21\ \mu21}{\delta1} + \frac{e12\ v12\ v22\ \mu22}{\delta2} + \frac{e13\ v13\ v23\ \mu23}{\delta3}\right)\right);$$

$$\alpha21 = \frac{1}{r2}\left(c21 + e21\ v21\ v11\ \tau11 + e22\ v22\ v12\ \tau12 + e23\ v23\ v13\ \tau13 - \left(\frac{e21\ v21\ v11\ \mu11}{\delta1} + \frac{e22\ v22\ v12\ \mu12}{\delta2} + \frac{e23\ v23\ v13\ \mu13}{\delta3}\right)\right);$$

$$\alpha11 = \frac{1}{r1}\left(c11 + e11\ v11\ v11\ \tau11 + e12\ v12\ v12\ \tau12 + e13\ v13\ v13\ \tau13 -\right.$$

$$\left(\frac{\text{e11 v11 v11 }\mu 11}{\delta 1}+\frac{\text{e12 v12 v12 }\mu 12}{\delta 2}+\frac{\text{e13 v13 v13 }\mu 13}{\delta 3}\right)\Bigg);$$

$$\alpha 22=\frac{1}{\text{r2}}\Bigg(\text{c22}+\text{e21 v21 v21 }\tau 21+\text{e22 v22 v22 }\tau 22+\text{e23 v23 v23 }\tau 23-\left(\frac{\text{e21 v21 v21 }\mu 21}{\delta 1}+\frac{\text{e22 v22 v22 }\mu 22}{\delta 2}+\frac{\text{e23 v23 v23 }\mu 23}{\delta 3}\right)\Bigg);$$

```
(* Niche and fitness difference *)
```

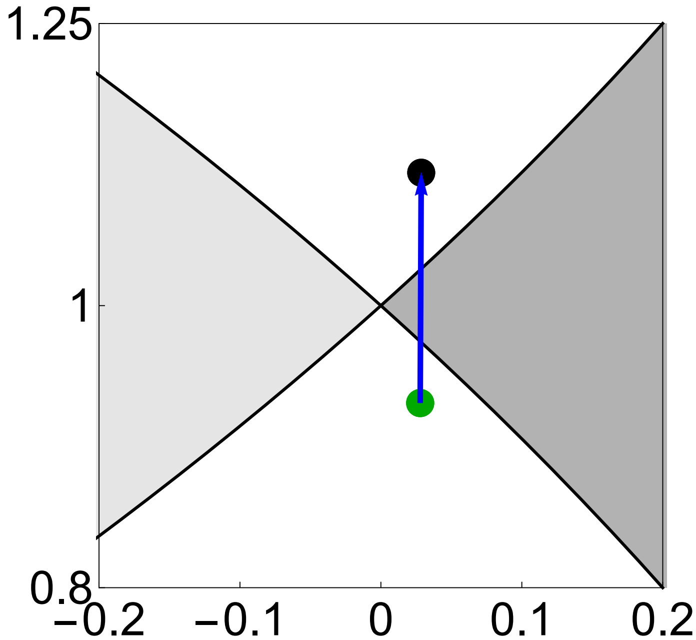
$$\rho=\sqrt{\frac{\alpha 12\ \alpha 21}{\alpha 11\ \alpha 22}};$$

$$\kappa 2\kappa 1=\sqrt{\frac{\alpha 12\ \alpha 11}{\alpha 21\ \alpha 22}};$$

```
(* Plots *)
(* Region plot showing interaction outcomes *)
RP1 =
  Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{-0.2, 0.2}, {0.8, 1.25}},
    Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
   LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
    FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
   AxesOrigin → {-1, Log[0.4]}, Frame → True, AspectRatio → 1,
   FrameTicks → {{{{Log[0.8], 0.8}, {Log[1], 1}, {Log[1.25], 1.25}}, None},
     {{-0.2, -0.1, 0, 0.1, 0.2}, None}}, FrameStyle →
    {{Black, Black}, {Black, Black}}, LabelStyle → Directive[FontSize → 24]];
(* Points and arrows *)
pS1b = Show[
   (* Resource competition *)
   ListPlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param},
    PlotMarkers → {Graphics[{EdgeForm[{Darker[Green], Thick}],
        FaceForm[Darker[Green]], Disk[]}], 0.05}],
   (* Competition and mutualism *)
   ListPlot[{{1 - ρ, Log[κ2κ1]} /. param}, PlotMarkers →
     {Graphics[{EdgeForm[{Black}], FaceForm[Black], Disk[]}], 0.05}], (* Arrow *)
   ListLinePlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param,
      {1 - ρ, Log[κ2κ1]} /. param}, PlotMarkers → None,
     PlotStyle → {Blue, Thickness[0.01]}] /. Line → Arrow];
Show[
```

```
RP1,
pS1b]
```

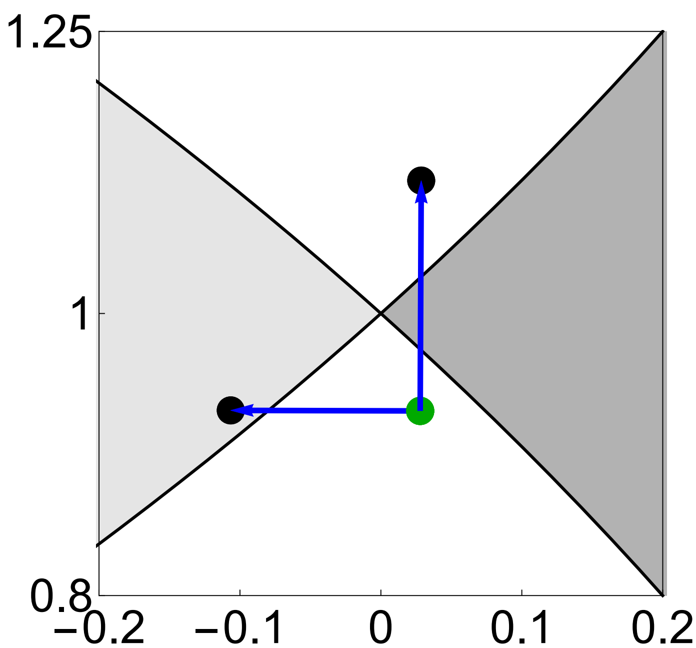
Out[1189]=




## Figure 2a

In[1190]:=
```
Show[RP1, pS1a, pS1b]
```

Out[1190]=




In[1191]:=

## Scenario 2: Mutualisms can promote coexistence even when they do not stabilize plant competition

In[1192]:=

```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 1.1, e11 → 0.5, e12 → 0.5,
   e13 → 0.5, v11 → 11, v12 → 9, v13 → 0, τ11 → 0.2, τ12 → 0.2, τ13 → 0.2,
   b2 → 1, c21 → 0.7, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1,
   v21 → 22.1, v22 → 25.5, v23 → 51, τ21 → 0.2, τ22 → 0.2, τ23 → 0.2,
   β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1, δ3 → 1, μ11 → 0.25,
   μ21 → 0.1, μ12 → 0.1, μ22 → 0.3, μ13 → 0, μ23 → 0.19};

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta 1}{\delta 1} + \frac{e12\ v12\ \beta 2}{\delta 2} + \frac{e13\ v13\ bM3}{\delta 3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta 1}{\delta 1} + \frac{e22\ v22\ \beta 2}{\delta 2} + \frac{e23\ v23\ bM3}{\delta 3};$$

$$\alpha 12 = \frac{1}{r1} \left( c12 + e11\ v11\ v21\ \tau 21 + e12\ v12\ v22\ \tau 22 + e13\ v13\ v23\ \tau 23 - \left( \frac{e11\ v11\ v21\ \mu 21}{\delta 1} + \frac{e12\ v12\ v22\ \mu 22}{\delta 2} + \frac{e13\ v13\ v23\ \mu 23}{\delta 3} \right) \right);$$

$$\alpha 21 = \frac{1}{r2} \left( c21 + e21\ v21\ v11\ \tau 11 + e22\ v22\ v12\ \tau 12 + e23\ v23\ v13\ \tau 13 - \left( \frac{e21\ v21\ v11\ \mu 11}{\delta 1} + \frac{e22\ v22\ v12\ \mu 12}{\delta 2} + \frac{e23\ v23\ v13\ \mu 13}{\delta 3} \right) \right);$$

$$\alpha 11 = \frac{1}{r1} \left( c11 + e11\ v11\ v11\ \tau 11 + e12\ v12\ v12\ \tau 12 + e13\ v13\ v13\ \tau 13 - \left( \frac{e11\ v11\ v11\ \mu 11}{\delta 1} + \frac{e12\ v12\ v12\ \mu 12}{\delta 2} + \frac{e13\ v13\ v13\ \mu 13}{\delta 3} \right) \right);$$

$$\alpha 22 = \frac{1}{r2} \left( c22 + e21\ v21\ v21\ \tau 21 + e22\ v22\ v22\ \tau 22 + e23\ v23\ v23\ \tau 23 - \left( \frac{e21\ v21\ v21\ \mu 21}{\delta 1} + \frac{e22\ v22\ v22\ \mu 22}{\delta 2} + \frac{e23\ v23\ v23\ \mu 23}{\delta 3} \right) \right);$$

