Portfolio Optimisation

A Comparison of the Mean-Variance and Mean-Conditional Value-at-Risk Portfolio Optimisation Techniques

Dissertation in BA Economics

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Declaration

This dissertation is the result of my own work. Material from the published or unpublished work of others, which is referred to in the dissertation, is credited to the author in question in the text. The dissertation is approximately 12,000 words in length.

Abbreviations

CVaR- Conditional Value-at-Risk

ES- Expected Shortfall

MAD- Mean-Absolute Deviation

MCVaR- Mean-Conditional Value-at-Risk

MV- Mean-Variance

VaR- Value-at-Risk

Abstract

How should one allocate funds to meet their investment objectives? The following is a microeconomic investigation into utility maximisation under uncertainty through the use of portfolio optimisation techniques. The analysis focuses on the mean-variance (MV) and mean-conditional value-at-risk (MCVaR) optimisation techniques for market representatives of the UK, USA, India, Europe and the World. Performance is considered in both the presence of perfect predictive posterior return distributions and with the use of historical distributions alone to predict future performance. It is found that both techniques are able to offer strong returns against most risk metrics tested, though MCVaR-optimal portfolios more often offer a better risk-adjusted return, whilst remaining more stable in presenting the risk-return trade-off. Against the MV optimisation method, MCVaR optimisation gives much greater control over investor preferences regarding the skewness but not kurtosis of return distributions. Historical returns alone prove themselves to offer very little predictive power for future returns, though risk metrics such as semi-variance demonstrate stability over time. Of the markets considered, it appears that with more opportunity for diversity, the optimisations are able to derive more return at the expense of more risk but are far less stable without perfect predictive return distributions. Further, the MV optimisation approach provides little consideration of potential extreme losses, an important factor in investor utility. The approach of equally weighting large gains and large losses proves to seriously lack risk consideration when compared with MCVaR-optimal portfolios. Thus, the MCVaR portfolio optimisation method overall proves to outperform the MV method in all environments tested, with the only exception being the presence of strict computing constraints.

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1. Introduction

> "Finally, I would like to add a comment concerning portfolio theory as a part of the microeconomics of action under uncertainty. It has not always been considered so. For example, when I defended my dissertation as a student in the Economics Department of the University of Chicago, Professor Milton Friedman argued that portfolio theory was not Economics, and that they could not award me a Ph.D. degree in Economics for a dissertation which was not in Economics. I assume that he was only half serious, since they did award me the degree without long debate. As to the merits of his arguments, at this point I am quite willing to concede: at the time I defended my dissertation, portfolio theory was not part of Economics. But now it is."
>
> Harry M. Markowitz (1991, p.476)

The objective of portfolio optimisation is to allocate the finances available to an investor in a way that offers as high a return as possible, whilst having the risk to this investment minimised. This field was, until fairly recently, not believed to hold much value above simply assessing the returns of assets individually and simply choosing to hold the best ones simultaneously.

The field of portfolio theory has gone through a great deal of change over the last century, starting with Harry Markowitz's concept of mean-variance (MV) optimisation. Since, the metric of conditional value-at-risk (CVaR) has become prominent as a risk measure for financial regulation and investment appraisal. This paper seeks to test whether either model is able to outperform the other when assessed against a variety of risk measures and for multiple different markets. Previous investigations have sought to prove that portfolio optimisation models outperform one another but only deduce this by comparing them against the risk measure through which one is optimised (Krokhmal et al., 2002). Given there is no perfect risk measure for all purposes, this is a flawed methodology as these methods are destined to perform optimally against their respective objectives. To provide a more thorough comparison, the MV and mean-CVaR (MCVaR) portfolio optimisation methods will be assessed against a variety of risk measures to advise which is better.

Much of the testing of newly proposed models often only assesses their performance for a single, small group of assets over a short period of time, with the assumption of perfect posterior predictive return distributions (Krokhmal et al., 2002, Alexander et al., 2007, Angelelli et al., 2008, Alem et al., 2017). This investigation uses 11 years of daily returns data for five different markets, with a minimum of 500 assets considered in each and for two periods; one in which return distributions are predicted perfectly and another where historical return distributions are used to predict future returns, referred to as the historical and applied periods. The intention is to gain a thorough understanding of their relative strengths and performance under imperfect return predictions.

The intent of this study is to first present this flaw in previous assessments but also provide guidance as to the best passive investment strategy to undertake given the investment environment, the set of assets considered and also the specific preferences that an investor may hold in the risk-return trade-off. There is no single perfect universal risk measure against which investor preferences can be set- otherwise the field of investment analysis would be far simpler - and so the consideration of multiple metrics, each with their own distinct features, allows a more complete evaluation of the risk each portfolio poses. The hope is to inspire others to recognise this shortcoming and adopt the framework presented to gain a better understanding of passive investment techniques, applying it to many of the alternative portfolio optimisation techniques and choices of risk measure. The resulting improvements would benefit a wide range of investors through better asset allocation and increased utility in both individual and commercial investing, providing motivation to investigate this potential.

Chapters 2 to 4 will cover the MV and MCVaR methods of portfolio optimisation and investor utility frameworks against which they will be assessed. Then, chapter 5 will explain the methodology and risk metrics used to assess the optimisations. Next, the results will be assessed in chapter 6 and finally chapter 7 will summarise the implications of these results for future research and investment advice.

2. Mean-Variance Analysis

Mean-variance optimisation was pioneered in Markowitz's seminal PhD, 'Portfolio Selection' (1952). In his model, investors are assumed to be risk-averse, with a higher variance of returns seen to be less desirable. Variance indicates the likelihood of gaining a return at or around that expected from a security. Its major contribution was realising that the returns of a portfolio are linear, but its variance is not linear with regard to the variances of its component securities. This allows a higher return to be gained with a lower risk being undertaken than would be if investing in the securities individually. This was simply elaborating on the concept of diversification that had been used for centuries (Bernoulli, 1738 in Rubinstein, 2002), but without any mathematical proof of concept or true benefit. The proof of such a concept is shown below (adjusted from Markowitz, 1952).

The portfolio return is given by:

$$\mu_p = \sum_{i=1}^{N} \omega_i \mu_i \tag{2.1}$$

Where p refers to the portfolio, μ_i refers to the expected return of asset i, ω_i is the weighting of asset i in the portfolio and N assets are considered for the portfolio.
The variance of the portfolio's returns can be rearranged to give:

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \omega_i \omega_j \sigma_{ij} \tag{2.2}$$

Where σ_i^2 is the variance of the returns of asset i and σ_{ij} is the covariance between the returns of assets i and j

Then, incorporating correlation for easier interpretation.

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \rho_{ij} \omega_i \omega_j \sigma_i \sigma_j \tag{2.3}$$

Here the variance of a portfolio will always be below the weighted sum of its components' variances, as long as they are not all perfectly positively correlated with one another. This is written as:

$$\sum_{i=1}^{N} \omega_i \sigma_i^2 > \sigma_p^2$$

Given $\exists \rho_{ij} \neq\ 1$ for any $i, j \in \{1,2,\dots,N\}$ where $i \neq j$ and $\omega_i \in [0,1) \quad \forall i \in \{1,2,\dots,N\}$

The expected return of the portfolio will still be equal to the weighted sum of its component asset returns.

The MV portfolio optimisation problem can then be set out as:

$$\min_{\omega_i} \sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \omega_i \omega_j \sigma_{ij}$$

$$s.t$$

$$\mu_p \geq \sum_{i=1}^{N} \mu_i \omega_i$$

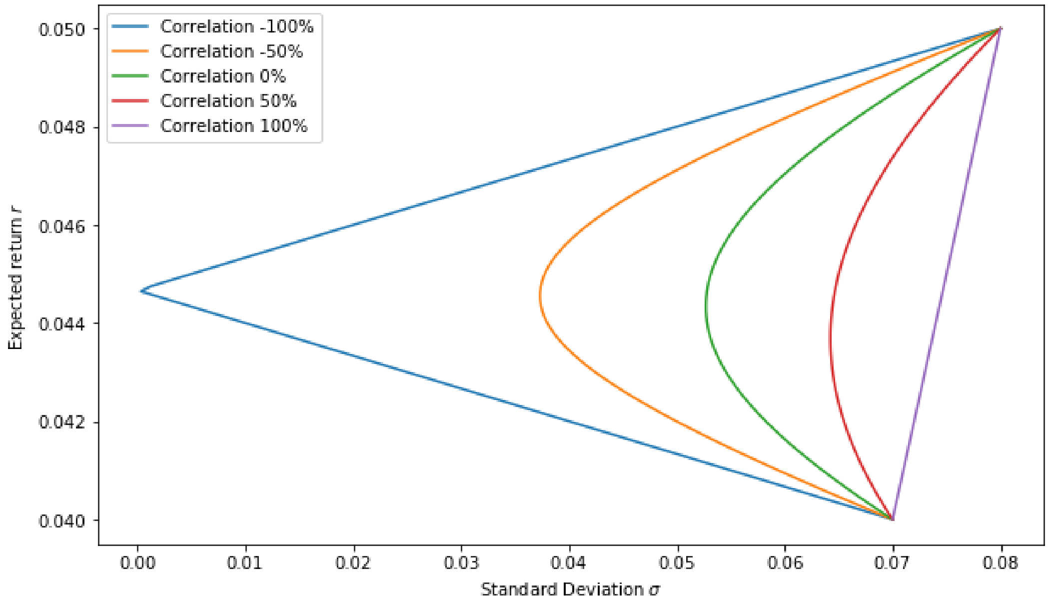
$$\sum_{i=1}^{N} \omega_i = 1$$

From here an efficient frontier can be generated, along which the minimum risk is undertaken for each possible level of return, by adjusting μ_p.

The shape of the efficient frontier varies depending on the correlation between the returns of each pair of assets. It can vary from a straight line between two perfectly correlated assets and two straight lines meeting along the expected returns axis. With a non-perfect correlation between the asset returns, the efficient frontier follows an egg shape that grows more curved the lower the correlation. This concept is illustrated in Figure 2.1.

Figure 2.1

Efficient Frontier Shape Depending on the Correlation Between Two Assets



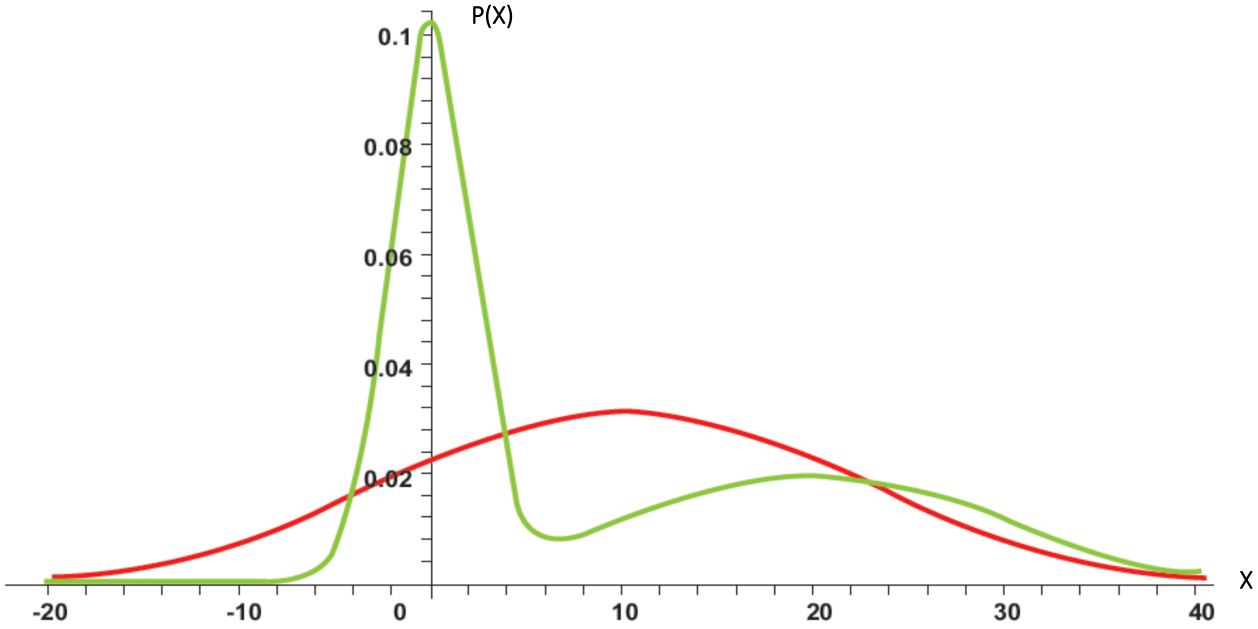
Source: Ondrej Martinsky, 2018

This process of portfolio optimisation brought much more interest to the topic and pushed its evolution over the past 67 years. One of the more significant suggestions to adjust the model included Markowitz (1991) himself considering the use of semi-variance rather than variance. This measured only the variance of the returns below the expected return since investors are not disappointed by returns being above expectations. Another alteration made to the model was by Beasley et al. (2013). Their model looked to generate a dynamic Markowitz portfolio which included transactions costs. This means that the portfolio could be considered for rebalancing each period to maintain a choice of optimal portfolio once costs and the investment horizon are taken into consideration. This had been a major drawback to the original Markowitz methodology and hence allowed for a more realistic application of the concept.

Another of the drawbacks of mean-variance analysis is that it fails to account for higher central moments in the distributions of the returns for the securities considered in the portfolio optimisation problem. All asset and portfolio returns are assumed to be normal, a very rare occurrence (see Introduction), which creates substantial inaccuracy in estimating portfolio risk as the probably densities at each standardised z-value $\left(z_i = \frac{x_i - \mu}{\sigma_i}\right)$ are not as dictated by the normal distribution, which assumes the central moments above the second take a constant value. For example, the third is assumed to always be zero and the fourth to be three. These central moments were found to have an effect on investor preferences (Horvath and Scott, 1980). Consider the two distributions in Figure 2.2, for example.

Figure 2.2

Both Distributions Have the Same Mean and Variance



Source: Keating and Shadwick, 2002, pp.12 in Evestment, 2018

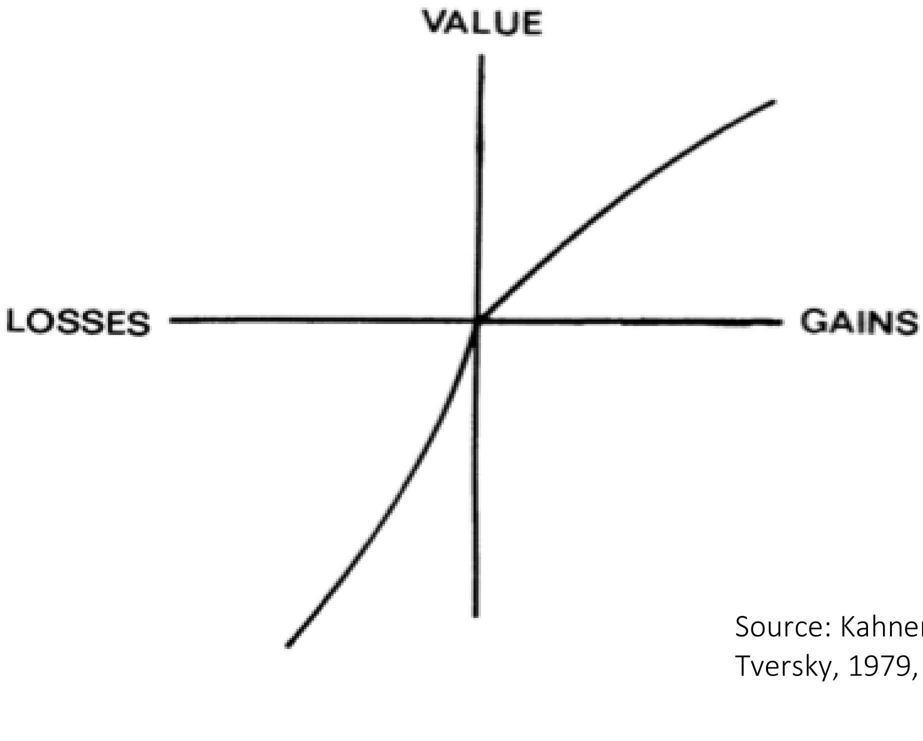
It is clear that investors would have different preferences over the distributions of returns that they offer, suggesting the need to take higher moments into account. The distribution in green is clearly not normal but could represent an asset's true return distribution.

Markowitz only ever investigated how to use stock data that correctly predicts the future behaviour of all the associated assets. He also did not specify how these were to be collected and calculated. With this concept and many others in the field of portfolio optimisation, this is where further research is needed to produce posterior estimates given that historical returns are often found to offer little use for predicting future returns (Cocco et al., 2007).

3. Value-at-Risk and Conditional Value-at-Risk

Another method of portfolio optimisation is through the minimisation of the value-at-risk (VaR) for a given time frame and confidence interval. The VaR metric was introduced during the late 1980s in a time of high market volatility and uncertainty. It was not named as such but was estimated by the US government in its capital requirements for financial corporations (Exchange Act, 1991 in Jamroz, 1992 in Holton, 2010). VaR gives the expected loss at a chosen percentile lower bound when an asset is held for a given length of time. For example, on 31/10/2018 the one-week 95% VaR of Apple's equity gave a 5% chance of losing more than 5.53% of the value invested if the stock was bought at that time and held for a week (Bloomberg Risk Model, 2018a). The premise behind VaR is that investors dislike large losses far more than they enjoy large returns and hence their only objectives are to minimise large losses whilst maximising expected return, a concept supported with a wealth of evidence in prospect theory (Kahneman and Tversky, 1979) and elsewhere in behavioural economics (Kahneman et al., 1990 in Campbell et al., 2001, p.1790). See Figure 3.1 below for a general utility curve under prospect theory.

Figure 3.1
A General Utility (Value) Function According to Prospect Theory

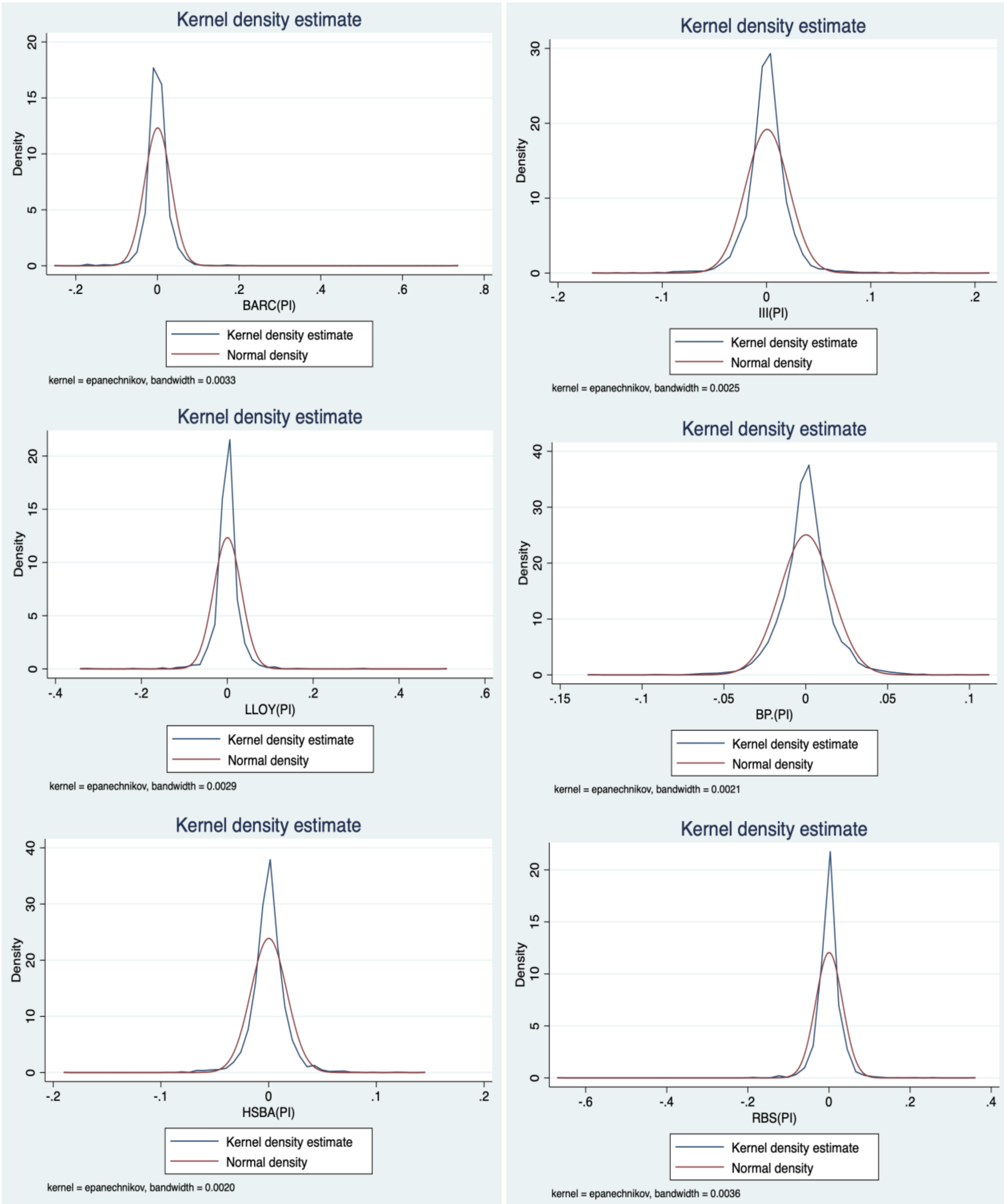


Source: Kahneman and Tversky, 1979, pp. 279

Through such an approach the expected return can be maximised for each VaR level to create an efficient frontier (Campbell et al., 2001). Another reason to instead look at VaR is that it is robust for non-normal returns distributions. It is widely acknowledged that the returns of equities, along with the returns of many other classes of assets, follow a non-normal distribution (Ahmadi-Javid, 2012). Hence, any risk measure based on a normal distribution will be suboptimal and potentially misleading. Specifically, equity returns very often have highly leptokurtic distributions, meaning that they have a much higher peak than the normal (Lechner and Ovaert, 2010).

