

# MATH 301 Homework 5

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2/24/2022

## Problem 4-4

+++ Use the method of direct proof to prove the following statment:

Suppose  $x, y \in \mathbb{Z}$ . If  $x$  and  $y$  are odd then  $xy$  is odd.

**Proof:**

Suppose  $x, y$  are odd therefore:

$$x = (2k + 1), y = (2m + 1)$$

$$k, m \in \mathbb{Z}$$

$$x(y) = (2k + 1)(2m + 1)$$

$$x(y) = 4km + 2k + 2m + 1$$

$$x(y) = 2(2km + k + m) + 1$$

$$k, m \in \mathbb{Z} \therefore (2km + k + m) \in \mathbb{Z}$$

$$\text{Let } (2km + k + m) = n, n \in \mathbb{Z} \therefore x(y) = 2n + 1$$

$\therefore x(y)$  is odd

This problem investigated the effect of multiplying two odd numbers together. I feel really confident with my work here, and fully understand the problem.

## Problem 4-6

+++ Use the method of direct proof to prove the following statment:

Suppose  $a, b, c \in \mathbb{Z}$ . If  $a|b$  and  $a|c$  then  $a|(b+c)$

**Proof:**

$$\begin{aligned} &\text{Suppose } a|b \text{ and } a|c \therefore \\ &b = a(m), m \in \mathbb{Z} \quad c = a(n), n \in \mathbb{Z} \end{aligned}$$

If  $a|(b+c)$  then we must show that  $(b+c) = a(k), k \in \mathbb{Z}$   
If  $b = a(m), m \in \mathbb{Z}$  and  $c = a(n), n \in \mathbb{Z}$  then

$$\begin{aligned} (b+c) &= a(m) + a(n) = a(m+n) \\ &\text{Let } m+n = k \therefore \\ (b+c) &= a(k) \end{aligned}$$

Since  $b+c$  can be expressed as a multiple of  $a$  when  $a|b$  and  $a|c$ , it is proved that  $a|(b+c)$ .  
This problem was my first exploring the concepts of division. I feel that I took a few notational liberties with my work, but that my take on this proof was largely justified. The bulk of this work went into demonstrating that  $b+c$  can be expressed as a multiple of  $a$ .

## Problem 4-14

+++ Use the method of direct proof to prove the following statment:

If  $n \in \mathbb{Z}$  then  $5n^2 + 3n + 7$  is odd (try cases)

For this problem we must test both parities of  $n$ : odd and even.

**Proof:  $n$  is odd**

$$\text{Suppose } n = 2k + 1, k \in \mathbb{Z}$$

Then the statement  $5n^2 + 3n + 7$  can be expressed as:

$$5(2k+1)^2 + 3(2k+1) + 7$$

$$\begin{aligned}
5(2k+1)^2 + 3(2k+1) + 7 &= 5(4k^2 + 4k + 1) + 6k + 10 \\
&= 20k^2 + 26k + 10 + 1 \\
&= 2(10k^2 + 13k + 5) + 1 \\
&\quad k \in \mathbb{Z} \therefore (10k^2 + 13k + 5) \in \mathbb{Z} \\
&\quad \text{let } (10k^2 + 13k + 5) = m \quad \therefore \\
&= \mathbf{2(m)+1} \therefore
\end{aligned}$$

**When n is odd  $5(n)^2 + 3(n) + 7$  is odd**

**Proof: n is even**

Suppose  $n = 2k, k \in \mathbb{Z}$

Then the statement  $5n^2 + 3n + 7$  can be expressed as:

$$5(2k)^2 + 3(2k) + 7$$

$$\begin{aligned}
5(2k)^2 + 3(2k) + 7 &= 5(4k^2) + 6k + 7 \\
&= 20k^2 + 6k + 6 + 1 \\
&= 2(10k^2 + 3k + 3) + 1 \\
&\quad k \in \mathbb{Z} (10k^2 + 3k + 3) \in \mathbb{Z} \\
&\quad \text{let } (10k^2 + 3k + 3) = m \\
&= 2(m) + 1 \therefore
\end{aligned}$$

**When n is even  $5(n)^2 + 3(n) + 7$  is odd**

Therefore regardless of the parity of n  $(5(n)^2 + 3(n) + 7)$  is odd

This question was difficult for me because it pushed me to consider all cases presented for the problem. Luckily the structure of the proof was very symmetric and I am confident in my work.

## Problem 5-2

+++ Prove the following statements with contrapositive proof. (In each case, think about how a direct proof would work. In most cases contrapositive is easier)

Suppose  $n \in \mathbb{Z}$ . If  $n^2$  is even then n is even

**Proof:**

Suppose n is odd. Then:

$$n = 2k + 1, k \in \mathbb{Z}$$

We now wish to prove that if  $n$  is odd  $n^2$  is odd.

$$\begin{aligned}
 (2k+1)^2 &= 4k^2 + 4k + 1 \\
 &= 2(2k^2 + 2k) + 1 \\
 k &\in \mathbb{Z} \therefore (2k^2 + 2k) \in \mathbb{Z} \\
 \text{let } 2k^2 + 2k &= m \therefore \\
 (2k+1)^2 &= 2(m) + 1
 \end{aligned}$$

Meaning that when  $n$  is odd  $n^2$  is odd. Through logical equivalency this implies that when  $n^2$  is even  $n$  is even.

This problem was a good introduction to the contrapositive proof method I am very confident with my result. It seems to me that the problem was made to investigate this new technique!

## Problem 5-16

+++ Prove the following statements using either direct or contrapositive proof.

Suppose  $x, y \in \mathbb{Z}$  If  $x+y$  is even, then  $x$  and  $y$  have the same parity.

Suppose that  $x$  and  $y$  do not have the same parity. This is to say that:

$$x = 2k \quad y = 2m + 1 \quad k, m \in \mathbb{Z}$$

or

$$x = 2m + 1 \quad y = 2k \quad k, m \in \mathbb{Z}$$

**Proof:  $x$  is even  $y$  is odd:** Suppose  $x = 2k \quad y = 2m + 1 \quad k, m \in \mathbb{Z}$ :

$$\begin{aligned}
 x + y &= 2k + 2m + 1 \\
 x + y &= 2(k + m) + 1 \\
 k, m &\in \mathbb{Z} \therefore k + m \in \mathbb{Z} \\
 \text{Let } n &= k + m : \\
 x + y &= 2(n) + 1
 \end{aligned}$$

Consequently  $x+y$  is odd .

Therefore  $x+y$  is not even.

**Proof:  $y$  is even  $x$  is odd:** Suppose  $y = 2k \quad x = 2m + 1 \quad k, m \in \mathbb{Z}$ :

$$\begin{aligned}
 y + x &= 2k + 2m + 1 \\
 y + x &= 2(k + m) + 1 \\
 k, m &\in \mathbb{Z} \therefore k + m \in \mathbb{Z} \\
 \text{Let } n &= k + m : \\
 y + x &= 2(n) + 1
 \end{aligned}$$

Consequently  $x+y$  is odd .

Therefore  $x+y$  is not even.

Since the negation of the original statement created two sub problems this problem had to be solved through cases and reminded of two previously solved in this assignment. I am confident in my answer.