# MATH 335 lecture 7

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equivilance relation
An equivilance relation$\sim$ on a set X is a relation where three properties are held:

1. $x \sim x \quad \forall x \in X$ Reflexive
2. $s \sim y \iff y \sim x \quad \forall x \in X$ Symmetric
3. $x \sim y \wedge y \sim z$ implies $x \sim z$ Transitivity.

equivilance realations are used to collect elements in a set for some intended purpose. It partitions the set by creating classifiers for the elements of the set.

Fundemental example:

We first start by fixing some m$\in \mathbb{Z}^+$ (this works for any positive integer). We define a realtion on all integers $\mathbb{Z}$ such that:
Given two integers a,b $a \sim b \iff n|a-b$ in other words: $a \equiv b \mod n$.

1. $x \sim x \quad \forall x \in X$ Reflexive

   *Proof.* $a \equiv bmodn$ since $n|a-a$ □

2. $s \sim y \iff y \sim x \quad \forall x \in X$ Symmetric

   *Proof.* $n|(x-y)$ so $n|-(x-y)$ so $n|y-x$ □

3. $x \sim y \wedge y \sim z$ implies $x \sim z$ Transitivity.

   *Proof.* $n|a-b$ and $n|b-c$ so $n|a-b+b = n|a-c$ so $a \sim c$ □

Equivilance classes
Let X be a set, and $\sim$ be an equivilance realtion on X. Let $x \in X$ Then:

$$[x] = \{y \in X : y \sim x\}$$

This set is called the equivilance class of x.The equivilance class of x always contains x.
The union of all equivilance classed mod n paritions the interges. :

$$\cup_{i=1}^{\infty}[x_i] = \mathbb{Z}$$

Also

$$x_i \cap x_j =, i \neq j$$