

MATH 301 Homework 4

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Problem +++1. Negate the following implications.

- (a)+++

If I finish my homework early, I will join you for a movie.

Let I finish my homework be P and I will join you for a movie be Q

$$P \rightarrow Q$$

The negation:

$$\neg(P \rightarrow Q) = P \wedge \neg Q$$

This negation makes sense to me and can be thought of as a lie! The negation encapsulates the concept of finishing homework early and not going to the movie.

- (b)+++

If n^2 is divisible by 12, then n is even or n is a multiple of three.

An interpretation of this statement learned from the in class activity is:

$$\forall n \in \mathbb{Z} : 12|n^2, (n \in \mathbb{E}) \vee (3|n)$$

The negation:

$$\begin{aligned} \neg(\forall n \in \mathbb{Z} : 12|n^2, (n \in \mathbb{E}) \vee (3|n)) &= \\ \exists n \in \mathbb{Z} : 12|n^2 \wedge (n \notin \mathbb{E}) \wedge (3 \nmid n) & \end{aligned}$$

Which can be understood as the statement there exists some integer n such that 12 divides n^2 and n is not even and 3 does not divide n. After our disussion in class about this problem it makes sense to me and is a good exercise on identifying situations where we are making universal claims instead of singular ones.

Problem 2. +++Consider the two statements:

P: For all integers x, there exists an integer y such that $y > x$. Q: There exists an integer x such that, for all integers y, we have $y > x$.

Let us first start by translating these statements to their logical forms:

$$P = \forall x \in \mathbb{Z} (\exists y \in \mathbb{Z} : y > x)$$

$$Q = (\exists x \in \mathbb{Z} : \forall y \in \mathbb{Z}, y > x)$$

- (a) One of these statements is true and the other is false. Determine which statement is true and justify your claim.

The first statement P is true. For all integers x you can always find some integer y such that the value of y is greater than the value of x. This is due to the infinite nature of the integers, meaning you can always look ahead past your selection of x to find a greater value.

- (b) Discuss why the other statement is false.

Q is false because $\nexists x : \forall y \in \mathbb{Z}, y > x$ This is to say that there is no single value x that is smaller than all other integers. Because the integers are infinite whatever value we pick for x there will always exist some integer y with a lower value.

- (c) Write down the negation of each statements.

$$\neg P = \neg(\forall x \in \mathbb{Z} (\exists y \in \mathbb{Z} : y > x)) = \exists x \in \mathbb{Z} (\forall y \in \mathbb{Z} : y \leq x)$$

$$\neg Q = \neg(\exists x \in \mathbb{Z} : \forall y \in \mathbb{Z}, y > x) = \forall x \in \mathbb{Z} : \exists y \in \mathbb{Z}, y \leq x$$

Overall this question was very fun and good practice with nested quantifiers. I think that I understand how to translate to the negated form the trickiest part for me is placing the such that at the end of the negation.

Problem 3. +++ Consider the statement:

“For every positive integer n, there exists a prime number between n and n + 10.”

- (a) Do you think the statement is true or false. Why?

I think that this statement is false due to a very compelling counterargument identified by my in class partner Ian which states that if n is equal to 200 there is not a prime number between 200 and 210. This can be verified computationally with most programming languages

- (b) Write down the negation of this statement.

The statement is first stated as follows:

$$\forall n \in \mathbb{Z}^+ (\exists p \in \mathbf{P} : n < p < n + 10)$$

where \mathbf{P} is the set of prime numbers.

The Negation:

$$\neg(\forall n \in \mathbb{Z}^+ (\exists p \in \mathbf{P} : n < p < n + 10)) = \exists n \in \mathbb{Z}^+ (\forall p \in \mathbf{P} : (n \geq p) \wedge (n + 10 \geq p))$$

This negation reads: there exists some positive integer n for all primes such that n is greater than or equal to a given prime p and n+10 is also greater than or equal to that same p

Problem 4

- (a) 2.10.2 +++

If x is prime then \sqrt{x} is not a rational number Let P be "x is prime"
Let Q be " \sqrt{x} is not rational"

$$\begin{aligned} P \rightarrow Q \\ \neg(P \rightarrow Q) = P \wedge \neg Q, \therefore \end{aligned}$$

The negation is: "x is prime and \sqrt{x} is rational"

I am confident with this problem and have built up an understanding that negating statement is a way of describing the opposite.

- (b) 2.10.4 +++

For every positive number ϵ , there is a positive number δ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \epsilon$

$$\forall \epsilon \in \mathbb{R}^+ (\exists \delta \in \mathbb{R}^+ : (|x - a| < \delta) \rightarrow (|f(x) - f(a)| < \epsilon))$$

The negation:

$$\exists \epsilon \in \mathbb{R}^+ : \forall \delta \in \mathbb{R}^+, (|x - a| < \delta) \wedge (|f(x) - f(a)| \geq \epsilon)$$

The negation of this statement reads there exists some positive real number ϵ (Here number is assumed to be real technically we could just pick some element of a field I suppose) such that for all real numbers δ , $|x - a| < \delta$ and $(|f(x) - f(a)| \geq \epsilon)$

Problem 5

- (a) 2.10.8+++

If x is a rational number and $x \neq 0$, then $\tan(x)$ is not a rational number:

$$(x \in \mathbb{Q} \wedge x \neq 0) \rightarrow (\tan(x) \notin \mathbb{Q})$$

The negation:

$$(x \in \mathbb{Q} \wedge x \neq 0) \wedge (\tan(x) \in \mathbb{Q})$$

For this problem i deployed the negation of the conditional statement to find my answer.
I am confident in this result!

- (b) 2.10.10 +++

If f is a polynomial and its degree is greater than 2 then f' is not constant.

$$(f \in \mathcal{P}_n \wedge n > 2) \rightarrow f' \text{ is not constant}$$

The negation:

$$(f \in \mathcal{P}_n \wedge n > 2) \wedge f' \text{ is constant}$$

This was a good problem that made me consider how to represent the set of polynomial over a certain degree. I chose the notation above from referencing a math stack overflow post. I find it to be an intuitive way of describing the set as there is a handy subscript at all times that denotes the degree of the polynomial family being sampled.