Figure 3.2

Kernel Density Plots Against the Normal Distribution for a Selection of FTSE All-Share Constituents



Source: Stata, 2018

The above is just a small representation that the returns distributions for equities rarely follow anything resembling a normal distribution. These are plots of the normal distribution against the returns of six stocks from the FTSE All-Share index between 2007 and 2018. All are very much leptokurtic, where RBS (RBSPI) has a kurtosis of 76.2 in this case, versus 3.0 for a normal distribution. Such a phenomenon is noted by Hull (2017, p.535): "A 5-standard-deviation daily move in a market variable is one such extreme event. Under the assumption of a normal distribution, it happens about once every 7,000 years, but, in practice, it is not uncommon to see a 5-standard-deviation daily move once or twice every 10 years.". This shows that such an estimation of the expected risk of an asset can be extremely misleading, creating a gap for an improved risk measure to avoid such a flaw. This extreme value estimation for a non-normal return distribution is the gap that VaR fills.

A mathematical representation of VaR is given as follows (adapted from MathWorks, 2019):

$$VaR_{\alpha}^{t}(x) = \min\{\gamma\colon \Pr[f_t(x, Y) \leq \gamma] \geq \alpha\} \tag{3.4}$$

Where α is the confidence level (e.g. 95%), x gives the portfolio under assessment, y is the portfolio's possible returns and $f_t(x, Y)$ is the loss function for x and y over the holding time t.

The metric of VaR can be measured for an individual asset or entire portfolio. This is hence where it is used by many financial firms either for its capital requirements or risk assessment for its various funds or potential investments (Jorion, 2007, Bank for International Settlements, 2016). Many funds offered display a VaR metric as their defining risk measure that can be compared against other funds, along with their past and predicted returns. Another such regulatory offshoot of VaR rulings is that of stressed VaR. This looks to give an estimate of extreme risk when sampled from a period of extreme market behaviour. A stressed VaR measure provides a safer estimate of the potential extremes faced in financial markets that are almost always underestimated by the normal distribution (Hull, 2017, p.517).

Some years after the adoption of VaR for investment management and portfolio generation, the concept of Conditional Value at Risk became equally popular as an improvement to the loss-aversion measure. Such a representation of this popularity is that the Basel Committee now uses expected shortfall (CVaR or ES) as its primary risk measure, rather than VaR (Bank for International Settlements, 2016). CVaR is very similar in its approach, assessing the risk associated with the lower tail of the distribution of returns but instead estimating the expected value of loss below a chosen confidence level (Hull, 2017, p.518). As a comparison to the above example with VaR, the equivalent 95% 1-week CVaR for Apple gives an expected extreme loss of 7.43% (Bloomberg Risk Model, 2018b).

Some of the benefits to the use of CVaR as a risk measure, as opposed to VaR, are that (1) CVaR gives a better measure of the downside risk of a portfolio and (2) CVaR is sub-additive, whilst VaR has been heavily criticised for not being (Acerbi and Tasche, 2002). This ensures that the CVaR of any combination of assets cannot be greater than the weighted sum of the components' CVaRs.

The CVaR of a portfolio can calculated from its VaR level, making it easy to switch between the two. This is especially helpful with the given regulatory changes made in recent years. CVaR can be calculated as shown below (adapted from MathWorks, 2019):

$$CVaR_{\alpha}^{t}(x) = \frac{1}{1-\alpha} \int\limits_{f_t(x,y) \geq VaR_{\alpha}^{t}(x)} f_t(x,y)p(y)dy \tag{3.2}$$

$$CVaR_{\alpha}^{t}(x) = VaR_{\alpha}^{t}(x) + \frac{1}{1-\alpha} \int\limits_{\mathbb{R}^n} \max\{0, \left(f_t(x,y) - VaR_{\alpha}^{t}(x)\right)\} p(y)dy \tag{3.3}$$

Where $p(y)$ is the probability density function of returns y and $\mathbb{R}^n$ is the space spanning the possible set of portfolios.

Figure 3.3 below illustrates how the concept of CVaR gives a better estimate of risk than does VaR (Hull, 2017), as also supported by (Cadle et al., 2008, p.136). In the case of Figure 3.4, there is the same VaR value, but a much higher expected loss given that some terrible event does occur for the returns of the portfolio. This is obviously an exaggerated and simplified example for the sake of explaining the difference between the two concepts.

Figure 3.3

Calculation of VaR from the probability distribution of the change in the portfolio value; confidence level is $X\%$. Gains in portfolio value are positive; losses are negative.

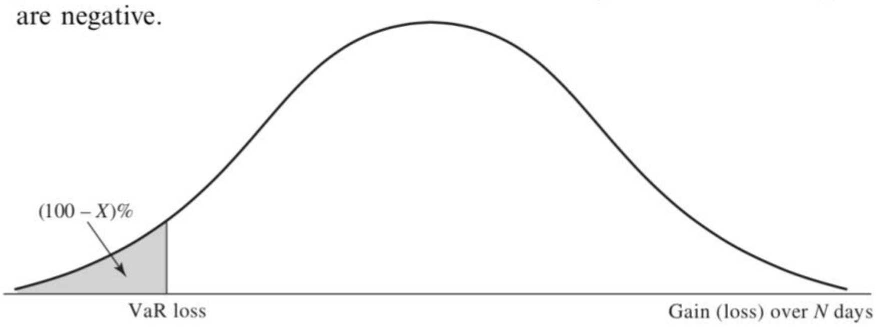
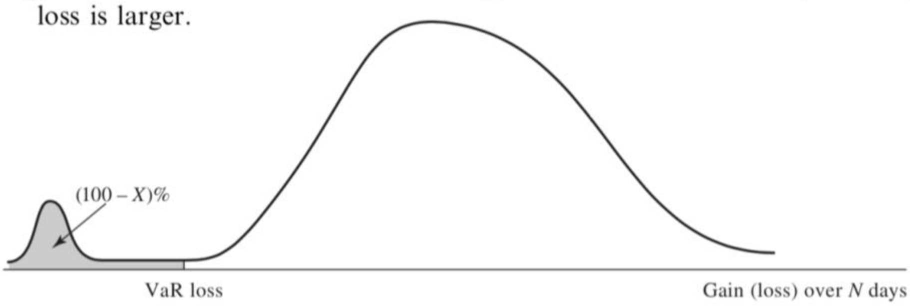


Figure 3.4

Alternative situation to Figure 3.3. VaR is the same, but the potential loss is larger.



Source: Hull, 2017, pp.518.

4. Higher Moments in Investor Utility

More recently, there have been adaptations and extensions of the original Markowitz (1952) optimisation to include investor utility in terms of more central moments of the portfolio's return distribution. The central moments of a distribution are moments of a distribution, taken about the mean. The nth central moment of random variable X can be given by the general formula:

$$\mu_n = E[(X - E[X])^n] \tag{4.5}$$

(Adapted from Severini, 2012, p.95)

The zeroth central moment and first central moment are clearly 1 and 0 respectively, whilst the second central moment is commonly known as the variance of X. The central moments above this will be referred to as the higher central moments in this paper, starting with the skewness. This is the third, whilst kurtosis is the fourth central moment. Skewness gives an estimation of the asymmetry of the distribution, estimating how much the density of the distribution lies on one side of the mean. Kurtosis then provides a numerical measure of the peakedness of the probability distribution of X. These are fairly well-known statistics, though above the fourth central moment there is comparatively little documentation. Such a lack of investigation comes partly from the decreasing significance of the central moments to describe the general shape of a distribution, and partly due to the huge increase in complexity in the definition and interpretation of these central moments. These points are all assessed by Darlington (1970), Moors (1986) and Ruppert (1987) in Dodge and Rousson (1999, p.269). These even cast great uncertainty over what is being measured by kurtosis, including whether it is instead a measure of whether a distribution is bimodal or unimodal, not necessarily its peakedness (Darlington, 1970, p.19). Dodge and Rousson (1999) begin to describe the mathematical complications brought out looking at the fifth central moment and higher, stating that even kurtosis cannot be rigorously defined. In the field of portfolio optimisation, it appears consideration almost stops at kurtosis (Horvath and Scott, 1980, pp.915-916, Homaifar and Graddy, 1988, Cheridito et al., 2014, Nalpas et al., 2017). It is stated that there would be benefits to consideration of these in improving the estimate of the true return distribution (Levy, 1969 in Chaudhry and Christie-David, 2001, p.56), though considering the mathematical complications discussed above it is likely to quickly become extremely complicated.

When choosing to consider a utility function which extends that used by Markowitz (1952) there must now be consideration of how the different central moments create investor utility and by what degrees respectively, since the Markowitz approach only required the minimisation of variance. An early finding by Horvath and Scott (1980) provided a simple proof as to the direction of preferences for any central moment in the utility function of investors. Utility comes from the returns gained from investing, so a utility function may be simply derived through expanding the Taylor series of the future wealth from investing, given by:

$$E(U) = \sum_{k=0}^{m} \frac{\mu_k}{k!} U^k(\mu) \tag{4.6}$$

Where μ_k is the kth central moment (1), $U^k(\mu)$ is the kth derivative of the utility function and m represents the chosen number of the first central moments to consider in the utility framework. This would be $m = 2$ in the Markowitz case, but can theoretically be set to infinity.
(Adapted from Horvath and Scott, 1980, p.915, using Garlappi and Skoulakis, 2011, p.124)

From here, Horvath and Scott (1980) were able to take the $2n$th and $(2n + 1)$th derivatives of the utility function in the Taylor form and prove that their signs are negative for all even central moments and positive for all odd central moments. This asserts that investors will prefer the even central moments to be smaller, but that the odd central moments be larger. This is demonstrated in the Markowitz optimisation, where the variance is minimised (the first even central moment). Though this does not describe the interaction between the interests in optimisation of each central moment, it does allow one to recognise preferred portfolios where one outperforms for all central moments considered. This idea will be considered in the later analysis and comparison of portfolio performance. Some models have been generated to use these preferences through iteration using Monte Carlo methods to keep improving the portfolio to find Pareto efficient frontiers (Nalpas et al., 2017). These run the risk of falling into local maxima for utility, presenting great difficulty in proving that the portfolio generated provides a global optimum, given the lack of smoothness in a multi-objective optimisation with objectives that are not strictly concave (Nalpas et al., 2017, p.309). Alternatively, Konno et al. (1993) used the above Taylor expansion of utility for the higher central moments to include the skewness in mean-absolute deviation (MAD) portfolio optimisation.

Other than the central moments of a distribution, there are mixed moments, which instead look at the relationships between variables within a compound probability function. The mixed moments corresponding to the second, third and fourth central moments respectively are covariance, coskewness and cokurtosis. The covariance is the most well-known statistically and, in this environment, measures how closely two asset returns move together, whilst the coskewness measures how three asset returns move together, with extreme simultaneous movements magnified, and cokurtosis measures whether two assets experience extreme returns at the same time (Dittmar, 2002, p.375). It should be expected that the MCVaR-optimisation technique can indirectly consider many of these mixed moments in the minimisation of extreme losses, considering these are metrics stressing consideration of extreme behaviour.

5. Investigation

Intentions

In this investigation, the performances of the MV and MCVaR portfolio generation techniques will be tested to compare which risk attributes and investor preferences are favoured by each and how they behave in different investment environments. The intention is to find the best method of passive investing given the different markets that an investor may consider for the portfolio. From these recommendations, one could make a decision on the portfolio generation and management method to be followed based on the investor's specific utility function and preferences for the ability to manipulate their portfolio's returns distribution, including an awareness of the assets under consideration. The investigation is orientated similarly to Markowitz's seminal paper (1952), concentrating more on the performance of the respective methods when posterior return distributions are predicted perfectly (the historical portfolios and frontiers). Their risk exposure will be measured using multiple metrics to give as clear an assessment of their relative strengths and weaknesses as possible.

Data Manipulation

To generate variance and CVaR-optimal portfolios, DataStream was used to download stock prices for components of major stock market indices from top stock exchanges by value. The indices chosen contained at least 500 assets to give a representation of the entire market, rather than for just the largest and often most internationally exposed companies in an economy, offering little comparability between different markets. Smaller indices also pose more risk of overconcentrating the portfolio into certain industries and exposing it to more systematic risk. These prices were adjusted for dividends so the true returns of each asset could be calculated for each trading day. Having returns data from multiple major exchanges from around the world, this brought the issue of compatibility, given their assets' prices were denominated in the national currency and in some cases multiple currencies. To translate these returns so that portfolios generated in different markets can be compared, all returns have been denominated in US Dollars. This was done with DataStream exchange rate data, using the daily exchange rate change against the dollar to calculate the daily return if the asset was held in dollars:

$$r_{it,\$} = \left(1 + r_{it,x}\right)(1 + \Delta e_{it}) - 1 \qquad (5.7)$$

Where:

$r_{it,\$}$ is the daily return of asset i on day t adjusted for holding the asset in USD

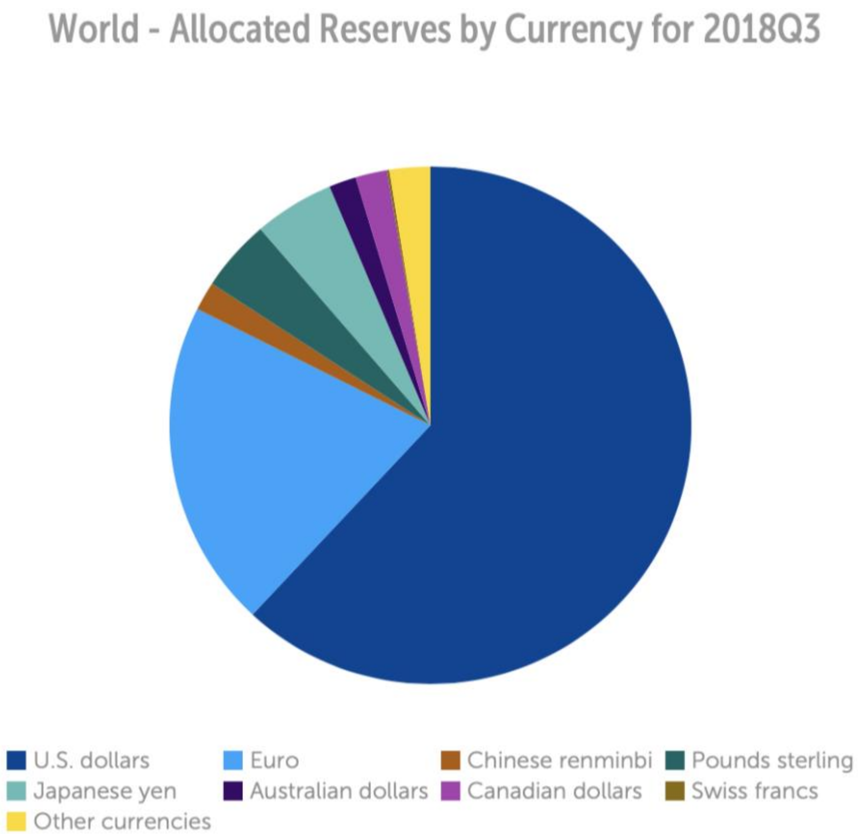
$r_{it,x}$ is the daily return of asset i on day t in its originally denominated currency, x

Δe_{it} is the percentage change in the exchange rate on day t between the dollar and the currency in which asset i is denominated. The exchange rate is of orientation $e = x/USD$

The reason for choosing the dollar is that it is seen as a base currency for many purposes, such as economic indicators, whilst it holds stability as the world's primary currency in terms of central bank reserves and trade (Wallace, 2018, BullMarketz, 2018 in Amadeo, 2019, IG, 2018, Foy, 2018, IMF, 2019a and The Economist, 2019). Figure 5.1 illustrates that the IMF holds 62% of its $10.7bn of foreign exchange reserves in the US Dollar, displaying its strength as the leading world currency for safety and dependency.

Figure 5.1

IMF Official Foreign Exchange Reserves by Total Value in Dollars



Source: IMF, 2019

This gives it great convertibility and should not pose a problem for comparing the frontiers and distributions as they will all be based on what is seen as the most stable currency and store of value. Hence, it should actually add stability to measuring the real purchasing power of the returns gained in any market. The currency risk posed when generating the efficient frontiers should be of little issue for translated returns, given that a ten-year historical dataset has been used to reduce the significance of exchange rate noise and instead closer estimate the relative value of holding shares denominated in different currencies over this time period. The exchange rate changes that affect returns should act to provide an incentive regarding the relative attractiveness of investing in the market through the strength of the economy, a factor known to affect asset returns (Baker et al., 2005). Another benefit of representing returns in dollars is that it tests the abilities of the models to handle the added currency risk and periods of higher volatility, a significant concern when investing. Most financial institutions will hold the bulk of their earnings in their home/domiciled currency, whilst inevitably investing in securities denominated in multiple different currencies. By basing the data in dollars, this gives commercial applicability to the assessment by replicating the behaviour of institutions transferring returns into their preferred currency for their balance sheets and repayments to investors (Avdjiev at al., 2019 and Krogstrup and Tille, 2019).

The next step taken to ensure that all stocks assessed were reliable for investment purposes, any assets with fewer than 200 trading days' returns data were removed. This ensured sufficient data was used for the statistics generated (for the optimisations) to be reliable and approach their statistical asymptotics in the true relationships; with a smaller sample set comes higher sample variance for these statistics. These sample variances would then be expected to fall throughout the investing period as more data was added, whilst all of the statistical parameters should approach their true value as the sample variance falls, displaying that the true relationships differed to a significant degree. This can be important when a high (low) correlation is found between asset returns, but only due to a lack of data that overstates (understates) the closeness of their relationship. This would create an issue of the over or underweighting of assets within the portfolio due to the lack of accuracy in estimating the relationships between asset returns over the investment horizon, giving a suboptimal portfolio.

