Assignment 05: Vectors and Matrices

Note: Please upload your solution as an ipynb file to the Canvas page.

The purpose of this assignment is to develop your skills in using Numpy arrays and doing vector and matrix operations.

1. Write custom individual functions to perform the following matrix operations, without using the in-built numpy functions. Add checks on suitability of the shape of matrices whenever possible

   (a) Add two matrices

   (b) Subtract two matrices

   (c) Scalar product of a matrix

   (d) Multiply two matrices

   (e) Sum of all elements in a matrix

   (f) Check if a given matrix is symmetric and square.

2. Let A be a 4 x 4 matrix and B a 2 x 1 matrix. The symbol $\cdot$ represents a dot product. Using the shape of the matrix as a guide, indicate if the following expressions are valid or not. Explain your reason.

   (a) $A + B$

   (b) $A \cdot B$

   (c) $A \cdot A$

   (d) $B \cdot B$

   (e) $B^T \cdot B \cdot A$

3. Using the following matrices and vectors. Compute the following using built-in numpy functions .

$$A = \begin{bmatrix} 1 & 4 & -2 \\ 4 & 8 & 6 \\ -2 & 6 & 12 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 2 & -2 \\ 4 & 8 & 3 \\ -1 & 6 & 9 \end{bmatrix}$$

$$c = [6, -4, 3]$$

$$d = [3, -1, 5]$$

   (a) $A - A^T$

   (b) $A \cdot B$

   (c) $8A - 4B$

(d) $B^T \cdot B$

(e) $c^T \cdot B$

(f) $A \cdot B \cdot c \cdot d^T$

(g) $c \times d$

(h) Inverse A^{-1} and check if $A * A^{-1} = I$

4. Find a a unit vector (i.e., vector of magnitude equal to 1) that is perpendicular to both c and d. Hint: *use the cross product definition.*

5. Using array slicing on matrix A

$$A = \begin{bmatrix} 4.0 & 7.0 & -2.43 & 67.1 \\ -4.0 & 64.0 & 54.7 & -3.33 \\ 2.43 & 23.2 & 3.64 & 4.11 \\ 1.2 & 2.5 & -113.2 & 323.22 \end{bmatrix}$$

(a) Extract the third column as a 1D array

(b) Extract the first two rows as a 2D sub-array

(c) Extract the bottom-right 2×2 block as a 2D sub-array

(d) Sum the last column