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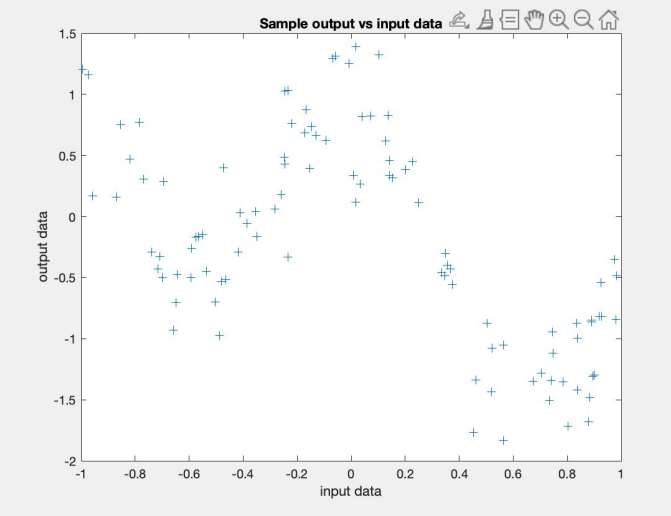
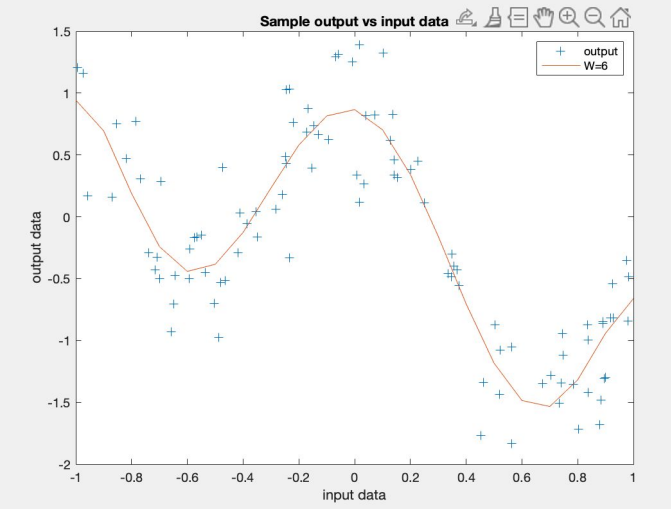
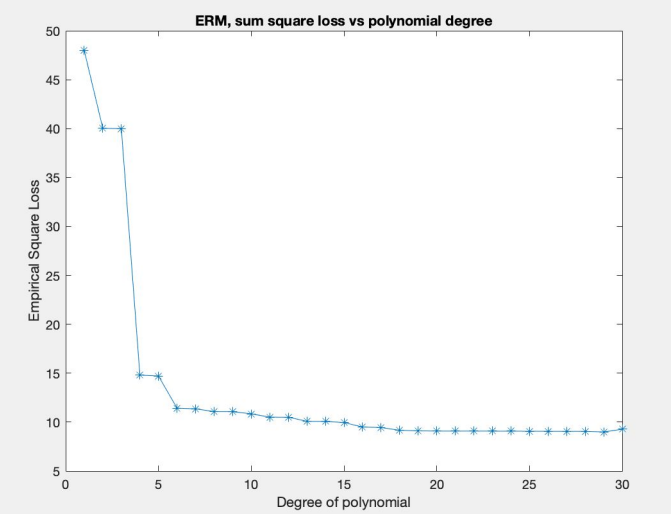
EECS 4404 Assignment 2: Linear Least Squares Regression

Step 1)



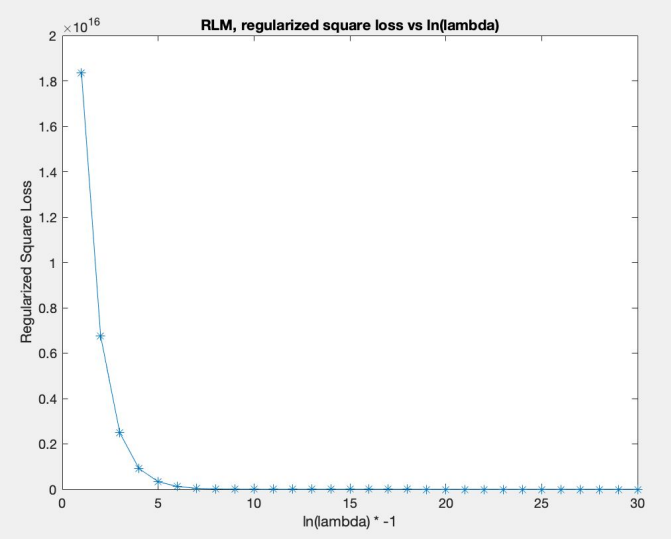


Step 2)



From the above plot, we can see that when the degree of the polynomial is 30 the square loss is minimized. However, this does not produce the most suitable value for W. The resultant polynomial is becoming over fit. Taking this into account I believe a polynomial of degree 6 offers the best combination of error minimization and simplicity of model.

