MATH 370 lecture 6

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$$lim_{n\mapsto\infty}a_n = L$$

1. outside every interval centered at L there is a finite number of $a_n' s$
2. Inside dvery interval containing L therae are infinite number of $a_n' s$.

$$lim_{n\mapsto\infty}a_n = \infty$$

1. Every bounded interval contains finite number of sequences
2. outside every bonded interval there are infinite sequences.

Midterm structure: in person
Example of a question that may appear on the midterm.

Probably about three weeks from now.

If $\lim_{n\to\infty} a_n = L_1$ and $\lim_{n\to\infty} a_n = L_2$ then $L_1 = L_2$

$\lim_{n\to\infty}(ka_nn) = k\lim_{n\to\infty} a_n \quad k \in \mathbb{Z}$

Consider the following:

$$\forall\mu > 0 \quad \exists N : \forall n > N : |a_n - L| < \mu$$

Given an arbitray $\varepsilon > 0$ let $\mu = \frac{\varepsilon}{10}$ So this gives that:

$$\forall\varepsilon > 0 \quad \exists N : \forall n > N|a_n - L| < \frac{\varepsilon}{10}$$

Proof of $\lim_{n\to\infty}(ka_nn) = k\lim_{n\to\infty} a_n \quad k \in \mathbb{Z}$

Proof.

$$\forall\varepsilon > 0 \quad \exists N : \forall n > N|a_n - L| < \frac{\varepsilon}{k}$$

$$\begin{aligned}
&k|a_n - L| < \varepsilon \\
&\forall \varepsilon > 0 \quad \exists N |ka_n - kL| < \varepsilon \\
&\lim_{n\to\infty}(ka_n) = kL \\
&\lim_{n\to\infty}(ka_n) = k \lim_{n\to\infty} a_n
\end{aligned}$$

$\square$

Lemma: If there exists a finite limit then the sequence must be bounded. In the situation where we have a finite number of points hat exist outside of the limit strip we can add the maximum outlier to the limit to correct the inequality.

$$\begin{aligned}
|a_n - L| &< 10 \\
|a_n| &< 10 + L
\end{aligned}$$

Let $\mathrm{M} = \max(|a_1|, |a_2|, ..., |a_{n-1}|, 10 + L)$ then for all a_n $a_n < M$ thus if the limit exists the the set is bounded.

Theorem: The limit of the prodcut of two sequences is equal to the limit of both as folows:

$$\lim_{n\to\infty} a_n b_n = \lim_{n\to\infty} a_n \lim_{n\to\infty} b_n$$

Additional Theorems:

$$\lim_{n\to\infty} b_n \neq 0 \to \lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$$

Additional Theorem:

$$\lim_{n\to\infty}\left(\frac{1}{n^p}\right) = 0 \text{ if} p > 0$$

Additional Theorem:

$$\lim_{n\to\infty} a^n = 0 \text{ if } |a| < 1$$

Additional Theorem:

$$\lim_{n\to\infty} n^{\frac{1}{n}} = 1$$

Additional Theorem:

$$\lim_{n\to\infty} a^{\frac{1}{n}} = 1, q \neq 0$$