# MATH 370 Lecture 8

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Geometric Series

$$\sum_{k=0}^{n} aq^k = \frac{a(1-q^{n+1})}{1-q} = \frac{a}{1-q}$$

Any element in Q can be represented as a finite expression.

A monotone sequence that is bounded by above converges.

**Definition 1.** Cauchy Sequence:
all Cauchy sequences converge, all cauchy sequences are monotone and bounded:

$$\forall \varepsilon > 0 \quad \exists N \forall n, m > N : |s_n - s_m| < \varepsilon$$

This is saying that for any two points in the sequence past M that the difference of their values when evaluated in the sequence is less tahn epsilon.
If this is true the sequence is referred to as being a cauchy sequence.

If $\lim_{n\to\infty}$ exits then the sequence is a cauchy sequence.

*Proof.*

$$\forall \varepsilon > 0 \quad \exists N \quad \forall n > N : |s_n - L| < \frac{\varepsilon}{2}$$

$$\forall \varepsilon > 0 \quad \exists N \quad \forall m > N : |s_n - L| < \frac{\varepsilon}{2}$$

$$|a_n - a_m| \leq |a_n - L| + |L - a_m| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

□

Being cacuchy and having a limit is equivilant to the completeness of the real numbers. (EXAM QUESTION)

**Definition 2.** : Let $b_m = \{sup\{a_m, a_{m+1}, ..\}$ then we know that $b_m$ converges since the sequence is decreasing. Thus:

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \sup a_n$$

$\lim_{n\to\infty} \sup a_n$ always exists: Example:

$$\lim_{n\to\infty} \sup \frac{(-1)^n}{n} = 0$$

$$\lim_{n\to\infty} \inf a_n = \lim_{n\to\infty} \inf\{a_m, a_{m+1}, \ldots$$

**Theorem 1.** If $\lim_{n\to\infty} \sup a_n$exists and $\lim_{n\to\infty} \inf a_n$ and $\lim a_n$ exists then :

$$\lim_{n\to\infty} \sup a_n = \lim_{n\to\infty} \inf a_n = \lim_{n\to\infty} a_n$$