

1. Produce a clear and clean proof of the following statement you have discovered in class: Let  $a$  and  $b$  two positive integers. Then  $ab = \gcd(a,b)\text{lcm}(a,b)$ .
2. We learned that if two integers  $a$  and  $b$  are relatively prime, then there exist integers  $t$  and  $u$  such that  $at + bu = 1$ . Prove the converse: if there are integers  $t$  and  $u$  such that  $at + bu = 1$  then  $a$  and  $b$  are relatively prime.
3. Let  $a$  and  $b$  be integers with  $b > 0$ . Using division algorithm, write  $a = bq + r$  where  $q, r \in \mathbb{Z}$  and  $0 \leq r < b$ . Show that  $\gcd(a,b) = \gcd(b,r)$ .
4. Show that for all positive integers  $n > 2$ ,  $\phi(n)$  is an even number.
5. Prove that if  $d$  divides  $n$  then  $\phi(d)$  divides  $\phi(n)$ .