```
(* Niche and fitness difference *)
```

$$\rho = \sqrt{\frac{\alpha 12\ \alpha 21}{\alpha 11\ \alpha 22}};$$

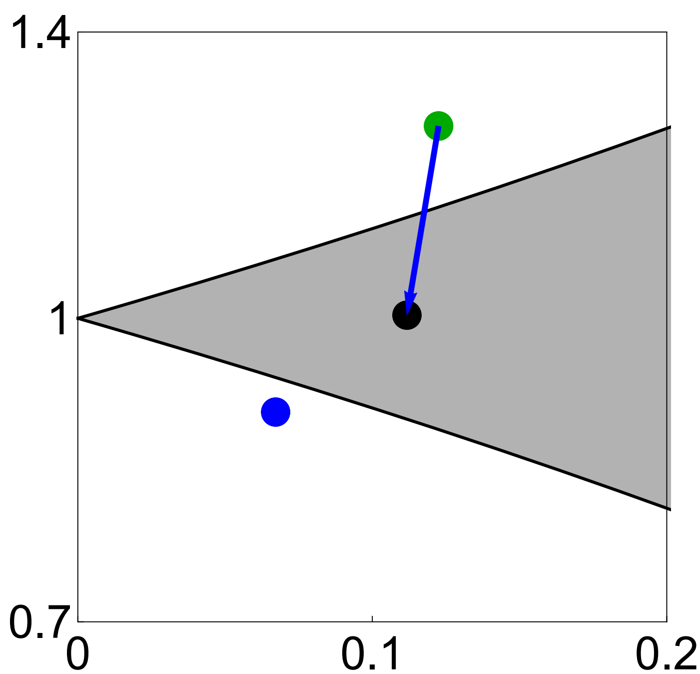
```
1 - ρ /. param;
```

$$\kappa 2\kappa 1 = \sqrt{\frac{\alpha 12\ \alpha 11}{\alpha 21\ \alpha 22}};$$

```
κ2κ1 /. param;

(* Plots *)
```

```
(* Region plot showing interaction outcomes *)
RP2 = LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1},
   PlotRange → {{0, 0.2}, {0.7, 1.4}}, Filling → 1, FillingStyle → GrayLevel[0.7],
   PlotStyle → {{Black}, {Black}}, Frame → True, AspectRatio → 1,
   FrameTicks → {{{0.7, 1, 1.4}, None}, {{0, 0.1, 0.2}, None}}, FrameStyle →
    {{Black, Black}, {Black, Black}}, LabelStyle → Directive[FontSize → 24]];
(* Points and arrows *)
pS2 = Show[
   (* Resource competition *)
   ListPlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param},
    PlotMarkers → {Graphics[{EdgeForm[{Darker[Green], Thick}],
        FaceForm[Darker[Green]], Disk[]}], 0.05}],
   (* Mutualism *)
   ListPlot[
    {{1 - ρ, Log[κ2κ1]} /. {b1 → 0, c11 → 1, c12 → 1, e11 → 0.5, e12 → 0.5, e13 → 0.5,
       v11 → 11, v12 → 9, v13 → 0, τ11 → 0.2, τ12 → 0.2, τ13 → 0, b2 → 0, c21 → 1,
       c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1, v21 → 22.1, v22 → 25.5, v23 → 51,
       τ21 → 0.2, τ22 → 0.2, τ23 → 0.2, β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1,
       δ3 → 1, μ11 → 0.25, μ21 → 0.1, μ12 → 0.1, μ22 → 0.3, μ13 → 0, μ23 → 0.19}},
    PlotMarkers → {Graphics[{EdgeForm[{Blue}], FaceForm[Blue], Disk[]}], 0.05}],
   (* Competition and mutualism *)
   ListPlot[{{1 - ρ, Log[κ2κ1]} /. param},
    PlotMarkers → {Graphics[{EdgeForm[{Black}], FaceForm[Black], Disk[]}], 0.05}],
   (* Arrows *)
   ListLinePlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param,
      {1 - ρ, Log[κ2κ1]} /. param}, PlotMarkers → None,
     PlotStyle → {Blue, Thickness[0.01]}] /. Line → Arrow];
Show[
 RP2,
 pS2]
```

Out[1205]=




In[1206]:=

## Scenario 3: Mutualisms drive competitive exclusion when plants would otherwise coexist

In[1207]:=

```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 0.6, e11 → 0.1, e12 → 0.1, e13 → 0.1,
   v11 → 32, v12 → 68, v13 → 0, τ11 → 0.12, τ12 → 0.12, τ13 → 0.12,
   b2 → 1, c21 → 0.9, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1,
   v21 → 3, v22 → 17, v23 → 9.5, τ21 → 0.2, τ22 → 0.2, τ23 → 0.2,
   β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1, δ3 → 1, μ11 → 0.2,
   μ21 → 0.1, μ12 → 0.1, μ22 → 0.2, μ13 → 0, μ23 → 0.1};

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta 1}{\delta 1} + \frac{e12\ v12\ \beta 2}{\delta 2} + \frac{e13\ v13\ bM3}{\delta 3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta 1}{\delta 1} + \frac{e22\ v22\ \beta 2}{\delta 2} + \frac{e23\ v23\ bM3}{\delta 3};$$

$$\alpha 12 = \frac{1}{r1} \left( c12 + e11\ v11\ v21\ \tau 21 + e12\ v12\ v22\ \tau 22 + e13\ v13\ v23\ \tau 23 - \left( \frac{e11\ v11\ v21\ \mu 21}{\delta 1} + \frac{e12\ v12\ v22\ \mu 22}{\delta 2} + \frac{e13\ v13\ v23\ \mu 23}{\delta 3} \right) \right);$$

$$\alpha 21 = \frac{1}{r2} \left( c21 + e21\ v21\ v11\ \tau 11 + e22\ v22\ v12\ \tau 12 + e23\ v23\ v13\ \tau 13 - \right.$$

