The path to true concurrency (Sculptures, ST-structures and HDAs)

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Abstract

The main purpose is to provide concrete relationships between highly expressive concurrency models coming from two different schools of thought: the higher dimensional automata, a state-based approach by Pratt and van Glabbeek; and the configuration structures and unrestricted event structures, an event-based approach by van Glabbeek and Plotkin. In this respect, we will define the method of sculpting, described by Pratt, in categorical terms to better understand the event-state duality defined by Pratt. These investigations of sculptures in category theory are intended to provide a better understanding of the relationship between ST-structures, HDAs and Sculptures in terms of expresiveness.

1 Introduction

One of the most commonly used models of concurrency is that of automata, also known as process graphs, state transition diagrams or labelled transition systems. In ordinary automata, the parallel composition of two actions a and b is displayed as a process P that executes a and b in either order, ending in the same state each way, such that a and b are mutually exclusive. There is a hidden assumption of excluded middle as described by Pratt in [3], such that the dual of a true concurrency schedule appears to be a false concurrency automaton.

This has lead to the geometric model of concurrency, studied by Pratt and van Glabbeek [3, 4, 9], which is of high expressive power, thus providing a general framework for studying the difference and common features of various other models of concurrency(as done in [9] and [8]). This model was named Higher Dimensional Automata(HDA) by Pratt[3]. HDAs retained the state-based view and models $a||b$ as the evident four-state "square" automaton accepting ab + ba, but with its square interior filled in(Section 2). This aspect is opposed to the event-based model of concurrency, like (prime, flow, (non-)stable) event structures[1, 2, 10] or configuration structures and unrestricted event structures[6, 7].

We are interested in studying such models based on sets of event, but in relation to the state-based model of higher dimensional automata. This study of event-state duality is argued by Pratt[5], and the model of chu spaces has been developed in response[13, 14].

Here the challenge of Pratt with insight from chu spaces and the models based on sets of event, in the spirit of van Glabbeek and Plotkin[7], are addressed in [ST-paper](Section 3). This model was named ST-structures(ST)[ST-paper]. The relationship between ST-structures and HDAs gives an intuition of the event-state duality described by Pratt, and the method of sculpting gives the relation between ST-structures and HDAs.

2 HDA

Definition 2.1. (higher dimensional automata [3, Def.1])

A cubical set H = (Q, $\bar{s}$, $\bar{t}$) is formed of a family of sets Q=$\cup_{n=0}^{\infty} Q_n$ with all sets Q_n disjoint, and for each n, a family of maps s_i, $t_i : Q_n \to Q_{n-1}$, with $1 \leq i \leq i \leq n$, which respects the following cubical laws:

$$\alpha_i \circ \beta_j = \beta_{j-1} \circ \alpha_i,\ 1 \leq i < j \leq n \ and \ \alpha, \beta \in \{s, t\}. \tag{1}$$

In H, the $\bar{s}$ and $\bar{t}$ denote the collection of all the maps from all the families (i.e., for all n).A higher dimensional automaton (Q, $\bar{s}$, $\bar{t}$, l, F) over an alphabet σ is a cubical set together with a labelling function l:$Q_1 \to \sigma$ which respects $l(s_i(q))$ = $l(t_i(q))$ for all $q \in Q_2$ and $i \in \{1, 2\}$; and with $I \in Q_0$ initial and $F \subset Q_0$ final cells.

The elements of Q_0 are called nodes and those of Q_1, Q_2 and Q_3 are *edges*, *square* and *cubes*, respectively. In general, the elements of Q_n are called *n-dimensional hypercubes*, or *n-cells*. An n-dimensional hypercube represents a state of a concurrent system in which n transitions are firing concurrently. Because the dimensions of the hypercube are numbered 1,...,n, these transitions are de facto stored as a list.

3 ST-structures

We define ST-structures, showing in Section 3 that they are a natural extension of configuration structures[9], and define related notions that stem from the latter. The classical notions of concurrency, causality and conflict are not interdefinable as in the case of event structures or stable configuration structures; but are more loose, as in the case with HDAs.

Definition 3.1. (ST-structures) An ST-configuration structure(also called ST-structure) is a tuple ST = (E, ST, l) with ST a set of ST-configurations over E satisfying the constraint:

$$if(S,\ T) \in ST \ then \ (S,\ S) \in ST, \tag{2}$$

and l:$E \to \sigma$ a labelling function with σ the set of labels. We often omit the set of events E from the notation when there is no danger of confusion.

The contraint(2) above is a closure, ensuring that we do not represent events that are started but never terminated. he set of all ST-structures is denoted ST.

4 Sculptures

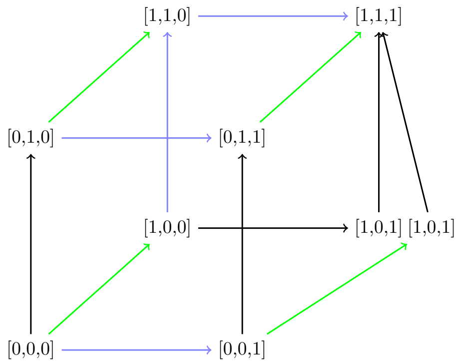
5 event structures / configuration structures

6 Results/just future plans

6.1 category theory relate sculptures

6.1.1 through ST structures

6.2 proof of non-sculpture



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