

MATH 301 Homework 8

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Chapter 7) 4,6,8,12,16,20

7.4

Given an integer a , then $a^3 + 4a + 5$ is odd if and only if a is even.

If and only if proof

$a^3 + 4a + 5$ is odd $\rightarrow a$ is even

Contrapositive proof: if a is odd $\rightarrow a^3 + 4a + 5$ is even

Let $a = 2k + 1, k \in \mathbb{Z}$.

$$\begin{aligned}a^3 + 4a + 5 &= (2k + 1)^2 + 4(2k + 1) + 5 = \\4k^2 + 4k + 1 + 8k + 4 + 5 &= \\4k^2 + 12k + 10 &= \\2(2k^2 + 6k + 5)\end{aligned}$$

Thus it can be shown that when a is an odd number the expression $a^3 + 4a + 5$ can be expressed as 2 times some integer of the form: $2k^2 + 6k + 5, k \in \mathbb{Z}$

a is even $\rightarrow a^3 + 4a + 5$ is odd

Let $a = 2k, k \in \mathbb{Z}$.

$$\begin{aligned}a^3 + 4a + 5 &= (2k)^3 + 4(2k) + 5 = \\8k^3 + 8k + 5 &= \\8k^3 + 8k + 4 + 1 &= \\2(4k^3 + 4k + 2) + 1\end{aligned}$$

Let $m = 4k^3 + 4k + 2$

$$a^3 + 4a + 5 = 2m + 1$$

Meaning when a is even $a^3 + 4a + 5$ is odd

7.6

Suppose $x, y \in \mathbb{R}$ Then $x^3 + x^2y = y^2 + xy$ if and only if $y = x^2$ or $y = -x$

If and only if proof

If $x^3 + x^2y = y^2 + xy \rightarrow y = x^2$ **or** $y = -x$

Direct Proof:

$$x^3 + x^2y = y^2 + xy$$

$$x^2(x + y) = y(x + y)$$

$$x^2(x + y) - y(x + y) = 0$$

$$(x^2 - y)(x + y) = 0$$

$$\therefore y = -x \vee y = x^2$$

If $y = x^2$ **or** $y = -x \rightarrow x^3 + x^2y = y^2 + xy$

Direct Proof:

Let $y = x^2 \therefore$

$$x^3 + x^2y = x^3 + x^4$$

$$y^2 + xy = (x^2)^2 + xx^2 = x^3 + x^4$$

$$\therefore x^3 + x^2y = y^2 + xy$$

7.8

Suppose $a, b \in \mathbb{Z}$ Prove that $a \equiv b \pmod{10}$ if and only if $a \equiv b \pmod{2}$ and $a \equiv b \pmod{5}$

7.12

There exists a positive real number x for which $x^2 < \sqrt{x}$ **Existential proof:** Consider the number $\frac{1}{2}$

$$\left(\frac{1}{2}\right)^2 < \sqrt{\frac{1}{2}}$$

$$.25 < .707107\dots$$

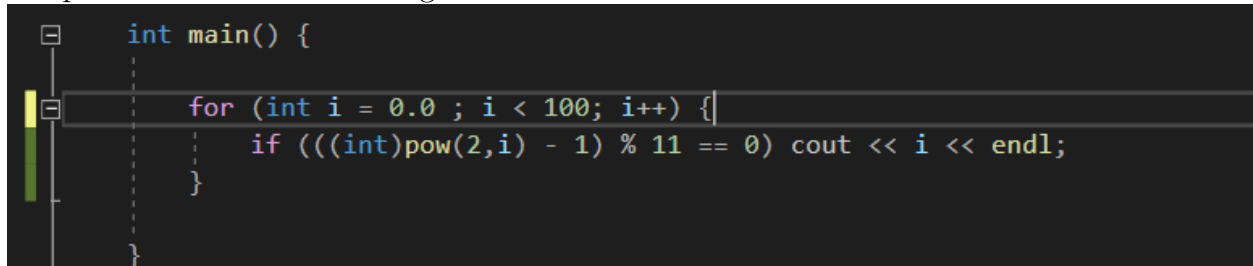
7.16

7.20

There exists an $n \in \mathbb{N}$ for which $11|(2^n - 1)$

Existential proof:

Figure 1: Consider the numbers of the form $10k$, $k \in \mathbb{W}$ (whole numbers) these are solutions to this problem, for example 10,20,30 are solutions. Answers generated using c++ computation with the following code:

A screenshot of a code editor with a dark background. The code is written in C++ and is enclosed in a code block. The code defines a main function that iterates over integers from 0 to 100. For each integer i, it checks if (2^i - 1) is divisible by 11. If it is, it prints the value of i. The code is as follows:

```
int main() {  
    for (int i = 0; i < 100; i++) {  
        if (((int)pow(2,i) - 1) % 11 == 0) cout << i << endl;  
    }  
}
```

$$2^{40} - 1 = 1099511627776 \quad \frac{1099511627776}{11} = 99955602525$$