

MATH 335 Lecture 22

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Definition: The Dihedral Group

The dihedral Group is a concept that emerges from the study of a regular convex n -gon, $n \geq 3$ in the cartesian plane. convexity in this context meaning the edges are all pointing outwards regularity indicating uniform edge length between vertices.

Typically the dihedral group is denoted as: D_{2n} where n is the degree of the n gon. The elements of D_{2n} are the rigid motions of the n gon.

Definition: Rigid motion

A rigid motion is a motion that enables a shape to be placed back in place after transformation.

Definition: $D_6 = \{S_3\} = \{r, s, r \circ s, s \circ r, e, r^2\}$

proposition: $|D_{2n}| = 2n$

Proof. Under a rigid motion of an n gon vertex 1 has to be placed at one of the n vertices and vertex 2 has 2 possible places to occupy. \square

Definition: $\langle r \rangle \subset D_{2n}$

We know this since $|\langle r \rangle| = n$ and $|\langle r \rangle \circ s| \circ s| = n$

$$D_{2n} = \langle r \rangle + s \circ \langle r \rangle$$

$$r^k s = s r^{-k}$$

$$D_{2n} = s \circ \langle r \rangle \cup \langle r \rangle$$