A Wasserstein Subsequence Kernel for Time Series

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ETH Colors

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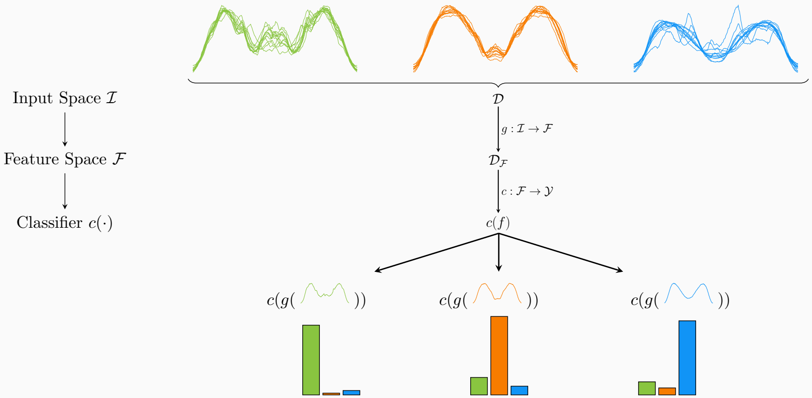
Motivation

Time Series Classification in a Nutshell

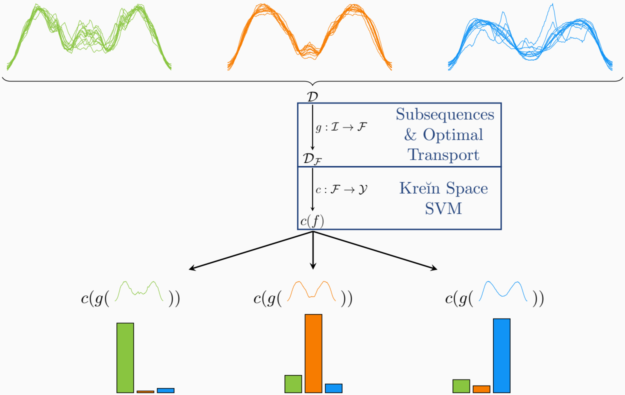


Time Series Classification in a Nutshell

Input Space $\mathcal{I}$

Feature Space $\mathcal{F}$

Classifier $c(\cdot)$

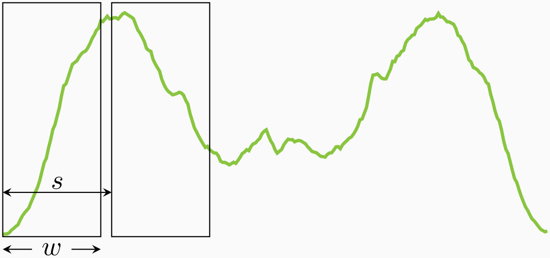


Feature Space: Subsequences and Optimal Transport

Subsequence Extraction

The Sliding Window Approach

- Window Size w
- Stride s



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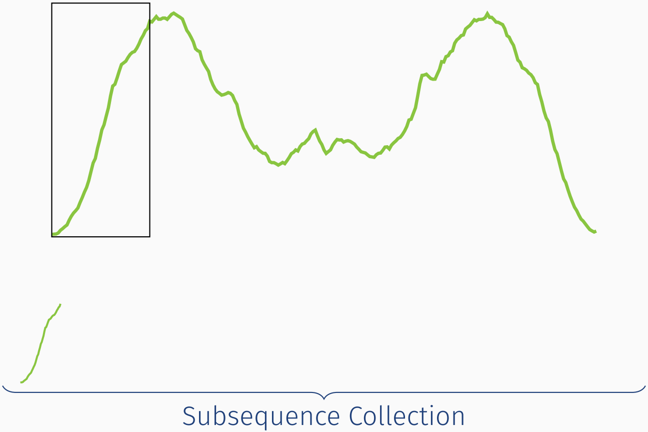

