MATH 370 lecture 9

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Analysis

Definition 1. : Tail of a sequence:
A tail of a sequence where a sequence a_n is seen as a function as follows:

$$a : \mathbb{N} \mapsto \mathbb{R}$$

is restricted to:

$$\mathbb{N} - \{1, ..., k-1\}$$

Let a_n be a sequence . A tail for this sequence can be expressed as :

$$(a_n)_{n=k}^{\infty}$$

Recall the definitions of

- $\lim_{n\to\infty} \inf a_n$ =$\lim_{k\to\infty} u_k$Where U_k is the sequence of infiumums of a_n.
- $\lim_{n\to\infty} \sup a_n$ =$\lim_{k\to\infty} v_k$Where v_k is the sequence of supermums of a_n.

If $\lim_{n\to\infty} \inf a_n = \lim_{n\to\infty} \sup a_n$ then $\lim_{n\to\infty} a_n$ exists

Let S=$\lim_{n\to\infty} a_n$ By definition of limit$\forall \varepsilon > 0 \exists N \forall n > N : |a_n - s| < \varepsilon$
we Know that $\lim_{n\to\infty} \sup a_n < V_N < s + \varepsilon$ and that we Know that $\lim_{n\to\infty} \inf a_n > U_N > s - \varepsilon$