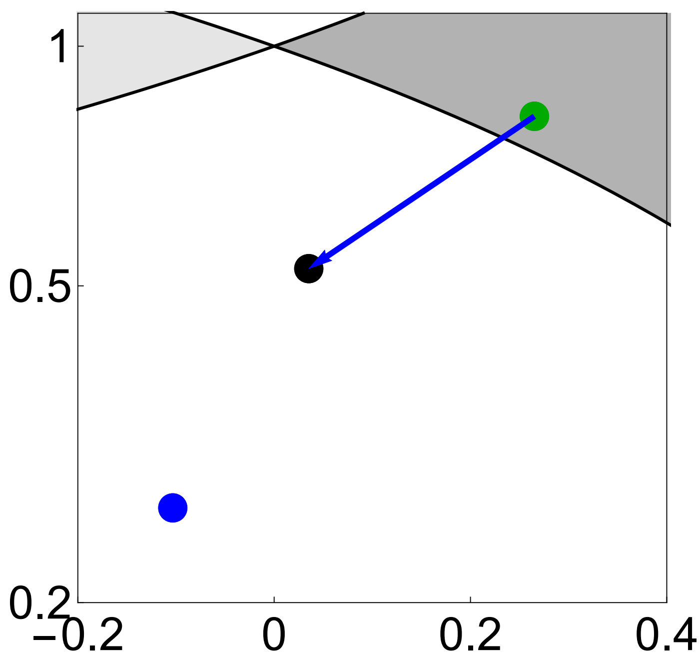
```
    ((e21 v21 v11 μ11)/δ1 + (e22 v22 v12 μ12)/δ2 + (e23 v23 v13 μ13)/δ3));
α11 = 1/r1 ( c11 + e11 v11 v11 τ11 + e12 v12 v12 τ12 + e13 v13 v13 τ13 -
    ((e11 v11 v11 μ11)/δ1 + (e12 v12 v12 μ12)/δ2 + (e13 v13 v13 μ13)/δ3));
α22 = 1/r2 (c22 + e21 v21 v21 τ21 + e22 v22 v22 τ22 + e23 v23 v23 τ23 -
    ((e21 v21 v21 μ21)/δ1 + (e22 v22 v22 μ22)/δ2 + (e23 v23 v23 μ23)/δ3));

(* Niche and fitness difference *)
ρ = Sqrt[(α12 α21)/(α11 α22)];
1 - ρ /. param
κ2κ1 = Sqrt[(α12 α11)/(α21 α22)];
κ2κ1 /. param

(* Plots *)
(* Region plot showing interaction outcomes *)
RP3 = Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{-0.2, 0.4}, {0.2, 1.1}},
     Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
    LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
     FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
    AxesOrigin → {-1, Log[0.2]}, Frame → True, AspectRatio → 1,
    FrameTicks → {{{{Log[0.2], 0.2}, {Log[0.5], 0.5}, {Log[1], 1}}, None},
      {{-0.2, 0, 0.2, 0.4}, None}}, FrameStyle →
     {{Black, Black}, {Black, Black}}, LabelStyle → Directive[FontSize → 24]];
(* Points and arrows *)
pS3 = Show[
    (* Resource competition *)
    ListPlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param},
     PlotMarkers → {Graphics[{EdgeForm[{Darker[Green], Thick}],
         FaceForm[Darker[Green]], Disk[]}], 0.05}],
    (* Mutualism *)
    ListPlot[
     {{1 - ρ, Log[κ2κ1]} /. {b1 → 0, c11 → 1, c12 → 1, e11 → 0.1, e12 → 0.1, e13 → 0.1,
        v11 → 32, v12 → 68, v13 → 0, τ11 → 0.12, τ12 → 0.12, τ13 → 0.12, b2 → 0, c21 → 1,
        c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1, v21 → 3, v22 → 17, v23 → 9.5,
```

```
      τ21 → 0.2, τ22 → 0.2, τ23 → 0.2, β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1,
      δ3 → 1, μ11 → 0.2, μ21 → 0.1, μ12 → 0.1, μ22 → 0.2, μ13 → 0, μ23 → 0.1}},
   PlotMarkers → {Graphics[{EdgeForm[{Blue}], FaceForm[Blue], Disk[]}], 0.05}],
  (* Competition and mutualism *)
  ListPlot[{{1 - ρ, Log[κ2κ1]} /. param},
   PlotMarkers → {Graphics[{EdgeForm[{Black}], FaceForm[Black], Disk[]}], 0.05}],
  (* Arrows *)
  ListLinePlot[{{1 - Sqrt[(c12 c21)/(c11 c22)], Log[Sqrt[(c12 c11)/(c21 c22)]]} /. param,
     {1 - ρ, Log[κ2κ1]} /. param}, PlotMarkers → None,
    PlotStyle → {Blue, Thickness[0.01]}] /. Line → Arrow];
Show[
 RP3,
 pS3]
```

Out[1215]= 0.035278

Out[1217]= 0.525489

Out[1220]=




In[1221]:=

## Scenario 4: Systematic changes in the parameters underlying the niche and fitness differences

### Scenario 4a (Figure 2d): Changes in v

In[1222]:=
```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 0.4, e11 → 0.1,
   e12 → 0.1, e13 → 0.1, v13 → 0, τ11 → 0.2, τ12 → 0.2, τ13 → 0.2,
   b2 → 1, c21 → 0.4, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1,
   v23 → 0, τ21 → 0.2, τ22 → 0.2, τ23 → 0.2,
   β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1, δ3 → 1, μ11 → 0.18,
   μ21 → 0.1, μ12 → 0.1, μ22 → 0.18, μ13 → 0, μ23 → 0};
steps = 10;

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta1}{\delta1} + \frac{e12\ v12\ \beta2}{\delta2} + \frac{e13\ v13\ bM3}{\delta3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta1}{\delta1} + \frac{e22\ v22\ \beta2}{\delta2} + \frac{e23\ v23\ bM3}{\delta3};$$

$$\alpha12 = \frac{1}{r1}\left(c12 + e11\ v11\ v21\ \tau21 + e12\ v12\ v22\ \tau22 + e13\ v13\ v23\ \tau23 - \left(\frac{e11\ v11\ v21\ \mu21}{\delta1} + \frac{e12\ v12\ v22\ \mu22}{\delta2} + \frac{e13\ v13\ v23\ \mu23}{\delta3}\right)\right);$$

$$\alpha21 = \frac{1}{r2}\left(c21 + e21\ v21\ v11\ \tau11 + e22\ v22\ v12\ \tau12 + e23\ v23\ v13\ \tau13 - \left(\frac{e21\ v21\ v11\ \mu11}{\delta1} + \frac{e22\ v22\ v12\ \mu12}{\delta2} + \frac{e23\ v23\ v13\ \mu13}{\delta3}\right)\right);$$

$$\alpha11 = \frac{1}{r1}\left(c11 + e11\ v11\ v11\ \tau11 + e12\ v12\ v12\ \tau12 + e13\ v13\ v13\ \tau13 - \left(\frac{e11\ v11\ v11\ \mu11}{\delta1} + \frac{e12\ v12\ v12\ \mu12}{\delta2} + \frac{e13\ v13\ v13\ \mu13}{\delta3}\right)\right);$$

$$\alpha22 = \frac{1}{r2}\left(c22 + e21\ v21\ v21\ \tau21 + e22\ v22\ v22\ \tau22 + e23\ v23\ v23\ \tau23 - \left(\frac{e21\ v21\ v21\ \mu21}{\delta1} + \frac{e22\ v22\ v22\ \mu22}{\delta2} + \frac{e23\ v23\ v23\ \mu23}{\delta3}\right)\right);$$

```
(* Niche and fitness difference *)
```

$$\rho = \sqrt{\frac{\alpha12\ \alpha21}{\alpha11\ \alpha22}};$$

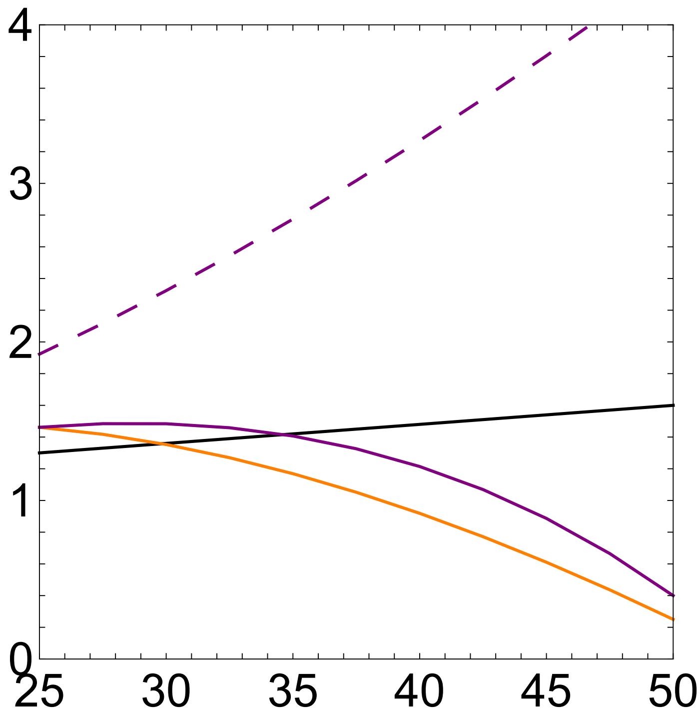
$$\kappa2\kappa1 = \sqrt{\frac{\alpha12\ \alpha11}{\alpha21\ \alpha22}};$$