Stock Data and Market Representatives

In this investigation, five very different markets or regions around the world will be assessed. These five markets are: The United Kingdom (UK), United States (US), India, Europe and the World. The markets of Europe and the World were chosen to explore the power of aggregating markets for investing and exploiting the benefits of diversification. This was explored by Solnik (1995), who found that with more nations considered for a portfolio came more opportunities for diversification, referred to as international diversification.

The UK will be represented by the FTSE All-Share index, containing 636 constituent companies and holding 98% of the market capitalisation listed on the London Stock Exchange (FTSE Russell, 2019).

The US will be represented by the S&P500, which is made up of the 500 largest companies on the New York Stock Exchange (NYSE) by market capitalisation. This index contains 505 constituents, given some companies included have multiple stock classes. Such an index is very similar to the FTSE All-Share and so should hence offer a strong comparison.

The Indian National Stock Exchange (NSE) will be represented by the Nifty 500, the extension of the NiftyFifty. This "represents about 95.2% of the free float market capitalization of the stocks listed on NSE" (NSE India, 2017), giving a good representative of the Indian equities accessible to most investors. There was much more emphasis required in choosing a market representative for India due to its emerging and high growth environment, with a current annual GDP growth rate of 7.3% and an average growth rate of 10.1% since 2002 (World Bank, 2019). This suggests that there will be much more company turnover where many smaller firms with huge growth quickly catch up to and often replace incumbent larger firms, giving rise to both higher return and higher non-systematic risk that should be priced into the market.

The European market will be represented by the STOXX All Europe Total Market Index, containing over 1200 stocks from a selection of European exchanges and covering around 95% of the free-floating market capitalisation in European exchanges. Stocks are selected from 35 different European exchanges, as listed and screened for the STOXX investable universe (STOXX, 2019). This market was investigated for its aspiration to promote a free trade area as the single European market, whilst offering a huge range of investment opportunities and diversity as a large collection of advanced economies.

To represent a good selection of opportunities around the entire world, with exposure to as many markets as possible with highly accessible stock exchanges, the S&P1200 was chosen. This contained 1215 securities at the time of investigation and spans 30 countries within 7 regions around the world (S&P Dow Jones Indices, 2019). These constituents are taken from the following indices and screened to promote industry diversity: S&P 500 (US), S&P Europe 350, S&P TOPIX 150 (Japan), S&P/TSX 60 (Canada), S&P/ASX All Australian 50, S&P Asia 50 and S&P Latin America 40. The European and World markets cover a much wider area both geographically and in terms of their diversity of countries and markets, so required larger representative indices than others by number and total value of constituents.

To display the depth and range of investment opportunities considered in this investigation- below in Figure 5.2 is an illustration of the regions included in the stock data used.

Figure 5.2
Regions Considered for Investment in this Investigation, Indicated by Blue Shading

Source: mapchart.net

It is clear that a majority of the developed world has been included. Research was intended to include China, being the second largest nation by total nominal GDP (IMF, 2019b) and set to soon become the largest-if not already by measures such as purchasing power parity (Federal Reserve Bank of St Louis in Smith, 2018, Standard Chartered, China in Martin, 2019). This conclusion is also clear with China's current annual GDP growth rate at 6.2% against the USA's 2.5%, with no expectation for a large change in either of these values in the near future (IMF, 2019c). There were concerns initially over the investment environment in China, given the relatively restrictive nature of the Chinese government. The Shanghai Stock Exchange (SSE) is composed of A-shares and B-shares, where B-shares are open to foreign investment, but A-shares are restricted to licensed international investors with a quota on maximum holdings (Shanghai Stock Exchange, 2017). This meant the market can be seen as less accessible, given free trading here can only truly be undertaken as a Chinese citizen and that stricter capital controls could be imposed given the uncertain trade situation with the US in particular. The Shanghai Composite Index was considered, containing 1350 constituents and the majority of the SSE (Bloomberg, 2019). The resulting portfolios and frontiers generated posed problems as there were many volatile stocks that resulted in close to full portfolio concentration into very few stocks, whilst this volatility was often found to be a result of recent M&A or floatation activity that gave an unrealistic representation of future returns behaviour. An example of such a stock was '360 Security Technology Inc.', which returned to the SSE via a back-door listing on 6 November 2017 (He, 2018) and consequently saw its share price increase six-fold within the month and then fall back towards its previous value for the next year- see Figure 5.3 below for its price chart. Due to the timing of the 10-year dataset, the stock was summarised by a huge positive expected return, which gave it an unrealistic majority weighting in any optimised portfolio. The resulting performance after November 2017 of such portfolios was terrible and so not eligible for assessment. These issues repeated as such stocks were removed as anomalies, so China was not considered further due to its unrealistic optimisation outcomes. It is suggested that a giant such as China should be investigated independently.

Figure 5.3

Stock Price (CNY) of 360 Security Technology between 2012 and 2019

Source: Yahoo Finance, 2019

Investigation Timing

There are two sets of portfolios and frontiers being generated for the purpose of this paper. The first will be generated using the ten years' historical returns data from December 2007 to December 2017. The other will be generated using the optimal asset weightings found using the historical data and applied over the 12 months from December 2017 to December 2018. This allows a realistic formulation where each portfolio is generated based solely on historical data when investing for the future. The historical distribution will be used as an estimate of the posterior distribution, though it has often been found to deviate from the realised posterior distribution (Cocco et al., 2007). In this case, a more accurate estimation of the historical distribution should add to its predictive power. Investment in the historical portfolios will take place from the first trading date in December 2007 and then closed at the end of the last trading day in November 2017. Investment in the set of applied portfolios will be completed on the first trading day of December 2017 and closed at the end of the last trading day of November 2018.

Model Restrictions

A restriction adopted regularly within such investigations (Markowitz, 1952, p.78, Harvey et al., 2010, p.477, Beasley et al., 2013, p.409, Nalpas et al., 2017, p.310) is the prevention of the ability to short-sell assets. Mathematically, this equates to restricting the weights of all assets in the portfolios to non-negative values. Allowing short-selling can often find mathematical anomalies and unrealistic portfolios in which some stocks are short sold in large quantities to allow a huge weighting into another due to their offering of a more attractive portfolio. In essence, the asset with the lowest return could be sold infinitely to buy an infinite amount of the asset with the highest return (Markowitz, 1952, p. 78). This obviously leaves the investor massively exposed to non-systematic risk, the feature that such a method is attempting to minimise through diversification. Hence, these are seen to be unrealistic recommendations and are avoided by restricting the investor to having only non-negative portfolio weightings. It is also dangerous in terms of model risk to allow certain assets within a portfolio to have such a high weighting as it becomes exposed to the risk of a crisis occurring that was not considered and weighted properly in the predictive return distribution.

Portfolio Generation

Optimisations were run using the MATLAB (R2018b) Financial Toolbox, making use of the Portfolio and PortfolioCVaR objects to generate MV and MCVaR-efficient portfolios. For each market and optimisation method, efficient frontiers were generated out of 10 optimal portfolios. These were then generated for both the historical and applied periods. These are the two previously specified periods before and after December 2017, where the historical portfolio returns are those for which the optimisation is run from December 2007 to December 2017, and the applied portfolio returns are those generated over the period December 2017 to December 2018. This gives an indication of the power of the optimisation techniques in using historical returns to manipulate returns where they are correctly predicted but also their ability to handle imperfect predictions to achieve the objectives set out in a realistic portfolio generation.

Mean-Variance Portfolio Generation

To generate efficient frontiers and the portfolios that make them up under the mean-variance method, MATLAB will be used to set up the problem with the historical return matrix and corresponding weights vector for all the assets considered. These are used to generate the expected return and variance for the portfolio made by any combination of the asset weightings. The optimisation in (5.1) will be run multiple times to plot out an efficient frontier over all possible minimum variance-mean return combinations. First, two different optimisations are run to compute the minimum variance and the maximum return portfolios, between which the optimisation can be run.

The optimisation when generating a portfolio of n assets is set up as presented (adapted from MathWorks, 2019):

The bold font is used to denote vectors and matrices.

Weights vector:

$$\boldsymbol{\omega} = \begin{pmatrix} \omega_1 \\ \vdots \\ \omega_n \end{pmatrix}$$

Expected returns vector:

$$\overline{\boldsymbol{R}} = \begin{pmatrix} E(R_1) \\ \vdots \\ E(R_n) \end{pmatrix}$$

Covariance matrix:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{n1} \\ \vdots & \ddots & \vdots \\ \sigma_{1n} & \cdots & \sigma_{nn} \end{pmatrix}$$

Where $\sigma_{ii} = \sigma_i^2$

Portfolio expected return: $E(R_p) = \boldsymbol{\omega}'.\overline{\boldsymbol{R}}$

Portfolio variance: $\sigma_p^2 = \boldsymbol{\omega}'.\boldsymbol{\sigma}.\boldsymbol{\omega}$

Optimisation:

$$
\begin{aligned}
&\min_{\boldsymbol{\omega}} \sigma_p^2 \\
s.t \quad &\mathbf{1}_n . \boldsymbol{\omega} = 1 \\
&E(R_p) \geq a \\
&\boldsymbol{\omega} \geq 0
\end{aligned}
\tag{5.1}
$$

Where $\mathbf{1}_n$ is a $1 \times n$ vector of ones and the portfolios are generated by altering the respective value of a between the expected return of the minimum variance and maximum return portfolios.

Conditional Value-at-Risk Portfolio Generation

In this investigation, the daily 1% CVaR over will be minimised in each portfolio with the default constraints in place, plus the minimum expected returns requirements, to generate an efficient frontier. This confidence level was chosen due to the large sample size used and hence the accuracy that can be expected in estimating extreme tail risk. Given the intention of the CVaR method to minimise extreme loss, the 1% expected loss gives a good proxy for this. By taking a large sample set of 10-years, the return distributions generated are smooth and accurately represent the true distribution using the historical method (this would require collecting all possible historical data for every asset, a huge extension to the dataset with little improvement to asset allocations).

The optimisation run by MATLAB can be set out simply as:

$$
\begin{aligned}
&\min_{\boldsymbol{\omega}} CVaR_{0.99}(x) \\
s.t \quad &\mathbf{1}_n . \boldsymbol{\omega} = 1 \\
&E(R_p) \geq a \\
&\boldsymbol{\omega} \geq 0
\end{aligned}
\tag{5.2}
$$

Where, again, $\mathbf{1}_n$ is a $1 \times n$ vector of ones and the portfolios are generated by altering the respective value of a.

Assessment Criteria and Assessment of Metrics

The models, efficient frontiers and historical and applied returns distributions will be assessed according to their real-world applicability and ability to provide desirable risk-return combinations. These will be related closely to investor utility maximisation and the factors required to achieve such, and how well each method achieves these. Stata (15.1) will be used to assess the return distributions of the portfolios. The attributes used to assess the return distributions and portfolio risk include the mean, variance, semi-variance, absolute deviation, skewness, kurtosis, VaR and CVaR. The first four central moments are calculated using Stata, while the CVaR and VaR are calculated with MATLAB and Excel using a discrete probability distribution. These attributes assess the ability of MV and MCVaR optimisation to meet the goals of more complex utility functions, taking into consideration the higher central moments of the return distribution (Horvath and Scott, 1980, Harvey et al., 2010, Nalpas et al., 2017, Bernardi and Catania, 2018). The reason for assessing so many different metrics relating to the risk-return criteria of each portfolio is the different attributes brought for analysis by each of these metrics.

Variance was one of the first choices of risk measure when considering investments and the risk associated with any given asset or portfolio. It is a very simplistic measure and can be easily understood by many beginners in the field given its significance elsewhere in statistics and mathematical models in economics, as stressed by Markowitz (1991, p.193). It is also easily calculated and available through any reasonable statistical program, making it very easily applied and interpreted. Another feature being its applicability to any possible returns, given at least one asset and at least two values for return. As an introductory risk metric, variance offers many opportunities to assess investment performance relatively realistically, though it does bring with it some drawbacks. One issue is that it is implausible to present above-average returns as undesirable. Being such a simple measure, one can be quickly misled by a high variance where the majority of the asset's variability lies above the mean return gained. Such a phenomenon was noticed in the performance of Warren Buffett's Berkshire Hathaway (Martin and Puthenpuackal, 2008, p.29), where the fund's incredible performance was being underrated due to its tendency to deliver exceptionally high returns and almost never low enough returns to be considered particularly disappointing. This issue was soon appreciated by Markowitz himself and led to his consideration of the adaptation of the original mean-variance method with the use of semi-variance instead. It was in this piece that he accepted the semi-variance is a more plausible risk measure than the variance (Markowitz, 1991, p.374). Although, he defended that the two were overall equally impressive and so did not suggest his model be changed before further research was conducted. A very similar metric to variance is absolute deviation, measuring the average absolute deviation from the mean, rather than the average squared difference from the mean. This will also be assessed as a risk measure in case there are any discrepancies and given that absolute deviation is often used for risk assessment and the generation of MAD-efficient portfolios (Simaan, 1997, Konno and Koshizuka, 2005, Alem et al., 2017).

When VaR was introduced as an advancement of variance and semi-variance, this provided a strong complement to expected return in assessing the risk-return attributes of an investment with respect to almost all investors. Behavioural economics (see chapter 3) stressed that such large losses are especially important to investors, whilst another defining feature of VaR lies in its ability to drop the very often unrealistic assumption of normal returns distributions for all securities and portfolios considered (see chapter 1)- an important flaw found when using variance to quantify risk. This allows value-at-risk to elegantly bring into consideration the higher moments of the return distribution, whilst also requiring no definition of the probability density function of returns. CVaR takes all these benefits of the VaR metric and enhances them to provide a more feasible and well-rounded estimate of risk. One of the few drawbacks of CVaR relative to VaR is its longer computing time. Unlike with variance, an extremely efficient computing time is not one of VaR's significant features, so extending the method for improved risk estimation is a less important factor in relation. Compared to Markowitz's first consideration of the portfolio optimisation problem, the cost of computing time and performance has diminished to an enormous degree. A popular measure of this can be derived using Moore's Law, which states that computing power doubles every one-to-two years (Moore, 1965 and Moore, 2005). Upon calculation, since Markowitz's study in 1952, it equates to a reduction in cost and timing to the magnitude of between 1.21×10^{10} and 1.48×10^{20} times.

An important characteristic highlighted in these first metrics considered is their ease of comprehension, being relatively easy to describe to and understand for those unfamiliar with the field of portfolio risk. The higher moments of the return distribution are then often much harder to interpret intuitively and require simultaneous consideration, where the investor's utility function will be assumed to have more than two dimensions, as in MV and MCVaR optimised portfolios. This begins to open a field of multidimensional portfolio optimisation, which is beyond the breadth of analysis considered in this paper (though provides many interesting challenges). The depth considered in this analysis will stop at assessing the signs of investor preferences for the skew and kurtosis and considering simple frontiers in the mean and first four central moments of the return distributions for MV and MCVaR-optimised portfolios.

The further benefits of considering the skew and kurtosis of a distribution include their ability to better describe its features and how it differs from the previously assumed normal distribution. They have their limits in ability to describe every part of the return distribution, though to do this would be near impossible and require the consideration of their every central moment. For reasons stated in chapter 4, this investigation also stops at the fourth central moment.

Hypotheses

Due to the nature of the approach taken by the CVaR model, it is to be expected the MCVaR-efficient portfolios will have much greater control over the skewness and kurtosis of their distributions. This is because CVaR seeks to minimise the lower tail of its distribution, while the kurtosis is especially concerned with the peakedness and size of the tails of a distribution. Since the mean-variance methods are restricted to assume an improbable normal distribution, it should also be expected that the CVaR method is much better able to manipulate and accurately estimate the probabilities of each potential outcome.

It should be expected that in more diverse or volatile markets, such as the world and India respectively, the abilities of the models to consider more advanced risk will be exposed as more investment opportunities arise. This provides a pivotal point for assessing the usefulness of these methods given current global uncertainty and the interests of risk averse investors. In the case of another global recession or crisis, the model that offers better performance with more instability would be hugely important as it can clearly dictate the preferred model for investor utility maximisation.

As for the application of portfolio weightings, it is to be expected that the relative risk and return for most portfolios will differ greatly from the historical results, though through testing the outcome, the required accuracy of posterior distributions can be assessed to test the sensitivity of the optimisation methods to model risk.

6. Results

Areas for Assessment

The following chapter will set out the comparisons of the frontiers and portfolios generated for each period and optimisation method against the risk metrics described in the last. Their implications will then be assessed for further research and investment strategy. Therefore, multiple potential forms of utility function and preferences will be considered to assess the ability of each portfolio generation method to meet such objectives, so as to first assess whether either method is able to outperform for all kinds of investor or investment, which is not to be expected. Then, if one does not strictly outperform, the methods can be assessed to advise which is likely to better match investor preferences for a certain market structure. The two will also be compared based on their performance within these utility frameworks. This has been done, considering when generating commercial or individual investment products, one would be interested in ensuring that the portfolios generated perform well against all measures of expected performance. For example, if a portfolio was set to perform very strongly under both the MV and MCVaR utility frameworks, yet gave a large negative skew, this suggests that although it has performed well under these two measures, it is far from optimal for many whose investment preferences will likely be affected by such a feature that leads to frequent above-average returns but an increased likelihood of a relatively large loss. It would also hence be important that the method chosen also performs well in applying such a historical dataset to maintaining strong future performance, though this would more likely be referred as an issue of altering the posterior returns distribution estimate.

Historical Frontier Comparison

Where not directly referenced, the frontiers used for these comparisons are presented in the appendix.

Standard Deviation

Relating closely to the methods set out by Krokhmal et al. (2002), the efficient frontiers generated by both techniques were plotted according to their expected return and standard deviation of returns. This allowed the comparison of the two methods according to the mean-variance setting's choice of risk measure. This ensures that the MV-efficient frontier will be above the CVaR frontier at all points. Thus, the MCVaR method can instead be assessed based on its ability to keep up with the MV method when risk is quantified by variance alone. Upon assessment of all their separate frontiers, the CVaR frontiers very closely follow the MV frontiers in general, being within a 0.2% difference in daily returns for any two portfolios with equal expected return- a relatively small difference given the range of returns offered by each of the sets of frontiers. One of the promising features possessed by every frontier is that they are increasing, though not necessarily strictly concave in all cases. This suggests that the MCVaR optimisation technique continues to offer the same risk-return trade-off within the MV framework and hence does not offer a contradiction that the MCVaR method is not able to offer support when considering alternative risk measures.

Europe and India have slightly less-smooth frontiers, whilst India in particular is able to offer higher returns across its frontiers than is found in any other market considered. This is understandable considering the average daily stock returns for the Indian market is almost double that in any other. This hence suggests there is an opportunity to generate much higher expected returns through each portfolio optimisation. The more unstable behaviour of the CVaR frontiers in the returns-standard deviation space may simply be explained by the shape of the tails of the distributions of some of the stocks in these two markets, a feature not considered when the distributions of each stock's returns are assumed to be normal in the generation of the MV-optimal portfolios. The CVaR minimisation will in this case have traded off more variance in returns for the sake of reducing potential extreme losses, as intuitively explained in Alexander and Baptista (2004, p.1268).

Another noticeable feature of each pair of frontiers is that they tend to one another at their extremes. This suggests that there is little difference to separate the two when looking to generate a very safe portfolio or a very high expected return portfolio; towards the middle of the frontier is where the MV and MCVaR methods appear to express their main differences. The portfolios contained within this area are more likely to be of interest to a majority of investors as these will be chosen by those without either an extreme risk aversion or extreme risk neutrality.

These observations suggest that the two methods perform relatively similarly in the mean-standard deviation space and hence offer similar solutions that do not set apart one far more than another.

CVaR

When comparing the frontiers generated in terms of the CVaR values of each portfolio, there were far more discrepancies and points for consideration. The first point that is easily noticeable when looking at the frontiers is that the variance-minimising portfolios performed far worse and much more unpredictably when assessed according to their ES, in contrast to how CVaR frontiers behaved under the mean-variance setting.

