Rule Sheet: Intuitionistic Sequent Calculus

Chris Martens

September 15, 2025

1 Inference Rules

Judgment $\Gamma \Rightarrow A$, where Γ is an unordered list of propositions A. Proposition forms A, B, C include conjunction $A \wedge B$, disjunction $A \vee B$, implication $A \supset B$, truth $\top$, falsehood $\bot$, and atomic propositions P.

Conjunction:

$$\frac{\Gamma \Rightarrow A \quad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \wedge B}\ \wedge R \qquad \frac{\Gamma, A \wedge B, A \Rightarrow C}{\Gamma, A \wedge B \Rightarrow C}\ \wedge L_1 \qquad \frac{\Gamma, A \wedge B, B \Rightarrow C}{\Gamma, A \wedge B \Rightarrow C}\ \wedge L_2$$

Disjunction:

$$\frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A \vee B}\ \vee R_1 \qquad \frac{\Gamma \Rightarrow B}{\Gamma \Rightarrow A \vee B}\ \vee R_2$$

$$\frac{\Gamma, A \vee B, A \Rightarrow C \quad \Gamma, A \vee B, B \Rightarrow C}{\Gamma, A \vee B \Rightarrow C}\ \vee L$$

Implication:

$$\frac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow A \supset B}\ \supset R \qquad \frac{\Gamma, A \supset B \Rightarrow A \quad \Gamma, A \supset B, B \Rightarrow C}{\Gamma, A \supset B \Rightarrow C}\ \supset L$$

Truth and Falsehood:

$$\frac{}{\Gamma \Rightarrow \top}\ \top R \qquad \text{(no } \top\text{L)} \qquad \text{(no } \bot\text{R)} \qquad \frac{}{\Gamma, \bot \Rightarrow C}\ \bot L$$

Negation $\neg A$ is, as before, defined as $A \supset \bot$.

Identity rule:

$$\frac{}{\Gamma, A \Rightarrow A}\ \mathsf{id}$$