```
(* Define matrices with niche and fitness
 differences as model parameter is varied *)
v1Matrix = v2Matrix = {{1 - ρ /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25,
     Log[κ2κ1] /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25}};

(* Define matrices with parameter being systematically
```

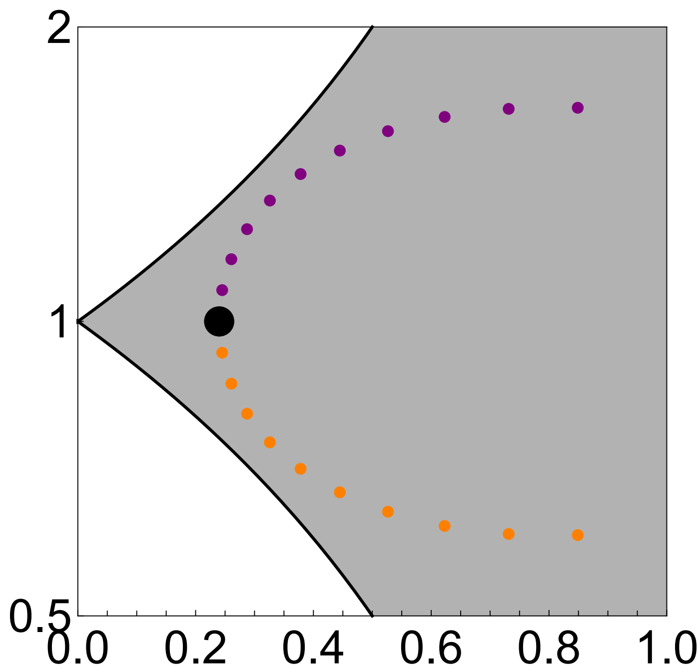
```
 varied and resulting r and α terms for Appendix S5 *)
v1r = {{v11 /. v11 → 25, r1 /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25}};
v1α12 = {{v11 /. v11 → 25, α12 /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25}};
v1α11 = {{v11 /. v11 → 25, α11 /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25}};
v1α21 = {{v11 /. v11 → 25, α21 /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25}};

(* Systematically vary model parameter by stepsize 0.1 t and update matrices
 of niche and fitness differences as well as matrices of r and α terms *)
For[t = 1, t ≤ steps, t++,
 (* Increase v1 *)
 v1Matrix = AppendTo[v1Matrix,
   {1 - ρ /. param /. v11 → 25 (1 + 0.1 t) /. v12 → 5 (1 + 0.1 t) /. v21 → 5 (1 - 0.1 t) /.
     v22 → 25 (1 - 0.1 t), Log[κ2κ1] /. param /. v11 → 25 (1 + 0.1 t) /. v12 → 5 (1 + 0.1 t) /.
      v21 → 5 (1 - 0.1 t) /. v22 → 25 (1 - 0.1 t)}];
 v1r = AppendTo[v1r, {v11 /. v11 → 25 (1 + 0.1 t), r1 /. param /. v11 → 25 (1 + 0.1 t) /.
       v12 → 5 (1 + 0.1 t) /. v21 → 5 (1 - 0.1 t) /. v22 → 25 (1 - 0.1 t)}];
 v1α12 = AppendTo[v1α12, {v11 /. v11 → 25 (1 + 0.1 t), α12 /. param /. v11 → 25
          (1 + 0.1 t) /. v12 → 5 (1 + 0.1 t) /. v21 → 5 (1 - 0.1 t) /. v22 → 25 (1 - 0.1 t)}];
 v1α11 = AppendTo[v1α11, {v11 /. v11 → 25 (1 + 0.1 t),
    α11 /. param /. v11 → 25 (1 + 0.1 t) /. v12 → 5 (1 + 0.1 t) /. v21 → 5 (1 - 0.1 t) /.
     v22 → 25 (1 - 0.1 t)}];
 v1α21 = AppendTo[v1α21, {v11 /. v11 → 25 (1 + 0.1 t),
    α21 /. param /. v11 → 25 (1 + 0.1 t) /. v12 → 5 (1 + 0.1 t) /. v21 → 5 (1 - 0.1 t) /.
     v22 → 25 (1 - 0.1 t)}];
 (* Increase v2 *)
 v2Matrix = AppendTo[v2Matrix,
   {1 - ρ /. param /. v11 → 25 (1 - 0.1 t) /. v12 → 5 (1 - 0.1 t) /. v21 → 5 (1 + 0.1 t) /.
     v22 → 25 (1 + 0.1 t), Log[κ2κ1] /. param /. v11 → 25 (1 - 0.1 t) /. v12 → 5 (1 - 0.1 t) /.
      v21 → 5 (1 + 0.1 t) /. v22 → 25 (1 + 0.1 t)}]]

(* See arrays if needed *)
v1Matrix // MatrixForm;

(* Plot r and alphas for Appendix S5: Figure 1a *)
ListLinePlot[{v1r, v1α12, v1α11, v1α21},
 PlotStyle → {Black, Orange, {Purple, Dashing[Large]}, Purple}, Frame → True,
 FrameStyle → {{Black, Black}, {Black, Black}}, PlotRange → {{25, 50}, {0, 4}},
 AxesOrigin → {25, 0}, AspectRatio → 1, LabelStyle → Directive[FontSize → 24]]

(* Plots *)
(* Region plot showing interaction outcomes *)
RP4a = Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{0, 1}, {0.5, 2}},
   Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
```

```
   LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
    FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
   AxesOrigin → {-1, Log[0.4]}, Frame → True, AspectRatio → 1, FrameTicks →
    {{{{Log[0.5], 0.5}, {Log[1], 1}, {Log[2], 2}}, None}, {Automatic, None}},
   FrameStyle → {{Black, Black}, {Black, Black}},
   LabelStyle → Directive[FontSize → 24]];
(* Competition and mutualism *)
pS4a =
  ListPlot[{{1 - ρ, Log[κ2κ1]} /. param /. v11 → 25 /. v12 → 5 /. v21 → 5 /. v22 → 25},
   PlotMarkers →
    {Graphics[{EdgeForm[{Black, Thick}], FaceForm[Black], Disk[]}], 0.051}];
(* Increase in v_1k *)
S4v1 = ListPlot[{v1Matrix}, PlotMarkers →
    {Graphics[{EdgeForm[{Purple, Thick}], FaceForm[Purple], Disk[]}], 0.02}];
(* Increase in v_2k *)
S4v2 = ListPlot[{v2Matrix}, PlotMarkers →
   {Graphics[{EdgeForm[{Orange, Thick}], FaceForm[Orange], Disk[]}], 0.02}];
(* Combine plots: orange and purple dots overlain with
   arrows in Adobe Illustrator to generate panel 2d *)
p4a = Show[RP4a, S4v1, S4v2, pS4a]
```

Out[1239]=



Out[1243]=




## Scenario 4b (Figure 2e): Changes in e

In[1245]:=
```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 0.4, v11 → 25, v12 → 5, v13 → 0, τ11 → 0.2,
   τ12 → 0.2, τ13 → 0.2, b2 → 1, c21 → 0.4, c22 → 1, v21 → 5, v22 → 25, v23 → 0,
   τ21 → 0.2, τ22 → 0.2, τ23 → 0.2, β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1,
   δ3 → 1, μ11 → 0.18, μ21 → 0.1, μ12 → 0.1, μ22 → 0.18, μ13 → 0, μ23 → 0};
steps = 10;