It is obvious when looking at the sets of frontiers that there are much larger gaps between the two frontiers overall, whilst none of the MV-optimal frontiers are concave in the mean-CVaR space. This indicates that, as expected, the MV method takes little consideration for potential extreme losses. This has proven to be an important factor in investor utility and hence displays that the MV method lacks performance in this area, especially given that the CVaR method also performed so well in the mean-variance space. Here there is clear evidence suggesting the CVaR optimisation method to be superior to that initially presented by Markowitz.

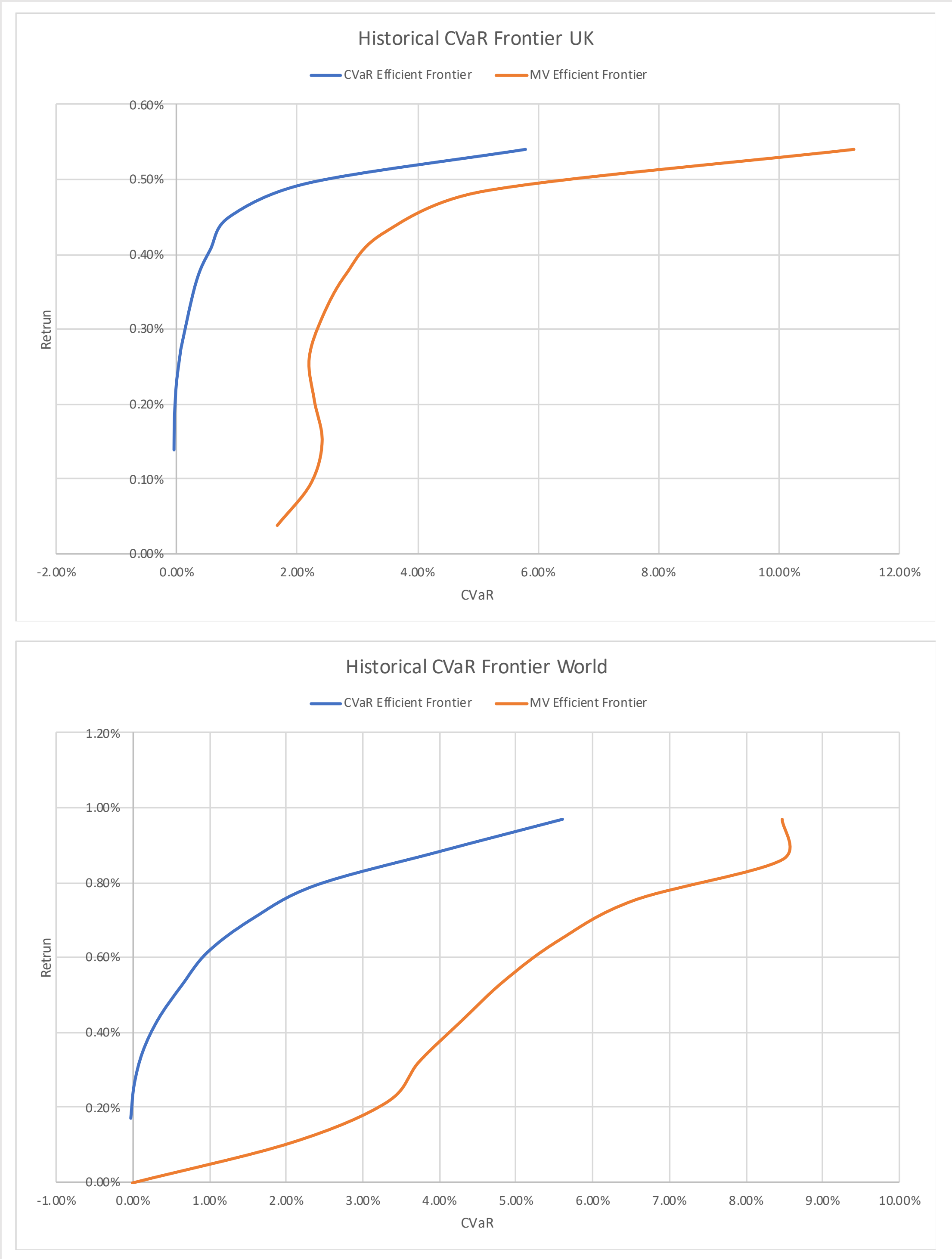
The performance of the MV frontier within the CVaR space appears to have been much stronger in the US and especially the UK, where the MV frontiers follow the CVaR-optimal frontier far more closely. These two sets of frontiers still differ more in the lower returns considered, where the MV frontiers are non-concave, with the sign of their gradients changing multiple times between the returns 0.10% and 0.30%. For returns above this, both MV frontiers for the US and UK track the CVaR-optimal frontier very closely, having a very similar gradient, but a difference in CVaR of around 3% and 4% respectively. This suggests that the MV method has little control over extreme losses, even in lower-risk environments.

For India, the MV frontier is far from smooth in the mean-CVaR space and shows signs of much more volatility than the other markets. The sign of the gradient of the MV frontier in the mean-CVaR space changes multiple times, only finding stability towards the higher return levels. These CVaR values are then all around the higher levels found by the CVaR-optimal approach, giving almost no protection from extreme losses by giving up return. This indicates the difficulty of the MV method to take extreme values of return into consideration in an environment of higher volatility. Such behaviour suggests a lack of consideration for the negative cokurtosis of returns between the assets included in the portfolio- assets that face extreme negative returns in the same time periods- generating large losses rather than more, smaller negative returns. Hence, in a more volatile environment, the MV method is unlikely to give such a fully well-rounded risk-optimised return and should be expected to give no indication as to the level of expected extreme losses.

Over a wider range of markets in the consideration of Europe, the ability of the MV method to minimise CVaR is much weaker, where the frontier is far less smooth than in the case of the US and UK, yet still follows the trend of tracking the CVaR-efficient frontier's gradient at higher returns. This suggests that with more diversity and opportunity to diversify across borders, the MV method struggles to control the potential for a hugely variable negative return. Both methods, however, are able to derive much higher expected returns than when fewer economies are considered together, where there should be far fewer opportunities for diversification (especially considering the UK market is included in the representation of Europe). These findings are further corroborated in the frontiers for the World. The MV frontier is even less smooth in the mean-CVaR space, displaying the increasing disparity between the abilities of the two methods to control the shape of their return distributions. The two pairs of frontiers are shown below in Figure 6.1 to show this difference as the size of the market considered increases. This provides further support for the MCVaR portfolio optimisation method to provide a better all-round risk optimised portfolio than variance-minimising portfolios.

Figure 6.1

Variance and CVaR-Efficient Frontiers in the Mean-CVaR Space for the UK and World

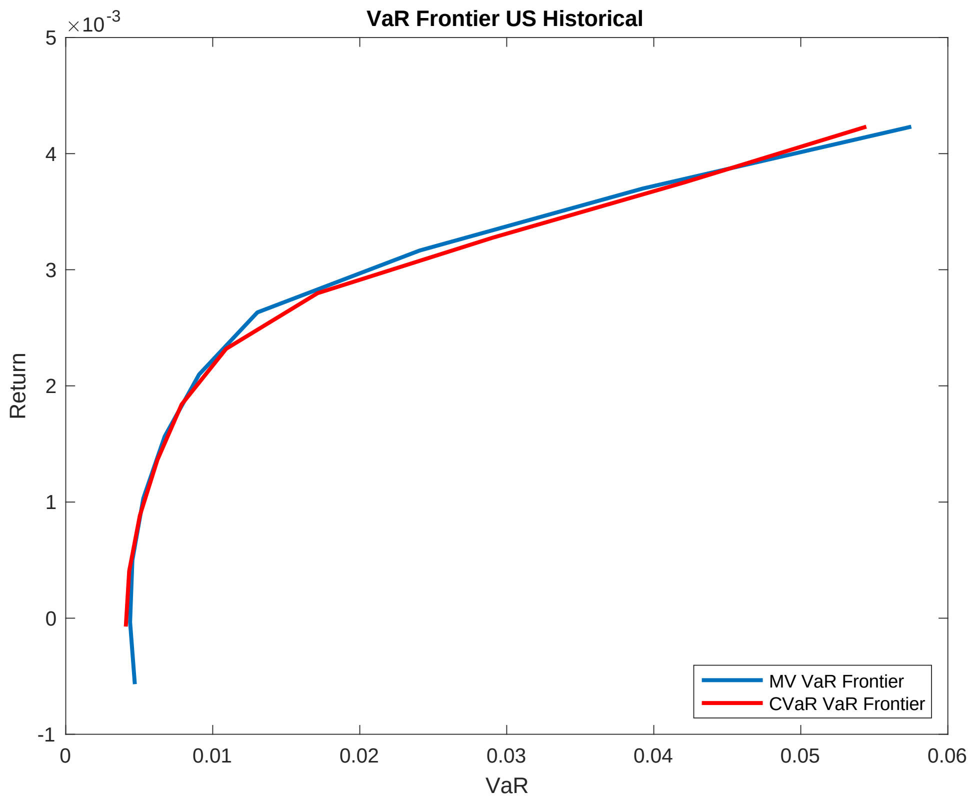


The finding in the mean-variance space that the two frontiers tend to one another at their two ends is clearly not supported in the mean-CVaR space.

VaR

Surprisingly, the MV-optimal portfolios in the mean-VaR space compete extremely well with the MCVaR-efficient portfolios. In each market the two frontiers follow one another very closely and even cross at least once in every case. The findings support that the MV method is actually able to offer higher returns to VaR more often than the CVaR method, shown where the 'MV VaR Frontier' is above the 'CVaR VaR Frontier'. The closest relationship is found in the US and shown in Figure 6.2 below.

Figure 6.2
Historical Variance and CVaR-Efficient Frontiers for the US in the Mean-VaR Space



It can be seen that, especially in this case, the two perform very similarly. This is a surprise, potentially displaying the power of the simplicity of variance to measure portfolio risk.

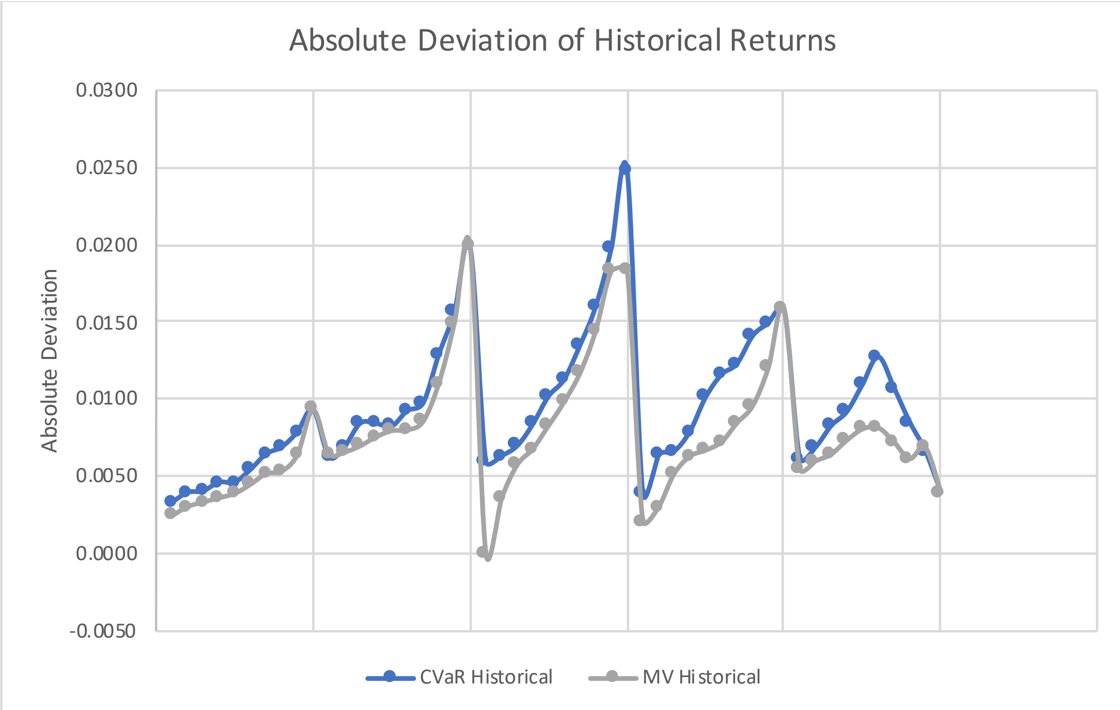
Given the expectation for the VaR method to be slightly biased towards showing the CVaR method as a better optimisation method, this a surprising and interesting finding, suggesting that for investors concerned with the tails of a portfolio's return distribution, the extra computing time and costs associated with CVaR versus mean-variance optimisation may not necessarily provide a great reduction in risk. Alternatively, this finding could instead be seen to support the significance of the shift from the use of VaR to CVaR, showing that VaR may have little ability to differentiate extreme loss risk when compared to the expectations when looking at variance. Given it was shown that the return distributions of the assets making up these portfolios almost never bare any resemblance to normal distributions, the predictions made by VaR in these cases made little difference in taking account of these factors; whilst the difference shown between the frontiers in the mean-CVaR space represent relatively large disparities in risk. Upon assessment of the normality of the return distributions for the portfolios generated, it is clear that none of the distributions can be said to have a normal distribution, with almost no portfolio having a p-value greater than 0.001 under the Shapiro-Francia test (Francia and Shapiro, 1972). Hence, there is little significance that the normality assumption holds when computing the risk of any portfolios, suggesting the results should be far from accurate. This, intriguingly, appears not to necessarily be the case here.

Semi-Variance and Absolute Deviation

Upon inspection, it is clear that the CVaR-optimal portfolios almost always outperform their variance-minimising equivalent portfolio in terms of both semi-variance and absolute deviation. In particular, the performance according to absolute deviation is better for almost every CVaR-optimal portfolio, as illustrated in Figure 6.3.

Figure 6.3

Absolute Deviation of Returns for all Variance and CVaR- Efficient Historical Portfolios



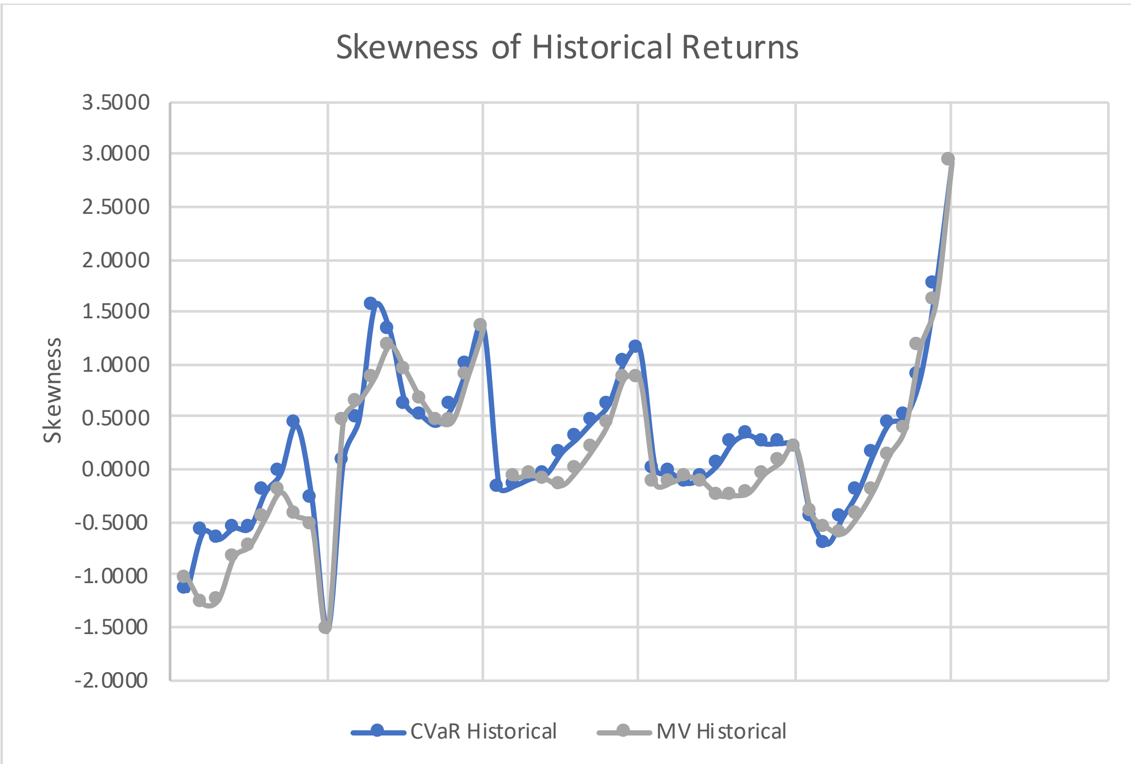
The frontiers are also generally smooth, showing an ability for the optimisation methods to take account of such risk and face a trade-off to gain a higher return. The only clear outlier being in the Indian market, where there is an inverted 'v' shape in both the MCVaR and MV-optimal frontiers for absolute deviation and semi-variance. This may well be down to the high volatility in the market, providing more variance of returns above the mean rather than below for portfolios higher up the efficient frontier. For semi-variance, this trend in absolute deviation may also be explained by such extreme positive returns, which causes a much higher variance in the returns, but not such a high absolute deviation as these extreme differences from the mean are not exaggerated compared to smaller differences by squaring.

These are intriguing results, considering the expectation stated previously that these metrics be biased towards illustrating the mean-variance portfolio optimisation method as the better of the two. Thus, the importance of the consideration of multiple risk metrics can be appreciated here, given that the MV-optimal portfolios were also found to frequently outperform the CVaR-optimal portfolios in the mean-VaR framework. This also exposes the issue that variance bares through penalising above-average returns.

Higher Moments

Upon inspection of the first four central moments of the return distributions for each set of portfolios, there are some clear and distinguishing factors. One further proof of the CVaR method's ability to better consider the higher moments in an investor's utility function is that the skewness of CVaR-optimal portfolios is very often higher than that of variance-efficient portfolios. Such a tendency is displayed below in Figure 6.4, where the skewnesses of the historical distributions are plotted for both sets of portfolios. The line plot for the CVaR portfolios is higher for the majority of the portfolios when considered against their equivalent MV-efficient portfolio.

Figure 6.4
Skewness of Returns of all Variance and CVaR-Efficient Historical Portfolios



It can be deduced that, given all other factors constant, the CVaR portfolios would be preferred as they offer extreme, high returns far more often than equivalent extreme losses. This makes complete sense given the objective of CVaR is to minimise expected extreme losses. The two methods do not appear to express a great difference in the levels of kurtosis observed for their portfolios. The average difference between the kurtoses of the portfolios using the CVaR and MV-efficient portfolios in both the historical and applied samples was only 0.59 and insignificant from zero to the 5% significance level. Given the variances of the MV-optimal portfolios will always be lower than those of the CVaR-optimal portfolios, upon assessment of the remaining first central moments, it appears that CVaR does show an awareness to improve investor utility in the presence of at least a third order utility function in terms of expected value. This does not fully satisfy the first hypothesis set out, given there is almost no difference in the kurtoses of these respective portfolios. For visualisation of the central moments of each portfolio and frontier, see the 3D plots of mean, variance, skewness and kurtosis in the appendix.

