Resource-aware control

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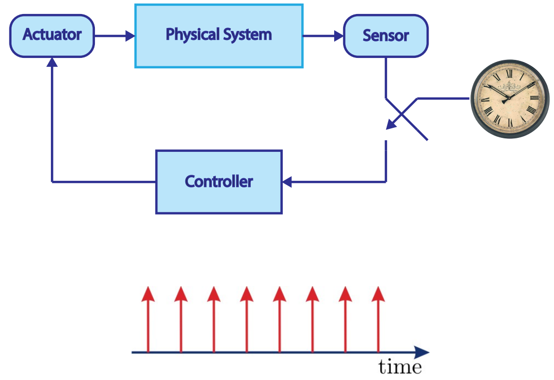
Introduction

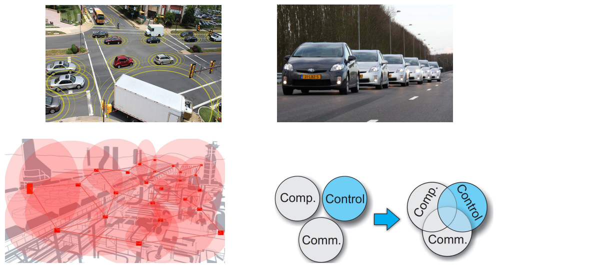
Standard digital control loop



⟶ All control tasks executed periodically and triggered by time

Introduction

Resource-aware control



- Resource-constrained control systems
  - Computation time on embedded systems
  - Network utilization in NCS
  - Battery power in WCS
- Time-triggered periodic control: Inefficient usage of resources

Introduction

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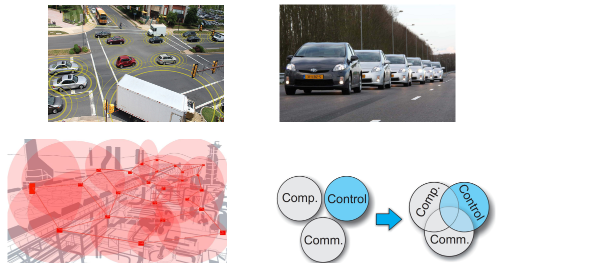
Resource-aware control



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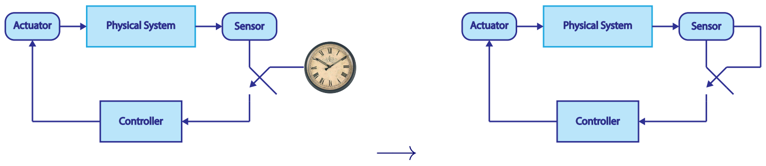
"Wise men speak because they have something to say, fools because they have to say something." – Plato

Introduction

Resource-aware control

- Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Only act when needed: bringing feedback in resource utilization



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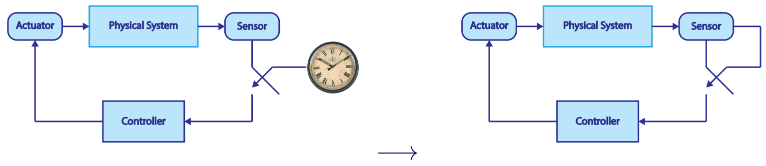

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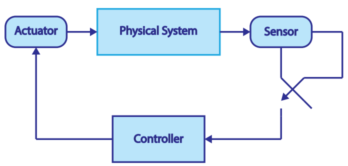
Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Event-triggered control:

$$u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} = \inf\{t > t_k \mid \mathcal{C}(x(t), x(t_k)) \geqslant 0\}$$



[1] Arzen, IFAC WC'99 [2] Astrom & Bernhardsson, IFAC WC'99 [3] Heemels *et al*, CEP'99

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Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

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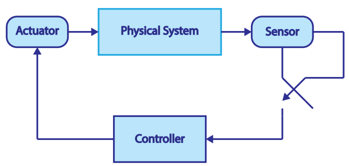

- Example event-triggering condition
  $\mathcal{C}(x(t), x(t_k)) \geqslant 0 \quad \Leftrightarrow \quad \|x(t) - \underbrace{x(t_k)}_{\hat{x}(t)}\| \geqslant \delta$

Introduction

Paradigm shift: Periodic control $\longrightarrow$ Aperiodic control

- Event-triggered control: reactive

$$u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} = \inf\{t > t_k \mid \mathcal{C}(x(t), x(t_k)) \geqslant 0\}$$

- Self-triggered control: proactive

$$u(t) = \mathcal{K}(x(t_k)), \text{ when } t \in [t_k, t_{k+1})$$

$$t_{k+1} = t_k + \mathcal{M}(x(t_k))$$

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Outline

- Basic setup state-feedback ETC: $\|x(t) - x(t_k)\| \geqslant \sigma\|x(t)\|$
- Hybrid systems
- Challenges
  - Performance/Robustness w.r.t. disturbances & Zeno-freeness
  - Output-based (& Decentralized)
- Alternative event-triggered controllers
  - Relative, absolute and mixed event generators
  - Periodic event-triggered control
  - Time regularisation
  - Dynamic event generators
- Application to vehicle platooning
- Conclusions & What's next?

(Dessert?)

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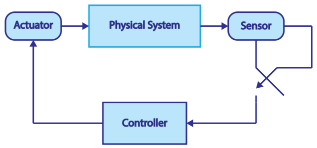
Basic ETC setup

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$

- Linear state feedback

$$u(t) = Kx(t), \quad t \in \mathbb{R}_{\geqslant 0}$$



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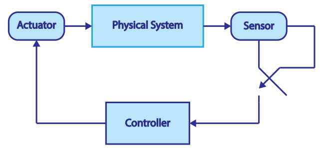
$$u(t) = Kx(t), \quad t \in \mathbb{R}_{\geqslant 0}$$

- Ideal loop: $\dot{x}(t) = (A + BK)x(t)$ $\quad A + BK$ Hurwitz



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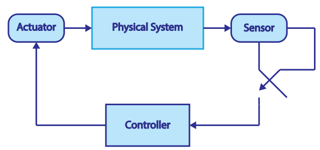

- Linear state feedback

$$u(t) = Kx(t), \quad t \in \mathbb{R}_{\geqslant 0}$$

- Ideal loop: $\dot{x}(t) = (A + BK)x(t)$
- Sampled-data control with execution times t_k, $k \in \mathbb{N}$ (ZOH)

$$u(t) = K\hat{x}(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007

Basic ETC setup

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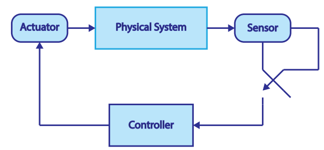

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- Ideal loop: $\dot{x}(t) = (A + BK)x(t)$
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$$u(t) = K\hat{x}(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Perturbation perspective: implementation-induced error

$$e(t) = x(t_k) - x(t) \text{ for } t \in [t_k, t_{k+1})$$

$$\boxed{\dot{x}(t) = Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)}$$

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Basic ETC setup

- Perturbation perspective:

$$\dot{x}(t) \;=\; Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)$$

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$ s.t.

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

Basic ETC setup

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$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Crux: Guarantee $\|e(t)\| \leqslant \rho a \cdot \|x(t)\|$ with $0 < \rho < 1$ s.t.

