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Introduction to Optimisation, GAMS and Electricity System Modelling

TEK5410 Lecture 2

Muhammad Shahzad Javed

m.s.javed@its.uio.no

Department of Technology Systems

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Contents

Learning outcome

After completing this course, you

- **know the basics of linear programming**
- are able to define research questions on electricity system and market problems and to build model formulations to answer them
- are able to collect and prepare input data using Python
- **are able to build your own electricity system model in GAMS** and conduct the analysis
- are able to interpret model results
- are able to describe your modelling approach and results including shortcomings
- can read and critically analyse state-of-the-art literature based on electricity models
- are able to write a research report and present it to a specialized audience

Contents

What is linear optimisation?

Linear optimisation is a method to achieve the best outcome (e.g. maximise revenue, minimise costs) in a mathematical model whose requirements are represented by linear relationships.

What is linear optimisation?

- Objective function
- Decision variables
- Constraints

What is linear optimisation?

Objective function:

$$\text{minimise } z(x_1, x_2) = c_1 x_1 + c_2 x_2$$

What is linear optimisation?

Objective function:

$$\text{minimise } z(x_1, x_2) = c_1x_1 + c_2x_2$$

Constraints $x_1 \geq 0$

$x_2 \geq 0$

Exercise (Mathematical Formulation)

Island power system linear optimisation example
Adopted from an example by Francesco Gardumi and Youssef Almulla at KTH Royal Institute of Technology

Exercise (Mathematical Formulation)

We want to plan the energy supply for a small isolated island in order to meet its annual electricity demand

Information

We want to plan the energy supply for a small isolated island in order to meet its annual electricity demand

- The annual electricity demand is 250 kWh per year
- The island has two available power plants, one coal-fired and one gas-fired.
- The cost of production for the coal power plant is €350/kWh and there is 200 kWh of coal available at the island
- The cost of production for the gas power plant is €300/kWh and there is 150 kWh of gas available at the island

Mathematical formulation

Objective function:

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Mathematical formulation

Objective function:
Minimise the cost for the power system
g_c = the amount of electricity [kWh] generated from coal
g_g = the amount of electricity [kWh] generated from gas

$$\min Z(g_c, g_g) = 350 \cdot g_c + 300 \cdot g_g$$

Mathematical formulation

Constraints
What limitations do we have to our optimisation problem?

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Mathematical formulation

Constraints

What limitations do we have to our optimisation problem?

$$g_c \leq 200 \text{ kWh - coal availability}$$

$$g_g \leq 150 \text{ kWh - gas availability}$$

$$g_c + g_g \geq 250 \text{ kWh - demand constraint}$$

... ???

Mathematical formulation

Constraints

What limitations do we have to our optimisation problem?

$$g_c \leq 200 \text{ kWh - coal availability}$$
$$g_g \leq 150 \text{ kWh - gas availability}$$
$$g_c + g_g \geq 250 \text{ kWh - demand constraint}$$
$$g_c \geq 0$$
$$g_g \geq 0$$

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Mathematical formulation

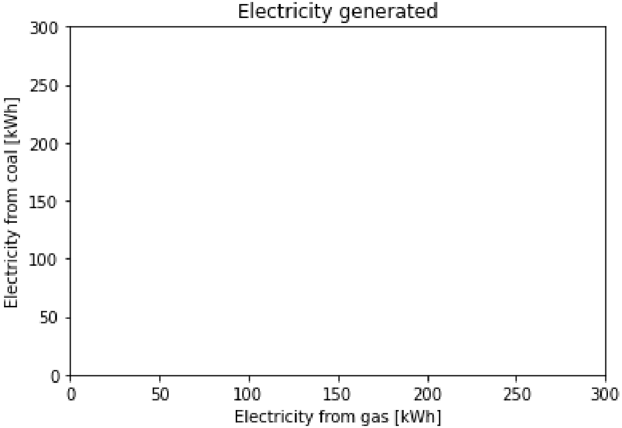
Objective:

$$\min Z(g_c, g_g) = 350 \cdot g_c + 300 \cdot g_g$$

Subject to:

$$g_c \leq 200 \text{ kWh - coal availability}$$
$$g_g \leq 150 \text{ kWh - gas availability}$$
$$g_c + g_g \geq 250 \text{ kWh - demand constraint}$$
$$g_c \geq 0$$
$$g_g \geq 0$$

