MATH 335 Lecture 25

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An example on Factor Groups

Let $G = \mathbb{Z}$Since this group is abelian ever subgroup of G is normal. Any subgroup of $\mathbb{Z}$ is of the form $< n >= n\mathbb{Z}$

We not consider the factor group $\mathbb{Z}/n\mathbb{Z}$ which is the set of all left cosets of $n\mathbb{Z}$ in $\mathbb{Z}$

The left cosets are of the form $m + n\mathbb{Z}$,

$$n\mathbb{Z} = \{kn : k \in \mathbb{Z}\} \quad \mathbb{Z}/n\mathbb{Z} = \{m + kn : k \in \mathbb{Z}\}$$

$$\mathbb{Z}/n\mathbb{Z} = \{n\mathbb{Z}, 1 + n\mathbb{Z}, 2 + n\mathbb{Z}, ..., (n-1) + n\mathbb{Z}\}$$

The factor group is a group under the operation:

$$(a + n\mathbb{Z}) + (b + n\mathbb{Z}) = (a + b \mod n) + n\mathbb{Z}$$

The groups $\mathbb{Z}/n\mathbb{Z}and\mathbb{Z}_n$ are isomorphic to one another since their elements are equivilant, but their construction is different.

homomorphism

A homomorphism between the groups $(G_1, *), (G_2, \circ)$ is a map (function) from the first group to the second one. Let

$$\phi : G_1 \mapsto G_2$$

, such that

$$a, b \in G_1$$

$$\phi(a * b) = \phi(a) \circ \phi(b)$$

Proving that a group homomorphism is bijective is equivilant to proving that two groups are isomorphic!!!!!!!!!

Proposition

Let $\phi : G_1 \mapsto G_2$ be a group homomorphism . Then :

1. $$\phi(e_{G_1}) = e_{G_2}$$

Proof.
$\phi(e_{G_1}) = \phi(e_{G_1} e_{G_1}) = \phi(e_{G_1})\phi(e_{G_2})\ e_{G_2} = \phi(e_{G_1})$ □

2. $\forall g \in G$

$$\phi(g^{-1}) = \phi(g)^{-1}$$

3. *Proof.*

$$\phi(e_{G_1}) = \phi(gg^{-1}) = \phi(g)\phi(g^{-1}) = e_{G_2}$$

□