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2 \leqslant -(1-\rho^2)a^2\|x(t)\|^2$$

- Guarantee for Global Exponential Stability

$$\boxed{t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k) - x(t)}_{=e(t)}\| \geqslant \rho a \cdot \|x(t)\|\}}$$

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Event-triggered control

- Summary of event-triggered setup:
  - Linear system
$$\dot{x}(t) = Ax(t) + Bu(t)$$
  - Execution times t_k, $k \in \mathbb{N}$
$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| \geqslant \rho a \cdot \|x(t)\|\}$$
  - Control law:
$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$
- Global exponential stability (GES)

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- Question: Which important issue should we still verify?

Basic ETC setup

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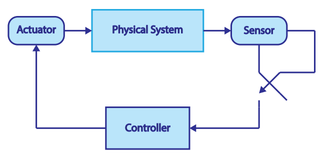
$$\dot{x}(t) = Ax(t) + Bu(t)$$



- Linear state feedback (ZOH)
$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$
- Execution times: $t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\|\}$
- Properties established in [1]:
  - Global exponential stability (GES) when σ suff. small
  - Zeno-free: Global positive lower bound on minimal inter-event time (MIET)
$$\inf\{t_{k+1} - t_k \mid k \in \mathbb{N}\} > \tau_{\min} > 0$$

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007

Basic ETC setup

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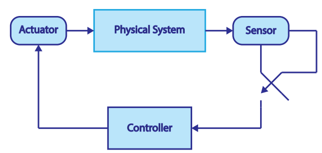

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- Properties established in [1]:
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  - Global positive lower bound on minimal inter-event time (MIET)
$$\inf\{t_{k+1} - t_k \mid k \in \mathbb{N}\} > \tau_{\min} > 0$$
- Improved designs for GES/$\mathcal{L}_\infty$-gain via hybrid system analysis [2]

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007
[2] Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain ...*, TAC 2012

Hybrid systems (side trip)

- Perturbation perspective:

$$\dot{x}(t) \;=\; Ax(t) + BKx(t_k) = (A+BK)x(t) + BKe(t)$$

- Execution times t_k, $k \in \mathbb{N}$

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- Hybrid system perspective [1,2] based on jump-flow models [3]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x \\ e \end{bmatrix} = \begin{bmatrix} (A+BK)x + BKe \\ -(A+BK)x - BKe \end{bmatrix}, \qquad \text{when } \|e\|^2 \leqslant \sigma^2\|x\|^2$$

$$\begin{bmatrix} x^+ \\ e^+ \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}, \qquad \text{when } \|e\|^2 \geqslant \sigma^2\|x\|^2$$

[1] Donkers, Heemels, *Output-Based Event-Triggered Control ...*, TAC 2012 & CDC 2010
[2] Postoyan, Anta, Nesic, Tabuada, *A unifying Lyapunov-based framework ...*, CDC-ECC 2011
[3] Goebel, Sanfelice, Teel, *Hybrid Dynamical Systems*, Princeton, 2012.

ETC based on feedback

Hybrid system perspective (side trip)

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$$\begin{bmatrix} x^+ \\ e^+ \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix} \qquad \text{when } \|e\|^2 \geqslant \sigma^2\|x\|^2$$

or compactly with $\xi = \begin{bmatrix} x \\ e \end{bmatrix}$ $\begin{cases} \dot{\xi} = \Phi\xi, \text{ when } \xi^\top Q\xi \leqslant 0 \\ \xi^+ = J\xi, \text{ when } \xi^\top Q\xi \geqslant 0 \end{cases}$

ETC based on feedback

Hybrid system perspective (side trip)

$$\begin{aligned} \dot{\xi} &= \Phi\xi \quad \text{when } \xi^\top Q\xi \leqslant 0 \\ \xi^+ &= J\xi \quad \text{when } \xi^\top Q\xi \geqslant 0 \end{aligned}$$

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- Stability analysis using hybrid tools [1,2]: $V(\xi) = \xi^\top P\xi$
  - $\frac{d}{dt}V(\xi) < 0$ when $\xi^\top Q\xi \leqslant 0$
  - $V(J\xi) \leqslant V(\xi)$ when $\xi^\top Q\xi \geqslant 0$

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  - $\frac{d}{dt}V(\xi) < 0$ when $\xi^\top Q\xi \leqslant 0$
  - $V(J\xi) \leqslant V(\xi)$ when $\xi^\top Q\xi \geqslant 0$
- Linear matrix inequalities: if there are $\alpha, \beta \geqslant 0$ s.t.
  - $\Phi^\top P + P\Phi - \alpha Q \prec 0$
  - $J^\top PJ - P + \beta Q \preceq 0$
- Guarantee for GES (extended ideas apply for $\mathcal{L}_\infty$-gains)
- Never more conservative than perturbation approach [1]

[1] Donkers, Heemels, *Output-based event-triggered control with guaranteed L_∞-gain*, TAC 2012 & CDC 2010
[2] Goebel, Sanfelice, Teel, *Hybrid Dynamical Systems: Modeling, Stability and Robustness*, Princeton, 2012.

Illustrative Example

Example 1: State feedback control

- Consider $\dot{x} = \begin{bmatrix}0 & 1\\ -2 & 3\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}u$ and $u(t) = \begin{bmatrix}1 & -4\end{bmatrix}x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geqslant 0.05\|x(t)\|$

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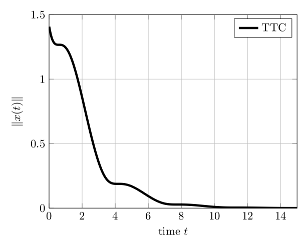

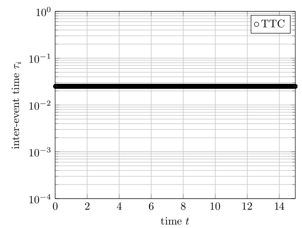

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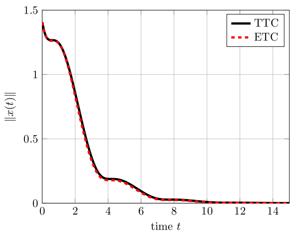

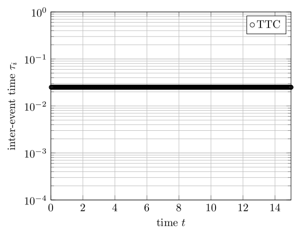

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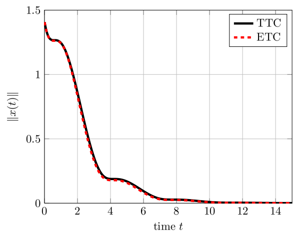

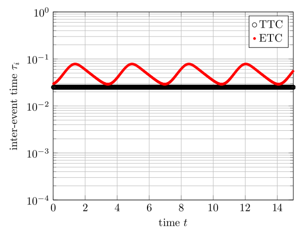

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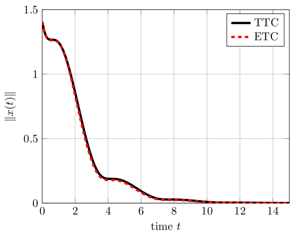

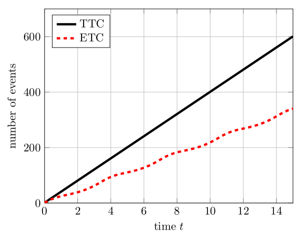

Illustrative Examples

Example 1: Comparison P and HS approach

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- Example taken from [1]
- We look for largest σ^2 giving GES: $\|e\|^2 \leqslant \sigma^2\|x\|^2$ [2]