(* Intrinsic per capita growth rates and interaction coefficients *)
r1 = b1 + e11 v11 β1/δ1 + e12 v12 β2/δ2 + e13 v13 bM3/δ3;
r2 = b2 + e21 v21 β1/δ1 + e22 v22 β2/δ2 + e23 v23 bM3/δ3;
α12 = 1/r1 (c12 + e11 v11 v21 τ21 + e12 v12 v22 τ22 +
     e13 v13 v23 τ23 - (e11 v11 v21 μ21/δ1 + e12 v12 v22 μ22/δ2 + e13 v13 v23 μ23/δ3));
α21 = 1/r2 (c21 + e21 v21 v11 τ11 + e22 v22 v12 τ12 + e23 v23 v13 τ13 -
     (e21 v21 v11 μ11/δ1 + e22 v22 v12 μ12/δ2 + e23 v23 v13 μ13/δ3));
α11 = 1/r1 ( c11 + e11 v11 v11 τ11 + e12 v12 v12 τ12 + e13 v13 v13 τ13 -
     (e11 v11 v11 μ11/δ1 + e12 v12 v12 μ12/δ2 + e13 v13 v13 μ13/δ3));
```

$$\alpha 22 = \frac{1}{r2}\left(c22 + e21\, v21\, v21\, \tau 21 + e22\, v22\, v22\, \tau 22 + e23\, v23\, v23\, \tau 23 - \left(\frac{e21\, v21\, v21\, \mu 21}{\delta 1} + \frac{e22\, v22\, v22\, \mu 22}{\delta 2} + \frac{e23\, v23\, v23\, \mu 23}{\delta 3}\right)\right);$$

```
(* Niche and fitness difference *)
```

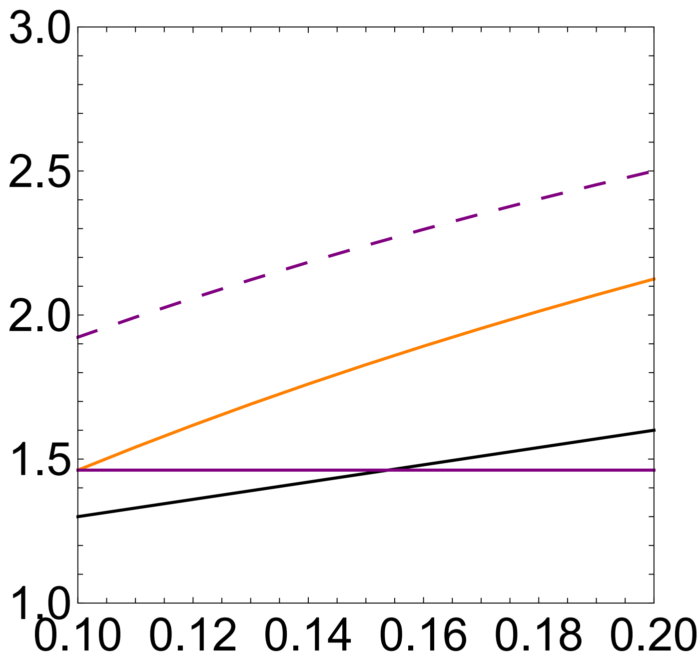
$$\rho = \sqrt{\frac{\alpha 12\, \alpha 21}{\alpha 11\, \alpha 22}};$$

$$\kappa 2\kappa 1 = \sqrt{\frac{\alpha 12\, \alpha 11}{\alpha 21\, \alpha 22}};$$

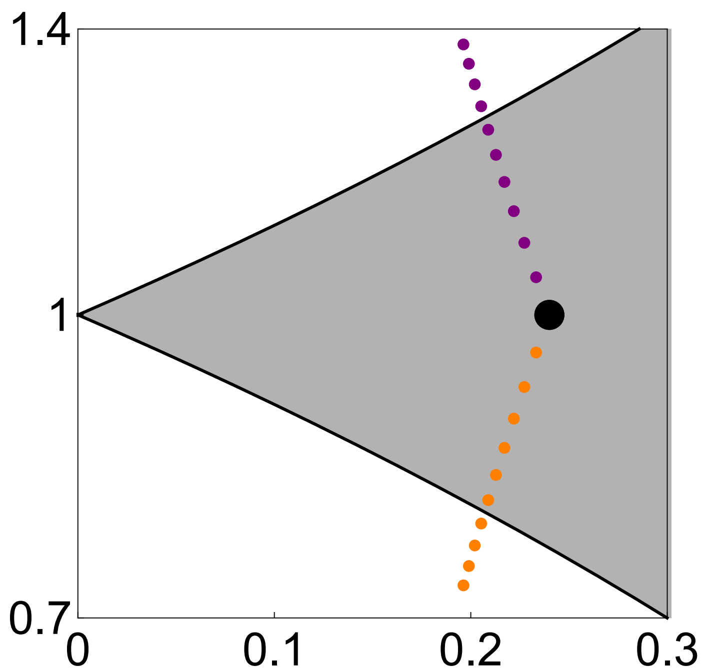
```
(* Define matrices with niche and fitness
 differences as model parameter is varied *)
e1Matrix = e2Matrix = {{1 - ρ /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /. e22 → 0.1,
     Log[κ2κ1] /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /. e22 → 0.1}};

(* Define matrices with parameter being systematically
 varied and resulting r and α terms for Appendix S5 *)
e1r = {{e11 /. e11 → 0.1, r1 /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /. e22 → 0.1}};
e1α12 =
  {{e11 /. e11 → 0.1, α12 /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /. e22 → 0.1}};
e1α11 = {{e11 /. e11 → 0.1,
    α11 /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /. e22 → 0.1}};
e1α21 = {{e11 /. e11 → 0.1, α21 /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /.
     e22 → 0.1}};

(* Systematically vary model parameter by stepsize 0.1 t and update matrices
 of niche and fitness differences as well as matrices of r and α terms *)
For[t = 1, t ≤ steps, t++,
 (* Increase e1k *)
 e1Matrix = AppendTo[e1Matrix,
   {1 - ρ /. param /. e11 → 0.1 (1 + 0.1 t) /. e21 → 0.1 /. e12 → 0.1 (1 + 0.1 t) /. e22 → 0.1,
    Log[κ2κ1] /. param /. e11 → 0.1 (1 + 0.1 t) /. e21 → 0.1 /.
      e12 → 0.1 (1 + 0.1 t) /. e22 → 0.1}];
 e1r = AppendTo[e1r, {e11 /. e11 → 0.1 (1 + 0.1 t),
    r1 /. param /. e11 → 0.1 (1 + 0.1 t) /. e21 → 0.1 /. e12 → 0.1 (1 + 0.1 t) /. e22 → 0.1}];
 e1α12 = AppendTo[e1α12, {e11 /. e11 → 0.1 (1 + 0.1 t), α12 /. param /.
       e11 → 0.1 (1 + 0.1 t) /. e21 → 0.1 /. e12 → 0.1 (1 + 0.1 t) /. e22 → 0.1}];
 e1α11 = AppendTo[e1α11, {e11 /. e11 → 0.1 (1 + 0.1 t),
    α11 /. param /. e11 → 0.1 (1 + 0.1 t) /. e21 → 0.1 /.
      e12 → 0.1 (1 + 0.1 t) /. e22 → 0.1}];
```

```
 e1α21 = AppendTo[e1α21, {e11 /. e11 → 0.1 (1 + 0.1 t),
    α21 /. param /. e11 → 0.1 (1 + 0.1 t) /. e21 → 0.1 /.
      e12 → 0.1 (1 + 0.1 t) /. e22 → 0.1}];
 (* Increase e2k *)
 e2Matrix = AppendTo[e2Matrix,
   {1 - ρ /. param /. e11 → 0.1 /. e21 → 0.1 (1 + 0.1 t) /. e12 → 0.1 /. e22 → 0.1 (1 + 0.1 t),
    Log[κ2κ1] /. param /. e11 → 0.1 /. e21 → 0.1 (1 + 0.1 t) /. e12 → 0.1 /.
     e22 → 0.1 (1 + 0.1 t)}]]

(* See arrays if needed *)
e1Matrix // MatrixForm;

