

MATH 301 Homework 5

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Problem 4-4

+++ Use the method of direct proof to prove the following statement:

Suppose $x, y \in \mathbb{Z}$. If x and y are odd then xy is odd.

Proof:

Suppose x, y are odd therefore:

$$x = (2k + 1), y = (2m + 1)$$

$$k, m \in \mathbb{Z}$$

$$x(y) = (2k + 1)(2m + 1)$$

$$x(y) = 4km + 2k + 2m + 1$$

$$x(y) = 2(2km + k + m) + 1$$

$$k, m \in \mathbb{Z} \therefore (2km + k + m) \in \mathbb{Z}$$

$$\begin{aligned} \text{Let } & (2km + k + m) = n, n \in \mathbb{Z} \therefore x(y) = 2n + 1 \\ \therefore & x(y) \text{ is odd} \end{aligned}$$

This problem investigated the effect of multiplying two odd numbers together. I feel really confident with my work here, and fully understand the problem.

Problem 4-6

+++ Use the method of direct proof to prove the following statement:

Suppose $a, b, c \in \mathbb{Z}$. If $a|b$ and $a|c$ then $a|(b + c)$

Proof:

$$\begin{aligned} & \text{Suppose } a|b \text{ and } a|c : \\ & b = a(m), m \in \mathbb{Z} \quad c = a(n), n \in \mathbb{Z} \end{aligned}$$

If $a|(b + c)$ then we must show that $(b + c) = a(k), k \in \mathbb{Z}$

If $b = a(m), m \in \mathbb{Z}$ and $c = a(n), n \in \mathbb{Z}$ then

$$\begin{aligned} (b + c) &= a(m) + a(n) = a(m + n) \\ \text{Let } m + n &= k : \\ (b + c) &= a(k) \end{aligned}$$

Since $b+c$ can be expressed as a multiple of a when $a|b$ and $a|c$, it is proved that $a|(b + c)$. This problem was my first exploring the concepts of division. I feel that I took a few notational liberties with my work, but that my take on this proof was largely justified. The bulk of this work went into demonstrating that $b+c$ can be expressed as a multiple of a .

Problem 4-14

+++ Use the method of direct proof to prove the following statement:

If $n \in \mathbb{Z}$ then $5n^2 + 3n + 7$ is odd (try cases)

For this problem we must test both parities of n : odd and even.

Proof: n is odd

Suppose $n = 2k + 1, k \in \mathbb{Z}$

Then the statement $5n^2 + 3n + 7$ can be expressed as:

$$5(2k + 1)^2 + 3(2k + 1) + 7$$

$$\begin{aligned}
5(2k+1)^2 + 3(2k+1) + 7 &= 5(4k^2 + 4k + 1) + 6k + 10 \\
&= 20k^2 + 26k + 10 + 1 \\
&= 2(10k^2 + 13k + 5) + 1 \\
&\quad k \in \mathbb{Z} \therefore (10k^2 + 13k + 5) \in \mathbb{Z} \\
&\quad \text{let } (10k^2 + 13k + 5) = m \quad \therefore \\
&= 2(m) + 1 \therefore
\end{aligned}$$

When n is odd $5(n)^2 + 3(n) + 7$ **is odd**

Proof: n is even

$$\text{Suppose } n = 2k, k \in \mathbb{Z}$$

Then the statement $5n^2 + 3n + 7$ can be expressed as:

$$5(2k)^2 + 3(2k) + 7$$

$$\begin{aligned}
5(2k)^2 + 3(2k) + 7 &= 5(4k^2) + 6k + 7 \\
&= 10k^2 + 6k + 6 + 1 \\
&= 2(5k^2 + 3k + 3) + 1 \\
&\quad k \in \mathbb{Z} (5k^2 + 3k + 3) \in \mathbb{Z} \\
&\quad \text{let } (5k^2 + 3k + 3) = m \\
&= 2(m) + 1 \therefore
\end{aligned}$$

When n is even $5(n)^2 + 3(n) + 7$ **is odd**

Therefore regardless of the parity of n ($5(n)^2 + 3(n) + 7$) is odd

This question was difficult for me because it pushed me to consider all cases presented for the problem. Luckily the structure of the proof was very symmetric and I am confident in my work.

Problem 5-2

+++ Prove the following statements with contrapositive proof. (In each case, think about how a direct proof would work. In most cases contrapositive is easier)

Suppose $n \in \mathbb{Z}$. If n^2 is even then n is even

Proof:

Suppose n is odd. Then:

$$n = 2k + 1, k \in \mathbb{Z}$$

We now wish to prove that if n is odd n^2 is odd.

$$\begin{aligned}(2k+1)^2 &= 4k^2 + 4k + 1 \\&= 2(2k^2 + 2k) + 1 \\k \in \mathbb{Z} \therefore (2k^2 + 2k) &\in \mathbb{Z} \\ \text{let } 2k^2 + 2k &= m \therefore \\(2k+1)^2 &= 2(m) + 1\end{aligned}$$

Meaning that when n is odd n^2 is odd. Through logical equivalency this implies that when n^2 is even n is even.

This problem was a good introduction to the contrapositive proof method I am very confident with my result. It seems to me that the problem was made to investigate this new technique!

Problem 5-16

+++ Prove the following statements using either direct or contrapositive proof.

Suppose $x, y \in \mathbb{Z}$ If $x+y$ is even, then x and y have the same parity.

Suppose that x and y do not have the same parity. This is to say that:

$$x = 2k \quad y = 2m + 1 \quad k, m \in \mathbb{Z}$$

or

$$x = 2m + 1 \quad y = 2k \quad k, m \in \mathbb{Z}$$

Proof: x is even y is odd: Suppose $x = 2k \quad y = 2m + 1 \quad k, m \in \mathbb{Z}$:

$$\begin{aligned}x + y &= 2k + 2m + 1 \\x + y &= 2(k + m) + 1 \\k, m \in \mathbb{Z} \therefore k + m &\in \mathbb{Z} \\ \text{Let } n &= k + m : \\x + y &= 2(n) + 1\end{aligned}$$

Consequently $x+y$ is odd .

Therfore $x+y$ is not even.

Proof: y is even x is odd: Suppose $y = 2k \quad x = 2m + 1 \quad k, m \in \mathbb{Z}$:

$$\begin{aligned}y + x &= 2k + 2m + 1 \\y + x &= 2(k + m) + 1 \\k, m \in \mathbb{Z} \therefore k + m &\in \mathbb{Z} \\ \text{Let } n &= k + m : \\y + x &= 2(n) + 1\end{aligned}$$

Consequently $x+y$ is odd .

Therfore $x+y$ is not even.

Since the negation of the original statement created two sub problems this problem had to be solved through cases and reminded of two previously solved in this assignment. I am confident in my answer.