Ran out of time for question 2 :(

2) a) $\max_{\lambda_i \geq 0} \min_{\omega, \alpha} \|\omega\|^2 - \sum_{i=1}^{n} \lambda_i (y_i (x_i \cdot \omega + \alpha) - 1)$

$$\max_{\lambda_i \geq 0} \sum_{i=1}^{n} \lambda_i - \sum_{i=1}^{n} \lambda_i (y_i (x_i \cdot \underline{\omega + \alpha) - 1)}$$

$$\max_{\lambda_i \geq 0} \sum_{i=1}^{n} \lambda_i - \sum_{i=1}^{n} \lambda_i y_i x_i \cdot \overset{\sum_{j=1}^{n}}{\overset{\downarrow}{\omega}} + \alpha - 1$$

$$\boxed{\max_{\lambda_i \geq 0} \sum_{i=1}^{n} \lambda_i - \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2} \sum_{j=1}^{n} \lambda_i y_i x_i \lambda_j y_j x_j}$$

b) $r(x) = \begin{cases} +1 & \text{if } \omega \cdot x + \alpha \geq 0 \\ -1 & \text{otherwise} \end{cases}$

weights of each λ, y, x

$$r(x) = \begin{cases} +1 & \text{if } \sum_{i=1}^{n} \lambda_i y_i x_i \cdot x + \alpha \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

$$\boxed{r(x) = \begin{cases} +1 & \text{if } \alpha + \frac{1}{2} \sum_{i=1}^{n} \lambda_i y_i x_i \cdot x \geq 0 \\ -1 & \text{otherwise} \end{cases}}$$

c) Looking at the condition, for all $i > 0$, the condition goes to 0, therefore for points corresponding to $\lambda_i^* > 0$ the other variables y_i, x_i, α, ω must result in 0.

d) Support vectors are the only training points needed because they add meaningful information to the training set, while other training sets may not.

e) The support vectors are the points on the graph closest to the margin

f) Use contradiction: Let's assume there are no support vectors for each class.

New weight vector: $\omega' = \frac{\omega}{1+\epsilon/2}$ with bias α
$\epsilon > 0$

Symmetric argument: For the class when it does fit, a decision boundary has to be present, per the definition of a support vector, it is the closest to the decision boundary, therefore there must be at least one support vector.