(* Plot r and alphas for Appendix S5: Figure 1b *)
ListLinePlot[{e1r, e1α12, e1α11, e1α21},
 PlotStyle → {Black, Orange, {Purple, Dashing[Large]}, Purple}, Frame → True,
 FrameStyle → {{Black, Black}, {Black, Black}}, PlotRange → {{0.1, 0.2}, {1, 3}},
 AxesOrigin → {0.1, 1}, AspectRatio → 1, LabelStyle → Directive[FontSize → 24]]

(* Plots *)
(* Region plot showing interaction outcomes *)
RP4b = Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{0, 0.3}, {0.7, 1.4}},
     Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
    LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
     FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
    AxesOrigin → {-1, Log[0.4]}, Frame → True, AspectRatio → 1,
    FrameTicks → {{{{Log[0.7], 0.7}, {Log[1], 1}, {Log[1.4], 1.4}}, None},
      {{0, 0.1, 0.2, 0.3}, None}}, FrameStyle → {{Black, Black}, {Black, Black}},
    LabelStyle → Directive[FontSize → 24]];
(* Competition and mutualism *)
pS4b = ListPlot[
    {{1 - ρ, Log[κ2κ1]} /. param /. e11 → 0.1 /. e21 → 0.1 /. e12 → 0.1 /. e22 → 0.1},
    PlotMarkers →
     {Graphics[{EdgeForm[{Black, Thick}], FaceForm[Black], Disk[]}], 0.051}];
(* Increase in e_1k *)
S4e1 = ListPlot[{e1Matrix}, PlotMarkers →
     {Graphics[{EdgeForm[{Purple, Thick}], FaceForm[Purple], Disk[]}], 0.02}];
(* Increase in e_2k *)
S4e2 = ListPlot[{e2Matrix}, PlotMarkers →
    {Graphics[{EdgeForm[{Orange, Thick}], FaceForm[Orange], Disk[]}], 0.02}];
(* Combine plots: orange and purple dots overlain with
   arrows in Adobe Illustrator to generate panel 2d *)
p4b = Show[RP4b, S4e1, S4e2, pS4b]
```

Out[1262]=




Out[1266]=




## Scenario 4c (Figure 2f): Changes in μ

In[1268]:=

```
(* Parameters *)
param = {b1 → 1, c11 → 1, c12 → 0.4, e11 → 0.1, e12 → 0.1,
   e13 → 0.1, v11 → 25, v12 → 5, v13 → 0, τ11 → 0.2, τ12 → 0.2, τ13 → 0.2,
   b2 → 1, c21 → 0.4, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1,
   v21 → 5, v22 → 25, v23 → 0, τ21 → 0.2, τ22 → 0.2, τ23 → 0.2,
   β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1, δ3 → 1, μ13 → 0, μ23 → 0};
```

```
steps = 11;

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta 1}{\delta 1} + \frac{e12\ v12\ \beta 2}{\delta 2} + \frac{e13\ v13\ bM3}{\delta 3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta 1}{\delta 1} + \frac{e22\ v22\ \beta 2}{\delta 2} + \frac{e23\ v23\ bM3}{\delta 3};$$

$$\alpha 12 = \frac{1}{r1}\left(c12 + e11\ v11\ v21\ \tau 21 + e12\ v12\ v22\ \tau 22 + e13\ v13\ v23\ \tau 23 - \left(\frac{e11\ v11\ v21\ \mu 21}{\delta 1} + \frac{e12\ v12\ v22\ \mu 22}{\delta 2} + \frac{e13\ v13\ v23\ \mu 23}{\delta 3}\right)\right);$$

$$\alpha 21 = \frac{1}{r2}\left(c21 + e21\ v21\ v11\ \tau 11 + e22\ v22\ v12\ \tau 12 + e23\ v23\ v13\ \tau 13 - \left(\frac{e21\ v21\ v11\ \mu 11}{\delta 1} + \frac{e22\ v22\ v12\ \mu 12}{\delta 2} + \frac{e23\ v23\ v13\ \mu 13}{\delta 3}\right)\right);$$

$$\alpha 11 = \frac{1}{r1}\left(c11 + e11\ v11\ v11\ \tau 11 + e12\ v12\ v12\ \tau 12 + e13\ v13\ v13\ \tau 13 - \left(\frac{e11\ v11\ v11\ \mu 11}{\delta 1} + \frac{e12\ v12\ v12\ \mu 12}{\delta 2} + \frac{e13\ v13\ v13\ \mu 13}{\delta 3}\right)\right);$$

$$\alpha 22 = \frac{1}{r2}\left(c22 + e21\ v21\ v21\ \tau 21 + e22\ v22\ v22\ \tau 22 + e23\ v23\ v23\ \tau 23 - \left(\frac{e21\ v21\ v21\ \mu 21}{\delta 1} + \frac{e22\ v22\ v22\ \mu 22}{\delta 2} + \frac{e23\ v23\ v23\ \mu 23}{\delta 3}\right)\right);$$

```
(* Niche and fitness difference *)
```

$$\rho = \sqrt{\frac{\alpha 12\ \alpha 21}{\alpha 11\ \alpha 22}};$$

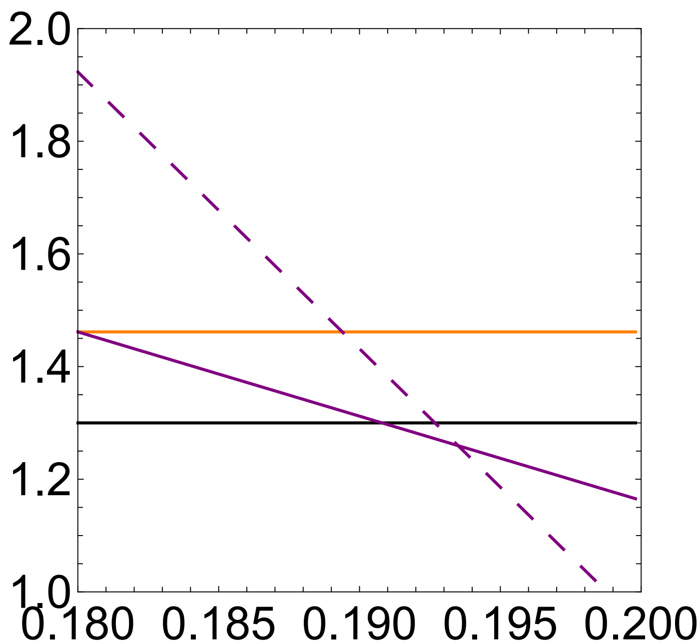
$$\kappa 2\kappa 1 = \sqrt{\frac{\alpha 12\ \alpha 11}{\alpha 21\ \alpha 22}};$$

```
(* Define matrices with niche and fitness
 differences as model parameter is varied *)
μ1Matrix = μ2Matrix =
  {{1 - ρ /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /. μ22 → 0.18,
    Log[κ2κ1] /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /. μ22 → 0.18}};

(* Define matrices with parameter being systematically
 varied and resulting r and α terms for Appendix S5 *)
μ1r = {{μ11 /. μ11 → 0.18,
    r1 /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /. μ22 → 0.18}};
μ1α12 = {{μ11 /. μ11 → 0.18, α12 /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /.
    μ22 → 0.18}};
μ1α11 = {{μ11 /. μ11 → 0.18, α11 /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /.
```