	σ^2	MIET
P: Results from [1]	0.0030	0.0318
P: By minimising the $\mathcal{L}_2$-gain	0.0273	0.0840
Hybrid System	0.0588	0.1136

[1] Tabuada, TAC '07 [2] Donkers, Heemels, CDC10 & TAC12

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Hybrid System	0.0588	0.1136

- PS: via minimising $\mathcal{L}_2$-gain: maximise a (note $\sigma = \rho a$)

$$\dot{V} \leqslant -a^2 \|x(t)\|^2 + \|e(t)\|^2 \quad \text{for } \dot{x} = (A + BK)x + BKe$$

- ETM:

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \rho a \cdot \|x(t)\| \}$$

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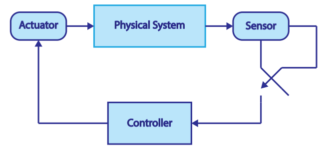
Summary

- Linear system

$$\dot{x}(t) = Ax(t) + Bu(t)$$



- Linear state feedback (ZOH)

$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

- Execution times: $t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\| \}$
- Properties established in [1]:
  - Global exponential stability (GES) when σ suff. small
  - Global positive lower bound on minimal inter-event time (MIET)

$$\inf\{t_{k+1} - t_k \mid k \in \mathbb{N}\} > \tau_{\min} > 0$$

- Improved designs for GES/$\mathcal{L}_\infty$-gain via hybrid system analysis [2]

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007
[2] Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed $\mathcal{L}_\infty$-gain ...*, TAC 2012

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Challenges

- Performance/Robustness w.r.t. disturbances
- Output-based (& Decentralized)

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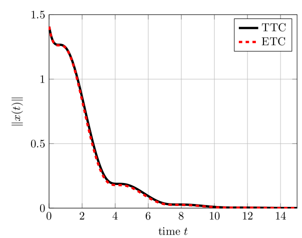
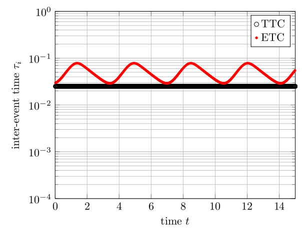
Disturbances in ETC

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geqslant 0.05 \|x(t)\|$





Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

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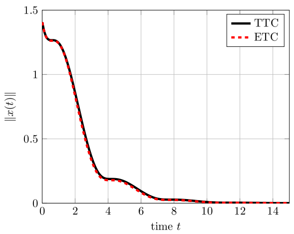
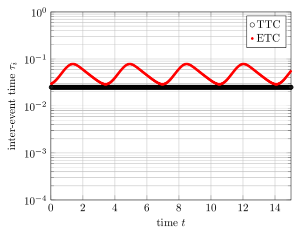
Disturbances in ETC

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
- ETC: $t_k = t \iff \|e(t)\| \geqslant 0.05\|x(t)\|$





Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Disturbances in ETC

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
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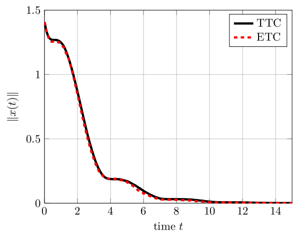

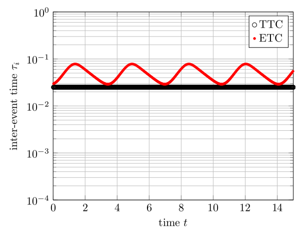

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Disturbances in ETC

Illustrative example

- Consider $\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$ and $u(t) = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$
- TTC: $t_k = k \cdot 0.025$
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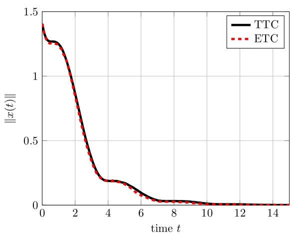
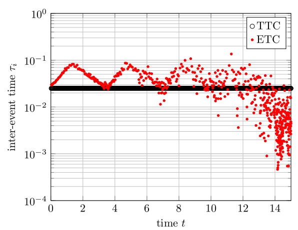
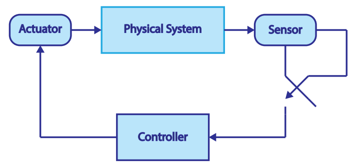




Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Output-based ETC

Illustrative example



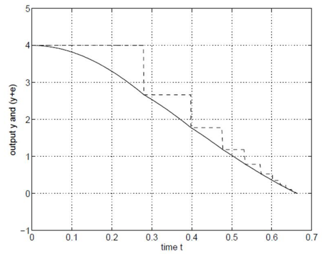
- Consider

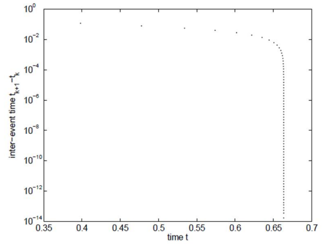
$$\begin{cases} \dot{x}_p = \begin{bmatrix} 1 & -1 \\ 10 & -1 \end{bmatrix} x_p + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p \end{cases} \qquad u(t) = -2y(t_k)$$

- ETM: $\|y(t) - y(t_k)\|^2 > \sigma^2 \|y(t)\|^2$
- Parameter: $\sigma^2 = 0.5$

Output-based ETC

Illustrative example





- Minimal inter-event time (MIET) is zero! (Zeno behavior)

Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain and Improved and Decentralised Event-Triggering*, TAC 2012

Event-triggered control schemes

- Relative: $\|y - \hat{y}\| \geqslant \sigma\|y\|$ [1]
- Absolute: $\|y - \hat{y}\| \geqslant \delta$ [2-4]
- Mixed: $\|y - \hat{y}\| \geqslant \sigma\|y\| + \delta$ [5]

[1] Tabuada, *Event-triggered real-time scheduling of stabilizing control tasks*, TAC 2007
[2] Yook, Tilbury, Soparkar, *Trading computation for bandwidth: Reducing communication in distributed control systems using state estimators*, TCST 2002
[3] Miskowicz, *Send-on-delta concept: An event-based data-reporting strategy*, Sensors, 2006
[4] Lunze and Lehmann, *A state-feedback approach to event-based control*, Automatica, 2010
[5] Donkers, Heemels, *Output-Based Event-Triggered Control with Guaranteed L_∞-gain ...*, TAC 2012

Movie ETC in action

Inverted pendulum

Disturbances in ETC

Event-separation properties / Zeno-freeness

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t_k) = K(x(t) + e(t))$
- Execution times:

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma\|x(t)\| + \delta\}$$

$\rightarrow$ MIET $\tau(x_0, w)$ dependent on x_0 and w: $\tau(x_0, w) = \inf_{k\in\mathbb{N}} (t_{k+1} - t_k)$

Disturbances in ETC

Event-separation properties / Zeno-freeness

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t_k) = K(x(t) + e(t))$
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$\rightarrow$ MIET $\tau(x_0, w)$ dependent on x_0 and w: $\tau(x_0, w) = \inf_{k \in \mathbb{N}} (t_{k+1} - t_k)$

