Virginia Tech
ECE 5484: Fundamentals of Computer Systems

Example of Deriving a Truth Table

Consider the following Boolean function of three variables.

$$F(a, b, c) = (a + b)' + a'c$$

Since there are three variables, the truth table will have eight ($2^3 = 8$) rows. There will be three columns for input variables (a, b, and c) and one column for the output variable (F). The general format for a truth table for a function of three variables is as follows, regardless of the function.

a	b	c	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

One could go through the eight input combinations and ascertain the value of F for each combination. For example, for (a, b, c) = 000, the value of F is determined as follows.

$$F(0, 0, 0) = (0 + 0)' + 0'\cdot 0 = (0)' + 1\cdot 0 = 1 + 0 = 1$$

The first row of the truth table would be as follows.

a	b	c	F
0	0	0	1
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Alternatively, one can do some algebraic manipulation to simplify the equation and then map terms to rows in the table. The expression can be simplified as follows using DeMorgan's theorem.

$$F(a, b, c) = (a + b)' + a'c = a'b' + a'c$$

We can see that the expression will be true anytime a = 0 and b = 0, or if a =0 and c = 1. The rows with the pattern 00- and 0-1, where "-" indicates the value can be 0 or 1, have output F = 1. Other rows have output F = 0. The resulting truth table is as follows.

a	B	c	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0