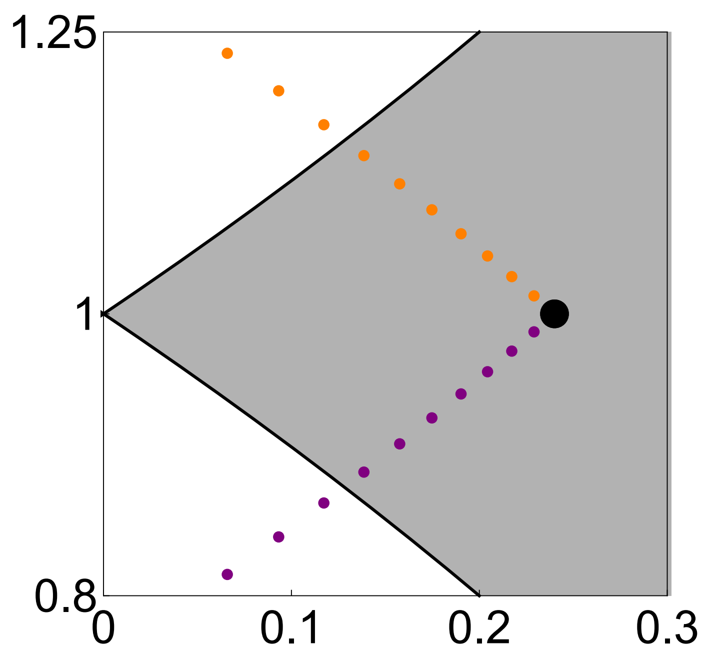
```
      μ22 → 0.18}};
μ1α21 = {{μ11 /. μ11 → 0.18, α21 /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /.
      μ22 → 0.18}};

(* Systematically vary model parameter by stepsize 0.1 t and update matrices
 of niche and fitness differences as well as matrices of r and α terms *)
For[t = 1, t ≤ steps, t++,
 (* Increase μ1k *)
 μ1Matrix = AppendTo[μ1Matrix,
   {1 - ρ /. param /. μ11 → 0.18 (1 + 0.01 t) /. μ21 → 0.1 /. μ12 → 0.1 (1 + 0.01 t) /.
     μ22 → 0.18, Log[κ2κ1] /. param /. μ11 → 0.18 (1 + 0.01 t) /. μ21 → 0.1 /.
      μ12 → 0.1 (1 + 0.01 t) /. μ22 → 0.18}];
 μ1r = AppendTo[μ1r, {μ11 /. μ11 → 0.18 (1 + 0.01 t),
    r1 /. param /. μ11 → 0.18 (1 + 0.01 t) /. μ21 → 0.1 /.
      μ12 → 0.1 (1 + 0.01 t) /. μ22 → 0.18}];
 μ1α12 = AppendTo[μ1α12, {μ11 /. μ11 → 0.18 (1 + 0.01 t), α12 /. param /.
        μ11 → 0.18 (1 + 0.01 t) /. μ21 → 0.1 /. μ12 → 0.1 (1 + 0.01 t) /. μ22 → 0.18}];
 μ1α11 = AppendTo[μ1α11, {μ11 /. μ11 → 0.18 (1 + 0.01 t),
    α11 /. param /. μ11 → 0.18 (1 + 0.01 t) /. μ21 → 0.1 /.
      μ12 → 0.1 (1 + 0.01 t) /. μ22 → 0.18}];
 μ1α21 = AppendTo[μ1α21, {μ11 /. μ11 → 0.18 (1 + 0.01 t),
    α21 /. param /. μ11 → 0.18 (1 + 0.01 t) /. μ21 → 0.1 /.
      μ12 → 0.1 (1 + 0.01 t) /. μ22 → 0.18}];
 (* Increase μ2k *)
 μ2Matrix =
  AppendTo[μ2Matrix, {1 - ρ /. param /. μ11 → 0.18 /. μ21 → 0.1 (1 + 0.01 t) /. μ12 → 0.1 /.
     μ22 → 0.18 (1 + 0.01 t), Log[κ2κ1] /. param /. μ11 → 0.18 /. μ21 → 0.1 (1 + 0.01 t) /.
      μ12 → 0.1 /. μ22 → 0.18 (1 + 0.01 t)}]]

(* See arrays if needed *)
μ1Matrix // MatrixForm;

(* Plot r and alphas for Appendix S5: Figure 1c *)
ListLinePlot[{μ1r, μ1α12, μ1α11, μ1α21},
 PlotStyle → {Black, Orange, {Purple, Dashing[Large]}, Purple}, Frame → True,
 FrameStyle → {{Black, Black}, {Black, Black}}, PlotRange → {{0.18, 0.2}, {1, 2}},
 AxesOrigin → {0.18, 1}, AspectRatio → 1, LabelStyle → Directive[FontSize → 24]]

(* Plots *)
(* Region plot showing interaction outcomes *)
RP4c = Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{0, 0.3}, {0.8, 1.25}},
    Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
   LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
```

```
    FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
   AxesOrigin → {-1, Log[0.4]}, Frame → True, AspectRatio → 1,
   FrameTicks → {{{{Log[0.8], 0.8}, {Log[1], 1}, {Log[1.25], 1.25}}, None},
     {{0, 0.1, 0.2, 0.3}, None}}, FrameStyle → {{Black, Black}, {Black, Black}},
   LabelStyle → Directive[FontSize → 24]];
(* Competition and mutualism *)
pS4c = ListPlot[
   {{1 - ρ, Log[κ2κ1]} /. param /. μ11 → 0.18 /. μ21 → 0.1 /. μ12 → 0.1 /. μ22 → 0.18},
   PlotMarkers →
    {Graphics[{EdgeForm[{Black, Thick}], FaceForm[Black], Disk[]}], 0.051}];
(* Increase in μ_1k *)
S4μ1 = ListPlot[{μ1Matrix}, PlotMarkers →
    {Graphics[{EdgeForm[{Purple, Thick}], FaceForm[Purple], Disk[]}], 0.02}];
(* Increase in μ_2k *)
S4μ2 = ListPlot[{μ2Matrix}, PlotMarkers →
   {Graphics[{EdgeForm[{Orange, Thick}], FaceForm[Orange], Disk[]}], 0.02}];
(* Combine plots: orange and purple dots overlain with
   arrows in Adobe Illustrator to generate panel 2d *)
p4c = Show[RP4c, S4μ1, S4μ2, pS4c]
```

Out[1285]=



Out[1289]=




## Scenario 4d (Figure 2g): Changes in τ

In[1291]:=

```
(* Parameters *)
param =
  {b1 → 1, c11 → 1, c12 → 0.4, e11 → 0.1, e12 → 0.1, e13 → 0.1, v11 → 25, v12 → 5, v13 → 0,
   b2 → 1, c21 → 0.4, c22 → 1, e21 → 0.1, e22 → 0.1, e23 → 0.1, v21 → 5, v22 → 25, v23 → 0,
   β1 → 0.1, β2 → 0.1, bM3 → 0.1, δ1 → 1, δ2 → 1, δ3 → 1,
   μ11 → 0.18, μ21 → 0.1, μ12 → 0.1, μ22 → 0.18, μ13 → 0, μ23 → 0};
steps = 10;

(* Intrinsic per capita growth rates and interaction coefficients *)
```

$$r1 = b1 + \frac{e11\ v11\ \beta 1}{\delta 1} + \frac{e12\ v12\ \beta 2}{\delta 2} + \frac{e13\ v13\ bM3}{\delta 3};$$

$$r2 = b2 + \frac{e21\ v21\ \beta 1}{\delta 1} + \frac{e22\ v22\ \beta 2}{\delta 2} + \frac{e23\ v23\ bM3}{\delta 3};$$

$$\alpha 12 = \frac{1}{r1}\left(c12 + e11\ v11\ v21\ \tau 21 + e12\ v12\ v22\ \tau 22 + e13\ v13\ v23\ \tau 23 - \left(\frac{e11\ v11\ v21\ \mu 21}{\delta 1} + \frac{e12\ v12\ v22\ \mu 22}{\delta 2} + \frac{e13\ v13\ v23\ \mu 23}{\delta 3}\right)\right);$$

$$\alpha 21 = \frac{1}{r2}\left(c21 + e21\ v21\ v11\ \tau 11 + e22\ v22\ v12\ \tau 12 + e23\ v23\ v13\ \tau 13 - \left(\frac{e21\ v21\ v11\ \mu 11}{\delta 1} + \frac{e22\ v22\ v12\ \mu 12}{\delta 2} + \frac{e23\ v23\ v13\ \mu 13}{\delta 3}\right)\right);$$

$$\alpha 11 = \frac{1}{r1}\left(c11 + e11\ v11\ v11\ \tau 11 + e12\ v12\ v12\ \tau 12 + e13\ v13\ v13\ \tau 13 - \left(\frac{e11\ v11\ v11\ \mu 11}{\delta 1} + \frac{e12\ v12\ v12\ \mu 12}{\delta 2} + \frac{e13\ v13\ v13\ \mu 13}{\delta 3}\right)\right);$$

$$\alpha 22 = \frac{1}{r2}\left(c22 + e21\ v21\ v21\ \tau 21 + e22\ v22\ v22\ \tau 22 + e23\ v23\ v23\ \tau 23 - \left(\frac{e21\ v21\ v21\ \mu 21}{\delta 1} + \frac{e22\ v22\ v22\ \mu 22}{\delta 2} + \frac{e23\ v23\ v23\ \mu 23}{\delta 3}\right)\right);$$

```
(* Niche and fitness difference *)
```