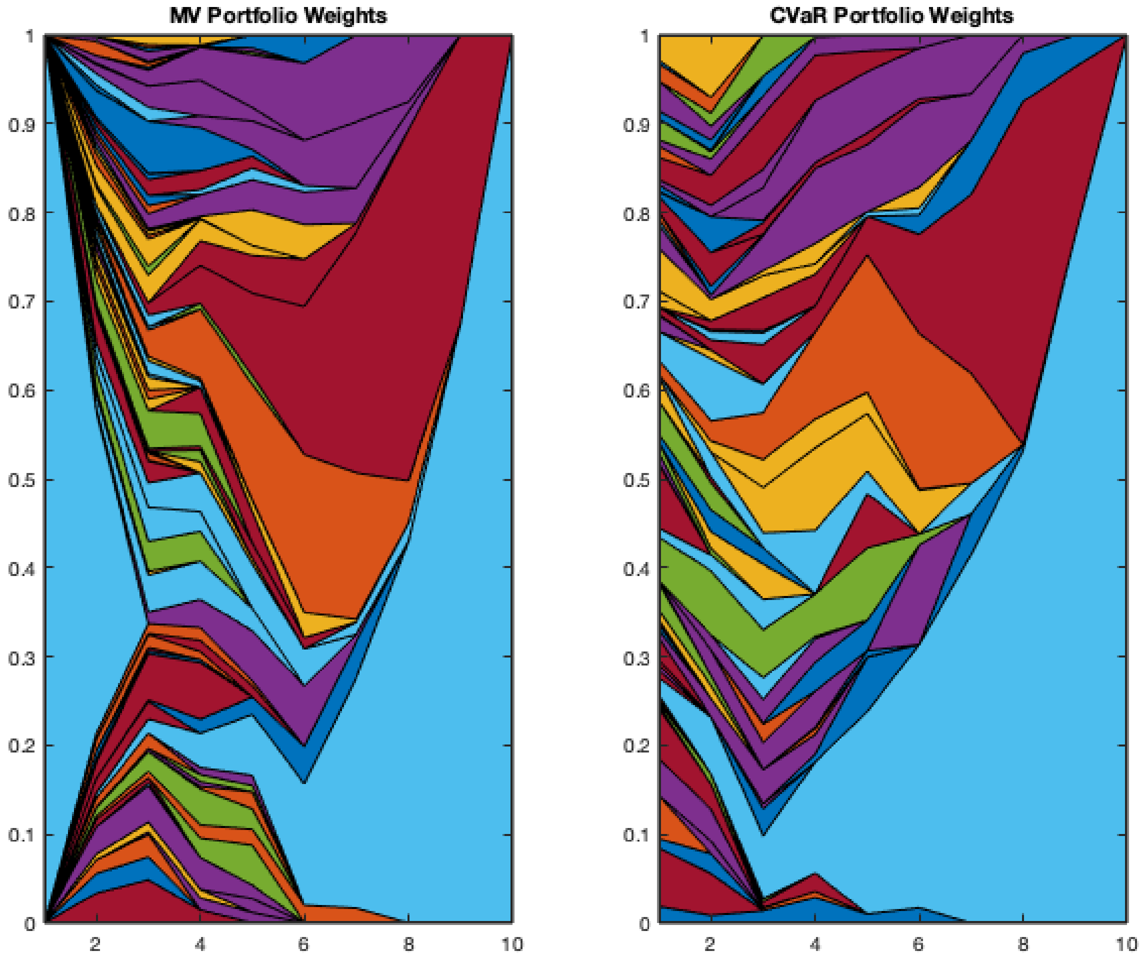
Historical v Applied Frontier Comparison

Little resemblance is found between the frontiers in the historical and applied period overall. Most frontiers actually have a negative slope in the applied period, as opposed to their prediction using historical data. This could be a sign of a winner's curse and reverse of fortune in the choice of assets in the optimised portfolios, where the fortunes of the best performers historically switch (Thaler, 1988 and Biais et al., 2010). This is largely undocumented in the literature, where reversals of fortune are generally only discovered for short term returns or after large negative returns (Bremer and Sweeney, 1991 and Cox and Peterson, 1994).

The frontiers in the applied period, though appearing to move randomly, follow one another closely. This results partly from the similarities in the portfolios held in each market with regards to asset weightings, especially at the extremes of their frontiers. A good example of this tendency is displayed for India in Figure 6.5, where the portfolios both become completely concentrated in one asset for the portfolio with the highest return.

Figure 6.5

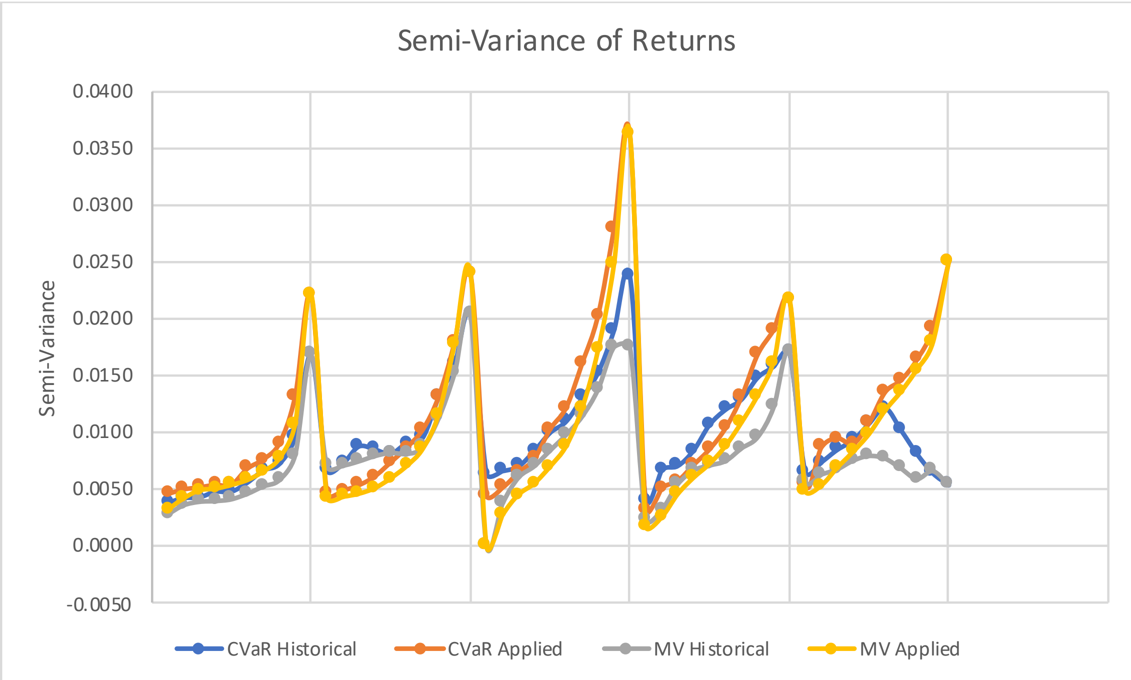
Portfolio Weightings in the Indian Market for the Variance and CVaR-Efficient Frontiers



An important feature that may be able to cast some confidence in the ability to choose a risk level for investing is shown in the semi-variance and absolute deviation in Figure 6.6. In both cases, the historical and applied frontiers are relatively smooth.

Figure 6.6

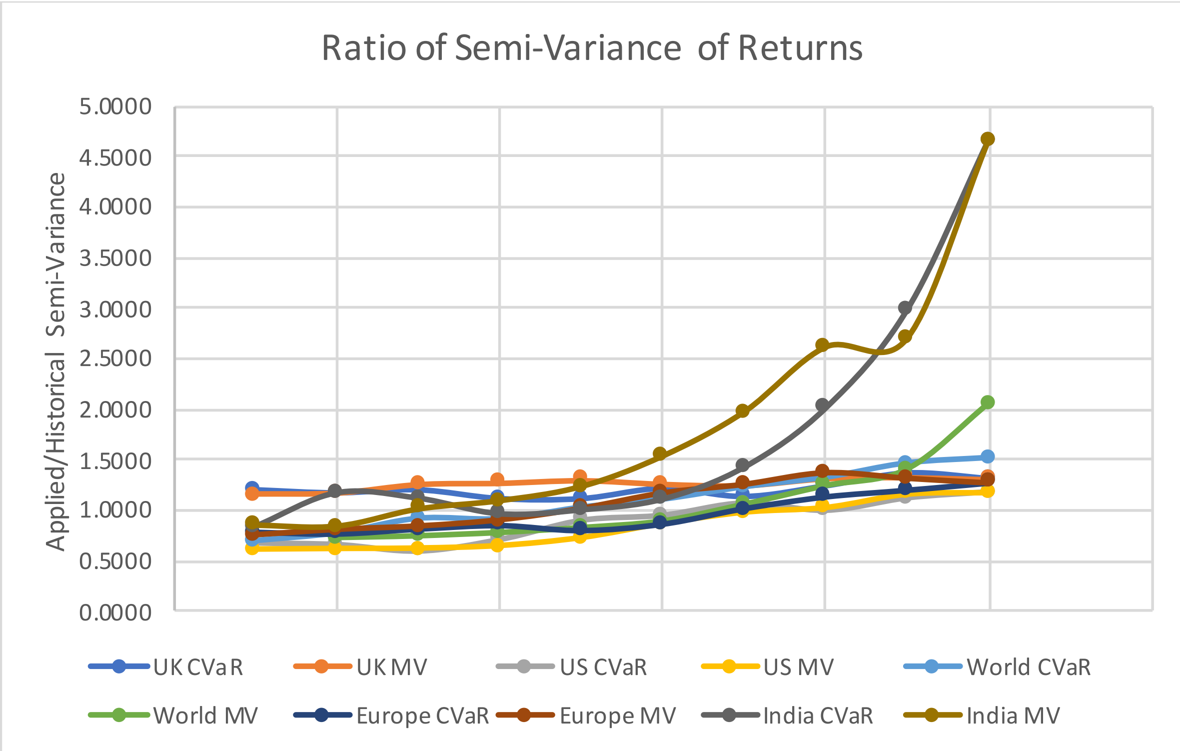
Semi-Variance of Returns of all Variance and CVaR-Efficient Historical and Applied Portfolios



Most risk metrics are found to be fairly consistent over time and hence give a reliable objective to set when considering a portfolio, with VaR and CVaR appearing to be the only inconsistent ones. For example, an investor could choose the level of semi-variance or absolute deviation they are willing to accept and make a good estimate of this value using the equivalent metric from the historical frontier. This could be done by simply fitting a straight line through the ratios of historical to applied semi-variance along the efficient frontier for a chosen market given below in Figure 6.7, allowing one to find the multiplier to estimate the applied semi-variance of a chosen portfolio along the frontier for another future period. Then, the investor could choose the portfolio along the historical frontier they would like to invest in based on their preferred level of return semi-variance. Most of the metrics have remained stable, with a clear trend between the historical and applied periods for most portfolios, though begin to show less stability when investing in the entire World and especially the Indian market.

Figure 6.7

Ratio of Applied to Historical Semi-Variance for all Portfolios Generated



These features cast considerable doubt over the applicability of the models regarding the use of historical data only. There is clear space for the introduction of predictive returns distributions, which has been applied for years but will never be able to perfectly predict returns like assessed in the historical sample. A reason for such a difference in results may partly due to a lack of applied data, given the sample of only one versus ten years of returns in the historical sample. This has unsurprisingly given especially temperamental results for the VaR and CVaR estimates, since they are made for extreme returns and hence require an abundance of data.

Given the inconsistency of VaR and CVaR between the two periods, it should also be tested as to whether the use of stressed VaR or stressed CVaR instead provides an improved measure of future risk, given the correct period of stressed asset behaviour could be identified to provide a better prediction of expected extreme risk. Some suggestions specifically within the sample collected are in the peak of the global financial crisis from 2007 to 2010, which could be applied to all of the regions considered. For more country-specific stressed samples, the periods immediately following the Brexit vote in the UK or appointment of President Trump for the US.

7. Discussion and Conclusion

Research Implications

The results obtained for the five chosen markets display, most importantly, the difficulty brought about in predicting returns. There have been many methods proposed for generating posterior return distributions, though to the knowledge of the author there are very few, if any, proposed to include the consideration of a winner's curse or reversal of fortune. This ensures that the stocks chosen from historical data are not just companies with inflated prices or those that have simply enjoyed good times recently. This would be very difficult to differentiate. To end this point, it is worth considering: is it even possible to generate a reliable predictor of future return distributions? Nevertheless, an investigation into how best to use perfect data provides crucial investment advice.

Given the importance of ES for current regulatory purposes (Bank for International Settlements, 2016), there are many research questions brought for the estimation of CVaR, given the instability in the CVaR-values for MV-efficient portfolios during the applied period, even relative to the unstable values given in historical performance. This unpredictability suggests that far more work is required for these regulators to correctly estimate the extreme risk of portfolios generated with the objective to optimise some metric other than ES. This is highly important given the value of financial markets worldwide and the range of alternate metrics against which portfolios are optimised.

The next step in the utilisation of diversification opportunities would be to expand the consideration to multiple asset classes and take advantage of the different return distributions offered by each, such as those of options considered by Black and Scholes (1973) and Capiński (2015).

Investment Implications

Upon investigation of these two popular portfolio generation techniques, it appears that both are able to provide guidance and superior performance under different investment objectives, though the MCVaR model proves itself to almost always outperform MV model overall.

Many of the risk metrics have proven themselves to be fairly stable between the historical and applied periods, suggesting that historical data can give a close estimate of future portfolio risk with very little need for manipulation. The issue lies in the ability to predict average returns, which is very unstable in the frontiers assessed. Uncertainty in any metrics appears to be magnified in volatile emerging markets, as shown by the results for India and inability to feasibly undertake such an investigation into China. When presented with opportunity for diversification, the models appear to generate more unpredictable results. This is not to say that the optimisation methods are unable to extract more return, but this is offset with more risk.

One of the more drastic findings is the lack of ability for historical returns alone to predict future returns. This backs the need for more advanced methods for return distribution prediction, which alone supports the value of fund managers in often forming expectations and predictions for future stock performance. This would hence require human interaction even in the most passive investment vehicles that do not simply track an index, such as 'closet indexes', a field in which many are attempting to minimise transaction costs to appeal to the growing passive investing community (Sushko and Turner, 2018, p.113). It does not therefore appear there is space for such intelligent portfolio generation methods to replace active investors without huge improvements in return predictions, though these methods do show themselves to be very powerful in the presence of accurate posterior distribution estimates and perfect capital markets.

An extension of the investigation that should be considered for applied portfolio optimisation is the consideration of transaction costs associated with investing in the imperfect conditions of the real world. These can include both fixed and variable components but also be considered for rebalancing in each period, as in Beasley et al. (2013). This ensures that rebalancing is only done when it is profitable and so allows better dynamic performance. There is a possibility that by rebalancing based on new historical data, predictions of future performance could be improved. Though from the findings of this investigation, performance could actually worsen through the purchase of 'winners' that then experience a reversal of fortune in the near future once their weightings have been increased by the optimisation. It is therefore apparent that the flawed use of historical data alone must be corrected before applying these concepts to making investments and expecting reliable, predictable performance.

In choosing where these optimisation methods perform best, there is clear evidence that, when presented with a sufficiently large market representative, both MV and MCVaR-optimal portfolios are able to offer strong returns at each risk level in developed capital markets with more stability relative to emerging markets. The UK in particular showed very strong and stable performance in both the historical and applied portfolios for both optimisation methods. Whilst diversifying more in the larger markets of the US and Europe provided strong performance, difficulties very much began to show themselves when applying these methods to include less-stable markets within the whole world and India, including the task being too problematic to generate sensible portfolios in China. Performance historically was often very strong but gave misleading expectations as to future performance.

When choosing the preferred portfolio optimisation technique, it is worth considering that the MV method is incapable of controlling extreme risk in the presence of more diversification opportunities, whilst the MCVaR method is able to offer similar risk-return combinations in the mean-variance space. Thus, it is not recommended to use the MV over the MCVaR method in emerging and diverse markets. The only situation in which the MV optimisation technique would be recommended over MCVaR is where strict computing constraints and a concentrated set of assets internationally from highly developed economies is considered. In all other conditions tested, the MCVaR optimisation technique is able to offer better overall risk-return characteristics.

Conclusion

This investigation into the performances of the MV and MCVaR methods of portfolio optimisation in five major regions has uncovered many interesting recommendations and even some questions. The previously held consensus that with more assets, industries and regions considered comes further opportunities to benefit from diversification is supported, though further risk is brought on and shown by alternative risk metrics.

Through optimisation, portfolios with far higher returns and much lower risk can be achieved compared with their respective benchmark indices. The only portfolio offering a lower historical return than its benchmark was the interesting minimum-variance portfolio for Europe, which recommended not investing at all for a guaranteed return of zero. The problem presented was then that the actual index was able to provide a higher return than most portfolios in the applied period, given the apparent reversal of fortune for these portfolios when applied for future performance. A summary of the performance of the indices used is given in the appendix for reference. This stems from the issues discussed regarding the use of only historical returns.

It should always be considered in financial markets that the unexpected will inevitably happen and that returns are unpredictable at best. As such, a limitation to this investigation is that these characteristics observed may not persist and could simply be a result of current market conditions. Such an abundance of data was collected to reduce the likelihood that misleading trends or market behaviour be discovered. The only way to test these observations is through analysis of more markets and time periods, providing more confidence in the robustness of recommendations that can be made and applied to other environments. The same is the case for the deductions made regarding the different market structures described in the results. With only five different samples to test for a general feature such as market size and growth prospects and their relation to risk, there is only a small amount of evidence to support these claims. The fact that there is a clear trend in the behaviour of the frontiers across the markets, where indicated, helps support the significance of these claims. It is encouraged that these be further investigated to test the significance with which these recur across other similar markets and for different periods.

There is so much scope for advancement in this topic, though the author would particularly advocate next comparing whether the CVaR method is able to offer a set of more comprehensive risk-return combinations than some of the models that extend the Markowitz model to include a concern for some of the higher central moments of the return distribution of each portfolio. This paper has shown that the CVaR method is able to indirectly take an interest in the skewness of the portfolio for utility maximisation, though it is unclear to what degree this is able to compete with a method such as those in Konno et al. (1993) and Lam et al. (2013), which directly optimise the skewness. A comparison then with the model in Lam et al. (2013) could also discover whether a consideration of kurtosis improves performance. A comparison of these with the model of Alexander et al. (2007, p.3766) would be very thought-provoking in examining whether combining mean, variance and CVaR optimisation offers a significant improvement.

