MATH 310 Homework 1.1

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1

Prove that

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$P(k) = \sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

$$P(k+1):$$

$$\sum_{i=1}^{k+1} i^2 = \frac{k+1(k+1+1)(2(k+1)+1)}{6}$$

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k+1(k+2)(2k+3)}{6}$$

$$\frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+9k^2+13k+6}{6}$$

2

Prove that :

$$\sum_{i=0}^{n} i^3 = (\sum_{i=1}^{n} i)^2$$

$$\sum_{i=0}^{n} i^3 = (\frac{n(n+1)}{2})^2$$

$$\sum_{i=0}^{n} i^3 = (\frac{n^2+n}{2})^2$$

$$\sum_{i=0}^{n} i^3 = \frac{(n^2+n)^2}{4}$$

$$P(k) = \sum_{i=0}^{k} i^3 = \frac{(k^2+k)^2}{4}$$

$$P(k+1):$$

$$\sum_{i=0}^{k+1} i^3 = (\frac{((k+1)^2+k+1)^2}{4})$$

$$\frac{(k^2+k)^2}{4} + (k+1)^3 = \frac{(k^2+3k+2)^2}{4}$$

$$\frac{k^4+6k^3+13k^2+12k+4}{4} = \frac{k^4+6k^3+13k^2+12k+4}{4}$$

3

Prove that:

$$x^n - y^n = (x-y)\sum_{i=1}^{n} x^{n-i}y^{i-1}$$

Another representation of the sum that is easier to work with would be:

$$x^n - y^n = (x-y)\sum_{i=0}^{k-1} x^{i}y^{k-1-i}$$

$$P(k) = x^k - y^k = (x-y)\sum_{i=0}^{k-1} x^i y^{k-1-i}$$

$$P(k) = \frac{x^k - y^k}{(x-y)} = \sum_{i=0}^{k-1} x^i y^{k-1-i}$$

$$P(k+1):$$

$$P(k+1) = \frac{x^{k+1} - y^{k+1}}{(x-y)} = \sum_{i=0}^{k} x^i y^{k-i}$$

$$\frac{x^{k+1} - y^{k+1}}{(x-y)} = x^k + \sum_{i=0}^{k-1} x^i y^{k-i}$$

$$\frac{x^{k+1} - y^{k+1}}{(x-y)} = x^k + y\sum_{i=0}^{k-1} x^i y^{k-1-i}$$

$$\frac{x^{k+1} - y^{k+1}}{(x-y)} = x^k + y\,\frac{x^k - y^k}{(x-y)}$$

$$\frac{x^{k+1} - y^{k+1}}{(x-y)} = \frac{x^{k+1} - y^{k+1}}{(x-y)}$$

4

Prove that :

$$\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$$

$$P(k) := \sum_{i=1}^{k} i(i+1) = \frac{k(k+1)(k+2)}{3}$$

$P(k+1):$

$$= \sum_{i=1}^{k+1} i(i+1) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$= \sum_{i=1}^{k} i(i+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$$

$$\frac{k^3+6k^2+11k+6}{3} = \frac{k^3+6k^2+11k+6}{3}$$