- Event-separation properties (nominal)
  - Global ESP: $\inf_{x_0 \in \mathbb{R}^n} \tau(x_0, 0) > 0$
  - Semi-global ESP: for compact $X_0 \subset \mathbb{R}^n$: $\inf_{x_0 \in X_0} \tau(x_0, 0) > 0$
  - Local ESP: for each $x_0 \in \mathbb{R}^n$: $\tau(x_0, 0) > 0$

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Disturbances in ETC

Event-separation properties / Zeno-freeness

- Consider $\dot{x} = Ax + Bu + w$ and $u(t) = Kx(t_k) = K(x(t) + e(t))$
- Execution times:

$$t_{k+1} = \inf\{t > t_k \mid \| \underbrace{x(t_k) - x(t)}_{=e(t)} \| \geqslant \sigma \|x(t)\| + \delta\}$$

$\rightarrow$ MIET $\tau(x_0, w)$ dependent on x_0 and w: $\tau(x_0, w) = \inf_{k \in \mathbb{N}} (t_{k+1} - t_k)$

- Event-separation properties (robust): there is $\varepsilon > 0$
  - Robust global: $\inf_{x_0 \in \mathbb{R}^n,\ \|w\|_\infty < \varepsilon} \tau(x_0, w) > 0$
  - Robust semi-global: compact X_0: $\inf_{x_0 \in X_0,\ \|w\|_\infty < \varepsilon} \tau(x_0, w) > 0$
  - Robust local: for each $x_0 \in \mathbb{R}^n$ and $\|w\|_\infty < \varepsilon$: $\tau(x_0, w) > 0$

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Overview

State-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	✓	×	✓	×	✓
absolute	×	×	✓	✓	✓	✓
mixed	✓	✓	✓	✓	✓	✓

Overview

State-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	✓	×	✓	×	✓
absolute	×	×	✓	✓	✓	✓
mixed	✓	✓	✓	✓	✓	✓

Output-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	×	×	×	×	×
absolute	×	×	✓	✓	✓	✓
mixed	×	×	✓	✓	✓	✓

- Relative triggering fragile. Zero robustness
- Mixed or absolute effective (semi-global)
- However, only practical stability / ultimate boundedness (no GAS)

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Overview

State-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	✓	×	✓	×	✓
absolute	×	×	✓	✓	✓	✓
mixed	✓	✓	✓	✓	✓	✓

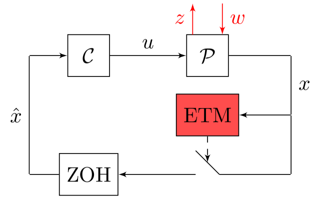
Output-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	×	×	×	×	×
absolute	×	×	✓	✓	✓	✓
mixed	×	×	✓	✓	✓	✓

- Relative triggering fragile. Zero robustness
- Mixed or absolute effective (semi-global)
- However, only practical stability / ultimate boundedness (no GAS)
- Challenge: What about robust global ESP and GAS/$\mathcal{L}_2$-gains?

Borgers, Heemels, *Event-Separation Properties of Event-Triggered Control Systems*, TAC 2014

Objectives



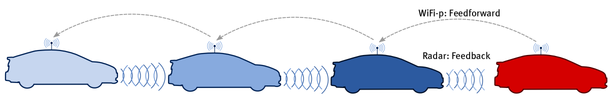
- Guaranteed control performance ($\mathcal{L}_2$−gain) from disturbance w to output $z = q(x, w)$:

$$\|z\|_{\mathcal{L}_2} \leqslant \beta(|\xi_0|) + \gamma\|w\|_{\mathcal{L}_2} \text{ with } \|z\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt}$$

- Global asymptotic stability (GAS) in absence of disturbances
- Robust positive "minimal inter-event time" (τ_{miet})
- Reduced communication w.r.t. time-triggered control

Motivation

Cooperative Adaptive Cruise Control



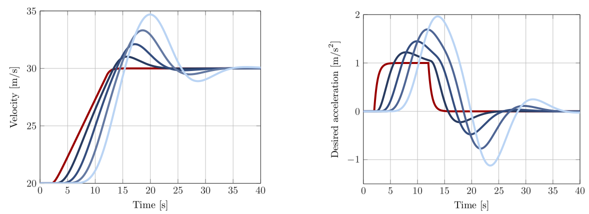
- String stability: disturbance attenuation along the vehicle string $\gamma \leqslant 1$

$$\|z\|_{\mathcal{L}_2} \leqslant \beta(|\xi_0|) + \gamma\|w\|_{\mathcal{L}_2} \text{ with } \|z\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt}$$

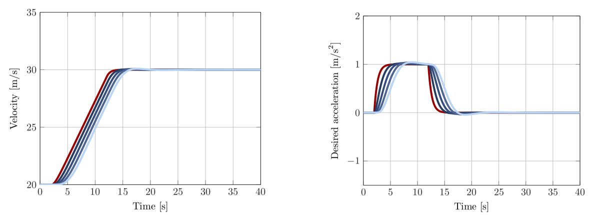
- Communication resources limited $\rightarrow$ event-triggered communication

String (in)stability

String unstable (no communication)



String stable (with communication)



Event-triggered control schemes

Time regularisation:

- Periodic Event-Triggered Control (PETC) [6-9,12]

$$t_{k+1} = \inf\{t > t_k \mid \|y(t) - \hat{y}(t)\| > \sigma\|y(t)\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Enforcing minimal inter-event time [7,9-13]

$$t_{k+1} = \inf\{t > t_k + T \mid \|y(t) - \hat{y}(t)\| > \sigma\|y(t)\|\}$$

[6] Arzen, *A simple event-based PID controller*, IFAC 1999
[7] Heemels, Sandee, van den Bosch, *Analysis of event-driven controllers for linear systems*, IJC 2008
[8] Heemels, Donkers, Teel, *Periodic Event-Triggered Control for Linear Systems*, TAC 2013
[9] Henningsson, Johannesson, Cervin, *Sporadic event-based control of first-order linear stochastic ..*, Aut. 2008
[10] Tallapragada, Chopra, *Event-triggered dynamic output feedback control for LTI systems*, CDC 2012
[11] Tallapragada, Chopra, *Event-triggered decentralized dynamic output .. LTI systems*, NECSYS 2012
[12] Borgers, Dolk, Heemels, *Riccati-based design of ETCs for Linear Systems ...* , HSCC 17 + TAC18
[13] Dolk, Borgers, Heemels, *Output-based and Decentralized Dynamic ETC ...* , TAC 2017

Time regularized ETC

- Periodic Event-Triggered Control (PETC)

$$t_{k+1} = \inf\{t > t_k \mid \|y - \hat{y}\| > \sigma\|y\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Enforcing minimal inter-event time

$$t_{k+1} = \inf\{t > t_k + T \mid \|y - \hat{y}\| > \sigma\|y\|\}$$

Output-feedback case

ETM	robust global	global	robust semi-global	semi-global	robust local	local
relative	×	×	×	×	×	×
absolute	×	×	✓	✓	✓	✓
mixed	×	×	✓	✓	✓	✓
time-regu	✓	✓	✓	✓	✓	✓

PETC

- Periodic Event-Triggered Control (PETC)

$$t_{k+1} = \inf\{t > t_k \mid \|y - \hat{y}\| > \sigma\|y\| \wedge t = kh,\ k \in \mathbb{N}\}$$