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List of Figures

Appendix

MATLAB and Stata Code

Stata Code

```
* Mean and Variance of all portfolios
summarize

* First four moments of returns distributions of all portfolios
summarize, detail

* Plot returns distribution of portfolio 'n' against the normal
kdensity n, normal

* Perform the Shapiro-Francia test of return distribution normality for portfolio or asset 'n'
sfrancia n
```

MATLAB Code

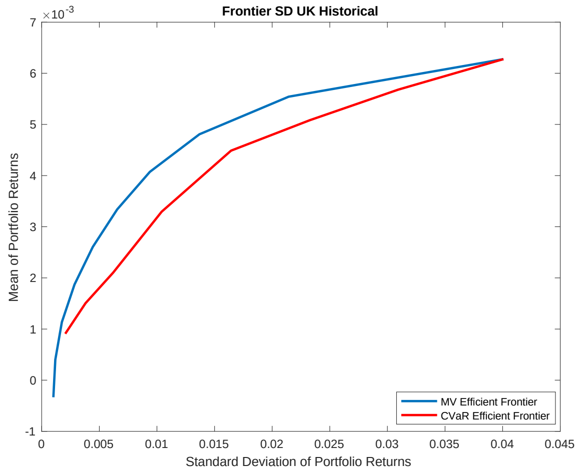
```
% Generation of a mean-variance efficient frontier

mv=Portfolio;
mv=mv.setAssetList(ticker);
mv=mv.estimateAssetMoments(returns);
mv=mv.setDefaultConstraints;

mvwgts=mv.estimateFrontier(10);

% Generation of a CVaR efficient frontier

cvar=PortfolioCVaR;
cvar=cvar.setAssetList(ticker);
```

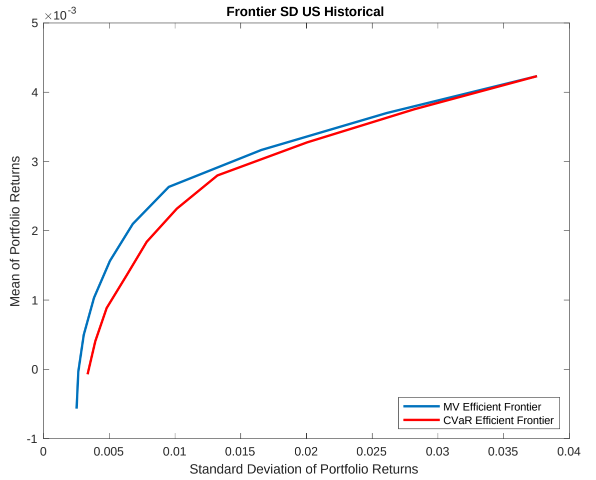
```
cvar=cvar.setScenarios(returns);
cvar=cvar.setDefaultConstraints;
cvar=cvar.setProbabilityLevel(0.99);

% Plotting the two frontiers against one another according to standard deviation

figure; [cvarRisk, cvarReturns]=cvar.plotFrontier(10);
cvarwgts=cvar.estimateFrontier(10);
cvarSD=cvar.estimatePortStd(cvarwgts);

figure;
mv.plotFrontier(10);
hold on
plot(cvarSD,cvarReturns,'-r','LineWidth',2);
legend('MV Efficient Frontier', 'CVaR Efficient Frontier', 'Location', 'SouthEast');

% Generation and plotting of the frontiers according to VaR

figure; [mvRisk, mvReturns]=mv.plotFrontier(10);

mvVaR = portvrisk(mvReturns, mvRisk);
cvarVaR = estimatePortVaR(cvar, cvarwgts);

figure; plot(mvVaR, mvReturns, cvarVaR, cvarReturns, '-r', 'LineWidth', 2);
legend('MV VaR Frontier', 'CVaR VaR Frontier', 'Location', 'SouthEast');
xlabel('VaR');
ylabel('Return');
```

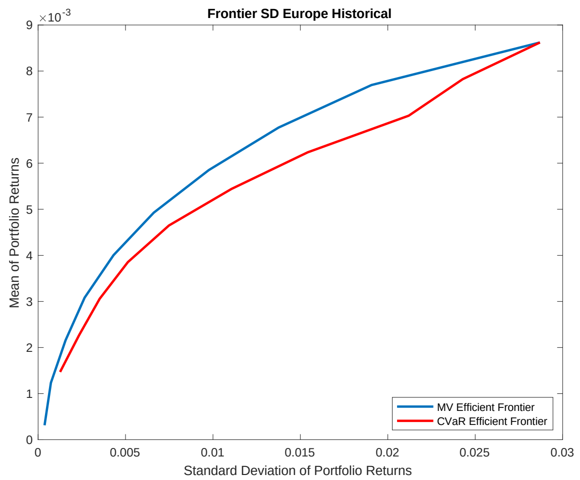
Frontiers

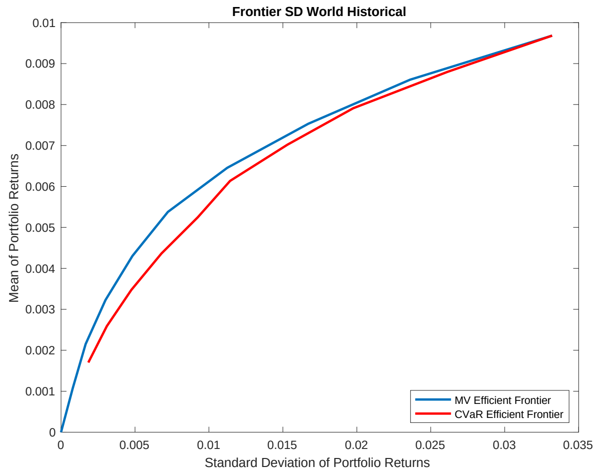
Mean-Variance space

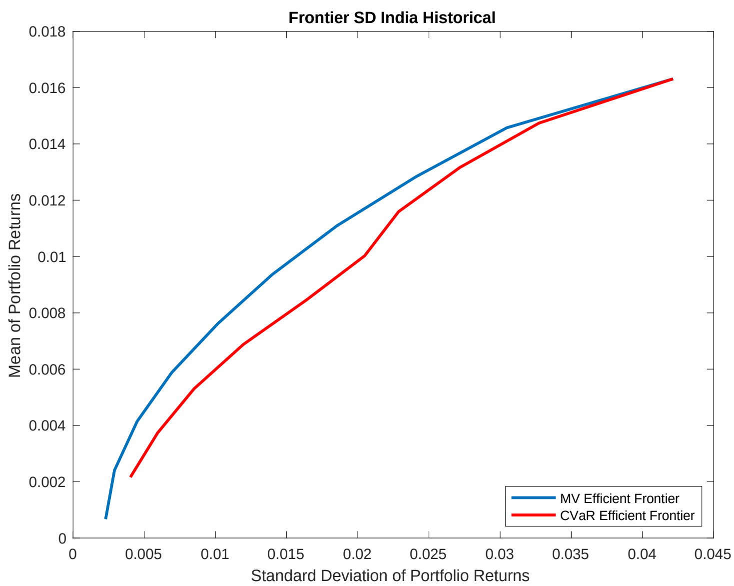
Historical



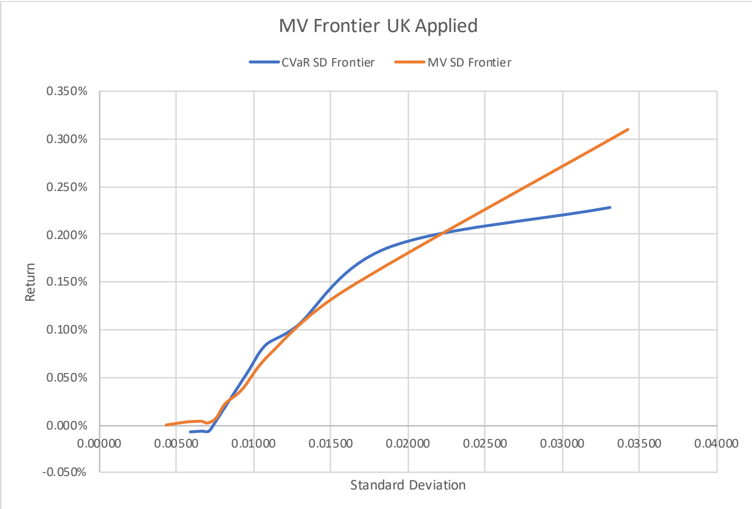


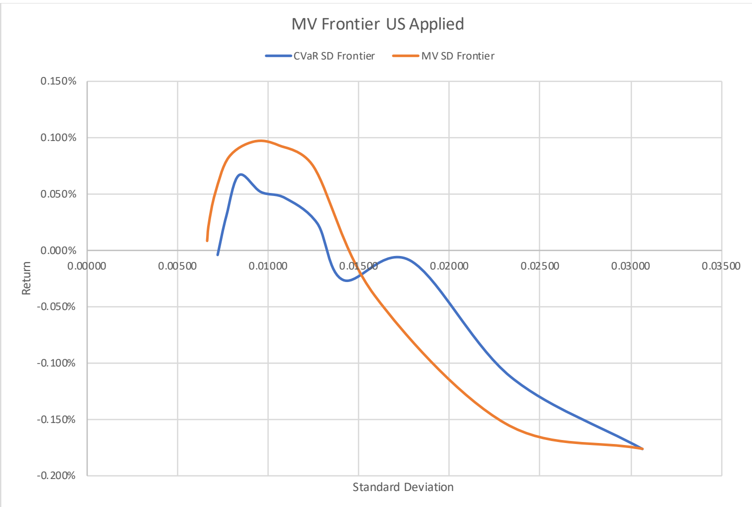


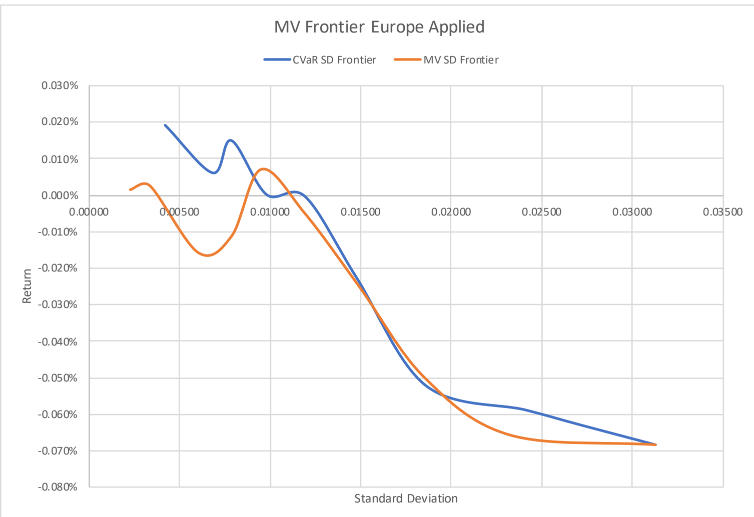


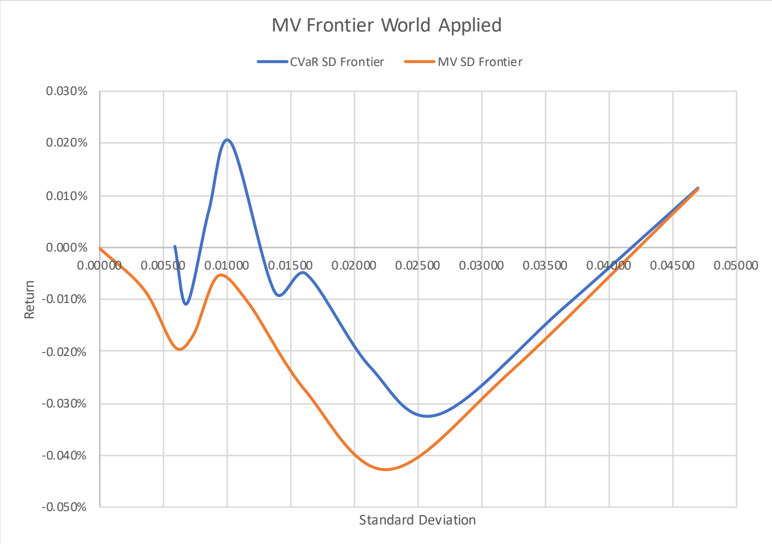


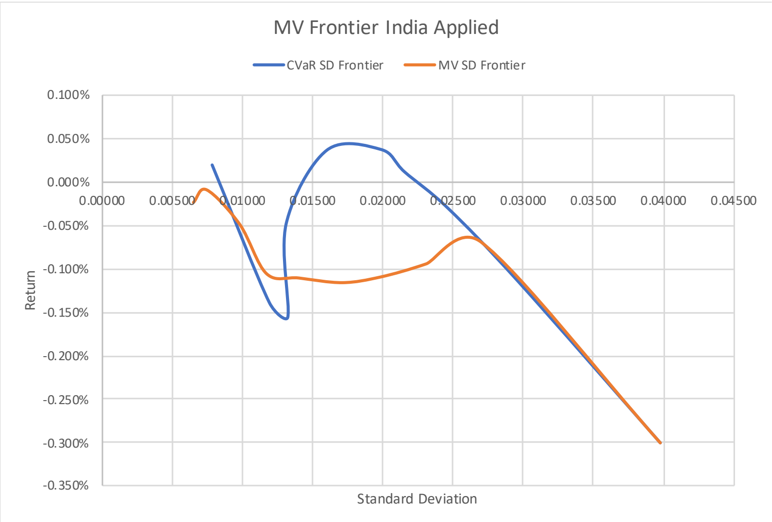
Applied


MV Frontier UK Applied
CVaR SD Frontier
MV SD Frontier
0.350%
0.300%
0.250%
0.200%
0.150%
0.100%
0.050%
0.000%
-0.050%
Return
0.00000
0.00500
0.01000
0.01500
0.02000
0.02500
0.03000
0.03500
0.04000
Standard Deviation


MV Frontier US Applied
CVaR SD Frontier
MV SD Frontier
0.150%
0.100%
0.050%
0.000%
-0.050%
-0.100%
-0.150%
-0.200%
Return
0.00000
0.00500
0.01000
0.01500
0.02000
0.02500
0.03000
0.03500
Standard Deviation

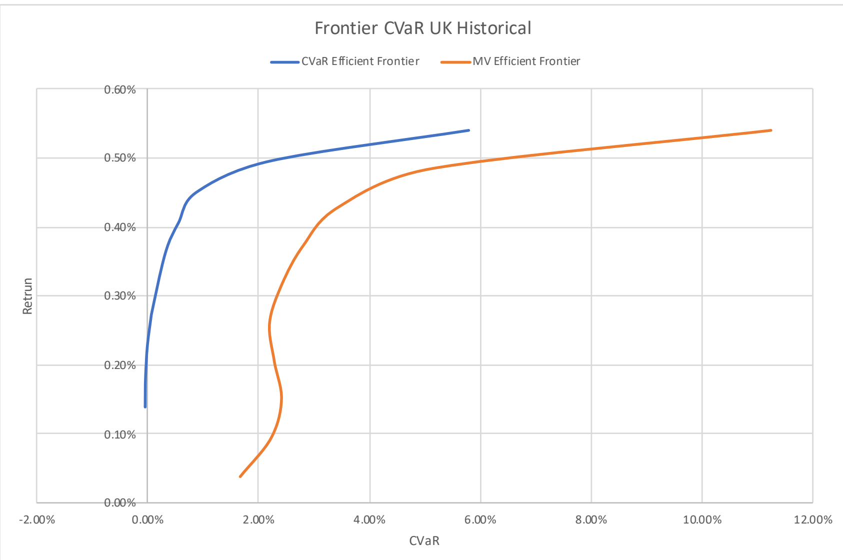

MV Frontier Europe Applied
CVaR SD Frontier
MV SD Frontier
0.030%
0.020%
0.010%
0.000%
-0.010%
-0.020%
-0.030%
-0.040%
-0.050%
-0.060%
-0.070%
-0.080%
Return
0.00000
0.00500
0.01000
0.01500
0.02000
0.02500
0.03000
0.03500
Standard Deviation

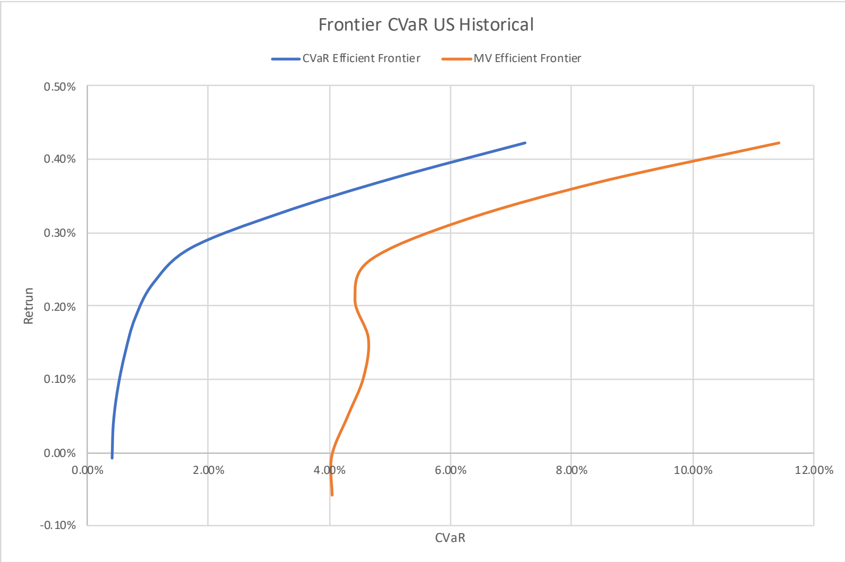


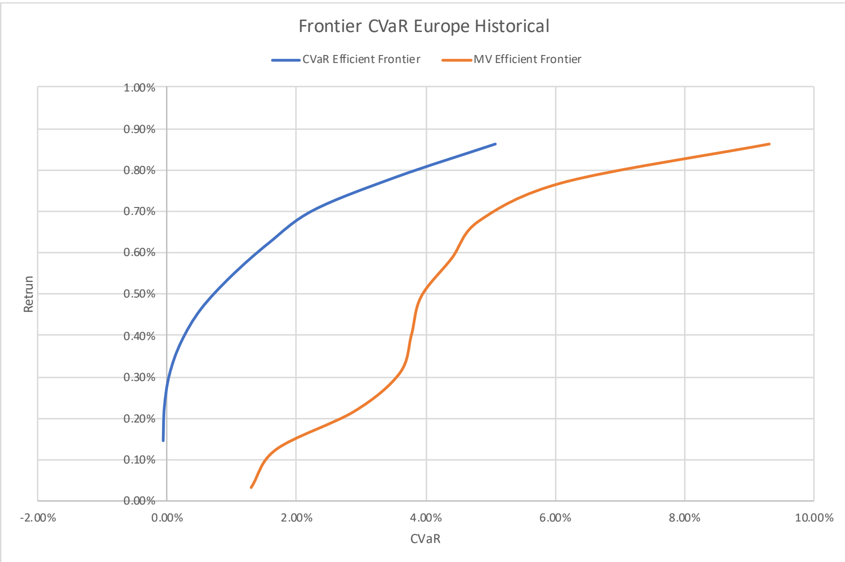


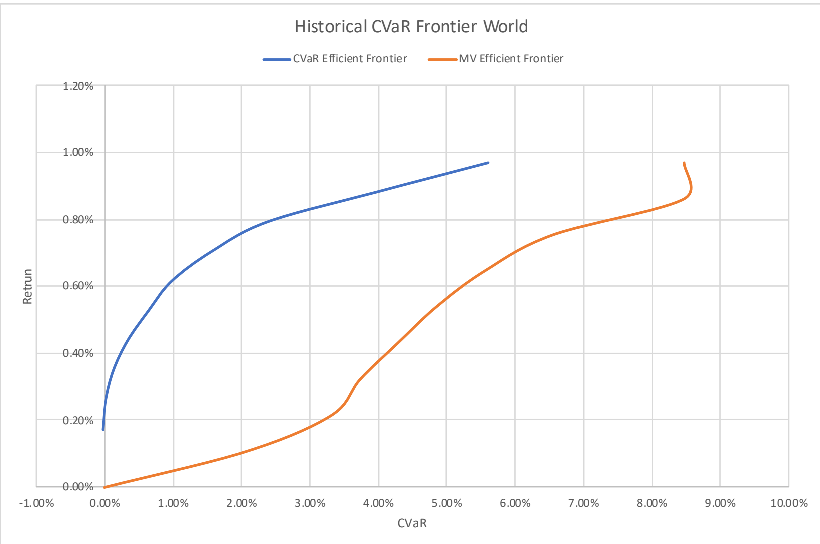
Mean-CVaR space

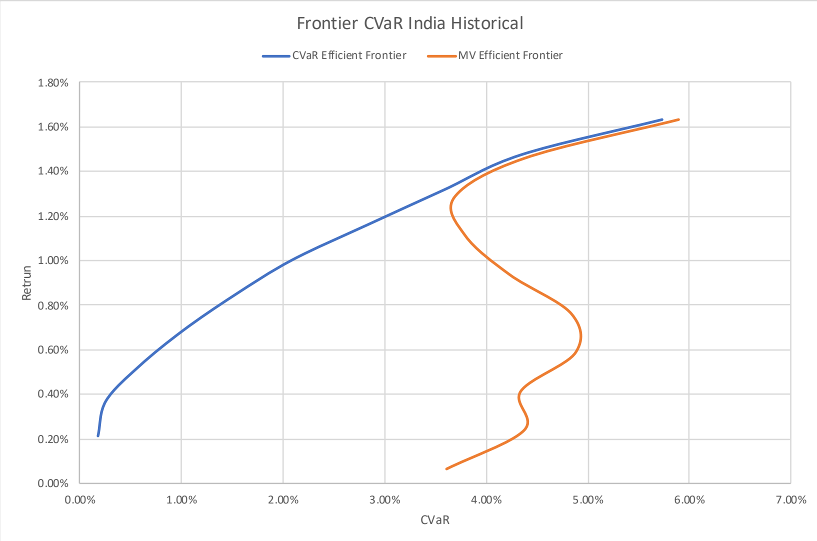
Historical



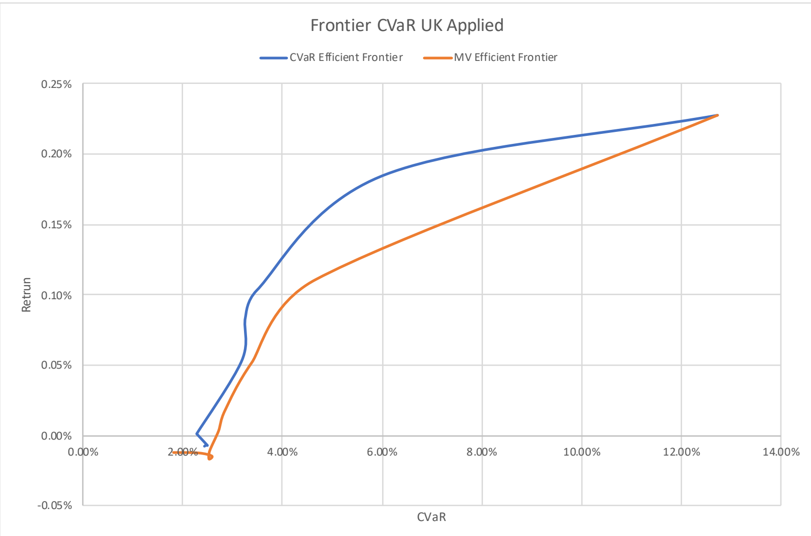

Frontier CVaR US Historical
CVaR Efficient Frontier
MV Efficient Frontier
0.50%
0.40%
0.30%
0.20%
0.10%
0.00%
-0.10%
Retrun
0.00%
2.00%
4.00%
6.00%
8.00%
10.00%
12.00%
CVaR

