MATH 335 Homework 3

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1. Deterimine which one of the following sets is a group under addition: Addition is a binary operator by the proof provided in class, therefore we will validate the properties of each group accepting that addition is binary operator in these contexts.

   a) the set of rational numbers in lowest terms whose denominators are odd

   Proof. (a) Proof of associativity over operator
   (b) Proof of existence of identity element
   (c) Proof of existence of inverse of operator

   □

   b) the set of rational numbers in lowest terms whose denominators are even

   Proof. (a) Proof of associativity over operator
   (b) Proof of existence of identity element
   (c) Proof of existence of inverse of operator

   □

   c) the set of rational numbers of absolute value < 1

   Proof. (a) Proof of associativity over operator
   (b) Proof of existence of identity element
   (c) Proof of existence of inverse of operator

   □

   d) the set of rational numbers of absolute value ≥ 1 together with 0.

   Proof. (a) Proof of associativity over operator
   (b) Proof of existence of identity element
   (c) Proof of existence of inverse of operator

   □

2. Let $G = \{x \in \mathbb{R} : 0 \leq x < 1\}$ and for $x, y \in G$ let $x \cdot y$ be the fractional part of $x + y$ (i.e. $x \cdot y = x + y - [x + y]$ where $[a]$ is the greatest integer less than or equal to a). Prove that $\cdot$ is a binary operation on G and that G is a group.

Proof. Proof that $\cdot$ is a binary operator

$\square$

Proof. Proof that G is a group

(a) Proof of associativity over operator

(b) Proof of existence of identity element

(c) Proof of existence of inverse of operator

$\square$

3. Let $G = \{z \in \mathbb{C} : z^n = 1 \text{ for some nonnegative integer } n\}$. Prove that G is a group under multiplication (called the groups of *roots of unity* in $\mathbb{C}$).

Proof. (a) Proof of associativity over operator

(b) Proof of existence of identity element

(c) Proof of existence of inverse of operator

$\square$

4. Let $G = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$.

a) Prove that G is a group under addition.

Proof. (a) Proof of associativity over operator

(b) Proof of existence of identity element

(c) Proof of existence of inverse of operator

$\square$

b) Prove that the nonzero elements of G are a group under multiplication ["Rationalize the denominators" to find the inverses].

Proof. (a) Proof of associativity over operator

(b) Proof of existence of identity element

(c) Proof of existence of inverse of operator

$\square$