Graphical representation



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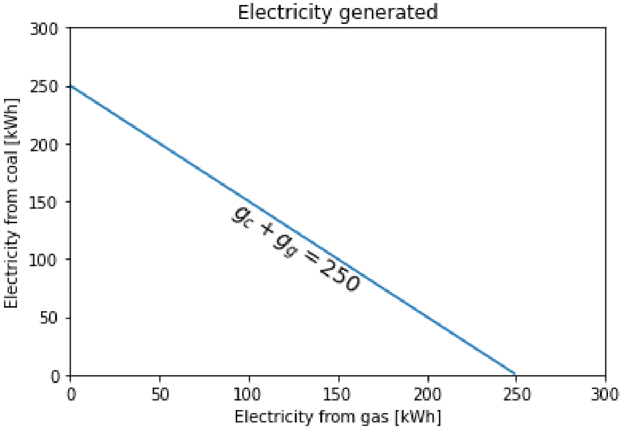
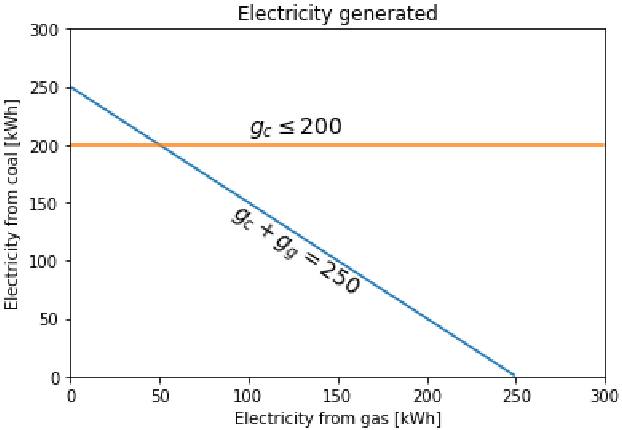
Graphical representation

The demand constraint



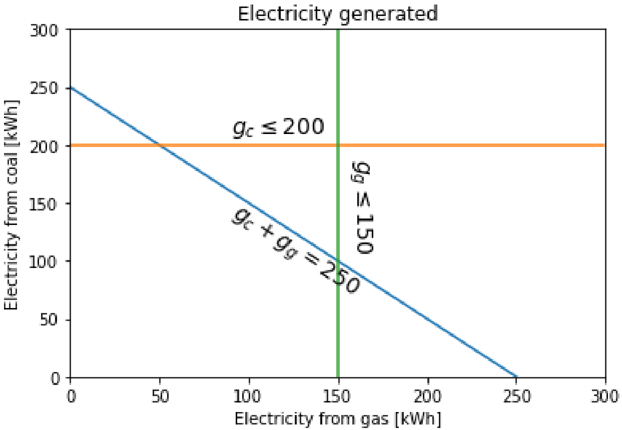
Graphical representation

The coal availability constraint



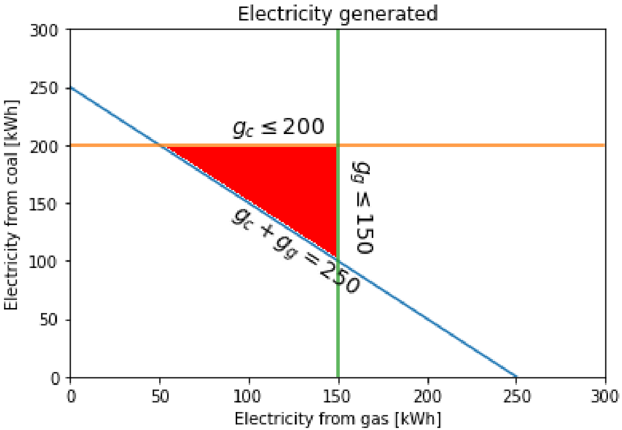
Graphical representation

The gas availability constraint



Graphical representation

The final solution space



Exercise 2

The Norwegian Water Resources and Energy Directorate (NVE) has tasked you with designing the techno-economic optimal Norwegian electricity system for **2050**. To do so, you need to use a linear optimisation model to find the mathematical solution to how much of different renewable electricity generating technologies need to be built.

There is no more potential to build more hydropower than there currently is today. Therefore, the additional demand of electricity has to be met either by new onshore & offshore wind power, ground-mounted solar PV systems or import of electricity.