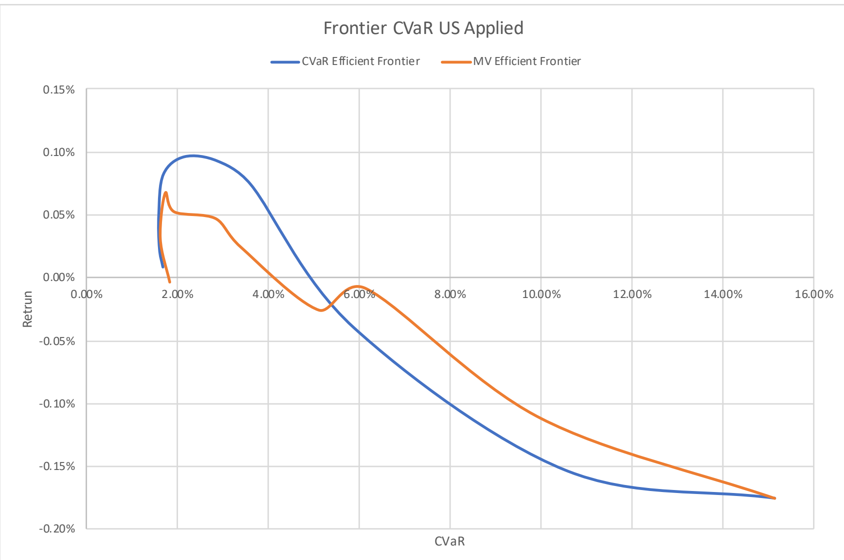

Frontier CVaR Europe Historical
CVaR Efficient Frontier
MV Efficient Frontier
1.00%
0.90%
0.80%
0.70%
0.60%
0.50%
0.40%
0.30%
0.20%
0.10%
0.00%
Retrun
-2.00%
0.00%
2.00%
4.00%
6.00%
8.00%
10.00%
CVaR

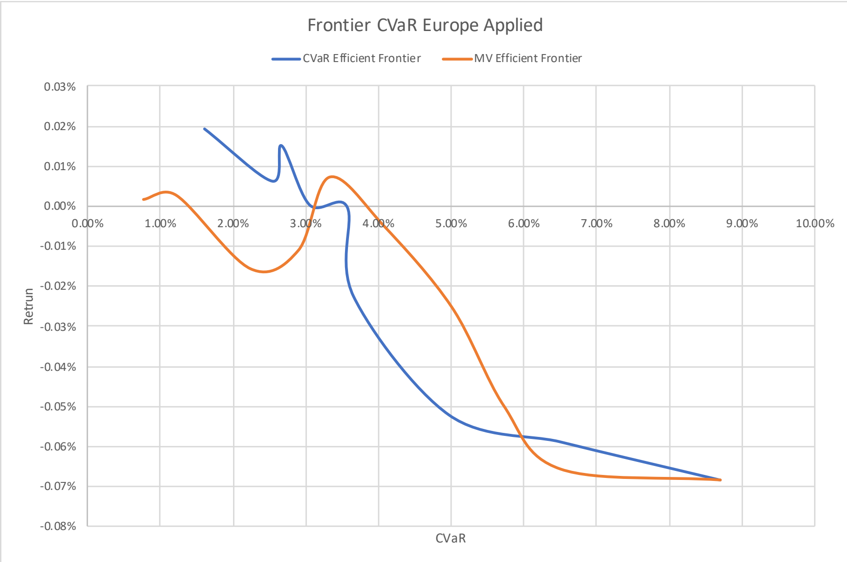

Historical CVaR Frontier World
CVaR Efficient Frontier
MV Efficient Frontier
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
Retrun
-1.00%
0.00%
1.00%
2.00%
3.00%
4.00%
5.00%
6.00%
7.00%
8.00%
9.00%
10.00%
CVaR

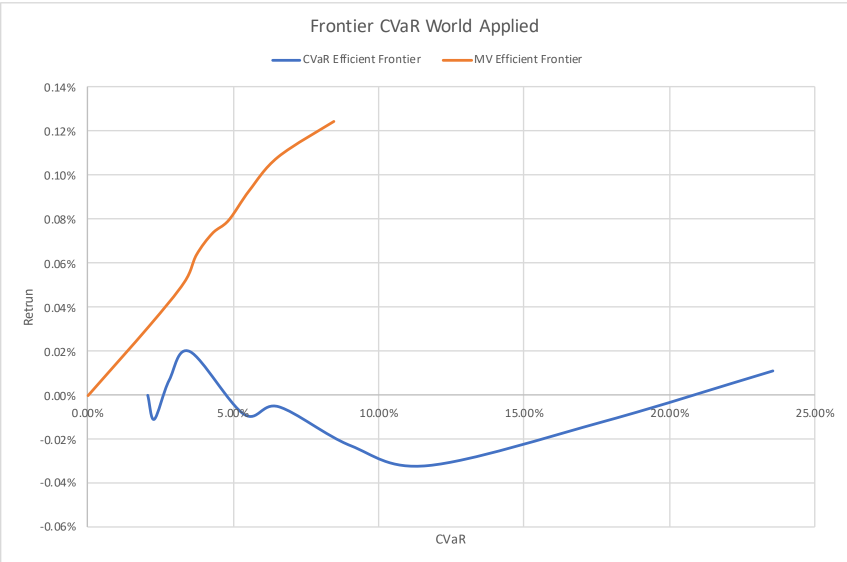

Frontier CVaR India Historical
CVaR Efficient Frontier
MV Efficient Frontier
Retrun
CVaR
1.80%
1.60%
1.40%
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
0.00%
1.00%
2.00%
3.00%
4.00%
5.00%
6.00%
7.00%

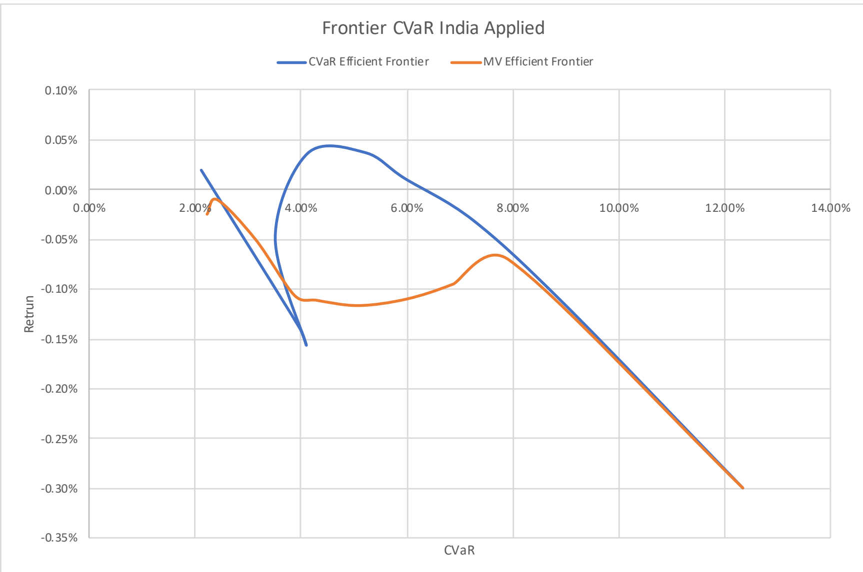
Applied


Frontier CVaR UK Applied
CVaR Efficient Frontier
MV Efficient Frontier
Retrun
CVaR
0.25%
0.20%
0.15%
0.10%
0.05%
0.00%
-0.05%
0.00%
2.00%
4.00%
6.00%
8.00%
10.00%
12.00%
14.00%


Frontier CVaR US Applied
CVaR Efficient Frontier
MV Efficient Frontier
Retrun
CVaR
0.15%
0.10%
0.05%
0.00%
-0.05%
-0.10%
-0.15%
-0.20%
0.00%
2.00%
4.00%
6.00%
8.00%
10.00%
12.00%
14.00%
16.00%

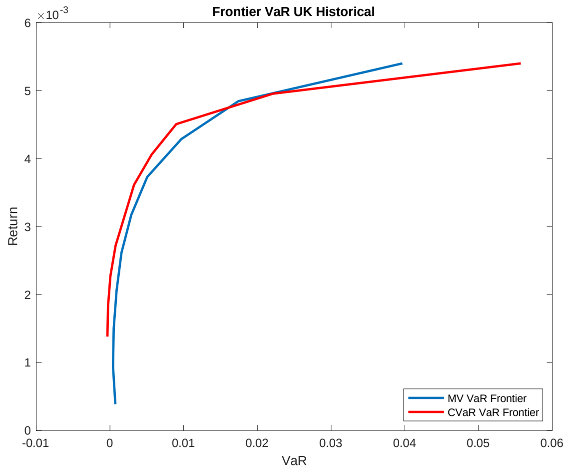

Frontier CVaR Europe Applied
CVaR Efficient Frontier
MV Efficient Frontier
0.03%
0.02%
0.01%
0.00%
-0.01%
-0.02%
-0.03%
-0.04%
-0.05%
-0.06%
-0.07%
-0.08%
0.00%
1.00%
2.00%
3.00%
4.00%
5.00%
6.00%
7.00%
8.00%
9.00%
10.00%
Retrun
CVaR

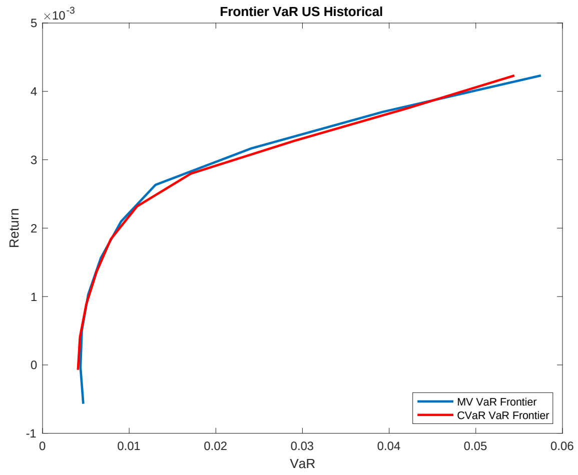

Frontier CVaR World Applied
CVaR Efficient Frontier
MV Efficient Frontier
0.14%
0.12%
0.10%
0.08%
0.06%
0.04%
0.02%
0.00%
-0.02%
-0.04%
-0.06%
0.00%
5.00%
10.00%
15.00%
20.00%
25.00%
Retrun
CVaR

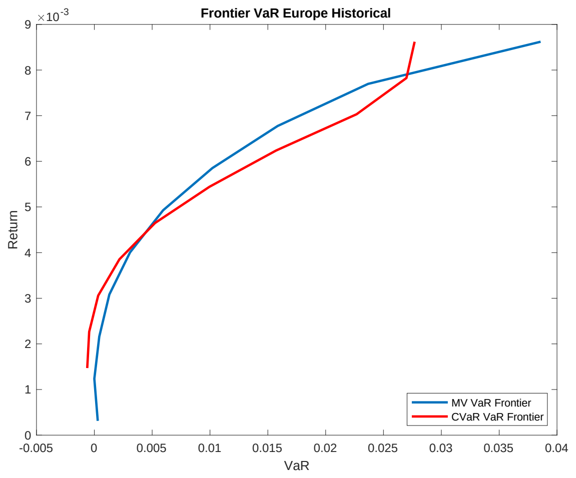

Frontier CVaR India Applied
CVaR Efficient Frontier
MV Efficient Frontier
0.10%
0.05%
0.00%
-0.05%
-0.10%
-0.15%
-0.20%
-0.25%
-0.30%
-0.35%
0.00%
2.00%
4.00%
6.00%
8.00%
10.00%
12.00%
14.00%
Retrun
CVaR

