MATH 335 lecture 18

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Refresher of last lecture

Left Coset:
Let G be a group and H be a subgroup of G. For g the left coset of H with representative g is the set which is denoted as follows:

$$gH = \{gh : h \in H\}$$

Right Coset:
Let G be a group and H be a subgroup of G. For g the left coset of H with representative g is the set which is denoted as follows:

$$Hg = \{hg : h \in H\}$$

Cosets are a means of relating the size of the subgroup H to the size of the group G, more to follow:

Original Example

Let:

$$G = S_3 = \{e, \tau, \tau^2, \sigma_1, \sigma_2, \sigma_3\}$$

$$H = <\tau> = \{e, \tau, \tau^2\}$$

$$H = <\sigma_1> = \{e, \sigma_1\}$$

$$\tau = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\sigma_1 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}$$

Compute all left cosets of H.
Compute all left cosets of K and all right cosets of K

For the left cosets of H we obtain only two unique cosets
The elements of a cosets generate the same coset if used as a representative.

A coset is a set of its own representatives

Coset observations

1. $$g_1H = g_2H \iff g_2 \in g_1H$$

2. $$\frac{|G|}{|H|} = \text{ number of Unique left cosets of H } = \text{ number of Unique right cosets of H}$$

3. $$|H| \mid |G|$$

4. In general left coset is not equal to the right coset.

5. order of each coset is the order of H.

6. The distinct cosets partition the group