Assignment: Codes: Definitions and Theorems, Due Monday, 11:59pm

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1. Non-Perfect Code

Make up an example of a linear code with 16 code words that isn't perfect. List the 16 code words, and give a generating matrix.

```
M <- matrix(c(1,0,0,0,1,1,0, 0,1,0,0,1,1,1, 0,0,1,0,0,1,1, 0,0,0,1,1,1,1), nrow=4, byrow=TRUE)
M
```

```
     [,1] [,2] [,3] [,4] [,5] [,6] [,7]
[1,]    1    0    0    0    1    1    0
[2,]    0    1    0    0    1    1    1
[3,]    0    0    1    0    0    1    1
[4,]    0    0    0    1    1    1    1
```

```
C <- generateCode(M)
C
```

```
      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
 [1,]    1    0    0    0    1    1    0
 [2,]    0    1    0    0    1    1    1
 [3,]    1    1    0    0    0    0    1
 [4,]    0    0    1    0    0    1    1
 [5,]    1    0    1    0    1    0    1
 [6,]    0    1    1    0    1    0    0
 [7,]    1    1    1    0    0    1    0
 [8,]    0    0    0    1    1    1    1
 [9,]    1    0    0    1    0    0    1
[10,]    0    1    0    1    0    0    0
[11,]    1    1    0    1    1    1    0
[12,]    0    0    1    1    1    0    0
[13,]    1    0    1    1    0    1    0
[14,]    0    1    1    1    0    1    1
[15,]    1    1    1    1    1    0    1
[16,]    0    0    0    0    0    0    0
```

```
isPerfect(C)
```

```
[1] FALSE
```

2. Check bounds

Consider the code generated by the following matrix.

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

```r
M <- matrix(c(1,0,0,1,1,0, 0,1,0,0,1,1, 0,0,1,1,0,1), nrow=3, byrow=TRUE)
```

a. List all the code words.

```r
C <- generateCode(M)
C
```

```
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    0    0    1    1    0
[2,]    0    1    0    0    1    1
[3,]    1    1    0    1    0    1
[4,]    0    0    1    1    0    1
[5,]    1    0    1    0    1    1
[6,]    0    1    1    1    1    0
[7,]    1    1    1    0    0    0
[8,]    0    0    0    0    0    0
```

b. Calculate the code rate of this code.

```r
log2(nrow(C))/length(C[1, ])
```

```
[1] 0.5
```

c. Is this code a maximum distance separable (MDS) code? Prove or disprove.

```r
M <- nrow(C)
n <- length(C[1, ])
d <- codeDistance(C)
isMDS <-function(){
if(M == 2^(n-d+1))
  return(TRUE)
else{
    return(FALSE)
}
}
isMDS()
```

```
[1] FALSE
```

d. Is this code a perfect code? Prove or disprove.

```r
isPerfect(C)
```

```
[1] FALSE
```

3. Perfect repetition codes

Prove that a k repetition code is perfect if and only if k is odd.

Repetition codes have two code words, all 0's or all 1's thus $M = 2$.

A code is perfect if $M = \frac{2^k}{\sum_{j=0}^{t} \binom{n}{j}}$

Since $M = 2$, the numerator must be twice the denominator for a repetition code to be perfect.

They have a minimum distance $d = k$

Thus, if k is odd $t = \frac{k-1}{2}$, and if k is even $t = \frac{k}{2} - 1$

So if k is odd, $\sum_{j=0}^{t} \binom{k}{j} = 2^{t-1}$ by the binomial theorem.

Thus if, and only if, k is odd, $2 = \frac{2^k}{2^{k-1}}$

4. 8×8 Hadamard code

Here's an 8×8 Hadamard matrix:

$$B = \begin{bmatrix} 1 & 1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

```
B <- matrix(c(1,1,1,0,1,0,0,0, 1,1,0,0,0,1,0,1, 1,0,1,1,0,0,0,1, 1,0,1,0,0,1,1,0, 1,0,0,0,1,0,1,1, 1,1,
```

a. Is the corresponding 8×8 Hadamard code (with -1's replaced by 0's) a linear code? Explain why or why not.

```
isLinear <- function(B){
  vec3 <- vector()
  nrowB <- nrow(B)
  inv <- vector()
  for(l in 1:length(B[1, ])){
    inv <- c(inv, 1)
  }
  for(j in 1:(nrowB-1)){
    vec1 <- B[j, ]
  for(i in (j+1):nrowB){
    match <- FALSE
    vec2 <- B[i, ]
    vec3 <- as.numeric(xor(vec1, vec2))
    for(k in 1:nrowB){
      if(all(vec3 == B[k, ]) || all((vec3 == xor(inv, B[k, ]) )))
        match <- TRUE
    }
    if(match == FALSE)
```

```
      return(FALSE)
  }
  }
  return(TRUE)
}

isLinear(B)
```

```
[1] TRUE
```

This is a linear code, because any two code words added together produce another code word.

b. Is this Hadamard code perfect? Prove or disprove.

```
C <- generateCode(B)
isPerfect(C)
```

```
[1] FALSE
```