MATH 370 Homework 4

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1. 9.8 (no proofs needed)

   (a)
   $$\lim_{n\to\infty} n^3 = \infty$$

   (b)
   $$\lim_{n\to\infty} (-n^3) = -\infty$$

   (c)
   $$\lim_{n\to\infty} (-n^n) = NOT\,EXIST$$

   (d)
   $$\lim_{n\to\infty} (1.01)^n = \infty$$

   (e)
   $$\lim_{n\to\infty} (n^n) = \infty$$

2. 9.17 (proof needed):
   Give a formal proof that
   $$\lim_{n\to\infty} (n^2) = +\infty$$

   using definition 9.8: **Definition 9.8**
   For a squence(s_n) we write
   $$\lim_{n\to\infty} (s_n) = +\infty$$

   provided:
   $$\forall_{M>0} \exists_N : n > N \to s_n > M$$

   Proof. To prove this fact we must choose some value for N such that the sequence s_n for any n greater than N s_n is greater than M. To do this let us consider the relationship between M and N. To satisfy the requirement under the definition of the function, let N=$\sqrt{m}$ then we have the following for n greater than our choice:

   $$n > N$$
   $$n > \sqrt{M}$$
   $$n^2 > M$$
   $$s_n < M$$

   So we have proven that with a selection of $N = \sqrt{M}$ then $n > N$ implies $s_n > M$ $\qquad\square$

3. 9.18

Proof. (a) Verify:

$$\sum_{k=0}^{n} a^k = \frac{1 - a^{n+1}}{1 - a}, a \neq 1$$

Proof. Base case: n=1:

$$a^0 + a^1 = \frac{1 - a^2}{1 - a}$$

$$1 + a = \frac{1 - a^2}{1 - a}$$

$$1 + a = \frac{(1 + a)(1 - a)}{1 - a}$$

$$1 + a = 1 + a$$

$$P(k) : \sum_{i=0}^{k} a^i = \frac{1 - a^{k+1}}{1 - a}, a \neq 1$$

P(k+1)

$$\sum_{i=0}^{k+1} a^i = \frac{1 - a^{k+2}}{1 - a}$$

$$\sum_{i=0}^{k} a^i + a^{k+1} = \frac{1 - a^{k+2}}{1 - a}$$

$$\frac{1 - a^{k+1}}{1 - a} + a^{k+1} = \frac{1 - a^{k+2}}{1 - a}$$

$$\frac{1 - a^{k+1} + a^{k+1}(1 - a)}{1 - a} = \frac{1 - a^{k+2}}{1 - a}$$

$$\frac{1 - a^{k+1} + a^{k+1} - a^{k+2}}{1 - a} = \frac{1 - a^{k+2}}{1 - a}$$

$$\frac{1 - a^{k+2}}{1 - a} = \frac{1 - a^{k+2}}{1 - a}$$

□

(b) **Find** $\lim_{n \to \infty} \left(\sum_{k=0}^{n} a^k \right), |a| < 1$

(c) **Calculate** $\lim_{n \to \infty} \left(\sum_{k=0}^{n} \frac{1}{3^k} \right)$

(d) **What is** $\lim_{n \to \infty} \left(\sum_{k=0}^{n} a^k \right), a \geq 1$

□

4. 10.1

   Proof. Which of the sequences are increasing? decreasing? Bounded?

   (a)
   $$s_n = \frac{1}{n}$$

   (b)
   $$s_n = \frac{(-1^n)}{n^2}$$

   (c)
   $$s_n = n^5$$

   (d)
   $$s_n = \sin\left(\frac{n\pi}{7}\right)$$

   (e)
   $$s_n = (-2)^n$$

   (f)
   $$s_n = \frac{n}{3^n}$$

   □

5. 10.6

   Proof.

   Let (s_n) be a sequence such that:
   $$|s_{n+1} - s_n| < s^{-n} \forall n \in \mathbb{N}$$

   Prove s_n is a Cauchy sequence and hence a convergent sequence.

   Is the result in (a) true if we only assume $|s_{n+1} - s_n| < \frac{1}{n}$?

   □