MATH 370 lecture 7

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Formalization of Zeno's paradox:

$$a_n = 1 - \frac{1}{2^n}$$

A random fact:

$$\lim_{x\to\infty}\sum_{i=1}^{n}\frac{1}{k} = \infty$$

For all series of the form:

$$\lim_{x\to\infty}\sum_{i=1}^{n}\frac{1}{i^p}$$

The limit is ∞ unless p is one.

$$a_n = (1+\frac{1}{n})^n$$

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = e$$

Theorem 1. A monotone sequence that is bounded converges: Assume we have an increasing sequence . Since we have a bound on the sequence denoted as M we then have:

$$\exists_M \forall_n a_n < M$$

We want to show that the limit of a_n exists, in order to do this we consider the nature of what it means to be bounded. The fact that the set it bounded implies the existince of a supremum for the sequence. This gives the following.
Due to the completness of real numbers there exists some number S such that S acts as the supremum for the sequence. S is then by definition of supremum the least upper bound for all elements in the sequence.
We show that the limit of the sequence as $n \to \infty$ is equal to S. To show this we need to show that:

$$\forall_{\varepsilon>0}\exists_N : \forall_{n>N}|a_n - S| < \varepsilon$$

$$s - a_n < \varepsilon$$
$$s < \varepsilon + a_n$$

$s - \varepsilon$ is not an upper bound for a_n since S is the supremum which implies there exists N such that $a_N > S - \varepsilon$ so:

$$\forall_{n>N} a_n \geq a_N > s - \varepsilon$$

Since the sequence is increasing. so we have proven that $a_n \geq s - \varepsilon$

Other theorems from the textbook:
Given s_n, t_n where $\lim_{n\to\infty} s_n = s, \lim_{n\to\infty} t_n = t$ then

$$\lim_{n\to\infty} s_n t_n = st$$

Additional Theorem;

$$(s_n \to \infty) \to \frac{1}{s_n} \to \infty$$