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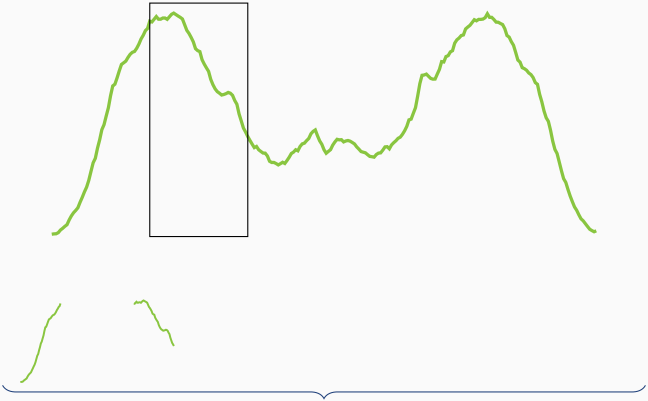
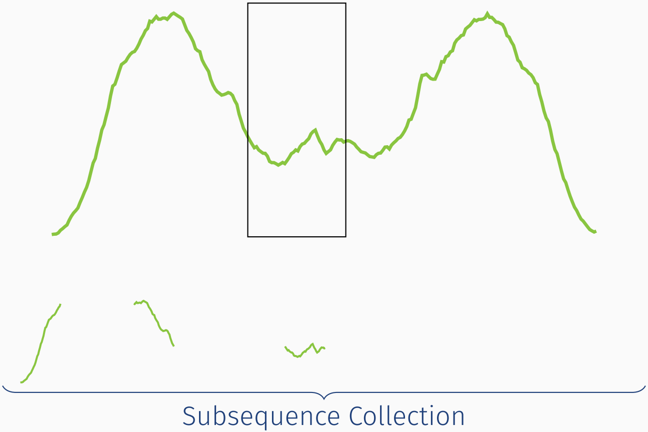


Subsequence Collection

Subsequence Extraction

The Sliding Window Approach

- Window Size w
- Stride s



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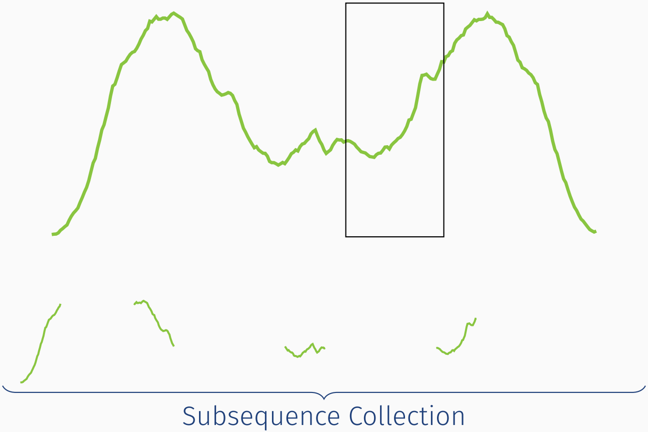

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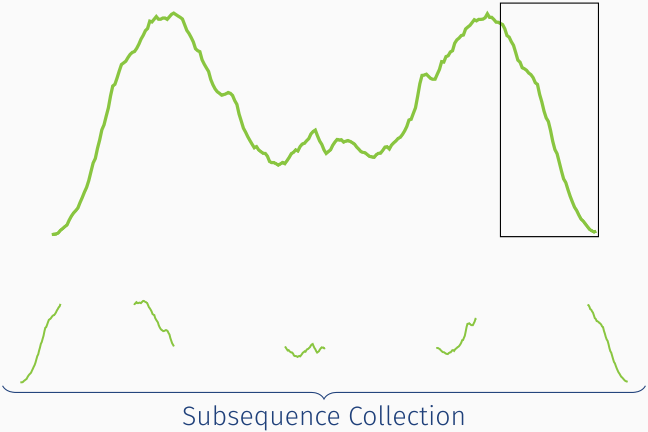
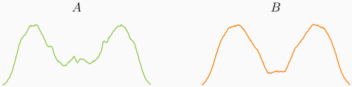
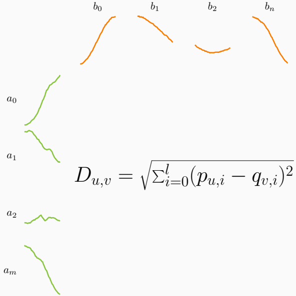