With an almost fully electrified Norwegian society, projections indicate that there will be an increase in the annual demand of 40 TWh. The cost of installing onshore & offshore wind power is 1355 & 1750 EUR/kW installed capacity and 400 EUR/kW for solar PV. However, due to the resistance towards onshore wind power, NVE has informed that the new system cannot contain more than 5000 MW of onshore wind power.

Develop a formal mathematical description for the optimisation problem.

Contents

Introduction to GAMS

- Initially developed by the World Bank
- GAMS is a modelling language used to transform real world optimisation problems into computer code and make it more manageable.
- The structure and organisation of GAMS programs
  - consist of statements (data structures, initial values, equations)
  - all statements end with a semicolon ";"

Data entries

Table: Scalars, parameters and tables (see GAMS documentation)

Keyword	Description
Scalar	Single data entry (one number)
Parameter	List oriented data, defined over one or more sets
Tables	Tabular data for higher-dimensional parameters

Variables

Variables in GAMS is what we call decision variables and is what the model can decide. They are unknown until the model has been solved.

Table: Some variable types

Type	Description
free (default)	no bounds
positive	no negative values
binary	0 or 1
integer	discrete - 1,2,3...

Our objective function must always be a free variable in GAMS.

Equations

In GAMS, equations are algebraic relationships which are used to generate constraints for the model.
It must first be declared before it can be defined and used in the model.

Table: Equation operators

Type	Description
=E=	Equality (LHS = RHS)
=G=	Greater than ($\geq$)
=L=	Less than ($\leq$)

Model and solve statement

The model statement is used to collect equations into groups and label them so that they can be solved. The simplest form is to use the keyword: "all". Once a model has been defined, the solve statement prompts GAMS to call on one of the solvers.

Syntax:

solve model_name maximizing|minimizing var_name using model_type

Exercise 3

We want to plan the electricity supply for a small isolated island in order to meet its annual electricity demand

Objective:

$$\min Z = 350 \cdot g_c + 300 \cdot g_g$$

Subject to:

$$g_c \leq 200 \text{ kWh - coal availability}$$
$$g_g \leq 150 \text{ kWh - gas availability}$$
$$g_c + g_g \geq 250 \text{ kWh - demand constraint}$$
$$g_c \geq 0$$
$$g_g \geq 0$$

Island power system linear optimisation example

```
scalar
* The scalar statement is used to declare and (optionally) initialize a GAMS parameter of dimensionality zero.
* This means that there are no associated sets, so there is exactly one number associated with the parameter.
;

positive variables
* decision variables

;

free variables
* default so we could just write "variables"
* the thing we maximise/minimise need to be a free variable (although it is a positive variable).
;

equations

;


model optimal_generation /
                            all/;

solve optimal_generation minimizing var_system_cost using lp;
```

Syntax: scalar

```
scalar
scalar_name [text] [/numerical_value/]
demand "electricity demand [kWh]" /250/
;
```

Syntax: Variables

```
var_type
variable_name ["explanatory text"]
;
```

```
free variables
var_system _cost "Total system cost"
;
```

Syntax: Equations

Declaring equations
equations
equation_name [explanatory text]
eq_objective "The objective function"
;

Defining equations
equation_name.. expression eqn_type expression ;
eq_objective.. var_system_cost =E= generation_coal * cost_coal ;

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Sets in GAMS

Sets are important in GAMS and help structure a model. A set is a collection of elements and is defined in GAMS similarly to its mathematical notation:

$$Set = \{a, b, c\}$$

Syntax:

Set

set_name ["optional text"] [/element$_1$, element$_2$,...element$_n$/]

$$Set\ S/a, b, c/;$$

$$Set\ technology/coal, gas/$$

Syntax: parameter

Syntax:

```
Parameter parameter_name (index_list) /
element_name numerical_value
element_name numerical_value
/;
```

e.g.

```
parameter capacity(technology)/
coal 200
gas 150
/;
```

Loading input data

Dollar Control Options ($include)

Syntax:
$include external_file

e.g.
$include tech_costs.tsv

More uses of the dollar operator

if statements

Condition

if (b > 1.5), then a = 5

```
if b > 5:
    a = 5
```

In GAMS

```
a $ (b > 1.5) = 2 ;
```

What is next?

- Office hour 17/9
- Assignment 2 due 24/9
- Data preparation with Python 24/9 (Tobias)

Assignment 2

The Future Energy System of Liberland

Muhammad Shahzad Javed
E-mail: `m.s.javed@its.uio.no`
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