Rule Sheet: Linear Sequent Calculus

Chris Martens

October 8, 2025

1 Inference Rules

Tensor:

$$\frac{\Delta_1 \Rightarrow A \quad \Delta_2 \Rightarrow B}{\Delta_1, \Delta_2 \Rightarrow A \otimes B}\ \otimes R \qquad \frac{\Delta, A, B \Rightarrow C}{\Delta, A \otimes B \Rightarrow C}\ \otimes L$$

With:

$$\frac{\Delta \Rightarrow A \quad \Delta \Rightarrow B}{\Delta \Rightarrow A \& B}\ \& R \qquad \frac{\Delta, A \Rightarrow C}{\Delta, A \& B \Rightarrow C}\ \& L_1 \qquad \frac{\Delta, B \Rightarrow C}{\Delta, A \& B \Rightarrow C}\ \& L_2$$

Disjunction (oplus):

$$\frac{\Delta \Rightarrow A}{\Delta \Rightarrow A \oplus B}\ \oplus R_1 \qquad \frac{\Delta \Rightarrow B}{\Delta \Rightarrow A \oplus B}\ \oplus R_2 \qquad \frac{\Delta, A \Rightarrow C \quad \Delta, B \Rightarrow C}{\Delta, A \oplus B \Rightarrow C}\ \oplus L$$

Implication (lolli):

$$\frac{\Delta, A \Rightarrow B}{\Delta \Rightarrow A \multimap B}\ \multimap R \qquad \frac{\Delta_1 \Rightarrow A \quad \Delta_2, B \Rightarrow C}{\Delta_1, \Delta_2, A \multimap B \Rightarrow C}\ \multimap L$$

Positive unit:

$$\frac{\Delta = \cdot}{\Delta \Rightarrow \mathbb{1}}\ \mathbb{1}R \qquad \frac{\Delta \Rightarrow C}{\Delta, \mathbb{1} \Rightarrow C}\ \mathbb{1}L$$

Negative and positive zero:

$$\frac{}{\Delta \Rightarrow \top}\ \top R \qquad (\text{no } \top\text{L}) \qquad (\text{no } \mathbb{0}\text{R}) \qquad \frac{}{\Delta, \mathbb{0} \Rightarrow C}\ \mathbb{0}L$$

Identity rule:

$$\frac{}{A \Rightarrow A}\ \mathsf{id}$$

2 Syntax

Judgment $\Delta \Rightarrow A$, where Δ is an unordered list of propositions A. We may also write Γ as a context metavariable when it is clear that this does not represent an unrestricted context.

Proposition forms A, B, C include:

- $A \otimes B$ (positive conjunction)
- $A \& B$ (negative conjunction)
- $A \oplus B$ (positive disjunction)
- $\mathbb{1}$ (unit of $\otimes$/positive unit)
- $\mathbb{0}$ (unit of $\oplus$/positive zero)
- $\top$ (unit of $\&$/negative zero)
- p (atomic propositions/placeholders for arbitrary propositions)