Given Two Time Series



we Construct the Subsequence Distance Matrix D



$$D_{u,v} = \sqrt{\Sigma_{i=0}^{l} (p_{u,i} - q_{v,i})^2}$$

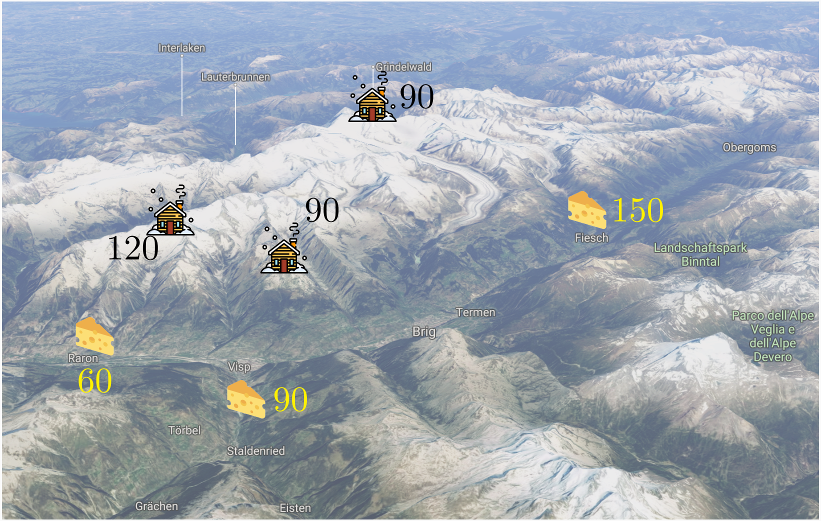
and obtain $\quad W_1(A, B) = \min_{P \in \Gamma(A,B)} \langle D, P \rangle_{\mathrm{F}}$

Optimal Transport and the Wasserstein Distance

Transportation Theory

- Study of optimal transportation and allocation of resources
- Gaspard Monge (1746–1818)
  Morph one probability distribution into a second one with minmal cost.
- Frank Lauren Hitchcock (1875 – 1957)
  Transport a commodity from different sources to different “sinks”.
- Leonid Kantorovich (1912 - 1986)
  Distribute soldiers to the frontline.

The Swiss Kantorovich Problem



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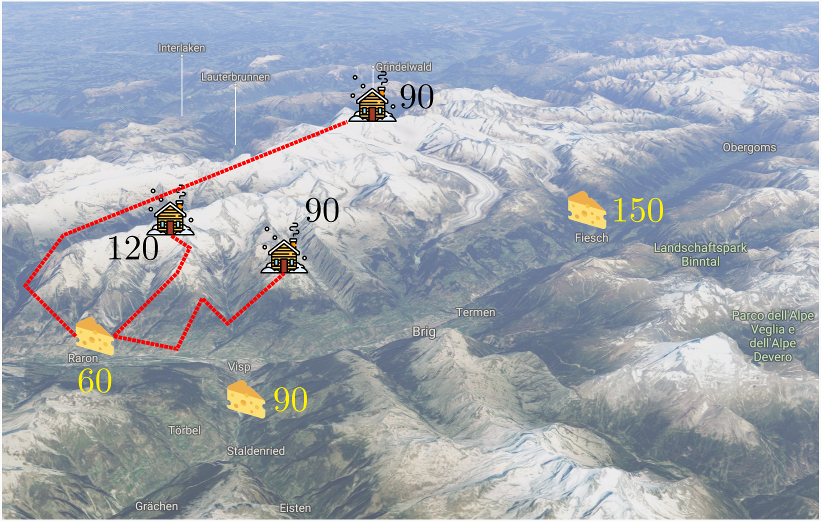

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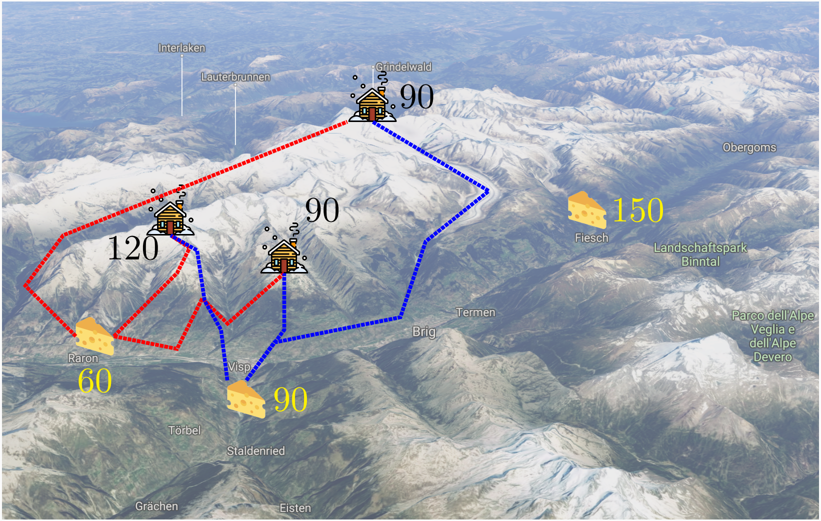