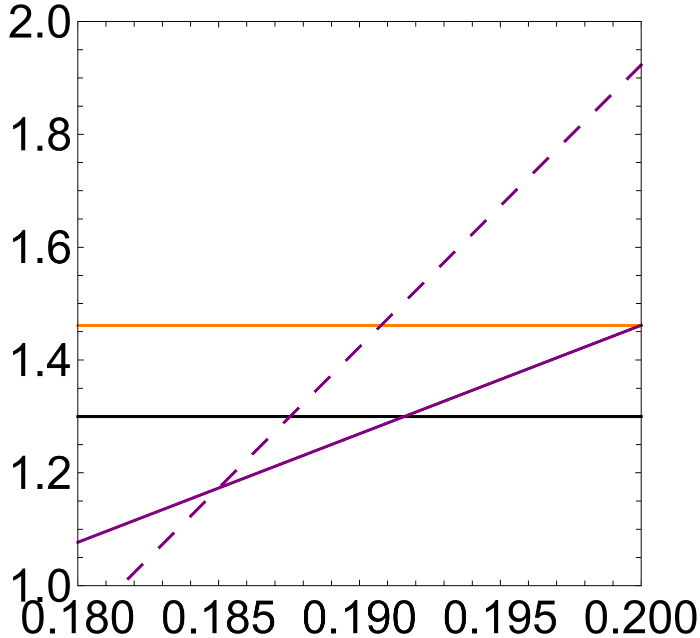
$$\rho = \sqrt{\frac{\alpha 12\ \alpha 21}{\alpha 11\ \alpha 22}};$$

$$\kappa 2\kappa 1 = \sqrt{\frac{\alpha 12\ \alpha 11}{\alpha 21\ \alpha 22}};$$

```
(* Define matrices with niche and fitness
 differences as model parameter is varied *)
τ1Matrix = τ2Matrix = {{1 - ρ /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /. τ22 → 0.2,
     Log[κ2κ1] /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /. τ22 → 0.2}};

(* Define matrices with parameter being systematically
 varied and resulting r and α terms for Appendix S5 *)
τ1r = {{τ11 /. τ11 → 0.2, r1 /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /. τ22 → 0.2}};
τ1α12 =
  {{τ11 /. τ11 → 0.2, α12 /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /. τ22 → 0.2}};
τ1α11 = {{τ11 /. τ11 → 0.2,
    α11 /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /. τ22 → 0.2}};
τ1α21 = {{τ11 /. τ11 → 0.2, α21 /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /.
     τ22 → 0.2}};

(* Systematically vary model parameter by stepsize 0.01 t and update matrices
 of niche and fitness differences as well as matrices of r and α terms *)
For[t = 1, t ≤ steps, t++,
 (* Increase τ1k *)
 τ1Matrix = AppendTo[τ1Matrix,
   {1 - ρ /. param /. τ11 → 0.2 (1 - 0.01 t) /. τ21 → 0.2 /. τ12 → 0.2 (1 - 0.01 t) /.
     τ22 → 0.2, Log[κ2κ1] /. param /. τ11 → 0.2 (1 - 0.01 t) /. τ21 → 0.2 /.
      τ12 → 0.2 (1 - 0.01 t) /. τ22 → 0.2}];
 τ1r = AppendTo[τ1r, {τ11 /. τ11 → 0.2 (1 - 0.01 t), r1 /. param /. τ11 → 0.2 (1 - 0.01 t) /.
       τ21 → 0.2 /. τ12 → 0.2 (1 - 0.01 t) /. τ22 → 0.2}];
 τ1α12 = AppendTo[τ1α12, {τ11 /. τ11 → 0.2 (1 - 0.01 t), α12 /. param /.
        τ11 → 0.2 (1 - 0.01 t) /. τ21 → 0.2 /. τ12 → 0.2 (1 - 0.01 t) /. τ22 → 0.2}];
 τ1α11 = AppendTo[τ1α11, {τ11 /. τ11 → 0.2 (1 - 0.01 t),
    α11 /. param /. τ11 → 0.2 (1 - 0.01 t) /. τ21 → 0.2 /.
      τ12 → 0.2 (1 - 0.01 t) /. τ22 → 0.2}];
```

```
 τ1α21 = AppendTo[τ1α21, {τ11 /. τ11 → 0.2 (1 - 0.01 t),
    α21 /. param /. τ11 → 0.2 (1 - 0.01 t) /. τ21 → 0.2 /.
      τ12 → 0.2 (1 - 0.01 t) /. τ22 → 0.2}];
 (* Increase τ2k *)
 τ2Matrix =
  AppendTo[τ2Matrix, {1 - ρ /. param /. τ11 → 0.2 /. τ21 → 0.2 (1 - 0.01 t) /. τ12 → 0.2 /.
     τ22 → 0.2 (1 - 0.01 t), Log[κ2κ1] /. param /. τ11 → 0.2 /. τ21 → 0.2 (1 - 0.01 t) /.
      τ12 → 0.2 /. τ22 → 0.2 (1 - 0.01 t)}]]

(* See arrays if needed *)
τ1Matrix // MatrixForm;

(* Plot r and alphas for Appendix S5: Figure 1d *)
ListLinePlot[{τ1r, τ1α12, τ1α11, τ1α21},
 PlotStyle → {Black, Orange, {Purple, Dashing[Large]}, Purple}, Frame → True,
 FrameStyle → {{Black, Black}, {Black, Black}}, PlotRange → {{0.18, 0.2}, {1, 2}},
 AxesOrigin → {0.18, 1}, AspectRatio → 1, LabelStyle → Directive[FontSize → 24]]

(* Plots *)
(* Region plot showing interaction outcomes *)
RP4d = Show[LogPlot[{1 - x, 1 / (1 - x)}, {x, 0, 1}, PlotRange → {{0, 0.3}, {0.7, 1.4}},
     Filling → 1, FillingStyle → GrayLevel[0.7], PlotStyle → {{Black}, {Black}}],
   LogPlot[{1 - x, 1 / (1 - x)}, {x, -1, 0}, Filling → 1,
     FillingStyle → GrayLevel[0.9], PlotStyle → {{Black}, {Black}}],
   AxesOrigin → {-1, Log[0.4]}, Frame → True, AspectRatio → 1,
   FrameTicks → {{{{Log[0.7], 0.7}, {Log[1], 1}, {Log[1.4], 1.4}}, None},
     {{0, 0.1, 0.2, 0.3}, None}}, FrameStyle → {{Black, Black}, {Black, Black}},
   LabelStyle → Directive[FontSize → 24]];
(* Competition and mutualism *)
pS4d = ListPlot[
   {{1 - ρ, Log[κ2κ1]} /. param /. τ11 → 0.2 /. τ21 → 0.2 /. τ12 → 0.2 /. τ22 → 0.2},
   PlotMarkers →
    {Graphics[{EdgeForm[{Black, Thick}], FaceForm[Black], Disk[]}], 0.051}];
(* Increase in τ_1k *)
S4τ1 = ListPlot[{τ1Matrix}, PlotMarkers →
    {Graphics[{EdgeForm[{Purple, Thick}], FaceForm[Purple], Disk[]}], 0.02}];
(* Increase in τ_2k *)
S4τ2 = ListPlot[{τ2Matrix}, PlotMarkers →
   {Graphics[{EdgeForm[{Orange, Thick}], FaceForm[Orange], Disk[]}], 0.02}];
(* Combine plots: orange and purple dots overlain with
   arrows in Adobe Illustrator to generate panel 2d *)
p4c = Show[RP4d, S4τ1, S4τ2, pS4d]
```

Out[1308]=




Out[1312]=