- Hybrid system analysis: GAS & finite $\mathcal{L}_2$-gains [1,2]
- Implementation advantages:
  - Guaranteed (reasonable) minimal inter-event time
  - Only time-periodic verification of event-triggering conditions
  - More in line with time-sliced architectures

[1] Heemels, Donkers, Teel, *Periodic Event-Triggered Control for Linear Systems*, TAC 2013
[2] Heemels, Donkers, *Model-based Periodic Event-Triggered Control for Linear Systems*, Automatica 2013

PETC

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix},\ \text{when } \tau \in [0,h],$$

$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$

$$z = C\xi + Dw$$

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$
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$$z = C\xi + Dw$$

- In case $w = 0$ and interested in stability only
- Discretize at kh, $k \in \mathbb{N}$ (just before jump) leading to discrete-time PWL system [1,2,3]

$$\xi_{k+1} = \begin{cases}e^{Ah}J_1\xi_k, & \text{when } \xi_k^\top Q\xi_k > 0\\ e^{Ah}J_2\xi_k, & \text{when } \xi_k^\top Q\xi_k \leqslant 0\end{cases}$$

[1] Heemels, Donkers, Teel, *Periodic Event-Triggered Control for Linear Systems*, TAC 2013
[2] Heemels, Donkers, *Model-based Periodic Event-Triggered Control for Linear Systems*, Automatica 2013
[3] Heemels, Sandee, van den Bosch, *Analysis of event-driven controllers for linear systems*, IJC 2008

Hybrid systems formulation

- Including intersample-behavior, e.g., for $\mathcal{L}_2$-gain analysis

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h],$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{cases}\begin{bmatrix}J_1\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi > 0,\ \tau = h\\ \begin{bmatrix}J_2\xi\\ 0\end{bmatrix}, & \text{when } \xi^\top Q\xi \leqslant 0,\ \tau = h\end{cases}$$
$$z = C\xi + Dw$$

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \qquad \text{when } \tau = h$$
$$z = C\xi + Dw$$

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \text{ when } \tau \in [0,h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \qquad \text{when } \tau = h$$
$$z = C\xi + Dw$$

Other applications for this framework:

- Reset controllers with testing for reset only at kh, $k \in \mathbb{N}$ [1]
- Linear systems/controllers with one sensor/actuator node transmitting at kh, $k \in \mathbb{N}$ determined by quadratic protocol [1]
- Linear systems controlled by arbitrarily switching sampled-data controllers (in this case ϕ setvalued) [2]
- Linear systems controlled by saturating sampled-data controllers [1]

[1] Heemels, Dullerud, Teel, *$\mathcal{L}_2$-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach*, TAC'16
[2] CDC'15 version of above

PETC

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0,h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$
$$z = C\xi + Dw$$

PETC

34/56

Hybrid systems formulation

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0,h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$
$$z = C\xi + Dw$$

- $\mathcal{L}_2$-contractive: There are $\gamma_0 \in [0,1)$ and a $\mathcal{K}$-function β s.t.

$$\|z\|_{\mathcal{L}_2} \leqslant \beta(|\xi_0|) + \gamma_0\|w\|_{\mathcal{L}_2} \text{ with } \|z\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty \|z(t)\|^2 dt}$$

Lifting-based approach

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0,h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$
$$z = C\xi + Dw$$

$$\begin{aligned}\bar{\xi}_{k+1} &= A_d\phi(\bar{\xi}_k) + B_d v_k\\ r_k &= C_d\phi(\bar{\xi}_k)\end{aligned}$$

Main result: The hybrid system is internally stable and $\mathcal{L}_2$-contractive iff the discrete-time nonlinear system is internally stable and ℓ_2-contractive.

[1] Heemels, Dullerud, Teel, *$\mathcal{L}_2$-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach*, TAC'16

Lifting-based approach

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0,h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$
$$z = C\xi + Dw$$

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Main result: The hybrid system is internally stable and $\mathcal{L}_2$-contractive iff the discrete-time nonlinear system is internally stable and ℓ_2-contractive.

- ℓ_2-contractive: there is $\gamma_0 \in [0,1)$ s.t.

$$\|r\|_{\ell_2} \leqslant \beta(|\bar{\xi}_0|) + \gamma_0\|v\|_{\ell_2} \text{ with } \|r\|_{\ell_2}^2 = \sum_{k=0}^{\infty} |r_k|^2$$

[1] Heemels, Dullerud, Teel, *$\mathcal{L}_2$-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach*, TAC'16

Lifting-based approach

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\xi\\ \tau\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\end{bmatrix}, \quad \text{when } \tau \in [0, h]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\end{bmatrix} = \begin{bmatrix}\phi(\xi)\\ 0\end{bmatrix}, \quad \text{when } \tau = h$$
$$z = C\xi + Dw$$

$$\bar{\xi}_{k+1} = A_d\phi(\bar{\xi}_k) + B_d v_k$$
$$r_k = C_d\phi(\bar{\xi}_k)$$

Main result: The hybrid system is internally stable and $\mathcal{L}_2$-contractive iff the discrete-time nonlinear system is internally stable and ℓ_2-contractive.

- Lifting with verifiable conditions without linearity
- For PETC *piecewise linear* system $\longrightarrow$ contractivity/stability analysis via LMIs using piecewise quadratic Lyapunov functions

[1] Heemels, Dullerud, Teel, *$\mathcal{L}_2$-gain Analysis for a Class of Hybrid Systems with Applications to Reset and Event-triggered Control: A Lifting Approach*, TAC'16

Self-triggered control (Side trip)

- Linear system
$$\dot{x}(t) = Ax(t) + Bu(t)$$
- When? Execution times t_k, $k \in \mathbb{N}$
$$t_{k+1} = t_k + \mathcal{M}(x_k)$$
with $\mathcal{M} : \mathbb{R}^n \to \{h_1, h_2, \ldots, h_N\} \subset \mathbb{R}_{>0}$
- What? Control law:
$$u(t) = Kx(t_k), \quad t \in [t_k, t_{k+1})$$

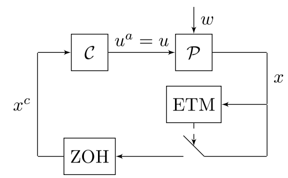
Extensions

$$\frac{d}{dt}\begin{bmatrix}\xi\\ \tau\\ \ell\end{bmatrix} = \begin{bmatrix}A\xi + Bw\\ 1\\ 0\end{bmatrix}, \quad \text{when } \tau \in [0, h_\ell]$$
$$\begin{bmatrix}\xi^+\\ \tau^+\\ \ell^+\end{bmatrix} \in \phi(\xi) \times \{0\} \times L(\ell, \xi), \quad \text{when } \tau = h_\ell$$
$$z = C\xi + Dw$$

- Self-triggered control: Select on event time t_k the next event time in a state-dependent fashion:
$$t_{k+1} = t_k + h_{\ell^+} \text{ with } \ell^+ \in L(\ell, \xi)$$

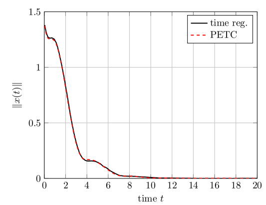
Strijbosch, Dullerud, Teel, Heemels, CDC 2017??