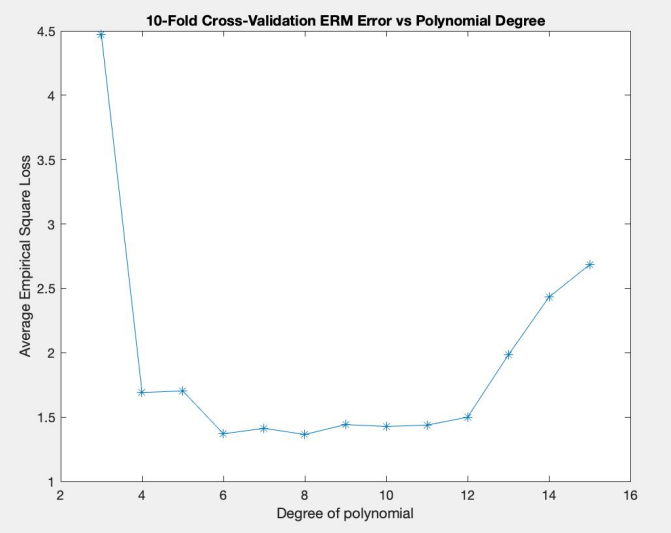
Step 3)



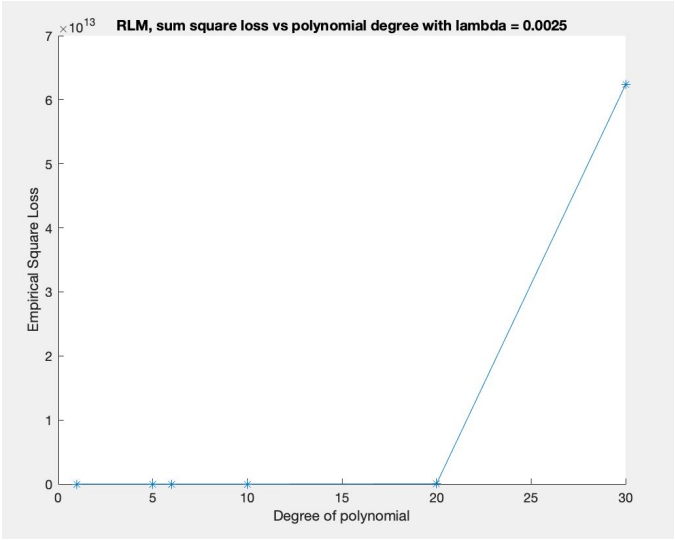
From the size of the error one can see that the regularization parameter is not capable of overcoming the large degree of the trained polynomial.

Compared to the error curve seen in step 2, where the minimal error occurs with a polynomial of degree 30, we can infer that a regularization factor is necessary to prevent a model of this complexity from being selected.

Step 4)

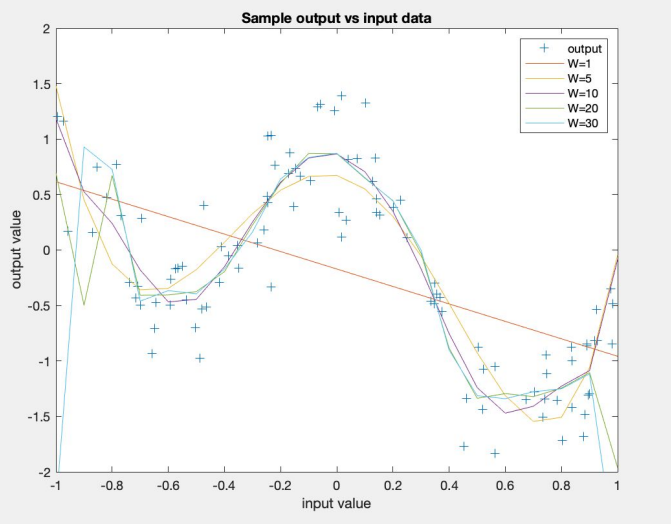


Using a W range of [3,15] shows a drop in error rate going from W=5 to W=6 and staying fairly consistent until error rates start to rise after W=12. From this I would state that W=6 is the most degree for the trained polynomial.

