

# MATH 335 lecture 12

Chris Camano: ccamano@sfsu.edu

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## 1 Opening Notes

The final exam will only be an inclass exam without a take home component. Hosten will be out of the office during the final so we will only be doing an in class exam. This means that we will have two hours to complete a test. The difficulty will be somewhere between the regular in class exam and the take home exam.

The final exam will not be curved since the general grading system is generous to begin with.

## 2 New Material

$S_3 = \{\text{The set of bijections of } \{1,2,3\}\}$

$$S_3 = \left\{ \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} \right\}$$

Operation on  $S_3$ : Composition of bijections. Under composition it is not abelian. Verify this is a group

1. Closure under operator

2. Associativity

In general function composition is associative therefore:

$$\forall \sigma, \mu, \tau \in S_3 : \sigma \circ (\tau \circ \mu) = (\sigma \circ \tau) \circ \mu$$

3. Identity element

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix},$$

4. Inverse

for the fixed point functions you can compose with itself to get back to identity. For rotations compose with opposite rotation. There exists an inverse for all bijections, since  $S_3$  is the set of all bijections it is implied that the inverse is an element of  $S_3$

**Theorem 1.** :  $S_3$  is a non abelian cyclic group under function composition.

**Theorem 2.** : Let  $S_n$  be the set of bijections of  $\{1, 2, 3, 4, \dots, n\}$  Then  $S_n$  is a non abelian group under function composition.

**Theorem 3.**  $|S_n|$  is  $n!$ , Since this the number of permutations for a set of  $n$  elements.