1. Let r represent the rigid motion of a regular n-gon ($n \geq 3$) which rotates the figure counterclockwise by $2\pi/n$. Let s represent the rigid motion which reflects the figure along the axis of symmetry passing through the vertex labeled by 1.

   a) Carefully prove that $rs = sr^{-1}$ by computing the image of each vertex under the rigid motions rs and sr^{-1}. Conclude that D_n is a non-abelian group.

   b) Using induction prove that $r^k s = sr^{-k}$ for all positive integers k.

   c) We saw that $D_n = \{e, r, r^2, \ldots, r^{n-1}, s, sr, sr^2, \ldots, sr^{n-1}\}$. Show that $|sr^k| = 2$ for all $k = 0, 1, \ldots, n-1$.

   d) Compute the index of the subgroup $\langle r \rangle$ in D_n. Do the same for $\langle s \rangle$.

2. Let Δ be a regular tetrahedron and let G be the group of rigid motions of Δ [we are allowed to move Δ around in $\mathbb{R}^3$, but note that we cannot do reflections anymore]. Show that $|G| = 12$. [Hint: consider the argument with which we showed $|D_n| = 2n$]

3. Determine all subgroups of D_4 and decide which ones are normal. Then find two subgroups H_1 and H_2 in D_4 such that $H_1 \subset H_2$, H_1 is normal in H_2, H_2 is normal in D_4 but H_1 is not normal in D_4. This show that "is a normal subgroup of" is not a transitive relation.

4. Let G be a group and $Z(G) = \{g \in G : ga = ag \text{ for all } a \in G\}$. We have shown that $Z(G)$ is a subgroup of G. Now show that $Z(G)$ is a normal subgroup of G.

5. Let H be a subgroup of G.

   a) For any $g \in G$, prove that gHg^{-1} is a subgroup of G.

   b) Now suppose that H is the unique subgroup of order k in G. Prove that H is a normal subgroup.