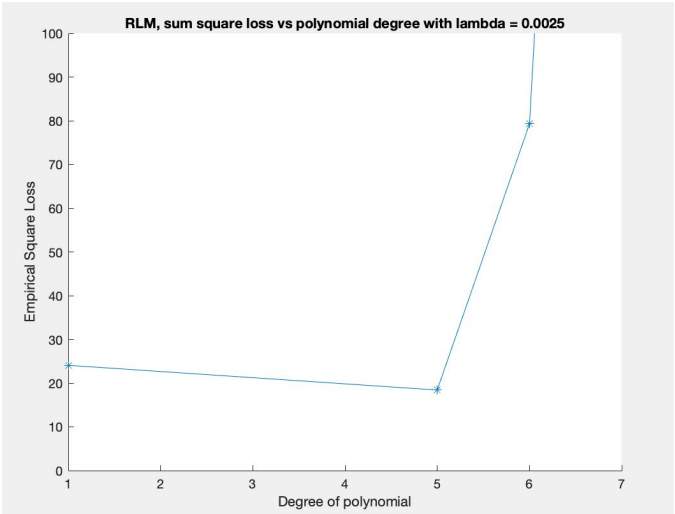


Using W=[1,5,10,20,30] with a regularization parameter of λ = 0.0025 the error rate quickly gets astronomical.

Step 5)

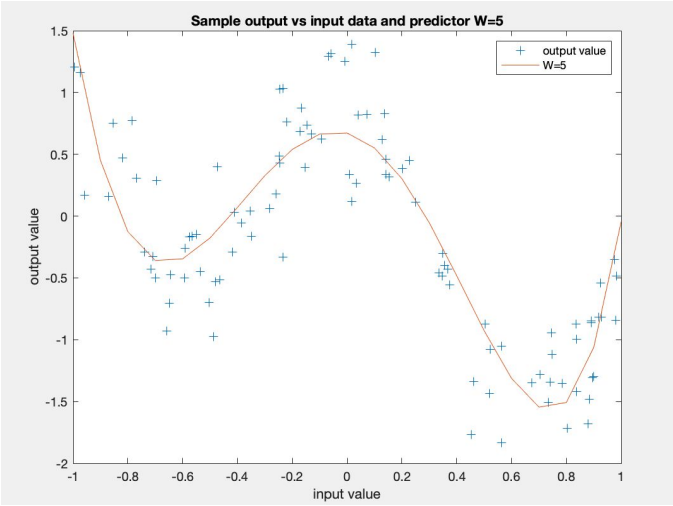


Comparison of W=1,5,10,20,30



If we limit the max error shown on the y-axis to 100, the graph now shows the error climbing sharply after W=5.

Taking everything shown on all of the graphs of prediction error, with and without regularization, I would now update my recommendation for the degree of polynomial used from W=6 to W=5. The regularization parameter begins to negatively influence a choice of W=6 much more than the slight decrease of prediction error in selecting W=6 over W=5 shown in the 10-fold cross validation.



Final model predictor polynomial with degree 5 shown compared to the actual output values.

MATLAB Code

ERM

```
x = load('dataset1_inputs.txt');
t = load('dataset1_outputs.txt');
w = 30;
errors = zeros(1,w);
for i=1:w
    %polynomial curve fitting using my function designmatrix
    p = designmatrix(x,t,i);
    %evaluate polynomial at each input value in x
    results = evalpoly(p,x);
    %find square error difference
    errors(i) = sum((results - t).^2);
end
%plot errors against the matching value of W
figure;
plot([1:1:w],errors,'-*');
title('ERM, sum square loss vs polynomial degree');
xlabel('Degree of polynomial');
ylabel('Empirical Square Loss');
%step 5 visualization
%plot of learned polynomials
figure;
x2 = [-1:.1:1];
plot(x,t,'+');
hold on;
plot(x2,evalpoly(designmatrix(x,t,1),x2));
plot(x2,evalpoly(designmatrix(x,t,5),x2));
plot(x2,evalpoly(designmatrix(x,t,10),x2));
plot(x2,evalpoly(designmatrix(x,t,20),x2));
plot(x2,evalpoly(designmatrix(x,t,30),x2));
ylim([-2 2]);
title('Sample output vs input data');
xlabel('input value');
ylabel('output value');
legend('output','W=1','W=5','W=10','W=20','W=30');
hold off;
```

```
%plot of errors for W=1,5,10,20,30
data = [errors(1),errors(5),errors(10),errors(20),errors(30)];
figure;
hold on;
plot([1,5,10,20,30],data,'-*');
title('ERM, sum square loss vs polynomial degree');
xlabel('Degree of polynomial');
ylabel('Empirical Square Loss');
hold off;

%plot of expected output versus input and
%predictor polynomial degree 5
figure;
plot(x,t,'+',[-1:.1:1],evalpoly(designmatrix(x,t,5),x2));
title('Sample output vs input data and predictor W=5');
xlabel('input value');
ylabel('output value');
legend('output value','W=5');
```