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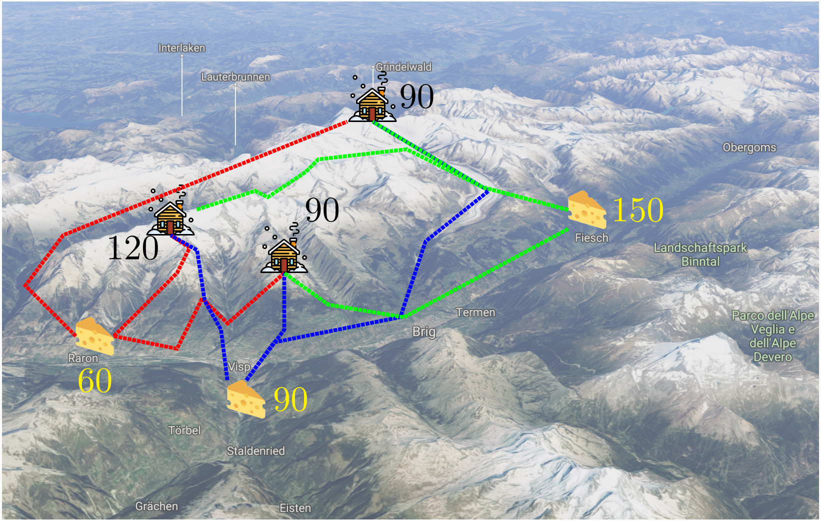

The Swiss Kantorovich Problem

Distance Matrix D

	A	B	C
1	$D_{1,A}$	$D_{1,B}$	$D_{1,C}$
2	$D_{2,A}$	$D_{2,B}$	$D_{2,C}$
3	$D_{3,A}$	$D_{3,B}$	$D_{3,C}$

Transportation Matrix P

	120	90	90
60	?	?	?
90	?	?	?
150	?	?	?

The Swiss Kantorovich Problem

Distance Matrix D

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Transportation Matrix P

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60	?	?	?
90	?	?	?
150	?	?	?

Find P

	b_A	b_B	b_C
a_1	$P_{1,A}$	$P_{1,B}$	$P_{1,C}$
a_2	$P_{2,A}$	$P_{2,B}$	$P_{2,C}$
a_3	$P_{3,A}$	$P_{3,B}$	$P_{3,C}$

such that

$$\sum_{j \in \{A,B,C\}} P_{i,j} = a_i, \forall i \in \{1,2,3\}$$

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that minimises the cost function

$$C(P) = \sum_i \sum_j D_{i,j} P_{i,j} = \langle D, P \rangle_{\mathbf{F}}$$

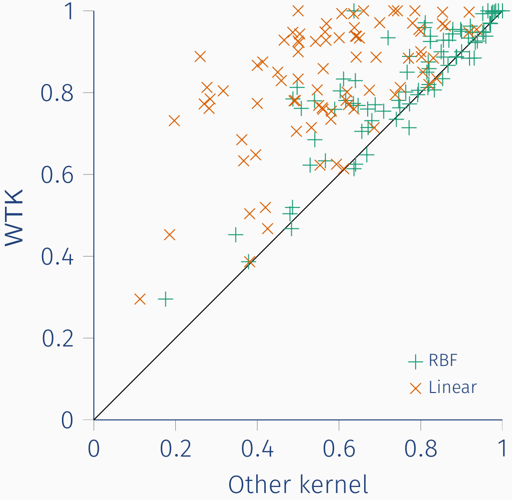
A Wasserstein Kernel for Time Series

Results

Experiments

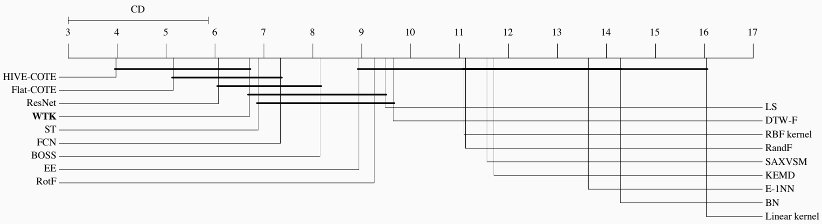
Datasets	*UCR Time Series Archive* 85 datasets predetermined train/test splits
Hyperparameters	Selected via a 5-fold cross-validation on the training set
Evaluation metric	Classification accuracy
Comparison partners	– Other kernels – DTW-1NN – State-of-the-art methods

Comparison with Other Kernels



Critical Difference Plot

The classification performances of methods sharing horizontal bars are not significantly different.



Take aways

Questions?

References