MATH 335 lecture 10

Chris Camano: ccamano@sfsu.edu

September 27, 2022

Summary from last lecture
If we have a sequence: $_n \in \mathbb{R}$ if the lower limit of $\liminf a_n = \limsup a_n$ then the limit of a_n exists and:

$$\liminf a_n = \limsup a_n = \lim a_n$$

In $(L+\varepsilon,\infty)$ There are finitley many a_n
In $(-\infty, L-\varepsilon,)$ There are finitley many a_n
In $(L-\varepsilon, L+\varepsilon)$ There are almost all a_n which means there is some number N such that for all n¿N $n \in (L-\varepsilon, L+\varepsilon)$
Thus the limit of the sequence exists

Definition of Cauchy Sequence:

$$\forall \varepsilon > 0 \exists N : \forall m,n > N \quad |a_m - a_n| < \varepsilon$$

a_n is cauchy if and only if a_n converges to a number.

$$a_m - \varepsilon \leq \liminf a_n \leq \limsup a_n \leq a_m + \varepsilon$$

since $a_m - \varepsilon \leq \liminf a_n \leq \limsup a_n \leq a_m + \varepsilon$ holds for all ε then $\liminf a_N = \limsup a_n$ which implies a_n converges