RLM

```
x = load('dataset1_inputs.txt');
t = load('dataset1_outputs.txt');

p = designmatrix(x,t,30);
results = evalpoly(p,x);
errors = zeros(1,30);
n = norm(p)^2;
%get error for each lambda value [1,30]
for i=1:30
    errors(i) = sum((results - t).^2 + (1/exp(i) * n));
end
%plot errors for each lambda value
figure;
plot([1:1:30],errors,'-*');
title('RLM, regularized square loss vs ln(lambda)');
xlabel('ln(lambda) * -1');
ylabel('Regularized Square Loss');

%step 5 visualization
w = [1,5,6,10,20,30];
disp_errors = zeros(1,6);
for j=1:6
    p = designmatrix(x,t,w(j));
    disp_data = evalpoly(p,x);
    disp_norm = norm(p)^2;
    disp_errors(j) = sum(0.5 .* (disp_data - t).^2 + (0.0025/2 * disp_norm));
end

%plot of errors for W=1,5,10,20,30
figure;
hold on;
plot(w,disp_errors,'-*');
ylim([0,100]);
title('RLM, sum square loss vs polynomial degree with lambda = 0.0025');
xlabel('Degree of polynomial');
ylabel('Empirical Square Loss');
```

Cross-Validation

```
x = load('dataset1_inputs.txt');
t = load('dataset1_outputs.txt');

m = [x t];
errors = zeros(1,15);
table = array2table(m,'VariableNames',{'input','output'});
% Nonstratified partition
hpartition = cvpartition(100,'KFold',10);

%loop through each degree of polynomial for model
for w=3:15
   %loop through using each of 10 subsets as testing data, while
   %remaining 9 subsets are all used as training data
   for i=1:10
      %get a column vector, 1 indicates include row in training
      %idxTrain is the ith training set
      idxTrain = training(hpartition,i);
      %get rows identified for training from table
      tblTrain = table(idxTrain,:);
      %use remaining rows of table for test partition
      idxNew = test(hpartition,i);
      tblTest = table(idxNew,:);

      %fit polynomial
      p = designmatrix(tblTrain.input,tblTrain.output,w);
      %get predicted output for training data
      pred = evalpoly(p, tblTest.input);
      errors(w) =  errors(w) + sum((pred - tblTest.output).^2);
   end
   errors(w) = errors(w) / 10;
end

%plot average errors for each of w=[3,8] against w=[3,8]
figure;
plot([3:1:15],errors(3:15),'-*');
title('10-Fold Cross-Validation ERM Error vs Polynomial Degree');
xlabel('Degree of polynomial');
ylabel('Average Empirical Square Loss');
```

designmatrix

```
function coeff = designmatrix(input,output,deg)
%populates the design matrix as specified in lecture8.pdf
%input: vector of input values
%output: vector of expected output values
%deg: degree of polynomial to create design matrix for
%coeff: returns a column vector of size deg+1, with constant term
%       in the first index, then increasing degree row by row
%Note: does not verify input, not tested for degrees <1 or >30
dmat = zeros(numel(input),deg+1);
%set first column equal to 1
dmat(:,1) = 1;

if deg == 1
   dmat(:,2) = input;
else
   for i=1:deg
      dmat(:,i+1) = input .^ i;
   end
end
%transpose for ease of evaluation with evalpoly
coeff = ((dmat' * dmat) \ (dmat'*output))';
end
```

evalpoly

```
function results = evalpoly(poly, x)
%evaluates the given polynomial, poly, at value x
%
%poly:column vector of polynomial coefficients with constant term in
%     the first index, increasing in power from there
%x   :value to evaluate polynomial with
%results: column vector same number of elements as x
%
%Note: no input verification

pow = [0:1:numel(poly)-1];
results = zeros(numel(x),1);
for i=1:numel(x)
results(i) = sum(poly .* x(i).^pow);
end
```