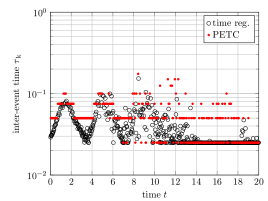
Time regularisation: Example



$$\mathcal{P} : \dot{x} = \begin{bmatrix}0 & 1\\ -2 & 3\end{bmatrix} x + \begin{bmatrix}0\\ 1\end{bmatrix} u + w$$
$$\mathcal{C} : u = \begin{bmatrix}1 & -4\end{bmatrix} x(t_k)$$

- Periodic Event-Triggered Control (PETC)
$$t_{k+1} = \inf\{t > t_k \mid \|x(t_k) - x(t)\| > \sigma\|x(t)\| \wedge t = kh, \ k \in \mathbb{N}\}$$
- Enforcing minimal inter-event time
$$t_{k+1} = \inf\{t > t_k + T \mid \|x(t_k) - x(t)\| > \sigma\|x(t)\|\}$$





Dynamic event-triggered control

- Static event generator: $t_{k+1} := \inf\{t > t_k + T \mid \mathcal{C}(x(t), e(t)) > 0\}$

Dynamic event generator [1,2,3]

$$\dot{\eta} = \Psi(x, e, \eta)$$
$$t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$$

[1] Postoyan et al., "Event-triggered and self-triggered stabilization ...," CDC 2011
[2] Girard, "Dynamic triggering mechanisms for event-triggered control," TAC 2015
[3] Dolk, Borgers, Heemels, *Output-based and Decentralized Dynamic ETC ...* , CDC14+ TAC 17
[4] Borgers, Dolk, Heemels, *Riccati-based design of ETCs for Linear Systems ...* , HSCC 17 + TAC18

Dynamic event-triggered control

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$$t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$$

- How to find Ψ and T?

[1] Postoyan et al., "Event-triggered and self-triggered stabilization ...," CDC 2011
[2] Girard, "Dynamic triggering mechanisms for event-triggered control," TAC 2015
[3] Dolk, Borgers, Heemels, *Output-based and Decentralized Dynamic ETC ...* , CDC14+ TAC 2017
[4] Borgers, Dolk, Heemels, *Riccati-based design of ETCs for Linear Systems ...* , HSCC 17 + TAC 18

Recap: Design relative triggering

- Perturbation perspective:

$$\dot{x}(t) = Ax(t) + BKx(t_k) = (A + BK)x(t) + BKe(t)$$

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Crux: Guarantee $\|e(t)\| \leqslant \rho a \cdot \|x(t)\|$ with $0 < \rho < 1$ s.t.

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2 \leqslant -(1-\rho^2)a^2\|x(t)\|^2$$

- Guarantee for Global Exponential Stability

$$t_{k+1} = \inf\{t > t_k \mid \|\underbrace{x(t_k) - x(t)}_{=e(t)}\| \geqslant \rho a \cdot \|x(t)\|\}$$

- Zeno-free: There is $T > 0$ such that $t_{k+1} - t_k \geqslant T$ for all $k \in \mathbb{N}$.

Basic design dETM

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

Basic design dETM

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Now consider $\dot{\eta} = \Psi(x, e, \eta)$ and LF $U(x, \eta) = V(x) + \eta$ [2] :

$$\frac{d}{dt}U \leqslant -a^2\|x\|^2 + \|e\|^2 + \Psi$$

Basic design dETM

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

- Now consider $\dot{\eta} = \Psi(x, e, \eta)$ and LF $U(x, \eta) = V(x) + \eta$ [2] :

$$\frac{d}{dt}U \leqslant -a^2\|x\|^2 + \|e\|^2 + \Psi$$

- To get $\frac{d}{dt}U \leqslant -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$ for some $\varepsilon > 0$ we require

$$-a^2\|x\|^2 + \|e\|^2 + \Psi = -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$$

  and thus $\dot{\eta} = \Psi(x, e, \eta) = \rho^2 a^2\|x\|^2 - \varepsilon\eta - \|e\|^2$

Basic design dETM

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

$$\frac{d}{dt}V \leqslant -a^2\|x(t)\|^2 + \|e(t)\|^2$$

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- Now $t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$, $\eta(0) = 0$ and $t_0 = 0$:
  - $\eta(t) \geqslant 0$ for $t \in \mathbb{R}_{\geqslant 0}$ and thus U positive definite
  - $\frac{d}{dt}U \leqslant -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$ and thus GES

Basic design dETM

- Since $A + BK$ Hurwitz, quadratic Lyapunov function $V(x) = x^\top Px$

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$$\frac{d}{dt}U \leqslant -a^2\|x\|^2 + \|e\|^2 + \Psi$$

- To get $\frac{d}{dt}U \leqslant -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$ for some $\varepsilon > 0$ we require

$$-a^2\|x\|^2 + \|e\|^2 + \Psi = -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$$

  and thus $\dot{\eta} = \Psi(x, e, \eta) = \rho^2 a^2\|x\|^2 - \varepsilon\eta - \|e\|^2$
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  - $\eta(t) \geqslant 0$ for $t \in \mathbb{R}_{\geqslant 0}$ and thus U positive definite
  - $\frac{d}{dt}U \leqslant -(1-\rho^2)a^2\|x\|^2 - \varepsilon\eta$ and thus GES
- Never triggers before the static version!!

Dynamic event-triggered control

- Static event generator: $t_{k+1} := \inf\{t > t_k + T | C(x(t), e(t)) > 0\}$

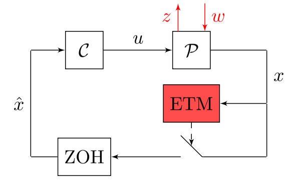
Dynamic event generator [1,2,3]

$$\dot{\eta} = \Psi(x, e, \eta)$$
$$t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$$

- [1,2] design for $w = 0$ (no disturbances)
- Recently, [3] new design methodology for output-based decentralized triggering under disturbances ($\mathcal{L}_p$-gain)
- New "non-conservative" designs tailored for linear systems using Riccati-based designs (output-based / disturbances ($\mathcal{L}_2$-gain)) [4]

[1] Postoyan et al., "Event-triggered and self-triggered stabilization ...," CDC 2011
[2] Girard, "Dynamic triggering mechanisms for event-triggered control," TAC 2015
[3] Dolk, Borgers, Heemels, "Dynamic Event-triggered Control...," CDC 2014 and TAC 2017
[4] Borgers, Dolk, Heemels, *Riccati-based design of ETCs for Linear Systems ...*, HSCC 17 + TAC 18

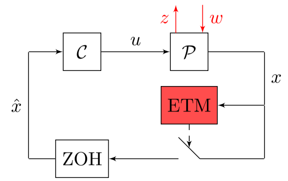
Dynamic ETC: Example



$$\mathcal{P} : \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$$
$$\mathcal{C} : u = \begin{bmatrix} 1 & -4 \end{bmatrix} x(t_k)$$

Case study: $\mathcal{L}_2$-gain $\theta = 4$ from input w to state x: $T = 9.1 \cdot 10^{-3}$

Dynamic ETC: Example



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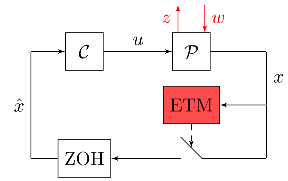
Case study: $\mathcal{L}_2$-gain $\theta = 4$ from input w to state x: $T = 9.1 \cdot 10^{-3}$

