MATH 335 Lecture 1

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Number systems

The Integers

$$\mathbb{Z} = \{..., -1, 0, 1, ...\}$$

The Natural Numbers

$$\mathbb{N} = \{1, 2, 3, ...\}$$

Definition: Well ordered
A non empty subset S of integers is wellordered if and only if S contains a smallest or least element.

Well ordering of $\mathbb{N}$ Every non empty subset of $\mathbb{N}$ is well ordered. This is to say it has a smallest element since it will always contain positive integer values.

Divisibility

Let $A \neq 0, b \in \mathbb{Z}$ a divides b if and only if b can be expressed in the following way:

$$b = ak, k \in \mathbb{Z}$$

a is a divisor of b and is often denoted in the following way, read as a divides b:

$$a|b$$

Properties of divisibility
Division is transitive, therefore:

$$\text{if } a|b \wedge a|c \rightarrow a|b \pm c$$

if $a|n$ and $a|(n+m)$ then $a|m$.

Prime Numbers

Definition:
A positive integer p $>$ 1 is said to be a prime number if and only if the one only positive integer divisiors are 1 and p.

Fundemental Theorem of Arithmetic
Every positive integer $n > 1$ is equal to:

$$n = P_1^{\alpha_1} P_2^{\alpha_2} \cdots P_{k-1}^{\alpha_{k-1}} P_k^{\alpha_k} = \prod_{i=1}^{k} P_i^{\alpha_i}$$

Where $P_1, .., P_k$ are distinct primes and $\alpha_1, .., \alpha_k$ are positive integers. These two sets of primes and positive integers are unique to the prime factorization of a given number.

Proposition:
Let p be a prime number and a and b $in \mathbb{Z}$ if $p|ab$ then $p|a$ or $p|b$

Greatest Common Denominator:
Let d be a positive integer, d is the greatest common divisor(gcd) of a,b$\in$ $\mathbb{Z}$ if and only if two criteria are satisfied:

$$(1) \quad d|a \text{ and } d|b$$

$$(2) \text{ if } e|a \text{ and } e|b \text{ then } e|d$$

Statement one states that the given positive integer must in fact divide both a and b.
Statement two says that if there exists another comon divisor then it must also divide d. This is a statement asserting that no greater common divisor exists.