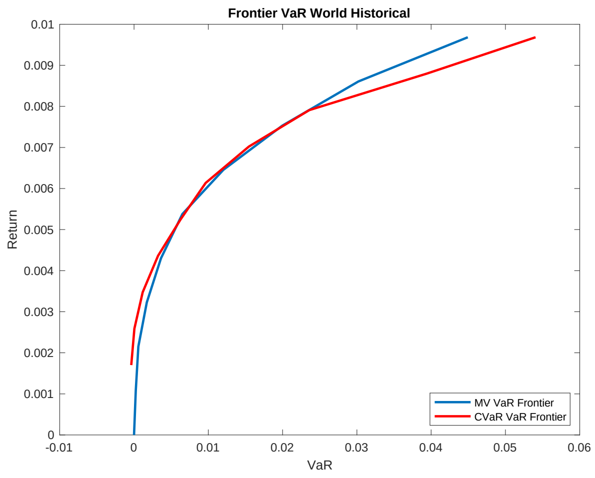
Mean-VaR space

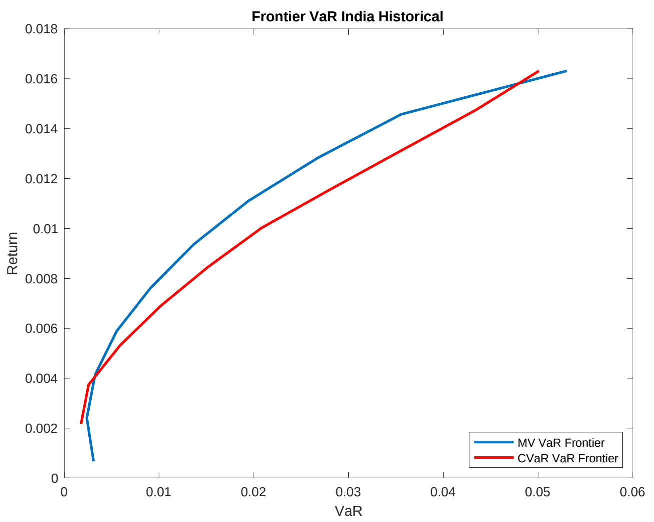
Historical



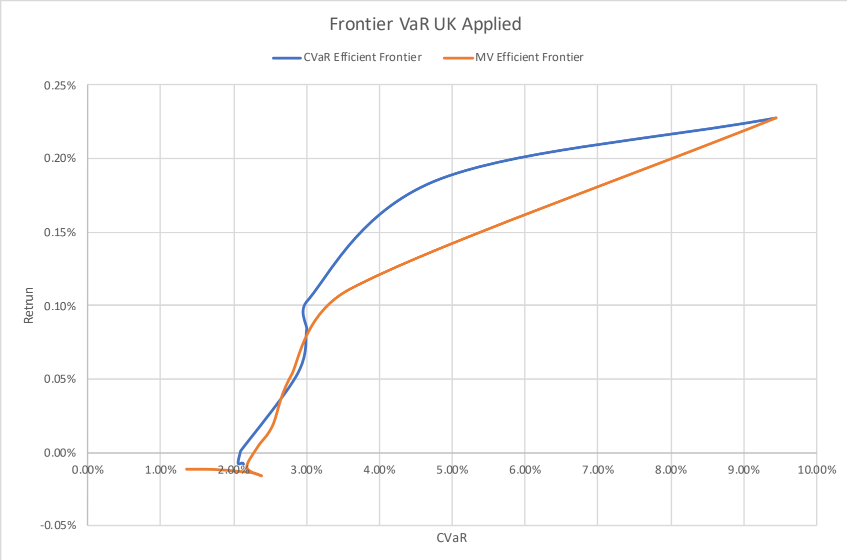


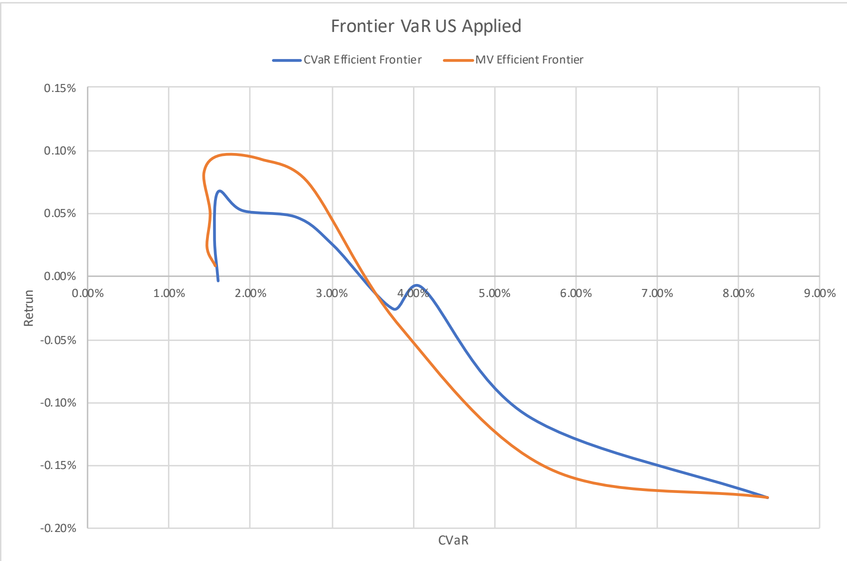


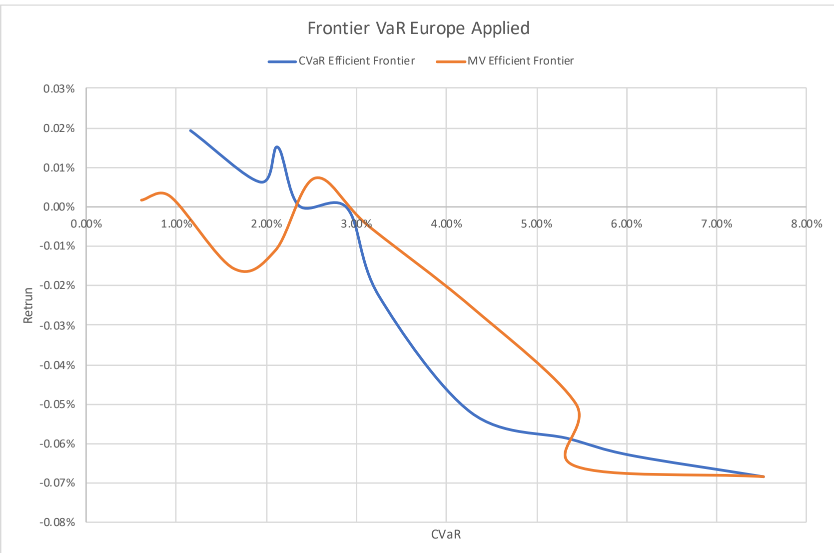


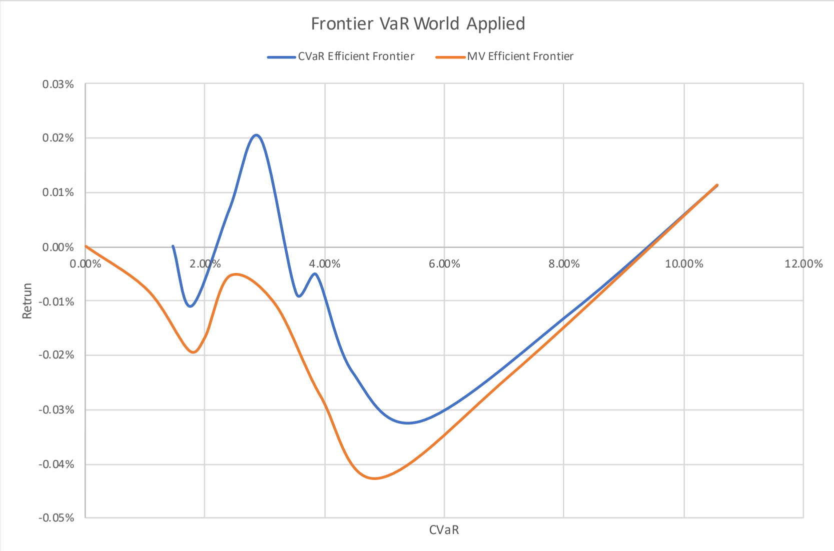


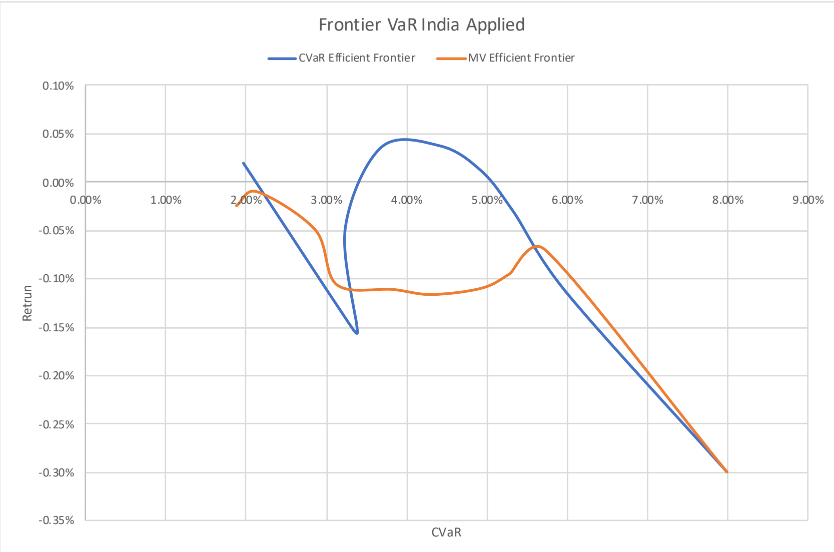
Applied


Frontier VaR UK Applied
CVaR Efficient Frontier
MV Efficient Frontier
Retrun
CVaR
0.25%
0.20%
0.15%
0.10%
0.05%
0.00%
-0.05%
0.00%
1.00%
2.00%
3.00%
4.00%
5.00%
6.00%
7.00%
8.00%
9.00%
10.00%


Frontier VaR US Applied
CVaR Efficient Frontier
MV Efficient Frontier
Retrun
CVaR
0.15%
0.10%
0.05%
0.00%
-0.05%
-0.10%
-0.15%
-0.20%
0.00%
1.00%
2.00%
3.00%
4.00%
5.00%
6.00%
7.00%
8.00%
9.00%

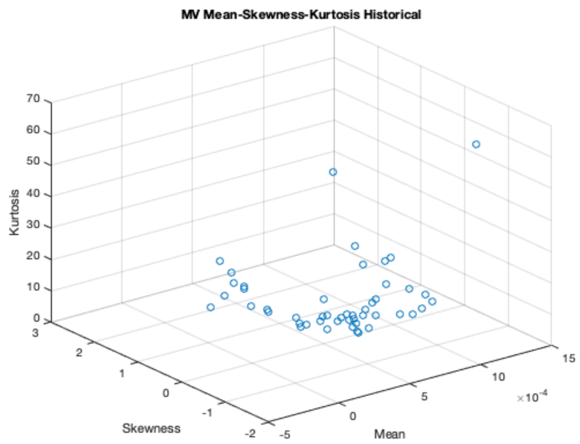

Frontier VaR Europe Applied
CVaR Efficient Frontier
MV Efficient Frontier
Retrun
CVaR
0.03%
0.02%
0.01%
0.00%
-0.01%
-0.02%
-0.03%
-0.04%
-0.05%
-0.06%
-0.07%
-0.08%
0.00%
1.00%
2.00%
3.00%
4.00%
5.00%
6.00%
7.00%
8.00%

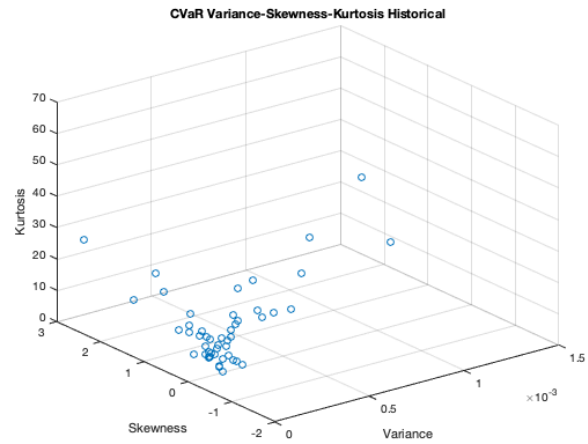


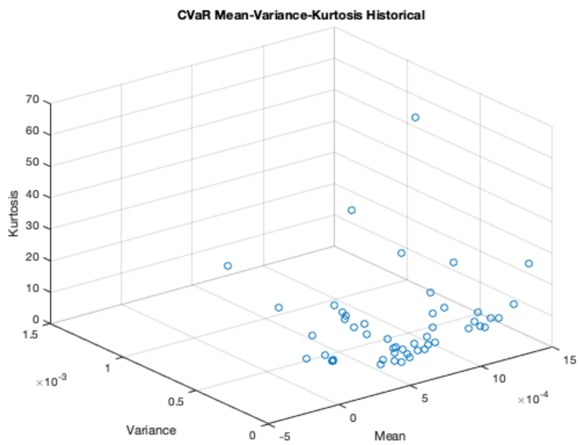


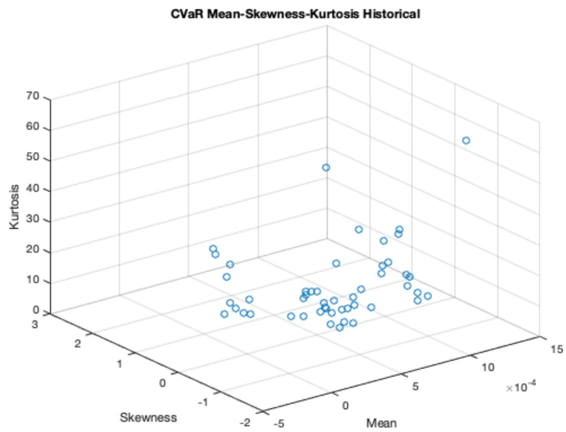
3D Plots

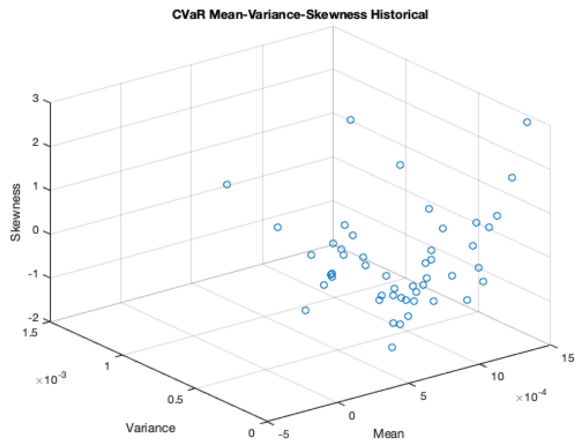
Historical

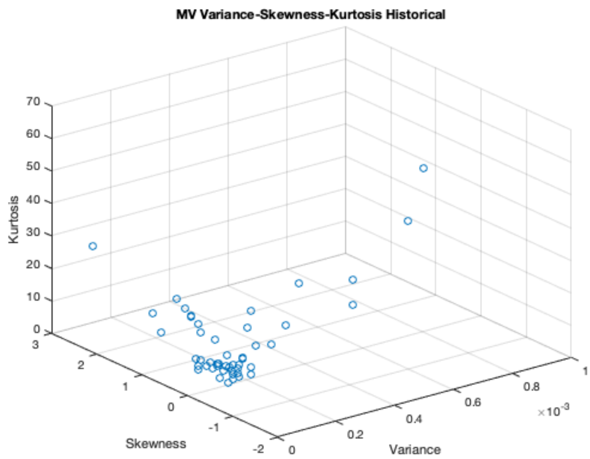

MV Mean-Skewness-Kurtosis Historical
Kurtosis
Skewness
Mean

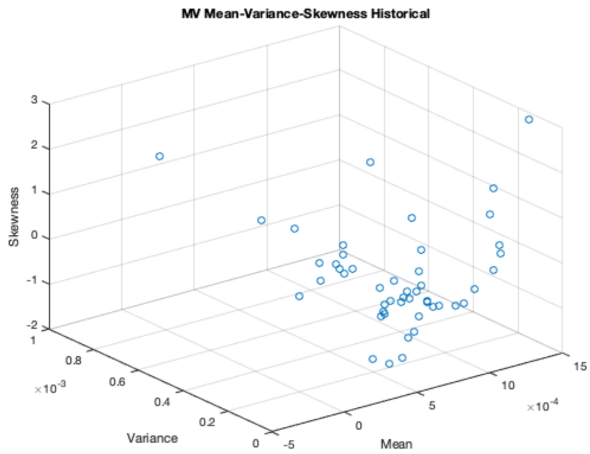

CVaR Variance-Skewness-Kurtosis Historical
Kurtosis
Skewness
Variance

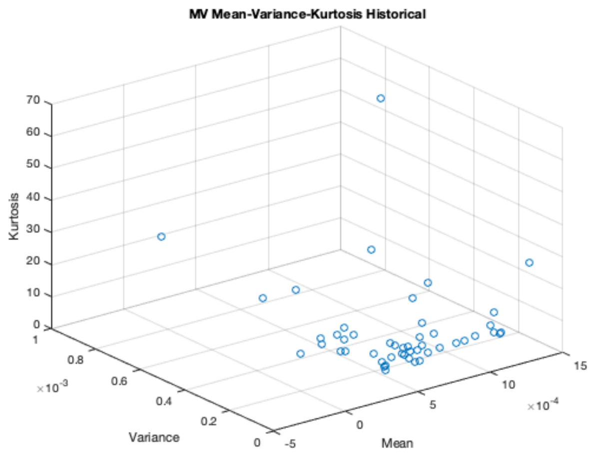

CVaR Mean-Variance-Kurtosis Historical
Kurtosis
Variance
Mean


CVaR Mean-Skewness-Kurtosis Historical
Kurtosis
Skewness
Mean

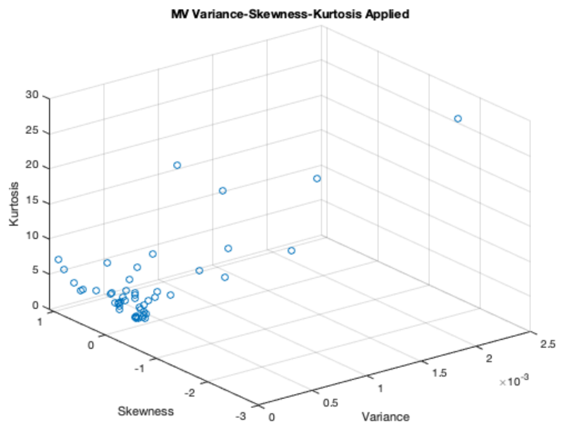

CVaR Mean-Variance-Skewness Historical
Skewness
Variance
Mean

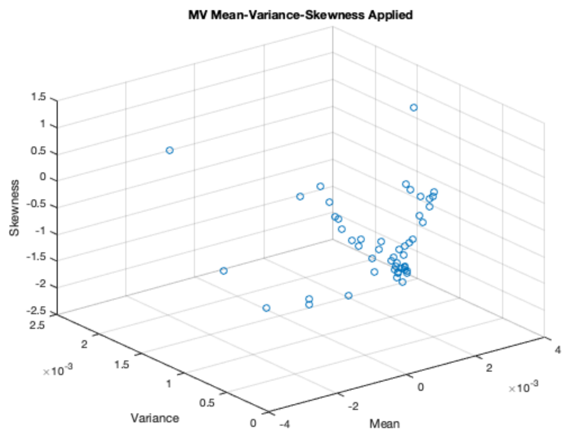

MV Variance-Skewness-Kurtosis Historical
Kurtosis
Skewness
Variance

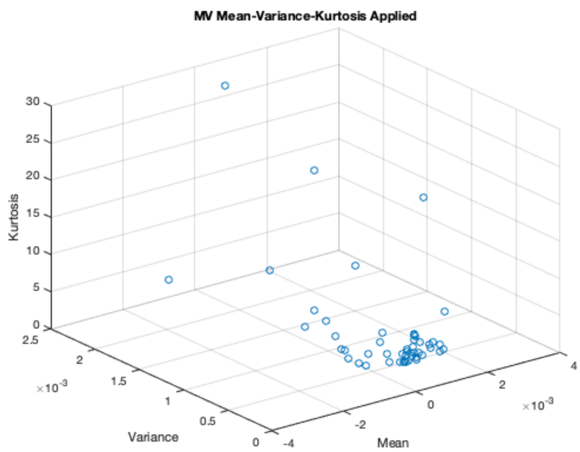

MV Mean-Variance-Skewness Historical
Skewness
Variance
Mean

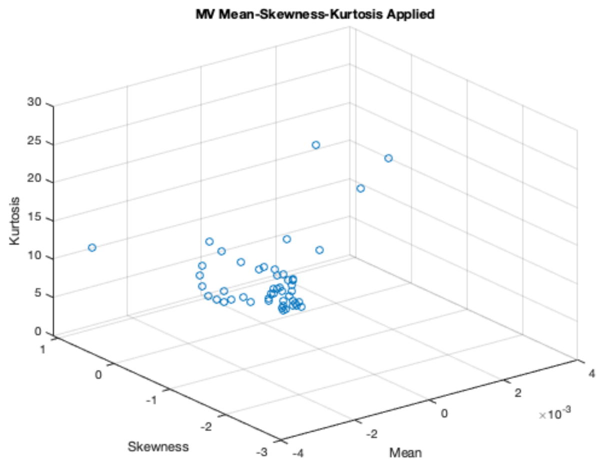

MV Mean-Variance-Kurtosis Historical
Kurtosis
Variance
Mean

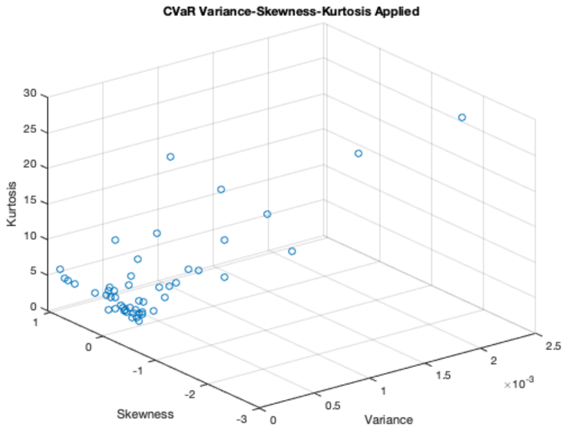
Applied

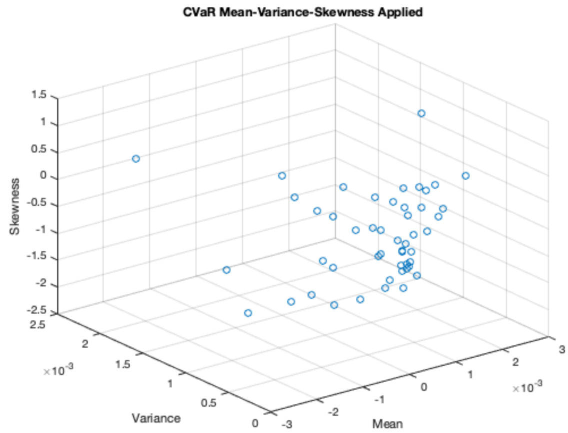

MV Variance-Skewness-Kurtosis Applied
Kurtosis
Skewness
Variance

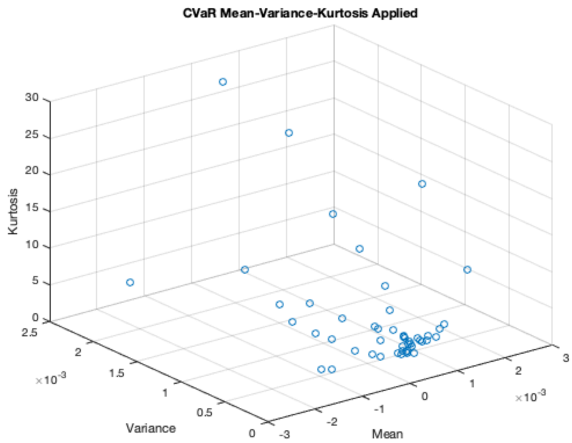

MV Mean-Variance-Skewness Applied
Skewness
Variance
Mean

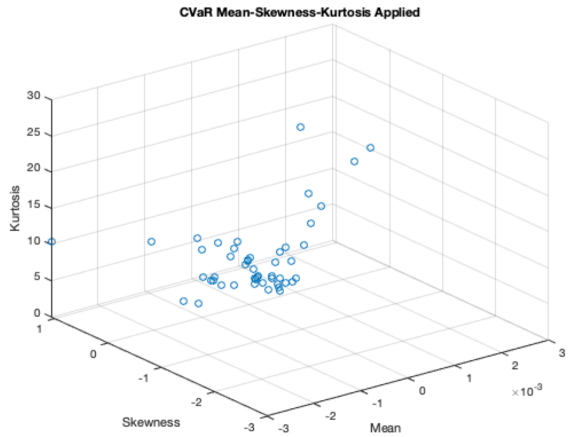

MV Mean-Variance-Kurtosis Applied
Kurtosis
Variance
Mean


MV Mean-Skewness-Kurtosis Applied
Kurtosis
Skewness
Mean









Index Returns and Risk Metrics

	Market	US	UK	Europe	World	India
Historical	Return	0.030%	0.018%	0.011%	0.016%	0.052%
	SD	0.0126735	0.0190373	0.0166011	0.0111553	0.0280780
	CVaR	5.49%	7.79%	6.65%	4.88%	12.05%
	VaR	3.90%	5.49%	4.90%	3.58%	8.39%
	SV	0.0000000	0.0597286	0.0352054	0.0137159	0.0204497
	Abs Dev	0.0078382	0.0123981	0.0110592	0.0071480	0.0172777
Applied	Return	0.020%	-0.023%	-0.031%	-0.005%	0.006%
	Variance	0.0092602	0.0092564	0.0083990	0.0072005	0.0115591
	CVaR	3.71%	2.94%	2.62%	2.69%	3.67%
	VaR	3.17%	2.34%	2.32%	2.27%	2.90%
	SV	0.0071105	0.0068322	0.0061902	0.0055482	0.0083951
	Abs Dev	0.0063553	0.0069495	0.0063428	0.0052058	0.0089127