- *Dynamic* event generator $t_{k+1} := \inf\{t > t_k + T \mid \eta(t) < 0\}$
- *Static* event generator: $t_{k+1} := \inf\{t > t_k + T \mid \Psi(x, e, \tau, \eta) < 0\}$

Dynamic ETC: Example



$$\mathcal{P} : \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + w$$
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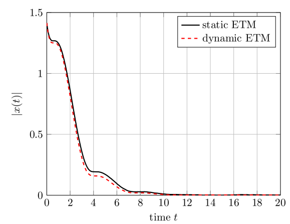
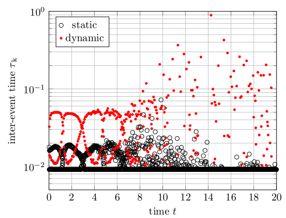
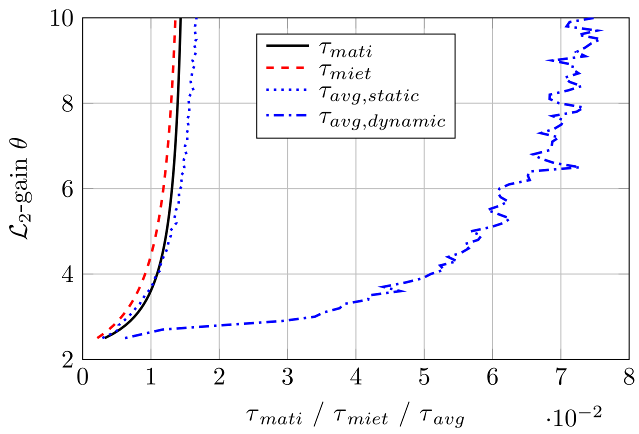


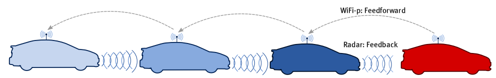


Dynamic ETC: Example

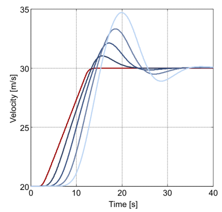


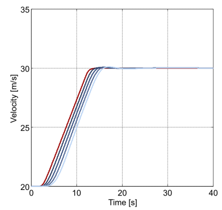
Real-life application

Cooperative Adaptive Cruise Control

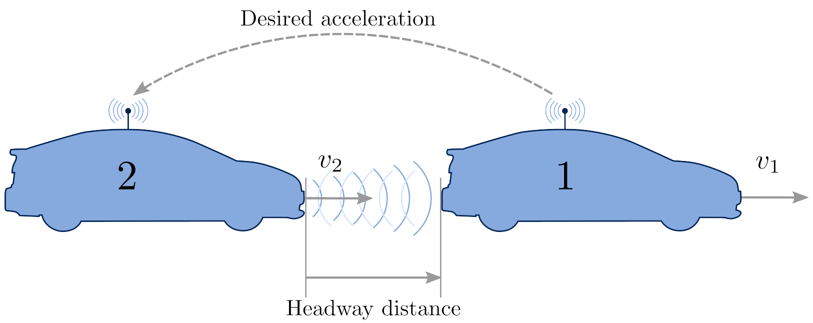


- String stability: disturbance attenuation along the vehicle string
  - $\mathcal{L}_p$-gain$\leqslant 1$
- Communication resources limited $\rightarrow$ event-triggered communication

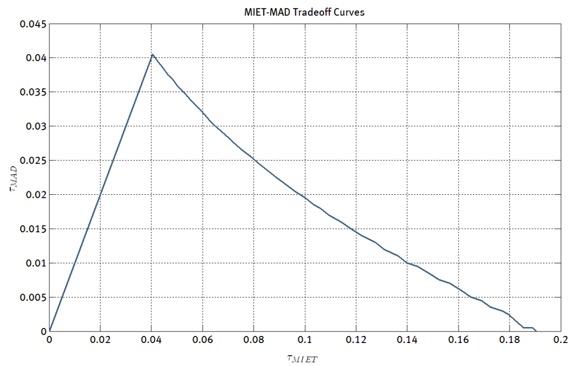




Cooperative Adaptive Cruise Control



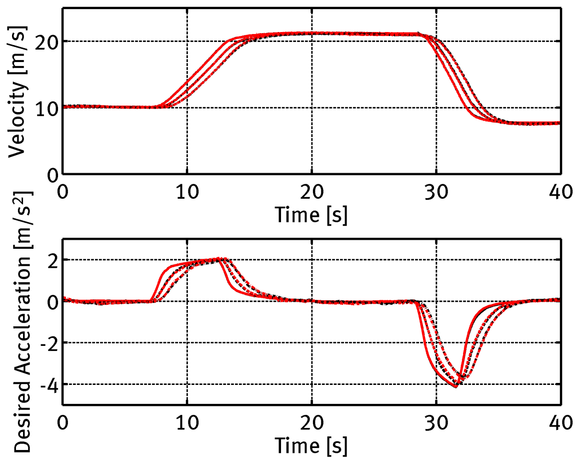
CACC



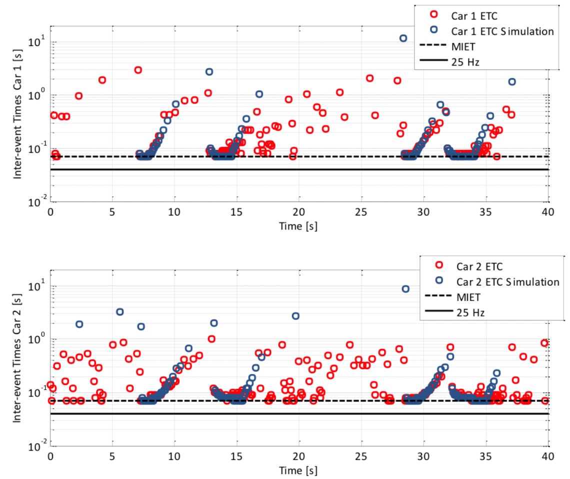
- Headway time: 0.6 seconds
- MIET: 0.07 seconds

$\longrightarrow$ MOVIE

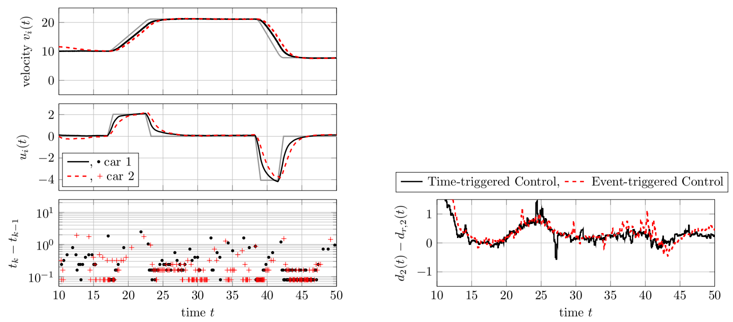
CACC



CACC



Event-triggered CACC



- Similar behaviour as time-triggered communication
- 60-70% less communication

[1] Dolk, Ploeg, Heemels, *Event-triggered Control for String-Stable Vehicle Platooning*, submitted IEEE Trans. ITS [HSCC'17]

Conclusions

- Event-triggered control: A new resource-aware control paradigm
- Several ETC algorithms discussed with their own tools (hybrid)
- Challenges
  - Performance / Robustness w.r.t. disturbances
  - Output-based & decentralized event generators
  - Constrained systems (MPC)
  - Implementation and Applications
  - Better than periodic time-triggered control
  - Improved analysis and design tools: MIET, average inter-execution times, $\mathcal{L}_p$-gain, etc.
- Many interesting practical and theoretical issues open in this appealing research field

"Wise men speak because they have something to say, fools because they have to say something." – Plato

- More info: http://www.heemels.tue.nl

Acknowledgements

- Collaborators
  - Duarte Antunes, Mahmoud Abdelrahim, Behnam Asadi, Hadi Balaghi, Niek Borgers, Florian Brunner, Victor Dolk, Tijs Donkers, Tom Gommans, Stefan Heijmans, Heico Sandee, Nard Strijbosch, ...
  - Frank Allgöwer, Adolfo Anta, Jamal Daafouz, Geir Dullerud, Kalle Johansson, Dragan Nesic, Romain Postoyan, Paulo Tabuada, Andy Teel, Paul van den Bosch, ...
- Financial support

STW Enabling new technology

NWO Netherlands Organisation for Scientific Research

- More info: http://www.heemels.tue.nl

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Recent overviews:

- W.P.M.H. Heemels, K.H. Johansson, and P. Tabuada, *An introduction to event-triggered and self-triggered control*, 51st IEEE Conference on Decision and Control 2012, Hawaii, USA, p. 3270-3285
- W.P.M.H. Heemels, K.H. Johansson, and P. Tabuada, *Event-Triggered and Self-Triggered Control*, Encyclopedia of Systems and Control, Springer-Verlag London 2014.

Pointers for "better than periodic time-triggered control:"

- D. Antunes and W.P.M.H. Heemels, *Rollout Event-Triggered Control: Beyond Periodic Control Performance*, IEEE Transactions on Automatic Control 59(12), p. 3296-3311, 2014.
- Astrom, Bernhardsson *Comparison of periodic and event based sampling for first order stochastic systems*, IFAC World Congress 1999
- T.M.P. Gommans, D. Antunes, M.C.F. Donkers, P. Tabuada, W.P.M.H. Heemels, Self-Triggered Linear Quadratic Control, Automatica 50(4), p. 1279-1287, 2014.

$\longrightarrow$ More info: http://www.heemels.tue.nl

Dessert:

Approximate dynamic programming approach to resource-aware control

$\longrightarrow$ Connection to lecture/work by Prof. Dimitri Bertsekas

Roll-out LQR

- Linear system:

$$x_{t+1} = Ax_t + Bu_t$$

- Control costs:

$$J_{\text{cont}} = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t$$

- Communication costs:

$$J_{\text{comm}} \sim f_{\text{ave}} = \frac{1}{h_{\text{ave}}}$$

Three variants

(A) $\min J_{\text{cont}} + \rho J_{\text{comm}}$

(B) $\min J_{\text{comm}}$ s.t. $J_{\text{cont}} \leqslant c_{\text{cont}}$

(C) $\min J_{\text{cont}}$ s.t. $J_{\text{comm}} \leqslant c_{\text{comm}}$ [1,2]

[1] D. Antunes, W. P. M. H. Heemels, P. Tabuada, CDC12
[2] D. Antunes, W. P. M. H. Heemels, *Rollout Event-Triggered Control: Beyond Periodic Control Performance*, TAC 14

Roll-out LQR

- Control costs:

$$J_{\text{cont}} = \sum_{t=0}^{\infty} x_t^\top Q x_t + u_t^\top R u_t$$

- Communication costs: $J_{\text{comm}} \sim f_{\text{ave}} = \frac{1}{h_{\text{ave}}}$

$$x_{t+1} = \begin{cases} Ax_t + Bu_t, & \text{when } u_t \text{ transmitted at time } t\ (\sigma_t = 1) \\ Ax_t + Bu_{t-1} & \text{when } u_t \text{ not transmitted at time } t\ (\sigma_t = 0) \end{cases}$$

Problem: Design control/scheduling policy $\pi = \{(\mu_t^\sigma, \mu_t^u)\}_{t\in\mathbb{N}}$ with

$$(\sigma_t, u_t) = (\mu_t^\sigma(x_t), \mu_t^u(x_t))$$

minimizing J_{cont} s.t. $J_{\text{comm}} \leqslant c_{\text{comm}} = \frac{1}{q}$ (i.e., $h_{\text{ave}} \geqslant q$)

Roll-out LQR: The main idea

- Receding horizon: Scheduling times $t_k = kh$ with h sched. period
- Optimal control problem: based on state $x_{t_k} = x$ minimize J_{cont} over admissible schedules $\{\sigma_t^j\}_{t\in\mathbb{N}}$, $j = 1, 2, \ldots, J$, and inputs $\{u_t\}_{t\in\mathbb{N}}$

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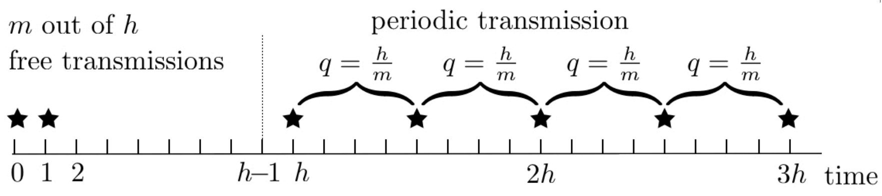
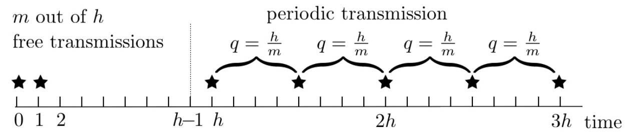
  - Free choice: $m = \frac{h}{q}$ transmissions at $0, 1, 2, \ldots, h-1$
  - Roll-out algorithm: after time h periodic transmission with period $q = \frac{h}{m}$



Roll-out LQR



- Resulting scheduling/control policy: At scheduling time $t_k = kh$ and state $x_{t_k} = x$

$$j^* = \text{arg min}\{x^\top P_j x \mid j = 1, 2, \ldots, J\}$$

with $x^\top P_j x$ the optimal costs corresponding to schedule $\{\sigma_t^j\}_{t\in\mathbb{N}}$

$$\sigma_t = \sigma_t^{j^*}$$

$$u_t = K_{t,j^*} x_t,\ t \in \mathbb{N}_{[kh,(k+1)h)}$$

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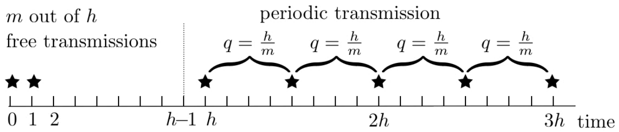

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- This policy results in $h_{ave} = q$ and thus $J_{\text{comm}} = \frac{1}{q}$
- Outperforms periodic schedule with period q!

Roll-out LQR: Numerical example

$$\dot{x} = A_C x + B_C u$$

with

$$A_C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\kappa_m & \kappa_m & 0 & 0 \\ \kappa_m & -\kappa_m & 0 & 0 \end{bmatrix}, \quad B_C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \kappa_m = 2\pi^2$$

- Control costs: $\int_0^\infty (x_1^2 + x_2^2 + 0.1 u_C^2) dt$

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